Computer algebra independent integration tests

6-Hyperbolic-functions/6.3-Hyperbolic-tangent/6.3.7-d-hyper-^m-a+b-c-tanh-^n-^p

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [263]. This is test number [173].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (263)	% 0. (0)
Mathematica	% 100. (263)	% 0. (0)
Maple	% 94.68 (249)	% 5.32 (14)
Maxima	% 51.33 (135)	% 48.67 (128)
Fricas	% 93.54 (246)	% 6.46 (17)
Sympy	% 15.21 (40)	% 84.79 (223)
Giac	% 81.75 (215)	% 18.25 (48)

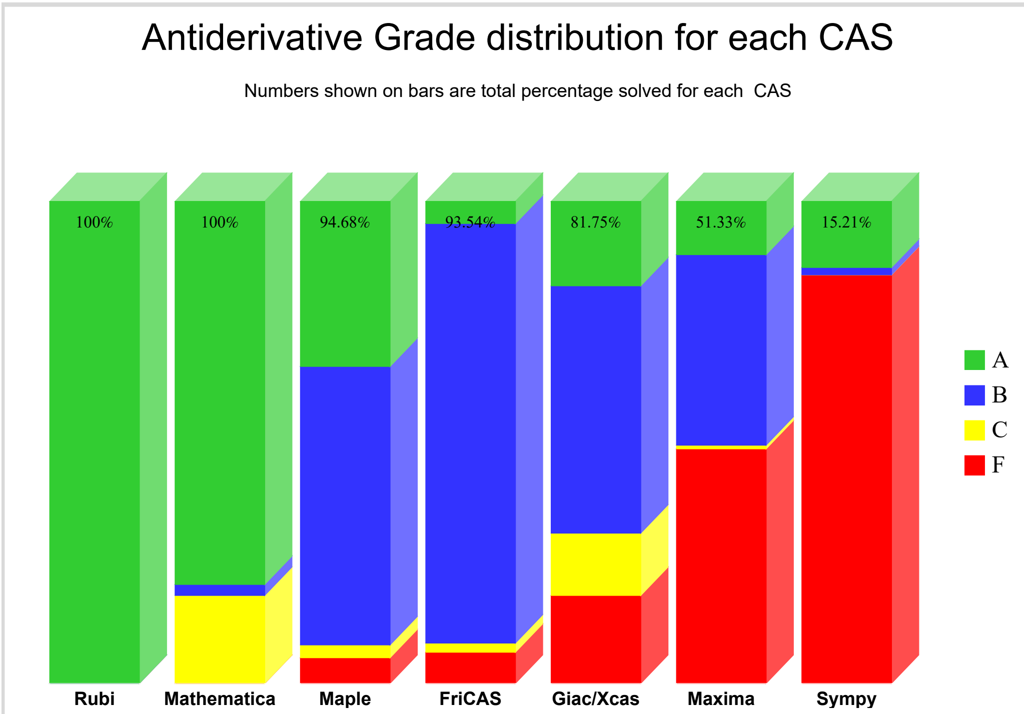
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

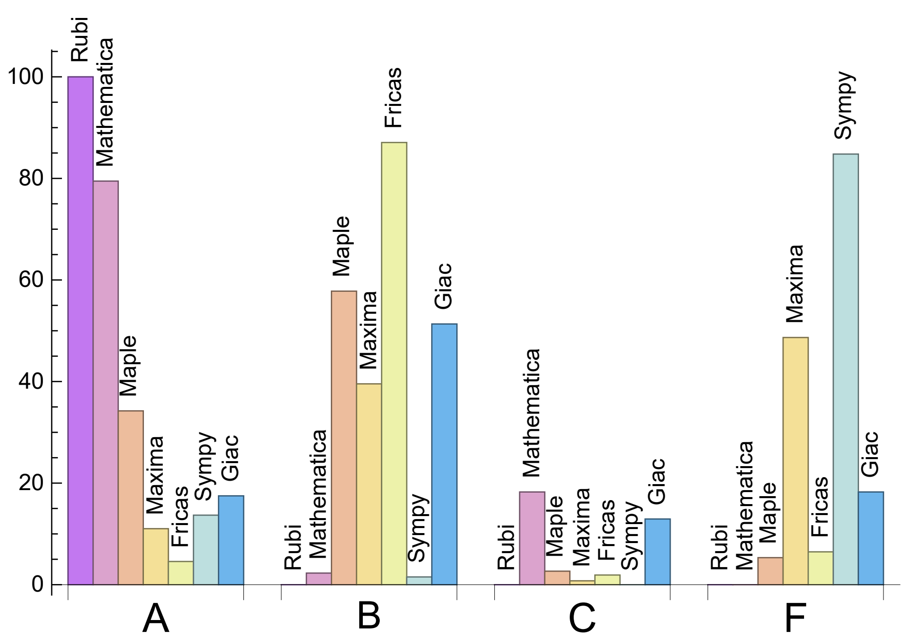
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	79.47	2.28	18.25	0.
Maple	34.22	57.79	2.66	5.32
Maxima	11.03	39.54	0.76	48.67
Fricas	4.56	87.07	1.9	6.46
Sympy	13.69	1.52	0.	84.79
Giac	17.49	51.33	12.93	18.25

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.12	91.19	0.99	78.	1.
Mathematica	1.2	125.37	1.54	86.	1.
Maple	0.06	367.34	3.73	196.	2.34
Maxima	1.29	349.2	4.07	265.	3.58
Fricas	3.07	6970.59	70.04	4360.5	56.52
Sympy	15.74	296.15	4.05	124.	1.96
Giac	1.63	1176.31	12.5	294.	3.44

1.4 list of integrals that has no closed form antiderivative

{74, 76, 77, 79}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {74, 76, 77, 79}

Maple {74, 76, 77, 79}

Maxima {}

Fricas {}

Sympy {}

Giac {74, 76, 77}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {93, 95, 98, 101, 236, 243, 250, 252}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

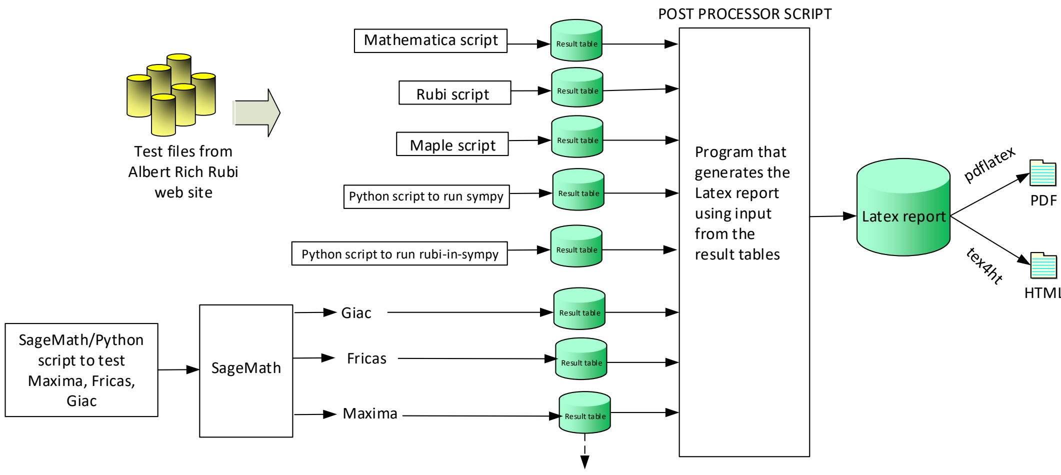
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 30, 32, 33, 35, 38, 40, 41, 43, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 76, 77, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 96, 97, 99, 100, 102, 103, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 141, 143, 145, 147, 148, 149, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 203, 204, 205, 206, 207, 208, 210, 212, 214, 216, 218, 219, 221, 222, 223, 225, 226, 227, 228, 229, 231, 233, 234, 235, 237, 240, 248, 255, 256, 257, 258, 259, 260, 261, 262, 263 }

B grade: { 104, 144, 146, 202, 213, 241 }

C grade: { 26, 28, 29, 31, 34, 36, 37, 39, 42, 44, 45, 47, 73, 75, 78, 80, 93, 95, 98, 101, 133, 140, 142, 150, 154, 209, 211, 215, 217, 220, 224, 230, 232, 236, 238, 239, 242, 243, 244, 245, 246, 247, 249, 250, 251, 252, 253, 254 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 13, 14, 15, 16, 17, 19, 23, 24, 29, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 71, 74, 76, 77, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 139, 140, 141, 142, 143, 149, 150, 151, 152, 153, 154, 155, 161, 162, 163, 164, 165, 166, 167, 170, 171, 172, 173, 174, 180, 182, 184, 202, 203, 204, 205, 206, 207, 257, 258, 260, 261 }

B grade: { 10, 12, 18, 20, 21, 22, 25, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 62, 70, 72, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 144, 145, 146, 147, 148, 156, 157, 158, 159, 160, 168, 169, 175, 176, 177, 178, 179, 181, 183, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 208, 209, 210, 211, 212, 213, 219, 220, 221, 222, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 238, 239, 240, 241, 242, 243, 246, 247, 248, 249, 250, 251, 252, 255, 256 }

C grade: { 73, 75, 78, 80, 259, 262, 263 }

F grade: { 214, 215, 216, 217, 218, 223, 224, 235, 236, 237, 244, 245, 253, 254 }

2.1.4 Maxima

A grade: { 5, 6, 49, 50, 51, 52, 53, 54, 66, 68, 74, 76, 77, 79, 81, 86, 94, 102, 138, 139, 140, 150, 172, 174, 176, 178, 202, 204, 206

B grade: { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 69, 70, 71, 72, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 95, 96, 97, 98, 99, 100, 101, 103, 104, 134, 135, 136, 137, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 180, 182, 184, 186, 188, 191, 193, 195, 197, 199, 207, 257, 258 }

C grade: { 203, 205 }

F grade: { 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 73, 75, 78, 80, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 171, 173, 175, 177, 179, 181, 183, 185, 187, 189, 190, 192, 194, 196, 198, 200, 201, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 259, 260, 261, 262, 263 }

2.1.5 FriCAS

A grade: { 1, 3, 4, 81, 82, 83, 89, 138, 203, 205, 206, 207 }

B grade: { 2, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 43, 44, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 78, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 202, 204, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 221, 222, 223, 224, 225, 227, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 247, 248, 249, 250, 251, 252, 254, 255, 257, 258, 260, 261, 262 }

C grade: { 75, 80, 226, 228, 256 }

F grade: { 41, 42, 47, 48, 65, 73, 74, 76, 77, 79, 200, 201, 220, 246, 253, 259, 263 }

2.1.6 Sympy

A grade: { 77, 79, 134, 135, 136, 137, 138, 144, 145, 146, 147, 148, 156, 157, 158, 159, 160, 168, 169, 170, 171, 172, 173, 174, 175, 181, 183, 185, 206, 208, 210, 212, 233, 242, 251, 257 }

B grade: { 140, 219, 221, 258 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 139, 141, 142, 143, 149, 150, 151, 152, 153, 154, 155, 161, 162, 163, 164, 165, 166, 167, 176, 177, 178, 179, 180, 182, 184, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 207, 209, 211, 213, 214, 215, 216, 217, 218, 220, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 234, 235, 236, 237, 238, 239, 240, 241, 243, 244, 245, 246, 247, 248, 249, 250, 252, 253, 254, 255, 256, 259, 260, 261, 262, 263 }

2.1.7 Giac

A grade: { 5, 6, 8, 14, 23, 30, 49, 50, 51, 52, 53, 54, 58, 60, 61, 63, 66, 68, 73, 74, 75, 76, 77, 78, 79, 80, 81, 85, 93, 110, 112, 138, 139, 140, 153, 171, 173, 174, 175, 176, 177, 178, 202, 206, 257, 258 }

B grade: { 1, 2, 3, 4, 7, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 27, 32, 33, 35, 38, 40, 41, 43, 46, 48, 55, 56, 57, 59, 62, 64, 65, 67, 69, 70, 71, 72, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 107, 114, 116, 119, 121, 123, 125, 128, 130, 132, 134, 135, 136, 137, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 204, 225, 227, 255 }

C grade: { 26, 28, 29, 31, 34, 36, 37, 39, 42, 44, 45, 47, 106, 108, 109, 111, 113, 115, 117, 118, 120, 122, 124, 126, 127, 129, 131, 133, 203, 205, 207, 226, 228, 256 }

F grade: { 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 259, 260, 261, 262, 263 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	56	96	208	327	0	196
normalized size	1	1.	0.77	1.32	2.85	4.48	0.	2.68
time (sec)	N/A	0.076	0.341	0.041	1.202	1.85	0.	1.323

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	73	73	184	246	0	162
normalized size	1	1.	1.55	1.55	3.91	5.23	0.	3.45
time (sec)	N/A	0.055	0.05	0.034	1.152	1.999	0.	1.302

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	41	66	136	193	0	147
normalized size	1	1.	0.93	1.5	3.09	4.39	0.	3.34
time (sec)	N/A	0.051	0.189	0.033	1.126	1.959	0.	1.27

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	45	43	90	115	0	107
normalized size	1	1.	1.8	1.72	3.6	4.6	0.	4.28
time (sec)	N/A	0.032	0.045	0.033	1.108	1.905	0.	1.206

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	52	44	54	483	0	70
normalized size	1	1.	2.	1.69	2.08	18.58	0.	2.69
time (sec)	N/A	0.033	0.027	0.037	1.08	2.084	0.	1.33

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	53	238	0	61
normalized size	1	1.	1.	0.96	2.21	9.92	0.	2.54
time (sec)	N/A	0.033	0.02	0.036	1.141	1.886	0.	1.264

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	87	50	205	2480	0	153
normalized size	1	1.	1.71	0.98	4.02	48.63	0.	3.
time (sec)	N/A	0.059	0.046	0.044	1.101	2.219	0.	1.208

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	61	55	153	657	0	108
normalized size	1	1.	1.39	1.25	3.48	14.93	0.	2.45
time (sec)	N/A	0.043	0.068	0.046	1.164	2.04	0.	1.215

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	94	166	398	1021	0	398
normalized size	1	1.	0.8	1.41	3.37	8.65	0.	3.37
time (sec)	N/A	0.133	1.374	0.049	1.118	2.068	0.	1.726

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	71	162	358	664	0	392
normalized size	1	1.	0.92	2.1	4.65	8.62	0.	5.09
time (sec)	N/A	0.098	0.517	0.052	1.192	2.055	0.	1.646

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	70	118	293	747	0	290
normalized size	1	1.	0.89	1.49	3.71	9.46	0.	3.67
time (sec)	N/A	0.111	0.85	0.05	1.243	2.059	0.	1.465

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	46	113	231	424	0	219
normalized size	1	1.	0.94	2.31	4.71	8.65	0.	4.47
time (sec)	N/A	0.053	0.308	0.046	1.137	2.058	0.	1.371

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	50	97	265	2331	0	170
normalized size	1	1.	0.98	1.9	5.2	45.71	0.	3.33
time (sec)	N/A	0.065	0.155	0.05	1.042	2.003	0.	1.335

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	43	68	184	683	0	116
normalized size	1	1.	0.93	1.48	4.	14.85	0.	2.52
time (sec)	N/A	0.057	0.449	0.048	1.054	1.859	0.	1.35

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	96	103	244	6384	0	227
normalized size	1	1.	1.17	1.26	2.98	77.85	0.	2.77
time (sec)	N/A	0.119	1.54	0.062	1.057	2.32	0.	1.373

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	59	81	284	1011	0	193
normalized size	1	1.	0.82	1.12	3.94	14.04	0.	2.68
time (sec)	N/A	0.079	0.48	0.059	1.051	1.932	0.	1.417

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	125	246	648	2273	0	684
normalized size	1	1.	0.69	1.35	3.56	12.49	0.	3.76
time (sec)	N/A	0.222	3.978	0.053	1.077	2.075	0.	2.748

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	91	287	593	1388	0	597
normalized size	1	1.	0.87	2.73	5.65	13.22	0.	5.69
time (sec)	N/A	0.123	0.291	0.052	1.101	1.967	0.	2.437

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	139	95	180	509	1854	0	533
normalized size	1	1.14	0.78	1.48	4.17	15.2	0.	4.37
time (sec)	N/A	0.185	2.159	0.053	1.1	2.102	0.	2.176

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	63	219	433	959	0	435
normalized size	1	1.	0.9	3.13	6.19	13.7	0.	6.21
time (sec)	N/A	0.068	0.829	0.049	1.092	1.984	0.	1.774

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	79	186	756	5895	0	354
normalized size	1	1.	0.94	2.21	9.	70.18	0.	4.21
time (sec)	N/A	0.084	0.302	0.052	1.05	2.251	0.	1.643

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	70	141	470	1449	0	273
normalized size	1	1.	1.09	2.2	7.34	22.64	0.	4.27
time (sec)	N/A	0.064	0.679	0.053	1.113	1.875	0.	1.713

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	127	174	544	12984	0	379
normalized size	1	1.	0.84	1.14	3.58	85.42	0.	2.49
time (sec)	N/A	0.225	6.175	0.063	1.093	2.476	0.	1.77

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	87	136	666	2385	0	347
normalized size	1	1.	0.89	1.39	6.8	24.34	0.	3.54
time (sec)	N/A	0.097	1.218	0.058	1.158	2.14	0.	1.783

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	93	865	0	5158	0	406
normalized size	1	1.	0.79	7.33	0.	43.71	0.	3.44
time (sec)	N/A	0.172	0.254	0.108	0.	2.314	0.	2.395

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	135	202	0	3729	0	5603
normalized size	1	1.	1.8	2.69	0.	49.72	0.	74.71
time (sec)	N/A	0.128	0.539	0.069	0.	2.451	0.	1.888

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	67	605	0	2475	0	238
normalized size	1	1.	0.86	7.76	0.	31.73	0.	3.05
time (sec)	N/A	0.104	0.142	0.076	0.	2.441	0.	1.658

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	107	104	0	1906	0	6017
normalized size	1	1.	2.02	1.96	0.	35.96	0.	113.53
time (sec)	N/A	0.063	0.242	0.055	0.	2.201	0.	1.559

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	123	69	0	1673	0	5242
normalized size	1	1.	2.24	1.25	0.	30.42	0.	95.31
time (sec)	N/A	0.078	0.199	0.063	0.	2.395	0.	1.526

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	413	0	1636	0	93
normalized size	1	1.	1.	8.6	0.	34.08	0.	1.94
time (sec)	N/A	0.063	0.115	0.076	0.	2.204	0.	1.42

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	170	181	0	4872	0	7173
normalized size	1	1.	2.	2.13	0.	57.32	0.	84.39
time (sec)	N/A	0.119	0.638	0.077	0.	2.651	0.	1.822

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	71	750	0	4307	0	178
normalized size	1	1.	1.01	10.71	0.	61.53	0.	2.54
time (sec)	N/A	0.088	0.289	0.089	0.	2.386	0.	1.419

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	132	1246	0	17565	0	709
normalized size	1	1.	0.69	6.49	0.	91.48	0.	3.69
time (sec)	N/A	0.252	0.933	0.119	0.	3.304	0.	3.392

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	160	267	0	12417	0	8694
normalized size	1	1.	1.29	2.15	0.	100.14	0.	70.11
time (sec)	N/A	0.218	1.22	0.095	0.	3.063	0.	2.472

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	105	1128	0	9441	0	540
normalized size	1	1.	0.8	8.55	0.	71.52	0.	4.09
time (sec)	N/A	0.165	0.65	0.099	0.	2.677	0.	2.253

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	133	167	0	5933	0	6742
normalized size	1	1.	1.45	1.82	0.	64.49	0.	73.28
time (sec)	N/A	0.075	0.684	0.079	0.	2.585	0.	1.698

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	175	331	0	6604	0	7329
normalized size	1	1.	1.7	3.21	0.	64.12	0.	71.16
time (sec)	N/A	0.144	0.663	0.092	0.	3.284	0.	1.704

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	86	552	0	6218	0	306
normalized size	1	1.	1.05	6.73	0.	75.83	0.	3.73
time (sec)	N/A	0.076	0.42	0.102	0.	2.409	0.	1.653

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	203	367	0	15776	0	5650
normalized size	1	1.	1.44	2.6	0.	111.89	0.	40.07
time (sec)	N/A	0.209	4.089	0.108	0.	3.304	0.	1.973

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	114	1012	0	12585	0	298
normalized size	1	1.	1.01	8.96	0.	111.37	0.	2.64
time (sec)	N/A	0.154	0.775	0.119	0.	2.786	0.	1.61

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	184	2366	0	0	0	1246
normalized size	1	1.	0.77	9.86	0.	0.	0.	5.19
time (sec)	N/A	0.345	0.763	0.128	0.	0.	0.	4.662

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F(-1)	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	227	341	0	0	0	8107
normalized size	1	1.	1.37	2.05	0.	0.	0.	48.84
time (sec)	N/A	0.286	1.812	0.109	0.	0.	0.	3.143

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	158	2110	0	29831	0	788
normalized size	1	1.	0.85	11.41	0.	161.25	0.	4.26
time (sec)	N/A	0.251	1.209	0.115	0.	3.844	0.	3.103

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	157	252	0	17194	0	8412
normalized size	1	1.	1.25	2.	0.	136.46	0.	66.76
time (sec)	N/A	0.097	1.737	0.089	0.	3.084	0.	2.003

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	236	1132	0	25141	0	9281
normalized size	1	1.	1.51	7.26	0.	161.16	0.	59.49
time (sec)	N/A	0.254	1.288	0.104	0.	4.358	0.	1.981

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	109	816	0	19301	0	474
normalized size	1	1.	0.97	7.29	0.	172.33	0.	4.23
time (sec)	N/A	0.087	0.882	0.116	0.	3.142	0.	1.916

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	269	1083	0	0	0	8479
normalized size	1	1.	1.37	5.53	0.	0.	0.	43.26
time (sec)	N/A	0.334	4.095	0.118	0.	0.	0.	2.445

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	149	1416	0	0	0	549
normalized size	1	1.	0.99	9.38	0.	0.	0.	3.64
time (sec)	N/A	0.201	1.244	0.135	0.	0.	0.	1.918

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	92	122	262	4209	0	278
normalized size	1	1.	0.7	0.92	1.98	31.89	0.	2.11
time (sec)	N/A	0.172	0.19	0.047	1.608	2.49	0.	1.374

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	104	129	235	2942	0	192
normalized size	1	1.	1.06	1.32	2.4	30.02	0.	1.96
time (sec)	N/A	0.12	0.257	0.043	1.541	2.334	0.	1.3

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	69	79	190	2453	0	192
normalized size	1	1.	0.69	0.79	1.9	24.53	0.	1.92
time (sec)	N/A	0.117	0.107	0.041	1.584	2.351	0.	1.266

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	72	85	142	1473	0	128
normalized size	1	1.	1.14	1.35	2.25	23.38	0.	2.03
time (sec)	N/A	0.076	0.121	0.04	1.611	2.361	0.	1.217

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	75	65	112	1453	0	100
normalized size	1	1.	1.53	1.33	2.29	29.65	0.	2.04
time (sec)	N/A	0.074	0.031	0.049	1.516	2.634	0.	1.217

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	34	59	375	0	61
normalized size	1	1.	1.	1.17	2.03	12.93	0.	2.1
time (sec)	N/A	0.034	0.025	0.05	1.043	2.445	0.	1.212

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	95	62	211	3216	0	193
normalized size	1	1.	1.34	0.87	2.97	45.3	0.	2.72
time (sec)	N/A	0.09	0.026	0.059	1.531	2.795	0.	1.222

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	74	60	248	4702	0	201
normalized size	1	1.	1.32	1.07	4.43	83.96	0.	3.59
time (sec)	N/A	0.058	0.197	0.061	1.577	2.454	0.	1.236

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	206	156	243	512	13666	0	508
normalized size	1	1.21	0.92	1.43	3.01	80.39	0.	2.99
time (sec)	N/A	0.297	2.809	0.058	1.586	2.942	0.	2.273

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	121	296	470	9211	0	405
normalized size	1	1.	0.66	1.63	2.58	50.61	0.	2.23
time (sec)	N/A	0.225	0.72	0.058	1.555	2.777	0.	2.041

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	159	137	173	406	9671	0	402
normalized size	1	1.23	1.06	1.34	3.15	74.97	0.	3.12
time (sec)	N/A	0.205	1.55	0.056	1.578	2.871	0.	1.836

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	90	225	342	5998	0	289
normalized size	1	1.	0.73	1.83	2.78	48.76	0.	2.35
time (sec)	N/A	0.147	0.393	0.054	1.584	2.512	0.	1.617

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	106	180	603	6643	0	240
normalized size	1	1.	1.08	1.84	6.15	67.79	0.	2.45
time (sec)	N/A	0.136	0.157	0.072	1.535	2.673	0.	1.511

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	94	105	346	1334	0	165
normalized size	1	1.	2.	2.23	7.36	28.38	0.	3.51
time (sec)	N/A	0.058	0.221	0.067	1.055	2.178	0.	1.613

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	138	154	510	12523	0	259
normalized size	1	1.	1.29	1.44	4.77	117.04	0.	2.42
time (sec)	N/A	0.16	0.151	0.085	1.561	2.992	0.	1.593

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	147	146	632	11420	0	336
normalized size	1	1.	1.52	1.51	6.52	117.73	0.	3.46
time (sec)	N/A	0.098	0.186	0.083	1.582	2.815	0.	1.693

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	306	294	385	873	0	0	956
normalized size	1	1.11	1.07	1.4	3.17	0.	0.	3.48
time (sec)	N/A	0.494	6.239	0.076	1.807	0.	0.	4.874

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	291	554	815	23883	0	811
normalized size	1	1.	0.83	1.58	2.32	68.04	0.	2.31
time (sec)	N/A	0.35	6.545	0.13	1.787	3.607	0.	3.952

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	241	244	289	734	25608	0	799
normalized size	1	1.1	1.11	1.31	3.34	116.4	0.	3.63
time (sec)	N/A	0.315	6.22	0.073	1.751	4.052	0.	3.473

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	233	458	653	17484	0	655
normalized size	1	1.	0.87	1.7	2.43	65.	0.	2.43
time (sec)	N/A	0.249	6.289	0.067	1.747	3.985	0.	2.633

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	154	387	883	19499	0	570
normalized size	1	1.	0.7	1.77	4.03	89.04	0.	2.6
time (sec)	N/A	0.264	2.886	0.096	1.72	3.6	0.	2.286

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	113	223	917	3071	0	420
normalized size	1	1.	1.59	3.14	12.92	43.25	0.	5.92
time (sec)	N/A	0.065	0.687	0.086	1.195	2.234	0.	2.492

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	243	334	791	30318	0	586
normalized size	1	1.	1.05	1.44	3.41	130.68	0.	2.53
time (sec)	N/A	0.316	6.316	0.099	1.778	3.905	0.	2.564

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	213	275	1346	25978	0	590
normalized size	1	1.	1.54	1.99	9.75	188.25	0.	4.28
time (sec)	N/A	0.115	0.169	0.106	1.81	3.63	0.	2.877

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	491	491	645	603	0	0	0	489
normalized size	1	1.	1.31	1.23	0.	0.	0.	1.
time (sec)	N/A	0.892	4.377	0.128	0.	0.	0.	2.33

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	32	0	826	346	0	0	0	473
normalized size	1	0.	25.81	10.81	0.	0.	0.	14.78
time (sec)	N/A	0.046	0.484	0.11	0.	0.	0.	2.058

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	384	384	423	356	0	22579	0	301
normalized size	1	1.	1.1	0.93	0.	58.8	0.	0.78
time (sec)	N/A	0.629	3.391	0.109	0.	16.062	0.	1.784

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	30	0	409	164	0	0	0	263
normalized size	1	0.	13.63	5.47	0.	0.	0.	8.77
time (sec)	N/A	0.028	0.226	0.108	0.	0.	0.	1.621

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	30	0	319	98	0	0	0	207
normalized size	1	0.	10.63	3.27	0.	0.	0.	6.9
time (sec)	N/A	0.041	0.158	0.105	0.	0.	0.	1.453

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	190	121	0	1801	0	28
normalized size	1	1.	1.21	0.77	0.	11.47	0.	0.18
time (sec)	N/A	0.14	0.135	0.112	0.	2.665	0.	1.33

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	32	0	201	144	0	0	0	0
normalized size	1	0.	6.28	4.5	0.	0.	0.	0.
time (sec)	N/A	0.047	0.362	0.125	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	C	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	322	187	0	4415	0	243
normalized size	1	1.	1.5	0.87	0.	20.53	0.	1.13
time (sec)	N/A	0.239	3.157	0.131	0.	17.102	0.	1.423

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	44	82	140	169	0	144
normalized size	1	1.	0.7	1.3	2.22	2.68	0.	2.29
time (sec)	N/A	0.05	0.163	0.039	1.144	1.825	0.	1.279

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	44	53	112	119	0	127
normalized size	1	1.	1.47	1.77	3.73	3.97	0.	4.23
time (sec)	N/A	0.036	0.016	0.036	1.097	1.813	0.	1.246

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	54	93	80	0	109
normalized size	1	1.	0.97	1.64	2.82	2.42	0.	3.3
time (sec)	N/A	0.039	0.057	0.033	1.11	1.861	0.	1.219

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	47	37	74	296	0	76
normalized size	1	1.	1.74	1.37	2.74	10.96	0.	2.81
time (sec)	N/A	0.032	0.029	0.034	1.61	1.865	0.	1.265

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	48	65	108	883	0	85
normalized size	1	1.	1.2	1.62	2.7	22.08	0.	2.12
time (sec)	N/A	0.031	0.023	0.026	1.65	1.96	0.	1.211

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	53	46	424	0	80
normalized size	1	1.	1.	1.89	1.64	15.14	0.	2.86
time (sec)	N/A	0.029	0.011	0.036	1.133	1.818	0.	1.189

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	93	103	244	2826	0	178
normalized size	1	1.	1.41	1.56	3.7	42.82	0.	2.7
time (sec)	N/A	0.046	0.029	0.044	1.616	2.012	0.	1.226

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	86	75	501	934	0	128
normalized size	1	1.	1.79	1.56	10.44	19.46	0.	2.67
time (sec)	N/A	0.04	0.048	0.044	1.108	1.92	0.	1.343

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	63	124	231	234	0	255
normalized size	1	1.	0.74	1.46	2.72	2.75	0.	3.
time (sec)	N/A	0.087	0.293	0.04	1.035	1.96	0.	1.933

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	71	117	217	1314	0	221
normalized size	1	1.	1.31	2.17	4.02	24.33	0.	4.09
time (sec)	N/A	0.06	0.443	0.043	1.674	1.926	0.	1.791

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	54	96	189	261	0	230
normalized size	1	1.	1.06	1.88	3.71	5.12	0.	4.51
time (sec)	N/A	0.076	0.371	0.04	1.205	2.008	0.	1.689

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	54	122	205	1967	0	185
normalized size	1	1.	0.9	2.03	3.42	32.78	0.	3.08
time (sec)	N/A	0.082	0.179	0.043	1.749	1.993	0.	1.666

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	427	173	269	3433	0	212
normalized size	1	1.	4.69	1.9	2.96	37.73	0.	2.33
time (sec)	N/A	0.085	7.22	0.036	1.615	2.191	0.	1.396

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	126	72	1023	0	228
normalized size	1	1.	1.	2.57	1.47	20.88	0.	4.65
time (sec)	N/A	0.053	0.18	0.052	1.165	1.862	0.	1.411

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	125	125	792	236	466	7337	0	393
normalized size	1	1.	6.34	1.89	3.73	58.7	0.	3.14
time (sec)	N/A	0.152	8.641	0.056	1.72	2.294	0.	1.489

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	83	158	1253	1829	0	321
normalized size	1	1.	1.09	2.08	16.49	24.07	0.	4.22
time (sec)	N/A	0.07	0.515	0.057	1.211	1.904	0.	1.456

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	81	184	360	539	0	385
normalized size	1	1.	0.89	2.02	3.96	5.92	0.	4.23
time (sec)	N/A	0.13	0.668	0.042	1.167	2.04	0.	2.677

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	87	87	494	227	383	4523	0	377
normalized size	1	1.	5.68	2.61	4.4	51.99	0.	4.33
time (sec)	N/A	0.105	6.901	0.044	1.665	2.207	0.	2.42

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	69	148	346	903	0	360
normalized size	1	1.	0.88	1.9	4.44	11.58	0.	4.62
time (sec)	N/A	0.09	0.85	0.047	1.155	2.042	0.	2.061

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	89	257	398	5933	0	347
normalized size	1	1.	0.9	2.6	4.02	59.93	0.	3.51
time (sec)	N/A	0.116	0.363	0.048	1.701	2.189	0.	1.832

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	149	149	1341	334	489	8768	0	433
normalized size	1	1.	9.	2.24	3.28	58.85	0.	2.91
time (sec)	N/A	0.154	17.289	0.041	1.687	2.322	0.	1.589

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	67	227	96	2034	0	468
normalized size	1	1.	1.	3.39	1.43	30.36	0.	6.99
time (sec)	N/A	0.061	0.167	0.065	1.054	1.99	0.	1.705

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	158	421	747	16058	0	698
normalized size	1	1.	0.8	2.13	3.77	81.1	0.	3.53
time (sec)	N/A	0.239	11.677	0.071	1.668	2.568	0.	1.704

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	218	269	2493	3218	0	603
normalized size	1	1.	2.14	2.64	24.44	31.55	0.	5.91
time (sec)	N/A	0.089	0.826	0.124	1.172	2.014	0.	1.827

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	115	857	0	5515	0	440
normalized size	1	1.	0.96	7.14	0.	45.96	0.	3.67
time (sec)	N/A	0.168	0.275	0.088	0.	2.574	0.	2.795

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	79	468	0	4604	0	6807
normalized size	1	1.	0.99	5.85	0.	57.55	0.	85.09
time (sec)	N/A	0.109	0.396	0.087	0.	2.374	0.	2.115

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	77	608	0	2520	0	232
normalized size	1	1.	1.	7.9	0.	32.73	0.	3.01
time (sec)	N/A	0.103	0.148	0.083	0.	2.285	0.	1.853

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	315	0	2040	0	6017
normalized size	1	1.	1.	5.94	0.	38.49	0.	113.53
time (sec)	N/A	0.07	0.092	0.071	0.	2.462	0.	1.584

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	235	0	1385	0	6607
normalized size	1	1.	1.	6.53	0.	38.47	0.	183.53
time (sec)	N/A	0.044	0.037	0.059	0.	2.398	0.	1.57

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	363	0	1199	0	59
normalized size	1	1.	1.	11.34	0.	37.47	0.	1.84
time (sec)	N/A	0.056	0.053	0.07	0.	2.396	0.	1.401

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	494	0	1494	0	4938
normalized size	1	1.	1.	8.98	0.	27.16	0.	89.78
time (sec)	N/A	0.073	0.195	0.073	0.	2.584	0.	1.793

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	648	0	1750	0	113
normalized size	1	1.	1.	12.96	0.	35.	0.	2.26
time (sec)	N/A	0.07	0.133	0.068	0.	2.422	0.	1.38

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	79	836	0	4316	0	5747
normalized size	1	1.	0.92	9.72	0.	50.19	0.	66.83
time (sec)	N/A	0.12	0.557	0.076	0.	2.825	0.	1.844

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	71	1077	0	5189	0	207
normalized size	1	1.	0.95	14.36	0.	69.19	0.	2.76
time (sec)	N/A	0.092	0.341	0.078	0.	2.65	0.	1.449

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	111	875	0	16069	0	9686
normalized size	1	1.	0.87	6.84	0.	125.54	0.	75.67
time (sec)	N/A	0.182	1.053	0.105	0.	3.215	0.	2.513

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	110	1146	0	10118	0	606
normalized size	1	1.	0.79	8.19	0.	72.27	0.	4.33
time (sec)	N/A	0.194	0.748	0.107	0.	2.861	0.	2.307

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	84	729	0	8197	0	8317
normalized size	1	1.	0.83	7.22	0.	81.16	0.	82.35
time (sec)	N/A	0.139	0.697	0.096	0.	2.532	0.	1.88

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	78	666	0	4917	0	6892
normalized size	1	1.	0.94	8.02	0.	59.24	0.	83.04
time (sec)	N/A	0.071	0.275	0.082	0.	2.302	0.	1.681

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	63	498	0	3663	0	186
normalized size	1	1.	0.95	7.55	0.	55.5	0.	2.82
time (sec)	N/A	0.066	0.239	0.096	0.	2.308	0.	1.618

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	69	375	0	4026	0	4585
normalized size	1	1.	0.96	5.21	0.	55.92	0.	63.68
time (sec)	N/A	0.077	0.138	0.101	0.	2.274	0.	1.938

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	83	746	0	3452	0	211
normalized size	1	1.	1.08	9.69	0.	44.83	0.	2.74
time (sec)	N/A	0.079	0.279	0.095	0.	2.221	0.	1.668

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	203	1007	0	5292	0	5392
normalized size	1	1.	1.99	9.87	0.	51.88	0.	52.86
time (sec)	N/A	0.127	0.483	0.099	0.	2.587	0.	2.16

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	102	1283	0	6589	0	335
normalized size	1	1.	1.05	13.23	0.	67.93	0.	3.45
time (sec)	N/A	0.129	0.585	0.092	0.	2.597	0.	1.665

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	265	1477	0	15050	0	6431
normalized size	1	1.	1.71	9.53	0.	97.1	0.	41.49
time (sec)	N/A	0.249	1.303	0.102	0.	3.933	0.	2.158

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	164	2132	0	31513	0	803
normalized size	1	1.	0.83	10.77	0.	159.16	0.	4.06
time (sec)	N/A	0.31	1.279	0.121	0.	4.53	0.	3.207

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	136	1570	0	25858	0	10338
normalized size	1	1.	0.88	10.19	0.	167.91	0.	67.13
time (sec)	N/A	0.223	1.857	0.111	0.	3.783	0.	2.144

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	134	1676	0	18148	0	7417
normalized size	1	1.	0.93	11.64	0.	126.03	0.	51.51
time (sec)	N/A	0.141	0.943	0.086	0.	3.184	0.	1.896

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	77	764	0	13154	0	432
normalized size	1	1.	0.8	7.96	0.	137.02	0.	4.5
time (sec)	N/A	0.075	0.748	0.108	0.	3.048	0.	1.917

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	123	1226	0	15408	0	7308
normalized size	1	1.	0.95	9.5	0.	119.44	0.	56.65
time (sec)	N/A	0.12	0.742	0.12	0.	2.877	0.	2.428

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	115	1270	0	12604	0	448
normalized size	1	1.	1.	11.04	0.	109.6	0.	3.9
time (sec)	N/A	0.097	0.963	0.111	0.	2.974	0.	1.916

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	88	634	0	12057	0	5954
normalized size	1	1.	0.85	6.1	0.	115.93	0.	57.25
time (sec)	N/A	0.091	0.295	0.116	0.	2.782	0.	2.297

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	128	1776	0	11889	0	456
normalized size	1	1.	0.98	13.56	0.	90.76	0.	3.48
time (sec)	N/A	0.124	0.894	0.114	0.	2.869	0.	1.938

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	317	1907	0	18573	0	6677
normalized size	1	1.	2.03	12.22	0.	119.06	0.	42.8
time (sec)	N/A	0.228	3.171	0.112	0.	3.353	0.	2.466

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	97	128	269	938	82	181
normalized size	1	1.	1.8	2.37	4.98	17.37	1.52	3.35
time (sec)	N/A	0.052	0.034	0.006	1.036	1.953	0.933	1.194

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	43	104	227	3336	88	130
normalized size	1	1.	0.88	2.12	4.63	68.08	1.8	2.65
time (sec)	N/A	0.059	0.236	0.004	1.665	2.154	0.64	1.214

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	65	100	142	428	54	116
normalized size	1	1.	1.81	2.78	3.94	11.89	1.5	3.22
time (sec)	N/A	0.04	0.025	0.005	1.138	1.83	0.381	1.166

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	41	76	103	1114	60	82
normalized size	1	1.	1.32	2.45	3.32	35.94	1.94	2.65
time (sec)	N/A	0.032	0.023	0.004	1.698	1.9	0.32	1.165

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	28	47	42	96	20	46
normalized size	1	1.	1.47	2.47	2.21	5.05	1.05	2.42
time (sec)	N/A	0.013	0.007	0.002	1.092	2.054	0.191	1.166

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	33	26	47	178	0	66
normalized size	1	1.	1.32	1.04	1.88	7.12	0.	2.64
time (sec)	N/A	0.041	0.04	0.041	1.103	2.029	0.	1.176

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	32	28	42	97	49	43
normalized size	1	1.	1.78	1.56	2.33	5.39	2.72	2.39
time (sec)	N/A	0.029	0.026	0.033	1.041	1.934	60.368	1.217

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	39	40	143	1114	0	84
normalized size	1	1.	1.26	1.29	4.61	35.94	0.	2.71
time (sec)	N/A	0.042	0.104	0.046	1.079	2.106	0.	1.184

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	61	46	142	408	0	116
normalized size	1	1.	1.69	1.28	3.94	11.33	0.	3.22
time (sec)	N/A	0.039	0.036	0.042	1.172	1.915	0.	1.198

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	51	68	278	3336	0	131
normalized size	1	1.	1.04	1.39	5.67	68.08	0.	2.67
time (sec)	N/A	0.062	0.313	0.046	1.165	2.172	0.	1.201

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	190	236	498	2140	165	405
normalized size	1	1.	2.29	2.84	6.	25.78	1.99	4.88
time (sec)	N/A	0.079	0.077	0.006	1.161	1.975	1.518	1.314

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	66	196	450	8982	170	262
normalized size	1	1.	0.87	2.58	5.92	118.18	2.24	3.45
time (sec)	N/A	0.11	0.408	0.004	1.79	2.342	1.217	1.308

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	137	189	312	1245	117	294
normalized size	1	1.	2.17	3.	4.95	19.76	1.86	4.67
time (sec)	N/A	0.076	0.048	0.005	1.163	2.092	0.92	1.248

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	50	149	251	4165	122	162
normalized size	1	1.	0.88	2.61	4.4	73.07	2.14	2.84
time (sec)	N/A	0.073	0.307	0.004	1.723	2.166	0.704	1.268

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	65	144	154	509	68	139
normalized size	1	1.	1.51	3.35	3.58	11.84	1.58	3.23
time (sec)	N/A	0.031	0.707	0.006	1.058	1.849	0.42	1.191

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	48	60	140	1715	0	198
normalized size	1	1.	0.98	1.22	2.86	35.	0.	4.04
time (sec)	N/A	0.077	0.125	0.053	1.529	2.093	0.	1.265

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	64	49	86	243	0	99
normalized size	1	1.	1.78	1.36	2.39	6.75	0.	2.75
time (sec)	N/A	0.067	0.096	0.043	1.074	1.901	0.	1.308

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	50	60	181	1715	0	198
normalized size	1	1.	0.96	1.15	3.48	32.98	0.	3.81
time (sec)	N/A	0.095	0.153	0.058	1.018	2.036	0.	1.295

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	65	59	154	489	0	139
normalized size	1	1.	1.51	1.37	3.58	11.37	0.	3.23
time (sec)	N/A	0.071	0.571	0.05	1.067	2.067	0.	1.274

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	58	91	319	4165	0	165
normalized size	1	1.	0.81	1.26	4.43	57.85	0.	2.29
time (sec)	N/A	0.099	0.391	0.054	1.036	2.184	0.	1.315

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	98	87	312	1206	0	294
normalized size	1	1.	1.56	1.38	4.95	19.14	0.	4.67
time (sec)	N/A	0.077	0.079	0.056	1.069	2.043	0.	1.289

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	74	138	527	8982	0	263
normalized size	1	1.	0.8	1.5	5.73	97.63	0.	2.86
time (sec)	N/A	0.113	0.484	0.059	1.072	2.455	0.	1.307

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	123	365	787	4166	260	721
normalized size	1	1.	1.08	3.2	6.9	36.54	2.28	6.32
time (sec)	N/A	0.101	1.432	0.007	1.151	2.19	2.827	1.453

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	98	307	729	19047	279	421
normalized size	1	1.	0.92	2.87	6.81	178.01	2.61	3.93
time (sec)	N/A	0.152	0.289	0.007	1.592	3.118	2.4	1.464

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	108	299	540	2693	192	564
normalized size	1	1.	1.15	3.18	5.74	28.65	2.04	6.
time (sec)	N/A	0.093	1.613	0.005	1.069	1.977	1.698	1.461

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	76	241	474	10689	211	296
normalized size	1	1.	0.92	2.9	5.71	128.78	2.54	3.57
time (sec)	N/A	0.101	0.214	0.006	1.6	2.623	1.404	1.386

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	95	235	323	1412	126	325
normalized size	1	1.	1.28	3.18	4.36	19.08	1.7	4.39
time (sec)	N/A	0.046	0.57	0.004	1.107	2.044	0.904	1.187

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	67	111	289	5854	0	366
normalized size	1	1.	0.93	1.54	4.01	81.31	0.	5.08
time (sec)	N/A	0.098	0.518	0.055	1.706	2.221	0.	1.413

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	81	80	198	811	0	186
normalized size	1	1.	1.37	1.36	3.36	13.75	0.	3.15
time (sec)	N/A	0.08	2.181	0.049	1.158	1.951	0.	1.428

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	63	94	274	4113	0	375
normalized size	1	1.	0.88	1.31	3.81	57.12	0.	5.21
time (sec)	N/A	0.108	0.437	0.06	1.573	2.173	0.	1.421

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	82	80	198	811	0	186
normalized size	1	1.	1.39	1.36	3.36	13.75	0.	3.15
time (sec)	N/A	0.081	1.187	0.05	1.074	2.006	0.	1.428

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	67	111	356	5854	0	366
normalized size	1	1.	0.81	1.34	4.29	70.53	0.	4.41
time (sec)	N/A	0.117	0.555	0.062	1.076	2.37	0.	1.502

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	100	100	323	1372	0	325
normalized size	1	1.	1.35	1.35	4.36	18.54	0.	4.39
time (sec)	N/A	0.089	1.672	0.065	1.07	2.101	0.	1.511

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	76	161	567	10689	0	297
normalized size	1	1.	0.74	1.56	5.5	103.78	0.	2.88
time (sec)	N/A	0.129	0.238	0.062	1.074	2.878	0.	1.53

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	128	344	554	2954	209	603
normalized size	1	1.	1.16	3.13	5.04	26.85	1.9	5.48
time (sec)	N/A	0.07	1.673	0.006	1.118	2.525	1.608	1.194

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	170	472	842	5516	308	973
normalized size	1	1.	1.06	2.95	5.26	34.48	1.92	6.08
time (sec)	N/A	0.094	2.148	0.006	1.205	2.671	2.828	1.227

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	60	93	180	1817	425	190
normalized size	1	1.	0.91	1.41	2.73	27.53	6.44	2.88
time (sec)	N/A	0.115	0.161	0.018	1.576	3.057	39.448	1.267

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	66	95	0	2043	495	126
normalized size	1	1.	1.12	1.61	0.	34.63	8.39	2.14
time (sec)	N/A	0.108	0.172	0.02	0.	2.239	21.121	1.208

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	42	75	111	319	316	136
normalized size	1	1.	0.91	1.63	2.41	6.93	6.87	2.96
time (sec)	N/A	0.104	0.034	0.017	1.548	2.435	18.753	1.212

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	47	77	0	1249	294	92
normalized size	1	1.	1.02	1.67	0.	27.15	6.39	2.
time (sec)	N/A	0.083	0.029	0.016	0.	1.913	11.628	1.176

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	35	71	78	220	156	84
normalized size	1	1.	0.83	1.69	1.86	5.24	3.71	2.
time (sec)	N/A	0.061	0.024	0.018	1.08	1.75	11.396	1.19

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	65	76	0	1249	280	90
normalized size	1	1.	1.44	1.69	0.	27.76	6.22	2.
time (sec)	N/A	0.074	0.077	0.017	0.	1.959	11.581	1.161

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	54	121	136	319	0	138
normalized size	1	1.	0.9	2.02	2.27	5.32	0.	2.3
time (sec)	N/A	0.101	0.054	0.076	1.091	2.057	0.	1.215

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	67	494	0	2043	0	127
normalized size	1	1.	1.12	8.23	0.	34.05	0.	2.12
time (sec)	N/A	0.106	0.17	0.084	0.	1.88	0.	1.249

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	60	180	215	1817	0	190
normalized size	1	1.	0.71	2.12	2.53	21.38	0.	2.24
time (sec)	N/A	0.137	0.164	0.086	1.081	2.487	0.	1.199

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	91	580	0	5846	0	203
normalized size	1	1.	1.11	7.07	0.	71.29	0.	2.48
time (sec)	N/A	0.177	0.605	0.096	0.	2.219	0.	1.2

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	69	156	293	2678	0	275
normalized size	1	1.	0.83	1.88	3.53	32.27	0.	3.31
time (sec)	N/A	0.15	0.474	0.027	1.638	2.616	0.	1.267

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	90	172	0	4703	2179	274
normalized size	1	1.	1.01	1.93	0.	52.84	24.48	3.08
time (sec)	N/A	0.117	0.484	0.026	0.	2.101	101.773	1.213

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	57	118	230	1611	0	208
normalized size	1	1.	0.79	1.64	3.19	22.38	0.	2.89
time (sec)	N/A	0.118	0.439	0.024	1.089	1.797	0.	1.233

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	86	162	0	4747	2144	251
normalized size	1	1.	1.01	1.91	0.	55.85	25.22	2.95
time (sec)	N/A	0.108	0.376	0.023	0.	2.147	83.268	1.221

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	55	113	230	1611	0	208
normalized size	1	1.	0.81	1.66	3.38	23.69	0.	3.06
time (sec)	N/A	0.086	0.371	0.027	1.092	1.956	0.	1.223

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	97	172	0	4703	2086	274
normalized size	1	1.	1.09	1.93	0.	52.84	23.44	3.08
time (sec)	N/A	0.088	0.495	0.024	0.	2.092	151.002	1.19

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	83	325	317	2678	0	279
normalized size	1	1.	0.87	3.42	3.34	28.19	0.	2.94
time (sec)	N/A	0.149	1.858	0.102	1.128	3.142	0.	1.218

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	111	1061	0	8676	0	464
normalized size	1	1.	0.93	8.92	0.	72.91	0.	3.9
time (sec)	N/A	0.194	1.559	0.108	0.	2.731	0.	1.252

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	93	383	543	7919	0	450
normalized size	1	1.	0.75	3.09	4.38	63.86	0.	3.63
time (sec)	N/A	0.189	0.8	0.113	1.165	4.111	0.	1.283

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	139	1137	0	19919	0	393
normalized size	1	1.	0.87	7.15	0.	125.28	0.	2.47
time (sec)	N/A	0.283	1.327	0.122	0.	3.247	0.	1.269

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	144	352	0	17204	0	566
normalized size	1	1.	1.	2.44	0.	119.47	0.	3.93
time (sec)	N/A	0.208	1.182	0.027	0.	3.475	0.	1.266

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	91	234	508	6044	0	339
normalized size	1	1.	0.83	2.15	4.66	55.45	0.	3.11
time (sec)	N/A	0.17	0.984	0.027	1.241	2.45	0.	1.265

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	135	340	0	17383	0	533
normalized size	1	1.	0.99	2.48	0.	126.88	0.	3.89
time (sec)	N/A	0.195	1.06	0.029	0.	3.193	0.	1.257

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	80	196	518	6130	0	339
normalized size	1	1.	0.82	2.	5.29	62.55	0.	3.46
time (sec)	N/A	0.138	0.453	0.027	1.313	2.437	0.	1.268

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	137	340	0	17383	0	539
normalized size	1	1.	1.	2.48	0.	126.88	0.	3.93
time (sec)	N/A	0.157	1.069	0.026	0.	3.33	0.	1.264

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	77	193	510	6044	0	339
normalized size	1	1.	0.82	2.05	5.43	64.3	0.	3.61
time (sec)	N/A	0.105	0.514	0.029	1.37	2.547	0.	1.24

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	147	352	0	17204	0	567
normalized size	1	1.	1.04	2.48	0.	121.15	0.	3.99
time (sec)	N/A	0.163	0.277	0.031	0.	3.367	0.	1.221

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	117	952	672	10714	0	417
normalized size	1	1.	0.85	6.9	4.87	77.64	0.	3.02
time (sec)	N/A	0.205	1.569	0.117	1.308	5.282	0.	1.289

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	166	2045	0	27774	0	609
normalized size	1	1.	0.93	11.49	0.	156.03	0.	3.42
time (sec)	N/A	0.291	5.76	0.125	0.	3.825	0.	1.334

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	138	1020	1040	24520	0	656
normalized size	1	1.	0.81	5.96	6.08	143.39	0.	3.84
time (sec)	N/A	0.262	1.719	0.128	1.401	7.58	0.	1.412

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	194	2139	0	0	0	683
normalized size	1	1.	0.85	9.38	0.	0.	0.	3.
time (sec)	N/A	0.368	3.418	0.14	0.	0.	0.	1.458

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	203	608	0	0	0	1030
normalized size	1	1.	1.01	3.02	0.	0.	0.	5.12
time (sec)	N/A	0.277	0.547	0.034	0.	0.	0.	1.315

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	19	4	7	39	0	7
normalized size	1	1.	6.33	1.33	2.33	13.	0.	2.33
time (sec)	N/A	0.017	0.008	0.033	1.687	2.161	0.	1.169

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	21	15	7	4	0	23
normalized size	1	1.	1.31	0.94	0.44	0.25	0.	1.44
time (sec)	N/A	0.021	0.007	0.036	1.678	2.234	0.	1.203

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	29	21	38	504	0	61
normalized size	1	1.	1.32	0.95	1.73	22.91	0.	2.77
time (sec)	N/A	0.02	0.018	0.03	1.682	2.147	0.	1.297

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	28	28	43	4	0	84
normalized size	1	1.	0.8	0.8	1.23	0.11	0.	2.4
time (sec)	N/A	0.024	0.019	0.035	1.724	2.355	0.	1.207

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	14	15	12	12	15
normalized size	1	1.	1.	1.27	1.36	1.09	1.09	1.36
time (sec)	N/A	0.021	0.007	0.01	1.54	2.222	0.593	1.139

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	34	4	0	15
normalized size	1	1.	1.	0.92	2.62	0.31	0.	1.15
time (sec)	N/A	0.021	0.007	0.014	1.56	2.29	0.	1.185

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	85	288	0	12513	97	0
normalized size	1	1.	0.98	3.31	0.	143.83	1.11	0.
time (sec)	N/A	0.161	0.466	0.071	0.	6.511	7.927	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	580	337	0	25867	0	0
normalized size	1	1.	4.79	2.79	0.	213.78	0.	0.
time (sec)	N/A	0.2	6.167	0.049	0.	7.357	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	60	253	0	6839	71	0
normalized size	1	1.	0.95	4.02	0.	108.56	1.13	0.
time (sec)	N/A	0.119	0.161	0.045	0.	3.917	4.425	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	193	276	0	14340	0	0
normalized size	1	1.	2.27	3.25	0.	168.71	0.	0.
time (sec)	N/A	0.125	3.262	0.042	0.	4.622	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	238	0	4456	51	0
normalized size	1	1.	1.	5.41	0.	101.27	1.16	0.
time (sec)	N/A	0.077	0.027	0.049	0.	2.857	2.281	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	137	238	0	9993	0	0
normalized size	1	1.	2.28	3.97	0.	166.55	0.	0.
time (sec)	N/A	0.047	0.213	0.046	0.	3.689	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	0	0	9991	0	0
normalized size	1	1.	1.	0.	0.	178.41	0.	0.
time (sec)	N/A	0.112	0.028	0.168	0.	3.595	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	42	0	0	4456	0	0
normalized size	1	1.	0.88	0.	0.	92.83	0.	0.
time (sec)	N/A	0.093	0.105	0.153	0.	2.885	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	83	0	0	14338	0	0
normalized size	1	1.	1.	0.	0.	172.75	0.	0.
time (sec)	N/A	0.155	0.206	0.155	0.	4.447	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	235	0	0	6839	0	0
normalized size	1	1.	3.01	0.	0.	87.68	0.	0.
time (sec)	N/A	0.146	8.836	0.152	0.	3.914	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	111	0	0	25865	0	0
normalized size	1	1.	0.92	0.	0.	213.76	0.	0.
time (sec)	N/A	0.213	0.578	0.153	0.	8.328	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	86	593	0	13874	175	0
normalized size	1	1.	1.05	7.23	0.	169.2	2.13	0.
time (sec)	N/A	0.152	0.412	0.027	0.	7.374	31.263	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	584	633	0	0	0	0
normalized size	1	1.	4.75	5.15	0.	0.	0.	0.
time (sec)	N/A	0.244	6.202	0.022	0.	0.	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	59	578	0	7218	128	0
normalized size	1	1.	0.94	9.17	0.	114.57	2.03	0.
time (sec)	N/A	0.101	0.16	0.023	0.	3.241	16.326	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	161	578	0	14453	0	0
normalized size	1	1.	1.83	6.57	0.	164.24	0.	0.
time (sec)	N/A	0.085	0.338	0.023	0.	4.087	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	71	0	0	11954	0	0
normalized size	1	1.	1.	0.	0.	168.37	0.	0.
time (sec)	N/A	0.136	0.06	0.116	0.	3.569	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	197	0	0	11507	0	0
normalized size	1	1.	2.56	0.	0.	149.44	0.	0.
time (sec)	N/A	0.127	2.763	0.115	0.	3.639	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	51	97	0	2319	0	140
normalized size	1	1.	1.65	3.13	0.	74.81	0.	4.52
time (sec)	N/A	0.027	0.058	0.059	0.	1.91	0.	1.216

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	C	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	53	142	0	710	0	136
normalized size	1	1.	1.18	3.16	0.	15.78	0.	3.02
time (sec)	N/A	0.035	0.033	0.056	0.	1.878	0.	1.27

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	74	158	0	3526	0	273
normalized size	1	1.	1.48	3.16	0.	70.52	0.	5.46
time (sec)	N/A	0.039	0.134	0.025	0.	2.044	0.	1.256

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	C	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	76	211	0	1103	0	273
normalized size	1	1.	1.13	3.15	0.	16.46	0.	4.07
time (sec)	N/A	0.047	0.067	0.027	0.	1.841	0.	1.216

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	68	164	0	7692	0	0
normalized size	1	1.	0.97	2.34	0.	109.89	0.	0.
time (sec)	N/A	0.136	0.482	0.053	0.	4.371	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	208	178	0	15340	0	0
normalized size	1	1.	2.36	2.02	0.	174.32	0.	0.
time (sec)	N/A	0.123	4.434	0.048	0.	4.58	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	129	0	4629	0	0
normalized size	1	1.	1.	2.74	0.	98.49	0.	0.
time (sec)	N/A	0.106	0.092	0.045	0.	2.686	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	101	137	0	9661	0	0
normalized size	1	1.	1.68	2.28	0.	161.02	0.	0.
time (sec)	N/A	0.093	0.376	0.044	0.	3.38	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	114	0	3792	31	0
normalized size	1	1.	1.	3.93	0.	130.76	1.07	0.
time (sec)	N/A	0.064	0.014	0.05	0.	2.146	1.035	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	114	0	3568	0	0
normalized size	1	1.	1.	3.68	0.	115.1	0.	0.
time (sec)	N/A	0.027	0.021	0.046	0.	2.23	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	0	0	10107	0	0
normalized size	1	1.	1.	0.	0.	180.48	0.	0.
time (sec)	N/A	0.113	0.039	0.15	0.	3.477	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	51	51	123	0	0	4405	0	0
normalized size	1	1.	2.41	0.	0.	86.37	0.	0.
time (sec)	N/A	0.095	5.579	0.151	0.	2.519	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	107	0	0	15786	0	0
normalized size	1	1.	1.22	0.	0.	179.39	0.	0.
time (sec)	N/A	0.166	0.423	0.154	0.	4.626	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	67	322	0	9975	0	0
normalized size	1	1.	0.93	4.47	0.	138.54	0.	0.
time (sec)	N/A	0.163	0.102	0.029	0.	5.041	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	188	328	0	18425	0	0
normalized size	1	1.	2.24	3.9	0.	219.35	0.	0.
time (sec)	N/A	0.13	2.253	0.027	0.	5.325	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	287	0	6730	0	0
normalized size	1	1.	1.	5.52	0.	129.42	0.	0.
time (sec)	N/A	0.123	0.119	0.023	0.	3.074	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	112	289	0	6395	0	0
normalized size	1	1.	2.11	5.45	0.	120.66	0.	0.
time (sec)	N/A	0.099	1.456	0.021	0.	3.099	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	41	273	0	6395	51	0
normalized size	1	1.	0.84	5.57	0.	130.51	1.04	0.
time (sec)	N/A	0.086	0.035	0.02	0.	2.989	12.259	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	56	56	223	272	0	6730	0	0
normalized size	1	1.	3.98	4.86	0.	120.18	0.	0.
time (sec)	N/A	0.043	4.215	0.026	0.	3.097	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	70	0	0	18423	0	0
normalized size	1	1.	0.9	0.	0.	236.19	0.	0.
time (sec)	N/A	0.146	0.077	0.115	0.	5.605	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	230	0	0	9975	0	0
normalized size	1	1.	2.71	0.	0.	117.35	0.	0.
time (sec)	N/A	0.161	7.811	0.121	0.	4.994	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	231	549	0	0	0	0
normalized size	1	1.	1.96	4.65	0.	0.	0.	0.
time (sec)	N/A	0.22	1.862	0.032	0.	0.	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	68	469	0	16629	0	0
normalized size	1	1.	0.81	5.58	0.	197.96	0.	0.
time (sec)	N/A	0.181	0.111	0.027	0.	9.055	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	132	491	0	14436	0	0
normalized size	1	1.	1.47	5.46	0.	160.4	0.	0.
time (sec)	N/A	0.138	2.556	0.025	0.	8.557	0.	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	63	435	0	15965	0	0
normalized size	1	1.	0.85	5.88	0.	215.74	0.	0.
time (sec)	N/A	0.144	0.087	0.021	0.	8.759	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	193	454	0	15741	0	0
normalized size	1	1.	2.19	5.16	0.	178.88	0.	0.
time (sec)	N/A	0.132	7.519	0.02	0.	8.763	0.	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	43	420	0	14660	73	0
normalized size	1	1.	0.61	6.	0.	209.43	1.04	0.
time (sec)	N/A	0.103	0.041	0.02	0.	8.4	20.681	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	976	420	0	16405	0	0
normalized size	1	1.	10.49	4.52	0.	176.4	0.	0.
time (sec)	N/A	0.085	7.488	0.024	0.	8.35	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	73	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.208	0.072	0.122	0.	0.	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	246	0	0	25307	0	0
normalized size	1	1.	1.88	0.	0.	193.18	0.	0.
time (sec)	N/A	0.242	7.456	0.119	0.	18.786	0.	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	35	62	0	1831	0	78
normalized size	1	1.	1.4	2.48	0.	73.24	0.	3.12
time (sec)	N/A	0.019	0.025	0.047	0.	2.546	0.	1.308

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	C	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	37	66	0	551	0	96
normalized size	1	1.	1.37	2.44	0.	20.41	0.	3.56
time (sec)	N/A	0.02	0.021	0.049	0.	2.325	0.	1.33

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	112	95	163	262	5516	100	196
normalized size	1	1.26	1.07	1.83	2.94	61.98	1.12	2.2
time (sec)	N/A	0.075	0.673	0.006	1.522	2.676	0.773	1.348

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	40	41	99	340	102	34
normalized size	1	1.	1.05	1.08	2.61	8.95	2.68	0.89
time (sec)	N/A	0.066	0.065	0.024	1.507	2.461	0.822	1.298

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	166	620	0	0	0	0
normalized size	1	1.	1.34	5.	0.	0.	0.	0.
time (sec)	N/A	0.242	4.395	0.082	0.	0.	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	86	116	0	15323	0	0
normalized size	1	1.	0.97	1.3	0.	172.17	0.	0.
time (sec)	N/A	0.128	0.057	0.052	0.	4.128	0.	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	37	0	3553	0	0
normalized size	1	1.	1.	0.92	0.	88.82	0.	0.
time (sec)	N/A	0.078	0.016	0.054	0.	4.294	0.	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	73	431	0	10031	0	0
normalized size	1	1.	0.99	5.82	0.	135.55	0.	0.
time (sec)	N/A	0.123	0.524	0.041	0.	5.86	0.	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	113	637	0	0	0	0
normalized size	1	1.	0.96	5.4	0.	0.	0.	0.
time (sec)	N/A	0.199	0.786	0.053	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [258] had the largest ratio of [0.625]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	5	1.	21	0.238
2	A	3	2	1.	21	0.095
3	A	4	4	1.	21	0.19
4	A	3	2	1.	19	0.105
5	A	3	3	1.	19	0.158
6	A	3	2	1.	21	0.095
7	A	4	4	1.	21	0.19
8	A	3	2	1.	21	0.095
9	A	6	5	1.	23	0.217
10	A	3	2	1.	23	0.087
11	A	5	5	1.	23	0.217
12	A	3	2	1.	21	0.095
13	A	4	3	1.	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
14	A	3	2	1.	23	0.087
15	A	5	5	1.	23	0.217
16	A	3	2	1.	23	0.087
17	A	6	5	1.	23	0.217
18	A	3	2	1.	23	0.087
19	A	6	5	1.14	23	0.217
20	A	3	2	1.	21	0.095
21	A	4	3	1.	21	0.143
22	A	3	2	1.	23	0.087
23	A	6	5	1.	23	0.217
24	A	3	2	1.	23	0.087
25	A	6	6	1.	23	0.261
26	A	4	4	1.	23	0.174
27	A	5	5	1.	23	0.217
28	A	3	3	1.	21	0.143
29	A	4	4	1.	21	0.19
30	A	3	3	1.	23	0.13
31	A	5	5	1.	23	0.217
32	A	4	4	1.	23	0.174
33	A	7	6	1.	23	0.261
34	A	5	4	1.	23	0.174
35	A	6	6	1.	23	0.261
36	A	4	4	1.	21	0.19
37	A	5	5	1.	21	0.238
38	A	4	4	1.	23	0.174
39	A	6	6	1.	23	0.261
40	A	5	4	1.	23	0.174
41	A	8	6	1.	23	0.261
42	A	6	5	1.	23	0.217
43	A	7	6	1.	23	0.261
44	A	5	4	1.	21	0.19
45	A	6	6	1.	21	0.286
46	A	5	4	1.	23	0.174
47	A	7	6	1.	23	0.261
48	A	6	5	1.	23	0.217
49	A	8	5	1.	21	0.238
50	A	9	6	1.	21	0.286
51	A	7	5	1.	21	0.238
52	A	7	6	1.	19	0.316
53	A	5	3	1.	19	0.158
54	A	3	2	1.	21	0.095
55	A	6	3	1.	21	0.143
56	A	3	2	1.	21	0.095
57	A	8	5	1.21	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
58	A	12	8	1.	23	0.348
59	A	7	5	1.23	23	0.217
60	A	10	8	1.	21	0.381
61	A	8	5	1.	21	0.238
62	A	3	2	1.	23	0.087
63	A	9	5	1.	23	0.217
64	A	3	2	1.	23	0.087
65	A	8	5	1.11	23	0.217
66	A	20	8	1.	23	0.348
67	A	7	5	1.1	23	0.217
68	A	17	8	1.	21	0.381
69	A	13	5	1.	21	0.238
70	A	3	2	1.	23	0.087
71	A	14	6	1.	23	0.261
72	A	3	2	1.	23	0.087
73	A	11	10	1.	23	0.435
74	A	0	0	0.	0	0.
75	A	11	10	1.	23	0.435
76	A	0	0	0.	0	0.
77	A	0	0	0.	0	0.
78	A	8	8	1.	23	0.348
79	A	0	0	0.	0	0.
80	A	12	11	1.	23	0.478
81	A	4	4	1.	21	0.19
82	A	2	1	1.	21	0.048
83	A	3	3	1.	21	0.143
84	A	3	3	1.	19	0.158
85	A	3	3	1.	19	0.158
86	A	2	1	1.	21	0.048
87	A	4	4	1.	21	0.19
88	A	3	2	1.	21	0.095
89	A	4	4	1.	23	0.174
90	A	4	3	1.	23	0.13
91	A	5	4	1.	23	0.174
92	A	5	4	1.	21	0.19
93	A	4	4	1.	21	0.19
94	A	3	2	1.	23	0.087
95	A	5	5	1.	23	0.217
96	A	3	2	1.	23	0.087
97	A	6	5	1.	23	0.217
98	A	5	4	1.	23	0.174
99	A	5	4	1.	23	0.174
100	A	6	5	1.	21	0.238
101	A	5	5	1.	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
102	A	3	2	1.	23	0.087
103	A	6	6	1.	23	0.261
104	A	3	2	1.	23	0.087
105	A	6	6	1.	23	0.261
106	A	4	3	1.	23	0.13
107	A	5	5	1.	23	0.217
108	A	3	3	1.	21	0.143
109	A	2	2	1.	21	0.095
110	A	2	2	1.	23	0.087
111	A	4	4	1.	23	0.174
112	A	3	3	1.	23	0.13
113	A	5	5	1.	23	0.217
114	A	4	3	1.	23	0.13
115	A	5	4	1.	23	0.174
116	A	6	6	1.	23	0.261
117	A	5	4	1.	21	0.19
118	A	3	3	1.	21	0.143
119	A	3	3	1.	23	0.13
120	A	3	3	1.	23	0.13
121	A	3	3	1.	23	0.13
122	A	5	5	1.	23	0.217
123	A	5	4	1.	23	0.174
124	A	6	6	1.	23	0.261
125	A	7	6	1.	23	0.261
126	A	6	5	1.	21	0.238
127	A	4	4	1.	21	0.19
128	A	4	3	1.	23	0.13
129	A	4	4	1.	23	0.174
130	A	4	4	1.	23	0.174
131	A	4	3	1.	23	0.13
132	A	4	4	1.	23	0.174
133	A	6	6	1.	23	0.261
134	A	4	3	1.	21	0.143
135	A	3	3	1.	21	0.143
136	A	3	3	1.	21	0.143
137	A	2	2	1.	19	0.105
138	A	3	2	1.	12	0.167
139	A	3	2	1.	19	0.105
140	A	2	2	1.	21	0.095
141	A	3	3	1.	21	0.143
142	A	4	4	1.	21	0.19
143	A	4	4	1.	21	0.19
144	A	4	3	1.	23	0.13
145	A	4	3	1.	23	0.13

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
146	A	4	3	1.	23	0.13
147	A	4	3	1.	21	0.143
148	A	4	3	1.	14	0.214
149	A	4	3	1.	21	0.143
150	A	4	3	1.	23	0.13
151	A	4	3	1.	23	0.13
152	A	4	3	1.	23	0.13
153	A	4	3	1.	23	0.13
154	A	4	3	1.	23	0.13
155	A	4	3	1.	23	0.13
156	A	4	3	1.	23	0.13
157	A	4	3	1.	23	0.13
158	A	4	3	1.	23	0.13
159	A	4	3	1.	21	0.143
160	A	4	3	1.	14	0.214
161	A	4	3	1.	21	0.143
162	A	4	3	1.	23	0.13
163	A	4	3	1.	23	0.13
164	A	4	3	1.	23	0.13
165	A	4	3	1.	23	0.13
166	A	4	3	1.	23	0.13
167	A	4	3	1.	23	0.13
168	A	4	3	1.	14	0.214
169	A	4	3	1.	14	0.214
170	A	4	3	1.	23	0.13
171	A	5	5	1.	23	0.217
172	A	4	3	1.	23	0.13
173	A	4	4	1.	23	0.174
174	A	5	4	1.	21	0.19
175	A	3	3	1.	14	0.214
176	A	4	3	1.	21	0.143
177	A	5	5	1.	23	0.217
178	A	4	3	1.	23	0.13
179	A	6	6	1.	23	0.261
180	A	4	3	1.	23	0.13
181	A	5	5	1.	23	0.217
182	A	4	3	1.	23	0.13
183	A	5	5	1.	23	0.217
184	A	4	3	1.	21	0.143
185	A	5	5	1.	14	0.357
186	A	4	3	1.	21	0.143
187	A	6	6	1.	23	0.261
188	A	4	3	1.	23	0.13
189	A	7	6	1.	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
190	A	6	6	1.	23	0.261
191	A	4	3	1.	23	0.13
192	A	6	6	1.	23	0.261
193	A	4	3	1.	23	0.13
194	A	6	6	1.	23	0.261
195	A	4	3	1.	21	0.143
196	A	6	6	1.	14	0.429
197	A	4	3	1.	21	0.143
198	A	7	7	1.	23	0.304
199	A	4	3	1.	23	0.13
200	A	8	7	1.	23	0.304
201	A	7	6	1.	14	0.429
202	A	3	3	1.	12	0.25
203	A	4	4	1.	10	0.4
204	A	4	4	1.	12	0.333
205	A	5	5	1.	10	0.5
206	A	3	3	1.	12	0.25
207	A	3	3	1.	10	0.3
208	A	7	6	1.	17	0.353
209	A	8	7	1.	17	0.412
210	A	6	6	1.	17	0.353
211	A	7	6	1.	17	0.353
212	A	5	5	1.	15	0.333
213	A	6	5	1.	12	0.417
214	A	7	5	1.	15	0.333
215	A	5	5	1.	17	0.294
216	A	8	6	1.	17	0.353
217	A	6	6	1.	17	0.353
218	A	9	7	1.	17	0.412
219	A	7	6	1.	17	0.353
220	A	8	7	1.	17	0.412
221	A	6	5	1.	15	0.333
222	A	7	6	1.	12	0.5
223	A	8	6	1.	15	0.4
224	A	7	6	1.	17	0.353
225	A	5	5	1.	10	0.5
226	A	6	5	1.	12	0.417
227	A	6	6	1.	10	0.6
228	A	7	6	1.	12	0.5
229	A	6	5	1.	17	0.294
230	A	7	6	1.	17	0.353
231	A	5	5	1.	17	0.294
232	A	6	5	1.	17	0.294
233	A	4	4	1.	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
234	A	3	3	1.	12	0.25
235	A	7	5	1.	15	0.333
236	A	5	5	1.	17	0.294
237	A	8	6	1.	17	0.353
238	A	6	5	1.	17	0.294
239	A	7	6	1.	17	0.353
240	A	5	5	1.	17	0.294
241	A	4	4	1.	17	0.235
242	A	5	5	1.	15	0.333
243	A	4	4	1.	12	0.333
244	A	8	6	1.	15	0.4
245	A	6	6	1.	17	0.353
246	A	8	7	1.	17	0.412
247	A	6	5	1.	17	0.294
248	A	6	6	1.	17	0.353
249	A	6	6	1.	17	0.353
250	A	6	6	1.	17	0.353
251	A	6	5	1.	15	0.333
252	A	6	6	1.	12	0.5
253	A	9	7	1.	15	0.467
254	A	7	7	1.	17	0.412
255	A	3	3	1.	10	0.3
256	A	3	3	1.	12	0.25
257	A	6	4	1.26	14	0.286
258	A	6	5	1.	8	0.625
259	A	9	8	1.	15	0.533
260	A	8	7	1.	15	0.467
261	A	4	4	1.	15	0.267
262	A	6	6	1.	15	0.4
263	A	7	7	1.	15	0.467

Chapter 3

Listing of integrals

3.1 $\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=73

$$\frac{(a+b)\sinh(c+dx)\cosh^3(c+dx)}{4d}-\frac{(5a+9b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{3}{8}x(a+5b)-\frac{b\tanh(c+dx)}{d}$$

```
[Out] (3*(a + 5*b)*x)/8 - ((5*a + 9*b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + ((a +
 b)*Cosh[c + d*x]^3*Sinh[c + d*x])/(4*d) - (b*Tanh[c + d*x])/d
```

Rubi [A] time = 0.0757167, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3663, 455, 1157, 388, 206}

$$\frac{(a+b)\sinh(c+dx)\cosh^3(c+dx)}{4d}-\frac{(5a+9b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{3}{8}x(a+5b)-\frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (3*(a + 5*b)*x)/8 - ((5*a + 9*b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + ((a +
 b)*Cosh[c + d*x]^3*Sinh[c + d*x])/(4*d) - (b*Tanh[c + d*x])/d
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+bx^2\right)}{\left(1-x^2\right)^3}dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{(a+b)\cosh^3(c+dx)\sinh(c+dx)}{4d} + \frac{\text{Subst}\left(\int \frac{-a-b-4(a+b)x^2-4bx^4}{\left(1-x^2\right)^2}dx, x, \text{tanh}\right.}{4d}$$

$$= -\frac{(5a+9b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{(a+b)\cosh^3(c+dx)\sinh(c+dx)}{4d}$$

$$= -\frac{(5a+9b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{(a+b)\cosh^3(c+dx)\sinh(c+dx)}{4d}$$

$$= \frac{3}{8}(a+5b)x - \frac{(5a+9b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{(a+b)\cosh^3(c+dx)\text{si}}{4d}$$

Mathematica [A] time = 0.340941, size = 56, normalized size = 0.77

$$\frac{12(a+5b)(c+dx)-8(a+2b)\sinh(2(c+dx))+(a+b)\sinh(4(c+dx))-32b\tanh(c+dx)}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (12*(a + 5*b)*(c + d*x) - 8*(a + 2*b)*Sinh[2*(c + d*x)] + (a + b)*Sinh[4*(c
 + d*x)] - 32*b*Tanh[c + d*x])/(32*d)
```

Maple [A] time = 0.041, size = 96, normalized size = 1.3

$$\frac{1}{d}\left(a\left(\left(\frac{(\sinh(dx+c))^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(\frac{(\sinh(dx+c))^5}{4\cosh(dx+c)}-\frac{5(\sinh(dx+c))^3}{8\cosh(dx+c)}+\frac{15d}{8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x)
```

```
[Out] 1/d*(a*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+b*(1
/4*sinh(d*x+c)^5/cosh(d*x+c)-5/8*sinh(d*x+c)^3/cosh(d*x+c)+15/8*d*x+15/8*c-
15/8*tanh(d*x+c)))
```

Maxima [B] time = 1.20197, size = 208, normalized size = 2.85

$$\frac{1}{64}\,a\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)+\frac{1}{64}\,b\left(\frac{120\,(dx+c)}{d}+\frac{16\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}}{d}-\frac{15}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/64*a*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)
/d - e^(-4*d*x - 4*c)/d) + 1/64*b*(120*(d*x + c)/d + (16*e^(-2*d*x - 2*c) -
 e^(-4*d*x - 4*c))/d - (15*e^(-2*d*x - 2*c) + 144*e^(-4*d*x - 4*c) - 1)/(d*
(e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))))
```

Fricas [A] time = 1.84961, size = 327, normalized size = 4.48

$$\frac{(a+b)\sinh(dx+c)^5+\left(10\,(a+b)\cosh(dx+c)^2-7\,a-15\,b\right)\sinh(dx+c)^3+8\,(3\,(a+5\,b)dx+8\,b)\cosh(dx+c)+}{64\,d\cosh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/64*((a + b)*sinh(d*x + c)^5 + (10*(a + b)*cosh(d*x + c)^2 - 7*a - 15*b)*s
inh(d*x + c)^3 + 8*(3*(a + 5*b)*d*x + 8*b)*cosh(d*x + c) + (5*(a + b)*cosh(
d*x + c)^4 - 3*(7*a + 15*b)*cosh(d*x + c)^2 - 8*a - 80*b)*sinh(d*x + c))/(d
*cosh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tanh^2(c+dx)\right)\sinh^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*sinh(c + d*x)**4, x)
```

Giac [B] time = 1.32276, size = 196, normalized size = 2.68

$$\frac{24\,(a+5\,b)dx-\left(18\,ae^{(4\,dx+4\,c)}+90\,be^{(4\,dx+4\,c)}-8\,ae^{(2\,dx+2\,c)}-16\,be^{(2\,dx+2\,c)}+a+b\right)e^{(-4\,dx-4\,c)}+\left(ae^{(4\,dx+20\,c)}+be^{(4\,dx+2}\right.}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/64*(24*(a + 5*b)*d*x - (18*a*e^(4*d*x + 4*c) + 90*b*e^(4*d*x + 4*c) - 8*a
*e^(2*d*x + 2*c) - 16*b*e^(2*d*x + 2*c) + a + b)*e^(-4*d*x - 4*c) + (a*e^(4
*d*x + 20*c) + b*e^(4*d*x + 20*c) - 8*a*e^(2*d*x + 18*c) - 16*b*e^(2*d*x +
18*c))*e^(-16*c) + 128*b/(e^(2*d*x + 2*c) + 1))/d
```

3.2 $\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=47

$$\frac{(a+b)\cosh^3(c+dx)}{3d}-\frac{(a+2b)\cosh(c+dx)}{d}-\frac{b\text{sech}(c+dx)}{d}$$

```
[Out] -(((a + 2*b)*Cosh[c + d*x])/d) + ((a + b)*Cosh[c + d*x]^3)/(3*d) - (b*Sech[
c + d*x])/d
```

Rubi [A] time = 0.0546025, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3664, 448}

$$\frac{(a+b)\cosh^3(c+dx)}{3d}-\frac{(a+2b)\cosh(c+dx)}{d}-\frac{b\text{sech}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -(((a + 2*b)*Cosh[c + d*x])/d) + ((a + b)*Cosh[c + d*x]^3)/(3*d) - (b*Sech[
c + d*x])/d
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)\left(a+b-bx^2\right)}{x^4}\,dx,x,\text{sech}(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(-b+\frac{-a-b}{x^4}+\frac{a+2b}{x^2}\right)\,dx,x,\text{sech}(c+dx)\right)}{d}\\
&= -\frac{(a+2b)\cosh(c+dx)}{d}+\frac{(a+b)\cosh^3(c+dx)}{3d}-\frac{b\text{sech}(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.0503273, size = 73, normalized size = 1.55

$$-\frac{3a\cosh(c+dx)}{4d}+\frac{a\cosh(3(c+dx))}{12d}-\frac{7b\cosh(c+dx)}{4d}+\frac{b\cosh(3(c+dx))}{12d}-\frac{b\text{sech}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (-3*a*Cosh[c + d*x])/(4*d) - (7*b*Cosh[c + d*x])/(4*d) + (a*Cosh[3*(c + d*x
)])/(12*d) + (b*Cosh[3*(c + d*x)])/(12*d) - (b*Sech[c + d*x])/d
```

Maple [A] time = 0.034, size = 73, normalized size = 1.6

$$\frac{1}{d}\left(a\left(-\frac{2}{3}+\frac{(\sinh(dx+c))^2}{3}\right)\cosh(dx+c)+b\left(\frac{(\sinh(dx+c))^4}{3\cosh(dx+c)}+\frac{4(\sinh(dx+c))^2}{3\cosh(dx+c)}-\frac{8\cosh(dx+c)}{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x)
```

```
[Out] 1/d*(a*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+b*(1/3*sinh(d*x+c)^4/cosh(d*x+c
)+4/3*sinh(d*x+c)^2/cosh(d*x+c)-8/3*cosh(d*x+c)))
```

Maxima [B] time = 1.15218, size = 184, normalized size = 3.91

$$-\frac{1}{24}b\left(\frac{21e^{(-dx-c)}-e^{(-3dx-3c)}}{d}+\frac{20e^{(-2dx-2c)}+69e^{(-4dx-4c)}-1}{d\left(e^{(-3dx-3c)}+e^{(-5dx-5c)}\right)}\right)+\frac{1}{24}a\left(\frac{e^{(3dx+3c)}}{d}-\frac{9e^{(dx+c)}}{d}-\frac{9e^{(-dx-c)}}{d}+\frac{e^{(-3dx-3c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/24*b*((21*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + (20*e^(-2*d*x - 2*c) + 69
*e^(-4*d*x - 4*c) - 1)/(d*(e^(-3*d*x - 3*c) + e^(-5*d*x - 5*c)))) + 1/24*a*
(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/
d)
```

Fricas [B] time = 1.99928, size = 246, normalized size = 5.23

$$\frac{(a+b)\cosh(dx+c)^4+(a+b)\sinh(dx+c)^4-4(2a+5b)\cosh(dx+c)^2+2\left(3(a+b)\cosh(dx+c)^2-4a-10b\right)\sinh}{24d\cosh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/24*((a + b)*cosh(d*x + c)^4 + (a + b)*sinh(d*x + c)^4 - 4*(2*a + 5*b)*cos
h(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 - 4*a - 10*b)*sinh(d*x + c)^2 -
 9*a - 45*b)/(d*cosh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tanh^2(c+dx)\right)\sinh^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*sinh(c + d*x)**3, x)
```

Giac [B] time = 1.30233, size = 162, normalized size = 3.45

$$-\frac{\left(9\,ae^{(2\,dx+2\,c)} + 21\,be^{(2\,dx+2\,c)} - a - b\right)e^{(-3\,dx-3\,c)} - \left(ae^{(3\,dx+24\,c)} + be^{(3\,dx+24\,c)} - 9\,ae^{(dx+22\,c)} - 21\,be^{(dx+22\,c)}\right)e^{(-21\,c)} + \frac{48\,be}{e^{(2\,dx-}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/24*((9*a*e^(2*d*x + 2*c) + 21*b*e^(2*d*x + 2*c) - a - b)*e^(-3*d*x - 3*c
) - (a*e^(3*d*x + 24*c) + b*e^(3*d*x + 24*c) - 9*a*e^(d*x + 22*c) - 21*b*e^
(d*x + 22*c))*e^(-21*c) + 48*b*e^(d*x + c)/(e^(2*d*x + 2*c) + 1))/d
```

3.3 $\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)dx$

Optimal. Leaf size=44

$$\frac{(a+b)\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{1}{2}x(a+3b)+\frac{b\tanh(c+dx)}{d}$$

```
[Out] -((a + 3*b)*x)/2 + ((a + b)*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + (b*Tanh[c
+ d*x])/d
```

Rubi [A] time = 0.0507793, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3663, 455, 388, 206}

$$\frac{(a+b)\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{1}{2}x(a+3b)+\frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -((a + 3*b)*x)/2 + ((a + b)*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + (b*Tanh[c
+ d*x])/d
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx^2)}{(1-x^2)^2}dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{(a+b)\cosh(c+dx)\sinh(c+dx)}{2d} - \frac{\text{Subst}\left(\int \frac{a+b+2bx^2}{1-x^2}dx, x, \tanh(c+dx\right.}{2d}$$
$$= \frac{(a+b)\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{b\tanh(c+dx)}{d} - \frac{(a+3b)\,\text{Subst}\left(\int\right.}{}$$
$$= -\frac{1}{2}(a+3b)x + \frac{(a+b)\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{b\tanh(c+dx)}{d}$$

Mathematica [A] time = 0.189214, size = 41, normalized size = 0.93

$$\frac{-2(a+3b)(c+dx)+(a+b)\sinh(2(c+dx))+4b\tanh(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (-2*(a + 3*b)*(c + d*x) + (a + b)*Sinh[2*(c + d*x)] + 4*b*Tanh[c + d*x])/(4
*d)
```

Maple [A] time = 0.033, size = 66, normalized size = 1.5

$$\frac{1}{d}\left(a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b\left(\frac{(\sinh(dx+c))^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x)
```

```
[Out] 1/d*(a*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+b*(1/2*sinh(d*x+c)^3/cos
h(d*x+c)-3/2*d*x-3/2*c+3/2*tanh(d*x+c)))
```

Maxima [B] time = 1.12602, size = 136, normalized size = 3.09

$$-\frac{1}{8}a\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)-\frac{1}{8}b\left(\frac{12(dx+c)}{d}+\frac{e^{(-2dx-2c)}}{d}-\frac{17e^{(-2dx-2c)}+1}{d\left(e^{(-2dx-2c)}+e^{(-4dx-4c)}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/8*a*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/8*b*(12*(d*x + c)
/d + e^(-2*d*x - 2*c)/d - (17*e^(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x - 2*c) +
e^(-4*d*x - 4*c))))
```

Fricas [A] time = 1.95945, size = 193, normalized size = 4.39

$$\frac{(a+b)\sinh(dx+c)^3-4((a+3b)dx+2b)\cosh(dx+c)+\left(3(a+b)\cosh(dx+c)^2+a+9b\right)\sinh(dx+c)}{8d\cosh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/8*((a + b)*sinh(d*x + c)^3 - 4*((a + 3*b)*d*x + 2*b)*cosh(d*x + c) + (3*(
a + b)*cosh(d*x + c)^2 + a + 9*b)*sinh(d*x + c))/(d*cosh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tanh^2(c+dx)\right)\sinh^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*sinh(c + d*x)**2, x)
```

Giac [B] time = 1.2702, size = 147, normalized size = 3.34

$$-\frac{4(a+3b)dx-\left(ae^{(2dx+8c)}+be^{(2dx+8c)}\right)e^{(-6c)}-\frac{\left(ae^{(4dx+4c)}+3be^{(4dx+4c)}-14be^{(2dx+2c)}-a-b\right)e^{(-2c)}}{e^{(2dx)}+e^{(4dx+2c)}}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/8*(4*(a + 3*b)*d*x - (a*e^(2*d*x + 8*c) + b*e^(2*d*x + 8*c))*e^(-6*c) -
(a*e^(4*d*x + 4*c) + 3*b*e^(4*d*x + 4*c) - 14*b*e^(2*d*x + 2*c) - a - b)*e^
(-2*c)/(e^(2*d*x) + e^(4*d*x + 2*c)))/d
```

3.4 $\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=25

$$\frac{(a+b)\cosh(c+dx)}{d}+\frac{b\text{sech}(c+dx)}{d}$$

```
[Out] ((a + b)*Cosh[c + d*x])/d + (b*Sech[c + d*x])/d
```

Rubi [A] time = 0.0320672, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3664, 14}

$$\frac{(a+b)\cosh(c+dx)}{d}+\frac{b\text{sech}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((a + b)*Cosh[c + d*x])/d + (b*Sech[c + d*x])/d
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx&=-\frac{\text{Subst}\left(\int \frac{a+b-bx^2}{x^2}\,dx,x,\text{sech}(c+dx)\right)}{d}\\&=-\frac{\text{Subst}\left(\int \left(-b+\frac{a+b}{x^2}\right)\,dx,x,\text{sech}(c+dx)\right)}{d}\\&=\frac{(a+b)\cosh(c+dx)}{d}+\frac{b\text{sech}(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.04502, size = 45, normalized size = 1.8

$$\frac{a\sinh(c)\sinh(dx)}{d}+\frac{a\cosh(c)\cosh(dx)}{d}+\frac{b\cosh(c+dx)}{d}+\frac{b\text{sech}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a*Cosh[c]*Cosh[d*x])/d + (b*Cosh[c + d*x])/d + (b*Sech[c + d*x])/d + (a*Si
nh[c]*Sinh[d*x])/d
```

Maple [A] time = 0.033, size = 43, normalized size = 1.7

$$\frac{1}{d}\left(a\cosh\left(dx+c\right)+b\left(-\frac{\left(\sinh\left(dx+c\right)\right)^{2}}{\cosh\left(dx+c\right)}+2\,\cosh\left(dx+c\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*(a+b*tanh(d*x+c)^2),x)
```

```
[Out] 1/d*(a*cosh(d*x+c)+b*(-sinh(d*x+c)^2/cosh(d*x+c)+2*cosh(d*x+c)))
```

Maxima [B] time = 1.1077, size = 90, normalized size = 3.6

$$\frac{1}{2}\,b\left(\frac{e^{(-dx-c)}}{d}+\frac{5\,e^{(-2\,dx-2\,c)}+1}{d\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}\right)+\frac{a\cosh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*b*(e^(-d*x - c)/d + (5*e^(-2*d*x - 2*c) + 1)/(d*(e^(-d*x - c) + e^(-3*d
*x - 3*c)))) + a*cosh(d*x + c)/d
```

Fricas [A] time = 1.90525, size = 115, normalized size = 4.6

$$\frac{(a+b)\cosh\left(dx+c\right)^{2}+(a+b)\sinh\left(dx+c\right)^{2}+a+3\,b}{2\,d\cosh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a + 3*b)/(d*cosh(d
*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tanh^{2}\left(c+dx\right)\right)\sinh\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*sinh(c + d*x), x)
```

Giac [B] time = 1.20609, size = 107, normalized size = 4.28

$$\frac{\left(ae^{(dx+6\,c)} + be^{(dx+6\,c)}\right)e^{(-5\,c)} + \frac{\left(ae^{(2\,dx+2\,c)}+5\,be^{(2\,dx+2\,c)}+a+b\right)e^{(-c)}}{e^{(3\,dx+2\,c)}+e^{(dx)}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*((a*e^(d*x + 6*c) + b*e^(d*x + 6*c))*e^(-5*c) + (a*e^(2*d*x + 2*c) + 5*
b*e^(2*d*x + 2*c) + a + b)*e^(-c)/(e^(3*d*x + 2*c) + e^(d*x)))/d
```

3.5 $\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)dx$

Optimal. Leaf size=26

$$-\frac{a\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b\operatorname{sech}(c+dx)}{d}$$

```
[Out] -((a*ArcTanh[Cosh[c + d*x]])/d) - (b*Sech[c + d*x])/d
```

Rubi [A] time = 0.0332959, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3664, 388, 207}

$$-\frac{a\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b\operatorname{sech}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -((a*ArcTanh[Cosh[c + d*x]])/d) - (b*Sech[c + d*x])/d
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)dx &= \frac{\operatorname{Subst}\left(\int \frac{a+b-bx^2}{-1+x^2}\,dx, x, \operatorname{sech}(c+dx)\right)}{d}\\ &= -\frac{b\operatorname{sech}(c+dx)}{d}+\frac{a\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \operatorname{sech}(c+dx)\right)}{d}\\ &= -\frac{a\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b\operatorname{sech}(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0267322, size = 52, normalized size = 2.

$$\frac{a\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{a\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{b\operatorname{sech}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -((a*Log[Cosh[c/2 + (d*x)/2]])/d) + (a*Log[Sinh[c/2 + (d*x)/2]])/d - (b*Sec
h[c + d*x])/d
```

Maple [A] time = 0.037, size = 44, normalized size = 1.7

$$\frac{1}{d}\left(-2\,a\operatorname{Artanh}\left(e^{dx+c}\right)+b\left(\frac{(\sinh(dx+c))^2}{\cosh(dx+c)}-\cosh(dx+c)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*tanh(d*x+c)^2),x)
```

```
[Out] 1/d*(-2*a*arctanh(exp(d*x+c))+b*(sinh(d*x+c)^2/cosh(d*x+c)-cosh(d*x+c)))
```

Maxima [A] time = 1.08009, size = 54, normalized size = 2.08

$$\frac{a\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}-\frac{2\,b}{d\left(e^{(dx+c)}+e^{(-dx-c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] a*log(tanh(1/2*d*x + 1/2*c))/d - 2*b/(d*(e^(d*x + c) + e^(-d*x - c)))
```

Fricas [B] time = 2.0839, size = 483, normalized size = 18.58

$$-\frac{2\,b\cosh(dx+c)+\left(a\cosh(dx+c)^2+2\,a\cosh(dx+c)\sinh(dx+c)+a\sinh(dx+c)^2+a\right)\log(\cosh(dx+c)+\sin\ldots}{d\cosh(dx+c)^2+\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -(2*b*cosh(d*x + c) + (a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c)
+ a*sinh(d*x + c)^2 + a)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (a*cosh(d
*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a)*log(co
sh(d*x + c) + sinh(d*x + c) - 1) + 2*b*sinh(d*x + c))/(d*cosh(d*x + c)^2 +
2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\tanh^2\left(c+dx\right)\right)\operatorname{csch}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*csch(c + d*x), x)
```

Giac [A] time = 1.32969, size = 70, normalized size = 2.69

$$-\frac{a\log\left(e^{(dx+c)}+1\right)-a\log\left(\left|e^{(dx+c)}-1\right|\right)+\frac{2\,be^{(dx+c)}}{e^{(2\,dx+2\,c)}+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -(a*log(e^(d*x + c) + 1) - a*log(abs(e^(d*x + c) - 1)) + 2*b*e^(d*x + c)/(e
^(2*d*x + 2*c) + 1))/d
```

3.6 $\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)dx$

Optimal. Leaf size=24

$$\frac{b\tanh(c+dx)}{d}-\frac{a\coth(c+dx)}{d}$$

```
[Out] -((a*Coth[c + d*x])/d) + (b*Tanh[c + d*x])/d
```

Rubi [A] time = 0.0327757, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3663, 14}

$$\frac{b\tanh(c+dx)}{d}-\frac{a\coth(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]

[Out] -((a*Coth[c + d*x])/d) + (b*Tanh[c + d*x])/d
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)dx &= \frac{\operatorname{Subst}\left(\int \frac{a+bx^2}{x^2}\,dx,x,\tanh(c+dx)\right)}{d}\\
&= \frac{\operatorname{Subst}\left(\int \left(b+\frac{a}{x^2}\right)dx,x,\tanh(c+dx)\right)}{d}\\
&= -\frac{a\coth(c+dx)}{d}+\frac{b\tanh(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.0197248, size = 24, normalized size = 1.

$$\frac{b\tanh(c+dx)}{d}-\frac{a\coth(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -((a*Coth[c + d*x])/d) + (b*Tanh[c + d*x])/d
```

Maple [A] time = 0.036, size = 23, normalized size = 1.

$$\frac{-\coth(dx+c)\,a+b\tanh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2),x)
```

```
[Out] 1/d*(-coth(d*x+c)*a+b*tanh(d*x+c))
```

Maxima [A] time = 1.14149, size = 53, normalized size = 2.21

$$\frac{2\,b}{d\left(e^{(-2\,dx-2\,c)}+1\right)}+\frac{2\,a}{d\left(e^{(-2\,dx-2\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 2*b/(d*(e^(-2*d*x - 2*c) + 1)) + 2*a/(d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [B] time = 1.8861, size = 238, normalized size = 9.92

$$-\frac{4\,(a\cosh(dx+c)+b\sinh(dx+c))}{d\cosh(dx+c)^3+3\,d\cosh(dx+c)\sinh(dx+c)^2+d\sinh(dx+c)^3-d\cosh(dx+c)+\left(3\,d\cosh(dx+c)^2+d\right)\sinh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -4*(a*cosh(d*x + c) + b*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x +
c)*sinh(d*x + c)^2 + d*sinh(d*x + c)^3 - d*cosh(d*x + c) + (3*d*cosh(d*x +
c)^2 + d)*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tanh^2(c+dx)\right)\operatorname{csch}^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*csch(c + d*x)**2, x)
```

Giac [A] time = 1.26439, size = 61, normalized size = 2.54

$$-\frac{2\left(ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b\right)}{d\left(e^{(4\,dx+4\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/(d*(e^(4*d*x + 4*c) - 1)
)
```

3.7 $\int \mathbf{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=51

$$\frac{(a-2b)\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a\coth(c+dx)\mathbf{csch}(c+dx)}{2d}+\frac{b\mathbf{sech}(c+dx)}{d}$$

```
[Out] ((a - 2*b)*ArcTanh[Cosh[c + d*x]])/(2*d) - (a*Coth[c + d*x]*Csch[c + d*x])/
(2*d) + (b*Sech[c + d*x])/d
```

Rubi [A] time = 0.0588961, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3664, 455, 388, 207}

$$\frac{(a-2b)\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a\coth(c+dx)\mathbf{csch}(c+dx)}{2d}+\frac{b\mathbf{sech}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((a - 2*b)*ArcTanh[Cosh[c + d*x]])/(2*d) - (a*Coth[c + d*x]*Csch[c + d*x])/
(2*d) + (b*Sech[c + d*x])/d
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)dx = -\frac{\text{Subst}\left(\int \frac{x^2(a+b-bx^2)}{(-1+x^2)^2}\,dx, x, \operatorname{sech}(c+dx)\right)}{d}$$

$$= -\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d} + \frac{\text{Subst}\left(\int \frac{-a+2bx^2}{-1+x^2}\,dx, x, \operatorname{sech}(c+dx)\right)}{2d}$$

$$= -\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d} + \frac{b\operatorname{sech}(c+dx)}{d} - \frac{(a-2b)\,\text{Subst}\left(\int \frac{1}{-1+x}\right.}{2c}$$

$$= \frac{(a-2b)\tanh^{-1}(\cosh(c+dx))}{2d} - \frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d} + \frac{b\operatorname{sech}(c}{d}$$

Mathematica [A] time = 0.0457941, size = 87, normalized size = 1.71

$$-\frac{a\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{b\operatorname{sech}(c+dx)}{d} + \frac{b\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -(a*Csch[(c + d*x)/2]^2)/(8*d) - (a*Log[Tanh[(c + d*x)/2]])/(2*d) + (b*Log[
Tanh[(c + d*x)/2]])/d - (a*Sech[(c + d*x)/2]^2)/(8*d) + (b*Sech[c + d*x])/d
```

Maple [A] time = 0.044, size = 50, normalized size = 1.

$$\frac{1}{d}\left(a\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2} + \text{Artanh}\left(e^{dx+c}\right)\right) + b\left((\cosh(dx+c))^{-1} - 2\,\text{Artanh}\left(e^{dx+c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2),x)
```

```
[Out] 1/d*(a*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+b*(1/cosh(d*x+c)-
2*arctanh(exp(d*x+c))))
```

Maxima [B] time = 1.1012, size = 205, normalized size = 4.02

$$\frac{1}{2}a\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d\left(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1\right)}\right) - b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{\log\left(e^{(-dx-c)}-1\right)}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*a*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c)
+ e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - b*(l
og(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d - 2*e^(-d*x - c)/(d*(e^(-2
*d*x - 2*c) + 1)))
```

Fricas [B] time = 2.21944, size = 2480, normalized size = 48.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a - 2*b)*cosh(d*x + c)^5 + 10*(a - 2*b)*cosh(d*x + c)*sinh(d*x + c
)^4 + 2*(a - 2*b)*sinh(d*x + c)^5 + 4*(a + 2*b)*cosh(d*x + c)^3 + 4*(5*(a -
 2*b)*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^3 + 4*(5*(a - 2*b)*cosh(d*x
+ c)^3 + 3*(a + 2*b)*cosh(d*x + c))*sinh(d*x + c)^2 + 2*(a - 2*b)*cosh(d*x
+ c) - ((a - 2*b)*cosh(d*x + c)^6 + 6*(a - 2*b)*cosh(d*x + c)*sinh(d*x + c)
^5 + (a - 2*b)*sinh(d*x + c)^6 - (a - 2*b)*cosh(d*x + c)^4 + (15*(a - 2*b)*
cosh(d*x + c)^2 - a + 2*b)*sinh(d*x + c)^4 + 4*(5*(a - 2*b)*cosh(d*x + c)^3
 - (a - 2*b)*cosh(d*x + c))*sinh(d*x + c)^3 - (a - 2*b)*cosh(d*x + c)^2 + (
15*(a - 2*b)*cosh(d*x + c)^4 - 6*(a - 2*b)*cosh(d*x + c)^2 - a + 2*b)*sinh(
d*x + c)^2 + 2*(3*(a - 2*b)*cosh(d*x + c)^5 - 2*(a - 2*b)*cosh(d*x + c)^3 -
 (a - 2*b)*cosh(d*x + c))*sinh(d*x + c) + a - 2*b)*log(cosh(d*x + c) + sinh
(d*x + c) + 1) + ((a - 2*b)*cosh(d*x + c)^6 + 6*(a - 2*b)*cosh(d*x + c)*sin
h(d*x + c)^5 + (a - 2*b)*sinh(d*x + c)^6 - (a - 2*b)*cosh(d*x + c)^4 + (15*
(a - 2*b)*cosh(d*x + c)^2 - a + 2*b)*sinh(d*x + c)^4 + 4*(5*(a - 2*b)*cosh(
d*x + c)^3 - (a - 2*b)*cosh(d*x + c))*sinh(d*x + c)^3 - (a - 2*b)*cosh(d*x
+ c)^2 + (15*(a - 2*b)*cosh(d*x + c)^4 - 6*(a - 2*b)*cosh(d*x + c)^2 - a +
2*b)*sinh(d*x + c)^2 + 2*(3*(a - 2*b)*cosh(d*x + c)^5 - 2*(a - 2*b)*cosh(d*
x + c)^3 - (a - 2*b)*cosh(d*x + c))*sinh(d*x + c) + a - 2*b)*log(cosh(d*x +
 c) + sinh(d*x + c) - 1) + 2*(5*(a - 2*b)*cosh(d*x + c)^4 + 6*(a + 2*b)*cos
h(d*x + c)^2 + a - 2*b)*sinh(d*x + c))/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x +
c)*sinh(d*x + c)^5 + d*sinh(d*x + c)^6 - d*cosh(d*x + c)^4 + (15*d*cosh(d*x
 + c)^2 - d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*si
nh(d*x + c)^3 - d*cosh(d*x + c)^2 + (15*d*cosh(d*x + c)^4 - 6*d*cosh(d*x +
c)^2 - d)*sinh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^5 - 2*d*cosh(d*x + c)^3 -
d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tanh^2(c + dx)\right)\operatorname{csch}^3(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*csch(c + d*x)**3, x)
```

Giac [B] time = 1.20788, size = 153, normalized size = 3.

$$\frac{(ae^c - 2\,be^c)e^{(-c)}\log\left(e^{(dx+c)} + 1\right) - (ae^c - 2\,be^c)e^{(-c)}\log\left(\left|e^{(dx+c)} - 1\right|\right) + \frac{4\,be^{(dx+c)}}{e^{(2\,dx+2\,c)}+1} - \frac{2\left(ae^{(3\,dx+3\,c)}+ae^{(dx+c)}\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*((a*e^c - 2*b*e^c)*e^(-c)*log(e^(d*x + c) + 1) - (a*e^c - 2*b*e^c)*e^(-
c)*log(abs(e^(d*x + c) - 1)) + 4*b*e^(d*x + c)/(e^(2*d*x + 2*c) + 1) - 2*(a
*e^(3*d*x + 3*c) + a*e^(d*x + c))/(e^(2*d*x + 2*c) - 1)^2)/d
```

3.8 $\int \text{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=44

$$\frac{(a-b)\coth(c+dx)}{d}-\frac{a\coth^3(c+dx)}{3d}-\frac{b\tanh(c+dx)}{d}$$

```
[Out] ((a - b)*Coth[c + d*x])/d - (a*Coth[c + d*x]^3)/(3*d) - (b*Tanh[c + d*x])/d
```

Rubi [A] time = 0.0431187, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3663, 448}

$$\frac{(a-b)\coth(c+dx)}{d}-\frac{a\coth^3(c+dx)}{3d}-\frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((a - b)*Coth[c + d*x])/d - (a*Coth[c + d*x]^3)/(3*d) - (b*Tanh[c + d*x])/d
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\int \text{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx &= \frac{\text{Subst}\left(\int \frac{(1-x^2)(a+bx^2)}{x^4}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(-b+\frac{a}{x^4}+\frac{-a+b}{x^2}\right)dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{(a-b)\coth(c+dx)}{d}-\frac{a\coth^3(c+dx)}{3d}-\frac{b\tanh(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.0678965, size = 61, normalized size = 1.39

$$\frac{2a\coth(c+dx)}{3d}-\frac{a\coth(c+dx)\text{csch}^2(c+dx)}{3d}-\frac{b\tanh(c+dx)}{d}-\frac{b\coth(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (2*a*Coth[c + d*x])/(3*d) - (b*Coth[c + d*x])/d - (a*Coth[c + d*x]*Csch[c +
 d*x]^2)/(3*d) - (b*Tanh[c + d*x])/d
```

Maple [A] time = 0.046, size = 55, normalized size = 1.3

$$\frac{1}{d}\left(a\left(\frac{2}{3}-\frac{(\operatorname{csch}(dx+c))^2}{3}\right)\coth(dx+c)+b\left(-\frac{1}{\cosh(dx+c)\sinh(dx+c)}-2\tanh(dx+c)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2),x)
```

```
[Out] 1/d*(a*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+b*(-1/sinh(d*x+c)/cosh(d*x+c)-2*
tanh(d*x+c)))
```

Maxima [B] time = 1.16375, size = 153, normalized size = 3.48

$$\frac{4}{3}a\left(\frac{3e^{(-2dx-2c)}}{d\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1\right)}-\frac{1}{d\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1\right)}\right)+\frac{4b}{d\left(e^{(-4dx-4c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 4/3*a*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6
*d*x - 6*c) - 1))) + 4*b/(d*(e^(-4*d*x - 4*c) - 1))
```

Fricas [B] time = 2.04028, size = 657, normalized size = 14.93

$$-\frac{}{3\left(d\cosh(dx+c)^6+6d\cosh(dx+c)\sinh(dx+c)^5+d\sinh(dx+c)^6-2d\cosh(dx+c)^4+\left(15d\cosh(dx+c)^2-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -8/3*((a + 3*b)*cosh(d*x + c)^2 + 4*a*cosh(d*x + c)*sinh(d*x + c) + (a + 3*
b)*sinh(d*x + c)^2 + a - 3*b)/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh(d
*x + c)^5 + d*sinh(d*x + c)^6 - 2*d*cosh(d*x + c)^4 + (15*d*cosh(d*x + c)^2
 - 2*d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - 2*d*cosh(d*x + c))*sinh(
d*x + c)^3 - d*cosh(d*x + c)^2 + (15*d*cosh(d*x + c)^4 - 12*d*cosh(d*x + c)
^2 - d)*sinh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^5 - 4*d*cosh(d*x + c)^3 + d*
cosh(d*x + c))*sinh(d*x + c) + 2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b \tanh^2 (c + dx)\right) \operatorname{csch}^4 (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*csch(c + d*x)**4, x)
```

Giac [A] time = 1.2148, size = 108, normalized size = 2.45

$$\frac{2\left(\frac{3\,b}{e^{(2\,dx+2\,c)}+1} - \frac{3\,be^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}-6\,be^{(2\,dx+2\,c)}-2\,a+3\,b}{\left(e^{(2\,dx+2\,c)}-1\right)^3}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 2/3*(3*b/(e^(2*d*x + 2*c) + 1) - (3*b*e^(4*d*x + 4*c) + 6*a*e^(2*d*x + 2*c)
 - 6*b*e^(2*d*x + 2*c) - 2*a + 3*b)/(e^(2*d*x + 2*c) - 1)^3)/d
```

3.9 $\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=118

$$-\frac{\left(a^2+10ab+13b^2\right)\tanh(c+dx)}{4d}+\frac{1}{8}x\left(3a^2+30ab+35b^2\right)+\frac{(a+b)^2\sinh^4(c+dx)\tanh(c+dx)}{4d}-\frac{(a+b)(a+9b)}{}$$

```
[Out] ((3*a^2 + 30*a*b + 35*b^2)*x)/8 - ((a + b)*(a + 9*b)*Cosh[c + d*x]*Sinh[c +
 d*x])/(8*d) - ((a^2 + 10*a*b + 13*b^2)*Tanh[c + d*x])/(4*d) + ((a + b)^2*S
inh[c + d*x]^4*Tanh[c + d*x])/(4*d) - (b^2*Tanh[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.132579, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3663, 463, 455, 1153, 206}

$$-\frac{\left(a^2+10ab+13b^2\right)\tanh(c+dx)}{4d}+\frac{1}{8}x\left(3a^2+30ab+35b^2\right)+\frac{(a+b)^2\sinh^4(c+dx)\tanh(c+dx)}{4d}-\frac{(a+b)(a+9b)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((3*a^2 + 30*a*b + 35*b^2)*x)/8 - ((a + b)*(a + 9*b)*Cosh[c + d*x]*Sinh[c +
 d*x])/(8*d) - ((a^2 + 10*a*b + 13*b^2)*Tanh[c + d*x])/(4*d) + ((a + b)^2*S
inh[c + d*x]^4*Tanh[c + d*x])/(4*d) - (b^2*Tanh[c + d*x]^3)/(3*d)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+bx^2\right)^2}{\left(1-x^2\right)^3}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$= \frac{(a+b)^2\sinh^4(c+dx)\tanh(c+dx)}{4d} - \frac{\text{Subst}\left(\int \frac{x^4\left(a^2+10ab+5b^2+4b^2x^2\right)}{\left(1-x^2\right)^2}\,dx,x,\right.}{4d}$$

$$= -\frac{(a+b)(a+9b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{(a+b)^2\sinh^4(c+dx)\tanh(}{4d}$$

$$= -\frac{(a+b)(a+9b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{(a+b)^2\sinh^4(c+dx)\tanh(}{4d}$$

$$= -\frac{(a+b)(a+9b)\cosh(c+dx)\sinh(c+dx)}{8d} - \frac{\left(a^2+10ab+13b^2\right)\tanh(c+}{4d}$$

$$= \frac{1}{8}\left(3a^2+30ab+35b^2\right)x - \frac{(a+b)(a+9b)\cosh(c+dx)\sinh(c+dx)}{8d} - \frac{\left(a^2\right.}{}$$

Mathematica [A] time = 1.37418, size = 94, normalized size = 0.8

$$\frac{12\left(3a^2+30ab+35b^2\right)(c+dx) - 24\left(a^2+4ab+3b^2\right)\sinh(2(c+dx)) + 3(a+b)^2\sinh(4(c+dx)) + 32b\tanh(c+dx)\left(-\right.}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (12*(3*a^2 + 30*a*b + 35*b^2)*(c + d*x) - 24*(a^2 + 4*a*b + 3*b^2)*Sinh[2*(
c + d*x)] + 3*(a + b)^2*Sinh[4*(c + d*x)] + 32*b*(-6*a - 10*b + b*Sech[c +
d*x]^2)*Tanh[c + d*x])/(96*d)
```

Maple [A] time = 0.049, size = 166, normalized size = 1.4

$$\frac{1}{d}\left(a^2\left(\left(\frac{(\sinh(dx+c))^3}{4} - \frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c) + \frac{3dx}{8} + \frac{3c}{8}\right) + 2ab\left(1/4\frac{(\sinh(dx+c))^5}{\cosh(dx+c)} - 5/8\frac{(\sinh(dx+c))}{\cosh(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+2*
a*b*(1/4*sinh(d*x+c)^5/cosh(d*x+c)-5/8*sinh(d*x+c)^3/cosh(d*x+c)+15/8*d*x+1
```

```
5/8*c-15/8*tanh(d*x+c))+b^2*(1/4*sinh(d*x+c)^7/cosh(d*x+c)^3-7/8*sinh(d*x+c
)^5/cosh(d*x+c)^3+35/8*d*x+35/8*c-35/8*tanh(d*x+c)-35/24*tanh(d*x+c)^3))
```

Maxima [B] time = 1.11824, size = 398, normalized size = 3.37

$$\frac{1}{64}a^2\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)+\frac{1}{192}b^2\left(\frac{840(dx+c)}{d}+\frac{3\left(24e^{(-2dx-2c)}-e^{(-4dx-4c)}\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/64*a^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + 1/192*b^2*(840*(d*x + c)/d + 3*(24*e^(-2*d*x -
 2*c) - e^(-4*d*x - 4*c))/d - (63*e^(-2*d*x - 2*c) + 1487*e^(-4*d*x - 4*c)
+ 2517*e^(-6*d*x - 6*c) + 1608*e^(-8*d*x - 8*c) - 3)/(d*(e^(-4*d*x - 4*c) +
 3*e^(-6*d*x - 6*c) + 3*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c)))) + 1/32*a*b
*(120*(d*x + c)/d + (16*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c))/d - (15*e^(-2*
d*x - 2*c) + 144*e^(-4*d*x - 4*c) - 1)/(d*(e^(-4*d*x - 4*c) + e^(-6*d*x - 6
*c))))
```

Fricas [B] time = 2.06828, size = 1021, normalized size = 8.65

$$\frac{3\left(a^2+2ab+b^2\right)\sinh(dx+c)^7+3\left(21\left(a^2+2ab+b^2\right)\cosh(dx+c)^2-5a^2-26ab-21b^2\right)\sinh(dx+c)^5+8\left(3\left(3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/192*(3*(a^2 + 2*a*b + b^2)*sinh(d*x + c)^7 + 3*(21*(a^2 + 2*a*b + b^2)*co
sh(d*x + c)^2 - 5*a^2 - 26*a*b - 21*b^2)*sinh(d*x + c)^5 + 8*(3*(3*a^2 + 30
*a*b + 35*b^2)*d*x + 48*a*b + 80*b^2)*cosh(d*x + c)^3 + 24*(3*(3*a^2 + 30*a
*b + 35*b^2)*d*x + 48*a*b + 80*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + (105*(a
^2 + 2*a*b + b^2)*cosh(d*x + c)^4 - 30*(5*a^2 + 26*a*b + 21*b^2)*cosh(d*x +
 c)^2 - 63*a^2 - 654*a*b - 847*b^2)*sinh(d*x + c)^3 + 24*(3*(3*a^2 + 30*a*b
 + 35*b^2)*d*x + 48*a*b + 80*b^2)*cosh(d*x + c) + 3*(7*(a^2 + 2*a*b + b^2)*
cosh(d*x + c)^6 - 5*(5*a^2 + 26*a*b + 21*b^2)*cosh(d*x + c)^4 - (63*a^2 + 6
54*a*b + 847*b^2)*cosh(d*x + c)^2 - 15*a^2 - 190*a*b - 175*b^2)*sinh(d*x +
c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x +
 c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.72564, size = 398, normalized size = 3.37

$$24\left(3a^2+30ab+35b^2\right)dx-3\left(18a^2e^{(4dx+4c)}+180abe^{(4dx+4c)}+210b^2e^{(4dx+4c)}-8a^2e^{(2dx+2c)}-32abe^{(2dx+2c)}-24b^2e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/192*(24*(3*a^2 + 30*a*b + 35*b^2)*d*x - 3*(18*a^2*e^(4*d*x + 4*c) + 180*a
*b*e^(4*d*x + 4*c) + 210*b^2*e^(4*d*x + 4*c) - 8*a^2*e^(2*d*x + 2*c) - 32*a
*b*e^(2*d*x + 2*c) - 24*b^2*e^(2*d*x + 2*c) + a^2 + 2*a*b + b^2)*e^(-4*d*x
- 4*c) + 3*(a^2*e^(4*d*x + 28*c) + 2*a*b*e^(4*d*x + 28*c) + b^2*e^(4*d*x +
28*c) - 8*a^2*e^(2*d*x + 26*c) - 32*a*b*e^(2*d*x + 26*c) - 24*b^2*e^(2*d*x
+ 26*c))*e^(-24*c) + 256*(3*a*b*e^(4*d*x + 4*c) + 6*b^2*e^(4*d*x + 4*c) + 6
*a*b*e^(2*d*x + 2*c) + 9*b^2*e^(2*d*x + 2*c) + 3*a*b + 5*b^2)/(e^(2*d*x + 2
*c) + 1)^3)/d
```

3.10 $\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=77

$$\frac{(a+b)^2\cosh^3(c+dx)}{3d}-\frac{(a+b)(a+3b)\cosh(c+dx)}{d}-\frac{b(2a+3b)\text{sech}(c+dx)}{d}+\frac{b^2\text{sech}^3(c+dx)}{3d}$$

```
[Out] -(((a + b)*(a + 3*b)*Cosh[c + d*x])/d) + ((a + b)^2*Cosh[c + d*x]^3)/(3*d)
- (b*(2*a + 3*b)*Sech[c + d*x])/d + (b^2*Sech[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0978984, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3664, 448}

$$\frac{(a+b)^2\cosh^3(c+dx)}{3d}-\frac{(a+b)(a+3b)\cosh(c+dx)}{d}-\frac{b(2a+3b)\text{sech}(c+dx)}{d}+\frac{b^2\text{sech}^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -(((a + b)*(a + 3*b)*Cosh[c + d*x])/d) + ((a + b)^2*Cosh[c + d*x]^3)/(3*d)
- (b*(2*a + 3*b)*Sech[c + d*x])/d + (b^2*Sech[c + d*x]^3)/(3*d)
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx&=\frac{\text{Subst}\left(\int\frac{(-1+x^2)(a+b-bx^2)^2}{x^4}\,dx,x,\text{sech}(c+dx)\right)}{d}\\&=\frac{\text{Subst}\left(\int\left(-b(2a+3b)-\frac{(a+b)^2}{x^4}+\frac{(a+b)(a+3b)}{x^2}+b^2x^2\right)dx,x,\text{sech}(c+dx)\right)}{d}\\&=-\frac{(a+b)(a+3b)\cosh(c+dx)}{d}+\frac{(a+b)^2\cosh^3(c+dx)}{3d}-\frac{b(2a+3b)\text{se}}{d}\end{aligned}$$

Mathematica [A] time = 0.516529, size = 71, normalized size = 0.92

$$\frac{-3\left(3a^2+14ab+11b^2\right)\cosh(c+dx)+(a+b)^2\cosh(3(c+dx))+4b\,\text{sech}(c+dx)\left(-6a+b\,\text{sech}^2(c+dx)-9b\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (-3*(3*a^2 + 14*a*b + 11*b^2)*Cosh[c + d*x] + (a + b)^2*Cosh[3*(c + d*x)] +
 4*b*Sech[c + d*x]*(-6*a - 9*b + b*Sech[c + d*x]^2))/(12*d)
```

Maple [B] time = 0.052, size = 162, normalized size = 2.1

$$\frac{1}{d}\left(a^2\left(-\frac{2}{3}+\frac{(\sinh(dx+c))^2}{3}\right)\cosh(dx+c)+2ab\left(1/3\,\frac{(\sinh(dx+c))^4}{\cosh(dx+c)}+4/3\,\frac{(\sinh(dx+c))^2}{\cosh(dx+c)}-8/3\cosh(dx+c)\right)+b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+2*a*b*(1/3*sinh(d*x+c)^4/cosh
(d*x+c)+4/3*sinh(d*x+c)^2/cosh(d*x+c)-8/3*cosh(d*x+c))+b^2*(1/3*sinh(d*x+c)
^6/cosh(d*x+c)^3-2*sinh(d*x+c)^4/cosh(d*x+c)^3-8/3*sinh(d*x+c)^2/cosh(d*x+c
)^3+16/3*sinh(d*x+c)^2/cosh(d*x+c)-16/3*cosh(d*x+c)))
```

Maxima [B] time = 1.19186, size = 358, normalized size = 4.65

$$-\frac{1}{24}b^2\left(\frac{33\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}+\frac{30\,e^{(-2\,dx-2\,c)}+240\,e^{(-4\,dx-4\,c)}+322\,e^{(-6\,dx-6\,c)}+177\,e^{(-8\,dx-8\,c)}-1}{d\left(e^{(-3\,dx-3\,c)}+3\,e^{(-5\,dx-5\,c)}+3\,e^{(-7\,dx-7\,c)}+e^{(-9\,dx-9\,c)}\right)}\right)-\frac{1}{12}ab\left(\frac{21\,e^{(-dx-c)}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/24*b^2*((33*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + (30*e^(-2*d*x - 2*c) +
240*e^(-4*d*x - 4*c) + 322*e^(-6*d*x - 6*c) + 177*e^(-8*d*x - 8*c) - 1)/(d*
(e^(-3*d*x - 3*c) + 3*e^(-5*d*x - 5*c) + 3*e^(-7*d*x - 7*c) + e^(-9*d*x - 9
*c)))) - 1/12*a*b*((21*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + (20*e^(-2*d*x -
 2*c) + 69*e^(-4*d*x - 4*c) - 1)/(d*(e^(-3*d*x - 3*c) + e^(-5*d*x - 5*c)))
 + 1/24*a^2*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3
*d*x - 3*c)/d)
```

Fricas [B] time = 2.05485, size = 664, normalized size = 8.62

$$\frac{\left(a^2+2\,ab+b^2\right)\cosh(dx+c)^6+\left(a^2+2\,ab+b^2\right)\sinh(dx+c)^6-6\left(a^2+6\,ab+5\,b^2\right)\cosh(dx+c)^4+3\left(5\left(a^2+2\,ab+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + (a^2 + 2*a*b + b^2)*sinh(d*x +
c)^6 - 6*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^4 + 3*(5*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)^2 - 2*a^2 - 12*a*b - 10*b^2)*sinh(d*x + c)^4 - 3*(11*a^2 + 86*
```

```
a*b + 91*b^2)*cosh(d*x + c)^2 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 -
12*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^2 - 11*a^2 - 86*a*b - 91*b^2)*sinh(d
*x + c)^2 - 26*a^2 - 220*a*b - 210*b^2)/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x +
 c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.64637, size = 392, normalized size = 5.09

$$\left(a^2e^{(3\,dx+36\,c)} + 2\,abe^{(3\,dx+36\,c)} + b^2e^{(3\,dx+36\,c)} - 9\,a^2e^{(dx+34\,c)} - 42\,abe^{(dx+34\,c)} - 33\,b^2e^{(dx+34\,c)}\right)e^{(-33\,c)} - \frac{\left(9\,a^2e^{(8\,dx+8\,c)}+138\,abe^{(}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*((a^2*e^(3*d*x + 36*c) + 2*a*b*e^(3*d*x + 36*c) + b^2*e^(3*d*x + 36*c)
 - 9*a^2*e^(d*x + 34*c) - 42*a*b*e^(d*x + 34*c) - 33*b^2*e^(d*x + 34*c))*e^
(-33*c) - (9*a^2*e^(8*d*x + 8*c) + 138*a*b*e^(8*d*x + 8*c) + 177*b^2*e^(8*d
*x + 8*c) + 26*a^2*e^(6*d*x + 6*c) + 316*a*b*e^(6*d*x + 6*c) + 322*b^2*e^(6
*d*x + 6*c) + 24*a^2*e^(4*d*x + 4*c) + 216*a*b*e^(4*d*x + 4*c) + 240*b^2*e^
(4*d*x + 4*c) + 6*a^2*e^(2*d*x + 2*c) + 36*a*b*e^(2*d*x + 2*c) + 30*b^2*e^(
2*d*x + 2*c) - a^2 - 2*a*b - b^2)*e^(-3*c)/(e^(3*d*x + 2*c) + e^(d*x))^3)/d
```

3.11 $\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=79

$$\frac{(a+b)(a+5b)\tanh(c+dx)}{2d}+\frac{(a+b)^2\sinh^2(c+dx)\tanh(c+dx)}{2d}-\frac{1}{2}x(a+b)(a+5b)+\frac{b^2\tanh^3(c+dx)}{3d}$$

```
[Out] -((a + b)*(a + 5*b)*x)/2 + ((a + b)*(a + 5*b)*Tanh[c + d*x])/(2*d) + ((a +
b)^2*Sinh[c + d*x]^2*Tanh[c + d*x])/(2*d) + (b^2*Tanh[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.110672, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3663, 463, 459, 321, 206}

$$\frac{(a+b)(a+5b)\tanh(c+dx)}{2d}+\frac{(a+b)^2\sinh^2(c+dx)\tanh(c+dx)}{2d}-\frac{1}{2}x(a+b)(a+5b)+\frac{b^2\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```


```
[Out] -((a + b)*(a + 5*b)*x)/2 + ((a + b)*(a + 5*b)*Tanh[c + d*x])/(2*d) + ((a +
b)^2*Sinh[c + d*x]^2*Tanh[c + d*x])/(2*d) + (b^2*Tanh[c + d*x]^3)/(3*d)
```


Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+bx^2\right)^2}{\left(1-x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{(a+b)^2\sinh^2(c+dx)\tanh(c+dx)}{2d} - \frac{\text{Subst}\left(\int \frac{x^2\left(a^2+6ab+3b^2+2b^2x^2\right)}{1-x^2}\,dx, \right.}{2d}$$

$$= \frac{(a+b)^2\sinh^2(c+dx)\tanh(c+dx)}{2d} + \frac{b^2\tanh^3(c+dx)}{3d} - \frac{((a+b)(a+}{}$$

$$= \frac{(a+b)(a+5b)\tanh(c+dx)}{2d} + \frac{(a+b)^2\sinh^2(c+dx)\tanh(c+dx)}{2d} + \frac{b}{}$$

$$= -\frac{1}{2}(a+b)(a+5b)x + \frac{(a+b)(a+5b)\tanh(c+dx)}{2d} + \frac{(a+b)^2\sinh^2(c+}{2}$$

Mathematica [A] time = 0.849773, size = 70, normalized size = 0.89

$$\frac{-6\left(a^2+6ab+5b^2\right)(c+dx)+3(a+b)^2\sinh(2(c+dx))+4b\tanh(c+dx)\left(6a-b\,\text{sech}^2(c+dx)+7b\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (-6*(a^2 + 6*a*b + 5*b^2)*(c + d*x) + 3*(a + b)^2*Sinh[2*(c + d*x)] + 4*b*(
6*a + 7*b - b*Sech[c + d*x]^2)*Tanh[c + d*x])/(12*d)
```

Maple [A] time = 0.05, size = 118, normalized size = 1.5

$$\frac{1}{d}\left(a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2} - \frac{dx}{2} - \frac{c}{2}\right) + 2\,ab\left(1/2\,\frac{(\sinh(dx+c))^3}{\cosh(dx+c)} - 3/2\,dx - 3/2\,c + 3/2\,\tanh(dx+c)\right) + b^2\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+2*a*b*(1/2*sinh(d*x+c)
^3/cosh(d*x+c)-3/2*d*x-3/2*c+3/2*tanh(d*x+c))+b^2*(1/2*sinh(d*x+c)^5/cosh(d
*x+c)^3-5/2*d*x-5/2*c+5/2*tanh(d*x+c)+5/6*tanh(d*x+c)^3))
```

Maxima [B] time = 1.24277, size = 293, normalized size = 3.71

$$-\frac{1}{8}a^2\left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d}\right) - \frac{1}{24}b^2\left(\frac{60(dx+c)}{d} + \frac{3e^{(-2dx-2c)}}{d} - \frac{121e^{(-2dx-2c)} + 201e^{(-4dx-4c)} + 147e^{(-6dx-}}{d\left(e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + 3e^{(-6dx-6c)} + e^{(-}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/8*a^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/24*b^2*(60*(d*x
 + c)/d + 3*e^(-2*d*x - 2*c)/d - (121*e^(-2*d*x - 2*c) + 201*e^(-4*d*x - 4*
c) + 147*e^(-6*d*x - 6*c) + 3)/(d*(e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) +
3*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c)))) - 1/4*a*b*(12*(d*x + c)/d + e^(-2*
d*x - 2*c)/d - (17*e^(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x - 2*c) + e^(-4*d*x -
 4*c))))
```

Fricas [B] time = 2.05911, size = 747, normalized size = 9.46

$$3\left(a^2+2\,ab+b^2\right)\sinh\left(dx+c\right)^5-4\left(3\left(a^2+6\,ab+5\,b^2\right)dx+12\,ab+14\,b^2\right)\cosh\left(dx+c\right)^3-12\left(3\left(a^2+6\,ab+5\,b^2\right)dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(a^2 + 2*a*b + b^2)*sinh(d*x + c)^5 - 4*(3*(a^2 + 6*a*b + 5*b^2)*d*
x + 12*a*b + 14*b^2)*cosh(d*x + c)^3 - 12*(3*(a^2 + 6*a*b + 5*b^2)*d*x + 12
*a*b + 14*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + (30*(a^2 + 2*a*b + b^2)*cosh
(d*x + c)^2 + 9*a^2 + 66*a*b + 65*b^2)*sinh(d*x + c)^3 - 12*(3*(a^2 + 6*a*b
 + 5*b^2)*d*x + 12*a*b + 14*b^2)*cosh(d*x + c) + 3*(5*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)^4 + (9*a^2 + 66*a*b + 65*b^2)*cosh(d*x + c)^2 + 2*a^2 + 20*a*b
 + 10*b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x +
 c)^2 + 3*d*cosh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tanh^2\left(c+dx\right)\right)^2\sinh^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*sinh(c + d*x)**2, x)
```

Giac [B] time = 1.4646, size = 290, normalized size = 3.67

$$-\frac{12\left(a^2+6\,ab+5\,b^2\right)dx-3\left(2\,a^2e^{(2\,dx+2\,c)}+12\,abe^{(2\,dx+2\,c)}+10\,b^2e^{(2\,dx+2\,c)}-a^2-2\,ab-b^2\right)e^{(-2\,dx-2\,c)}-3\left(a^2e^{(2\,dx+12\,c)}+\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/24*(12*(a^2 + 6*a*b + 5*b^2)*d*x - 3*(2*a^2*e^(2*d*x + 2*c) + 12*a*b*e^(
2*d*x + 2*c) + 10*b^2*e^(2*d*x + 2*c) - a^2 - 2*a*b - b^2)*e^(-2*d*x - 2*c)
```

```
 - 3*(a^2*e^(2*d*x + 12*c) + 2*a*b*e^(2*d*x + 12*c) + b^2*e^(2*d*x + 12*c))
*e^(-10*c) + 16*(6*a*b*e^(4*d*x + 4*c) + 9*b^2*e^(4*d*x + 4*c) + 12*a*b*e^(
2*d*x + 2*c) + 12*b^2*e^(2*d*x + 2*c) + 6*a*b + 7*b^2)/(e^(2*d*x + 2*c) + 1
)^3)/d
```

3.12 $\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=49

$$\frac{(a+b)^2\cosh(c+dx)}{d}+\frac{2b(a+b)\text{sech}(c+dx)}{d}-\frac{b^2\text{sech}^3(c+dx)}{3d}$$

```
[Out] ((a + b)^2*Cosh[c + d*x])/d + (2*b*(a + b)*Sech[c + d*x])/d - (b^2*Sech[c +
 d*x]^3)/(3*d)
```

Rubi [A] time = 0.0532368, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3664, 270}

$$\frac{(a+b)^2\cosh(c+dx)}{d}+\frac{2b(a+b)\text{sech}(c+dx)}{d}-\frac{b^2\text{sech}^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((a + b)^2*Cosh[c + d*x])/d + (2*b*(a + b)*Sech[c + d*x])/d - (b^2*Sech[c +
 d*x]^3)/(3*d)
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(a+b-bx^2\right)^2}{x^2}\,dx,x,\text{sech}(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \left(-2b(a+b)+\frac{(a+b)^2}{x^2}+b^2x^2\right)\,dx,x,\text{sech}(c+dx)\right)}{d}\\ &= \frac{(a+b)^2\cosh(c+dx)}{d}+\frac{2b(a+b)\text{sech}(c+dx)}{d}-\frac{b^2\text{sech}^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.308327, size = 46, normalized size = 0.94

$$\frac{3(a+b)^2\cosh(c+dx)+b\,\text{sech}(c+dx)\left(6(a+b)-b\,\text{sech}^2(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (3*(a + b)^2*Cosh[c + d*x] + b*Sech[c + d*x]*(6*(a + b) - b*Sech[c + d*x]^2
))/(3*d)
```

Maple [B] time = 0.046, size = 113, normalized size = 2.3

$$\frac{1}{d}\left(a^2\cosh(dx+c)+2ab\left(-\frac{(\sinh(dx+c))^2}{\cosh(dx+c)}+2\cosh(dx+c)\right)+b^2\left(\frac{(\sinh(dx+c))^4}{(\cosh(dx+c))^3}+\frac{4(\sinh(dx+c))^2}{3(\cosh(dx+c))^3}-\frac{8(\sin}{3\cos}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*cosh(d*x+c)+2*a*b*(-sinh(d*x+c)^2/cosh(d*x+c)+2*cosh(d*x+c))+b^2*(
sinh(d*x+c)^4/cosh(d*x+c)^3+4/3*sinh(d*x+c)^2/cosh(d*x+c)^3-8/3*sinh(d*x+c)
^2/cosh(d*x+c)+8/3*cosh(d*x+c)))
```

Maxima [B] time = 1.13728, size = 231, normalized size = 4.71

$$\frac{1}{6}b^2\left(\frac{3e^{(-dx-c)}}{d}+\frac{33e^{(-2dx-2c)}+41e^{(-4dx-4c)}+27e^{(-6dx-6c)}+3}{d\left(e^{(-dx-c)}+3e^{(-3dx-3c)}+3e^{(-5dx-5c)}+e^{(-7dx-7c)}\right)}\right)+ab\left(\frac{e^{(-dx-c)}}{d}+\frac{5e^{(-2dx-2c)}+1}{d\left(e^{(-dx-c)}+e^{(-3dx-3c)}\right)}\right)+\frac{a^2\cosh}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*(3*e^(-d*x - c)/d + (33*e^(-2*d*x - 2*c) + 41*e^(-4*d*x - 4*c) + 27
*e^(-6*d*x - 6*c) + 3)/(d*(e^(-d*x - c) + 3*e^(-3*d*x - 3*c) + 3*e^(-5*d*x
- 5*c) + e^(-7*d*x - 7*c)))) + a*b*(e^(-d*x - c)/d + (5*e^(-2*d*x - 2*c) +
1)/(d*(e^(-d*x - c) + e^(-3*d*x - 3*c)))) + a^2*cosh(d*x + c)/d
```

Fricas [B] time = 2.05838, size = 424, normalized size = 8.65

$$\frac{3\left(a^2+2ab+b^2\right)\cosh(dx+c)^4+3\left(a^2+2ab+b^2\right)\sinh(dx+c)^4+12\left(a^2+4ab+3b^2\right)\cosh(dx+c)^2+6\left(3\left(a^2+\right.\right.}{6\left(d\cosh(dx+c)^3+3d\cosh(dx+c)\sinh(dx+c)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 3*(a^2 + 2*a*b + b^2)*sinh(d*x
 + c)^4 + 12*(a^2 + 4*a*b + 3*b^2)*cosh(d*x + c)^2 + 6*(3*(a^2 + 2*a*b + b^
2)*cosh(d*x + c)^2 + 2*a^2 + 8*a*b + 6*b^2)*sinh(d*x + c)^2 + 9*a^2 + 42*a*
b + 25*b^2)/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*co
sh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\tanh^2\left(c+dx\right)\right)^2\sinh\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*sinh(c + d*x), x)
```

Giac [B] time = 1.37067, size = 219, normalized size = 4.47

$$\frac{3\left(a^2+2\,ab+b^2\right)e^{(-dx-c)}+3\left(a^2e^{(dx+10\,c)}+2\,abe^{(dx+10\,c)}+b^2e^{(dx+10\,c)}\right)e^{(-9\,c)}+\frac{8\left(3\,abe^{(5\,dx+5\,c)}+3\,b^2e^{(5\,dx+5\,c)}+6\,abe^{(3\,dx+3\,c)}+4\,b^2e^{(3\,dx+3\,c)}+\cdots\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*(a^2 + 2*a*b + b^2)*e^(-d*x - c) + 3*(a^2*e^(d*x + 10*c) + 2*a*b*e^(
d*x + 10*c) + b^2*e^(d*x + 10*c))*e^(-9*c) + 8*(3*a*b*e^(5*d*x + 5*c) + 3*b
^2*e^(5*d*x + 5*c) + 6*a*b*e^(3*d*x + 3*c) + 4*b^2*e^(3*d*x + 3*c) + 3*a*b*
e^(d*x + c) + 3*b^2*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^3)/d
```

3.13 $\int \text{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=51

$$-\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b(2a+b)\text{sech}(c+dx)}{d}+\frac{b^2\text{sech}^3(c+dx)}{3d}$$

```
[Out] -((a^2*ArcTanh[Cosh[c + d*x]])/d) - (b*(2*a + b)*Sech[c + d*x])/d + (b^2*Se
ch[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0645143, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3664, 390, 207}

$$-\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b(2a+b)\text{sech}(c+dx)}{d}+\frac{b^2\text{sech}^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -((a^2*ArcTanh[Cosh[c + d*x]])/d) - (b*(2*a + b)*Sech[c + d*x])/d + (b^2*Se
ch[c + d*x]^3)/(3*d)
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b-bx^2\right)^2}{-1+x^2}\,dx, x, \operatorname{sech}(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-b(2a+b)+b^2x^2+\frac{a^2}{-1+x^2}\right)dx, x, \operatorname{sech}(c+dx)\right)}{d}$$
$$= -\frac{b(2a+b)\operatorname{sech}(c+dx)}{d}+\frac{b^2\operatorname{sech}^3(c+dx)}{3d}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \operatorname{sech}\right.}{d}$$
$$= -\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b(2a+b)\operatorname{sech}(c+dx)}{d}+\frac{b^2\operatorname{sech}^3(c+dx)}{3d}$$

Mathematica [A] time = 0.155003, size = 50, normalized size = 0.98

$$\frac{3a^2\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)-3b(2a+b)\operatorname{sech}(c+dx)+b^2\operatorname{sech}^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (3*a^2*Log[Tanh[(c + d*x)/2]] - 3*b*(2*a + b)*Sech[c + d*x] + b^2*Sech[c +
d*x]^3)/(3*d)
```

Maple [A] time = 0.05, size = 97, normalized size = 1.9

$$\frac{1}{d}\left(-2\,a^2\text{Artanh}\left(e^{dx+c}\right)+2\,ab\left(\frac{(\sinh(dx+c))^2}{\cosh(dx+c)}-\cosh(dx+c)\right)+b^2\left(-\frac{(\sinh(dx+c))^2}{3\,(\cosh(dx+c))^3}+\frac{2\,(\sinh(dx+c))^2}{3\,\cosh(dx+c)}-\frac{2\,\text{co}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(-2*a^2*arctanh(exp(d*x+c))+2*a*b*(sinh(d*x+c)^2/cosh(d*x+c)-cosh(d*x+c
))+b^2*(-1/3*sinh(d*x+c)^2/cosh(d*x+c)^3+2/3*sinh(d*x+c)^2/cosh(d*x+c)-2/3*
cosh(d*x+c)))
```

Maxima [B] time = 1.04153, size = 265, normalized size = 5.2

$$-\frac{2}{3}b^2\left(\frac{3\,e^{(-dx-c)}}{d\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1\right)}+\frac{2\,e^{(-3\,dx-3\,c)}}{d\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1\right)}+\frac{3}{d\left(3\,e^{(-2\,dx-2\,c)}+3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -2/3*b^2*(3*e^(-d*x - c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-
6*d*x - 6*c) + 1)) + 2*e^(-3*d*x - 3*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*
x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 3*e^(-5*d*x - 5*c)/(d*(3*e^(-2*d*x - 2*
c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + a^2*log(tanh(1/2*d*x +
```

```
1/2*c))/d - 4*a*b/(d*(e^(d*x + c) + e^(-d*x - c)))
```

Fricas [B] time = 2.00274, size = 2331, normalized size = 45.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(6*(2*a*b + b^2)*cosh(d*x + c)^5 + 30*(2*a*b + b^2)*cosh(d*x + c)*sinh
(d*x + c)^4 + 6*(2*a*b + b^2)*sinh(d*x + c)^5 + 4*(6*a*b + b^2)*cosh(d*x +
c)^3 + 4*(15*(2*a*b + b^2)*cosh(d*x + c)^2 + 6*a*b + b^2)*sinh(d*x + c)^3 +
 12*(5*(2*a*b + b^2)*cosh(d*x + c)^3 + (6*a*b + b^2)*cosh(d*x + c))*sinh(d*
x + c)^2 + 6*(2*a*b + b^2)*cosh(d*x + c) + 3*(a^2*cosh(d*x + c)^6 + 6*a^2*c
osh(d*x + c)*sinh(d*x + c)^5 + a^2*sinh(d*x + c)^6 + 3*a^2*cosh(d*x + c)^4
+ 3*(5*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^4 + 3*a^2*cosh(d*x + c)^2 +
 4*(5*a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*a^2
*cosh(d*x + c)^4 + 6*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^2 + a^2 + 6*(
a^2*cosh(d*x + c)^5 + 2*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x +
 c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 3*(a^2*cosh(d*x + c)^6 + 6*a^
2*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*sinh(d*x + c)^6 + 3*a^2*cosh(d*x + c)
^4 + 3*(5*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^4 + 3*a^2*cosh(d*x + c)^
2 + 4*(5*a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*
a^2*cosh(d*x + c)^4 + 6*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^2 + a^2 +
6*(a^2*cosh(d*x + c)^5 + 2*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*
x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 6*(5*(2*a*b + b^2)*cosh(d*
x + c)^4 + 2*(6*a*b + b^2)*cosh(d*x + c)^2 + 2*a*b + b^2)*sinh(d*x + c))/(d
*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c)^6 +
3*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 4*(5*d*
cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*d*cosh(d*x + c)^2
+ 3*(5*d*cosh(d*x + c)^4 + 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 6*(d*
cosh(d*x + c)^5 + 2*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tanh^2(c + dx)\right)^2 \operatorname{csch}(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*csch(c + d*x), x)
```

Giac [B] time = 1.33456, size = 170, normalized size = 3.33

$$-\frac{3\,a^2\log\left(e^{(dx+c)}+1\right)-3\,a^2\log\left(\left|e^{(dx+c)}-1\right|\right)+\frac{2\left(6\,abe^{(5\,dx+5\,c)}+3\,b^2e^{(5\,dx+5\,c)}+12\,abe^{(3\,dx+3\,c)}+2\,b^2e^{(3\,dx+3\,c)}+6\,abe^{(dx+c)}+3\,b^2e^{(dx+c)}\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*a^2*log(e^(d*x + c) + 1) - 3*a^2*log(abs(e^(d*x + c) - 1)) + 2*(6*a
*b*e^(5*d*x + 5*c) + 3*b^2*e^(5*d*x + 5*c) + 12*a*b*e^(3*d*x + 3*c) + 2*b^2
*e^(3*d*x + 3*c) + 6*a*b*e^(d*x + c) + 3*b^2*e^(d*x + c))/(e^(2*d*x + 2*c)
+ 1)^3)/d
```

3.14 $\int \text{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=46

$$-\frac{a^2\coth(c+dx)}{d}+\frac{2ab\tanh(c+dx)}{d}+\frac{b^2\tanh^3(c+dx)}{3d}$$

```
[Out] -((a^2*Coth[c + d*x])/d) + (2*a*b*Tanh[c + d*x])/d + (b^2*Tanh[c + d*x]^3)/
(3*d)
```

Rubi [A] time = 0.0567826, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3663, 270}

$$-\frac{a^2\coth(c+dx)}{d}+\frac{2ab\tanh(c+dx)}{d}+\frac{b^2\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -((a^2*Coth[c + d*x])/d) + (2*a*b*Tanh[c + d*x])/d + (b^2*Tanh[c + d*x]^3)/
(3*d)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \text{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{x^2}\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(2ab+\frac{a^2}{x^2}+b^2x^2\right)dx,x,\tanh(c+dx)\right)}{d}\\ &= -\frac{a^2\coth(c+dx)}{d}+\frac{2ab\tanh(c+dx)}{d}+\frac{b^2\tanh^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.448521, size = 43, normalized size = 0.93

$$\frac{b\tanh(c+dx)\left(6a-b\text{sech}^2(c+dx)+b\right)-3a^2\coth(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (-3*a^2*Coth[c + d*x] + b*(6*a + b - b*Sech[c + d*x]^2)*Tanh[c + d*x])/(3*d
)
```

Maple [A] time = 0.048, size = 68, normalized size = 1.5

$$\frac{1}{d}\left(-a^2\coth(dx+c)+2\,ab\tanh(dx+c)+b^2\left(-\frac{\sinh(dx+c)}{2\,(\cosh(dx+c))^3}+\frac{\tanh(dx+c)}{2}\left(\frac{2}{3}+\frac{(\mathrm{sech}\,(dx+c))^2}{3}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(-a^2*coth(d*x+c)+2*a*b*tanh(d*x+c)+b^2*(-1/2*sinh(d*x+c)/cosh(d*x+c)^3
+1/2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)))
```

Maxima [B] time = 1.05402, size = 184, normalized size = 4.

$$\frac{2}{3}\,b^2\left(\frac{3\,e^{(-4\,dx-4\,c)}}{d\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1\right)}+\frac{1}{d\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1\right)}\right)+\frac{4\,ab}{d\left(e^{(-2\,dx-2\,c)}+1\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 2/3*b^2*(3*e^(-4*d*x - 4*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e
^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) + 1))) + 4*a*b/(d*(e^(-2*d*x - 2*c) + 1)) + 2*a^2/(d*(e^(-2*d
*x - 2*c) - 1))
```

Fricas [B] time = 1.85856, size = 683, normalized size = 14.85

$$-\frac{4\left(\left(3\,a^2+b^2\right)\cosh(dx+c)^3+3\left(3\,a^2+b^2\right)\cosh(dx+c)\sinh(dx+c)^2+2\left(\right.\right.}{3\left(d\cosh(dx+c)^5+5\,d\cosh(dx+c)\sinh(dx+c)^4+d\sinh(dx+c)^5+d\cosh(dx+c)^3+\left(10\,d\cosh(dx+c)^2+3\,d\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -4/3*((3*a^2 + b^2)*cosh(d*x + c)^3 + 3*(3*a^2 + b^2)*cosh(d*x + c)*sinh(d*
x + c)^2 + 2*(3*a*b + b^2)*sinh(d*x + c)^3 + (9*a^2 - b^2)*cosh(d*x + c) +
2*(3*(3*a*b + b^2)*cosh(d*x + c)^2 + 3*a*b - b^2)*sinh(d*x + c))/(d*cosh(d*
x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + d*sinh(d*x + c)^5 + d*cosh(d
*x + c)^3 + (10*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^3 + (10*d*cosh(d*x +
 c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 - 2*d*cosh(d*x + c) + (5*d*cosh(
d*x + c)^4 + 9*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tanh^2(c + dx)\right)^2 \operatorname{csch}^2(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*csch(c + d*x)**2, x)
```

Giac [A] time = 1.34978, size = 116, normalized size = 2.52

$$-\frac{2\left(\frac{3\,a^2}{e^{(2\,dx+2\,c)}-1}+\frac{6\,abe^{(4\,dx+4\,c)}+3\,b^2e^{(4\,dx+4\,c)}+12\,abe^{(2\,dx+2\,c)}+6\,ab+b^2}{\left(e^{(2\,dx+2\,c)}+1\right)^3}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*a^2/(e^(2*d*x + 2*c) - 1) + (6*a*b*e^(4*d*x + 4*c) + 3*b^2*e^(4*d*x
 + 4*c) + 12*a*b*e^(2*d*x + 2*c) + 6*a*b + b^2)/(e^(2*d*x + 2*c) + 1)^3)/d
```

3.15 $\int \mathrm{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=82

$$-\frac{a^2\mathrm{csch}^2(c+dx)\mathrm{sech}(c+dx)}{2d}-\frac{a(a-4b)\mathrm{sech}(c+dx)}{2d}+\frac{a(a-4b)\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{b^2\mathrm{sech}^3(c+dx)}{3d}$$

```
[Out] (a*(a - 4*b)*ArcTanh[Cosh[c + d*x]])/(2*d) - (a*(a - 4*b)*Sech[c + d*x])/(2
*d) - (a^2*Csch[c + d*x]^2*Sech[c + d*x])/(2*d) - (b^2*Sech[c + d*x]^3)/(3*
d)
```

Rubi [A] time = 0.11943, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3664, 463, 459, 321, 207}

$$-\frac{a^2\mathrm{csch}^2(c+dx)\mathrm{sech}(c+dx)}{2d}-\frac{a(a-4b)\mathrm{sech}(c+dx)}{2d}+\frac{a(a-4b)\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{b^2\mathrm{sech}^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```


```
[Out] (a*(a - 4*b)*ArcTanh[Cosh[c + d*x]])/(2*d) - (a*(a - 4*b)*Sech[c + d*x])/(2
*d) - (a^2*Csch[c + d*x]^2*Sech[c + d*x])/(2*d) - (b^2*Sech[c + d*x]^3)/(3*
d)
```


Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{\operatorname{Subst}\left(\int \frac{x^2\left(a+b-bx^2\right)^2}{\left(-1+x^2\right)^2}\,dx, x, \operatorname{sech}(c+dx)\right)}{d}$$

$$= -\frac{a^2\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{2d} - \frac{\operatorname{Subst}\left(\int \frac{x^2\left(3a^2-2(a+b)^2+2b^2x^2\right)}{-1+x^2}\,dx, x, \operatorname{se}\right.}{2d}$$

$$= -\frac{a^2\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{2d} - \frac{b^2\operatorname{sech}^3(c+dx)}{3d} - \frac{(a(a-4b))\operatorname{Subst}}{}$$

$$= -\frac{a(a-4b)\operatorname{sech}(c+dx)}{2d} - \frac{a^2\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{2d} - \frac{b^2\operatorname{sech}^3(c+}{3d}$$

$$= \frac{a(a-4b)\tanh^{-1}(\cosh(c+dx))}{2d} - \frac{a(a-4b)\operatorname{sech}(c+dx)}{2d} - \frac{a^2\operatorname{csch}^2(c+}{}$$

Mathematica [A] time = 1.54012, size = 96, normalized size = 1.17

$$-\frac{3a^2\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)+3a^2\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)+12a^2\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)-48ab\operatorname{sech}(c+dx)-48ab\log\left(\tanh\left(\right.\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -(3*a^2*Csch[(c + d*x)/2]^2 + 12*a^2*Log[Tanh[(c + d*x)/2]] - 48*a*b*Log[Ta
nh[(c + d*x)/2]] + 3*a^2*Sech[(c + d*x)/2]^2 - 48*a*b*Sech[c + d*x] + 8*b^2
*Sech[c + d*x]^3)/(24*d)
```

Maple [A] time = 0.062, size = 103, normalized size = 1.3

$$\frac{1}{d}\left(a^2\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{Artanh}\left(e^{dx+c}\right)\right)+2\,ab\left((\cosh(dx+c))^{-1}-2\operatorname{Artanh}\left(e^{dx+c}\right)\right)+b^2\left(\frac{(\sinh(d}{3(\cosh(}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+2*a*b*(1/cosh(d
*x+c)-2*arctanh(exp(d*x+c)))+b^2*(1/3*sinh(d*x+c)^2/cosh(d*x+c)^3+1/3*sinh(
d*x+c)^2/cosh(d*x+c)-1/3*cosh(d*x+c)))
```

Maxima [B] time = 1.05745, size = 244, normalized size = 2.98

$$\frac{1}{2}a^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d\left(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1\right)}\right)-2\,ab\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c
) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - 2*
a*b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d - 2*e^(-d*x - c)/(d*
(e^(-2*d*x - 2*c) + 1))) - 8/3*b^2/(d*(e^(d*x + c) + e^(-d*x - c))^3)
```

Fricas [B] time = 2.31955, size = 6384, normalized size = 77.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(6*(a^2 - 4*a*b)*cosh(d*x + c)^9 + 54*(a^2 - 4*a*b)*cosh(d*x + c)*sinh
(d*x + c)^8 + 6*(a^2 - 4*a*b)*sinh(d*x + c)^9 + 8*(3*a^2 + 2*b^2)*cosh(d*x
+ c)^7 + 8*(27*(a^2 - 4*a*b)*cosh(d*x + c)^2 + 3*a^2 + 2*b^2)*sinh(d*x + c)
^7 + 56*(9*(a^2 - 4*a*b)*cosh(d*x + c)^3 + (3*a^2 + 2*b^2)*cosh(d*x + c))*s
inh(d*x + c)^6 + 4*(9*a^2 + 12*a*b - 8*b^2)*cosh(d*x + c)^5 + 4*(189*(a^2 -
 4*a*b)*cosh(d*x + c)^4 + 42*(3*a^2 + 2*b^2)*cosh(d*x + c)^2 + 9*a^2 + 12*a
*b - 8*b^2)*sinh(d*x + c)^5 + 4*(189*(a^2 - 4*a*b)*cosh(d*x + c)^5 + 70*(3*
a^2 + 2*b^2)*cosh(d*x + c)^3 + 5*(9*a^2 + 12*a*b - 8*b^2)*cosh(d*x + c))*si
nh(d*x + c)^4 + 8*(3*a^2 + 2*b^2)*cosh(d*x + c)^3 + 8*(63*(a^2 - 4*a*b)*cos
h(d*x + c)^6 + 35*(3*a^2 + 2*b^2)*cosh(d*x + c)^4 + 5*(9*a^2 + 12*a*b - 8*b
^2)*cosh(d*x + c)^2 + 3*a^2 + 2*b^2)*sinh(d*x + c)^3 + 8*(27*(a^2 - 4*a*b)*
cosh(d*x + c)^7 + 21*(3*a^2 + 2*b^2)*cosh(d*x + c)^5 + 5*(9*a^2 + 12*a*b -
8*b^2)*cosh(d*x + c)^3 + 3*(3*a^2 + 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 +
 6*(a^2 - 4*a*b)*cosh(d*x + c) - 3*((a^2 - 4*a*b)*cosh(d*x + c)^10 + 10*(a^
2 - 4*a*b)*cosh(d*x + c)*sinh(d*x + c)^9 + (a^2 - 4*a*b)*sinh(d*x + c)^10 +
 (a^2 - 4*a*b)*cosh(d*x + c)^8 + (45*(a^2 - 4*a*b)*cosh(d*x + c)^2 + a^2 -
4*a*b)*sinh(d*x + c)^8 + 8*(15*(a^2 - 4*a*b)*cosh(d*x + c)^3 + (a^2 - 4*a*b
)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(a^2 - 4*a*b)*cosh(d*x + c)^6 + 2*(105
*(a^2 - 4*a*b)*cosh(d*x + c)^4 + 14*(a^2 - 4*a*b)*cosh(d*x + c)^2 - a^2 + 4
*a*b)*sinh(d*x + c)^6 + 4*(63*(a^2 - 4*a*b)*cosh(d*x + c)^5 + 14*(a^2 - 4*a
*b)*cosh(d*x + c)^3 - 3*(a^2 - 4*a*b)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(a
^2 - 4*a*b)*cosh(d*x + c)^4 + 2*(105*(a^2 - 4*a*b)*cosh(d*x + c)^6 + 35*(a^
2 - 4*a*b)*cosh(d*x + c)^4 - 15*(a^2 - 4*a*b)*cosh(d*x + c)^2 - a^2 + 4*a*b
)*sinh(d*x + c)^4 + 8*(15*(a^2 - 4*a*b)*cosh(d*x + c)^7 + 7*(a^2 - 4*a*b)*c
osh(d*x + c)^5 - 5*(a^2 - 4*a*b)*cosh(d*x + c)^3 - (a^2 - 4*a*b)*cosh(d*x +
 c))*sinh(d*x + c)^3 + (a^2 - 4*a*b)*cosh(d*x + c)^2 + (45*(a^2 - 4*a*b)*co
sh(d*x + c)^8 + 28*(a^2 - 4*a*b)*cosh(d*x + c)^6 - 30*(a^2 - 4*a*b)*cosh(d*
x + c)^4 - 12*(a^2 - 4*a*b)*cosh(d*x + c)^2 + a^2 - 4*a*b)*sinh(d*x + c)^2
+ a^2 - 4*a*b + 2*(5*(a^2 - 4*a*b)*cosh(d*x + c)^9 + 4*(a^2 - 4*a*b)*cosh(d
*x + c)^7 - 6*(a^2 - 4*a*b)*cosh(d*x + c)^5 - 4*(a^2 - 4*a*b)*cosh(d*x + c)
^3 + (a^2 - 4*a*b)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d
*x + c) + 1) + 3*((a^2 - 4*a*b)*cosh(d*x + c)^10 + 10*(a^2 - 4*a*b)*cosh(d*
```

```
x + c)*sinh(d*x + c)^9 + (a^2 - 4*a*b)*sinh(d*x + c)^10 + (a^2 - 4*a*b)*cos
h(d*x + c)^8 + (45*(a^2 - 4*a*b)*cosh(d*x + c)^2 + a^2 - 4*a*b)*sinh(d*x +
c)^8 + 8*(15*(a^2 - 4*a*b)*cosh(d*x + c)^3 + (a^2 - 4*a*b)*cosh(d*x + c))*s
inh(d*x + c)^7 - 2*(a^2 - 4*a*b)*cosh(d*x + c)^6 + 2*(105*(a^2 - 4*a*b)*cos
h(d*x + c)^4 + 14*(a^2 - 4*a*b)*cosh(d*x + c)^2 - a^2 + 4*a*b)*sinh(d*x + c
)^6 + 4*(63*(a^2 - 4*a*b)*cosh(d*x + c)^5 + 14*(a^2 - 4*a*b)*cosh(d*x + c)^
3 - 3*(a^2 - 4*a*b)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(a^2 - 4*a*b)*cosh(d
*x + c)^4 + 2*(105*(a^2 - 4*a*b)*cosh(d*x + c)^6 + 35*(a^2 - 4*a*b)*cosh(d*
x + c)^4 - 15*(a^2 - 4*a*b)*cosh(d*x + c)^2 - a^2 + 4*a*b)*sinh(d*x + c)^4
+ 8*(15*(a^2 - 4*a*b)*cosh(d*x + c)^7 + 7*(a^2 - 4*a*b)*cosh(d*x + c)^5 - 5
*(a^2 - 4*a*b)*cosh(d*x + c)^3 - (a^2 - 4*a*b)*cosh(d*x + c))*sinh(d*x + c)
^3 + (a^2 - 4*a*b)*cosh(d*x + c)^2 + (45*(a^2 - 4*a*b)*cosh(d*x + c)^8 + 28
*(a^2 - 4*a*b)*cosh(d*x + c)^6 - 30*(a^2 - 4*a*b)*cosh(d*x + c)^4 - 12*(a^2
 - 4*a*b)*cosh(d*x + c)^2 + a^2 - 4*a*b)*sinh(d*x + c)^2 + a^2 - 4*a*b + 2*
(5*(a^2 - 4*a*b)*cosh(d*x + c)^9 + 4*(a^2 - 4*a*b)*cosh(d*x + c)^7 - 6*(a^2
 - 4*a*b)*cosh(d*x + c)^5 - 4*(a^2 - 4*a*b)*cosh(d*x + c)^3 + (a^2 - 4*a*b)
*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(
27*(a^2 - 4*a*b)*cosh(d*x + c)^8 + 28*(3*a^2 + 2*b^2)*cosh(d*x + c)^6 + 10*
(9*a^2 + 12*a*b - 8*b^2)*cosh(d*x + c)^4 + 12*(3*a^2 + 2*b^2)*cosh(d*x + c)
^2 + 3*a^2 - 12*a*b)*sinh(d*x + c))/(d*cosh(d*x + c)^10 + 10*d*cosh(d*x + c
)*sinh(d*x + c)^9 + d*sinh(d*x + c)^10 + d*cosh(d*x + c)^8 + (45*d*cosh(d*x
 + c)^2 + d)*sinh(d*x + c)^8 + 8*(15*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*s
inh(d*x + c)^7 - 2*d*cosh(d*x + c)^6 + 2*(105*d*cosh(d*x + c)^4 + 14*d*cosh
(d*x + c)^2 - d)*sinh(d*x + c)^6 + 4*(63*d*cosh(d*x + c)^5 + 14*d*cosh(d*x
+ c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^5 - 2*d*cosh(d*x + c)^4 + 2*(105*
d*cosh(d*x + c)^6 + 35*d*cosh(d*x + c)^4 - 15*d*cosh(d*x + c)^2 - d)*sinh(d
*x + c)^4 + 8*(15*d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)^5 - 5*d*cosh(d*x +
c)^3 - d*cosh(d*x + c))*sinh(d*x + c)^3 + d*cosh(d*x + c)^2 + (45*d*cosh(d*
x + c)^8 + 28*d*cosh(d*x + c)^6 - 30*d*cosh(d*x + c)^4 - 12*d*cosh(d*x + c)
^2 + d)*sinh(d*x + c)^2 + 2*(5*d*cosh(d*x + c)^9 + 4*d*cosh(d*x + c)^7 - 6*
d*cosh(d*x + c)^5 - 4*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) +
d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tanh^2(c + dx)\right)^2 \operatorname{csch}^3(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*csch(c + d*x)**3, x)
```

Giac [B] time = 1.373, size = 227, normalized size = 2.77

$$\frac{3\left(a^2e^c - 4\,abe^c\right)e^{(-c)}\log\left(e^{(dx+c)} + 1\right) - 3\left(a^2e^c - 4\,abe^c\right)e^{(-c)}\log\left(\left|e^{(dx+c)} - 1\right|\right) - \frac{6\left(a^2e^{(3\,dx+3\,c)} + a^2e^{(dx+c)}\right)}{\left(e^{(2\,dx+2\,c)} - 1\right)^2} + \frac{8\left(3\,abe^{(5\,dx+5\,c)} + 6\,ab\right.}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*(a^2*e^c - 4*a*b*e^c)*e^(-c)*log(e^(d*x + c) + 1) - 3*(a^2*e^c - 4*a
*b*e^c)*e^(-c)*log(abs(e^(d*x + c) - 1)) - 6*(a^2*e^(3*d*x + 3*c) + a^2*e^(
```

```
d*x + c))/(e^(2*d*x + 2*c) - 1)^2 + 8*(3*a*b*e^(5*d*x + 5*c) + 6*a*b*e^(3*d
*x + 3*c) - 2*b^2*e^(3*d*x + 3*c) + 3*a*b*e^(d*x + c))/(e^(2*d*x + 2*c) + 1
)^3)/d
```

3.16 $\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=72

$$-\frac{a^2\coth^3(c+dx)}{3d}-\frac{b(2a-b)\tanh(c+dx)}{d}+\frac{a(a-2b)\coth(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}$$

```
[Out] (a*(a - 2*b)*Coth[c + d*x])/d - (a^2*Coth[c + d*x]^3)/(3*d) - ((2*a - b)*b*
Tanh[c + d*x])/d - (b^2*Tanh[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0789086, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3663, 448}

$$-\frac{a^2\coth^3(c+dx)}{3d}-\frac{b(2a-b)\tanh(c+dx)}{d}+\frac{a(a-2b)\coth(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (a*(a - 2*b)*Coth[c + d*x])/d - (a^2*Coth[c + d*x]^3)/(3*d) - ((2*a - b)*b*
Tanh[c + d*x])/d - (b^2*Tanh[c + d*x]^3)/(3*d)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)(a+bx^2)^2}{x^4}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \left(b(-2a+b)+\frac{a^2}{x^4}-\frac{a(a-2b)}{x^2}-b^2x^2\right)dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{a(a-2b)\coth(c+dx)}{d}-\frac{a^2\coth^3(c+dx)}{3d}-\frac{(2a-b)b\tanh(c+dx)}{d}-\end{aligned}$$

Mathematica [A] time = 0.48016, size = 59, normalized size = 0.82

$$\frac{b\tanh(c+dx)\left(-6a+b\operatorname{sech}^2(c+dx)+2b\right)-a\coth(c+dx)\left(a\operatorname{csch}^2(c+dx)-2a+6b\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (-(a*Coth[c + d*x]*(-2*a + 6*b + a*Csch[c + d*x]^2)) + b*(-6*a + 2*b + b*Se
ch[c + d*x]^2)*Tanh[c + d*x])/(3*d)
```

Maple [A] time = 0.059, size = 81, normalized size = 1.1

$$\frac{1}{d}\left(a^2\left(\frac{2}{3}-\frac{(\operatorname{csch}(dx+c))^2}{3}\right)\coth(dx+c)+2\,ab\left(-\frac{1}{\cosh(dx+c)\sinh(dx+c)}-2\,\tanh(dx+c)\right)+b^2\left(\frac{2}{3}+\frac{(\operatorname{sech}(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+2*a*b*(-1/sinh(d*x+c)/cosh(d*x
+c)-2*tanh(d*x+c))+b^2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c))
```

Maxima [B] time = 1.05125, size = 284, normalized size = 3.94

$$\frac{4}{3}\,b^2\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1\right)}+\frac{1}{d\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1\right)}\right)+\frac{4}{3}\,a^2\left(\frac{}{d\left(3\,e^{(-2\,dx-2\,c)}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 4/3*b^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e
^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) + 1))) + 4/3*a^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) -
 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3
*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + 8*a*b/(d*(e^(-4*d*x - 4*c) -
1))
```

Fricas [B] time = 1.93208, size = 1011, normalized size = 14.04

$$-\frac{8\left(\left(a^2+6\,ab+b^2\right)\cosh(dx+c)^4+8\left(a^2+b^2\right)\cosh(dx+c)\sinh(dx+c\right.}{3\left(d\cosh(dx+c)^8+56\,d\cosh(dx+c)^3\sinh(dx+c)^5+28\,d\cosh(dx+c)^2\sinh(dx+c)^6+8\,d\cosh(dx+c)\sinh(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -8/3*((a^2 + 6*a*b + b^2)*cosh(d*x + c)^4 + 8*(a^2 + b^2)*cosh(d*x + c)*sin
h(d*x + c)^3 + (a^2 + 6*a*b + b^2)*sinh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d*x
 + c)^2 + 2*(3*(a^2 + 6*a*b + b^2)*cosh(d*x + c)^2 + 2*a^2 - 2*b^2)*sinh(d*
x + c)^2 + 3*a^2 - 6*a*b + 3*b^2 + 8*((a^2 + b^2)*cosh(d*x + c)^3 + (a^2 -
b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 56*d*cosh(d*x + c)^
3*sinh(d*x + c)^5 + 28*d*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*d*cosh(d*x + c
```

```
)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 - 4*d*cosh(d*x + c)^4 + 2*(35*d*cosh(
d*x + c)^4 - 2*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 - d*cosh(d*x + c
))*sinh(d*x + c)^3 + 4*(7*d*cosh(d*x + c)^6 - 6*d*cosh(d*x + c)^2)*sinh(d*x
 + c)^2 + 8*(d*cosh(d*x + c)^7 - d*cosh(d*x + c)^3)*sinh(d*x + c) + 3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tanh^2(c + dx)\right)^2 \operatorname{csch}^4(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*csch(c + d*x)**4, x)
```

Giac [B] time = 1.41702, size = 193, normalized size = 2.68

$$-\frac{4\left(3\,a^2e^{(8\,dx+8\,c)} + 6\,abe^{(8\,dx+8\,c)} + 3\,b^2e^{(8\,dx+8\,c)} + 8\,a^2e^{(6\,dx+6\,c)} - 8\,b^2e^{(6\,dx+6\,c)} + 6\,a^2e^{(4\,dx+4\,c)} - 12\,abe^{(4\,dx+4\,c)} + 6\,b^2e^{(4}}{3\,d\left(e^{(4\,dx+4\,c)} - 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -4/3*(3*a^2*e^(8*d*x + 8*c) + 6*a*b*e^(8*d*x + 8*c) + 3*b^2*e^(8*d*x + 8*c)
 + 8*a^2*e^(6*d*x + 6*c) - 8*b^2*e^(6*d*x + 6*c) + 6*a^2*e^(4*d*x + 4*c) -
12*a*b*e^(4*d*x + 4*c) + 6*b^2*e^(4*d*x + 4*c) - a^2 + 6*a*b - b^2)/(d*(e^(
4*d*x + 4*c) - 1)^3)
```

3.17 $\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=182

$$-\frac{b\left(6a^2+35ab+21b^2\right)\tanh^3(c+dx)}{8d}-\frac{3(a+b)\left(a^2+14ab+21b^2\right)\tanh(c+dx)}{8d}+\frac{3}{8}x(a+b)\left(a^2+14ab+21b^2\right)-\frac{3b^2}{}$$

```
[Out] (3*(a + b)*(a^2 + 14*a*b + 21*b^2)*x)/8 - (3*(a + b)*(a^2 + 14*a*b + 21*b^2
)*Tanh[c + d*x])/(8*d) - (b*(6*a^2 + 35*a*b + 21*b^2)*Tanh[c + d*x]^3)/(8*d
) - (3*b^2*(5*a + 21*b)*Tanh[c + d*x]^5)/(40*d) - (3*(a + 3*b)*Sinh[c + d*x
]^2*Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2)/(8*d) + (Cosh[c + d*x]*Sinh[c
+ d*x]^3*(a + b*Tanh[c + d*x]^2)^3)/(4*d)
```

Rubi [A] time = 0.22168, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3663, 467, 577, 570, 206}

$$-\frac{b\left(6a^2+35ab+21b^2\right)\tanh^3(c+dx)}{8d}-\frac{3(a+b)\left(a^2+14ab+21b^2\right)\tanh(c+dx)}{8d}+\frac{3}{8}x(a+b)\left(a^2+14ab+21b^2\right)-\frac{3b^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```


```
[Out] (3*(a + b)*(a^2 + 14*a*b + 21*b^2)*x)/8 - (3*(a + b)*(a^2 + 14*a*b + 21*b^2
)*Tanh[c + d*x])/(8*d) - (b*(6*a^2 + 35*a*b + 21*b^2)*Tanh[c + d*x]^3)/(8*d
) - (3*b^2*(5*a + 21*b)*Tanh[c + d*x]^5)/(40*d) - (3*(a + 3*b)*Sinh[c + d*x
]^2*Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2)/(8*d) + (Cosh[c + d*x]*Sinh[c
+ d*x]^3*(a + b*Tanh[c + d*x]^2)^3)/(4*d)
```


Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 577

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*b*g*n*(p + 1)), x] + Dist[1/(a*
b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b
*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m + n
*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n, 0
```

```
] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a
*f])
```

Rule 570

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+bx^2\right)^3}{\left(1-x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3}{4d} - \frac{\text{Subst}\left(\int \frac{x^2\left(a+bx^2\right)}{\left(1-\right.}\right.}{}$$

$$= -\frac{3(a+3b)\sinh^2(c+dx)\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2}{8d} + \frac{\cosh(}{}$$

$$= -\frac{3(a+3b)\sinh^2(c+dx)\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2}{8d} + \frac{\cosh(}{}$$

$$= -\frac{3(a+b)\left(a^2+14ab+21b^2\right)\tanh(c+dx)}{8d} - \frac{b\left(6a^2+35ab+21b^2\right)\tan}{8d}$$

$$= \frac{3}{8}(a+b)\left(a^2+14ab+21b^2\right)x - \frac{3(a+b)\left(a^2+14ab+21b^2\right)\tanh(c+d}{8d}$$

Mathematica [A] time = 3.97828, size = 125, normalized size = 0.69

$$\frac{60\left(15a^2b+a^3+35ab^2+21b^3\right)(c+dx) - 32b\tanh(c+dx)\left(15a^2-b(5a+7b)\text{sech}^2(c+dx)+50ab+b^2\text{sech}^4(c+d\right.}{160d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (60*(a^3 + 15*a^2*b + 35*a*b^2 + 21*b^3)*(c + d*x) - 40*(a + b)^2*(a + 4*b)
*Sinh[2*(c + d*x)] + 5*(a + b)^3*Sinh[4*(c + d*x)] - 32*b*(15*a^2 + 50*a*b
+ 36*b^2 - b*(5*a + 7*b)*Sech[c + d*x]^2 + b^2*Sech[c + d*x]^4)*Tanh[c + d*
x])/(160*d)
```

Maple [A] time = 0.053, size = 246, normalized size = 1.4

$$\frac{1}{d}\left(a^3\left(\left(\frac{(\sinh(dx+c))^3}{4} - \frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c) + \frac{3dx}{8} + \frac{3c}{8}\right) + 3a^2b\left(1/4\frac{(\sinh(dx+c))^5}{\cosh(dx+c)} - 5/8\frac{(\sinh(dx}{\cosh(dx}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+3*
a^2*b*(1/4*sinh(d*x+c)^5/cosh(d*x+c)-5/8*sinh(d*x+c)^3/cosh(d*x+c)+15/8*d*x
+15/8*c-15/8*tanh(d*x+c))+3*a*b^2*(1/4*sinh(d*x+c)^7/cosh(d*x+c)^3-7/8*sinh
(d*x+c)^5/cosh(d*x+c)^3+35/8*d*x+35/8*c-35/8*tanh(d*x+c)-35/24*tanh(d*x+c)^
3)+b^3*(1/4*sinh(d*x+c)^9/cosh(d*x+c)^5-9/8*sinh(d*x+c)^7/cosh(d*x+c)^5+63/
8*d*x+63/8*c-63/8*tanh(d*x+c)-21/8*tanh(d*x+c)^3-63/40*tanh(d*x+c)^5))
```

Maxima [B] time = 1.07719, size = 648, normalized size = 3.56

$$\frac{1}{64}a^3\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)+\frac{1}{320}b^3\left(\frac{2520(dx+c)}{d}+\frac{5\left(32e^{(-2dx-2c)}-e^{(-4dx-4c)}\right)}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/64*a^3*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + 1/320*b^3*(2520*(d*x + c)/d + 5*(32*e^(-2*d*x
- 2*c) - e^(-4*d*x - 4*c))/d - (135*e^(-2*d*x - 2*c) + 5358*e^(-4*d*x - 4*c
) + 18190*e^(-6*d*x - 6*c) + 28455*e^(-8*d*x - 8*c) + 19995*e^(-10*d*x - 10
*c) + 6560*e^(-12*d*x - 12*c) - 5)/(d*(e^(-4*d*x - 4*c) + 5*e^(-6*d*x - 6*c
) + 10*e^(-8*d*x - 8*c) + 10*e^(-10*d*x - 10*c) + 5*e^(-12*d*x - 12*c) + e^
(-14*d*x - 14*c)))) + 1/64*a*b^2*(840*(d*x + c)/d + 3*(24*e^(-2*d*x - 2*c)
- e^(-4*d*x - 4*c))/d - (63*e^(-2*d*x - 2*c) + 1487*e^(-4*d*x - 4*c) + 2517
*e^(-6*d*x - 6*c) + 1608*e^(-8*d*x - 8*c) - 3)/(d*(e^(-4*d*x - 4*c) + 3*e^(
-6*d*x - 6*c) + 3*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c)))) + 3/64*a^2*b*(12
0*(d*x + c)/d + (16*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c))/d - (15*e^(-2*d*x
- 2*c) + 144*e^(-4*d*x - 4*c) - 1)/(d*(e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))
))
```

Fricas [B] time = 2.07457, size = 2273, normalized size = 12.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/320*(5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^9 - 15*(a^3 + 11*a^2
*b + 19*a*b^2 + 9*b^3 - 12*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^7 + 8*(120*a^2*b + 400*a*b^2 + 288*b^3 + 15*(a^3 + 15*a^2*b
+ 35*a*b^2 + 21*b^3)*d*x)*cosh(d*x + c)^5 + 40*(120*a^2*b + 400*a*b^2 + 288
*b^3 + 15*(a^3 + 15*a^2*b + 35*a*b^2 + 21*b^3)*d*x)*cosh(d*x + c)*sinh(d*x
+ c)^4 + (630*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 - 150*a^3 - 2
010*a^2*b - 4850*a*b^2 - 3054*b^3 - 315*(a^3 + 11*a^2*b + 19*a*b^2 + 9*b^3)
*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 40*(120*a^2*b + 400*a*b^2 + 288*b^3 + 1
5*(a^3 + 15*a^2*b + 35*a*b^2 + 21*b^3)*d*x)*cosh(d*x + c)^3 + 5*(84*(a^3 +
3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 - 105*(a^3 + 11*a^2*b + 19*a*b^2 +
 9*b^3)*cosh(d*x + c)^4 - 62*a^3 - 978*a^2*b - 2282*a*b^2 - 1302*b^3 - 4*(7
5*a^3 + 1005*a^2*b + 2425*a*b^2 + 1527*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^
```

```
3 + 40*(2*(120*a^2*b + 400*a*b^2 + 288*b^3 + 15*(a^3 + 15*a^2*b + 35*a*b^2
+ 21*b^3)*d*x)*cosh(d*x + c)^3 + 3*(120*a^2*b + 400*a*b^2 + 288*b^3 + 15*(a
^3 + 15*a^2*b + 35*a*b^2 + 21*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2 + 80
*(120*a^2*b + 400*a*b^2 + 288*b^3 + 15*(a^3 + 15*a^2*b + 35*a*b^2 + 21*b^3)
*d*x)*cosh(d*x + c) + 5*(9*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8
- 21*(a^3 + 11*a^2*b + 19*a*b^2 + 9*b^3)*cosh(d*x + c)^6 - 2*(75*a^3 + 1005
*a^2*b + 2425*a*b^2 + 1527*b^3)*cosh(d*x + c)^4 - 36*a^3 - 612*a^2*b - 1372
*a*b^2 - 924*b^3 - 6*(31*a^3 + 489*a^2*b + 1141*a*b^2 + 651*b^3)*cosh(d*x +
 c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^
4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(
d*x + c)^2 + 10*d*cosh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.74754, size = 684, normalized size = 3.76

$$120\left(a^3+15\,a^2b+35\,ab^2+21\,b^3\right)dx-5\left(18\,a^3e^{(4\,dx+4\,c)}+270\,a^2be^{(4\,dx+4\,c)}+630\,ab^2e^{(4\,dx+4\,c)}+378\,b^3e^{(4\,dx+4\,c)}-8\,a^3e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/320*(120*(a^3 + 15*a^2*b + 35*a*b^2 + 21*b^3)*d*x - 5*(18*a^3*e^(4*d*x +
4*c) + 270*a^2*b*e^(4*d*x + 4*c) + 630*a*b^2*e^(4*d*x + 4*c) + 378*b^3*e^(4
*d*x + 4*c) - 8*a^3*e^(2*d*x + 2*c) - 48*a^2*b*e^(2*d*x + 2*c) - 72*a*b^2*e
^(2*d*x + 2*c) - 32*b^3*e^(2*d*x + 2*c) + a^3 + 3*a^2*b + 3*a*b^2 + b^3)*e^
(-4*d*x - 4*c) + 5*(a^3*e^(4*d*x + 36*c) + 3*a^2*b*e^(4*d*x + 36*c) + 3*a*b
^2*e^(4*d*x + 36*c) + b^3*e^(4*d*x + 36*c) - 8*a^3*e^(2*d*x + 34*c) - 48*a^
2*b*e^(2*d*x + 34*c) - 72*a*b^2*e^(2*d*x + 34*c) - 32*b^3*e^(2*d*x + 34*c))
*e^(-32*c) + 128*(15*a^2*b*e^(8*d*x + 8*c) + 60*a*b^2*e^(8*d*x + 8*c) + 50*
b^3*e^(8*d*x + 8*c) + 60*a^2*b*e^(6*d*x + 6*c) + 210*a*b^2*e^(6*d*x + 6*c)
+ 150*b^3*e^(6*d*x + 6*c) + 90*a^2*b*e^(4*d*x + 4*c) + 290*a*b^2*e^(4*d*x +
 4*c) + 210*b^3*e^(4*d*x + 4*c) + 60*a^2*b*e^(2*d*x + 2*c) + 190*a*b^2*e^(2
*d*x + 2*c) + 130*b^3*e^(2*d*x + 2*c) + 15*a^2*b + 50*a*b^2 + 36*b^3)/(e^(2
*d*x + 2*c) + 1)^5)/d
```

3.18 $\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=105

$$\frac{b^2(3a+4b)\text{sech}^3(c+dx)}{3d}+\frac{(a+b)^3\cosh^3(c+dx)}{3d}-\frac{(a+b)^2(a+4b)\cosh(c+dx)}{d}-\frac{3b(a+b)(a+2b)\text{sech}(c+dx)}{d}-b$$

```
[Out] -(((a + b)^2*(a + 4*b)*Cosh[c + d*x])/d) + ((a + b)^3*Cosh[c + d*x]^3)/(3*d
) - (3*b*(a + b)*(a + 2*b)*Sech[c + d*x])/d + (b^2*(3*a + 4*b)*Sech[c + d*x
]^3)/(3*d) - (b^3*Sech[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.12258, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3664, 448}

$$\frac{b^2(3a+4b)\text{sech}^3(c+dx)}{3d}+\frac{(a+b)^3\cosh^3(c+dx)}{3d}-\frac{(a+b)^2(a+4b)\cosh(c+dx)}{d}-\frac{3b(a+b)(a+2b)\text{sech}(c+dx)}{d}-b$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -(((a + b)^2*(a + 4*b)*Cosh[c + d*x])/d) + ((a + b)^3*Cosh[c + d*x]^3)/(3*d
) - (3*b*(a + b)*(a + 2*b)*Sech[c + d*x])/d + (b^2*(3*a + 4*b)*Sech[c + d*x
]^3)/(3*d) - (b^3*Sech[c + d*x]^5)/(5*d)
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx&=\frac{\text{Subst}\left(\int\frac{(-1+x^2)(a+b-bx^2)^3}{x^4}\,dx,x,\text{sech}(c+dx)\right)}{d}\\&=\frac{\text{Subst}\left(\int\left(3(-a-2b)b(a+b)-\frac{(a+b)^3}{x^4}+\frac{(a+b)^2(a+4b)}{x^2}+b^2(3a+4b)x^2-b^3x^4\right.\right.}{d}\\&=-\frac{(a+b)^2(a+4b)\cosh(c+dx)}{d}+\frac{(a+b)^3\cosh^3(c+dx)}{3d}-\frac{3b(a+b)(a+2}{d}\end{aligned}$$

Mathematica [A] time = 0.290715, size = 91, normalized size = 0.87

$$\frac{20b^2(3a+4b)\text{sech}^3(c+dx)-45(a+b)^2(a+5b)\cosh(c+dx)+5(a+b)^3\cosh(3(c+dx))-180b(a+b)(a+2b)\text{sech}(c+}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (-45*(a + b)^2*(a + 5*b)*Cosh[c + d*x] + 5*(a + b)^3*Cosh[3*(c + d*x)] - 18
0*b*(a + b)*(a + 2*b)*Sech[c + d*x] + 20*b^2*(3*a + 4*b)*Sech[c + d*x]^3 -
12*b^3*Sech[c + d*x]^5)/(60*d)
```

Maple [B] time = 0.052, size = 287, normalized size = 2.7

$$\frac{1}{d}\left(a^3\left(-\frac{2}{3}+\frac{(\sinh(dx+c))^2}{3}\right)\cosh(dx+c)+3a^2b\left(1/3\,\frac{(\sinh(dx+c))^4}{\cosh(dx+c)}+4/3\,\frac{(\sinh(dx+c))^2}{\cosh(dx+c)}-8/3\cosh(dx+c)\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+3*a^2*b*(1/3*sinh(d*x+c)^4/co
sh(d*x+c)+4/3*sinh(d*x+c)^2/cosh(d*x+c)-8/3*cosh(d*x+c))+3*a*b^2*(1/3*sinh(
d*x+c)^6/cosh(d*x+c)^3-2*sinh(d*x+c)^4/cosh(d*x+c)^3-8/3*sinh(d*x+c)^2/cosh
(d*x+c)^3+16/3*sinh(d*x+c)^2/cosh(d*x+c)-16/3*cosh(d*x+c))+b^3*(1/3*sinh(d*
x+c)^8/cosh(d*x+c)^5-8/3*sinh(d*x+c)^6/cosh(d*x+c)^5-16*sinh(d*x+c)^4/cosh(
d*x+c)^5-64/5*sinh(d*x+c)^2/cosh(d*x+c)^5+128/15*sinh(d*x+c)^2/cosh(d*x+c)^
3+128/15*sinh(d*x+c)^2/cosh(d*x+c)-128/15*cosh(d*x+c)))
```

Maxima [B] time = 1.10065, size = 593, normalized size = 5.65

$$-\frac{1}{120}b^3\left(\frac{5\left(45e^{(-dx-c)}-e^{(-3dx-3c)}\right)}{d}+\frac{200e^{(-2dx-2c)}+2515e^{(-4dx-4c)}+6680e^{(-6dx-6c)}+9073e^{(-8dx-8c)}+5600e^{(-10}}{d\left(e^{(-3dx-3c)}+5e^{(-5dx-5c)}+10e^{(-7dx-7c)}+10e^{(-9dx-9c)}+5e^{(-11dx-}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/120*b^3*(5*(45*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + (200*e^(-2*d*x - 2*c
) + 2515*e^(-4*d*x - 4*c) + 6680*e^(-6*d*x - 6*c) + 9073*e^(-8*d*x - 8*c) +
 5600*e^(-10*d*x - 10*c) + 1665*e^(-12*d*x - 12*c) - 5)/(d*(e^(-3*d*x - 3*c
) + 5*e^(-5*d*x - 5*c) + 10*e^(-7*d*x - 7*c) + 10*e^(-9*d*x - 9*c) + 5*e^(-
11*d*x - 11*c) + e^(-13*d*x - 13*c)))) - 1/8*a*b^2*((33*e^(-d*x - c) - e^(-
3*d*x - 3*c))/d + (30*e^(-2*d*x - 2*c) + 240*e^(-4*d*x - 4*c) + 322*e^(-6*d
*x - 6*c) + 177*e^(-8*d*x - 8*c) - 1)/(d*(e^(-3*d*x - 3*c) + 3*e^(-5*d*x -
5*c) + 3*e^(-7*d*x - 7*c) + e^(-9*d*x - 9*c)))) - 1/8*a^2*b*((21*e^(-d*x -
c) - e^(-3*d*x - 3*c))/d + (20*e^(-2*d*x - 2*c) + 69*e^(-4*d*x - 4*c) - 1)/
(d*(e^(-3*d*x - 3*c) + e^(-5*d*x - 5*c)))) + 1/24*a^3*(e^(3*d*x + 3*c)/d -
9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

Fricas [B] time = 1.96694, size = 1388, normalized size = 13.22

$$5\left(a^3+3a^2b+3ab^2+b^3\right)\cosh(dx+c)^8+5\left(a^3+3a^2b+3ab^2+b^3\right)\sinh(dx+c)^8-20\left(a^3+12a^2b+21ab^2+10b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/120*(5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 5*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*sinh(d*x + c)^8 - 20*(a^3 + 12*a^2*b + 21*a*b^2 + 10*b^3)
*cosh(d*x + c)^6 - 20*(a^3 + 12*a^2*b + 21*a*b^2 + 10*b^3 - 7*(a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 - 20*(11*a^3 + 123*a^2*
b + 249*a*b^2 + 137*b^3)*cosh(d*x + c)^4 + 10*(35*(a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*cosh(d*x + c)^4 - 22*a^3 - 246*a^2*b - 498*a*b^2 - 274*b^3 - 30*(a^3
 + 12*a^2*b + 21*a*b^2 + 10*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 425*a^3
 - 5235*a^2*b - 10395*a*b^2 - 5649*b^3 - 20*(31*a^3 + 372*a^2*b + 747*a*b^2
 + 390*b^3)*cosh(d*x + c)^2 + 20*(7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*
x + c)^6 - 15*(a^3 + 12*a^2*b + 21*a*b^2 + 10*b^3)*cosh(d*x + c)^4 - 31*a^3
 - 372*a^2*b - 747*a*b^2 - 390*b^3 - 6*(11*a^3 + 123*a^2*b + 249*a*b^2 + 13
7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2)/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x
+ c)*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*c
osh(d*x + c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.43728, size = 597, normalized size = 5.69

$$-\frac{5\left(9\,a^3e^{(2\,dx+2\,c)}+63\,a^2be^{(2\,dx+2\,c)}+99\,ab^2e^{(2\,dx+2\,c)}+45\,b^3e^{(2\,dx+2\,c)}-a^3-3\,a^2b-3\,ab^2-b^3\right)e^{(-3\,dx-3\,c)}-5\left(a^3e^{(3\,dx+48\,c)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/120*(5*(9*a^3*e^(2*d*x + 2*c) + 63*a^2*b*e^(2*d*x + 2*c) + 99*a*b^2*e^(2
*d*x + 2*c) + 45*b^3*e^(2*d*x + 2*c) - a^3 - 3*a^2*b - 3*a*b^2 - b^3)*e^(-3
*d*x - 3*c) - 5*(a^3*e^(3*d*x + 48*c) + 3*a^2*b*e^(3*d*x + 48*c) + 3*a*b^2*
e^(3*d*x + 48*c) + b^3*e^(3*d*x + 48*c) - 9*a^3*e^(d*x + 46*c) - 63*a^2*b*e
^(d*x + 46*c) - 99*a*b^2*e^(d*x + 46*c) - 45*b^3*e^(d*x + 46*c))*e^(-45*c)
+ 16*(45*a^2*b*e^(9*d*x + 9*c) + 135*a*b^2*e^(9*d*x + 9*c) + 90*b^3*e^(9*d*
x + 9*c) + 180*a^2*b*e^(7*d*x + 7*c) + 480*a*b^2*e^(7*d*x + 7*c) + 280*b^3*
e^(7*d*x + 7*c) + 270*a^2*b*e^(5*d*x + 5*c) + 690*a*b^2*e^(5*d*x + 5*c) + 4
28*b^3*e^(5*d*x + 5*c) + 180*a^2*b*e^(3*d*x + 3*c) + 480*a*b^2*e^(3*d*x + 3
*c) + 280*b^3*e^(3*d*x + 3*c) + 45*a^2*b*e^(d*x + c) + 135*a*b^2*e^(d*x + c
) + 90*b^3*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^5)/d
```

3.19 $\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=122

$$\frac{b^2(3a+2b)\tanh^3(c+dx)}{3d}+\frac{3b(a+b)^2\tanh(c+dx)}{d}+\frac{(a+b)^3}{4d(1-\tanh(c+dx))}-\frac{(a+b)^3}{4d(\tanh(c+dx)+1)}-\frac{1}{2}x(a+b)^2(a$$

```
[Out] -((a + b)^2*(a + 7*b)*x)/2 + (a + b)^3/(4*d*(1 - Tanh[c + d*x])) + (3*b*(a
+ b)^2*Tanh[c + d*x])/d + (b^2*(3*a + 2*b)*Tanh[c + d*x]^3)/(3*d) + (b^3*Ta
nh[c + d*x]^5)/(5*d) - (a + b)^3/(4*d*(1 + Tanh[c + d*x]))
```

Rubi [A] time = 0.185146, antiderivative size = 139, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3663, 467, 528, 388, 206}

$$\frac{b\left(81a^2+190ab+105b^2\right)\tanh(c+dx)}{30d}+\frac{7b\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2}{10d}+\frac{b(33a+35b)\tanh(c+dx)\left(a+b\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -((a + b)^2*(a + 7*b)*x)/2 + (b*(81*a^2 + 190*a*b + 105*b^2)*Tanh[c + d*x])
/(30*d) + (b*(33*a + 35*b)*Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2))/(30*d) +
(7*b*Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2)/(10*d) + (Cosh[c + d*x]*Sinh[
c + d*x]*(a + b*Tanh[c + d*x]^2)^3)/(2*d)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+bx^2\right)^3}{\left(1-x^2\right)^2}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3}{2d} - \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^2\left(a+7b\right.}{1-x^2}\right.}{2}$$

$$= \frac{7b\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2}{10d} + \frac{\cosh(c+dx)\sinh(c+dx)\left(a+b\right.}{2d}$$

$$= \frac{b(33a+35b)\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)}{30d} + \frac{7b\tanh(c+dx)\left(a+b\right.}{10d}$$

$$= \frac{b\left(81a^2+190ab+105b^2\right)\tanh(c+dx)}{30d} + \frac{b(33a+35b)\tanh(c+dx)\left(a+b\right.}{30d}$$

$$= -\frac{1}{2}(a+b)^2(a+7b)x + \frac{b\left(81a^2+190ab+105b^2\right)\tanh(c+dx)}{30d} + \frac{b(33a+3}{}$$

Mathematica [A] time = 2.15865, size = 95, normalized size = 0.78

$$\frac{4b\tanh(c+dx)\left(45a^2-b(15a+16b)\text{sech}^2(c+dx)+105ab+3b^2\text{sech}^4(c+dx)+58b^2\right)-30(a+7b)(a+b)^2(c+dx)+1}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (-30*(a + b)^2*(a + 7*b)*(c + d*x) + 15*(a + b)^3*Sinh[2*(c + d*x)] + 4*b*(
45*a^2 + 105*a*b + 58*b^2 - b*(15*a + 16*b)*Sech[c + d*x]^2 + 3*b^2*Sech[c
+ d*x]^4)*Tanh[c + d*x])/(60*d)
```

Maple [A] time = 0.053, size = 180, normalized size = 1.5

$$\frac{1}{d}\left(a^3\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3a^2b\left(1/2\,\frac{(\sinh(dx+c))^3}{\cosh(dx+c)}-3/2\,dx-3/2\,c+3/2\,\tanh(dx+c)\right)+3ab^2\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+3*a^2*b*(1/2*sinh(d*x+
c)^3/cosh(d*x+c)-3/2*d*x-3/2*c+3/2*tanh(d*x+c))+3*a*b^2*(1/2*sinh(d*x+c)^5/
cosh(d*x+c)^3-5/2*d*x-5/2*c+5/2*tanh(d*x+c)+5/6*tanh(d*x+c)^3)+b^3*(1/2*sin
h(d*x+c)^7/cosh(d*x+c)^5-7/2*d*x-7/2*c+7/2*tanh(d*x+c)+7/6*tanh(d*x+c)^3+7/
10*tanh(d*x+c)^5))
```

Maxima [B] time = 1.1, size = 509, normalized size = 4.17

$$-\frac{1}{8}a^3\left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d}\right) - \frac{1}{120}b^3\left(\frac{420(dx+c)}{d} + \frac{15e^{(-2dx-2c)}}{d} - \frac{1003e^{(-2dx-2c)} + 3350e^{(-4dx-4c)} + 5590}{d\left(e^{(-2dx-2c)} + 5e^{(-4dx-4c)} + 10e^{(-6dx-}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*a^3*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/120*b^3*(420*(d
*x + c)/d + 15*e^(-2*d*x - 2*c)/d - (1003*e^(-2*d*x - 2*c) + 3350*e^(-4*d*x
 - 4*c) + 5590*e^(-6*d*x - 6*c) + 3915*e^(-8*d*x - 8*c) + 1455*e^(-10*d*x -
 10*c) + 15)/(d*(e^(-2*d*x - 2*c) + 5*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c
) + 10*e^(-8*d*x - 8*c) + 5*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c)))) - 1/
8*a*b^2*(60*(d*x + c)/d + 3*e^(-2*d*x - 2*c)/d - (121*e^(-2*d*x - 2*c) + 20
1*e^(-4*d*x - 4*c) + 147*e^(-6*d*x - 6*c) + 3)/(d*(e^(-2*d*x - 2*c) + 3*e^(
-4*d*x - 4*c) + 3*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c)))) - 3/8*a^2*b*(12*(d
*x + c)/d + e^(-2*d*x - 2*c)/d - (17*e^(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x -
2*c) + e^(-4*d*x - 4*c))))
```

Fricas [B] time = 2.10247, size = 1854, normalized size = 15.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/120*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^7 - 4*(90*a^2*b + 2
10*a*b^2 + 116*b^3 + 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*d*x)*cosh(d*x +
c)^5 - 20*(90*a^2*b + 210*a*b^2 + 116*b^3 + 15*(a^3 + 9*a^2*b + 15*a*b^2 +
7*b^3)*d*x)*cosh(d*x + c)*sinh(d*x + c)^4 + (75*a^3 + 585*a^2*b + 1065*a*b^
2 + 539*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x
 + c)^5 - 20*(90*a^2*b + 210*a*b^2 + 116*b^3 + 15*(a^3 + 9*a^2*b + 15*a*b^2
 + 7*b^3)*d*x)*cosh(d*x + c)^3 + 5*(105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos
h(d*x + c)^4 + 27*a^3 + 297*a^2*b + 489*a*b^2 + 203*b^3 + 2*(75*a^3 + 585*a
^2*b + 1065*a*b^2 + 539*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 - 20*(2*(90*a
^2*b + 210*a*b^2 + 116*b^3 + 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*d*x)*cos
h(d*x + c)^3 + 3*(90*a^2*b + 210*a*b^2 + 116*b^3 + 15*(a^3 + 9*a^2*b + 15*a
*b^2 + 7*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2 - 40*(90*a^2*b + 210*a*b^
2 + 116*b^3 + 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*d*x)*cosh(d*x + c) + 5*
(21*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + (75*a^3 + 585*a^2*b +
 1065*a*b^2 + 539*b^3)*cosh(d*x + c)^4 + 15*a^3 + 189*a^2*b + 285*a*b^2 + 1
75*b^3 + 3*(27*a^3 + 297*a^2*b + 489*a*b^2 + 203*b^3)*cosh(d*x + c)^2)*sinh
(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cos
h(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2
 + 10*d*cosh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.1757, size = 533, normalized size = 4.37

$$-\frac{60\left(a^3+9\,a^2b+15\,ab^2+7\,b^3\right)dx-15\left(2\,a^3e^{(2\,dx+2\,c)}+18\,a^2be^{(2\,dx+2\,c)}+30\,ab^2e^{(2\,dx+2\,c)}+14\,b^3e^{(2\,dx+2\,c)}-a^3-3\,a^2b-3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/120*(60*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*d*x - 15*(2*a^3*e^(2*d*x + 2*
c) + 18*a^2*b*e^(2*d*x + 2*c) + 30*a*b^2*e^(2*d*x + 2*c) + 14*b^3*e^(2*d*x
+ 2*c) - a^3 - 3*a^2*b - 3*a*b^2 - b^3)*e^(-2*d*x - 2*c) - 15*(a^3*e^(2*d*x
 + 16*c) + 3*a^2*b*e^(2*d*x + 16*c) + 3*a*b^2*e^(2*d*x + 16*c) + b^3*e^(2*d
*x + 16*c))*e^(-14*c) + 16*(45*a^2*b*e^(8*d*x + 8*c) + 135*a*b^2*e^(8*d*x +
 8*c) + 90*b^3*e^(8*d*x + 8*c) + 180*a^2*b*e^(6*d*x + 6*c) + 450*a*b^2*e^(6
*d*x + 6*c) + 240*b^3*e^(6*d*x + 6*c) + 270*a^2*b*e^(4*d*x + 4*c) + 600*a*b
^2*e^(4*d*x + 4*c) + 340*b^3*e^(4*d*x + 4*c) + 180*a^2*b*e^(2*d*x + 2*c) +
390*a*b^2*e^(2*d*x + 2*c) + 200*b^3*e^(2*d*x + 2*c) + 45*a^2*b + 105*a*b^2
+ 58*b^3)/(e^(2*d*x + 2*c) + 1)^5)/d
```

3.20 $\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=70

$$-\frac{b^2(a+b)\text{sech}^3(c+dx)}{d}+\frac{(a+b)^3\cosh(c+dx)}{d}+\frac{3b(a+b)^2\text{sech}(c+dx)}{d}+\frac{b^3\text{sech}^5(c+dx)}{5d}$$

```
[Out] ((a + b)^3*Cosh[c + d*x])/d + (3*b*(a + b)^2*Sech[c + d*x])/d - (b^2*(a + b
)*Sech[c + d*x]^3)/d + (b^3*Sech[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0684337, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3664, 270}

$$-\frac{b^2(a+b)\text{sech}^3(c+dx)}{d}+\frac{(a+b)^3\cosh(c+dx)}{d}+\frac{3b(a+b)^2\text{sech}(c+dx)}{d}+\frac{b^3\text{sech}^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((a + b)^3*Cosh[c + d*x])/d + (3*b*(a + b)^2*Sech[c + d*x])/d - (b^2*(a + b
)*Sech[c + d*x]^3)/d + (b^3*Sech[c + d*x]^5)/(5*d)
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=-\frac{\text{Subst}\left(\int\frac{\left(a+b-bx^2\right)^3}{x^2}\,dx,x,\text{sech}(c+dx)\right)}{d}$$

$$=-\frac{\text{Subst}\left(\int\left(-3b(a+b)^2+\frac{(a+b)^3}{x^2}+3b^2(a+b)x^2-b^3x^4\right)dx,x,\text{sech}(c+\right.}{d}$$

$$=\frac{(a+b)^3\cosh(c+dx)}{d}+\frac{3b(a+b)^2\text{sech}(c+dx)}{d}-\frac{b^2(a+b)\text{sech}^3(c+dx}{d}$$

Mathematica [A] time = 0.828759, size = 63, normalized size = 0.9

$$\frac{b\,\text{sech}(c+dx)\left(-5b(a+b)\text{sech}^2(c+dx)+15(a+b)^2+b^2\text{sech}^4(c+dx)\right)+5(a+b)^3\cosh(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (5*(a + b)^3*Cosh[c + d*x] + b*Sech[c + d*x]*(15*(a + b)^2 - 5*b*(a + b)*Se
ch[c + d*x]^2 + b^2*Sech[c + d*x]^4))/(5*d)
```

Maple [B] time = 0.049, size = 219, normalized size = 3.1

$$\frac{1}{d}\left(a^3\cosh(dx+c)+3a^2b\left(-\frac{(\sinh(dx+c))^2}{\cosh(dx+c)}+2\cosh(dx+c)\right)+3ab^2\left(\frac{(\sinh(dx+c))^4}{(\cosh(dx+c))^3}+4/3\frac{(\sinh(dx+c))^2}{(\cosh(dx+c))^3}-8/3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*cosh(d*x+c)+3*a^2*b*(-sinh(d*x+c)^2/cosh(d*x+c)+2*cosh(d*x+c))+3*a
*b^2*(sinh(d*x+c)^4/cosh(d*x+c)^3+4/3*sinh(d*x+c)^2/cosh(d*x+c)^3-8/3*sinh(
d*x+c)^2/cosh(d*x+c)+8/3*cosh(d*x+c))+b^3*(sinh(d*x+c)^6/cosh(d*x+c)^5+6*si
nh(d*x+c)^4/cosh(d*x+c)^5+24/5*sinh(d*x+c)^2/cosh(d*x+c)^5-16/5*sinh(d*x+c)
^2/cosh(d*x+c)^3-16/5*sinh(d*x+c)^2/cosh(d*x+c)+16/5*cosh(d*x+c)))
```

Maxima [B] time = 1.09181, size = 433, normalized size = 6.19

$$\frac{1}{10}b^3\left(\frac{5e^{(-dx-c)}}{d}+\frac{85e^{(-2dx-2c)}+210e^{(-4dx-4c)}+314e^{(-6dx-6c)}+185e^{(-8dx-8c)}+65e^{(-10dx-10c)}+5}{d\left(e^{(-dx-c)}+5e^{(-3dx-3c)}+10e^{(-5dx-5c)}+10e^{(-7dx-7c)}+5e^{(-9dx-9c)}+e^{(-11dx-11c)}\right)}\right)+\frac{1}{2}ab^2\left(\frac{3e^{(-dx-c)}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/10*b^3*(5*e^(-d*x - c)/d + (85*e^(-2*d*x - 2*c) + 210*e^(-4*d*x - 4*c) +
314*e^(-6*d*x - 6*c) + 185*e^(-8*d*x - 8*c) + 65*e^(-10*d*x - 10*c) + 5)/(d
*(e^(-d*x - c) + 5*e^(-3*d*x - 3*c) + 10*e^(-5*d*x - 5*c) + 10*e^(-7*d*x -
7*c) + 5*e^(-9*d*x - 9*c) + e^(-11*d*x - 11*c)))) + 1/2*a*b^2*(3*e^(-d*x -
c)/d + (33*e^(-2*d*x - 2*c) + 41*e^(-4*d*x - 4*c) + 27*e^(-6*d*x - 6*c) + 3
)/(d*(e^(-d*x - c) + 3*e^(-3*d*x - 3*c) + 3*e^(-5*d*x - 5*c) + e^(-7*d*x -
7*c)))) + 3/2*a^2*b*(e^(-d*x - c)/d + (5*e^(-2*d*x - 2*c) + 1)/(d*(e^(-d*x
- c) + e^(-3*d*x - 3*c)))) + a^3*cosh(d*x + c)/d
```

Fricas [B] time = 1.98396, size = 959, normalized size = 13.7

$$\frac{5\left(a^3+3a^2b+3ab^2+b^3\right)\cosh(dx+c)^6+5\left(a^3+3a^2b+3ab^2+b^3\right)\sinh(dx+c)^6+30\left(a^3+5a^2b+7ab^2+3b^3\right)\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/10*(5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 5*(a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*sinh(d*x + c)^6 + 30*(a^3 + 5*a^2*b + 7*a*b^2 + 3*b^3)*cos
```

```
h(d*x + c)^4 + 15*(2*a^3 + 10*a^2*b + 14*a*b^2 + 6*b^3 + 5*(a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 50*a^3 + 330*a^2*b + 430
*a*b^2 + 182*b^3 + 5*(15*a^3 + 93*a^2*b + 125*a*b^2 + 47*b^3)*cosh(d*x + c)
^2 + 5*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 15*a^3 + 93*a^
2*b + 125*a*b^2 + 47*b^3 + 36*(a^3 + 5*a^2*b + 7*a*b^2 + 3*b^3)*cosh(d*x +
c)^2)*sinh(d*x + c)^2)/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)
^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh
(d*x + c)^2 + 10*d*cosh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.77435, size = 435, normalized size = 6.21

$$5\left(a^3 + 3\,a^2b + 3\,ab^2 + b^3\right)e^{(-dx-c)} + 5\left(a^3e^{(dx+14\,c)} + 3\,a^2be^{(dx+14\,c)} + 3\,ab^2e^{(dx+14\,c)} + b^3e^{(dx+14\,c)}\right)e^{(-13\,c)} + \frac{4\left(15\,a^2be^{(9\,dx+9\,c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/10*(5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*e^(-d*x - c) + 5*(a^3*e^(d*x + 14*c
) + 3*a^2*b*e^(d*x + 14*c) + 3*a*b^2*e^(d*x + 14*c) + b^3*e^(d*x + 14*c))*e
^(-13*c) + 4*(15*a^2*b*e^(9*d*x + 9*c) + 30*a*b^2*e^(9*d*x + 9*c) + 15*b^3*
e^(9*d*x + 9*c) + 60*a^2*b*e^(7*d*x + 7*c) + 100*a*b^2*e^(7*d*x + 7*c) + 40
*b^3*e^(7*d*x + 7*c) + 90*a^2*b*e^(5*d*x + 5*c) + 140*a*b^2*e^(5*d*x + 5*c)
 + 66*b^3*e^(5*d*x + 5*c) + 60*a^2*b*e^(3*d*x + 3*c) + 100*a*b^2*e^(3*d*x +
 3*c) + 40*b^3*e^(3*d*x + 3*c) + 15*a^2*b*e^(d*x + c) + 30*a*b^2*e^(d*x + c
) + 15*b^3*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^5)/d
```

3.21 $\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=84

$$-\frac{b\left(3a^2+3ab+b^2\right)\operatorname{sech}(c+dx)}{d}-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b^2(3a+2b)\operatorname{sech}^3(c+dx)}{3d}-\frac{b^3\operatorname{sech}^5(c+dx)}{5d}$$

```
[Out] -((a^3*ArcTanh[Cosh[c + d*x]])/d) - (b*(3*a^2 + 3*a*b + b^2)*Sech[c + d*x])
/d + (b^2*(3*a + 2*b)*Sech[c + d*x]^3)/(3*d) - (b^3*Sech[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0839635, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3664, 390, 207}

$$-\frac{b\left(3a^2+3ab+b^2\right)\operatorname{sech}(c+dx)}{d}-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b^2(3a+2b)\operatorname{sech}^3(c+dx)}{3d}-\frac{b^3\operatorname{sech}^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -((a^3*ArcTanh[Cosh[c + d*x]])/d) - (b*(3*a^2 + 3*a*b + b^2)*Sech[c + d*x])
/d + (b^2*(3*a + 2*b)*Sech[c + d*x]^3)/(3*d) - (b^3*Sech[c + d*x]^5)/(5*d)
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int \frac{\left(a+b-bx^2\right)^3}{-1+x^2}dx, x, \operatorname{sech}(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-b\left(3a^2+3ab+b^2\right)+b^2(3a+2b)x^2-b^3x^4+\frac{a^3}{-1+x^2}\right)dx, x, \text{se}\right.}{d}$$
$$= -\frac{b\left(3a^2+3ab+b^2\right)\operatorname{sech}(c+dx)}{d}+\frac{b^2(3a+2b)\operatorname{sech}^3(c+dx)}{3d}-\frac{b^3\text{sech}}{5}$$
$$= -\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b\left(3a^2+3ab+b^2\right)\operatorname{sech}(c+dx)}{d}+\frac{b^2(3a+}{}$$

Mathematica [A] time = 0.302332, size = 79, normalized size = 0.94

$$\frac{-15b\left(3a^2+3ab+b^2\right)\operatorname{sech}(c+dx)+15a^3\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+5b^2(3a+2b)\operatorname{sech}^3(c+dx)-3b^3\operatorname{sech}^5(c+dx)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (15*a^3*Log[Tanh[(c + d*x)/2]] - 15*b*(3*a^2 + 3*a*b + b^2)*Sech[c + d*x] +
 5*b^2*(3*a + 2*b)*Sech[c + d*x]^3 - 3*b^3*Sech[c + d*x]^5)/(15*d)
```

Maple [B] time = 0.052, size = 186, normalized size = 2.2

$$\frac{1}{d}\left(-2a^3\text{Artanh}\left(e^{dx+c}\right)+3a^2b\left(\frac{(\sinh(dx+c))^2}{\cosh(dx+c)}-\cosh(dx+c)\right)+3ab^2\left(-1/3\frac{(\sinh(dx+c))^2}{(\cosh(dx+c))^3}+2/3\frac{(\sinh(dx+c)}{\cosh(dx+c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(-2*a^3*arctanh(exp(d*x+c))+3*a^2*b*(sinh(d*x+c)^2/cosh(d*x+c)-cosh(d*x
+c))+3*a*b^2*(-1/3*sinh(d*x+c)^2/cosh(d*x+c)^3+2/3*sinh(d*x+c)^2/cosh(d*x+c
)-2/3*cosh(d*x+c))+b^3*(-sinh(d*x+c)^4/cosh(d*x+c)^5-4/5*sinh(d*x+c)^2/cosh
(d*x+c)^5+8/15*sinh(d*x+c)^2/cosh(d*x+c)^3+8/15*sinh(d*x+c)^2/cosh(d*x+c)-8
/15*cosh(d*x+c)))
```

Maxima [B] time = 1.05016, size = 756, normalized size = 9.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -2/15*b^3*(15*e^(-d*x - c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 1
0*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 20*e^(
-3*d*x - 3*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x -
```

```
6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 58*e^(-5*d*x - 5*c)
/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(
-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 20*e^(-7*d*x - 7*c)/(d*(5*e^(-2*
d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c)
 + e^(-10*d*x - 10*c) + 1)) + 15*e^(-9*d*x - 9*c)/(d*(5*e^(-2*d*x - 2*c) +
10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x
 - 10*c) + 1))) - 2*a*b^2*(3*e^(-d*x - c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*
d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 2*e^(-3*d*x - 3*c)/(d*(3*e^(-2*d*x -
2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 3*e^(-5*d*x - 5*c)/(d*
(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + a^3*lo
g(tanh(1/2*d*x + 1/2*c))/d - 6*a^2*b/(d*(e^(d*x + c) + e^(-d*x - c)))
```

Fricas [B] time = 2.25076, size = 5895, normalized size = 70.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/15*(30*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^9 + 270*(3*a^2*b + 3*a*b^
2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^8 + 30*(3*a^2*b + 3*a*b^2 + b^3)*sinh(
d*x + c)^9 + 40*(9*a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^7 + 40*(9*a^2*b + 6
*a*b^2 + b^3 + 27*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^
7 + 280*(9*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (9*a^2*b + 6*a*b^2 +
 b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 4*(135*a^2*b + 75*a*b^2 + 29*b^3)*co
sh(d*x + c)^5 + 4*(945*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 135*a^2*
b + 75*a*b^2 + 29*b^3 + 210*(9*a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh
(d*x + c)^5 + 20*(189*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 70*(9*a^2
*b + 6*a*b^2 + b^3)*cosh(d*x + c)^3 + (135*a^2*b + 75*a*b^2 + 29*b^3)*cosh(
d*x + c))*sinh(d*x + c)^4 + 40*(9*a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^3 +
40*(63*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 35*(9*a^2*b + 6*a*b^2 +
b^3)*cosh(d*x + c)^4 + 9*a^2*b + 6*a*b^2 + b^3 + (135*a^2*b + 75*a*b^2 + 29
*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 40*(27*(3*a^2*b + 3*a*b^2 + b^3)*c
osh(d*x + c)^7 + 21*(9*a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^5 + (135*a^2*b
+ 75*a*b^2 + 29*b^3)*cosh(d*x + c)^3 + 3*(9*a^2*b + 6*a*b^2 + b^3)*cosh(d*x
 + c))*sinh(d*x + c)^2 + 30*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c) + 15*(a
^3*cosh(d*x + c)^10 + 10*a^3*cosh(d*x + c)*sinh(d*x + c)^9 + a^3*sinh(d*x +
 c)^10 + 5*a^3*cosh(d*x + c)^8 + 10*a^3*cosh(d*x + c)^6 + 5*(9*a^3*cosh(d*x
 + c)^2 + a^3)*sinh(d*x + c)^8 + 40*(3*a^3*cosh(d*x + c)^3 + a^3*cosh(d*x +
 c))*sinh(d*x + c)^7 + 10*a^3*cosh(d*x + c)^4 + 10*(21*a^3*cosh(d*x + c)^4
+ 14*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^6 + 4*(63*a^3*cosh(d*x + c)^5
 + 70*a^3*cosh(d*x + c)^3 + 15*a^3*cosh(d*x + c))*sinh(d*x + c)^5 + 5*a^3*c
osh(d*x + c)^2 + 10*(21*a^3*cosh(d*x + c)^6 + 35*a^3*cosh(d*x + c)^4 + 15*a
^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^4 + 40*(3*a^3*cosh(d*x + c)^7 + 7*a
^3*cosh(d*x + c)^5 + 5*a^3*cosh(d*x + c)^3 + a^3*cosh(d*x + c))*sinh(d*x +
c)^3 + a^3 + 5*(9*a^3*cosh(d*x + c)^8 + 28*a^3*cosh(d*x + c)^6 + 30*a^3*cos
h(d*x + c)^4 + 12*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^2 + 10*(a^3*cosh
(d*x + c)^9 + 4*a^3*cosh(d*x + c)^7 + 6*a^3*cosh(d*x + c)^5 + 4*a^3*cosh(d*
x + c)^3 + a^3*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x +
 c) + 1) - 15*(a^3*cosh(d*x + c)^10 + 10*a^3*cosh(d*x + c)*sinh(d*x + c)^9
+ a^3*sinh(d*x + c)^10 + 5*a^3*cosh(d*x + c)^8 + 10*a^3*cosh(d*x + c)^6 + 5
*(9*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^8 + 40*(3*a^3*cosh(d*x + c)^3
+ a^3*cosh(d*x + c))*sinh(d*x + c)^7 + 10*a^3*cosh(d*x + c)^4 + 10*(21*a^3*
cosh(d*x + c)^4 + 14*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^6 + 4*(63*a^3
*cosh(d*x + c)^5 + 70*a^3*cosh(d*x + c)^3 + 15*a^3*cosh(d*x + c))*sinh(d*x
+ c)^5 + 5*a^3*cosh(d*x + c)^2 + 10*(21*a^3*cosh(d*x + c)^6 + 35*a^3*cosh(d
```

```
*x + c)^4 + 15*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^4 + 40*(3*a^3*cosh(
d*x + c)^7 + 7*a^3*cosh(d*x + c)^5 + 5*a^3*cosh(d*x + c)^3 + a^3*cosh(d*x +
 c))*sinh(d*x + c)^3 + a^3 + 5*(9*a^3*cosh(d*x + c)^8 + 28*a^3*cosh(d*x + c
)^6 + 30*a^3*cosh(d*x + c)^4 + 12*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^
2 + 10*(a^3*cosh(d*x + c)^9 + 4*a^3*cosh(d*x + c)^7 + 6*a^3*cosh(d*x + c)^5
 + 4*a^3*cosh(d*x + c)^3 + a^3*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x +
 c) + sinh(d*x + c) - 1) + 10*(27*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8
 + 28*(9*a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^6 + 2*(135*a^2*b + 75*a*b^2 +
 29*b^3)*cosh(d*x + c)^4 + 9*a^2*b + 9*a*b^2 + 3*b^3 + 12*(9*a^2*b + 6*a*b^
2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^10 + 10*d*cosh(d*
x + c)*sinh(d*x + c)^9 + d*sinh(d*x + c)^10 + 5*d*cosh(d*x + c)^8 + 5*(9*d*
cosh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 40*(3*d*cosh(d*x + c)^3 + d*cosh(d*x
 + c))*sinh(d*x + c)^7 + 10*d*cosh(d*x + c)^6 + 10*(21*d*cosh(d*x + c)^4 +
14*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 4*(63*d*cosh(d*x + c)^5 + 70*d*
cosh(d*x + c)^3 + 15*d*cosh(d*x + c))*sinh(d*x + c)^5 + 10*d*cosh(d*x + c)^
4 + 10*(21*d*cosh(d*x + c)^6 + 35*d*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2
+ d)*sinh(d*x + c)^4 + 40*(3*d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)^5 + 5*d*
cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^3 + 5*d*cosh(d*x + c)^2 +
5*(9*d*cosh(d*x + c)^8 + 28*d*cosh(d*x + c)^6 + 30*d*cosh(d*x + c)^4 + 12*d
*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 10*(d*cosh(d*x + c)^9 + 4*d*cosh(d*
x + c)^7 + 6*d*cosh(d*x + c)^5 + 4*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sin
h(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tanh^2\left(c + dx\right)\right)^3 \operatorname{csch}\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**3*csch(c + d*x), x)
```

Giac [B] time = 1.64338, size = 354, normalized size = 4.21

$$-\frac{15\,a^3\log\left(e^{(dx+c)}+1\right)-15\,a^3\log\left(\left|e^{(dx+c)}-1\right|\right)+\frac{2\left(45\,a^2be^{(9\,dx+9\,c)}+45\,ab^2e^{(9\,dx+9\,c)}+15\,b^3e^{(9\,dx+9\,c)}+180\,a^2be^{(7\,dx+7\,c)}+120\,ab^2e^{(7\,dx+7\,c)}+\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/15*(15*a^3*log(e^(d*x + c) + 1) - 15*a^3*log(abs(e^(d*x + c) - 1)) + 2*(
45*a^2*b*e^(9*d*x + 9*c) + 45*a*b^2*e^(9*d*x + 9*c) + 15*b^3*e^(9*d*x + 9*c
) + 180*a^2*b*e^(7*d*x + 7*c) + 120*a*b^2*e^(7*d*x + 7*c) + 20*b^3*e^(7*d*x
 + 7*c) + 270*a^2*b*e^(5*d*x + 5*c) + 150*a*b^2*e^(5*d*x + 5*c) + 58*b^3*e^
(5*d*x + 5*c) + 180*a^2*b*e^(3*d*x + 3*c) + 120*a*b^2*e^(3*d*x + 3*c) + 20*
b^3*e^(3*d*x + 3*c) + 45*a^2*b*e^(d*x + c) + 45*a*b^2*e^(d*x + c) + 15*b^3*
e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^5)/d
```

3.22 $\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=64

$$\frac{3a^2b\tanh(c+dx)}{d}-\frac{a^3\coth(c+dx)}{d}+\frac{ab^2\tanh^3(c+dx)}{d}+\frac{b^3\tanh^5(c+dx)}{5d}$$

```
[Out] -((a^3*Coth[c + d*x])/d) + (3*a^2*b*Tanh[c + d*x])/d + (a*b^2*Tanh[c + d*x]
^3)/d + (b^3*Tanh[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0639957, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3663, 270}

$$\frac{3a^2b\tanh(c+dx)}{d}-\frac{a^3\coth(c+dx)}{d}+\frac{ab^2\tanh^3(c+dx)}{d}+\frac{b^3\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -((a^3*Coth[c + d*x])/d) + (3*a^2*b*Tanh[c + d*x])/d + (a*b^2*Tanh[c + d*x]
^3)/d + (b^3*Tanh[c + d*x]^5)/(5*d)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\left(a+bx^2\right)^3}{x^2}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \left(3a^2b+\frac{a^3}{x^2}+3ab^2x^2+b^3x^4\right)dx, x, \tanh(c+dx)\right)}{d}\\ &= -\frac{a^3\coth(c+dx)}{d}+\frac{3a^2b\tanh(c+dx)}{d}+\frac{ab^2\tanh^3(c+dx)}{d}+\frac{b^3\tanh^5(c}{5d}\end{aligned}$$

Mathematica [A] time = 0.678704, size = 70, normalized size = 1.09

$$\frac{b\tanh(c+dx)\left(15a^2-b(5a+2b)\operatorname{sech}^2(c+dx)+5ab+b^2\operatorname{sech}^4(c+dx)+b^2\right)-5a^3\coth(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (-5*a^3*Coth[c + d*x] + b*(15*a^2 + 5*a*b + b^2 - b*(5*a + 2*b)*Sech[c + d*
x]^2 + b^2*Sech[c + d*x]^4)*Tanh[c + d*x])/(5*d)
```

Maple [B] time = 0.053, size = 141, normalized size = 2.2

$$\frac{1}{d}\left(-a^3\coth(dx+c)+3a^2b\tanh(dx+c)+3ab^2\left(-1/2\,\frac{\sinh(dx+c)}{(\cosh(dx+c))^3}+1/2\,\left(2/3+1/3\,(\mathrm{sech}\,(dx+c))^2\right)\tanh(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(-a^3*coth(d*x+c)+3*a^2*b*tanh(d*x+c)+3*a*b^2*(-1/2*sinh(d*x+c)/cosh(d*
x+c)^3+1/2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c))+b^3*(-1/2*sinh(d*x+c)^3/cos
h(d*x+c)^5-3/8*sinh(d*x+c)/cosh(d*x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+4/15*s
ech(d*x+c)^2)*tanh(d*x+c)))
```

Maxima [B] time = 1.11298, size = 470, normalized size = 7.34

$$\frac{2}{5}\,b^3\left(\frac{10\,e^{(-4\,dx-4\,c)}}{d\left(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1\right)}+\frac{}{d\left(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+1\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 2/5*b^3*(10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) +
 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 5*e^
(-8*d*x - 8*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
- 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x
- 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e
^(-10*d*x - 10*c) + 1))) + 2*a*b^2*(3*e^(-4*d*x - 4*c)/(d*(3*e^(-2*d*x - 2*
c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c)
 + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 6*a^2*b/(d*(e^(-2*d*x - 2
*c) + 1)) + 2*a^3/(d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [B] time = 1.87471, size = 1449, normalized size = 22.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -4/5*((5*a^3 + 5*a*b^2 + 2*b^3)*cosh(d*x + c)^5 + 5*(5*a^3 + 5*a*b^2 + 2*b^
3)*cosh(d*x + c)*sinh(d*x + c)^4 + (15*a^2*b + 10*a*b^2 + 3*b^3)*sinh(d*x +
 c)^5 + (25*a^3 + 5*a*b^2 - 2*b^3)*cosh(d*x + c)^3 + (45*a^2*b + 10*a*b^2 -
```

```
 3*b^3 + 10*(15*a^2*b + 10*a*b^2 + 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3
+ (10*(5*a^3 + 5*a*b^2 + 2*b^3)*cosh(d*x + c)^3 + 3*(25*a^3 + 5*a*b^2 - 2*b
^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(5*a^3 - a*b^2)*cosh(d*x + c) + (5*
(15*a^2*b + 10*a*b^2 + 3*b^3)*cosh(d*x + c)^4 + 30*a^2*b + 10*b^3 + 3*(45*a
^2*b + 10*a*b^2 - 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^7
 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x + c)^7 + 3*d*cosh(d*x + c
)^5 + (21*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^5 + 5*(7*d*cosh(d*x + c)^3
 + 3*d*cosh(d*x + c))*sinh(d*x + c)^4 + d*cosh(d*x + c)^3 + (35*d*cosh(d*x
+ c)^4 + 50*d*cosh(d*x + c)^2 + 9*d)*sinh(d*x + c)^3 + 3*(7*d*cosh(d*x + c)
^5 + 10*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^2 - 5*d*cosh(d*x
 + c) + (7*d*cosh(d*x + c)^6 + 25*d*cosh(d*x + c)^4 + 27*d*cosh(d*x + c)^2
+ 5*d)*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tanh^2(c + dx)\right)^3 \operatorname{csch}^2(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**3*csch(c + d*x)**2, x)
```

Giac [B] time = 1.71288, size = 273, normalized size = 4.27

$$-\frac{2\left(\frac{5\,a^3}{e^{(2\,dx+2\,c)}-1} + \frac{15\,a^2be^{(8\,dx+8\,c)}+15\,ab^2e^{(8\,dx+8\,c)}+5\,b^3e^{(8\,dx+8\,c)}+60\,a^2be^{(6\,dx+6\,c)}+30\,ab^2e^{(6\,dx+6\,c)}+90\,a^2be^{(4\,dx+4\,c)}+20\,ab^2e^{(4\,dx+4\,c)}+10\,b^3e^{(4\,dx+4\,c)}+60\,a^2b}{\left(e^{(2\,dx+2\,c)}+1\right)^5}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -2/5*(5*a^3/(e^(2*d*x + 2*c) - 1) + (15*a^2*b*e^(8*d*x + 8*c) + 15*a*b^2*e^
(8*d*x + 8*c) + 5*b^3*e^(8*d*x + 8*c) + 60*a^2*b*e^(6*d*x + 6*c) + 30*a*b^2
*e^(6*d*x + 6*c) + 90*a^2*b*e^(4*d*x + 4*c) + 20*a*b^2*e^(4*d*x + 4*c) + 10
*b^3*e^(4*d*x + 4*c) + 60*a^2*b*e^(2*d*x + 2*c) + 10*a*b^2*e^(2*d*x + 2*c)
+ 15*a^2*b + 5*a*b^2 + b^3)/(e^(2*d*x + 2*c) + 1)^5)/d
```

3.23 $\int \mathrm{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=152

$$\frac{b\left(81a^2-28ab-4b^2\right)\operatorname{sech}(c+dx)}{30d}+\frac{a^2(a-6b)\tanh^{-1}(\cosh(c+dx))}{2d}+\frac{7b\operatorname{sech}(c+dx)\left(a-b\operatorname{sech}^2(c+dx)+b\right)^2}{10d}+$$

```
[Out] (a^2*(a - 6*b)*ArcTanh[Cosh[c + d*x]])/(2*d) + (b*(81*a^2 - 28*a*b - 4*b^2)
*Sech[c + d*x])/(30*d) + ((33*a - 2*b)*b*Sech[c + d*x]*(a + b - b*Sech[c +
d*x]^2))/(30*d) + (7*b*Sech[c + d*x]*(a + b - b*Sech[c + d*x]^2)^2)/(10*d)
- (Coth[c + d*x]*Csch[c + d*x]*(a + b - b*Sech[c + d*x]^2)^3)/(2*d)
```

Rubi [A] time = 0.225033, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3664, 467, 528, 388, 207}

$$\frac{b\left(81a^2-28ab-4b^2\right)\operatorname{sech}(c+dx)}{30d}+\frac{a^2(a-6b)\tanh^{-1}(\cosh(c+dx))}{2d}+\frac{7b\operatorname{sech}(c+dx)\left(a-b\operatorname{sech}^2(c+dx)+b\right)^2}{10d}+$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (a^2*(a - 6*b)*ArcTanh[Cosh[c + d*x]])/(2*d) + (b*(81*a^2 - 28*a*b - 4*b^2)
*Sech[c + d*x])/(30*d) + ((33*a - 2*b)*b*Sech[c + d*x]*(a + b - b*Sech[c +
d*x]^2))/(30*d) + (7*b*Sech[c + d*x]*(a + b - b*Sech[c + d*x]^2)^2)/(10*d)
- (Coth[c + d*x]*Csch[c + d*x]*(a + b - b*Sech[c + d*x]^2)^3)/(2*d)
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=-\frac{\operatorname{Subst}\left(\int\frac{x^2\left(a+b-bx^2\right)^3}{\left(-1+x^2\right)^2}\,dx,x,\operatorname{sech}(c+dx)\right)}{d}$$

$$=-\frac{\coth(c+dx)\operatorname{csch}(c+dx)\left(a+b-b\operatorname{sech}^2(c+dx)\right)^3}{2d}-\frac{\operatorname{Subst}\left(\int\frac{\left(a+b-7bx\right.}{-}\right.}{}$$

$$=\frac{7b\operatorname{sech}(c+dx)\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2}{10d}-\frac{\coth(c+dx)\operatorname{csch}(c+dx)\left(a-\right.}{2d}$$

$$=\frac{(33a-2b)b\operatorname{sech}(c+dx)\left(a+b-b\operatorname{sech}^2(c+dx)\right)}{30d}+\frac{7b\operatorname{sech}(c+dx)\left(a+b\right.}{10}$$

$$=\frac{b\left(81a^2-28ab-4b^2\right)\operatorname{sech}(c+dx)}{30d}+\frac{(33a-2b)b\operatorname{sech}(c+dx)\left(a+b-b\operatorname{se}\right.}{30d}$$

$$=\frac{a^2(a-6b)\tanh^{-1}(\cosh(c+dx))}{2d}+\frac{b\left(81a^2-28ab-4b^2\right)\operatorname{sech}(c+dx)}{30d}+$$

Mathematica [A] time = 6.17533, size = 127, normalized size = 0.84

$$\frac{3a^2b\operatorname{sech}(c+dx)}{d}-\frac{a^2(a-6b)\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a^3\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a^3\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{b^2(3a+b)\operatorname{sech}^3(}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -(a^3*Csch[(c + d*x)/2]^2)/(8*d) - (a^2*(a - 6*b)*Log[Tanh[(c + d*x)/2]])/(
2*d) - (a^3*Sech[(c + d*x)/2]^2)/(8*d) + (3*a^2*b*Sech[c + d*x])/d - (b^2*(
3*a + b)*Sech[c + d*x]^3)/(3*d) + (b^3*Sech[c + d*x]^5)/(5*d)
```

Maple [A] time = 0.063, size = 174, normalized size = 1.1

$$\frac{1}{d}\left(a^3\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{Artanh}\left(e^{dx+c}\right)\right)+3a^2b\left((\cosh(dx+c))^{-1}-2\operatorname{Artanh}\left(e^{dx+c}\right)\right)+3ab^2\left(1/3\frac{(\sinh}{(\cosh}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+3*a^2*b*(1/cosh
(d*x+c)-2*arctanh(exp(d*x+c)))+3*a*b^2*(1/3*sinh(d*x+c)^2/cosh(d*x+c)^3+1/3
*sinh(d*x+c)^2/cosh(d*x+c)-1/3*cosh(d*x+c))+b^3*(-1/5*sinh(d*x+c)^2/cosh(d*
x+c)^5+2/15*sinh(d*x+c)^2/cosh(d*x+c)^3+2/15*sinh(d*x+c)^2/cosh(d*x+c)-2/15
*cosh(d*x+c)))
```

Maxima [B] time = 1.0929, size = 544, normalized size = 3.58

$$\frac{1}{2}a^3\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d\left(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1\right)}\right)-3\,a^2b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^3*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c
) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - 3*
a^2*b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d - 2*e^(-d*x - c)/(
d*(e^(-2*d*x - 2*c) + 1))) - 8/15*b^3*(5*e^(-3*d*x - 3*c)/(d*(5*e^(-2*d*x -
 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^
(-10*d*x - 10*c) + 1)) - 2*e^(-5*d*x - 5*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(
-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*
c) + 1)) + 5*e^(-7*d*x - 7*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c)
+ 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) - 8*
a*b^2/(d*(e^(d*x + c) + e^(-d*x - c))^3)
```

Fricas [B] time = 2.47568, size = 12984, normalized size = 85.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/30*(30*(a^3 - 6*a^2*b)*cosh(d*x + c)^13 + 390*(a^3 - 6*a^2*b)*cosh(d*x +
 c)*sinh(d*x + c)^12 + 30*(a^3 - 6*a^2*b)*sinh(d*x + c)^13 + 20*(9*a^3 - 18
*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c)^11 + 20*(9*a^3 - 18*a^2*b + 12*a*b
^2 + 4*b^3 + 117*(a^3 - 6*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^11 + 220*(3
9*(a^3 - 6*a^2*b)*cosh(d*x + c)^3 + (9*a^3 - 18*a^2*b + 12*a*b^2 + 4*b^3)*c
osh(d*x + c))*sinh(d*x + c)^10 + 6*(75*a^3 + 30*a^2*b - 32*b^3)*cosh(d*x +
c)^9 + 2*(10725*(a^3 - 6*a^2*b)*cosh(d*x + c)^4 + 225*a^3 + 90*a^2*b - 96*b
^3 + 550*(9*a^3 - 18*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d*x +
c)^9 + 6*(6435*(a^3 - 6*a^2*b)*cosh(d*x + c)^5 + 550*(9*a^3 - 18*a^2*b + 12
*a*b^2 + 4*b^3)*cosh(d*x + c)^3 + 9*(75*a^3 + 30*a^2*b - 32*b^3)*cosh(d*x +
 c))*sinh(d*x + c)^8 + 8*(75*a^3 + 90*a^2*b - 60*a*b^2 + 28*b^3)*cosh(d*x +
 c)^7 + 8*(6435*(a^3 - 6*a^2*b)*cosh(d*x + c)^6 + 825*(9*a^3 - 18*a^2*b + 1
2*a*b^2 + 4*b^3)*cosh(d*x + c)^4 + 75*a^3 + 90*a^2*b - 60*a*b^2 + 28*b^3 +
27*(75*a^3 + 30*a^2*b - 32*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 8*(6435*
(a^3 - 6*a^2*b)*cosh(d*x + c)^7 + 1155*(9*a^3 - 18*a^2*b + 12*a*b^2 + 4*b^3
)*cosh(d*x + c)^5 + 63*(75*a^3 + 30*a^2*b - 32*b^3)*cosh(d*x + c)^3 + 7*(75
*a^3 + 90*a^2*b - 60*a*b^2 + 28*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 6*(75
```

```
*a^3 + 30*a^2*b - 32*b^3)*cosh(d*x + c)^5 + 6*(6435*(a^3 - 6*a^2*b)*cosh(d*
x + c)^8 + 1540*(9*a^3 - 18*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c)^6 + 126
*(75*a^3 + 30*a^2*b - 32*b^3)*cosh(d*x + c)^4 + 75*a^3 + 30*a^2*b - 32*b^3
+ 28*(75*a^3 + 90*a^2*b - 60*a*b^2 + 28*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)
^5 + 2*(10725*(a^3 - 6*a^2*b)*cosh(d*x + c)^9 + 3300*(9*a^3 - 18*a^2*b + 12
*a*b^2 + 4*b^3)*cosh(d*x + c)^7 + 378*(75*a^3 + 30*a^2*b - 32*b^3)*cosh(d*x
 + c)^5 + 140*(75*a^3 + 90*a^2*b - 60*a*b^2 + 28*b^3)*cosh(d*x + c)^3 + 15*
(75*a^3 + 30*a^2*b - 32*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 20*(9*a^3 - 1
8*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c)^3 + 4*(2145*(a^3 - 6*a^2*b)*cosh(
d*x + c)^10 + 825*(9*a^3 - 18*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c)^8 + 1
26*(75*a^3 + 30*a^2*b - 32*b^3)*cosh(d*x + c)^6 + 70*(75*a^3 + 90*a^2*b - 6
0*a*b^2 + 28*b^3)*cosh(d*x + c)^4 + 45*a^3 - 90*a^2*b + 60*a*b^2 + 20*b^3 +
 15*(75*a^3 + 30*a^2*b - 32*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 4*(585*
(a^3 - 6*a^2*b)*cosh(d*x + c)^11 + 275*(9*a^3 - 18*a^2*b + 12*a*b^2 + 4*b^3
)*cosh(d*x + c)^9 + 54*(75*a^3 + 30*a^2*b - 32*b^3)*cosh(d*x + c)^7 + 42*(7
5*a^3 + 90*a^2*b - 60*a*b^2 + 28*b^3)*cosh(d*x + c)^5 + 15*(75*a^3 + 30*a^2
*b - 32*b^3)*cosh(d*x + c)^3 + 15*(9*a^3 - 18*a^2*b + 12*a*b^2 + 4*b^3)*cos
h(d*x + c))*sinh(d*x + c)^2 + 30*(a^3 - 6*a^2*b)*cosh(d*x + c) - 15*((a^3 -
 6*a^2*b)*cosh(d*x + c)^14 + 14*(a^3 - 6*a^2*b)*cosh(d*x + c)*sinh(d*x + c)
^13 + (a^3 - 6*a^2*b)*sinh(d*x + c)^14 + 3*(a^3 - 6*a^2*b)*cosh(d*x + c)^12
 + (3*a^3 - 18*a^2*b + 91*(a^3 - 6*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^12
 + 4*(91*(a^3 - 6*a^2*b)*cosh(d*x + c)^3 + 9*(a^3 - 6*a^2*b)*cosh(d*x + c))
*sinh(d*x + c)^11 + (a^3 - 6*a^2*b)*cosh(d*x + c)^10 + (1001*(a^3 - 6*a^2*b
)*cosh(d*x + c)^4 + a^3 - 6*a^2*b + 198*(a^3 - 6*a^2*b)*cosh(d*x + c)^2)*si
nh(d*x + c)^10 + 2*(1001*(a^3 - 6*a^2*b)*cosh(d*x + c)^5 + 330*(a^3 - 6*a^2
*b)*cosh(d*x + c)^3 + 5*(a^3 - 6*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^9 - 5*
(a^3 - 6*a^2*b)*cosh(d*x + c)^8 + (3003*(a^3 - 6*a^2*b)*cosh(d*x + c)^6 + 1
485*(a^3 - 6*a^2*b)*cosh(d*x + c)^4 - 5*a^3 + 30*a^2*b + 45*(a^3 - 6*a^2*b)
*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(429*(a^3 - 6*a^2*b)*cosh(d*x + c)^7
+ 297*(a^3 - 6*a^2*b)*cosh(d*x + c)^5 + 15*(a^3 - 6*a^2*b)*cosh(d*x + c)^3
- 5*(a^3 - 6*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^7 - 5*(a^3 - 6*a^2*b)*cosh
(d*x + c)^6 + (3003*(a^3 - 6*a^2*b)*cosh(d*x + c)^8 + 2772*(a^3 - 6*a^2*b)*
cosh(d*x + c)^6 + 210*(a^3 - 6*a^2*b)*cosh(d*x + c)^4 - 5*a^3 + 30*a^2*b -
140*(a^3 - 6*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 2*(1001*(a^3 - 6*a^2
*b)*cosh(d*x + c)^9 + 1188*(a^3 - 6*a^2*b)*cosh(d*x + c)^7 + 126*(a^3 - 6*a
^2*b)*cosh(d*x + c)^5 - 140*(a^3 - 6*a^2*b)*cosh(d*x + c)^3 - 15*(a^3 - 6*a
^2*b)*cosh(d*x + c))*sinh(d*x + c)^5 + (a^3 - 6*a^2*b)*cosh(d*x + c)^4 + (1
001*(a^3 - 6*a^2*b)*cosh(d*x + c)^10 + 1485*(a^3 - 6*a^2*b)*cosh(d*x + c)^8
 + 210*(a^3 - 6*a^2*b)*cosh(d*x + c)^6 - 350*(a^3 - 6*a^2*b)*cosh(d*x + c)^
4 + a^3 - 6*a^2*b - 75*(a^3 - 6*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4
*(91*(a^3 - 6*a^2*b)*cosh(d*x + c)^11 + 165*(a^3 - 6*a^2*b)*cosh(d*x + c)^9
 + 30*(a^3 - 6*a^2*b)*cosh(d*x + c)^7 - 70*(a^3 - 6*a^2*b)*cosh(d*x + c)^5
- 25*(a^3 - 6*a^2*b)*cosh(d*x + c)^3 + (a^3 - 6*a^2*b)*cosh(d*x + c))*sinh(
d*x + c)^3 + a^3 - 6*a^2*b + 3*(a^3 - 6*a^2*b)*cosh(d*x + c)^2 + (91*(a^3 -
 6*a^2*b)*cosh(d*x + c)^12 + 198*(a^3 - 6*a^2*b)*cosh(d*x + c)^10 + 45*(a^3
 - 6*a^2*b)*cosh(d*x + c)^8 - 140*(a^3 - 6*a^2*b)*cosh(d*x + c)^6 - 75*(a^3
 - 6*a^2*b)*cosh(d*x + c)^4 + 3*a^3 - 18*a^2*b + 6*(a^3 - 6*a^2*b)*cosh(d*x
 + c)^2)*sinh(d*x + c)^2 + 2*(7*(a^3 - 6*a^2*b)*cosh(d*x + c)^13 + 18*(a^3
- 6*a^2*b)*cosh(d*x + c)^11 + 5*(a^3 - 6*a^2*b)*cosh(d*x + c)^9 - 20*(a^3 -
 6*a^2*b)*cosh(d*x + c)^7 - 15*(a^3 - 6*a^2*b)*cosh(d*x + c)^5 + 2*(a^3 - 6
*a^2*b)*cosh(d*x + c)^3 + 3*(a^3 - 6*a^2*b)*cosh(d*x + c))*sinh(d*x + c))*l
og(cosh(d*x + c) + sinh(d*x + c) + 1) + 15*((a^3 - 6*a^2*b)*cosh(d*x + c)^1
4 + 14*(a^3 - 6*a^2*b)*cosh(d*x + c)*sinh(d*x + c)^13 + (a^3 - 6*a^2*b)*sin
h(d*x + c)^14 + 3*(a^3 - 6*a^2*b)*cosh(d*x + c)^12 + (3*a^3 - 18*a^2*b + 91
*(a^3 - 6*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^12 + 4*(91*(a^3 - 6*a^2*b)*
cosh(d*x + c)^3 + 9*(a^3 - 6*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^11 + (a^3
- 6*a^2*b)*cosh(d*x + c)^10 + (1001*(a^3 - 6*a^2*b)*cosh(d*x + c)^4 + a^3 -
 6*a^2*b + 198*(a^3 - 6*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 2*(1001*
(a^3 - 6*a^2*b)*cosh(d*x + c)^5 + 330*(a^3 - 6*a^2*b)*cosh(d*x + c)^3 + 5*(
```

```
a^3 - 6*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^9 - 5*(a^3 - 6*a^2*b)*cosh(d*x
+ c)^8 + (3003*(a^3 - 6*a^2*b)*cosh(d*x + c)^6 + 1485*(a^3 - 6*a^2*b)*cosh(
d*x + c)^4 - 5*a^3 + 30*a^2*b + 45*(a^3 - 6*a^2*b)*cosh(d*x + c)^2)*sinh(d*
x + c)^8 + 8*(429*(a^3 - 6*a^2*b)*cosh(d*x + c)^7 + 297*(a^3 - 6*a^2*b)*cos
h(d*x + c)^5 + 15*(a^3 - 6*a^2*b)*cosh(d*x + c)^3 - 5*(a^3 - 6*a^2*b)*cosh(
d*x + c))*sinh(d*x + c)^7 - 5*(a^3 - 6*a^2*b)*cosh(d*x + c)^6 + (3003*(a^3
- 6*a^2*b)*cosh(d*x + c)^8 + 2772*(a^3 - 6*a^2*b)*cosh(d*x + c)^6 + 210*(a^
3 - 6*a^2*b)*cosh(d*x + c)^4 - 5*a^3 + 30*a^2*b - 140*(a^3 - 6*a^2*b)*cosh(
d*x + c)^2)*sinh(d*x + c)^6 + 2*(1001*(a^3 - 6*a^2*b)*cosh(d*x + c)^9 + 118
8*(a^3 - 6*a^2*b)*cosh(d*x + c)^7 + 126*(a^3 - 6*a^2*b)*cosh(d*x + c)^5 - 1
40*(a^3 - 6*a^2*b)*cosh(d*x + c)^3 - 15*(a^3 - 6*a^2*b)*cosh(d*x + c))*sinh
(d*x + c)^5 + (a^3 - 6*a^2*b)*cosh(d*x + c)^4 + (1001*(a^3 - 6*a^2*b)*cosh(
d*x + c)^10 + 1485*(a^3 - 6*a^2*b)*cosh(d*x + c)^8 + 210*(a^3 - 6*a^2*b)*co
sh(d*x + c)^6 - 350*(a^3 - 6*a^2*b)*cosh(d*x + c)^4 + a^3 - 6*a^2*b - 75*(a
^3 - 6*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(91*(a^3 - 6*a^2*b)*cosh
(d*x + c)^11 + 165*(a^3 - 6*a^2*b)*cosh(d*x + c)^9 + 30*(a^3 - 6*a^2*b)*cos
h(d*x + c)^7 - 70*(a^3 - 6*a^2*b)*cosh(d*x + c)^5 - 25*(a^3 - 6*a^2*b)*cosh
(d*x + c)^3 + (a^3 - 6*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^3 + a^3 - 6*a^2*
b + 3*(a^3 - 6*a^2*b)*cosh(d*x + c)^2 + (91*(a^3 - 6*a^2*b)*cosh(d*x + c)^1
2 + 198*(a^3 - 6*a^2*b)*cosh(d*x + c)^10 + 45*(a^3 - 6*a^2*b)*cosh(d*x + c)
^8 - 140*(a^3 - 6*a^2*b)*cosh(d*x + c)^6 - 75*(a^3 - 6*a^2*b)*cosh(d*x + c)
^4 + 3*a^3 - 18*a^2*b + 6*(a^3 - 6*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^2
+ 2*(7*(a^3 - 6*a^2*b)*cosh(d*x + c)^13 + 18*(a^3 - 6*a^2*b)*cosh(d*x + c)^
11 + 5*(a^3 - 6*a^2*b)*cosh(d*x + c)^9 - 20*(a^3 - 6*a^2*b)*cosh(d*x + c)^7
 - 15*(a^3 - 6*a^2*b)*cosh(d*x + c)^5 + 2*(a^3 - 6*a^2*b)*cosh(d*x + c)^3 +
 3*(a^3 - 6*a^2*b)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d
*x + c) - 1) + 2*(195*(a^3 - 6*a^2*b)*cosh(d*x + c)^12 + 110*(9*a^3 - 18*a^
2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c)^10 + 27*(75*a^3 + 30*a^2*b - 32*b^3)*
cosh(d*x + c)^8 + 28*(75*a^3 + 90*a^2*b - 60*a*b^2 + 28*b^3)*cosh(d*x + c)^
6 + 15*(75*a^3 + 30*a^2*b - 32*b^3)*cosh(d*x + c)^4 + 15*a^3 - 90*a^2*b + 3
0*(9*a^3 - 18*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*
cosh(d*x + c)^14 + 14*d*cosh(d*x + c)*sinh(d*x + c)^13 + d*sinh(d*x + c)^14
 + 3*d*cosh(d*x + c)^12 + (91*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^12 + 4
*(91*d*cosh(d*x + c)^3 + 9*d*cosh(d*x + c))*sinh(d*x + c)^11 + d*cosh(d*x +
 c)^10 + (1001*d*cosh(d*x + c)^4 + 198*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)
^10 + 2*(1001*d*cosh(d*x + c)^5 + 330*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c)
)*sinh(d*x + c)^9 - 5*d*cosh(d*x + c)^8 + (3003*d*cosh(d*x + c)^6 + 1485*d*
cosh(d*x + c)^4 + 45*d*cosh(d*x + c)^2 - 5*d)*sinh(d*x + c)^8 + 8*(429*d*co
sh(d*x + c)^7 + 297*d*cosh(d*x + c)^5 + 15*d*cosh(d*x + c)^3 - 5*d*cosh(d*x
 + c))*sinh(d*x + c)^7 - 5*d*cosh(d*x + c)^6 + (3003*d*cosh(d*x + c)^8 + 27
72*d*cosh(d*x + c)^6 + 210*d*cosh(d*x + c)^4 - 140*d*cosh(d*x + c)^2 - 5*d)
*sinh(d*x + c)^6 + 2*(1001*d*cosh(d*x + c)^9 + 1188*d*cosh(d*x + c)^7 + 126
*d*cosh(d*x + c)^5 - 140*d*cosh(d*x + c)^3 - 15*d*cosh(d*x + c))*sinh(d*x +
 c)^5 + d*cosh(d*x + c)^4 + (1001*d*cosh(d*x + c)^10 + 1485*d*cosh(d*x + c)
^8 + 210*d*cosh(d*x + c)^6 - 350*d*cosh(d*x + c)^4 - 75*d*cosh(d*x + c)^2 +
 d)*sinh(d*x + c)^4 + 4*(91*d*cosh(d*x + c)^11 + 165*d*cosh(d*x + c)^9 + 30
*d*cosh(d*x + c)^7 - 70*d*cosh(d*x + c)^5 - 25*d*cosh(d*x + c)^3 + d*cosh(d
*x + c))*sinh(d*x + c)^3 + 3*d*cosh(d*x + c)^2 + (91*d*cosh(d*x + c)^12 + 1
98*d*cosh(d*x + c)^10 + 45*d*cosh(d*x + c)^8 - 140*d*cosh(d*x + c)^6 - 75*d
*cosh(d*x + c)^4 + 6*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^2 + 2*(7*d*cosh
(d*x + c)^13 + 18*d*cosh(d*x + c)^11 + 5*d*cosh(d*x + c)^9 - 20*d*cosh(d*x
+ c)^7 - 15*d*cosh(d*x + c)^5 + 2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*si
nh(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tanh^2(c + dx)\right)^3 \operatorname{csch}^3(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**3*csch(c + d*x)**3, x)
```

Giac [A] time = 1.76989, size = 379, normalized size = 2.49

$$15\left(a^3e^c - 6\,a^2be^c\right)e^{(-c)}\log\left(e^{(dx+c)} + 1\right) - 15\left(a^3e^c - 6\,a^2be^c\right)e^{(-c)}\log\left(\left|e^{(dx+c)} - 1\right|\right) - \frac{30\left(a^3e^{(3\,dx+3\,c)} + a^3e^{(dx+c)}\right)}{\left(e^{(2\,dx+2\,c)} - 1\right)^2} + \frac{4\left(45\,a^2be^{(9\,dx+9\,c)} \ldots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/30*(15*(a^3*e^c - 6*a^2*b*e^c)*e^(-c)*log(e^(d*x + c) + 1) - 15*(a^3*e^c
- 6*a^2*b*e^c)*e^(-c)*log(abs(e^(d*x + c) - 1)) - 30*(a^3*e^(3*d*x + 3*c) +
 a^3*e^(d*x + c))/(e^(2*d*x + 2*c) - 1)^2 + 4*(45*a^2*b*e^(9*d*x + 9*c) + 1
80*a^2*b*e^(7*d*x + 7*c) - 60*a*b^2*e^(7*d*x + 7*c) - 20*b^3*e^(7*d*x + 7*c
) + 270*a^2*b*e^(5*d*x + 5*c) - 120*a*b^2*e^(5*d*x + 5*c) + 8*b^3*e^(5*d*x
+ 5*c) + 180*a^2*b*e^(3*d*x + 3*c) - 60*a*b^2*e^(3*d*x + 3*c) - 20*b^3*e^(3
*d*x + 3*c) + 45*a^2*b*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^5)/d
```

3.24 $\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=98

$$\frac{a^2(a-3b)\coth(c+dx)}{d}-\frac{a^3\coth^3(c+dx)}{3d}-\frac{b^2(3a-b)\tanh^3(c+dx)}{3d}-\frac{3ab(a-b)\tanh(c+dx)}{d}-\frac{b^3\tanh^5(c+dx)}{5d}$$

```
[Out] (a^2*(a - 3*b)*Coth[c + d*x])/d - (a^3*Coth[c + d*x]^3)/(3*d) - (3*a*(a - b
)*b*Tanh[c + d*x])/d - ((3*a - b)*b^2*Tanh[c + d*x]^3)/(3*d) - (b^3*Tanh[c
+ d*x]^5)/(5*d)
```

Rubi [A] time = 0.0965718, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3663, 448}

$$\frac{a^2(a-3b)\coth(c+dx)}{d}-\frac{a^3\coth^3(c+dx)}{3d}-\frac{b^2(3a-b)\tanh^3(c+dx)}{3d}-\frac{3ab(a-b)\tanh(c+dx)}{d}-\frac{b^3\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (a^2*(a - 3*b)*Coth[c + d*x])/d - (a^3*Coth[c + d*x]^3)/(3*d) - (3*a*(a - b
)*b*Tanh[c + d*x])/d - ((3*a - b)*b^2*Tanh[c + d*x]^3)/(3*d) - (b^3*Tanh[c
+ d*x]^5)/(5*d)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)(a+bx^2)^3}{x^4}\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \left(-3a(a-b)b+\frac{a^3}{x^4}-\frac{a^2(a-3b)}{x^2}-(3a-b)b^2x^2-b^3x^4\right)dx,x,\tan\right.}{d}\\ &= \frac{a^2(a-3b)\coth(c+dx)}{d}-\frac{a^3\coth^3(c+dx)}{3d}-\frac{3a(a-b)b\tanh(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 1.21802, size = 87, normalized size = 0.89

$$\frac{b\tanh(c+dx)\left(-45a^2+b(15a+b)\operatorname{sech}^2(c+dx)+30ab-3b^2\operatorname{sech}^4(c+dx)+2b^2\right)-5a^2\coth(c+dx)\left(a\operatorname{csch}^2(c+a\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (-5*a^2*Coth[c + d*x]*(-2*a + 9*b + a*Csch[c + d*x]^2) + b*(-45*a^2 + 30*a*
b + 2*b^2 + b*(15*a + b)*Sech[c + d*x]^2 - 3*b^2*Sech[c + d*x]^4)*Tanh[c +
d*x])/(15*d)
```

Maple [A] time = 0.058, size = 136, normalized size = 1.4

$$\frac{1}{d}\left(a^3\left(\frac{2}{3}-\frac{(\operatorname{csch}(dx+c))^2}{3}\right)\coth(dx+c)+3a^2b\left(-\frac{1}{\cosh(dx+c)\sinh(dx+c)}-2\tanh(dx+c)\right)+3ab^2\left(2/3+1/3\ (\mathrm{s}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+3*a^2*b*(-1/sinh(d*x+c)/cosh(d
*x+c)-2*tanh(d*x+c))+3*a*b^2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+b^3*(-1/4*
sinh(d*x+c)/cosh(d*x+c)^5+1/4*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*t
anh(d*x+c)))
```

Maxima [B] time = 1.15839, size = 666, normalized size = 6.8

$$\frac{4}{15}b^3\left(\frac{5\,e^{(-2\,dx-2\,c)}}{d\left(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1\right)}-\frac{5\,e}{d\left(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 4/15*b^3*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) +
 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) - 5*e^
(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
- 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*e^(-6*d*x - 6*c
)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^
(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e
^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 1
0*c) + 1))) + 4*a*b^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*
d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*
x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 4/3*a^3*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-
2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*
d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + 12*a^2*b/(d*(e^
(-4*d*x - 4*c) - 1))
```

Fricas [B] time = 2.1397, size = 2385, normalized size = 24.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -8/15*((5*a^3 + 45*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^6 + 12*(5*a^3 +
15*a*b^2 + 4*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (5*a^3 + 45*a^2*b + 15*a*
b^2 + 7*b^3)*sinh(d*x + c)^6 + 2*(15*a^3 + 45*a^2*b - 15*a*b^2 - 13*b^3)*co
sh(d*x + c)^4 + (30*a^3 + 90*a^2*b - 30*a*b^2 - 26*b^3 + 15*(5*a^3 + 45*a^2
*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(5*(5*a^3 + 15*
a*b^2 + 4*b^3)*cosh(d*x + c)^3 + 4*(5*a^3 - 3*b^3)*cosh(d*x + c))*sinh(d*x
+ c)^3 + 50*a^3 - 90*a^2*b + 30*a*b^2 - 22*b^3 + (75*a^3 - 45*a^2*b - 15*a*
b^2 + 41*b^3)*cosh(d*x + c)^2 + (15*(5*a^3 + 45*a^2*b + 15*a*b^2 + 7*b^3)*c
osh(d*x + c)^4 + 75*a^3 - 45*a^2*b - 15*a*b^2 + 41*b^3 + 12*(15*a^3 + 45*a^
2*b - 15*a*b^2 - 13*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(3*(5*a^3 + 1
5*a*b^2 + 4*b^3)*cosh(d*x + c)^5 + 8*(5*a^3 - 3*b^3)*cosh(d*x + c)^3 + (25*
a^3 - 45*a*b^2 + 12*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^10
+ 10*d*cosh(d*x + c)*sinh(d*x + c)^9 + d*sinh(d*x + c)^10 + 2*d*cosh(d*x +
c)^8 + (45*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^8 + 8*(15*d*cosh(d*x + c)
^3 + 2*d*cosh(d*x + c))*sinh(d*x + c)^7 - 3*d*cosh(d*x + c)^6 + (210*d*cosh
(d*x + c)^4 + 56*d*cosh(d*x + c)^2 - 3*d)*sinh(d*x + c)^6 + 2*(126*d*cosh(d
*x + c)^5 + 56*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^5 - 8*d
*cosh(d*x + c)^4 + (210*d*cosh(d*x + c)^6 + 140*d*cosh(d*x + c)^4 - 45*d*co
sh(d*x + c)^2 - 8*d)*sinh(d*x + c)^4 + 4*(30*d*cosh(d*x + c)^7 + 28*d*cosh(
d*x + c)^5 - 5*d*cosh(d*x + c)^3 - 4*d*cosh(d*x + c))*sinh(d*x + c)^3 + 2*d
*cosh(d*x + c)^2 + (45*d*cosh(d*x + c)^8 + 56*d*cosh(d*x + c)^6 - 45*d*cosh
(d*x + c)^4 - 48*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^2 + 2*(5*d*cosh(d*x
 + c)^9 + 8*d*cosh(d*x + c)^7 - 3*d*cosh(d*x + c)^5 - 8*d*cosh(d*x + c)^3 -
 2*d*cosh(d*x + c))*sinh(d*x + c) + 6*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tanh^2(c + dx)\right)^3 \operatorname{csch}^4(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**3*csch(c + d*x)**4, x)
```

Giac [B] time = 1.78319, size = 347, normalized size = 3.54

$$-\frac{2\left(\frac{5\left(9a^2be^{(4dx+4c)}+6a^3e^{(2dx+2c)}-18a^2be^{(2dx+2c)}-2a^3+9a^2b\right)}{\left(e^{(2dx+2c)}-1\right)^3}-\frac{45a^2be^{(8dx+8c)}+180a^2be^{(6dx+6c)}-90ab^2e^{(6dx+6c)}-30b^3e^{(6dx+6c)}+270a^2be^{(4dx+4c)}}{}\right.}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -2/15*(5*(9*a^2*b*e^(4*d*x + 4*c) + 6*a^3*e^(2*d*x + 2*c) - 18*a^2*b*e^(2*d
*x + 2*c) - 2*a^3 + 9*a^2*b)/(e^(2*d*x + 2*c) - 1)^3 - (45*a^2*b*e^(8*d*x +
 8*c) + 180*a^2*b*e^(6*d*x + 6*c) - 90*a*b^2*e^(6*d*x + 6*c) - 30*b^3*e^(6*
d*x + 6*c) + 270*a^2*b*e^(4*d*x + 4*c) - 210*a*b^2*e^(4*d*x + 4*c) + 10*b^3
*e^(4*d*x + 4*c) + 180*a^2*b*e^(2*d*x + 2*c) - 150*a*b^2*e^(2*d*x + 2*c) -
10*b^3*e^(2*d*x + 2*c) + 45*a^2*b - 30*a*b^2 - 2*b^3)/(e^(2*d*x + 2*c) + 1)
^5)/d
```

3.25 $\int \frac{\sinh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=118

$$\frac{x(3a^2-6ab-b^2)}{8(a+b)^3}+\frac{a^{3/2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{d(a+b)^3}+\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d(a+b)}-\frac{(5a+b)\sinh(c+dx)\cosh(c+dx)}{8d(a+b)^2}$$

```
[Out] ((3*a^2 - 6*a*b - b^2)*x)/(8*(a + b)^3) + (a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[b]*
Tanh[c + d*x])/Sqrt[a]])/((a + b)^3*d) - ((5*a + b)*Cosh[c + d*x]*Sinh[c +
d*x])/(8*(a + b)^2*d) + (Cosh[c + d*x]^3*Sinh[c + d*x])/(4*(a + b)*d)
```

Rubi [A] time = 0.171706, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3663, 470, 527, 522, 206, 205}

$$\frac{x(3a^2-6ab-b^2)}{8(a+b)^3}+\frac{a^{3/2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{d(a+b)^3}+\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d(a+b)}-\frac{(5a+b)\sinh(c+dx)\cosh(c+dx)}{8d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((3*a^2 - 6*a*b - b^2)*x)/(8*(a + b)^3) + (a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[b]*
Tanh[c + d*x])/Sqrt[a]])/((a + b)^3*d) - ((5*a + b)*Cosh[c + d*x]*Sinh[c +
d*x])/(8*(a + b)^2*d) + (Cosh[c + d*x]^3*Sinh[c + d*x])/(4*(a + b)*d)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^3(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d} - \frac{\text{Subst}\left(\int \frac{a+(4a+b)x^2}{(1-x^2)^2(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{4(a+b)d}$$

$$= -\frac{(5a+b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d} - \frac{\text{Subst}\left(\int \frac{-a(3a-b)+b}{(1-x^2)(a}\right.}{8}$$

$$= -\frac{(5a+b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d} + \frac{(a^2b)\,\text{Subst}\left(\int \frac{1}{a+b}\right.}{(a}$$

$$= \frac{(3a^2-6ab-b^2)\,x}{8(a+b)^3} + \frac{a^{3/2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{(a+b)^3d} - \frac{(5a+b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d} +$$

Mathematica [A] time = 0.253554, size = 93, normalized size = 0.79

$$\frac{4(3a^2-6ab-b^2)(c+dx)+32a^{3/2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)+(a+b)^2\sinh(4(c+dx))-8a(a+b)\sinh(2(c+dx))}{32d(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (4*(3*a^2 - 6*a*b - b^2)*(c + d*x) + 32*a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan
h[c + d*x])/Sqrt[a]] - 8*a*(a + b)*Sinh[2*(c + d*x)] + (a + b)^2*Sinh[4*(c
+ d*x)])/(32*(a + b)^3*d)
```

Maple [B] time = 0.108, size = 865, normalized size = 7.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x)
```

```
[Out] -8/d/(32*a+32*b)/(tanh(1/2*d*x+1/2*c)+1)^4+32/d/(64*a+64*b)/(tanh(1/2*d*x+1
/2*c)+1)^3+1/8/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)+1)^2*a-3/8/d/(a+b)^2/(tanh(1/
2*d*x+1/2*c)+1)^2*b-3/8/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)+1)*a+1/8/d/(a+b)^2/(
tanh(1/2*d*x+1/2*c)+1)*b+3/8/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)+1)*a^2-3/4/d/
(a+b)^3*ln(tanh(1/2*d*x+1/2*c)+1)*a*b-1/8/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)+
1)*b^2-1/d*a^3*b/(a+b)^3/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2
)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/d*a^
2*b/(a+b)^3/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*
c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-1/d*a^2*b^2/(a+b)^3/(b*(a+b))^(1/2)
/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(
a+b))^(1/2)-a-2*b)*a)^(1/2))-1/d*a^3*b/(a+b)^3/(b*(a+b))^(1/2)/((2*(b*(a+b)
)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+
2*b)*a)^(1/2))-1/d*a^2*b/(a+b)^3/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan
(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/d*a^2*b^2/(a+
b)^3/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*
d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+8/d/(32*a+32*b)/(tanh(1/2*d
*x+1/2*c)-1)^4+32/d/(64*a+64*b)/(tanh(1/2*d*x+1/2*c)-1)^3-1/8/d/(a+b)^2/(ta
nh(1/2*d*x+1/2*c)-1)^2*a+3/8/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)^2*b-3/8/d/(a
+b)^2/(tanh(1/2*d*x+1/2*c)-1)*a+1/8/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)*b-3/8
/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)-1)*a^2+3/4/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*
c)-1)*a*b+1/8/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)-1)*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.31437, size = 5158, normalized size = 43.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/64*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x
 + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^8 + 8*(3*a^2 - 6*
a*b - b^2)*d*x*cosh(d*x + c)^4 - 8*(a^2 + a*b)*cosh(d*x + c)^6 + 4*(7*(a^2
+ 2*a*b + b^2)*cosh(d*x + c)^2 - 2*a^2 - 2*a*b)*sinh(d*x + c)^6 + 8*(7*(a^2
 + 2*a*b + b^2)*cosh(d*x + c)^3 - 6*(a^2 + a*b)*cosh(d*x + c))*sinh(d*x + c
)^5 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(3*a^2 - 6*a*b - b^2)*d
*x - 60*(a^2 + a*b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b +
b^2)*cosh(d*x + c)^5 + 4*(3*a^2 - 6*a*b - b^2)*d*x*cosh(d*x + c) - 20*(a^2
+ a*b)*cosh(d*x + c)^3)*sinh(d*x + c)^3 + 8*(a^2 + a*b)*cosh(d*x + c)^2 + 4
*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 12*(3*a^2 - 6*a*b - b^2)*d*x*cosh
```

```
(d*x + c)^2 - 30*(a^2 + a*b)*cosh(d*x + c)^4 + 2*a^2 + 2*a*b)*sinh(d*x + c)
^2 + 32*(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)^3*sinh(d*x + c) + 6*a*cosh(d
*x + c)^2*sinh(d*x + c)^2 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x
+ c)^4)*sqrt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*
b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^
4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^
2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)
*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*((a + b)*co
sh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x +
c)^2 + a - b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c
)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2
*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x
+ c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) - a^2 - 2*a*b - b^2
 + 8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 4*(3*a^2 - 6*a*b - b^2)*d*x*cos
h(d*x + c)^3 - 6*(a^2 + a*b)*cosh(d*x + c)^5 + 2*(a^2 + a*b)*cosh(d*x + c))
*sinh(d*x + c))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^4 + 4*(a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*d*sinh(d*x + c)^4), 1/64*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a
^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(
d*x + c)^8 + 8*(3*a^2 - 6*a*b - b^2)*d*x*cosh(d*x + c)^4 - 8*(a^2 + a*b)*co
sh(d*x + c)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 - 2*a^2 - 2*a*b)*s
inh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - 6*(a^2 + a*b)*c
osh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 +
 4*(3*a^2 - 6*a*b - b^2)*d*x - 60*(a^2 + a*b)*cosh(d*x + c)^2)*sinh(d*x + c
)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 4*(3*a^2 - 6*a*b - b^2)*d*
x*cosh(d*x + c) - 20*(a^2 + a*b)*cosh(d*x + c)^3)*sinh(d*x + c)^3 + 8*(a^2
+ a*b)*cosh(d*x + c)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 12*(3*a
^2 - 6*a*b - b^2)*d*x*cosh(d*x + c)^2 - 30*(a^2 + a*b)*cosh(d*x + c)^4 + 2*
a^2 + 2*a*b)*sinh(d*x + c)^2 + 64*(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)^3*
sinh(d*x + c) + 6*a*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*a*cosh(d*x + c)*sin
h(d*x + c)^3 + a*sinh(d*x + c)^4)*sqrt(a*b)*arctan(1/2*((a + b)*cosh(d*x +
c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a
- b)*sqrt(a*b)/(a*b)) - a^2 - 2*a*b - b^2 + 8*((a^2 + 2*a*b + b^2)*cosh(d*x
 + c)^7 + 4*(3*a^2 - 6*a*b - b^2)*d*x*cosh(d*x + c)^3 - 6*(a^2 + a*b)*cosh(
d*x + c)^5 + 2*(a^2 + a*b)*cosh(d*x + c))*sinh(d*x + c))/((a^3 + 3*a^2*b +
3*a*b^2 + b^3)*d*cosh(d*x + c)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh
(d*x + c)^3*sinh(d*x + c) + 6*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x +
c)^2*sinh(d*x + c)^2 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)*si
nh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*sinh(d*x + c)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.39497, size = 406, normalized size = 3.44

$$\frac{\frac{64\,a^2b\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\sqrt{ab}}+\frac{8\left(3\,a^2-6\,ab-b^2\right)dx}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{\left(18\,a^2e^{(4\,dx+4\,c)}-36\,abe^{(4\,dx+4\,c)}-6\,b^2e^{(4\,dx+4\,c)}-8\,a^2e^{(2\,dx+2\,c)}-8\,abe^{(2\,dx+2\,c)}+a^2+2\,ab+b^2\right)e^{(}}{a^3e^{(4\,c)}+3\,a^2be^{(4\,c)}+3\,ab^2e^{(4\,c)}+b^3e^{(4\,c)}}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/64*(64*a^2*b*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/s
qrt(a*b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b)) + 8*(3*a^2 - 6*a*b -
b^2)*d*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (18*a^2*e^(4*d*x + 4*c) - 36*a*b
*e^(4*d*x + 4*c) - 6*b^2*e^(4*d*x + 4*c) - 8*a^2*e^(2*d*x + 2*c) - 8*a*b*e^
(2*d*x + 2*c) + a^2 + 2*a*b + b^2)*e^(-4*d*x)/(a^3*e^(4*c) + 3*a^2*b*e^(4*c
) + 3*a*b^2*e^(4*c) + b^3*e^(4*c)) + (a*e^(4*d*x + 20*c) + b*e^(4*d*x + 20*
c) - 8*a*e^(2*d*x + 18*c))/(a^2*e^(16*c) + 2*a*b*e^(16*c) + b^2*e^(16*c)))/
d
```

3.26 $\int \frac{\sinh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=75

$$\frac{\cosh^3(c+dx)}{3d(a+b)}-\frac{a\cosh(c+dx)}{d(a+b)^2}+\frac{a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{d(a+b)^{5/2}}$$

```
[Out] (a*Sqrt[b]*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/((a + b)^(5/2)*d)
- (a*Cosh[c + d*x])/((a + b)^2*d) + Cosh[c + d*x]^3/(3*(a + b)*d)
```

Rubi [A] time = 0.128102, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3664, 453, 325, 208}

$$\frac{\cosh^3(c+dx)}{3d(a+b)}-\frac{a\cosh(c+dx)}{d(a+b)^2}+\frac{a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{d(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a*Sqrt[b]*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/((a + b)^(5/2)*d)
- (a*Cosh[c + d*x])/((a + b)^2*d) + Cosh[c + d*x]^3/(3*(a + b)*d)
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4(a+b-bx^2)}\,dx, x, \text{sech}(c+dx)\right)}{d}$$
$$= \frac{\cosh^3(c+dx)}{3(a+b)d} + \frac{a\,\text{Subst}\left(\int \frac{1}{x^2(a+b-bx^2)}\,dx, x, \text{sech}(c+dx)\right)}{(a+b)d}$$
$$= -\frac{a\cosh(c+dx)}{(a+b)^2d} + \frac{\cosh^3(c+dx)}{3(a+b)d} + \frac{(ab)\,\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \text{sech}(c+dx)\right)}{(a+b)^2d}$$
$$= \frac{a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}d} - \frac{a\cosh(c+dx)}{(a+b)^2d} + \frac{\cosh^3(c+dx)}{3(a+b)d}$$

Mathematica [C] time = 0.538918, size = 135, normalized size = 1.8

$$\frac{(a+b)^{3/2}\cosh(3(c+dx)) - 3(3a-b)\sqrt{a+b}\cosh(c+dx) + 12ia\sqrt{b}\left(\tan^{-1}\left(\frac{-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)-i\sqrt{a+b}}{\sqrt{b}}\right) + \tan^{-1}\left(\frac{\sqrt{a}\tanh\left(\frac{1}{2}(c+\right.}{\sqrt{l}}\right.\right.}{12d(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((12*I)*a*Sqrt[b]*(ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2]])/Sq
rt[b]] + ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2]])/Sqrt[b]]) -
3*(3*a - b)*Sqrt[a + b]*Cosh[c + d*x] + (a + b)^(3/2)*Cosh[3*(c + d*x)])/(1
2*(a + b)^(5/2)*d)
```

Maple [B] time = 0.069, size = 202, normalized size = 2.7

$$\frac{1}{d}\left(-8\frac{1}{(16a+16b)(\tanh(1/2dx+c/2)+1)^2} + \frac{16}{48a+48b}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3} - \frac{a-b}{2(a+b)^2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x)
```

```
[Out] 1/d*(-8/(16*a+16*b)/(tanh(1/2*d*x+1/2*c)+1)^2+16/3/(tanh(1/2*d*x+1/2*c)+1)^
3/(16*a+16*b)-1/2*(a-b)/(a+b)^2/(tanh(1/2*d*x+1/2*c)+1)+a*b/(a+b)^2/(a*b+b^
2)^(1/2)*arctanh(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2))-1
6/3/(tanh(1/2*d*x+1/2*c)-1)^3/(16*a+16*b)-8/(16*a+16*b)/(tanh(1/2*d*x+1/2*c
)-1)^2-1/2/(a+b)^2*(-a+b)/(tanh(1/2*d*x+1/2*c)-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(\left(ae^{(6c)}+be^{(6c)}\right)e^{(6dx)} - 3\left(3ae^{(4c)}-be^{(4c)}\right)e^{(4dx)} - 3\left(3ae^{(2c)}-be^{(2c)}\right)e^{(2dx)} + a + b\right)e^{(-3dx)}}{24\left(a^2de^{(3c)} + 2abde^{(3c)} + b^2de^{(3c)}\right)} - \frac{1}{8}\int \frac{}{a^3+3a^2b+3ab^2+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/24*((a*e^(6*c) + b*e^(6*c))*e^(6*d*x) - 3*(3*a*e^(4*c) - b*e^(4*c))*e^(4*
d*x) - 3*(3*a*e^(2*c) - b*e^(2*c))*e^(2*d*x) + a + b)*e^(-3*d*x)/(a^2*d*e^(
3*c) + 2*a*b*d*e^(3*c) + b^2*d*e^(3*c)) - 1/8*integrate(16*(a*b*e^(3*d*x +
3*c) - a*b*e^(d*x + c))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3 + (a^3*e^(4*c) + 3*a
^2*b*e^(4*c) + 3*a*b^2*e^(4*c) + b^3*e^(4*c))*e^(4*d*x) + 2*(a^3*e^(2*c) +
a^2*b*e^(2*c) - a*b^2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 2.45116, size = 3729, normalized size = 49.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/24*((a + b)*cosh(d*x + c)^6 + 6*(a + b)*cosh(d*x + c)*sinh(d*x + c)^5 +
(a + b)*sinh(d*x + c)^6 - 3*(3*a - b)*cosh(d*x + c)^4 + 3*(5*(a + b)*cosh(d
*x + c)^2 - 3*a + b)*sinh(d*x + c)^4 + 4*(5*(a + b)*cosh(d*x + c)^3 - 3*(3*
a - b)*cosh(d*x + c))*sinh(d*x + c)^3 - 3*(3*a - b)*cosh(d*x + c)^2 + 3*(5*
(a + b)*cosh(d*x + c)^4 - 6*(3*a - b)*cosh(d*x + c)^2 - 3*a + b)*sinh(d*x +
 c)^2 + 12*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)^2*sinh(d*x + c) + 3*a*cos
h(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3)*sqrt(b/(a + b))*log(((a + b
)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(
d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a
 + 3*b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + 3*b)*cosh(d*x +
 c))*sinh(d*x + c) + 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*s
inh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a + b)*cosh(d*x + c) + (3*(a +
b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))*sqrt(b/(a + b)) + a + b)/((a + b
)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(
d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a -
 b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*s
inh(d*x + c) + a + b)) + 6*((a + b)*cosh(d*x + c)^5 - 2*(3*a - b)*cosh(d*x
+ c)^3 - (3*a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)/((a^2 + 2*a*b + b^
2)*d*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2*sinh(d*x + c
) + 3*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)*sinh(d*x + c)^2 + (a^2 + 2*a*b +
b^2)*d*sinh(d*x + c)^3), 1/24*((a + b)*cosh(d*x + c)^6 + 6*(a + b)*cosh(d*x
 + c)*sinh(d*x + c)^5 + (a + b)*sinh(d*x + c)^6 - 3*(3*a - b)*cosh(d*x + c)
^4 + 3*(5*(a + b)*cosh(d*x + c)^2 - 3*a + b)*sinh(d*x + c)^4 + 4*(5*(a + b)
*cosh(d*x + c)^3 - 3*(3*a - b)*cosh(d*x + c))*sinh(d*x + c)^3 - 3*(3*a - b)
*cosh(d*x + c)^2 + 3*(5*(a + b)*cosh(d*x + c)^4 - 6*(3*a - b)*cosh(d*x + c)
^2 - 3*a + b)*sinh(d*x + c)^2 + 24*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)^2
*sinh(d*x + c) + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3)*sqr
t(-b/(a + b))*arctan(1/2*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)
*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a - 3*b)*cosh(d*x + c) + (3*(
a + b)*cosh(d*x + c)^2 + a - 3*b)*sinh(d*x + c))*sqrt(-b/(a + b))/b) - 24*(
a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)^2*sinh(d*x + c) + 3*a*cosh(d*x + c)*s
inh(d*x + c)^2 + a*sinh(d*x + c)^3)*sqrt(-b/(a + b))*arctan(1/2*((a + b)*co
sh(d*x + c) + (a + b)*sinh(d*x + c))*sqrt(-b/(a + b))/b) + 6*((a + b)*cosh(
d*x + c)^5 - 2*(3*a - b)*cosh(d*x + c)^3 - (3*a - b)*cosh(d*x + c))*sinh(d*
x + c) + a + b)/((a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + b
^2)*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)
*sinh(d*x + c)^2 + (a^2 + 2*a*b + b^2)*d*sinh(d*x + c)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(sinh(c + d*x)**3/(a + b*tanh(c + d*x)**2), x)
```

Giac [C] time = 1.88798, size = 5603, normalized size = 74.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/24*(12*(3*(2*a*b + sqrt(-a*b)*(a - b))*cos(1/2*real_part(arccos(-a/(a +
b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*s
in(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) - (2*a*b + sqrt(-a*b)*(a
- b))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_pa
rt(arccos(-a/(a + b) + b/(a + b))))^3 - 9*(2*a*b + sqrt(-a*b)*(a - b))*cos(
1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(
-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)
)))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(2*a*b + sqrt(-
a*b)*(a - b))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2
*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/
(a + b) + b/(a + b)))) + 9*(2*a*b + sqrt(-a*b)*(a - b))*cos(1/2*real_part(a
rccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/
(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag
_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(2*a*b + sqrt(-a*b)*(a - b))*c
osh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos
(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a +
b))))^2 - 3*(2*a*b + sqrt(-a*b)*(a - b))*cos(1/2*real_part(arccos(-a/(a + b
) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(
1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (2*a*b + sqrt(-a*b)*(a -
 b))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_par
t(arccos(-a/(a + b) + b/(a + b))))^3 - (2*a*b + sqrt(-a*b)*(a - b))*cosh(1/
2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a
 + b) + b/(a + b)))) + (2*a*b + sqrt(-a*b)*(a - b))*sin(1/2*real_part(arcco
s(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b
)))))*arctan((((a^3 + 3*a^2*b + 3*a*b^2 + b^3)/(a^3*e^(4*c) + 3*a^2*b*e^(4*
c) + 3*a*b^2*e^(4*c) + b^3*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b))
) + e^(d*x))/(((a^3 + 3*a^2*b + 3*a*b^2 + b^3)/(a^3*e^(4*c) + 3*a^2*b*e^(4*
c) + 3*a*b^2*e^(4*c) + b^3*e^(4*c)))^(1/4)*sin(1/2*arccos(-(a - b)/(a + b))
)))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 12*(3*(2*a*b + sqrt(-a*b)*(a - b))*co
s(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arcco
s(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a +
b)))) - (2*a*b + sqrt(-a*b)*(a - b))*cosh(1/2*imag_part(arccos(-a/(a + b) +
 b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3 - 9*(2
*a*b + sqrt(-a*b)*(a - b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))
))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_par
t(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b
/(a + b)))) + 3*(2*a*b + sqrt(-a*b)*(a - b))*cosh(1/2*imag_part(arccos(-a/(
```

```
a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^
3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(2*a*b + sqrt(-a*
b)*(a - b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*i
mag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a +
b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3
*(2*a*b + sqrt(-a*b)*(a - b))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a +
 b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_p
art(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(2*a*b + sqrt(-a*b)*(a - b))*cos
(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(
-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))
))^3 + (2*a*b + sqrt(-a*b)*(a - b))*sin(1/2*real_part(arccos(-a/(a + b) + b
/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 - (2*a*
b + sqrt(-a*b)*(a - b))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))
*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) + (2*a*b + sqrt(-a*b)*(
a - b))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_pa
rt(arccos(-a/(a + b) + b/(a + b)))))*arctan(-(((a^3 + 3*a^2*b + 3*a*b^2 + b
^3)/(a^3*e^(4*c) + 3*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c) + b^3*e^(4*c)))^(1/4)*
cos(1/2*arccos(-(a - b)/(a + b))) - e^(d*x))/(((a^3 + 3*a^2*b + 3*a*b^2 + b
^3)/(a^3*e^(4*c) + 3*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c) + b^3*e^(4*c)))^(1/4)*
sin(1/2*arccos(-(a - b)/(a + b)))))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + (9*a*
e^(2*d*x + 2*c) - 3*b*e^(2*d*x + 2*c) - a - b)*e^(-3*d*x)/(a^2*e^(3*c) + 2*
a*b*e^(3*c) + b^2*e^(3*c)) + 6*((2*a*b + sqrt(-a*b)*(a - b))*cos(1/2*real_p
art(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b)
 + b/(a + b))))^3 - 3*(2*a*b + sqrt(-a*b)*(a - b))*cos(1/2*real_part(arccos
(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)
)))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(2*a*b + sqr
t(-a*b)*(a - b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(
1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(
-a/(a + b) + b/(a + b)))) + 9*(2*a*b + sqrt(-a*b)*(a - b))*cos(1/2*real_par
t(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b
/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2
*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(2*a*b + sqrt(-a*b)*(a - b)
)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(a
rccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a
 + b))))^2 - 9*(2*a*b + sqrt(-a*b)*(a - b))*cos(1/2*real_part(arccos(-a/(a
+ b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin
(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos
(-a/(a + b) + b/(a + b))))^2 - (2*a*b + sqrt(-a*b)*(a - b))*cos(1/2*real_pa
rt(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b)
+ b/(a + b))))^3 + 3*(2*a*b + sqrt(-a*b)*(a - b))*cos(1/2*real_part(arccos(
-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))
)^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 - (2*a*b + sqrt(-
a*b)*(a - b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*i
mag_part(arccos(-a/(a + b) + b/(a + b)))) + (2*a*b + sqrt(-a*b)*(a - b))*co
s(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(
-a/(a + b) + b/(a + b)))))*log(2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)/(a^3*e^(4
*c) + 3*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c) + b^3*e^(4*c)))^(1/4)*cos(1/2*arcco
s(-(a - b)/(a + b)))*e^(d*x) + sqrt((a^3 + 3*a^2*b + 3*a*b^2 + b^3)/(a^3*e^
(4*c) + 3*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c) + b^3*e^(4*c))) + e^(2*d*x))/(a^3
 + 3*a^2*b + 3*a*b^2 + b^3) - 6*((2*a*b + sqrt(-a*b)*(a - b))*cos(1/2*real_
part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b
) + b/(a + b))))^3 - 3*(2*a*b + sqrt(-a*b)*(a - b))*cos(1/2*real_part(arcco
s(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b
))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(2*a*b + sq
rt(-a*b)*(a - b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh
(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos
(-a/(a + b) + b/(a + b)))) + 9*(2*a*b + sqrt(-a*b)*(a - b))*cos(1/2*real_pa
rt(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) +
b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/
```

```
2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(2*a*b + sqrt(-a*b)*(a - b
))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(
arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(
a + b))))^2 - 9*(2*a*b + sqrt(-a*b)*(a - b))*cos(1/2*real_part(arccos(-a/(a
 + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*si
n(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arcco
s(-a/(a + b) + b/(a + b))))^2 - (2*a*b + sqrt(-a*b)*(a - b))*cos(1/2*real_p
art(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b)
 + b/(a + b))))^3 + 3*(2*a*b + sqrt(-a*b)*(a - b))*cos(1/2*real_part(arccos
(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))
))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 - (2*a*b + sqrt(
-a*b)*(a - b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*
imag_part(arccos(-a/(a + b) + b/(a + b)))) + (2*a*b + sqrt(-a*b)*(a - b))*c
os(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos
(-a/(a + b) + b/(a + b)))))*log(-2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)/(a^3*e^
(4*c) + 3*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c) + b^3*e^(4*c)))^(1/4)*cos(1/2*arc
cos(-(a - b)/(a + b)))*e^(d*x) + sqrt((a^3 + 3*a^2*b + 3*a*b^2 + b^3)/(a^3*
e^(4*c) + 3*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c) + b^3*e^(4*c))) + e^(2*d*x))/(a
^3 + 3*a^2*b + 3*a*b^2 + b^3) - (a^2*e^(3*d*x + 24*c) + 2*a*b*e^(3*d*x + 24
*c) + b^2*e^(3*d*x + 24*c) - 9*a^2*e^(d*x + 22*c) - 6*a*b*e^(d*x + 22*c) +
3*b^2*e^(d*x + 22*c))/(a^3*e^(21*c) + 3*a^2*b*e^(21*c) + 3*a*b^2*e^(21*c) +
 b^3*e^(21*c)))/d
```

3.27 $\int \frac{\sinh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=78

$$-\frac{\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{d(a+b)^2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d(a+b)}-\frac{x(a-b)}{2(a+b)^2}$$

```
[Out] -((a - b)*x)/(2*(a + b)^2) - (Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x]
)/Sqrt[a]])/((a + b)^2*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*(a + b)*d)
```

Rubi [A] time = 0.104404, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3663, 471, 522, 206, 205}

$$-\frac{\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{d(a+b)^2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d(a+b)}-\frac{x(a-b)}{2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -((a - b)*x)/(2*(a + b)^2) - (Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x]
)/Sqrt[a]])/((a + b)^2*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*(a + b)*d)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)^2\left(a+bx^2\right)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d} - \frac{\text{Subst}\left(\int \frac{a-bx^2}{\left(1-x^2\right)\left(a+bx^2\right)}\,dx, x, \tanh(c+dx)\right)}{2(a+b)d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d} - \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{2(a+b)^2d} - \frac{(ab)\,\text{Subst}\left(\int \frac{1}{a+b \ldots}\right.}{(a \ldots}$$

$$= -\frac{(a-b)x}{2(a+b)^2} - \frac{\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{(a+b)^2d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d}$$

Mathematica [A] time = 0.142357, size = 67, normalized size = 0.86

$$\frac{-2(a-b)(c+dx)+(a+b)\sinh(2(c+dx))-4\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{4d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (-2*(a - b)*(c + d*x) - 4*Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sq
rt[a]] + (a + b)*Sinh[2*(c + d*x)])/(4*(a + b)^2*d)
```

Maple [B] time = 0.076, size = 605, normalized size = 7.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x)
```

```
[Out] -4/d/(8*a+8*b)/(tanh(1/2*d*x+1/2*c)+1)^2+8/d/(16*a+16*b)/(tanh(1/2*d*x+1/2*
c)+1)-1/2/d/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)+1)*a+1/2/d/(a+b)^2*ln(tanh(1/2*d
*x+1/2*c)+1)*b+1/d*a^2*b/(a+b)^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)
*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)
)-1/d*a*b/(a+b)^2/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*
x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/d*a*b^2/(a+b)^2/(b*(a+b))^(
1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*
(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/d*a^2*b/(a+b)^2/(b*(a+b))^(1/2)/((2*(b*(
a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2
)+a+2*b)*a)^(1/2))+1/d*a*b/(a+b)^2/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arct
an(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/d*a*b^2/(a+
b)^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*
d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+4/d/(8*a+8*b)/(tanh(1/2*d*x
```

```
+1/2*c)-1)^2+8/d/(16*a+16*b)/(tanh(1/2*d*x+1/2*c)-1)+1/2/d/(a+b)^2*ln(tanh(
1/2*d*x+1/2*c)-1)*a-1/2/d/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)-1)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 2.44108, size = 2475, normalized size = 31.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")

[Out] [-1/8*(4*(a - b)*d*x*cosh(d*x + c)^2 - (a + b)*cosh(d*x + c)^4 - 4*(a + b)*
cosh(d*x + c)*sinh(d*x + c)^3 - (a + b)*sinh(d*x + c)^4 + 2*(2*(a - b)*d*x
- 3*(a + b)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 4*sqrt(-a*b)*(cosh(d*x + c)^
2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)*log(((a^2 + 2*a*b + b^
2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 +
(a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*
(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*
a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x +
 c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*s
inh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b))/((a + b)*cosh(d
*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)
^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh
(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x
+ c) + a + b)) + 4*(2*(a - b)*d*x*cosh(d*x + c) - (a + b)*cosh(d*x + c)^3)*
sinh(d*x + c) + a + b)/((a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2 + 2*(a^2 + 2*
a*b + b^2)*d*cosh(d*x + c)*sinh(d*x + c) + (a^2 + 2*a*b + b^2)*d*sinh(d*x +
 c)^2), -1/8*(4*(a - b)*d*x*cosh(d*x + c)^2 - (a + b)*cosh(d*x + c)^4 - 4*(
a + b)*cosh(d*x + c)*sinh(d*x + c)^3 - (a + b)*sinh(d*x + c)^4 + 2*(2*(a -
b)*d*x - 3*(a + b)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*sqrt(a*b)*(cosh(d*x
 + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)*arctan(1/2*((a +
 b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(
d*x + c)^2 + a - b)*sqrt(a*b)/(a*b)) + 4*(2*(a - b)*d*x*cosh(d*x + c) - (a
+ b)*cosh(d*x + c)^3)*sinh(d*x + c) + a + b)/((a^2 + 2*a*b + b^2)*d*cosh(d*
x + c)^2 + 2*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)*sinh(d*x + c) + (a^2 + 2*a
*b + b^2)*d*sinh(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(sinh(c + d*x)**2/(a + b*tanh(c + d*x)**2), x)
```

Giac [B] time = 1.65799, size = 238, normalized size = 3.05

$$-\frac{\frac{4\,(a-b)dx}{a^2+2\,ab+b^2}+\frac{8\,ab\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\left(a^2+2\,ab+b^2\right)\sqrt{ab}}-\frac{\left(2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}-a-b\right)e^{(-2\,dx)}}{a^2e^{(2\,c)}+2\,abe^{(2\,c)}+b^2e^{(2\,c)}}-\frac{e^{(2\,dx+8\,c)}}{ae^{(6\,c)}+be^{(6\,c)}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/8*(4*(a - b)*d*x/(a^2 + 2*a*b + b^2) + 8*a*b*arctan(1/2*(a*e^(2*d*x + 2*
c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b))
- (2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) - a - b)*e^(-2*d*x)/(a^2*e^(2*
c) + 2*a*b*e^(2*c) + b^2*e^(2*c)) - e^(2*d*x + 8*c)/(a*e^(6*c) + b*e^(6*c))
)/d
```

3.28 $\int \frac{\sinh(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=53

$$\frac{\cosh(c+dx)}{d(a+b)}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{d(a+b)^{3/2}}$$

```
[Out] -((Sqrt[b]*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/((a + b)^(3/2)*d))
 + Cosh[c + d*x]/((a + b)*d)
```

Rubi [A] time = 0.0626703, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3664, 325, 208}

$$\frac{\cosh(c+dx)}{d(a+b)}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{d(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -((Sqrt[b]*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/((a + b)^(3/2)*d))
 + Cosh[c + d*x]/((a + b)*d)
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2(a+b-bx^2)}\,dx, x, \text{sech}(c+dx)\right)}{d}$$
$$= \frac{\cosh(c+dx)}{(a+b)d} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \text{sech}(c+dx)\right)}{(a+b)d}$$
$$= -\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}d} + \frac{\cosh(c+dx)}{(a+b)d}$$

Mathematica [C] time = 0.242032, size = 107, normalized size = 2.02

$$\frac{\sqrt{a+b}\cosh(c+dx) - i\sqrt{b}\left(\tan^{-1}\left(\frac{-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)-i\sqrt{a+b}}{\sqrt{b}}\right) + \tan^{-1}\left(\frac{\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)-i\sqrt{a+b}}{\sqrt{b}}\right)\right)}{d(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((-I)*Sqrt[b]*(ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b
]] + ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]]) + Sqrt
[a + b]*Cosh[c + d*x])/((a + b)^(3/2)*d)
```

Maple [B] time = 0.055, size = 104, normalized size = 2.

$$\frac{1}{d}\left(-\frac{b}{a+b}\text{Artanh}\left(\frac{1}{4}\left(2\left(\tanh(1/2\,dx+c/2)\right)^2a+2\,a+4\,b\right)\frac{1}{\sqrt{ab+b^2}}\right)\frac{1}{\sqrt{ab+b^2}} - 4\,\frac{1}{(4\,a+4\,b)\left(\tanh(1/2\,dx+c/2)-1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a+b*tanh(d*x+c)^2),x)
```

```
[Out] 1/d*(-b/(a+b)/(a*b+b^2)^(1/2)*arctanh(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*
b)/(a*b+b^2)^(1/2))-4/(4*a+4*b)/(tanh(1/2*d*x+1/2*c)-1)+4/(4*a+4*b)/(tanh(1
/2*d*x+1/2*c)+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(e^{(2\,dx+2\,c)}+1\right)e^{(-dx)}}{2\,(ade^c+bde^c)} + \frac{1}{2}\int \frac{4\left(be^{(3\,dx+3\,c)}-be^{(dx+c)}\right)}{a^2+2\,ab+b^2+\left(a^2e^{(4\,c)}+2\,abe^{(4\,c)}+b^2e^{(4\,c)}\right)e^{(4\,dx)}+2\left(a^2e^{(2\,c)}-b^2e^{(2\,c)}\right)e^{(2\,dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*d*x + 2*c) + 1)*e^(-d*x)/(a*d*e^c + b*d*e^c) + 1/2*integrate(4*(b
*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^2 + 2*a*b + b^2 + (a^2*e^(4*c) + 2*a*b
*e^(4*c) + b^2*e^(4*c))*e^(4*d*x) + 2*(a^2*e^(2*c) - b^2*e^(2*c))*e^(2*d*x)
```

```
), x)
```

Fricas [B] time = 2.20077, size = 1906, normalized size = 35.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b/(a + b))*(cosh(d*x + c) + sinh(d*x + c))*log(((a + b)*cosh(d*x
 + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4
 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a + 3*b)*si
nh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + 3*b)*cosh(d*x + c))*sinh(
d*x + c) - 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x +
c)^2 + (a + b)*sinh(d*x + c)^3 + (a + b)*cosh(d*x + c) + (3*(a + b)*cosh(d*
x + c)^2 + a + b)*sinh(d*x + c))*sqrt(b/(a + b)) + a + b)/((a + b)*cosh(d*x
 + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4
 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d
*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x +
c) + a + b)) + cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
 c)^2 + 1)/((a + b)*d*cosh(d*x + c) + (a + b)*d*sinh(d*x + c)), -1/2*(2*sqr
t(-b/(a + b))*(cosh(d*x + c) + sinh(d*x + c))*arctan(1/2*((a + b)*cosh(d*x
+ c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3
+ (a - 3*b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a - 3*b)*sinh(d*x
+ c))*sqrt(-b/(a + b))/b) - 2*sqrt(-b/(a + b))*(cosh(d*x + c) + sinh(d*x +
c))*arctan(1/2*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x + c))*sqrt(-b/(a +
 b))/b) - cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) - sinh(d*x + c)^2
 - 1)/((a + b)*d*cosh(d*x + c) + (a + b)*d*sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(sinh(c + d*x)/(a + b*tanh(c + d*x)**2), x)
```

Giac [C] time = 1.55928, size = 6017, normalized size = 113.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/4*(2*(3*(a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*cos(1/2*real_part
(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) +
b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) - (a^2*b*
```

```
e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a +
b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3 -
9*(a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*cos(1/2*real_part(arccos(
-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b
))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_par
t(arccos(-a/(a + b) + b/(a + b)))) + 3*(a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b
^3*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*r
eal_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a
 + b) + b/(a + b)))) + 9*(a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*co
s(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arcco
s(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)
)))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(a^2*b*e^(4*c
) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b
/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*i
mag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(a^2*b*e^(4*c) + 2*a*b^2*e^
(4*c) + b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*s
in(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos
(-a/(a + b) + b/(a + b))))^3 + (a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*
c))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part
(arccos(-a/(a + b) + b/(a + b))))^3 - (a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^
3*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real
_part(arccos(-a/(a + b) + b/(a + b)))) + (a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) +
 b^3*e^(4*c))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*i
mag_part(arccos(-a/(a + b) + b/(a + b)))))*arctan((((a^2 + 2*a*b + b^2)/(a^
2*e^(4*c) + 2*a*b*e^(4*c) + b^2*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a
+ b))) + e^(d*x))/(((a^2 + 2*a*b + b^2)/(a^2*e^(4*c) + 2*a*b*e^(4*c) + b^2*
e^(4*c)))^(1/4)*sin(1/2*arccos(-(a - b)/(a + b)))))/(2*(a*e^(2*c) + b*e^(2*
c))^2*a*b + (a^2*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b)*abs(-a*e^(2*c) - b*e^(2*
c))) + 2*(3*(a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*cos(1/2*real_pa
rt(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b)
+ b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) - (a^2*
b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a
+ b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3
- 9*(a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*cos(1/2*real_part(arcco
s(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a +
 b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_p
art(arccos(-a/(a + b) + b/(a + b)))) + 3*(a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) +
 b^3*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2
*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/
(a + b) + b/(a + b)))) + 9*(a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*
cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arc
cos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a +
b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(a^2*b*e^(4
*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) +
 b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2
*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(a^2*b*e^(4*c) + 2*a*b^2*
e^(4*c) + b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2
*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arcc
os(-a/(a + b) + b/(a + b))))^3 + (a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(
4*c))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_pa
rt(arccos(-a/(a + b) + b/(a + b))))^3 - (a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) +
b^3*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*re
al_part(arccos(-a/(a + b) + b/(a + b)))) + (a^2*b*e^(4*c) + 2*a*b^2*e^(4*c)
 + b^3*e^(4*c))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2
*imag_part(arccos(-a/(a + b) + b/(a + b)))))*arctan(-(((a^2 + 2*a*b + b^2)/
(a^2*e^(4*c) + 2*a*b*e^(4*c) + b^2*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/
(a + b))) - e^(d*x))/(((a^2 + 2*a*b + b^2)/(a^2*e^(4*c) + 2*a*b*e^(4*c) + b
^2*e^(4*c)))^(1/4)*sin(1/2*arccos(-(a - b)/(a + b)))))/(2*(a*e^(2*c) + b*e^
(2*c))^2*a*b + (a^2*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b)*abs(-a*e^(2*c) - b*e^
```

(2*c))) + ((a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*cos(1/2*real_par
t(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) +
 b/(a + b))))^3 - 3*(a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*cos(1/2
*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a
 + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2
 - 3*(a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*cos(1/2*real_part(arcc
os(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a
+ b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(a^2*b*e^
(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b)
+ b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/
2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a
/(a + b) + b/(a + b)))) + 3*(a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))
*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(ar
ccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a
+ b))))^2 - 9*(a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*cos(1/2*real_
part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b)
+ b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/
2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - (a^2*b*e^(4*c) + 2*a*b^2*e
^(4*c) + b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*
sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + 3*(a^2*b*e^(4*c) +
2*a*b^2*e^(4*c) + b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a +
 b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_p
art(arccos(-a/(a + b) + b/(a + b))))^3 - (a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) +
 b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*i
mag_part(arccos(-a/(a + b) + b/(a + b)))) + (a^2*b*e^(4*c) + 2*a*b^2*e^(4*c
) + b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/
2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*log(2*((a^2 + 2*a*b + b^2)/(a
^2*e^(4*c) + 2*a*b*e^(4*c) + b^2*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a
 + b)))*e^(d*x) + sqrt((a^2 + 2*a*b + b^2)/(a^2*e^(4*c) + 2*a*b*e^(4*c) + b
^2*e^(4*c))) + e^(2*d*x))/(2*(a*e^(2*c) + b*e^(2*c))^2*a*b + (a^2*e^(2*c) -
 b^2*e^(2*c))*sqrt(-a*b)*abs(-a*e^(2*c) - b*e^(2*c))) - ((a^2*b*e^(4*c) + 2
*a*b^2*e^(4*c) + b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a +
b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 - 3*(a^2*b*e^
(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b)
+ b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/
2*real_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(a^2*b*e^(4*c) + 2*a*b^2
*e^(4*c) + b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^
3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(
arccos(-a/(a + b) + b/(a + b)))) + 9*(a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3
*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_
part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b)
 + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(
a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/
(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))
)*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 9*(a^2*b*e^(4*c)
+ 2*a*b^2*e^(4*c) + b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a
 + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_p
art(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b)
 + b/(a + b))))^2 - (a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*cos(1/2
*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/
(a + b) + b/(a + b))))^3 + 3*(a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c)
)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arcc
os(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a
+ b))))^3 - (a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*cos(1/2*real_pa
rt(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) +
b/(a + b)))) + (a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*cos(1/2*real
_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b)
 + b/(a + b)))))*log(-2*((a^2 + 2*a*b + b^2)/(a^2*e^(4*c) + 2*a*b*e^(4*c) +
 b^2*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b)))*e^(d*x) + sqrt((a^2

```
+ 2*a*b + b^2)/(a^2*e^(4*c) + 2*a*b*e^(4*c) + b^2*e^(4*c))) + e^(2*d*x))/(2
*(a*e^(2*c) + b*e^(2*c))^2*a*b + (a^2*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b)*abs
(-a*e^(2*c) - b*e^(2*c))) + 2*e^(d*x + 6*c)/(a*e^(5*c) + b*e^(5*c)) + 2*e^(
-d*x)/(a*e^c + b*e^c))/d
```

3.29 $\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=55

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{ad\sqrt{a+b}}-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}$$

```
[Out] -(ArcTanh[Cosh[c + d*x]]/(a*d)) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sech[c + d*x])/
Sqrt[a + b]])/(a*Sqrt[a + b]*d)
```

Rubi [A] time = 0.078118, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3664, 391, 207, 208}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{ad\sqrt{a+b}}-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -(ArcTanh[Cosh[c + d*x]]/(a*d)) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sech[c + d*x])/
Sqrt[a + b]])/(a*Sqrt[a + b]*d)
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(-1+x^2)(a+b-bx^2)}\,dx, x, \operatorname{sech}(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \operatorname{sech}(c+dx)\right)}{ad} + \frac{b\operatorname{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \operatorname{sech}(c+dx)\right)}{ad}$$

$$= -\frac{\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}d}$$

Mathematica [C] time = 0.199405, size = 123, normalized size = 2.24

$$\frac{\sqrt{a+b}\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) + i\sqrt{b}\tan^{-1}\left(\frac{-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)-i\sqrt{a+b}}{\sqrt{b}}\right) + i\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)-i\sqrt{a+b}}{\sqrt{b}}\right)}{ad\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (I*Sqrt[b]*ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]] +
 I*Sqrt[b]*ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]] +
 Sqrt[a + b]*Log[Tanh[(c + d*x)/2]])/(a*Sqrt[a + b]*d)
```

Maple [A] time = 0.063, size = 69, normalized size = 1.3

$$\frac{b}{da}\operatorname{Artanh}\left(\frac{1}{4}\left(2\,(\tanh(1/2\,dx+c/2))^2\,a+2\,a+4\,b\right)\frac{1}{\sqrt{ab+b^2}}\right)\frac{1}{\sqrt{ab+b^2}} + \frac{1}{da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a+b*tanh(d*x+c)^2),x)
```

```
[Out] 1/d/a*b/(a*b+b^2)^(1/2)*arctanh(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*
b+b^2)^(1/2))+1/d/a*ln(tanh(1/2*d*x+1/2*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(\left(e^{(dx+c)}+1\right)e^{(-c)}\right)}{ad} + \frac{\log\left(\left(e^{(dx+c)}-1\right)e^{(-c)}\right)}{ad} - 2\int \frac{be^{(3\,dx+3\,c)} - be^{(dx+c)}}{a^2+ab+\left(a^2e^{(4\,c)}+abe^{(4\,c)}\right)e^{(4\,dx)}+2\left(a^2e^{(2\,c)}-abe^{(2\,c)}\right)e^{(2\,dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -log((e^(d*x + c) + 1)*e^(-c))/(a*d) + log((e^(d*x + c) - 1)*e^(-c))/(a*d)
- 2*integrate((b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^2 + a*b + (a^2*e^(4*c)
 + a*b*e^(4*c))*e^(4*d*x) + 2*(a^2*e^(2*c) - a*b*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 2.39541, size = 1673, normalized size = 30.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b/(a + b))*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c
)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2 +
 2*(3*(a + b)*cosh(d*x + c)^2 + a + 3*b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(
d*x + c)^3 + (a + 3*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a + b)*cosh(d*x +
 c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 +
 (a + b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))
*sqrt(b/(a + b)) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c
)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2
*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x
+ c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) - 2*log(cosh(d*x +
c) + sinh(d*x + c) + 1) + 2*log(cosh(d*x + c) + sinh(d*x + c) - 1))/(a*d),
(sqrt(-b/(a + b))*arctan(1/2*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x
+ c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a - 3*b)*cosh(d*x + c) +
(3*(a + b)*cosh(d*x + c)^2 + a - 3*b)*sinh(d*x + c))*sqrt(-b/(a + b))/b) -
sqrt(-b/(a + b))*arctan(1/2*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x + c))
*sqrt(-b/(a + b))/b) - log(cosh(d*x + c) + sinh(d*x + c) + 1) + log(cosh(d*
x + c) + sinh(d*x + c) - 1))/(a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(csch(c + d*x)/(a + b*tanh(c + d*x)**2), x)
```

Giac [C] time = 1.52559, size = 5242, normalized size = 95.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/4*(2*(3*(a^2*b*e^(2*c) + a*b^2*e^(2*c))*cos(1/2*real_part(arccos(-a/(a +
 b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*
sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) - (a^2*b*e^(2*c) + a*b^2
*e^(2*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*rea
l_part(arccos(-a/(a + b) + b/(a + b))))^3 - 9*(a^2*b*e^(2*c) + a*b^2*e^(2*c
))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(
arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/
```

```
(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(a^2*b*e
^(2*c) + a*b^2*e^(2*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))
^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(
arccos(-a/(a + b) + b/(a + b)))) + 9*(a^2*b*e^(2*c) + a*b^2*e^(2*c))*cos(1/
2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a
/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*
sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(a^2*b*e^(2*c) +
a*b^2*e^(2*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*
real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(
a + b) + b/(a + b))))^2 - 3*(a^2*b*e^(2*c) + a*b^2*e^(2*c))*cos(1/2*real_pa
rt(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) +
 b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (a^2*
b*e^(2*c) + a*b^2*e^(2*c))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))
))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 - (a^2*b*e^(2*c)
 + a*b^2*e^(2*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1
/2*real_part(arccos(-a/(a + b) + b/(a + b)))) + (a^2*b*e^(2*c) + a*b^2*e^(2
*c))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(
arccos(-a/(a + b) + b/(a + b)))))*arctan((((a^2 + a*b)/(a^2*e^(4*c) + a*b*e
^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b))) + e^(d*x))/(((a^2 + a*b)/(
a^2*e^(4*c) + a*b*e^(4*c)))^(1/4)*sin(1/2*arccos(-(a - b)/(a + b)))))/(2*a^
3*b*e^(2*c) + (a^2*e^(2*c) - a*b*e^(2*c))*sqrt(-a*b)*abs(a)) + 2*(3*(a^2*b*
e^(2*c) + a*b^2*e^(2*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))
^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(
arccos(-a/(a + b) + b/(a + b)))) - (a^2*b*e^(2*c) + a*b^2*e^(2*c))*cosh(1/2
*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(
a + b) + b/(a + b))))^3 - 9*(a^2*b*e^(2*c) + a*b^2*e^(2*c))*cos(1/2*real_pa
rt(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b)
+ b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/
2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(a^2*b*e^(2*c) + a*b^2*e^(
2*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_pa
rt(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b)
+ b/(a + b)))) + 9*(a^2*b*e^(2*c) + a*b^2*e^(2*c))*cos(1/2*real_part(arccos
(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a +
b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part
(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(a^2*b*e^(2*c) + a*b^2*e^(2*c))*cos
h(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-
a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)
)))^2 - 3*(a^2*b*e^(2*c) + a*b^2*e^(2*c))*cos(1/2*real_part(arccos(-a/(a +
b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh
(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (a^2*b*e^(2*c) + a*b^2*
e^(2*c))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag
_part(arccos(-a/(a + b) + b/(a + b))))^3 - (a^2*b*e^(2*c) + a*b^2*e^(2*c))*
cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arcco
s(-a/(a + b) + b/(a + b)))) + (a^2*b*e^(2*c) + a*b^2*e^(2*c))*sin(1/2*real_
part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b)
+ b/(a + b)))))*arctan(-(((a^2 + a*b)/(a^2*e^(4*c) + a*b*e^(4*c)))^(1/4)*co
s(1/2*arccos(-(a - b)/(a + b))) - e^(d*x))/(((a^2 + a*b)/(a^2*e^(4*c) + a*b
*e^(4*c)))^(1/4)*sin(1/2*arccos(-(a - b)/(a + b)))))/(2*a^3*b*e^(2*c) + (a^
2*e^(2*c) - a*b*e^(2*c))*sqrt(-a*b)*abs(a)) + ((a^2*b*e^(2*c) + a*b^2*e^(2*
c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part
(arccos(-a/(a + b) + b/(a + b))))^3 - 3*(a^2*b*e^(2*c) + a*b^2*e^(2*c))*cos
(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-
a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))
))^2 - 3*(a^2*b*e^(2*c) + a*b^2*e^(2*c))*cos(1/2*real_part(arccos(-a/(a + b
) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*si
nh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(a^2*b*e^(2*c) + a*b^
2*e^(2*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag
_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b
) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*
```

```
(a^2*b*e^(2*c) + a*b^2*e^(2*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a
+ b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag
_part(arccos(-a/(a + b) + b/(a + b))))^2 - 9*(a^2*b*e^(2*c) + a*b^2*e^(2*c)
)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arc
cos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a +
b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - (a^2*b*e^(2
*c) + a*b^2*e^(2*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*s
inh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + 3*(a^2*b*e^(2*c) + a
*b^2*e^(2*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*re
al_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a
+ b) + b/(a + b))))^3 - (a^2*b*e^(2*c) + a*b^2*e^(2*c))*cos(1/2*real_part(a
rccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a
 + b)))) + (a^2*b*e^(2*c) + a*b^2*e^(2*c))*cos(1/2*real_part(arccos(-a/(a +
 b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*log
(2*((a^2 + a*b)/(a^2*e^(4*c) + a*b*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/
(a + b)))*e^(d*x) + sqrt((a^2 + a*b)/(a^2*e^(4*c) + a*b*e^(4*c))) + e^(2*d*
x))/(2*a^3*b*e^(2*c) + (a^2*e^(2*c) - a*b*e^(2*c))*sqrt(-a*b)*abs(a)) - ((a
^2*b*e^(2*c) + a*b^2*e^(2*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a +
b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 - 3*(a^2*b*e^
(2*c) + a*b^2*e^(2*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*c
osh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arcc
os(-a/(a + b) + b/(a + b))))^2 - 3*(a^2*b*e^(2*c) + a*b^2*e^(2*c))*cos(1/2*
real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(
a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))
 + 9*(a^2*b*e^(2*c) + a*b^2*e^(2*c))*cos(1/2*real_part(arccos(-a/(a + b) +
b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*
real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(
a + b) + b/(a + b)))) + 3*(a^2*b*e^(2*c) + a*b^2*e^(2*c))*cos(1/2*real_part
(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) +
b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 9*(a^2
*b*e^(2*c) + a*b^2*e^(2*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b)
)))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(a
rccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/
(a + b))))^2 - (a^2*b*e^(2*c) + a*b^2*e^(2*c))*cos(1/2*real_part(arccos(-a/
(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))
)^3 + 3*(a^2*b*e^(2*c) + a*b^2*e^(2*c))*cos(1/2*real_part(arccos(-a/(a + b)
 + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1
/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 - (a^2*b*e^(2*c) + a*b^2*e^
(2*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_par
t(arccos(-a/(a + b) + b/(a + b)))) + (a^2*b*e^(2*c) + a*b^2*e^(2*c))*cos(1/
2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(
a + b) + b/(a + b)))))*log(-2*((a^2 + a*b)/(a^2*e^(4*c) + a*b*e^(4*c)))^(1/
4)*cos(1/2*arccos(-(a - b)/(a + b)))*e^(d*x) + sqrt((a^2 + a*b)/(a^2*e^(4*c
) + a*b*e^(4*c))) + e^(2*d*x))/(2*a^3*b*e^(2*c) + (a^2*e^(2*c) - a*b*e^(2*c
))*sqrt(-a*b)*abs(a)) + 4*log(e^(d*x + c) + 1)/a - 4*log(abs(e^(d*x + c) -
1))/a)/d
```

3.30 $\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=48

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{\coth(c+dx)}{ad}$$

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*d)) - Coth[c +
 d*x]/(a*d)
```

Rubi [A] time = 0.062818, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3663, 325, 205}

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{\coth(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*d)) - Coth[c +
 d*x]/(a*d)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{\coth(c+dx)}{ad} - \frac{b\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{ad}$$
$$= -\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\coth(c+dx)}{ad}$$

Mathematica [A] time = 0.115079, size = 48, normalized size = 1.

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\coth(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*d)) - Coth[c +
 d*x]/(a*d)
```

Maple [B] time = 0.076, size = 413, normalized size = 8.6

$$-\frac{1}{2da}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{b}{d}\operatorname{Artanh}\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\frac{1}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}\right)\frac{1}{\sqrt{b(a+b)}}\frac{1}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2),x)
```

```
[Out] -1/2/d/a*tanh(1/2*d*x+1/2*c)+1/d*b/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*
b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/
2))-1/d*b/a/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*
c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/d/(b*(a+b))^(1/2)/a/((2*(b*(a+b))
^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-
2*b)*a)^(1/2))*b^2+1/d*b/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2
)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/d*b/a
/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a
+b))^(1/2)+a+2*b)*a)^(1/2))+1/d/(b*(a+b))^(1/2)/a/((2*(b*(a+b))^(1/2)+a+2*b
)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)
)*b^2-1/2/d/a/tanh(1/2*d*x+1/2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.20358, size = 1636, normalized size = 34.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 -
1)*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b
^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2
*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a
^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh
(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a^2 + a*b)*cos
h(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh
(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*c
osh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x
 + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b
)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) - 4)/(a*
d*cosh(d*x + c)^2 + 2*a*d*cosh(d*x + c)*sinh(d*x + c) + a*d*sinh(d*x + c)^2
 - a*d), -((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 - 1)*sqrt(b/a)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x
+ c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(b/a)/b) + 2)/(a*
d*cosh(d*x + c)^2 + 2*a*d*cosh(d*x + c)*sinh(d*x + c) + a*d*sinh(d*x + c)^2
 - a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(csch(c + d*x)**2/(a + b*tanh(c + d*x)**2), x)
```

Giac [A] time = 1.42002, size = 93, normalized size = 1.94

$$-\frac{\frac{b\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}a}+\frac{2}{a\left(e^{(2\,dx+2\,c)}-1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -(b*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/(
sqrt(a*b)*a) + 2/(a*(e^(2*d*x + 2*c) - 1)))/d
```

3.31 $\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=85

$$\frac{(a+2b)\tanh^{-1}(\cosh(c+dx))}{2a^2d}-\frac{\sqrt{b}\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{a^2d}-\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}$$

```
[Out] ((a + 2*b)*ArcTanh[Cosh[c + d*x]])/(2*a^2*d) - (Sqrt[b]*Sqrt[a + b]*ArcTanh
[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(a^2*d) - (Coth[c + d*x]*Csch[c + d*
x])/(2*a*d)
```

Rubi [A] time = 0.118728, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3664, 471, 522, 207, 208}

$$\frac{(a+2b)\tanh^{-1}(\cosh(c+dx))}{2a^2d}-\frac{\sqrt{b}\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{a^2d}-\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((a + 2*b)*ArcTanh[Cosh[c + d*x]])/(2*a^2*d) - (Sqrt[b]*Sqrt[a + b]*ArcTanh
[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(a^2*d) - (Coth[c + d*x]*Csch[c + d*
x])/(2*a*d)
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{x^2}{\left(-1+x^2\right)^2\left(a+b-bx^2\right)}\,dx, x, \operatorname{sech}(c+dx)\right)}{d}$$

$$= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad} - \frac{\operatorname{Subst}\left(\int \frac{a+b+bx^2}{\left(-1+x^2\right)\left(a+b-bx^2\right)}\,dx, x, \operatorname{sech}(c+dx)\right)}{2ad}$$

$$= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad} - \frac{(b(a+b))\operatorname{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \operatorname{sech}(c+dx)\right)}{a^2d} - \frac{(a+2b)\operatorname{Su}}{}$$

$$= \frac{(a+2b)\tanh^{-1}(\cosh(c+dx))}{2a^2d} - \frac{\sqrt{b}\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{a^2d} - \frac{\coth(c+dx)\operatorname{csch}(c+}{2ad}$$

Mathematica [C] time = 0.638159, size = 170, normalized size = 2.

$$-\frac{8i\sqrt{b}\sqrt{a+b}\tan^{-1}\left(\frac{-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)-i\sqrt{a+b}}{\sqrt{b}}\right)+8i\sqrt{b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)-i\sqrt{a+b}}{\sqrt{b}}\right)+a\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)+a\operatorname{sech}^2}{8a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -((8*I)*Sqrt[b]*Sqrt[a + b]*ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*
x)/2])/Sqrt[b]] + (8*I)*Sqrt[b]*Sqrt[a + b]*ArcTan[((-I)*Sqrt[a + b] + Sqrt
[a]*Tanh[(c + d*x)/2])/Sqrt[b]] + a*Csch[(c + d*x)/2]^2 + 4*a*Log[Tanh[(c +
 d*x)/2]] + 8*b*Log[Tanh[(c + d*x)/2]] + a*Sech[(c + d*x)/2]^2)/(8*a^2*d)
```

Maple [B] time = 0.077, size = 181, normalized size = 2.1

$$\frac{1}{8da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 - \frac{b}{da}\operatorname{Artanh}\left(\frac{1}{4}\left(2\left(\tanh\left(1/2\,dx+c/2\right)\right)^2a+2\,a+4\,b\right)\frac{1}{\sqrt{ab+b^2}}\right)\frac{1}{\sqrt{ab+b^2}} - \frac{b^2}{da^2}\operatorname{Artanh}\left(\frac{1}{4}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2),x)
```

```
[Out] 1/8/d/a*tanh(1/2*d*x+1/2*c)^2-1/d/a*b/(a*b+b^2)^(1/2)*arctanh(1/4*(2*tanh(1
/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2))-1/d*b^2/a^2/(a*b+b^2)^(1/2)*arc
tanh(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2))-1/8/d/a/tanh(
1/2*d*x+1/2*c)^2-1/2/d/a*ln(tanh(1/2*d*x+1/2*c))-1/d/a^2*b*ln(tanh(1/2*d*x+
1/2*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{e^{(3\,dx+3\,c)}+e^{(dx+c)}}{ade^{(4\,dx+4\,c)}-2\,ade^{(2\,dx+2\,c)}+ad}+\frac{(a+2\,b)\log\left(\left(e^{(dx+c)}+1\right)e^{(-c)}\right)}{2\,a^2d}-\frac{(a+2\,b)\log\left(\left(e^{(dx+c)}-1\right)e^{(-c)}\right)}{2\,a^2d}+8\int\frac{}{4\left(a^3+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -(e^(3*d*x + 3*c) + e^(d*x + c))/(a*d*e^(4*d*x + 4*c) - 2*a*d*e^(2*d*x + 2*
c) + a*d) + 1/2*(a + 2*b)*log((e^(d*x + c) + 1)*e^(-c))/(a^2*d) - 1/2*(a +
2*b)*log((e^(d*x + c) - 1)*e^(-c))/(a^2*d) + 8*integrate(1/4*((a*b*e^(3*c)
+ b^2*e^(3*c))*e^(3*d*x) - (a*b*e^c + b^2*e^c)*e^(d*x))/(a^3 + a^2*b + (a^3
*e^(4*c) + a^2*b*e^(4*c))*e^(4*d*x) + 2*(a^3*e^(2*c) - a^2*b*e^(2*c))*e^(2*
d*x)), x)
```

Fricas [B] time = 2.6513, size = 4872, normalized size = 57.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*cosh(d*x + c)^3 + 6*a*cosh(d*x + c)*sinh(d*x + c)^2 + 2*a*sinh(d
*x + c)^3 - (cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x +
 c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(
cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(a*b + b^2)*log(((a
 + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*s
inh(d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2
 + a + 3*b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + 3*b)*cosh(d
*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)
^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 + 1)*sinh(d*x + c) + cosh(d*x + c
))*sqrt(a*b + b^2) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x +
 c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 +
 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*
x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 2*a*cosh(d*x +
c) - ((a + 2*b)*cosh(d*x + c)^4 + 4*(a + 2*b)*cosh(d*x + c)*sinh(d*x + c)^3
 + (a + 2*b)*sinh(d*x + c)^4 - 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*(a + 2*b)
*cosh(d*x + c)^2 - a - 2*b)*sinh(d*x + c)^2 + 4*((a + 2*b)*cosh(d*x + c)^3
- (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a + 2*b)*log(cosh(d*x + c) + sin
h(d*x + c) + 1) + ((a + 2*b)*cosh(d*x + c)^4 + 4*(a + 2*b)*cosh(d*x + c)*si
nh(d*x + c)^3 + (a + 2*b)*sinh(d*x + c)^4 - 2*(a + 2*b)*cosh(d*x + c)^2 + 2
*(3*(a + 2*b)*cosh(d*x + c)^2 - a - 2*b)*sinh(d*x + c)^2 + 4*((a + 2*b)*cos
h(d*x + c)^3 - (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a + 2*b)*log(cosh(d
*x + c) + sinh(d*x + c) - 1) + 2*(3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))/(
a^2*d*cosh(d*x + c)^4 + 4*a^2*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*d*sinh(
d*x + c)^4 - 2*a^2*d*cosh(d*x + c)^2 + a^2*d + 2*(3*a^2*d*cosh(d*x + c)^2 -
 a^2*d)*sinh(d*x + c)^2 + 4*(a^2*d*cosh(d*x + c)^3 - a^2*d*cosh(d*x + c))*s
inh(d*x + c)), -1/2*(2*a*cosh(d*x + c)^3 + 6*a*cosh(d*x + c)*sinh(d*x + c)^
2 + 2*a*sinh(d*x + c)^3 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c
)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(
d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(-a
*b - b^2)*arctan(1/2*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sin
h(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a - 3*b)*cosh(d*x + c) + (3*(a +
```

```
b)*cosh(d*x + c)^2 + a - 3*b)*sinh(d*x + c))*sqrt(-a*b - b^2)/(a*b + b^2))
- 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 +
2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x
 + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(-a*b - b^2)*arctan(1/2*sqr
t(-a*b - b^2)*(cosh(d*x + c) + sinh(d*x + c))/b) + 2*a*cosh(d*x + c) - ((a
+ 2*b)*cosh(d*x + c)^4 + 4*(a + 2*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + 2
*b)*sinh(d*x + c)^4 - 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*(a + 2*b)*cosh(d*x
 + c)^2 - a - 2*b)*sinh(d*x + c)^2 + 4*((a + 2*b)*cosh(d*x + c)^3 - (a + 2*
b)*cosh(d*x + c))*sinh(d*x + c) + a + 2*b)*log(cosh(d*x + c) + sinh(d*x + c
) + 1) + ((a + 2*b)*cosh(d*x + c)^4 + 4*(a + 2*b)*cosh(d*x + c)*sinh(d*x +
c)^3 + (a + 2*b)*sinh(d*x + c)^4 - 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*(a +
2*b)*cosh(d*x + c)^2 - a - 2*b)*sinh(d*x + c)^2 + 4*((a + 2*b)*cosh(d*x + c
)^3 - (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a + 2*b)*log(cosh(d*x + c) +
 sinh(d*x + c) - 1) + 2*(3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))/(a^2*d*cos
h(d*x + c)^4 + 4*a^2*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*d*sinh(d*x + c)^
4 - 2*a^2*d*cosh(d*x + c)^2 + a^2*d + 2*(3*a^2*d*cosh(d*x + c)^2 - a^2*d)*s
inh(d*x + c)^2 + 4*(a^2*d*cosh(d*x + c)^3 - a^2*d*cosh(d*x + c))*sinh(d*x +
 c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(csch(c + d*x)**3/(a + b*tanh(c + d*x)**2), x)
```

Giac [C] time = 1.82166, size = 7173, normalized size = 84.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/4*(2*(3*(2*a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 + (a^3*b + a^2*b^2 - a*b^3 - b^4
)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2
*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(
a + b) + b/(a + b)))) - (2*a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 + (a^3*b + a^2*b^2
 - a*b^3 - b^4)*sqrt(-a*b))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b
))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3 - 9*(2*a^3*b^2
+ 4*a^2*b^3 + 2*a*b^4 + (a^3*b + a^2*b^2 - a*b^3 - b^4)*sqrt(-a*b))*cos(1/2
*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/
(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))
*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(2*a^3*b^2 + 4*a^2
*b^3 + 2*a*b^4 + (a^3*b + a^2*b^2 - a*b^3 - b^4)*sqrt(-a*b))*cosh(1/2*imag_
part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b)
 + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(
2*a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 + (a^3*b + a^2*b^2 - a*b^3 - b^4)*sqrt(-a*b
))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(
arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a
```

```
 + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(2*a^3*b
^2 + 4*a^2*b^3 + 2*a*b^4 + (a^3*b + a^2*b^2 - a*b^3 - b^4)*sqrt(-a*b))*cosh
(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a
/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))
))^2 - 3*(2*a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 + (a^3*b + a^2*b^2 - a*b^3 - b^4)
*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*r
eal_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a +
 b) + b/(a + b))))^3 + (2*a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 + (a^3*b + a^2*b^2
- a*b^3 - b^4)*sqrt(-a*b))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))
))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 - (2*a^3*b^2 + 4
*a^2*b^3 + 2*a*b^4 + (a^3*b + a^2*b^2 - a*b^3 - b^4)*sqrt(-a*b))*cosh(1/2*i
mag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a +
b) + b/(a + b)))) + (2*a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 + (a^3*b + a^2*b^2 - a
*b^3 - b^4)*sqrt(-a*b))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*
sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*arctan((((a^3 + a^2*b)
/(a^3*e^(4*c) + a^2*b*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b))) + e
^(d*x))/(((a^3 + a^2*b)/(a^3*e^(4*c) + a^2*b*e^(4*c)))^(1/4)*sin(1/2*arccos
(-(a - b)/(a + b)))))/(a^5*b + 2*a^4*b^2 + a^3*b^3) + 2*(3*(2*a^3*b^2 + 4*a
^2*b^3 + 2*a*b^4 + (a^3*b + a^2*b^2 - a*b^3 - b^4)*sqrt(-a*b))*cos(1/2*real
_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a +
b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) - (2
*a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 + (a^3*b + a^2*b^2 - a*b^3 - b^4)*sqrt(-a*b)
)*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(a
rccos(-a/(a + b) + b/(a + b))))^3 - 9*(2*a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 + (a
^3*b + a^2*b^2 - a*b^3 - b^4)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a +
b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*s
in(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos
(-a/(a + b) + b/(a + b)))) + 3*(2*a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 + (a^3*b +
a^2*b^2 - a*b^3 - b^4)*sqrt(-a*b))*cosh(1/2*imag_part(arccos(-a/(a + b) + b
/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2
*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(2*a^3*b^2 + 4*a^2*b^3 + 2*
a*b^4 + (a^3*b + a^2*b^2 - a*b^3 - b^4)*sqrt(-a*b))*cos(1/2*real_part(arcco
s(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a +
 b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_par
t(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(2*a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 +
 (a^3*b + a^2*b^2 - a*b^3 - b^4)*sqrt(-a*b))*cosh(1/2*imag_part(arccos(-a/(
a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*
sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(2*a^3*b^2 + 4*a^
2*b^3 + 2*a*b^4 + (a^3*b + a^2*b^2 - a*b^3 - b^4)*sqrt(-a*b))*cos(1/2*real_
part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b)
 + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (2*
a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 + (a^3*b + a^2*b^2 - a*b^3 - b^4)*sqrt(-a*b))
*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(ar
ccos(-a/(a + b) + b/(a + b))))^3 - (2*a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 + (a^3*
b + a^2*b^2 - a*b^3 - b^4)*sqrt(-a*b))*cosh(1/2*imag_part(arccos(-a/(a + b)
 + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) + (2*a^3
*b^2 + 4*a^2*b^3 + 2*a*b^4 + (a^3*b + a^2*b^2 - a*b^3 - b^4)*sqrt(-a*b))*si
n(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(
-a/(a + b) + b/(a + b)))))*arctan(-(((a^3 + a^2*b)/(a^3*e^(4*c) + a^2*b*e^(
4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b))) - e^(d*x))/(((a^3 + a^2*b)/(
a^3*e^(4*c) + a^2*b*e^(4*c)))^(1/4)*sin(1/2*arccos(-(a - b)/(a + b)))))/(a^
5*b + 2*a^4*b^2 + a^3*b^3) + ((2*a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 + (a^3*b + a
^2*b^2 - a*b^3 - b^4)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(
a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 - 3*(2*a^
3*b^2 + 4*a^2*b^3 + 2*a*b^4 + (a^3*b + a^2*b^2 - a*b^3 - b^4)*sqrt(-a*b))*c
os(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos
(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b
))))^2 - 3*(2*a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 + (a^3*b + a^2*b^2 - a*b^3 - b^
4)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/
```

```
2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a
/(a + b) + b/(a + b)))) + 9*(2*a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 + (a^3*b + a^2
*b^2 - a*b^3 - b^4)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a
+ b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_
part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b
) + b/(a + b)))) + 3*(2*a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 + (a^3*b + a^2*b^2 -
a*b^3 - b^4)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))
^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(a
rccos(-a/(a + b) + b/(a + b))))^2 - 9*(2*a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 + (a
^3*b + a^2*b^2 - a*b^3 - b^4)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a +
b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1
/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-
a/(a + b) + b/(a + b))))^2 - (2*a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 + (a^3*b + a^
2*b^2 - a*b^3 - b^4)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a
 + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + 3*(2*a^3
*b^2 + 4*a^2*b^3 + 2*a*b^4 + (a^3*b + a^2*b^2 - a*b^3 - b^4)*sqrt(-a*b))*co
s(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-
a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)
)))^3 - (2*a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 + (a^3*b + a^2*b^2 - a*b^3 - b^4)*
sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*ima
g_part(arccos(-a/(a + b) + b/(a + b)))) + (2*a^3*b^2 + 4*a^2*b^3 + 2*a*b^4
+ (a^3*b + a^2*b^2 - a*b^3 - b^4)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(
a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*
log(2*((a^3 + a^2*b)/(a^3*e^(4*c) + a^2*b*e^(4*c)))^(1/4)*cos(1/2*arccos(-(
a - b)/(a + b)))*e^(d*x) + sqrt((a^3 + a^2*b)/(a^3*e^(4*c) + a^2*b*e^(4*c))
) + e^(2*d*x))/(a^5*b + 2*a^4*b^2 + a^3*b^3) - ((2*a^3*b^2 + 4*a^2*b^3 + 2*
a*b^4 + (a^3*b + a^2*b^2 - a*b^3 - b^4)*sqrt(-a*b))*cos(1/2*real_part(arcco
s(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a +
 b))))^3 - 3*(2*a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 + (a^3*b + a^2*b^2 - a*b^3 -
b^4)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/
2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/
(a + b) + b/(a + b))))^2 - 3*(2*a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 + (a^3*b + a^
2*b^2 - a*b^3 - b^4)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a
 + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*i
mag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(2*a^3*b^2 + 4*a^2*b^3 + 2*a*
b^4 + (a^3*b + a^2*b^2 - a*b^3 - b^4)*sqrt(-a*b))*cos(1/2*real_part(arccos(
-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))
))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_par
t(arccos(-a/(a + b) + b/(a + b)))) + 3*(2*a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 + (
a^3*b + a^2*b^2 - a*b^3 - b^4)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a +
 b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*si
nh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 9*(2*a^3*b^2 + 4*a^2*
b^3 + 2*a*b^4 + (a^3*b + a^2*b^2 - a*b^3 - b^4)*sqrt(-a*b))*cos(1/2*real_pa
rt(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) +
b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*
imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - (2*a^3*b^2 + 4*a^2*b^3 + 2*a
*b^4 + (a^3*b + a^2*b^2 - a*b^3 - b^4)*sqrt(-a*b))*cos(1/2*real_part(arccos
(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a +
b))))^3 + 3*(2*a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 + (a^3*b + a^2*b^2 - a*b^3 - b
^4)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*
real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(
a + b) + b/(a + b))))^3 - (2*a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 + (a^3*b + a^2*b
^2 - a*b^3 - b^4)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a +
b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + (2*a^3*b^2 + 4*
a^2*b^3 + 2*a*b^4 + (a^3*b + a^2*b^2 - a*b^3 - b^4)*sqrt(-a*b))*cos(1/2*rea
l_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b
) + b/(a + b)))))*log(-2*((a^3 + a^2*b)/(a^3*e^(4*c) + a^2*b*e^(4*c)))^(1/4
)*cos(1/2*arccos(-(a - b)/(a + b)))*e^(d*x) + sqrt((a^3 + a^2*b)/(a^3*e^(4*
c) + a^2*b*e^(4*c))) + e^(2*d*x))/(a^5*b + 2*a^4*b^2 + a^3*b^3) + 2*(a*e^c
```

```
+ 2*b*e^c)*e^(-c)*log(e^(d*x + c) + 1)/a^2 - 2*(a*e^c + 2*b*e^c)*e^(-c)*log
(abs(e^(d*x + c) - 1))/a^2 - 4*(e^(3*d*x + 3*c) + e^(d*x + c))/(a*(e^(2*d*x
 + 2*c) - 1)^2))/d
```

3.32 $\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=70

$$\frac{(a+b)\coth(c+dx)}{a^2d}+\frac{\sqrt{b}(a+b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d}-\frac{\coth^3(c+dx)}{3ad}$$

```
[Out] (Sqrt[b]*(a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(5/2)*d) + ((a
 + b)*Coth[c + d*x])/(a^2*d) - Coth[c + d*x]^3/(3*a*d)
```

Rubi [A] time = 0.0875878, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3663, 453, 325, 205}

$$\frac{(a+b)\coth(c+dx)}{a^2d}+\frac{\sqrt{b}(a+b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d}-\frac{\coth^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (Sqrt[b]*(a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(5/2)*d) + ((a
 + b)*Coth[c + d*x])/(a^2*d) - Coth[c + d*x]^3/(3*a*d)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{x^4(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{\coth^3(c+dx)}{3ad} - \frac{(a+b)\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{ad}$$
$$= \frac{(a+b)\coth(c+dx)}{a^2d} - \frac{\coth^3(c+dx)}{3ad} + \frac{(b(a+b))\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{a^2d}$$
$$= \frac{\sqrt{b}(a+b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{(a+b)\coth(c+dx)}{a^2d} - \frac{\coth^3(c+dx)}{3ad}$$

Mathematica [A] time = 0.289309, size = 71, normalized size = 1.01

$$\frac{3\sqrt{b}(a+b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right) + \sqrt{a}\coth(c+dx)\left(-a\operatorname{csch}^2(c+dx)+2a+3b\right)}{3a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (3*Sqrt[b]*(a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] + Sqrt[a]*Coth[c
 + d*x]*(2*a + 3*b - a*Csch[c + d*x]^2))/(3*a^(5/2)*d)
```

Maple [B] time = 0.089, size = 750, normalized size = 10.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2),x)
```

```
[Out] -1/24/d/a*tanh(1/2*d*x+1/2*c)^3+3/8/d/a*tanh(1/2*d*x+1/2*c)+1/2/d/a^2*tanh(
1/2*d*x+1/2*c)*b-1/d*b/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*
arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/d*b/a/
((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a
+b))^(1/2)-a-2*b)*a)^(1/2))-2/d/(b*(a+b))^(1/2)/a/((2*(b*(a+b))^(1/2)-a-2*b
)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2
))*b^2-1/d*b/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*t
anh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/d*b/a/((2*(b*(a+b
))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a
+2*b)*a)^(1/2))-2/d/(b*(a+b))^(1/2)/a/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*a
rctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))*b^2+1/d/a^
2*b^2/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2
*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-1/d/a^2*b^3/(b*(a+b))^(1/2)/((2*(b*(a+b))
^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-
2*b)*a)^(1/2))-1/d/a^2*b^2/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tan
h(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/d/a^2*b^3/(b*(a+b))
^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2
*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/24/d/a/tanh(1/2*d*x+1/2*c)^3+3/8/d/a/ta
```

```
nh(1/2*d*x+1/2*c)+1/2/d/a^2/tanh(1/2*d*x+1/2*c)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.38569, size = 4307, normalized size = 61.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(12*b*cosh(d*x + c)^4 + 48*b*cosh(d*x + c)*sinh(d*x + c)^3 + 12*b*sinh
(d*x + c)^4 - 24*(a + b)*cosh(d*x + c)^2 + 24*(3*b*cosh(d*x + c)^2 - a - b)
*sinh(d*x + c)^2 + 3*((a + b)*cosh(d*x + c)^6 + 6*(a + b)*cosh(d*x + c)*sin
h(d*x + c)^5 + (a + b)*sinh(d*x + c)^6 - 3*(a + b)*cosh(d*x + c)^4 + 3*(5*(
a + b)*cosh(d*x + c)^2 - a - b)*sinh(d*x + c)^4 + 4*(5*(a + b)*cosh(d*x + c
)^3 - 3*(a + b)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)^2
+ 3*(5*(a + b)*cosh(d*x + c)^4 - 6*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*
x + c)^2 + 6*((a + b)*cosh(d*x + c)^5 - 2*(a + b)*cosh(d*x + c)^3 + (a + b)
*cosh(d*x + c))*sinh(d*x + c) - a - b)*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*
cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^
2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^
2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b
 + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c)
)*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x +
 c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((
a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*
sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2
+ a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x +
c))*sinh(d*x + c) + a + b)) + 48*(b*cosh(d*x + c)^3 - (a + b)*cosh(d*x + c)
)*sinh(d*x + c) + 8*a + 12*b)/(a^2*d*cosh(d*x + c)^6 + 6*a^2*d*cosh(d*x + c
)*sinh(d*x + c)^5 + a^2*d*sinh(d*x + c)^6 - 3*a^2*d*cosh(d*x + c)^4 + 3*a^2
*d*cosh(d*x + c)^2 + 3*(5*a^2*d*cosh(d*x + c)^2 - a^2*d)*sinh(d*x + c)^4 +
4*(5*a^2*d*cosh(d*x + c)^3 - 3*a^2*d*cosh(d*x + c))*sinh(d*x + c)^3 - a^2*d
 + 3*(5*a^2*d*cosh(d*x + c)^4 - 6*a^2*d*cosh(d*x + c)^2 + a^2*d)*sinh(d*x +
 c)^2 + 6*(a^2*d*cosh(d*x + c)^5 - 2*a^2*d*cosh(d*x + c)^3 + a^2*d*cosh(d*x
 + c))*sinh(d*x + c)), 1/3*(6*b*cosh(d*x + c)^4 + 24*b*cosh(d*x + c)*sinh(d
*x + c)^3 + 6*b*sinh(d*x + c)^4 - 12*(a + b)*cosh(d*x + c)^2 + 12*(3*b*cosh
(d*x + c)^2 - a - b)*sinh(d*x + c)^2 + 3*((a + b)*cosh(d*x + c)^6 + 6*(a +
b)*cosh(d*x + c)*sinh(d*x + c)^5 + (a + b)*sinh(d*x + c)^6 - 3*(a + b)*cosh
(d*x + c)^4 + 3*(5*(a + b)*cosh(d*x + c)^2 - a - b)*sinh(d*x + c)^4 + 4*(5*
(a + b)*cosh(d*x + c)^3 - 3*(a + b)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a +
 b)*cosh(d*x + c)^2 + 3*(5*(a + b)*cosh(d*x + c)^4 - 6*(a + b)*cosh(d*x + c
)^2 + a + b)*sinh(d*x + c)^2 + 6*((a + b)*cosh(d*x + c)^5 - 2*(a + b)*cosh(
d*x + c)^3 + (a + b)*cosh(d*x + c))*sinh(d*x + c) - a - b)*sqrt(b/a)*arctan
(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a
```

```
+ b)*sinh(d*x + c)^2 + a - b)*sqrt(b/a)/b) + 24*(b*cosh(d*x + c)^3 - (a + b
)*cosh(d*x + c))*sinh(d*x + c) + 4*a + 6*b)/(a^2*d*cosh(d*x + c)^6 + 6*a^2*
d*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*d*sinh(d*x + c)^6 - 3*a^2*d*cosh(d*x
+ c)^4 + 3*a^2*d*cosh(d*x + c)^2 + 3*(5*a^2*d*cosh(d*x + c)^2 - a^2*d)*sinh
(d*x + c)^4 + 4*(5*a^2*d*cosh(d*x + c)^3 - 3*a^2*d*cosh(d*x + c))*sinh(d*x
+ c)^3 - a^2*d + 3*(5*a^2*d*cosh(d*x + c)^4 - 6*a^2*d*cosh(d*x + c)^2 + a^2
*d)*sinh(d*x + c)^2 + 6*(a^2*d*cosh(d*x + c)^5 - 2*a^2*d*cosh(d*x + c)^3 +
a^2*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(csch(c + d*x)**4/(a + b*tanh(c + d*x)**2), x)
```

Giac [B] time = 1.4188, size = 178, normalized size = 2.54

$$\frac{\frac{3\left(abe^{(2c)}+b^2e^{(2c)}\right)\arctan\left(\frac{ae^{(2dx+2c)}+be^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)e^{(-2c)}}{\sqrt{ab}a^2}+\frac{2\left(3be^{(4dx+4c)}-6ae^{(2dx+2c)}-6be^{(2dx+2c)}+2a+3b\right)}{a^2\left(e^{(2dx+2c)}-1\right)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*(a*b*e^(2*c) + b^2*e^(2*c))*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d
*x + 2*c) + a - b)/sqrt(a*b))*e^(-2*c)/(sqrt(a*b)*a^2) + 2*(3*b*e^(4*d*x +
4*c) - 6*a*e^(2*d*x + 2*c) - 6*b*e^(2*d*x + 2*c) + 2*a + 3*b)/(a^2*(e^(2*d*
x + 2*c) - 1)^3))/d
```

3.33 $\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=192

$$\frac{3x\left(a^2-6ab+b^2\right)}{8(a+b)^4}+\frac{3b(3a-b)\tanh(c+dx)}{8d(a+b)^3\left(a+b\tanh^2(c+dx)\right)}+\frac{3\sqrt{a}\sqrt{b}(a-b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2d(a+b)^4}+\frac{\sinh(c+dx)\cosh^3(c+}{4d(a+b)\left(a+b\tanh^2(c\right.}$$

```
[Out] (3*(a^2 - 6*a*b + b^2)*x)/(8*(a + b)^4) + (3*Sqrt[a]*(a - b)*Sqrt[b]*ArcTan
[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(2*(a + b)^4*d) - ((5*a - b)*Cosh[c + d*
x]*Sinh[c + d*x])/(8*(a + b)^2*d*(a + b*Tanh[c + d*x]^2)) + (Cosh[c + d*x]^
3*Sinh[c + d*x])/(4*(a + b)*d*(a + b*Tanh[c + d*x]^2)) + (3*(3*a - b)*b*Tan
h[c + d*x])/(8*(a + b)^3*d*(a + b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.252353, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3663, 470, 527, 522, 206, 205}

$$\frac{3x\left(a^2-6ab+b^2\right)}{8(a+b)^4}+\frac{3b(3a-b)\tanh(c+dx)}{8d(a+b)^3\left(a+b\tanh^2(c+dx)\right)}+\frac{3\sqrt{a}\sqrt{b}(a-b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2d(a+b)^4}+\frac{\sinh(c+dx)\cosh^3(c+}{4d(a+b)\left(a+b\tanh^2(c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (3*(a^2 - 6*a*b + b^2)*x)/(8*(a + b)^4) + (3*Sqrt[a]*(a - b)*Sqrt[b]*ArcTan
[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(2*(a + b)^4*d) - ((5*a - b)*Cosh[c + d*
x]*Sinh[c + d*x])/(8*(a + b)^2*d*(a + b*Tanh[c + d*x]^2)) + (Cosh[c + d*x]^
3*Sinh[c + d*x])/(4*(a + b)*d*(a + b*Tanh[c + d*x]^2)) + (3*(3*a - b)*b*Tan
h[c + d*x])/(8*(a + b)^3*d*(a + b*Tanh[c + d*x]^2))
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)^3\left(a+bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{a+(4a-b)x^2}{\left(1-x^2\right)^2\left(a+bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{4(a+b)d}$$

$$= -\frac{(5a-b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-}{}\right.}{}$$

$$= -\frac{(5a-b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)} + \frac{3(3a-}{8(a+b)^3d\,(}$$

$$= -\frac{(5a-b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)} + \frac{3(3a-}{8(a+b)^3d\,(}$$

$$= \frac{3\left(a^2-6ab+b^2\right)x}{8(a+b)^4} + \frac{3\sqrt{a}(a-b)\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2(a+b)^4d} - \frac{(5a-b)\cosh(c+dx)\sinh}{8(a+b)^2d\left(a+b\tanh^2(\right.}$$

Mathematica [A] time = 0.933154, size = 132, normalized size = 0.69

$$\frac{12\left(a^2-6ab+b^2\right)(c+dx)+(a+b)^2\sinh(4(c+dx))-8(a-b)(a+b)\sinh(2(c+dx))+48\sqrt{a}\sqrt{b}(a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan}{}\right.}{32d(a+b)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (12*(a^2 - 6*a*b + b^2)*(c + d*x) + 48*Sqrt[a]*(a - b)*Sqrt[b]*ArcTan[(Sqrt
[b]*Tanh[c + d*x])/Sqrt[a]] - 8*(a - b)*(a + b)*Sinh[2*(c + d*x)] + (16*a*b
*(a + b)*Sinh[2*(c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)]) + (a + b)^2
*Sinh[4*(c + d*x)])/(32*(a + b)^4*d)
```

Maple [B] time = 0.119, size = 1246, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*a^2*b/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh
(1/2*d*x+1/2*c)^2*b+a)*tanh(1/2*d*x+1/2*c)^3+1/d*a*b^2/(a+b)^4/(tanh(1/2*d*
x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*tanh(1/
2*d*x+1/2*c)^3+1/d*a^2*b/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/
2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*tanh(1/2*d*x+1/2*c)+1/d*a*b^2/(a+b)^4
/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2
*b+a)*tanh(1/2*d*x+1/2*c)+3/2/d*a^2*b/(a+b)^4/((2*(b*(a+b))^(1/2)-a-2*b)*a)
^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-3
/2/d*a^2*b/(a+b)^4/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*
x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-3/2/d*a*b^2/(a+b)^4/((2*(b*(a
+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2
)-a-2*b)*a)^(1/2))+3/2/d*a*b^2/(a+b)^4/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*
arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-3/2/d*a^3
*b/(a+b)^4/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tan
h(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+3/2/d*a*b^3/(a+b)^4/(
b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1
/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+3/2/d*a*b^3/(a+b)^4/(b*(a+b))^(1
/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b
*(a+b))^(1/2)+a+2*b)*a)^(1/2))-3/2/d*a^3*b/(a+b)^4/(b*(a+b))^(1/2)/((2*(b*(
a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/
2)-a-2*b)*a)^(1/2))-9/4/d/(a+b)^4*ln(tanh(1/2*d*x+1/2*c)+1)*a*b+1/2/d/(a+b)
^2/(tanh(1/2*d*x+1/2*c)-1)^3+1/2/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)+1)^3-1/4/d/
(a+b)^2/(tanh(1/2*d*x+1/2*c)+1)^4+1/4/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)^4-7
/8/d/(a+b)^3/(tanh(1/2*d*x+1/2*c)+1)^2*b-3/8/d/(a+b)^3/(tanh(1/2*d*x+1/2*c)
+1)*a+5/8/d/(a+b)^3/(tanh(1/2*d*x+1/2*c)+1)*b+3/8/d/(a+b)^4*ln(tanh(1/2*d*x
+1/2*c)+1)*a^2+3/8/d/(a+b)^4*ln(tanh(1/2*d*x+1/2*c)+1)*b^2-3/8/d/(a+b)^4*ln
(tanh(1/2*d*x+1/2*c)-1)*a^2-3/8/d/(a+b)^4*ln(tanh(1/2*d*x+1/2*c)-1)*b^2-1/8
/d/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)^2*a+7/8/d/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1
)^2*b-3/8/d/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)*a+5/8/d/(a+b)^3/(tanh(1/2*d*x+1
/2*c)-1)*b+1/8/d/(a+b)^3/(tanh(1/2*d*x+1/2*c)+1)^2*a+9/4/d/(a+b)^4*ln(tanh(
1/2*d*x+1/2*c)-1)*a*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.30391, size = 17565, normalized size = 91.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/64*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^12 + 12*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^11 + (a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*sinh(d*x + c)^12 - 6*(a^3 + a^2*b - a*b^2 - b^3)*cosh(d*x + c)^10 -
 6*(a^3 + a^2*b - a*b^2 - b^3 - 11*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x
 + c)^2)*sinh(d*x + c)^10 + 20*(11*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x
 + c)^3 - 3*(a^3 + a^2*b - a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c)^9 - (1
5*a^3 - 19*a^2*b - 19*a*b^2 + 15*b^3 - 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*d
*x)*cosh(d*x + c)^8 + (495*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4
- 15*a^3 + 19*a^2*b + 19*a*b^2 - 15*b^3 + 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3
)*d*x - 270*(a^3 + a^2*b - a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 +
8*(99*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 - 90*(a^3 + a^2*b - a
*b^2 - b^3)*cosh(d*x + c)^3 - (15*a^3 - 19*a^2*b - 19*a*b^2 + 15*b^3 - 24*(
a^3 - 5*a^2*b - 5*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 - 16*(4*
a^2*b - 4*a*b^2 - 3*(a^3 - 7*a^2*b + 7*a*b^2 - b^3)*d*x)*cosh(d*x + c)^6 +
4*(231*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 - 315*(a^3 + a^2*b -
 a*b^2 - b^3)*cosh(d*x + c)^4 - 16*a^2*b + 16*a*b^2 + 12*(a^3 - 7*a^2*b + 7
*a*b^2 - b^3)*d*x - 7*(15*a^3 - 19*a^2*b - 19*a*b^2 + 15*b^3 - 24*(a^3 - 5*
a^2*b - 5*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(99*(a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 - 189*(a^3 + a^2*b - a*b^2 - b^3)
*cosh(d*x + c)^5 - 7*(15*a^3 - 19*a^2*b - 19*a*b^2 + 15*b^3 - 24*(a^3 - 5*a
^2*b - 5*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 - 12*(4*a^2*b - 4*a*b^2 - 3*(a^3
 - 7*a^2*b + 7*a*b^2 - b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + (15*a^3 -
 83*a^2*b - 83*a*b^2 + 15*b^3 + 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*d*x)*cos
h(d*x + c)^4 + (495*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 - 1260*
(a^3 + a^2*b - a*b^2 - b^3)*cosh(d*x + c)^6 - 70*(15*a^3 - 19*a^2*b - 19*a*
b^2 + 15*b^3 - 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 + 15
*a^3 - 83*a^2*b - 83*a*b^2 + 15*b^3 + 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*d*
x - 240*(4*a^2*b - 4*a*b^2 - 3*(a^3 - 7*a^2*b + 7*a*b^2 - b^3)*d*x)*cosh(d*
x + c)^2)*sinh(d*x + c)^4 + 4*(55*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x
+ c)^9 - 180*(a^3 + a^2*b - a*b^2 - b^3)*cosh(d*x + c)^7 - 14*(15*a^3 - 19*
a^2*b - 19*a*b^2 + 15*b^3 - 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*d*x)*cosh(d*
x + c)^5 - 80*(4*a^2*b - 4*a*b^2 - 3*(a^3 - 7*a^2*b + 7*a*b^2 - b^3)*d*x)*c
osh(d*x + c)^3 + (15*a^3 - 83*a^2*b - 83*a*b^2 + 15*b^3 + 24*(a^3 - 5*a^2*b
 - 5*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 - a^3 - 3*a^2*b - 3*a
*b^2 - b^3 + 6*(a^3 + a^2*b - a*b^2 - b^3)*cosh(d*x + c)^2 + 2*(33*(a^3 + 3
*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^10 - 135*(a^3 + a^2*b - a*b^2 - b^3)*
cosh(d*x + c)^8 - 14*(15*a^3 - 19*a^2*b - 19*a*b^2 + 15*b^3 - 24*(a^3 - 5*a
^2*b - 5*a*b^2 + b^3)*d*x)*cosh(d*x + c)^6 - 120*(4*a^2*b - 4*a*b^2 - 3*(a^
3 - 7*a^2*b + 7*a*b^2 - b^3)*d*x)*cosh(d*x + c)^4 + 3*a^3 + 3*a^2*b - 3*a*b
^2 - 3*b^3 + 3*(15*a^3 - 83*a^2*b - 83*a*b^2 + 15*b^3 + 24*(a^3 - 5*a^2*b -
 5*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 48*((a^2 - b^2)*cos
h(d*x + c)^8 + 8*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 - b^2)*si
nh(d*x + c)^8 + 2*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^6 + 2*(14*(a^2 - b^2)*c
osh(d*x + c)^2 + a^2 - 2*a*b + b^2)*sinh(d*x + c)^6 + 4*(14*(a^2 - b^2)*cos
h(d*x + c)^3 + 3*(a^2 - 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + (a^2
- b^2)*cosh(d*x + c)^4 + (70*(a^2 - b^2)*cosh(d*x + c)^4 + 30*(a^2 - 2*a*b
+ b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^4 + 4*(14*(a^2 - b^2)*cos
h(d*x + c)^5 + 10*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*
x + c))*sinh(d*x + c)^3 + 2*(14*(a^2 - b^2)*cosh(d*x + c)^6 + 15*(a^2 - 2*a
*b + b^2)*cosh(d*x + c)^4 + 3*(a^2 - b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2
```

```
+ 4*(2*(a^2 - b^2)*cosh(d*x + c)^7 + 3*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^5
+ (a^2 - b^2)*cosh(d*x + c)^3)*sinh(d*x + c))*sqrt(-a*b)*log(((a^2 + 2*a*b
+ b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^
3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2
*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2
- 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d
*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x +
c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b))/((a + b)*co
sh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x
+ c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*
sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(
d*x + c) + a + b)) + 4*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^11
- 15*(a^3 + a^2*b - a*b^2 - b^3)*cosh(d*x + c)^9 - 2*(15*a^3 - 19*a^2*b - 1
9*a*b^2 + 15*b^3 - 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*d*x)*cosh(d*x + c)^7
- 24*(4*a^2*b - 4*a*b^2 - 3*(a^3 - 7*a^2*b + 7*a*b^2 - b^3)*d*x)*cosh(d*x +
 c)^5 + (15*a^3 - 83*a^2*b - 83*a*b^2 + 15*b^3 + 24*(a^3 - 5*a^2*b - 5*a*b^
2 + b^3)*d*x)*cosh(d*x + c)^3 + 3*(a^3 + a^2*b - a*b^2 - b^3)*cosh(d*x + c)
)*sinh(d*x + c))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)
*d*cosh(d*x + c)^8 + 8*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 +
 b^5)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^
2*b^3 + 5*a*b^4 + b^5)*d*sinh(d*x + c)^8 + 2*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2
*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^6 + 2*(14*(a^5 + 5*a^4*b + 10*a^3
*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^2 + (a^5 + 3*a^4*b + 2*a
^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d)*sinh(d*x + c)^6 + (a^5 + 5*a^4*b + 1
0*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^4 + 4*(14*(a^5 + 5*
a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^3 + 3*(a^5
 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c))*sinh(d
*x + c)^5 + (70*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d
*cosh(d*x + c)^4 + 30*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^
5)*d*cosh(d*x + c)^2 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 +
 b^5)*d)*sinh(d*x + c)^4 + 4*(14*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 +
 5*a*b^4 + b^5)*d*cosh(d*x + c)^5 + 10*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b
^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^3 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^
2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(14*(a^5 + 5*a^
4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^6 + 15*(a^5
+ 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^4 + 3*(a
^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^2)*
sinh(d*x + c)^2 + 4*(2*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 +
 b^5)*d*cosh(d*x + c)^7 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^
4 - b^5)*d*cosh(d*x + c)^5 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a
*b^4 + b^5)*d*cosh(d*x + c)^3)*sinh(d*x + c)), 1/64*((a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*cosh(d*x + c)^12 + 12*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x +
c)*sinh(d*x + c)^11 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^12 - 6*
(a^3 + a^2*b - a*b^2 - b^3)*cosh(d*x + c)^10 - 6*(a^3 + a^2*b - a*b^2 - b^3
 - 11*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 2
0*(11*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 - 3*(a^3 + a^2*b - a*
b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c)^9 - (15*a^3 - 19*a^2*b - 19*a*b^2 +
 15*b^3 - 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*d*x)*cosh(d*x + c)^8 + (495*(a
^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 - 15*a^3 + 19*a^2*b + 19*a*b^
2 - 15*b^3 + 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*d*x - 270*(a^3 + a^2*b - a*
b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(99*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*cosh(d*x + c)^5 - 90*(a^3 + a^2*b - a*b^2 - b^3)*cosh(d*x + c)^3 -
 (15*a^3 - 19*a^2*b - 19*a*b^2 + 15*b^3 - 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3
)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 - 16*(4*a^2*b - 4*a*b^2 - 3*(a^3 - 7*
a^2*b + 7*a*b^2 - b^3)*d*x)*cosh(d*x + c)^6 + 4*(231*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*cosh(d*x + c)^6 - 315*(a^3 + a^2*b - a*b^2 - b^3)*cosh(d*x + c)^4
 - 16*a^2*b + 16*a*b^2 + 12*(a^3 - 7*a^2*b + 7*a*b^2 - b^3)*d*x - 7*(15*a^3
 - 19*a^2*b - 19*a*b^2 + 15*b^3 - 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*d*x)*c
osh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(99*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos
```

```
h(d*x + c)^7 - 189*(a^3 + a^2*b - a*b^2 - b^3)*cosh(d*x + c)^5 - 7*(15*a^3
- 19*a^2*b - 19*a*b^2 + 15*b^3 - 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*d*x)*co
sh(d*x + c)^3 - 12*(4*a^2*b - 4*a*b^2 - 3*(a^3 - 7*a^2*b + 7*a*b^2 - b^3)*d
*x)*cosh(d*x + c))*sinh(d*x + c)^5 + (15*a^3 - 83*a^2*b - 83*a*b^2 + 15*b^3
 + 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 + (495*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 - 1260*(a^3 + a^2*b - a*b^2 - b^3)*c
osh(d*x + c)^6 - 70*(15*a^3 - 19*a^2*b - 19*a*b^2 + 15*b^3 - 24*(a^3 - 5*a^
2*b - 5*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 + 15*a^3 - 83*a^2*b - 83*a*b^2 +
15*b^3 + 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*d*x - 240*(4*a^2*b - 4*a*b^2 -
3*(a^3 - 7*a^2*b + 7*a*b^2 - b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4
*(55*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^9 - 180*(a^3 + a^2*b - a
*b^2 - b^3)*cosh(d*x + c)^7 - 14*(15*a^3 - 19*a^2*b - 19*a*b^2 + 15*b^3 - 2
4*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 - 80*(4*a^2*b - 4*a*
b^2 - 3*(a^3 - 7*a^2*b + 7*a*b^2 - b^3)*d*x)*cosh(d*x + c)^3 + (15*a^3 - 83
*a^2*b - 83*a*b^2 + 15*b^3 + 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*d*x)*cosh(d
*x + c))*sinh(d*x + c)^3 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 6*(a^3 + a^2*b -
 a*b^2 - b^3)*cosh(d*x + c)^2 + 2*(33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(
d*x + c)^10 - 135*(a^3 + a^2*b - a*b^2 - b^3)*cosh(d*x + c)^8 - 14*(15*a^3
- 19*a^2*b - 19*a*b^2 + 15*b^3 - 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*d*x)*co
sh(d*x + c)^6 - 120*(4*a^2*b - 4*a*b^2 - 3*(a^3 - 7*a^2*b + 7*a*b^2 - b^3)*
d*x)*cosh(d*x + c)^4 + 3*a^3 + 3*a^2*b - 3*a*b^2 - 3*b^3 + 3*(15*a^3 - 83*a
^2*b - 83*a*b^2 + 15*b^3 + 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*d*x)*cosh(d*x
 + c)^2)*sinh(d*x + c)^2 + 96*((a^2 - b^2)*cosh(d*x + c)^8 + 8*(a^2 - b^2)*
cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 - b^2)*sinh(d*x + c)^8 + 2*(a^2 - 2*a*
b + b^2)*cosh(d*x + c)^6 + 2*(14*(a^2 - b^2)*cosh(d*x + c)^2 + a^2 - 2*a*b
+ b^2)*sinh(d*x + c)^6 + 4*(14*(a^2 - b^2)*cosh(d*x + c)^3 + 3*(a^2 - 2*a*b
 + b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + (a^2 - b^2)*cosh(d*x + c)^4 + (70*
(a^2 - b^2)*cosh(d*x + c)^4 + 30*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^2 + a^2
- b^2)*sinh(d*x + c)^4 + 4*(14*(a^2 - b^2)*cosh(d*x + c)^5 + 10*(a^2 - 2*a*
b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(
14*(a^2 - b^2)*cosh(d*x + c)^6 + 15*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^4 + 3
*(a^2 - b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(2*(a^2 - b^2)*cosh(d*x +
 c)^7 + 3*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^5 + (a^2 - b^2)*cosh(d*x + c)^3
)*sinh(d*x + c))*sqrt(a*b)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*
cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(a*b)/(a
*b)) + 4*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^11 - 15*(a^3 + a^
2*b - a*b^2 - b^3)*cosh(d*x + c)^9 - 2*(15*a^3 - 19*a^2*b - 19*a*b^2 + 15*b
^3 - 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*d*x)*cosh(d*x + c)^7 - 24*(4*a^2*b
- 4*a*b^2 - 3*(a^3 - 7*a^2*b + 7*a*b^2 - b^3)*d*x)*cosh(d*x + c)^5 + (15*a^
3 - 83*a^2*b - 83*a*b^2 + 15*b^3 + 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*d*x)*
cosh(d*x + c)^3 + 3*(a^3 + a^2*b - a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c
))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x +
c)^8 + 8*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d
*x + c)*sinh(d*x + c)^7 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^
4 + b^5)*d*sinh(d*x + c)^8 + 2*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a
*b^4 - b^5)*d*cosh(d*x + c)^6 + 2*(14*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*
b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^2 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2
*b^3 - 3*a*b^4 - b^5)*d)*sinh(d*x + c)^6 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10
*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^4 + 4*(14*(a^5 + 5*a^4*b + 10*a^3
*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^3 + 3*(a^5 + 3*a^4*b + 2
*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + (7
0*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)
^4 + 30*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x
+ c)^2 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d)*sinh(
d*x + c)^4 + 4*(14*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5
)*d*cosh(d*x + c)^5 + 10*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 -
 b^5)*d*cosh(d*x + c)^3 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^
4 + b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(14*(a^5 + 5*a^4*b + 10*a^3*b
^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^6 + 15*(a^5 + 3*a^4*b + 2*
```

```
a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^4 + 3*(a^5 + 5*a^4*b +
 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^2)*sinh(d*x + c)^
2 + 4*(2*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d
*x + c)^7 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cos
h(d*x + c)^5 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*
cosh(d*x + c)^3)*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.3925, size = 709, normalized size = 3.69

$$\frac{24\left(a^2-6ab+b^2\right)dx}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}+\frac{96\left(a^2be^{(2c)}-ab^2e^{(2c)}\right)\arctan\left(\frac{ae^{(2dx+2c)}+be^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)e^{(-2c)}}{\left(a^4+4a^3b+6a^2b^2+4ab^3+b^4\right)\sqrt{ab}}-\frac{\left(18a^2e^{(4dx+4c)}-108abe^{(4dx+4c)}+18b^2e^{(4dx+4c)}-8a^2e^{(2dx+2}\right.}{a^4e^{(4c)}+4a^3be^{(4c)}+6a^2b^2e^{(4c)}+4ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/64*(24*(a^2 - 6*a*b + b^2)*d*x/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4
) + 96*(a^2*b*e^(2*c) - a*b^2*e^(2*c))*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^
(2*d*x + 2*c) + a - b)/sqrt(a*b))*e^(-2*c)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*
a*b^3 + b^4)*sqrt(a*b)) - (18*a^2*e^(4*d*x + 4*c) - 108*a*b*e^(4*d*x + 4*c)
 + 18*b^2*e^(4*d*x + 4*c) - 8*a^2*e^(2*d*x + 2*c) + 8*b^2*e^(2*d*x + 2*c) +
 a^2 + 2*a*b + b^2)*e^(-4*d*x)/(a^4*e^(4*c) + 4*a^3*b*e^(4*c) + 6*a^2*b^2*e
^(4*c) + 4*a*b^3*e^(4*c) + b^4*e^(4*c)) + (a^2*e^(4*d*x + 28*c) + 2*a*b*e^(
4*d*x + 28*c) + b^2*e^(4*d*x + 28*c) - 8*a^2*e^(2*d*x + 26*c) + 8*b^2*e^(2*
d*x + 26*c))/(a^4*e^(24*c) + 4*a^3*b*e^(24*c) + 6*a^2*b^2*e^(24*c) + 4*a*b^
3*e^(24*c) + b^4*e^(24*c)) - 64*(a^2*b*e^(2*d*x + 2*c) - a*b^2*e^(2*d*x + 2
*c) + a^2*b + a*b^2)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(a*e^(4*d
*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) +
 a + b)))/d
```

3.34 $\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=124

$$\frac{\cosh^3(c+dx)}{3d(a+b)^2}-\frac{(a-b)\cosh(c+dx)}{d(a+b)^3}+\frac{ab\mathrm{sech}(c+dx)}{2d(a+b)^3\left(a-b\mathrm{sech}^2(c+dx)+b\right)}+\frac{\sqrt{b}(3a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\mathrm{sech}(c+dx)}{\sqrt{a+b}}\right)}{2d(a+b)^{7/2}}$$

```
[Out] ((3*a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(2*(a +
b)^(7/2)*d) - ((a - b)*Cosh[c + d*x])/((a + b)^3*d) + Cosh[c + d*x]^3/(3*(a
 + b)^2*d) + (a*b*Sech[c + d*x])/(2*(a + b)^3*d*(a + b - b*Sech[c + d*x]^2)
)
```

Rubi [A] time = 0.218372, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3664, 456, 1261, 208}

$$\frac{\cosh^3(c+dx)}{3d(a+b)^2}-\frac{(a-b)\cosh(c+dx)}{d(a+b)^3}+\frac{ab\mathrm{sech}(c+dx)}{2d(a+b)^3\left(a-b\mathrm{sech}^2(c+dx)+b\right)}+\frac{\sqrt{b}(3a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\mathrm{sech}(c+dx)}{\sqrt{a+b}}\right)}{2d(a+b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```



```
[Out] ((3*a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(2*(a +
b)^(7/2)*d) - ((a - b)*Cosh[c + d*x])/((a + b)^3*d) + Cosh[c + d*x]^3/(3*(a
 + b)^2*d) + (a*b*Sech[c + d*x])/(2*(a + b)^3*d*(a + b - b*Sech[c + d*x]^2)
)
```


Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4\left(a+b-bx^2\right)^2}\,dx, x, \text{sech}(c+dx)\right)}{d}$$

$$= \frac{ab\text{sech}(c+dx)}{2(a+b)^3d\left(a+b-b\text{sech}^2(c+dx)\right)} + \frac{b\,\text{Subst}\left(\int \frac{-\frac{2}{b(a+b)}+\frac{2ax^2}{b(a+b)^2}+\frac{ax^4}{(a+b)^3}}{x^4\left(a+b-bx^2\right)}\,dx, x, \text{sech}(c+dx)\right)}{2d}$$

$$= \frac{ab\text{sech}(c+dx)}{2(a+b)^3d\left(a+b-b\text{sech}^2(c+dx)\right)} + \frac{b\,\text{Subst}\left(\int\left(-\frac{2}{b(a+b)^2x^4}+\frac{2(a-b)}{b(a+b)^3x^2}+\frac{3a-2b}{(a+b)^3\left(a+b-bx^2\right)}\right.\right.}{2d}$$

$$= -\frac{(a-b)\cosh(c+dx)}{(a+b)^3d} + \frac{\cosh^3(c+dx)}{3(a+b)^2d} + \frac{ab\text{sech}(c+dx)}{2(a+b)^3d\left(a+b-b\text{sech}^2(c+dx)\right)} + \frac{((3a-2}{}$$

$$= \frac{(3a-2b)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{2(a+b)^{7/2}d} - \frac{(a-b)\cosh(c+dx)}{(a+b)^3d} + \frac{\cosh^3(c+dx)}{3(a+b)^2d} + \frac{a}{2(a+b)^3d}$$

Mathematica [C] time = 1.22047, size = 160, normalized size = 1.29

$$\frac{\frac{3\cosh(c+dx)\left(a\left(\frac{4b}{(a+b)\cosh(2(c+dx))+a-b}-3\right)+5b\right)}{(a+b)^3}+\frac{\cosh(3(c+dx))}{(a+b)^2}+\frac{6i\sqrt{b}(3a-2b)\left(\tan^{-1}\left(\frac{-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)-i\sqrt{a+b}}{\sqrt{b}}\right)+\tan^{-1}\left(\frac{\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)-i\sqrt{a+b}}{\sqrt{b}}\right)\right)}{(a+b)^{7/2}}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (((6*I)*(3*a - 2*b)*Sqrt[b]*(ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d
*x)/2])/Sqrt[b]] + ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sq
rt[b]]))/(a + b)^(7/2) + (3*Cosh[c + d*x]*(5*b + a*(-3 + (4*b)/(a - b + (a
+ b)*Cosh[2*(c + d*x)]))))/(a + b)^3 + Cosh[3*(c + d*x)]/(a + b)^2)/(12*d)
```

Maple [B] time = 0.095, size = 267, normalized size = 2.2

$$\frac{1}{d}\left(\frac{1}{3\,(a+b)^2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3}-\frac{1}{2\,(a+b)^2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}-\frac{a-3\,b}{2\,(a+b)^3}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}-4\,\frac{b}{(a+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(1/3/(a+b)^2/(tanh(1/2*d*x+1/2*c)+1)^3-1/2/(a+b)^2/(tanh(1/2*d*x+1/2*c)
+1)^2-1/2*(a-3*b)/(a+b)^3/(tanh(1/2*d*x+1/2*c)+1)-4*b/(a+b)^3*(((-1/4*a-1/2
```

```
*b)*tanh(1/2*d*x+1/2*c)^2-1/4*a)/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/
2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)-1/8*(3*a-2*b)/(a*b+b^2)^(1/2)*arctanh
(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2)))-1/3/(a+b)^2/(tan
h(1/2*d*x+1/2*c)-1)^3-1/2/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)^2-1/2/(a+b)^3*(-a
+3*b)/(tanh(1/2*d*x+1/2*c)-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/24*(a^2 + 2*a*b + b^2 + (a^2*e^(10*c) + 2*a*b*e^(10*c) + b^2*e^(10*c))*e^
(10*d*x) - (7*a^2*e^(8*c) - 6*a*b*e^(8*c) - 13*b^2*e^(8*c))*e^(8*d*x) - 2*(
13*a^2*e^(6*c) - 40*a*b*e^(6*c) + 7*b^2*e^(6*c))*e^(6*d*x) - 2*(13*a^2*e^(4
*c) - 40*a*b*e^(4*c) + 7*b^2*e^(4*c))*e^(4*d*x) - (7*a^2*e^(2*c) - 6*a*b*e^
(2*c) - 13*b^2*e^(2*c))*e^(2*d*x))/((a^4*d*e^(7*c) + 4*a^3*b*d*e^(7*c) + 6*
a^2*b^2*d*e^(7*c) + 4*a*b^3*d*e^(7*c) + b^4*d*e^(7*c))*e^(7*d*x) + 2*(a^4*d
*e^(5*c) + 2*a^3*b*d*e^(5*c) - 2*a*b^3*d*e^(5*c) - b^4*d*e^(5*c))*e^(5*d*x)
 + (a^4*d*e^(3*c) + 4*a^3*b*d*e^(3*c) + 6*a^2*b^2*d*e^(3*c) + 4*a*b^3*d*e^(
3*c) + b^4*d*e^(3*c))*e^(3*d*x)) - 1/8*integrate(8*((3*a*b*e^(3*c) - 2*b^2*
e^(3*c))*e^(3*d*x) - (3*a*b*e^c - 2*b^2*e^c)*e^(d*x))/(a^4 + 4*a^3*b + 6*a^
2*b^2 + 4*a*b^3 + b^4 + (a^4*e^(4*c) + 4*a^3*b*e^(4*c) + 6*a^2*b^2*e^(4*c)
+ 4*a*b^3*e^(4*c) + b^4*e^(4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) + 2*a^3*b*e^(2*
c) - 2*a*b^3*e^(2*c) - b^4*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 3.06339, size = 12417, normalized size = 100.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/24*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^10 + 10*(a^2 + 2*a*b + b^2)*cosh(d
*x + c)*sinh(d*x + c)^9 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^10 - (7*a^2 - 6
*a*b - 13*b^2)*cosh(d*x + c)^8 + (45*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 -
7*a^2 + 6*a*b + 13*b^2)*sinh(d*x + c)^8 + 8*(15*(a^2 + 2*a*b + b^2)*cosh(d*
x + c)^3 - (7*a^2 - 6*a*b - 13*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(13*
a^2 - 40*a*b + 7*b^2)*cosh(d*x + c)^6 + 2*(105*(a^2 + 2*a*b + b^2)*cosh(d*x
 + c)^4 - 14*(7*a^2 - 6*a*b - 13*b^2)*cosh(d*x + c)^2 - 13*a^2 + 40*a*b - 7
*b^2)*sinh(d*x + c)^6 + 4*(63*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 - 14*(7*a
^2 - 6*a*b - 13*b^2)*cosh(d*x + c)^3 - 3*(13*a^2 - 40*a*b + 7*b^2)*cosh(d*x
 + c))*sinh(d*x + c)^5 - 2*(13*a^2 - 40*a*b + 7*b^2)*cosh(d*x + c)^4 + 2*(1
05*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 - 35*(7*a^2 - 6*a*b - 13*b^2)*cosh(d
*x + c)^4 - 15*(13*a^2 - 40*a*b + 7*b^2)*cosh(d*x + c)^2 - 13*a^2 + 40*a*b
- 7*b^2)*sinh(d*x + c)^4 + 8*(15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 - 7*(7
*a^2 - 6*a*b - 13*b^2)*cosh(d*x + c)^5 - 5*(13*a^2 - 40*a*b + 7*b^2)*cosh(d
*x + c)^3 - (13*a^2 - 40*a*b + 7*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - (7*a
^2 - 6*a*b - 13*b^2)*cosh(d*x + c)^2 + (45*(a^2 + 2*a*b + b^2)*cosh(d*x + c
)^8 - 28*(7*a^2 - 6*a*b - 13*b^2)*cosh(d*x + c)^6 - 30*(13*a^2 - 40*a*b + 7
*b^2)*cosh(d*x + c)^4 - 12*(13*a^2 - 40*a*b + 7*b^2)*cosh(d*x + c)^2 - 7*a^
2 + 6*a*b + 13*b^2)*sinh(d*x + c)^2 - 6*((3*a^2 + a*b - 2*b^2)*cosh(d*x + c
```

)^7 + 7*(3*a^2 + a*b - 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^6 + (3*a^2 + a*b - 2*b^2)*sinh(d*x + c)^7 + 2*(3*a^2 - 5*a*b + 2*b^2)*cosh(d*x + c)^5 + (21*(3*a^2 + a*b - 2*b^2)*cosh(d*x + c)^2 + 6*a^2 - 10*a*b + 4*b^2)*sinh(d*x + c)^5 + 5*(7*(3*a^2 + a*b - 2*b^2)*cosh(d*x + c)^3 + 2*(3*a^2 - 5*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + (3*a^2 + a*b - 2*b^2)*cosh(d*x + c)^3 + (35*(3*a^2 + a*b - 2*b^2)*cosh(d*x + c)^4 + 20*(3*a^2 - 5*a*b + 2*b^2)*cosh(d*x + c)^2 + 3*a^2 + a*b - 2*b^2)*sinh(d*x + c)^3 + (21*(3*a^2 + a*b - 2*b^2)*cosh(d*x + c)^5 + 20*(3*a^2 - 5*a*b + 2*b^2)*cosh(d*x + c)^3 + 3*(3*a^2 + a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + (7*(3*a^2 + a*b - 2*b^2)*cosh(d*x + c)^6 + 10*(3*a^2 - 5*a*b + 2*b^2)*cosh(d*x + c)^4 + 3*(3*a^2 + a*b - 2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(b/(a + b))*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a + 3*b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + 3*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a + b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))*sqrt(b/(a + b)) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + a^2 + 2*a*b + b^2 + 2*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^9 - 4*(7*a^2 - 6*a*b - 13*b^2)*cosh(d*x + c)^7 - 6*(13*a^2 - 40*a*b + 7*b^2)*cosh(d*x + c)^5 - 4*(13*a^2 - 40*a*b + 7*b^2)*cosh(d*x + c)^3 - (7*a^2 - 6*a*b - 13*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^7 + 7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)*sinh(d*x + c)^6 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*sinh(d*x + c)^7 + 2*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d*cosh(d*x + c)^5 + (21*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^2 + 2*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d)*sinh(d*x + c)^5 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^3 + 5*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^3 + 2*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x + c)^4 + (35*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^4 + 20*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d*cosh(d*x + c)^2 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d)*sinh(d*x + c)^3 + (21*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^5 + 20*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d*cosh(d*x + c)^3 + 3*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c))*sinh(d*x + c)^2 + (7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^6 + 10*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d*cosh(d*x + c)^4 + 3*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^2)*sinh(d*x + c)), 1/24*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^10 + 10*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^9 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^10 - (7*a^2 - 6*a*b - 13*b^2)*cosh(d*x + c)^8 + (45*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 - 7*a^2 + 6*a*b + 13*b^2)*sinh(d*x + c)^8 + 8*(15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - (7*a^2 - 6*a*b - 13*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(13*a^2 - 40*a*b + 7*b^2)*cosh(d*x + c)^6 + 2*(105*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 - 14*(7*a^2 - 6*a*b - 13*b^2)*cosh(d*x + c)^2 - 13*a^2 + 40*a*b - 7*b^2)*sinh(d*x + c)^6 + 4*(63*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 - 14*(7*a^2 - 6*a*b - 13*b^2)*cosh(d*x + c)^3 - 3*(13*a^2 - 40*a*b + 7*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(13*a^2 - 40*a*b + 7*b^2)*cosh(d*x + c)^4 + 2*(105*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 - 35*(7*a^2 - 6*a*b - 13*b^2)*cosh(d*x + c)^4 - 15*(13*a^2 - 40*a*b + 7*b^2)*cosh(d*x + c)^2 - 13*a^2 + 40*a*b - 7*b^2)*sinh(d*x + c)^4 + 8*(15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 - 7*(7*a^2 - 6*a*b - 13*b^2)*cosh(d*x + c)^5 - 5*(13*a^2 - 40*a*b + 7*b^2)*cosh(d*x + c)^3 - (13*a^2 - 40*a*b + 7*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - (7*a^2 - 6*a*b - 13*b^2)*cosh(d*x + c)^2 + (45*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 - 28*(7*a^2 - 6*a*b - 13*b^2)*cosh(d*x + c)^6 - 30*(13*a^2 - 40*a*b + 7*b^2)*cosh(d*x + c)^4 - 12*(13*a^2 - 40*a*b + 7*b^2)*cosh(d*x + c)^2 - 7*a^2 + 6*a*b + 13*b^2)*sinh(d*x + c)^2 + 12*((3*a^2 + a*b - 2*b^2)*cosh(d*x + c)^7 + 7*(3*a^2 + a*b - 2*b^2)*cosh(d*x +

```
c)*sinh(d*x + c)^6 + (3*a^2 + a*b - 2*b^2)*sinh(d*x + c)^7 + 2*(3*a^2 - 5*a
*b + 2*b^2)*cosh(d*x + c)^5 + (21*(3*a^2 + a*b - 2*b^2)*cosh(d*x + c)^2 + 6
*a^2 - 10*a*b + 4*b^2)*sinh(d*x + c)^5 + 5*(7*(3*a^2 + a*b - 2*b^2)*cosh(d*
x + c)^3 + 2*(3*a^2 - 5*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + (3*a^
2 + a*b - 2*b^2)*cosh(d*x + c)^3 + (35*(3*a^2 + a*b - 2*b^2)*cosh(d*x + c)^
4 + 20*(3*a^2 - 5*a*b + 2*b^2)*cosh(d*x + c)^2 + 3*a^2 + a*b - 2*b^2)*sinh(
d*x + c)^3 + (21*(3*a^2 + a*b - 2*b^2)*cosh(d*x + c)^5 + 20*(3*a^2 - 5*a*b
+ 2*b^2)*cosh(d*x + c)^3 + 3*(3*a^2 + a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x
+ c)^2 + (7*(3*a^2 + a*b - 2*b^2)*cosh(d*x + c)^6 + 10*(3*a^2 - 5*a*b + 2*b
^2)*cosh(d*x + c)^4 + 3*(3*a^2 + a*b - 2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c
))*sqrt(-b/(a + b))*arctan(1/2*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*
x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a - 3*b)*cosh(d*x + c)
+ (3*(a + b)*cosh(d*x + c)^2 + a - 3*b)*sinh(d*x + c))*sqrt(-b/(a + b))/b)
- 12*((3*a^2 + a*b - 2*b^2)*cosh(d*x + c)^7 + 7*(3*a^2 + a*b - 2*b^2)*cosh(
d*x + c)*sinh(d*x + c)^6 + (3*a^2 + a*b - 2*b^2)*sinh(d*x + c)^7 + 2*(3*a^2
 - 5*a*b + 2*b^2)*cosh(d*x + c)^5 + (21*(3*a^2 + a*b - 2*b^2)*cosh(d*x + c)
^2 + 6*a^2 - 10*a*b + 4*b^2)*sinh(d*x + c)^5 + 5*(7*(3*a^2 + a*b - 2*b^2)*c
osh(d*x + c)^3 + 2*(3*a^2 - 5*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 +
 (3*a^2 + a*b - 2*b^2)*cosh(d*x + c)^3 + (35*(3*a^2 + a*b - 2*b^2)*cosh(d*x
 + c)^4 + 20*(3*a^2 - 5*a*b + 2*b^2)*cosh(d*x + c)^2 + 3*a^2 + a*b - 2*b^2)
*sinh(d*x + c)^3 + (21*(3*a^2 + a*b - 2*b^2)*cosh(d*x + c)^5 + 20*(3*a^2 -
5*a*b + 2*b^2)*cosh(d*x + c)^3 + 3*(3*a^2 + a*b - 2*b^2)*cosh(d*x + c))*sin
h(d*x + c)^2 + (7*(3*a^2 + a*b - 2*b^2)*cosh(d*x + c)^6 + 10*(3*a^2 - 5*a*b
 + 2*b^2)*cosh(d*x + c)^4 + 3*(3*a^2 + a*b - 2*b^2)*cosh(d*x + c)^2)*sinh(d
*x + c))*sqrt(-b/(a + b))*arctan(1/2*((a + b)*cosh(d*x + c) + (a + b)*sinh(
d*x + c))*sqrt(-b/(a + b))/b) + a^2 + 2*a*b + b^2 + 2*(5*(a^2 + 2*a*b + b^2
)*cosh(d*x + c)^9 - 4*(7*a^2 - 6*a*b - 13*b^2)*cosh(d*x + c)^7 - 6*(13*a^2
- 40*a*b + 7*b^2)*cosh(d*x + c)^5 - 4*(13*a^2 - 40*a*b + 7*b^2)*cosh(d*x +
c)^3 - (7*a^2 - 6*a*b - 13*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 4*a^3
*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^7 + 7*(a^4 + 4*a^3*b + 6*a^
2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)*sinh(d*x + c)^6 + (a^4 + 4*a^3*b + 6
*a^2*b^2 + 4*a*b^3 + b^4)*d*sinh(d*x + c)^7 + 2*(a^4 + 2*a^3*b - 2*a*b^3 -
b^4)*d*cosh(d*x + c)^5 + (21*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*
cosh(d*x + c)^2 + 2*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d)*sinh(d*x + c)^5 + (a
^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^3 + 5*(7*(a^4 + 4
*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^3 + 2*(a^4 + 2*a^3*b -
2*a*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x + c)^4 + (35*(a^4 + 4*a^3*b + 6*a^
2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^4 + 20*(a^4 + 2*a^3*b - 2*a*b^3 - b^
4)*d*cosh(d*x + c)^2 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d)*sinh(
d*x + c)^3 + (21*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c
)^5 + 20*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d*cosh(d*x + c)^3 + 3*(a^4 + 4*a^3
*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c))*sinh(d*x + c)^2 + (7*(a^4
+ 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^6 + 10*(a^4 + 2*a^3*
b - 2*a*b^3 - b^4)*d*cosh(d*x + c)^4 + 3*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b
^3 + b^4)*d*cosh(d*x + c)^2)*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a+b*tanh(d*x+c)**2)**2,x)

[Out] Timed out
```

Giac [C] time = 2.47194, size = 8694, normalized size = 70.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/24*(6*(3*(3*a^5*b*e^(4*c) + 10*a^4*b^2*e^(4*c) + 10*a^3*b^3*e^(4*c) - 5*
a*b^5*e^(4*c) - 2*b^6*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a +
 b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real
_part(arccos(-a/(a + b) + b/(a + b)))) - (3*a^5*b*e^(4*c) + 10*a^4*b^2*e^(4
*c) + 10*a^3*b^3*e^(4*c) - 5*a*b^5*e^(4*c) - 2*b^6*e^(4*c))*cosh(1/2*imag_p
art(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b)
+ b/(a + b))))^3 - 9*(3*a^5*b*e^(4*c) + 10*a^4*b^2*e^(4*c) + 10*a^3*b^3*e^(
4*c) - 5*a*b^5*e^(4*c) - 2*b^6*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b)
 + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin
(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-
a/(a + b) + b/(a + b)))) + 3*(3*a^5*b*e^(4*c) + 10*a^4*b^2*e^(4*c) + 10*a^3
*b^3*e^(4*c) - 5*a*b^5*e^(4*c) - 2*b^6*e^(4*c))*cosh(1/2*imag_part(arccos(-
a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))
))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(3*a^5*b*e^(4*
c) + 10*a^4*b^2*e^(4*c) + 10*a^3*b^3*e^(4*c) - 5*a*b^5*e^(4*c) - 2*b^6*e^(4
*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_par
t(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/
(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(3*a^5
*b*e^(4*c) + 10*a^4*b^2*e^(4*c) + 10*a^3*b^3*e^(4*c) - 5*a*b^5*e^(4*c) - 2*
b^6*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*re
al_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a
+ b) + b/(a + b))))^2 - 3*(3*a^5*b*e^(4*c) + 10*a^4*b^2*e^(4*c) + 10*a^3*b^
3*e^(4*c) - 5*a*b^5*e^(4*c) - 2*b^6*e^(4*c))*cos(1/2*real_part(arccos(-a/(a
 + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*s
inh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (3*a^5*b*e^(4*c) + 1
0*a^4*b^2*e^(4*c) + 10*a^3*b^3*e^(4*c) - 5*a*b^5*e^(4*c) - 2*b^6*e^(4*c))*s
in(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arcc
os(-a/(a + b) + b/(a + b))))^3 - (3*a^5*b*e^(4*c) + 10*a^4*b^2*e^(4*c) + 10
*a^3*b^3*e^(4*c) - 5*a*b^5*e^(4*c) - 2*b^6*e^(4*c))*cosh(1/2*imag_part(arcc
os(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b
)))) + (3*a^5*b*e^(4*c) + 10*a^4*b^2*e^(4*c) + 10*a^3*b^3*e^(4*c) - 5*a*b^5
*e^(4*c) - 2*b^6*e^(4*c))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))
)*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*arctan((((a^4 + 4*a^
3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)/(a^4*e^(4*c) + 4*a^3*b*e^(4*c) + 6*a^2*b^2
*e^(4*c) + 4*a*b^3*e^(4*c) + b^4*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a
 + b))) + e^(d*x))/(((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)/(a^4*e^(4*
c) + 4*a^3*b*e^(4*c) + 6*a^2*b^2*e^(4*c) + 4*a*b^3*e^(4*c) + b^4*e^(4*c)))^
(1/4)*sin(1/2*arccos(-(a - b)/(a + b)))))/(2*(a^3*e^(2*c) + 3*a^2*b*e^(2*c)
 + 3*a*b^2*e^(2*c) + b^3*e^(2*c))^2*a*b + (a^4*e^(2*c) + 2*a^3*b*e^(2*c) -
2*a*b^3*e^(2*c) - b^4*e^(2*c))*sqrt(-a*b)*abs(-a^3*e^(2*c) - 3*a^2*b*e^(2*c
) - 3*a*b^2*e^(2*c) - b^3*e^(2*c))) + 6*(3*(3*a^5*b*e^(4*c) + 10*a^4*b^2*e^
(4*c) + 10*a^3*b^3*e^(4*c) - 5*a*b^5*e^(4*c) - 2*b^6*e^(4*c))*cos(1/2*real_
part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b
) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) - (3*
a^5*b*e^(4*c) + 10*a^4*b^2*e^(4*c) + 10*a^3*b^3*e^(4*c) - 5*a*b^5*e^(4*c) -
 2*b^6*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1
/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3 - 9*(3*a^5*b*e^(4*c) + 10*a
^4*b^2*e^(4*c) + 10*a^3*b^3*e^(4*c) - 5*a*b^5*e^(4*c) - 2*b^6*e^(4*c))*cos(
1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(
-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)
)))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(3*a^5*b*e^(4*c
```

```
) + 10*a^4*b^2*e^(4*c) + 10*a^3*b^3*e^(4*c) - 5*a*b^5*e^(4*c) - 2*b^6*e^(4*
c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part
(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) +
b/(a + b)))) + 9*(3*a^5*b*e^(4*c) + 10*a^4*b^2*e^(4*c) + 10*a^3*b^3*e^(4*c)
 - 5*a*b^5*e^(4*c) - 2*b^6*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b
/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*r
eal_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a +
 b) + b/(a + b))))^2 - 3*(3*a^5*b*e^(4*c) + 10*a^4*b^2*e^(4*c) + 10*a^3*b^3
*e^(4*c) - 5*a*b^5*e^(4*c) - 2*b^6*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a
 + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*s
inh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(3*a^5*b*e^(4*c) +
 10*a^4*b^2*e^(4*c) + 10*a^3*b^3*e^(4*c) - 5*a*b^5*e^(4*c) - 2*b^6*e^(4*c))
*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arc
cos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a +
 b))))^3 + (3*a^5*b*e^(4*c) + 10*a^4*b^2*e^(4*c) + 10*a^3*b^3*e^(4*c) - 5*a
*b^5*e^(4*c) - 2*b^6*e^(4*c))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a +
b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 - (3*a^5*b*e^
(4*c) + 10*a^4*b^2*e^(4*c) + 10*a^3*b^3*e^(4*c) - 5*a*b^5*e^(4*c) - 2*b^6*e
^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_pa
rt(arccos(-a/(a + b) + b/(a + b)))) + (3*a^5*b*e^(4*c) + 10*a^4*b^2*e^(4*c)
 + 10*a^3*b^3*e^(4*c) - 5*a*b^5*e^(4*c) - 2*b^6*e^(4*c))*sin(1/2*real_part(
arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(
a + b)))))*arctan(-(((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)/(a^4*e^(4*
c) + 4*a^3*b*e^(4*c) + 6*a^2*b^2*e^(4*c) + 4*a*b^3*e^(4*c) + b^4*e^(4*c)))^
(1/4)*cos(1/2*arccos(-(a - b)/(a + b))) - e^(d*x))/(((a^4 + 4*a^3*b + 6*a^2
*b^2 + 4*a*b^3 + b^4)/(a^4*e^(4*c) + 4*a^3*b*e^(4*c) + 6*a^2*b^2*e^(4*c) +
4*a*b^3*e^(4*c) + b^4*e^(4*c)))^(1/4)*sin(1/2*arccos(-(a - b)/(a + b)))))/(
2*(a^3*e^(2*c) + 3*a^2*b*e^(2*c) + 3*a*b^2*e^(2*c) + b^3*e^(2*c))^2*a*b + (
a^4*e^(2*c) + 2*a^3*b*e^(2*c) - 2*a*b^3*e^(2*c) - b^4*e^(2*c))*sqrt(-a*b)*a
bs(-a^3*e^(2*c) - 3*a^2*b*e^(2*c) - 3*a*b^2*e^(2*c) - b^3*e^(2*c))) + (9*a*
e^(2*d*x + 2*c) - 15*b*e^(2*d*x + 2*c) - a - b)*e^(-3*d*x)/(a^3*e^(3*c) + 3
*a^2*b*e^(3*c) + 3*a*b^2*e^(3*c) + b^3*e^(3*c)) + 3*((3*a^5*b*e^(4*c) + 10*
a^4*b^2*e^(4*c) + 10*a^3*b^3*e^(4*c) - 5*a*b^5*e^(4*c) - 2*b^6*e^(4*c))*cos
(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos
(-a/(a + b) + b/(a + b))))^3 - 3*(3*a^5*b*e^(4*c) + 10*a^4*b^2*e^(4*c) + 10
*a^3*b^3*e^(4*c) - 5*a*b^5*e^(4*c) - 2*b^6*e^(4*c))*cos(1/2*real_part(arcco
s(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b
))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(3*a^5*b*e^
(4*c) + 10*a^4*b^2*e^(4*c) + 10*a^3*b^3*e^(4*c) - 5*a*b^5*e^(4*c) - 2*b^6*e
^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_
part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b
) + b/(a + b)))) + 9*(3*a^5*b*e^(4*c) + 10*a^4*b^2*e^(4*c) + 10*a^3*b^3*e^(
4*c) - 5*a*b^5*e^(4*c) - 2*b^6*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b)
 + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1
/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-
a/(a + b) + b/(a + b)))) + 3*(3*a^5*b*e^(4*c) + 10*a^4*b^2*e^(4*c) + 10*a^3
*b^3*e^(4*c) - 5*a*b^5*e^(4*c) - 2*b^6*e^(4*c))*cos(1/2*real_part(arccos(-a
/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))
))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 9*(3*a^5*b*e^(4*
c) + 10*a^4*b^2*e^(4*c) + 10*a^3*b^3*e^(4*c) - 5*a*b^5*e^(4*c) - 2*b^6*e^(4
*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(
arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a
 + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - (3*a^5*b
*e^(4*c) + 10*a^4*b^2*e^(4*c) + 10*a^3*b^3*e^(4*c) - 5*a*b^5*e^(4*c) - 2*b^
6*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*im
ag_part(arccos(-a/(a + b) + b/(a + b))))^3 + 3*(3*a^5*b*e^(4*c) + 10*a^4*b^
2*e^(4*c) + 10*a^3*b^3*e^(4*c) - 5*a*b^5*e^(4*c) - 2*b^6*e^(4*c))*cos(1/2*r
eal_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a +
b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 -
```

```
(3*a^5*b*e^(4*c) + 10*a^4*b^2*e^(4*c) + 10*a^3*b^3*e^(4*c) - 5*a*b^5*e^(4*
c) - 2*b^6*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh
(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + (3*a^5*b*e^(4*c) + 10*a^4
*b^2*e^(4*c) + 10*a^3*b^3*e^(4*c) - 5*a*b^5*e^(4*c) - 2*b^6*e^(4*c))*cos(1/
2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(
a + b) + b/(a + b)))))*log(2*((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)/(
a^4*e^(4*c) + 4*a^3*b*e^(4*c) + 6*a^2*b^2*e^(4*c) + 4*a*b^3*e^(4*c) + b^4*e
^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b)))*e^(d*x) + sqrt((a^4 + 4*a^
3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)/(a^4*e^(4*c) + 4*a^3*b*e^(4*c) + 6*a^2*b^2
*e^(4*c) + 4*a*b^3*e^(4*c) + b^4*e^(4*c))) + e^(2*d*x))/(2*(a^3*e^(2*c) + 3
*a^2*b*e^(2*c) + 3*a*b^2*e^(2*c) + b^3*e^(2*c))^2*a*b + (a^4*e^(2*c) + 2*a^
3*b*e^(2*c) - 2*a*b^3*e^(2*c) - b^4*e^(2*c))*sqrt(-a*b)*abs(-a^3*e^(2*c) -
3*a^2*b*e^(2*c) - 3*a*b^2*e^(2*c) - b^3*e^(2*c))) - 3*((3*a^5*b*e^(4*c) + 1
0*a^4*b^2*e^(4*c) + 10*a^3*b^3*e^(4*c) - 5*a*b^5*e^(4*c) - 2*b^6*e^(4*c))*c
os(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arcc
os(-a/(a + b) + b/(a + b))))^3 - 3*(3*a^5*b*e^(4*c) + 10*a^4*b^2*e^(4*c) +
10*a^3*b^3*e^(4*c) - 5*a*b^5*e^(4*c) - 2*b^6*e^(4*c))*cos(1/2*real_part(arc
cos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a +
 b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(3*a^5*b*
e^(4*c) + 10*a^4*b^2*e^(4*c) + 10*a^3*b^3*e^(4*c) - 5*a*b^5*e^(4*c) - 2*b^6
*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*ima
g_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a +
 b) + b/(a + b)))) + 9*(3*a^5*b*e^(4*c) + 10*a^4*b^2*e^(4*c) + 10*a^3*b^3*e
^(4*c) - 5*a*b^5*e^(4*c) - 2*b^6*e^(4*c))*cos(1/2*real_part(arccos(-a/(a +
b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin
(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos
(-a/(a + b) + b/(a + b)))) + 3*(3*a^5*b*e^(4*c) + 10*a^4*b^2*e^(4*c) + 10*a
^3*b^3*e^(4*c) - 5*a*b^5*e^(4*c) - 2*b^6*e^(4*c))*cos(1/2*real_part(arccos(
-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b
))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 9*(3*a^5*b*e^(
4*c) + 10*a^4*b^2*e^(4*c) + 10*a^3*b^3*e^(4*c) - 5*a*b^5*e^(4*c) - 2*b^6*e^
(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_par
t(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/
(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - (3*a^5
*b*e^(4*c) + 10*a^4*b^2*e^(4*c) + 10*a^3*b^3*e^(4*c) - 5*a*b^5*e^(4*c) - 2*
b^6*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*
imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + 3*(3*a^5*b*e^(4*c) + 10*a^4*
b^2*e^(4*c) + 10*a^3*b^3*e^(4*c) - 5*a*b^5*e^(4*c) - 2*b^6*e^(4*c))*cos(1/2
*real_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a
+ b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3
 - (3*a^5*b*e^(4*c) + 10*a^4*b^2*e^(4*c) + 10*a^3*b^3*e^(4*c) - 5*a*b^5*e^(
4*c) - 2*b^6*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*co
sh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + (3*a^5*b*e^(4*c) + 10*a
^4*b^2*e^(4*c) + 10*a^3*b^3*e^(4*c) - 5*a*b^5*e^(4*c) - 2*b^6*e^(4*c))*cos(
1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a
/(a + b) + b/(a + b)))))*log(-2*((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4
)/(a^4*e^(4*c) + 4*a^3*b*e^(4*c) + 6*a^2*b^2*e^(4*c) + 4*a*b^3*e^(4*c) + b^
4*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b)))*e^(d*x) + sqrt((a^4 + 4
*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)/(a^4*e^(4*c) + 4*a^3*b*e^(4*c) + 6*a^2*
b^2*e^(4*c) + 4*a*b^3*e^(4*c) + b^4*e^(4*c))) + e^(2*d*x))/(2*(a^3*e^(2*c)
+ 3*a^2*b*e^(2*c) + 3*a*b^2*e^(2*c) + b^3*e^(2*c))^2*a*b + (a^4*e^(2*c) + 2
*a^3*b*e^(2*c) - 2*a*b^3*e^(2*c) - b^4*e^(2*c))*sqrt(-a*b)*abs(-a^3*e^(2*c)
 - 3*a^2*b*e^(2*c) - 3*a*b^2*e^(2*c) - b^3*e^(2*c))) - (a^4*e^(3*d*x + 36*c
) + 4*a^3*b*e^(3*d*x + 36*c) + 6*a^2*b^2*e^(3*d*x + 36*c) + 4*a*b^3*e^(3*d*
x + 36*c) + b^4*e^(3*d*x + 36*c) - 9*a^4*e^(d*x + 34*c) - 12*a^3*b*e^(d*x +
 34*c) + 18*a^2*b^2*e^(d*x + 34*c) + 36*a*b^3*e^(d*x + 34*c) + 15*b^4*e^(d*
x + 34*c))/(a^6*e^(33*c) + 6*a^5*b*e^(33*c) + 15*a^4*b^2*e^(33*c) + 20*a^3*
b^3*e^(33*c) + 15*a^2*b^4*e^(33*c) + 6*a*b^5*e^(33*c) + b^6*e^(33*c)) - 24*
(a*b*e^(3*d*x + 3*c) + a*b*e^(d*x + c))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(a
```

```
*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x +
 2*c) + a + b)))/d
```

3.35 $\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=132

$$-\frac{b\tanh(c+dx)}{d(a+b)^2\left(a+b\tanh^2(c+dx)\right)}-\frac{\sqrt{b}(3a-b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}d(a+b)^3}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{x(a-3b)}{2(a+b)^3}$$

```
[Out] -((a - 3*b)*x)/(2*(a + b)^3) - ((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c +
d*x])/Sqrt[a]])/(2*Sqrt[a]*(a + b)^3*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*
(a + b)*d*(a + b*Tanh[c + d*x]^2)) - (b*Tanh[c + d*x])/((a + b)^2*d*(a + b*
Tanh[c + d*x]^2))
```

Rubi [A] time = 0.165343, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3663, 471, 527, 522, 206, 205}

$$-\frac{b\tanh(c+dx)}{d(a+b)^2\left(a+b\tanh^2(c+dx)\right)}-\frac{\sqrt{b}(3a-b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}d(a+b)^3}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{x(a-3b)}{2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```


```
[Out] -((a - 3*b)*x)/(2*(a + b)^3) - ((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c +
d*x])/Sqrt[a]])/(2*Sqrt[a]*(a + b)^3*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*
(a + b)*d*(a + b*Tanh[c + d*x]^2)) - (b*Tanh[c + d*x])/((a + b)^2*d*(a + b*
Tanh[c + d*x]^2))
```


Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)^2\left(a+bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{a-3bx^2}{\left(1-x^2\right)\left(a+bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{2(a+b)d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{b\tanh(c+dx)}{(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{-2a(a-\cdots}{\left(1-x^2\right)\cdots}\cdots\right.}{\cdots}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{b\tanh(c+dx)}{(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} - \frac{(a-3b)\,\text{Subst}\left(\cdots\right.}{\cdots}$$

$$= -\frac{(a-3b)x}{2(a+b)^3} - \frac{(3a-b)\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}(a+b)^3d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{\cdots}{(a\cdots}$$

Mathematica [A] time = 0.649926, size = 105, normalized size = 0.8

$$\frac{-2(a-3b)(c+dx)+(a+b)\sinh(2(c+dx))+\frac{2\sqrt{b}(b-3a)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2b(a+b)\sinh(2(c+dx))}{(a+b)\cosh(2(c+dx))+a-b}}{4d(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (-2*(a - 3*b)*(c + d*x) + (2*Sqrt[b]*(-3*a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*
x])/Sqrt[a]])/Sqrt[a] + (a + b)*Sinh[2*(c + d*x)] - (2*b*(a + b)*Sinh[2*(c
+ d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)]))/(4*(a + b)^3*d)
```

Maple [B] time = 0.099, size = 1128, normalized size = 8.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] -1/2/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)
+1)-1/2/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)+1)*a+3/2/d/(a+b)^3*ln(tanh(1/2*d*x
+1/2*c)+1)*b-1/d*b/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2
*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*tanh(1/2*d*x+1/2*c)^3*a-1/d*b^2/(a+b)^3/(ta
nh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a
)*tanh(1/2*d*x+1/2*c)^3-1/d*b/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d
*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*tanh(1/2*d*x+1/2*c)*a-1/d*b^2/(a
+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2
*c)^2*b+a)*tanh(1/2*d*x+1/2*c)+3/2/d*b/(a+b)^3*a^2/(b*(a+b))^(1/2)/((2*(b*(
a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/
2)-a-2*b)*a)^(1/2))-3/2/d*b/(a+b)^3*a/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*a
rctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/d*b^2/(
a+b)^3*a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh
(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+3/2/d*b/(a+b)^3*a^2/(b
*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2
*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+3/2/d*b/(a+b)^3*a/((2*(b*(a+b))^(1
/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)
*a)^(1/2))+1/d*b^2/(a+b)^3*a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^
(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/2
/d*b^2/(a+b)^3/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1
/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-1/2/d*b^3/(a+b)^3/(b*(a+b))^(1/2
)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*
(a+b))^(1/2)-a-2*b)*a)^(1/2))-1/2/d*b^2/(a+b)^3/((2*(b*(a+b))^(1/2)+a+2*b)*
a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-
1/2/d*b^3/(a+b)^3/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arcta
n(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/2/d/(a+b)^2/
(tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)+1/2/d/(a+b)
^3*ln(tanh(1/2*d*x+1/2*c)-1)*a-3/2/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)-1)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.67746, size = 9441, normalized size = 71.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x
+ c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^8 - 2*(2*(a^2 - 2*
a*b - 3*b^2)*d*x - a^2 + b^2)*cosh(d*x + c)^6 - 2*(2*(a^2 - 2*a*b - 3*b^2)*
d*x - 14*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 - a^2 + b^2)*sinh(d*x + c)^6 +
 4*(14*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - 3*(2*(a^2 - 2*a*b - 3*b^2)*d*x
 - a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 8*((a^2 - 4*a*b + 3*b^2)*d*x
 - a*b + b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 -
 4*(a^2 - 4*a*b + 3*b^2)*d*x - 15*(2*(a^2 - 2*a*b - 3*b^2)*d*x - a^2 + b^2)
*cosh(d*x + c)^2 + 4*a*b - 4*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2
)*cosh(d*x + c)^5 - 5*(2*(a^2 - 2*a*b - 3*b^2)*d*x - a^2 + b^2)*cosh(d*x +
c)^3 - 4*((a^2 - 4*a*b + 3*b^2)*d*x - a*b + b^2)*cosh(d*x + c))*sinh(d*x +
c)^3 - 2*(2*(a^2 - 2*a*b - 3*b^2)*d*x + a^2 - 4*a*b - 5*b^2)*cosh(d*x + c)^
2 + 2*(14*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 - 15*(2*(a^2 - 2*a*b - 3*b^2)
*d*x - a^2 + b^2)*cosh(d*x + c)^4 - 2*(a^2 - 2*a*b - 3*b^2)*d*x - 24*((a^2
- 4*a*b + 3*b^2)*d*x - a*b + b^2)*cosh(d*x + c)^2 - a^2 + 4*a*b + 5*b^2)*si
nh(d*x + c)^2 - 2*((3*a^2 + 2*a*b - b^2)*cosh(d*x + c)^6 + 6*(3*a^2 + 2*a*b
 - b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (3*a^2 + 2*a*b - b^2)*sinh(d*x + c)
^6 + 2*(3*a^2 - 4*a*b + b^2)*cosh(d*x + c)^4 + (15*(3*a^2 + 2*a*b - b^2)*co
sh(d*x + c)^2 + 6*a^2 - 8*a*b + 2*b^2)*sinh(d*x + c)^4 + 4*(5*(3*a^2 + 2*a*
b - b^2)*cosh(d*x + c)^3 + 2*(3*a^2 - 4*a*b + b^2)*cosh(d*x + c))*sinh(d*x
+ c)^3 + (3*a^2 + 2*a*b - b^2)*cosh(d*x + c)^2 + (15*(3*a^2 + 2*a*b - b^2)*
cosh(d*x + c)^4 + 12*(3*a^2 - 4*a*b + b^2)*cosh(d*x + c)^2 + 3*a^2 + 2*a*b
- b^2)*sinh(d*x + c)^2 + 2*(3*(3*a^2 + 2*a*b - b^2)*cosh(d*x + c)^5 + 4*(3*
a^2 - 4*a*b + b^2)*cosh(d*x + c)^3 + (3*a^2 + 2*a*b - b^2)*cosh(d*x + c))*s
inh(d*x + c))*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2
+ 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x
 + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x
 + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b
+ b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2
 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2
+ a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 +
4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a -
b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2
+ 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b
)) - a^2 - 2*a*b - b^2 + 4*(2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 - 3*(2*(a
^2 - 2*a*b - 3*b^2)*d*x - a^2 + b^2)*cosh(d*x + c)^5 - 8*((a^2 - 4*a*b + 3*
b^2)*d*x - a*b + b^2)*cosh(d*x + c)^3 - (2*(a^2 - 2*a*b - 3*b^2)*d*x + a^2
- 4*a*b - 5*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 4*a^3*b + 6*a^2*b^2
+ 4*a*b^3 + b^4)*d*cosh(d*x + c)^6 + 6*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3
 + b^4)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*
b^3 + b^4)*d*sinh(d*x + c)^6 + 2*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d*cosh(d*x
 + c)^4 + (15*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^2
 + 2*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d)*sinh(d*x + c)^4 + (a^4 + 4*a^3*b +
6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^2 + 4*(5*(a^4 + 4*a^3*b + 6*a^2*
b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^3 + 2*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*
d*cosh(d*x + c))*sinh(d*x + c)^3 + (15*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3
 + b^4)*d*cosh(d*x + c)^4 + 12*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d*cosh(d*x +
 c)^2 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d)*sinh(d*x + c)^2 + 2*
(3*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^5 + 4*(a^4 +
 2*a^3*b - 2*a*b^3 - b^4)*d*cosh(d*x + c)^3 + (a^4 + 4*a^3*b + 6*a^2*b^2 +
4*a*b^3 + b^4)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*((a^2 + 2*a*b + b^2)*co
sh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2
+ 2*a*b + b^2)*sinh(d*x + c)^8 - 2*(2*(a^2 - 2*a*b - 3*b^2)*d*x - a^2 + b^2
)*cosh(d*x + c)^6 - 2*(2*(a^2 - 2*a*b - 3*b^2)*d*x - 14*(a^2 + 2*a*b + b^2)
*cosh(d*x + c)^2 - a^2 + b^2)*sinh(d*x + c)^6 + 4*(14*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)^3 - 3*(2*(a^2 - 2*a*b - 3*b^2)*d*x - a^2 + b^2)*cosh(d*x + c))
*sinh(d*x + c)^5 - 8*((a^2 - 4*a*b + 3*b^2)*d*x - a*b + b^2)*cosh(d*x + c)^
4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 - 4*(a^2 - 4*a*b + 3*b^2)*d*x
 - 15*(2*(a^2 - 2*a*b - 3*b^2)*d*x - a^2 + b^2)*cosh(d*x + c)^2 + 4*a*b - 4
```

```
*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 - 5*(2*(a^
2 - 2*a*b - 3*b^2)*d*x - a^2 + b^2)*cosh(d*x + c)^3 - 4*((a^2 - 4*a*b + 3*b
^2)*d*x - a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*(2*(a^2 - 2*a*b - 3
*b^2)*d*x + a^2 - 4*a*b - 5*b^2)*cosh(d*x + c)^2 + 2*(14*(a^2 + 2*a*b + b^2
)*cosh(d*x + c)^6 - 15*(2*(a^2 - 2*a*b - 3*b^2)*d*x - a^2 + b^2)*cosh(d*x +
 c)^4 - 2*(a^2 - 2*a*b - 3*b^2)*d*x - 24*((a^2 - 4*a*b + 3*b^2)*d*x - a*b +
 b^2)*cosh(d*x + c)^2 - a^2 + 4*a*b + 5*b^2)*sinh(d*x + c)^2 - 4*((3*a^2 +
2*a*b - b^2)*cosh(d*x + c)^6 + 6*(3*a^2 + 2*a*b - b^2)*cosh(d*x + c)*sinh(d
*x + c)^5 + (3*a^2 + 2*a*b - b^2)*sinh(d*x + c)^6 + 2*(3*a^2 - 4*a*b + b^2)
*cosh(d*x + c)^4 + (15*(3*a^2 + 2*a*b - b^2)*cosh(d*x + c)^2 + 6*a^2 - 8*a*
b + 2*b^2)*sinh(d*x + c)^4 + 4*(5*(3*a^2 + 2*a*b - b^2)*cosh(d*x + c)^3 + 2
*(3*a^2 - 4*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + (3*a^2 + 2*a*b - b^
2)*cosh(d*x + c)^2 + (15*(3*a^2 + 2*a*b - b^2)*cosh(d*x + c)^4 + 12*(3*a^2
- 4*a*b + b^2)*cosh(d*x + c)^2 + 3*a^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 2*(
3*(3*a^2 + 2*a*b - b^2)*cosh(d*x + c)^5 + 4*(3*a^2 - 4*a*b + b^2)*cosh(d*x
+ c)^3 + (3*a^2 + 2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/a)*arct
an(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (
a + b)*sinh(d*x + c)^2 + a - b)*sqrt(b/a)/b) - a^2 - 2*a*b - b^2 + 4*(2*(a^
2 + 2*a*b + b^2)*cosh(d*x + c)^7 - 3*(2*(a^2 - 2*a*b - 3*b^2)*d*x - a^2 + b
^2)*cosh(d*x + c)^5 - 8*((a^2 - 4*a*b + 3*b^2)*d*x - a*b + b^2)*cosh(d*x +
c)^3 - (2*(a^2 - 2*a*b - 3*b^2)*d*x + a^2 - 4*a*b - 5*b^2)*cosh(d*x + c))*s
inh(d*x + c))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^
6 + 6*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)*sinh(d*x
+ c)^5 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*sinh(d*x + c)^6 + 2*
(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d*cosh(d*x + c)^4 + (15*(a^4 + 4*a^3*b + 6*
a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^2 + 2*(a^4 + 2*a^3*b - 2*a*b^3 - b
^4)*d)*sinh(d*x + c)^4 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh
(d*x + c)^2 + 4*(5*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x +
 c)^3 + 2*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3
+ (15*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^4 + 12*(a
^4 + 2*a^3*b - 2*a*b^3 - b^4)*d*cosh(d*x + c)^2 + (a^4 + 4*a^3*b + 6*a^2*b^
2 + 4*a*b^3 + b^4)*d)*sinh(d*x + c)^2 + 2*(3*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4
*a*b^3 + b^4)*d*cosh(d*x + c)^5 + 4*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d*cosh(
d*x + c)^3 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c))*s
inh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.25285, size = 540, normalized size = 4.09

$$-\frac{\frac{12\,(a-3\,b)dx}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{12\left(3\,abe^{(2\,c)}-b^2e^{(2\,c)}\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)e^{(-2\,c)}}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\sqrt{ab}}-\frac{3\,e^{(2\,dx+12\,c)}}{a^2e^{(10\,c)}+2\,abe^{(10\,c)}+b^2e^{(10\,c)}}-\frac{2\,a^2e^{(6\,dx+6\,c)}-4\,abe^{(6\,dx+6\,c)}-6\,b^2e^{(\cdots}}{\left(a^3e^{(2\,c)}+3\,a^2b\cdots\right.}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/24*(12*(a - 3*b)*d*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 12*(3*a*b*e^(2*c)
 - b^2*e^(2*c))*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/
sqrt(a*b))*e^(-2*c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b)) - 3*e^(2*d*
x + 12*c)/(a^2*e^(10*c) + 2*a*b*e^(10*c) + b^2*e^(10*c)) - (2*a^2*e^(6*d*x
+ 6*c) - 4*a*b*e^(6*d*x + 6*c) - 6*b^2*e^(6*d*x + 6*c) + a^2*e^(4*d*x + 4*c
) + 2*a*b*e^(4*d*x + 4*c) - 15*b^2*e^(4*d*x + 4*c) - 4*a^2*e^(2*d*x + 2*c)
+ 20*a*b*e^(2*d*x + 2*c) + 24*b^2*e^(2*d*x + 2*c) - 3*a^2 - 6*a*b - 3*b^2)/
((a^3*e^(2*c) + 3*a^2*b*e^(2*c) + 3*a*b^2*e^(2*c) + b^3*e^(2*c))*(a*e^(2*d*
x) + b*e^(2*d*x) + a*e^(6*d*x + 4*c) + b*e^(6*d*x + 4*c) + 2*a*e^(4*d*x + 2
*c) - 2*b*e^(4*d*x + 2*c))))/d
```

3.36 $\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=92

$$\frac{3\cosh(c+dx)}{2d(a+b)^2}-\frac{\cosh(c+dx)}{2d(a+b)\left(a-b\operatorname{sech}^2(c+dx)+b\right)}-\frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{2d(a+b)^{5/2}}$$

```
[Out] (-3*Sqrt[b]*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(2*(a + b)^(5/2)*
d) + (3*Cosh[c + d*x])/(2*(a + b)^2*d) - Cosh[c + d*x]/(2*(a + b)*d*(a + b
- b*Sech[c + d*x]^2))
```

Rubi [A] time = 0.0746356, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3664, 290, 325, 208}

$$\frac{3\cosh(c+dx)}{2d(a+b)^2}-\frac{\cosh(c+dx)}{2d(a+b)\left(a-b\operatorname{sech}^2(c+dx)+b\right)}-\frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{2d(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (-3*Sqrt[b]*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(2*(a + b)^(5/2)*
d) + (3*Cosh[c + d*x])/(2*(a + b)^2*d) - Cosh[c + d*x]/(2*(a + b)*d*(a + b
- b*Sech[c + d*x]^2))
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+b-bx^2\right)^2}\,dx, x, \text{sech}(c+dx)\right)}{d}$$

$$= -\frac{\cosh(c+dx)}{2(a+b)d\left(a+b-b\,\text{sech}^2(c+dx)\right)} - \frac{3\,\text{Subst}\left(\int \frac{1}{x^2\left(a+b-bx^2\right)}\,dx, x, \text{sech}(c+dx)\right)}{2(a+b)d}$$

$$= \frac{3\cosh(c+dx)}{2(a+b)^2 d} - \frac{\cosh(c+dx)}{2(a+b)d\left(a+b-b\,\text{sech}^2(c+dx)\right)} - \frac{(3b)\,\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \text{se}$$

$$= -\frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{2(a+b)^{5/2}d} + \frac{3\cosh(c+dx)}{2(a+b)^2 d} - \frac{\cosh(c+dx)}{2(a+b)d\left(a+b-b\,\text{sech}^2(c+dx)\right)}$$

Mathematica [C] time = 0.683644, size = 133, normalized size = 1.45

$$\frac{\frac{2\cosh(c+dx)\left(1-\frac{b}{(a+b)\cosh(2(c+dx))+a-b}\right)}{(a+b)^2} - \frac{3i\sqrt{b}\left(\tan^{-1}\left(\frac{-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)-i\sqrt{a+b}}{\sqrt{b}}\right)+\tan^{-1}\left(\frac{\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)-i\sqrt{a+b}}{\sqrt{b}}\right)\right)}{(a+b)^{5/2}}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (((-3*I)*Sqrt[b]*(ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2])/Sqr
t[b]] + ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]]))/(a
 + b)^(5/2) + (2*Cosh[c + d*x]*(1 - b/(a - b + (a + b)*Cosh[2*(c + d*x)])))
/(a + b)^2)/(2*d)
```

Maple [B] time = 0.079, size = 167, normalized size = 1.8

$$\frac{1}{d}\left(\frac{1}{(a+b)^2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1} + 2\frac{b}{(a+b)^2}\left(\frac{1}{(\tanh(1/2\,dx+c/2))^4 a + 2(\tanh(1/2\,dx+c/2))^2 a + 4(\tanh(1/2\,c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(1/(a+b)^2/(tanh(1/2*d*x+1/2*c)+1)+2*b/(a+b)^2*((-1/2*(a+2*b)/a*tanh(1/
2*d*x+1/2*c)^2-1/2)/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*ta
nh(1/2*d*x+1/2*c)^2*b+a)-3/4/(a*b+b^2)^(1/2)*arctanh(1/4*(2*tanh(1/2*d*x+1/
2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2)))-1/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(ae^{(6\,c)} + be^{(6\,c)}\right)e^{(6\,dx)} + 3\left(ae^{(4\,c)} - be^{(4\,c)}\right)e^{(4\,dx)} + 3\left(ae^{(2\,c)} - be^{(2\,c)}\right)e^{(2\,dx)} + a + b}{2\left(\left(a^3de^{(5\,c)} + 3\,a^2bde^{(5\,c)} + 3\,ab^2de^{(5\,c)} + b^3de^{(5\,c)}\right)e^{(5\,dx)} + 2\left(a^3de^{(3\,c)} + a^2bde^{(3\,c)} - ab^2de^{(3\,c)} - b^3de^{(3\,c)}\right)e^{(3\,dx)} + \left(a^3de^{c} + 3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((a*e^(6*c) + b*e^(6*c))*e^(6*d*x) + 3*(a*e^(4*c) - b*e^(4*c))*e^(4*d*x
) + 3*(a*e^(2*c) - b*e^(2*c))*e^(2*d*x) + a + b)/((a^3*d*e^(5*c) + 3*a^2*b*
d*e^(5*c) + 3*a*b^2*d*e^(5*c) + b^3*d*e^(5*c))*e^(5*d*x) + 2*(a^3*d*e^(3*c)
 + a^2*b*d*e^(3*c) - a*b^2*d*e^(3*c) - b^3*d*e^(3*c))*e^(3*d*x) + (a^3*d*e^
c + 3*a^2*b*d*e^c + 3*a*b^2*d*e^c + b^3*d*e^c)*e^(d*x)) + 1/2*integrate(6*(
b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3 + (a^3*e^
(4*c) + 3*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c) + b^3*e^(4*c))*e^(4*d*x) + 2*(a^3
*e^(2*c) + a^2*b*e^(2*c) - a*b^2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 2.58527, size = 5933, normalized size = 64.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a + b)*cosh(d*x + c)^6 + 12*(a + b)*cosh(d*x + c)*sinh(d*x + c)^5
+ 2*(a + b)*sinh(d*x + c)^6 + 6*(a - b)*cosh(d*x + c)^4 + 6*(5*(a + b)*cosh
(d*x + c)^2 + a - b)*sinh(d*x + c)^4 + 8*(5*(a + b)*cosh(d*x + c)^3 + 3*(a
- b)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(a - b)*cosh(d*x + c)^2 + 6*(5*(a +
 b)*cosh(d*x + c)^4 + 6*(a - b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 +
3*((a + b)*cosh(d*x + c)^5 + 5*(a + b)*cosh(d*x + c)*sinh(d*x + c)^4 + (a +
 b)*sinh(d*x + c)^5 + 2*(a - b)*cosh(d*x + c)^3 + 2*(5*(a + b)*cosh(d*x + c
)^2 + a - b)*sinh(d*x + c)^3 + 2*(5*(a + b)*cosh(d*x + c)^3 + 3*(a - b)*cos
h(d*x + c))*sinh(d*x + c)^2 + (a + b)*cosh(d*x + c) + (5*(a + b)*cosh(d*x +
 c)^4 + 6*(a - b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))*sqrt(b/(a + b))*l
og(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a
+ b)*sinh(d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x
+ c)^2 + a + 3*b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + 3*b)*
cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(
d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a + b)*cosh(d*x + c)
+ (3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))*sqrt(b/(a + b)) + a +
b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a
+ b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x +
c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d
*x + c))*sinh(d*x + c) + a + b)) + 12*((a + b)*cosh(d*x + c)^5 + 2*(a - b)*
cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + 2*a + 2*b)/((a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^5 + 5*(a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*d*cosh(d*x + c)*sinh(d*x + c)^4 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*si
nh(d*x + c)^5 + 2*(a^3 + a^2*b - a*b^2 - b^3)*d*cosh(d*x + c)^3 + 2*(5*(a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^2 + (a^3 + a^2*b - a*b^2 - b^3)
*d)*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c) + 2*(
5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^3 + 3*(a^3 + a^2*b - a*b^
2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c)^2 + (5*(a^3 + 3*a^2*b + 3*a*b^2 + b
^3)*d*cosh(d*x + c)^4 + 6*(a^3 + a^2*b - a*b^2 - b^3)*d*cosh(d*x + c)^2 + (
a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d)*sinh(d*x + c)), 1/2*((a + b)*cosh(d*x + c
```

```
)^6 + 6*(a + b)*cosh(d*x + c)*sinh(d*x + c)^5 + (a + b)*sinh(d*x + c)^6 + 3
*(a - b)*cosh(d*x + c)^4 + 3*(5*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x +
 c)^4 + 4*(5*(a + b)*cosh(d*x + c)^3 + 3*(a - b)*cosh(d*x + c))*sinh(d*x +
c)^3 + 3*(a - b)*cosh(d*x + c)^2 + 3*(5*(a + b)*cosh(d*x + c)^4 + 6*(a - b)
*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 - 3*((a + b)*cosh(d*x + c)^5 + 5*
(a + b)*cosh(d*x + c)*sinh(d*x + c)^4 + (a + b)*sinh(d*x + c)^5 + 2*(a - b)
*cosh(d*x + c)^3 + 2*(5*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^3 +
2*(5*(a + b)*cosh(d*x + c)^3 + 3*(a - b)*cosh(d*x + c))*sinh(d*x + c)^2 + (
a + b)*cosh(d*x + c) + (5*(a + b)*cosh(d*x + c)^4 + 6*(a - b)*cosh(d*x + c)
^2 + a + b)*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2*((a + b)*cosh(d*x +
c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 +
(a - 3*b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a - 3*b)*sinh(d*x +
c))*sqrt(-b/(a + b))/b) + 3*((a + b)*cosh(d*x + c)^5 + 5*(a + b)*cosh(d*x +
 c)*sinh(d*x + c)^4 + (a + b)*sinh(d*x + c)^5 + 2*(a - b)*cosh(d*x + c)^3 +
 2*(5*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^3 + 2*(5*(a + b)*cosh(
d*x + c)^3 + 3*(a - b)*cosh(d*x + c))*sinh(d*x + c)^2 + (a + b)*cosh(d*x +
c) + (5*(a + b)*cosh(d*x + c)^4 + 6*(a - b)*cosh(d*x + c)^2 + a + b)*sinh(d
*x + c))*sqrt(-b/(a + b))*arctan(1/2*((a + b)*cosh(d*x + c) + (a + b)*sinh(
d*x + c))*sqrt(-b/(a + b))/b) + 6*((a + b)*cosh(d*x + c)^5 + 2*(a - b)*cosh
(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)/((a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*d*cosh(d*x + c)^5 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*c
osh(d*x + c)*sinh(d*x + c)^4 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*sinh(d*x +
 c)^5 + 2*(a^3 + a^2*b - a*b^2 - b^3)*d*cosh(d*x + c)^3 + 2*(5*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^2 + (a^3 + a^2*b - a*b^2 - b^3)*d)*sinh
(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c) + 2*(5*(a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^3 + 3*(a^3 + a^2*b - a*b^2 - b^3)
*d*cosh(d*x + c))*sinh(d*x + c)^2 + (5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*co
sh(d*x + c)^4 + 6*(a^3 + a^2*b - a*b^2 - b^3)*d*cosh(d*x + c)^2 + (a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d)*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.69834, size = 6742, normalized size = 73.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*(6*(3*(2*a*b^2*e^(2*c) - (a*b*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cos(1/
2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a
/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))
) - (2*a*b^2*e^(2*c) - (a*b*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cosh(1/2*ima
g_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a +
b) + b/(a + b))))^3 - 9*(2*a*b^2*e^(2*c) - (a*b*e^(2*c) - b^2*e^(2*c))*sqrt
```

```
(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_
part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b)
 + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(2*
a*b^2*e^(2*c) - (a*b*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cosh(1/2*imag_part(
arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/
(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(2*a*b
^2*e^(2*c) - (a*b*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(arcc
os(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a
+ b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_pa
rt(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(2*a*b^2*e^(2*c) - (a*b*e^(2*c) -
 b^2*e^(2*c))*sqrt(-a*b))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))
))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(
arccos(-a/(a + b) + b/(a + b))))^2 - 3*(2*a*b^2*e^(2*c) - (a*b*e^(2*c) - b^
2*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2
*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arcc
os(-a/(a + b) + b/(a + b))))^3 + (2*a*b^2*e^(2*c) - (a*b*e^(2*c) - b^2*e^(2
*c))*sqrt(-a*b))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(
1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 - (2*a*b^2*e^(2*c) - (a*b*
e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cosh(1/2*imag_part(arccos(-a/(a + b) + b
/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) + (2*a*b^2*e
^(2*c) - (a*b*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*sin(1/2*real_part(arccos(-
a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))
))*arctan((((a^3 + 3*a^2*b + 3*a*b^2 + b^3)/(a^3*e^(4*c) + 3*a^2*b*e^(4*c)
+ 3*a*b^2*e^(4*c) + b^3*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b))) +
 e^(d*x))/(((a^3 + 3*a^2*b + 3*a*b^2 + b^3)/(a^3*e^(4*c) + 3*a^2*b*e^(4*c)
+ 3*a*b^2*e^(4*c) + b^3*e^(4*c)))^(1/4)*sin(1/2*arccos(-(a - b)/(a + b)))))
/(a^4*b*e^(2*c) + 3*a^3*b^2*e^(2*c) + 3*a^2*b^3*e^(2*c) + a*b^4*e^(2*c)) +
6*(3*(2*a*b^2*e^(2*c) - (a*b*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cos(1/2*rea
l_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a +
 b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) - (
2*a*b^2*e^(2*c) - (a*b*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cosh(1/2*imag_par
t(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) +
b/(a + b))))^3 - 9*(2*a*b^2*e^(2*c) - (a*b*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b
))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(
arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/
(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(2*a*b^2
*e^(2*c) - (a*b*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cosh(1/2*imag_part(arcco
s(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a +
b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(2*a*b^2*e^
(2*c) - (a*b*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(arccos(-a
/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))
))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(ar
ccos(-a/(a + b) + b/(a + b))))^2 - 3*(2*a*b^2*e^(2*c) - (a*b*e^(2*c) - b^2*
e^(2*c))*sqrt(-a*b))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*si
n(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arcco
s(-a/(a + b) + b/(a + b))))^2 - 3*(2*a*b^2*e^(2*c) - (a*b*e^(2*c) - b^2*e^(
2*c))*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(
1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a
/(a + b) + b/(a + b))))^3 + (2*a*b^2*e^(2*c) - (a*b*e^(2*c) - b^2*e^(2*c))*
sqrt(-a*b))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*i
mag_part(arccos(-a/(a + b) + b/(a + b))))^3 - (2*a*b^2*e^(2*c) - (a*b*e^(2*
c) - b^2*e^(2*c))*sqrt(-a*b))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a +
 b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) + (2*a*b^2*e^(2*c
) - (a*b*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*sin(1/2*real_part(arccos(-a/(a
+ b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*ar
ctan(-(((a^3 + 3*a^2*b + 3*a*b^2 + b^3)/(a^3*e^(4*c) + 3*a^2*b*e^(4*c) + 3*
a*b^2*e^(4*c) + b^3*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b))) - e^(
d*x))/(((a^3 + 3*a^2*b + 3*a*b^2 + b^3)/(a^3*e^(4*c) + 3*a^2*b*e^(4*c) + 3*
a*b^2*e^(4*c) + b^3*e^(4*c)))^(1/4)*sin(1/2*arccos(-(a - b)/(a + b)))))/(a^
```

```
4*b*e^(2*c) + 3*a^3*b^2*e^(2*c) + 3*a^2*b^3*e^(2*c) + a*b^4*e^(2*c)) + 3*((
2*a*b^2*e^(2*c) - (a*b*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part
(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) +
b/(a + b))))^3 - 3*(2*a*b^2*e^(2*c) - (a*b*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b
))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(ar
ccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a
 + b))))^2 - 3*(2*a*b^2*e^(2*c) - (a*b*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*c
os(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arcc
os(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a
+ b)))) + 9*(2*a*b^2*e^(2*c) - (a*b*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cos(
1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a
/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))
)^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(2*a*b^2*e^(2*c
) - (a*b*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a
+ b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*s
inh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 9*(2*a*b^2*e^(2*c) -
 (a*b*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b
) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/
2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a
/(a + b) + b/(a + b))))^2 - (2*a*b^2*e^(2*c) - (a*b*e^(2*c) - b^2*e^(2*c))*
sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*i
mag_part(arccos(-a/(a + b) + b/(a + b))))^3 + 3*(2*a*b^2*e^(2*c) - (a*b*e^(
2*c) - b^2*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a
+ b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_
part(arccos(-a/(a + b) + b/(a + b))))^3 - (2*a*b^2*e^(2*c) - (a*b*e^(2*c) -
 b^2*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))
)*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + (2*a*b^2*e^(2*c) -
(a*b*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b)
 + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*log(2*
((a^3 + 3*a^2*b + 3*a*b^2 + b^3)/(a^3*e^(4*c) + 3*a^2*b*e^(4*c) + 3*a*b^2*e
^(4*c) + b^3*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b)))*e^(d*x) + sq
rt((a^3 + 3*a^2*b + 3*a*b^2 + b^3)/(a^3*e^(4*c) + 3*a^2*b*e^(4*c) + 3*a*b^2
*e^(4*c) + b^3*e^(4*c))) + e^(2*d*x))/(a^4*b*e^(2*c) + 3*a^3*b^2*e^(2*c) +
3*a^2*b^3*e^(2*c) + a*b^4*e^(2*c)) - 3*((2*a*b^2*e^(2*c) - (a*b*e^(2*c) - b
^2*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^
3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 - 3*(2*a*b^2*e^(2*c
) - (a*b*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a
+ b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*s
in(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(2*a*b^2*e^(2*c) -
(a*b*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b)
 + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin
h(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(2*a*b^2*e^(2*c) - (a*
b*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) +
b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*
real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(
a + b) + b/(a + b)))) + 3*(2*a*b^2*e^(2*c) - (a*b*e^(2*c) - b^2*e^(2*c))*sq
rt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*ima
g_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b
) + b/(a + b))))^2 - 9*(2*a*b^2*e^(2*c) - (a*b*e^(2*c) - b^2*e^(2*c))*sqrt(
-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_par
t(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/
(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - (2*a*b
^2*e^(2*c) - (a*b*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(arcc
os(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a
+ b))))^3 + 3*(2*a*b^2*e^(2*c) - (a*b*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*co
s(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-
a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)
)))^3 - (2*a*b^2*e^(2*c) - (a*b*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cos(1/2*
real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a
```

```
+ b) + b/(a + b)))) + (2*a*b^2*e^(2*c) - (a*b*e^(2*c) - b^2*e^(2*c))*sqrt(-
a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part
(arccos(-a/(a + b) + b/(a + b)))))*log(-2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)/
(a^3*e^(4*c) + 3*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c) + b^3*e^(4*c)))^(1/4)*cos(
1/2*arccos(-(a - b)/(a + b)))*e^(d*x) + sqrt((a^3 + 3*a^2*b + 3*a*b^2 + b^3
)/(a^3*e^(4*c) + 3*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c) + b^3*e^(4*c))) + e^(2*d
*x))/(a^4*b*e^(2*c) + 3*a^3*b^2*e^(2*c) + 3*a^2*b^3*e^(2*c) + a*b^4*e^(2*c)
) + 4*e^(d*x + 10*c)/(a^2*e^(9*c) + 2*a*b*e^(9*c) + b^2*e^(9*c)) + 4*(a*e^(
4*d*x + 4*c) - b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 4*b*e^(2*d*x + 2*c
) + a + b)/((a^2*e^c + 2*a*b*e^c + b^2*e^c)*(a*e^(5*d*x + 4*c) + b*e^(5*d*x
 + 4*c) + 2*a*e^(3*d*x + 2*c) - 2*b*e^(3*d*x + 2*c) + a*e^(d*x) + b*e^(d*x)
)))/d
```

3.37 $\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=103

$$\frac{\sqrt{b}(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{2a^2d(a+b)^{3/2}}-\frac{\tanh^{-1}(\cosh(c+dx))}{a^2d}+\frac{b\operatorname{sech}(c+dx)}{2ad(a+b)\left(a-b\operatorname{sech}^2(c+dx)+b\right)}$$

```
[Out] -(ArcTanh[Cosh[c + d*x]]/(a^2*d)) + (Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*S
ech[c + d*x])/Sqrt[a + b]])/(2*a^2*(a + b)^(3/2)*d) + (b*Sech[c + d*x])/(2*
a*(a + b)*d*(a + b - b*Sech[c + d*x]^2))
```

Rubi [A] time = 0.143505, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3664, 414, 522, 207, 208}

$$\frac{\sqrt{b}(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{2a^2d(a+b)^{3/2}}-\frac{\tanh^{-1}(\cosh(c+dx))}{a^2d}+\frac{b\operatorname{sech}(c+dx)}{2ad(a+b)\left(a-b\operatorname{sech}^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -(ArcTanh[Cosh[c + d*x]]/(a^2*d)) + (Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*S
ech[c + d*x])/Sqrt[a + b]])/(2*a^2*(a + b)^(3/2)*d) + (b*Sech[c + d*x])/(2*
a*(a + b)*d*(a + b - b*Sech[c + d*x]^2))
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(-1+x^2)\left(a+b-bx^2\right)^2}\,dx, x, \operatorname{sech}(c+dx)\right)}{d}$$

$$= \frac{b\operatorname{sech}(c+dx)}{2a(a+b)d\left(a+b-b\operatorname{sech}^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{2a+b+bx^2}{(-1+x^2)\left(a+b-bx^2\right)}\,dx, x, \operatorname{sech}(c+dx)\right)}{2a(a+b)d}$$

$$= \frac{b\operatorname{sech}(c+dx)}{2a(a+b)d\left(a+b-b\operatorname{sech}^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \operatorname{sech}(c+dx)\right)}{a^2d} + \frac{(b(3a+2b}{}$$

$$= -\frac{\tanh^{-1}(\cosh(c+dx))}{a^2d} + \frac{\sqrt{b}(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}d} + \frac{b\operatorname{sech}(c+dx}{2a(a+b)d\left(a+b-b\operatorname{sec}\right.}$$

Mathematica [C] time = 0.663079, size = 175, normalized size = 1.7

$$\frac{\frac{2ab\cosh(c+dx)}{(a+b)((a+b)\cosh(2(c+dx))+a-b)} + \frac{i\sqrt{b}(3a+2b)\tan^{-1}\left(\frac{-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)-i\sqrt{a+b}}{\sqrt{b}}\right)}{(a+b)^{3/2}} + \frac{i\sqrt{b}(3a+2b)\tan^{-1}\left(\frac{\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)-i\sqrt{a+b}}{\sqrt{b}}\right)}{(a+b)^{3/2}} + 2\log\left(\tanh\left(\frac{1}{2}(c+\right.\right.}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((I*Sqrt[b]*(3*a + 2*b)*ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2
])/Sqrt[b]])/(a + b)^(3/2) + (I*Sqrt[b]*(3*a + 2*b)*ArcTan[((-I)*Sqrt[a + b
] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]])/(a + b)^(3/2) + (2*a*b*Cosh[c + d*
x])/((a + b)*(a - b + (a + b)*Cosh[2*(c + d*x)])) + 2*Log[Tanh[(c + d*x)/2]
])/(2*a^2*d)
```

Maple [B] time = 0.092, size = 331, normalized size = 3.2

$$\frac{b}{da\,(a+b)}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4 a+2\,(\tanh(1/2\,dx+c/2))^2\,a+4\,(\tanh(1/2\,dx+c/2))^2\,b+a\right)^{-1}+2\,\frac{}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*b/a/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1
/2*c)^2*b+a)/(a+b)*tanh(1/2*d*x+1/2*c)^2+2/d*b^2/a^2/(tanh(1/2*d*x+1/2*c)^4
```

```
*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/(a+b)*tanh(1/2*d*
x+1/2*c)^2+1/d*b/a/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tan
h(1/2*d*x+1/2*c)^2*b+a)/(a+b)+3/2/d*b/a/(a+b)/(a*b+b^2)^(1/2)*arctanh(1/4*(
2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2))+1/d*b^2/a^2/(a+b)/(a*b+
b^2)^(1/2)*arctanh(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2))
+1/d/a^2*ln(tanh(1/2*d*x+1/2*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{be^{(3\,dx+3\,c)}+be^{(dx+c)}}{a^3d+2\,a^2bd+ab^2d+\left(a^3de^{(4\,c)}+2\,a^2bde^{(4\,c)}+ab^2de^{(4\,c)}\right)e^{(4\,dx)}+2\left(a^3de^{(2\,c)}-ab^2de^{(2\,c)}\right)e^{(2\,dx)}}-\frac{\log\left(\left(e^{(dx+c)}+1\right)e^{(-c)}\right.}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] (b*e^(3*d*x + 3*c) + b*e^(d*x + c))/(a^3*d + 2*a^2*b*d + a*b^2*d + (a^3*d*e
^(4*c) + 2*a^2*b*d*e^(4*c) + a*b^2*d*e^(4*c))*e^(4*d*x) + 2*(a^3*d*e^(2*c)
- a*b^2*d*e^(2*c))*e^(2*d*x)) - log((e^(d*x + c) + 1)*e^(-c))/(a^2*d) + log
((e^(d*x + c) - 1)*e^(-c))/(a^2*d) - 2*integrate(1/2*((3*a*b*e^(3*c) + 2*b^
2*e^(3*c))*e^(3*d*x) - (3*a*b*e^c + 2*b^2*e^c)*e^(d*x))/(a^4 + 2*a^3*b + a^
2*b^2 + (a^4*e^(4*c) + 2*a^3*b*e^(4*c) + a^2*b^2*e^(4*c))*e^(4*d*x) + 2*(a^
4*e^(2*c) - a^2*b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 3.28411, size = 6604, normalized size = 64.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*b*cosh(d*x + c)^3 + 12*a*b*cosh(d*x + c)*sinh(d*x + c)^2 + 4*a*b*
sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) + ((3*a^2 + 5*a*b + 2*b^2)*cosh(d*x +
 c)^4 + 4*(3*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2 +
5*a*b + 2*b^2)*sinh(d*x + c)^4 + 2*(3*a^2 - a*b - 2*b^2)*cosh(d*x + c)^2 +
2*(3*(3*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^2 + 3*a^2 - a*b - 2*b^2)*sinh(d*
x + c)^2 + 3*a^2 + 5*a*b + 2*b^2 + 4*((3*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)
^3 + (3*a^2 - a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/(a + b))*lo
g(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a +
 b)*sinh(d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x +
 c)^2 + a + 3*b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + 3*b)*c
osh(d*x + c))*sinh(d*x + c) + 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d
*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a + b)*cosh(d*x + c) +
 (3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))*sqrt(b/(a + b)) + a + b
)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a +
 b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c
)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*
x + c))*sinh(d*x + c) + a + b)) - 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 +
4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*s
inh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 +
 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*l
og(cosh(d*x + c) + sinh(d*x + c) + 1) + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c
```

```
)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b +
b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b +
b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 4*(
(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x +
 c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 4*(3*a*b*cosh(d*x + c)^2 + a*
b)*sinh(d*x + c))/((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^4 + 4*(a^4 + 2
*a^3*b + a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^3*b + a^2*b^
2)*d*sinh(d*x + c)^4 + 2*(a^4 - a^2*b^2)*d*cosh(d*x + c)^2 + 2*(3*(a^4 + 2*
a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 + (a^4 - a^2*b^2)*d)*sinh(d*x + c)^2 + (
a^4 + 2*a^3*b + a^2*b^2)*d + 4*((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^3
 + (a^4 - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*a*b*cosh(d*x + c
)^3 + 6*a*b*cosh(d*x + c)*sinh(d*x + c)^2 + 2*a*b*sinh(d*x + c)^3 + 2*a*b*c
osh(d*x + c) + ((3*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^4 + 4*(3*a^2 + 5*a*b
+ 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2 + 5*a*b + 2*b^2)*sinh(d*x +
 c)^4 + 2*(3*a^2 - a*b - 2*b^2)*cosh(d*x + c)^2 + 2*(3*(3*a^2 + 5*a*b + 2*b
^2)*cosh(d*x + c)^2 + 3*a^2 - a*b - 2*b^2)*sinh(d*x + c)^2 + 3*a^2 + 5*a*b
+ 2*b^2 + 4*((3*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^3 + (3*a^2 - a*b - 2*b^2
)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2*((a + b)*cosh(d
*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)
^3 + (a - 3*b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a - 3*b)*sinh(d
*x + c))*sqrt(-b/(a + b))/b) - ((3*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^4 + 4
*(3*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2 + 5*a*b + 2
*b^2)*sinh(d*x + c)^4 + 2*(3*a^2 - a*b - 2*b^2)*cosh(d*x + c)^2 + 2*(3*(3*a
^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^2 + 3*a^2 - a*b - 2*b^2)*sinh(d*x + c)^2
+ 3*a^2 + 5*a*b + 2*b^2 + 4*((3*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^3 + (3*a
^2 - a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2
*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x + c))*sqrt(-b/(a + b))/b) - 2*((
a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*si
nh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*
x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x +
 c)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 -
 b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1)
+ 2*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x +
 c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*c
osh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh
(d*x + c)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 +
(a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c)
 - 1) + 2*(3*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c))/((a^4 + 2*a^3*b + a^
2*b^2)*d*cosh(d*x + c)^4 + 4*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)*sinh
(d*x + c)^3 + (a^4 + 2*a^3*b + a^2*b^2)*d*sinh(d*x + c)^4 + 2*(a^4 - a^2*b^
2)*d*cosh(d*x + c)^2 + 2*(3*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 + (
a^4 - a^2*b^2)*d)*sinh(d*x + c)^2 + (a^4 + 2*a^3*b + a^2*b^2)*d + 4*((a^4 +
 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^3 + (a^4 - a^2*b^2)*d*cosh(d*x + c))*si
nh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(csch(c + d*x)/(a + b*tanh(c + d*x)**2)**2, x)
```

Giac [C] time = 1.70439, size = 7329, normalized size = 71.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/8*(2*(3*(3*a^5*b*e^(4*c) + 8*a^4*b^2*e^(4*c) + 7*a^3*b^3*e^(4*c) + 2*a^2
*b^4*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2
*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(
a + b) + b/(a + b)))) - (3*a^5*b*e^(4*c) + 8*a^4*b^2*e^(4*c) + 7*a^3*b^3*e^
(4*c) + 2*a^2*b^4*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)
)))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3 - 9*(3*a^5*b*e^(
4*c) + 8*a^4*b^2*e^(4*c) + 7*a^3*b^3*e^(4*c) + 2*a^2*b^4*e^(4*c))*cos(1/2*r
eal_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a
 + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*s
inh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(3*a^5*b*e^(4*c) + 8
*a^4*b^2*e^(4*c) + 7*a^3*b^3*e^(4*c) + 2*a^2*b^4*e^(4*c))*cosh(1/2*imag_par
t(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) +
b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(3*a
^5*b*e^(4*c) + 8*a^4*b^2*e^(4*c) + 7*a^3*b^3*e^(4*c) + 2*a^2*b^4*e^(4*c))*c
os(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arcc
os(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b
))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(3*a^5*b*e^(
4*c) + 8*a^4*b^2*e^(4*c) + 7*a^3*b^3*e^(4*c) + 2*a^2*b^4*e^(4*c))*cosh(1/2*
imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a +
 b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2
- 3*(3*a^5*b*e^(4*c) + 8*a^4*b^2*e^(4*c) + 7*a^3*b^3*e^(4*c) + 2*a^2*b^4*e^
(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_pa
rt(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) +
b/(a + b))))^3 + (3*a^5*b*e^(4*c) + 8*a^4*b^2*e^(4*c) + 7*a^3*b^3*e^(4*c) +
 2*a^2*b^4*e^(4*c))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*si
nh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 - (3*a^5*b*e^(4*c) + 8*
a^4*b^2*e^(4*c) + 7*a^3*b^3*e^(4*c) + 2*a^2*b^4*e^(4*c))*cosh(1/2*imag_part
(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(
a + b)))) + (3*a^5*b*e^(4*c) + 8*a^4*b^2*e^(4*c) + 7*a^3*b^3*e^(4*c) + 2*a^
2*b^4*e^(4*c))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*
imag_part(arccos(-a/(a + b) + b/(a + b)))))*arctan((((a^4 + 2*a^3*b + a^2*b
^2)/(a^4*e^(4*c) + 2*a^3*b*e^(4*c) + a^2*b^2*e^(4*c)))^(1/4)*cos(1/2*arccos
(-(a - b)/(a + b))) + e^(d*x))/(((a^4 + 2*a^3*b + a^2*b^2)/(a^4*e^(4*c) + 2
*a^3*b*e^(4*c) + a^2*b^2*e^(4*c)))^(1/4)*sin(1/2*arccos(-(a - b)/(a + b))))
)/(2*(a^3*e^(2*c) + a^2*b*e^(2*c))^2*a*b - (a^4*e^(2*c) - a^2*b^2*e^(2*c))*
sqrt(-a*b)*abs(-a^3*e^(2*c) - a^2*b*e^(2*c))) + 2*(3*(3*a^5*b*e^(4*c) + 8*a
^4*b^2*e^(4*c) + 7*a^3*b^3*e^(4*c) + 2*a^2*b^4*e^(4*c))*cos(1/2*real_part(a
rccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/
(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) - (3*a^5*b*
e^(4*c) + 8*a^4*b^2*e^(4*c) + 7*a^3*b^3*e^(4*c) + 2*a^2*b^4*e^(4*c))*cosh(1
/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a
/(a + b) + b/(a + b))))^3 - 9*(3*a^5*b*e^(4*c) + 8*a^4*b^2*e^(4*c) + 7*a^3*
b^3*e^(4*c) + 2*a^2*b^4*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a
 + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*re
al_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a +
b) + b/(a + b)))) + 3*(3*a^5*b*e^(4*c) + 8*a^4*b^2*e^(4*c) + 7*a^3*b^3*e^(4
*c) + 2*a^2*b^4*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))
)^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part
(arccos(-a/(a + b) + b/(a + b)))) + 9*(3*a^5*b*e^(4*c) + 8*a^4*b^2*e^(4*c)
+ 7*a^3*b^3*e^(4*c) + 2*a^2*b^4*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b
```

```
) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(
1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a
/(a + b) + b/(a + b))))^2 - 3*(3*a^5*b*e^(4*c) + 8*a^4*b^2*e^(4*c) + 7*a^3*
b^3*e^(4*c) + 2*a^2*b^4*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(
a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*ima
g_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(3*a^5*b*e^(4*c) + 8*a^4*b^2*
e^(4*c) + 7*a^3*b^3*e^(4*c) + 2*a^2*b^4*e^(4*c))*cos(1/2*real_part(arccos(-
a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))
))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (3*a^5*b*e^(4*c)
 + 8*a^4*b^2*e^(4*c) + 7*a^3*b^3*e^(4*c) + 2*a^2*b^4*e^(4*c))*sin(1/2*real_
part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b
) + b/(a + b))))^3 - (3*a^5*b*e^(4*c) + 8*a^4*b^2*e^(4*c) + 7*a^3*b^3*e^(4*
c) + 2*a^2*b^4*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))
*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) + (3*a^5*b*e^(4*c) + 8*
a^4*b^2*e^(4*c) + 7*a^3*b^3*e^(4*c) + 2*a^2*b^4*e^(4*c))*sin(1/2*real_part(
arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(
a + b)))))*arctan(-(((a^4 + 2*a^3*b + a^2*b^2)/(a^4*e^(4*c) + 2*a^3*b*e^(4*
c) + a^2*b^2*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b))) - e^(d*x))/(
((a^4 + 2*a^3*b + a^2*b^2)/(a^4*e^(4*c) + 2*a^3*b*e^(4*c) + a^2*b^2*e^(4*c)
))^(1/4)*sin(1/2*arccos(-(a - b)/(a + b)))))/(2*(a^3*e^(2*c) + a^2*b*e^(2*c
))^2*a*b - (a^4*e^(2*c) - a^2*b^2*e^(2*c))*sqrt(-a*b)*abs(-a^3*e^(2*c) - a^
2*b*e^(2*c))) + ((3*a^5*b*e^(4*c) + 8*a^4*b^2*e^(4*c) + 7*a^3*b^3*e^(4*c) +
 2*a^2*b^4*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*co
sh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 - 3*(3*a^5*b*e^(4*c) +
8*a^4*b^2*e^(4*c) + 7*a^3*b^3*e^(4*c) + 2*a^2*b^4*e^(4*c))*cos(1/2*real_par
t(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b
/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(3*a
^5*b*e^(4*c) + 8*a^4*b^2*e^(4*c) + 7*a^3*b^3*e^(4*c) + 2*a^2*b^4*e^(4*c))*c
os(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arcc
os(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a
+ b)))) + 9*(3*a^5*b*e^(4*c) + 8*a^4*b^2*e^(4*c) + 7*a^3*b^3*e^(4*c) + 2*a^
2*b^4*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*
imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a
 + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))
+ 3*(3*a^5*b*e^(4*c) + 8*a^4*b^2*e^(4*c) + 7*a^3*b^3*e^(4*c) + 2*a^2*b^4*e^
(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_p
art(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) +
 b/(a + b))))^2 - 9*(3*a^5*b*e^(4*c) + 8*a^4*b^2*e^(4*c) + 7*a^3*b^3*e^(4*c
) + 2*a^2*b^4*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*c
osh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos
(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a +
b))))^2 - (3*a^5*b*e^(4*c) + 8*a^4*b^2*e^(4*c) + 7*a^3*b^3*e^(4*c) + 2*a^2*
b^4*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*
imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + 3*(3*a^5*b*e^(4*c) + 8*a^4*b
^2*e^(4*c) + 7*a^3*b^3*e^(4*c) + 2*a^2*b^4*e^(4*c))*cos(1/2*real_part(arcco
s(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)
)))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 - (3*a^5*b*e^(4
*c) + 8*a^4*b^2*e^(4*c) + 7*a^3*b^3*e^(4*c) + 2*a^2*b^4*e^(4*c))*cos(1/2*re
al_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a +
b) + b/(a + b)))) + (3*a^5*b*e^(4*c) + 8*a^4*b^2*e^(4*c) + 7*a^3*b^3*e^(4*c
) + 2*a^2*b^4*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*s
inh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*log(2*((a^4 + 2*a^3*b +
 a^2*b^2)/(a^4*e^(4*c) + 2*a^3*b*e^(4*c) + a^2*b^2*e^(4*c)))^(1/4)*cos(1/2*
arccos(-(a - b)/(a + b)))*e^(d*x) + sqrt((a^4 + 2*a^3*b + a^2*b^2)/(a^4*e^(
4*c) + 2*a^3*b*e^(4*c) + a^2*b^2*e^(4*c))) + e^(2*d*x))/(2*(a^3*e^(2*c) + a
^2*b*e^(2*c))^2*a*b - (a^4*e^(2*c) - a^2*b^2*e^(2*c))*sqrt(-a*b)*abs(-a^3*e
^(2*c) - a^2*b*e^(2*c))) - ((3*a^5*b*e^(4*c) + 8*a^4*b^2*e^(4*c) + 7*a^3*b^
3*e^(4*c) + 2*a^2*b^4*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a +
 b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 - 3*(3*a^5*b
```

```
*e^(4*c) + 8*a^4*b^2*e^(4*c) + 7*a^3*b^3*e^(4*c) + 2*a^2*b^4*e^(4*c))*cos(1
/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/
(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))
^2 - 3*(3*a^5*b*e^(4*c) + 8*a^4*b^2*e^(4*c) + 7*a^3*b^3*e^(4*c) + 2*a^2*b^4
*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*ima
g_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a +
 b) + b/(a + b)))) + 9*(3*a^5*b*e^(4*c) + 8*a^4*b^2*e^(4*c) + 7*a^3*b^3*e^(
4*c) + 2*a^2*b^4*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))
)*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(a
rccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/
(a + b)))) + 3*(3*a^5*b*e^(4*c) + 8*a^4*b^2*e^(4*c) + 7*a^3*b^3*e^(4*c) + 2
*a^2*b^4*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh
(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-
a/(a + b) + b/(a + b))))^2 - 9*(3*a^5*b*e^(4*c) + 8*a^4*b^2*e^(4*c) + 7*a^3
*b^3*e^(4*c) + 2*a^2*b^4*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(
a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_
part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b
) + b/(a + b))))^2 - (3*a^5*b*e^(4*c) + 8*a^4*b^2*e^(4*c) + 7*a^3*b^3*e^(4*
c) + 2*a^2*b^4*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^
3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + 3*(3*a^5*b*e^(4*c
) + 8*a^4*b^2*e^(4*c) + 7*a^3*b^3*e^(4*c) + 2*a^2*b^4*e^(4*c))*cos(1/2*real
_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b)
+ b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 - (3
*a^5*b*e^(4*c) + 8*a^4*b^2*e^(4*c) + 7*a^3*b^3*e^(4*c) + 2*a^2*b^4*e^(4*c))
*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arcc
os(-a/(a + b) + b/(a + b)))) + (3*a^5*b*e^(4*c) + 8*a^4*b^2*e^(4*c) + 7*a^3
*b^3*e^(4*c) + 2*a^2*b^4*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(
a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*log(-2*((a^4
 + 2*a^3*b + a^2*b^2)/(a^4*e^(4*c) + 2*a^3*b*e^(4*c) + a^2*b^2*e^(4*c)))^(1
/4)*cos(1/2*arccos(-(a - b)/(a + b)))*e^(d*x) + sqrt((a^4 + 2*a^3*b + a^2*b
^2)/(a^4*e^(4*c) + 2*a^3*b*e^(4*c) + a^2*b^2*e^(4*c))) + e^(2*d*x))/(2*(a^3
*e^(2*c) + a^2*b*e^(2*c))^2*a*b - (a^4*e^(2*c) - a^2*b^2*e^(2*c))*sqrt(-a*b
)*abs(-a^3*e^(2*c) - a^2*b*e^(2*c))) - 8*(b*e^(3*d*x + 3*c) + b*e^(d*x + c)
)/((a^2 + a*b)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c)
 - 2*b*e^(2*d*x + 2*c) + a + b)) + 8*log(e^(d*x + c) + 1)/a^2 - 8*log(abs(e
^(d*x + c) - 1))/a^2)/d
```

3.38 $\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=82

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}d}-\frac{3\coth(c+dx)}{2a^2d}+\frac{\coth(c+dx)}{2ad\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] (-3*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(5/2)*d) - (3*Cot
h[c + d*x])/(2*a^2*d) + Coth[c + d*x]/(2*a*d*(a + b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.0764964, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3663, 290, 325, 205}

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}d}-\frac{3\coth(c+dx)}{2a^2d}+\frac{\coth(c+dx)}{2ad\left(a+b\tanh^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (-3*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(5/2)*d) - (3*Cot
h[c + d*x])/(2*a^2*d) + Coth[c + d*x]/(2*a*d*(a + b*Tanh[c + d*x]^2))
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\coth(c+dx)}{2ad\left(a+b\tanh^2(c+dx)\right)} + \frac{3\operatorname{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)}\,dx, x, \tanh(c+dx)\right)}{2ad}$$

$$= -\frac{3\coth(c+dx)}{2a^2d} + \frac{\coth(c+dx)}{2ad\left(a+b\tanh^2(c+dx)\right)} - \frac{(3b)\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+d$$

$$= -\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}d} - \frac{3\coth(c+dx)}{2a^2d} + \frac{\coth(c+dx)}{2ad\left(a+b\tanh^2(c+dx)\right)}$$

Mathematica [A] time = 0.419541, size = 86, normalized size = 1.05

$$\frac{-3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right) - \frac{\sqrt{a}b\sinh(2(c+dx))}{(a+b)\cosh(2(c+dx))+a-b} - 2\sqrt{a}\coth(c+dx)}{2a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (-3*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] - 2*Sqrt[a]*Coth[c + d*
x] - (Sqrt[a]*b*Sinh[2*(c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)]))/(2*
a^(5/2)*d)
```

Maple [B] time = 0.102, size = 552, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] -1/2/d/a^2*tanh(1/2*d*x+1/2*c)-1/d*b/a^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/
2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*tanh(1/2*d*x+1/2*c)^3-1/d*b/a
^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)
^2*b+a)*tanh(1/2*d*x+1/2*c)+3/2/d/(b*(a+b))^(1/2)/a/((2*(b*(a+b))^(1/2)-a-2
*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1
/2))*b-3/2/d*b/a^2/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d
*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+3/2/d*b^2/a^2/(b*(a+b))^(1/2
)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*
(a+b))^(1/2)-a-2*b)*a)^(1/2))+3/2/d/(b*(a+b))^(1/2)/a/((2*(b*(a+b))^(1/2)+a
+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(
1/2))*b+3/2/d*b/a^2/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d
*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+3/2/d*b^2/a^2/(b*(a+b))^(1/2
)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(
a+b))^(1/2)+a+2*b)*a)^(1/2))-1/2/d/a^2/tanh(1/2*d*x+1/2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 2.40928, size = 6218, normalized size = 75.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [-1/4*(4*(2*a^2 + 3*a*b + 3*b^2)*cosh(d*x + c)^4 + 16*(2*a^2 + 3*a*b + 3*b^
2)*cosh(d*x + c)*sinh(d*x + c)^3 + 4*(2*a^2 + 3*a*b + 3*b^2)*sinh(d*x + c)^
4 + 8*(2*a^2 - 3*b^2)*cosh(d*x + c)^2 + 8*(3*(2*a^2 + 3*a*b + 3*b^2)*cosh(d
*x + c)^2 + 2*a^2 - 3*b^2)*sinh(d*x + c)^2 - 3*((a^2 + 2*a*b + b^2)*cosh(d*
x + c)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (a^2 + 2*a
*b + b^2)*sinh(d*x + c)^6 + (a^2 - 2*a*b - 3*b^2)*cosh(d*x + c)^4 + (15*(a^
2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - 2*a*b - 3*b^2)*sinh(d*x + c)^4 + 4
*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - 2*a*b - 3*b^2)*cosh(d*x +
c))*sinh(d*x + c)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(d*x + c)^2 + (15*(a^2 + 2*
a*b + b^2)*cosh(d*x + c)^4 + 6*(a^2 - 2*a*b - 3*b^2)*cosh(d*x + c)^2 - a^2
+ 2*a*b + 3*b^2)*sinh(d*x + c)^2 - a^2 - 2*a*b - b^2 + 2*(3*(a^2 + 2*a*b +
b^2)*cosh(d*x + c)^5 + 2*(a^2 - 2*a*b - 3*b^2)*cosh(d*x + c)^3 - (a^2 - 2*a
*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/a)*log(((a^2 + 2*a*b + b^
2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 +
(a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*
(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*
a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x +
 c))*sinh(d*x + c) - 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*
x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))
/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a +
b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)
^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x
 + c))*sinh(d*x + c) + a + b)) + 8*a^2 + 20*a*b + 12*b^2 + 16*((2*a^2 + 3*a
*b + 3*b^2)*cosh(d*x + c)^3 + (2*a^2 - 3*b^2)*cosh(d*x + c))*sinh(d*x + c))
/((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^6 + 6*(a^4 + 2*a^3*b + a^2*b^2)
*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^4 + 2*a^3*b + a^2*b^2)*d*sinh(d*x + c
)^6 + (a^4 - 2*a^3*b - 3*a^2*b^2)*d*cosh(d*x + c)^4 + (15*(a^4 + 2*a^3*b +
a^2*b^2)*d*cosh(d*x + c)^2 + (a^4 - 2*a^3*b - 3*a^2*b^2)*d)*sinh(d*x + c)^4
 - (a^4 - 2*a^3*b - 3*a^2*b^2)*d*cosh(d*x + c)^2 + 4*(5*(a^4 + 2*a^3*b + a^
2*b^2)*d*cosh(d*x + c)^3 + (a^4 - 2*a^3*b - 3*a^2*b^2)*d*cosh(d*x + c))*sin
h(d*x + c)^3 + (15*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^4 + 6*(a^4 - 2
*a^3*b - 3*a^2*b^2)*d*cosh(d*x + c)^2 - (a^4 - 2*a^3*b - 3*a^2*b^2)*d)*sinh
(d*x + c)^2 - (a^4 + 2*a^3*b + a^2*b^2)*d + 2*(3*(a^4 + 2*a^3*b + a^2*b^2)*
d*cosh(d*x + c)^5 + 2*(a^4 - 2*a^3*b - 3*a^2*b^2)*d*cosh(d*x + c)^3 - (a^4
- 2*a^3*b - 3*a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*(2*a^2 + 3*
a*b + 3*b^2)*cosh(d*x + c)^4 + 8*(2*a^2 + 3*a*b + 3*b^2)*cosh(d*x + c)*sinh
(d*x + c)^3 + 2*(2*a^2 + 3*a*b + 3*b^2)*sinh(d*x + c)^4 + 4*(2*a^2 - 3*b^2)
```

```
*cosh(d*x + c)^2 + 4*(3*(2*a^2 + 3*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*a^2 - 3
*b^2)*sinh(d*x + c)^2 + 3*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 6*(a^2 + 2
*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (a^2 + 2*a*b + b^2)*sinh(d*x +
c)^6 + (a^2 - 2*a*b - 3*b^2)*cosh(d*x + c)^4 + (15*(a^2 + 2*a*b + b^2)*cosh
(d*x + c)^2 + a^2 - 2*a*b - 3*b^2)*sinh(d*x + c)^4 + 4*(5*(a^2 + 2*a*b + b^
2)*cosh(d*x + c)^3 + (a^2 - 2*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 -
 (a^2 - 2*a*b - 3*b^2)*cosh(d*x + c)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(d*x +
 c)^4 + 6*(a^2 - 2*a*b - 3*b^2)*cosh(d*x + c)^2 - a^2 + 2*a*b + 3*b^2)*sinh
(d*x + c)^2 - a^2 - 2*a*b - b^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5
+ 2*(a^2 - 2*a*b - 3*b^2)*cosh(d*x + c)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(d*x
+ c))*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a +
 b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(b/a
)/b) + 4*a^2 + 10*a*b + 6*b^2 + 8*((2*a^2 + 3*a*b + 3*b^2)*cosh(d*x + c)^3
+ (2*a^2 - 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 2*a^3*b + a^2*b^2)*
d*cosh(d*x + c)^6 + 6*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)*sinh(d*x +
c)^5 + (a^4 + 2*a^3*b + a^2*b^2)*d*sinh(d*x + c)^6 + (a^4 - 2*a^3*b - 3*a^2
*b^2)*d*cosh(d*x + c)^4 + (15*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 +
 (a^4 - 2*a^3*b - 3*a^2*b^2)*d)*sinh(d*x + c)^4 - (a^4 - 2*a^3*b - 3*a^2*b^
2)*d*cosh(d*x + c)^2 + 4*(5*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^3 + (
a^4 - 2*a^3*b - 3*a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (15*(a^4 + 2*
a^3*b + a^2*b^2)*d*cosh(d*x + c)^4 + 6*(a^4 - 2*a^3*b - 3*a^2*b^2)*d*cosh(d
*x + c)^2 - (a^4 - 2*a^3*b - 3*a^2*b^2)*d)*sinh(d*x + c)^2 - (a^4 + 2*a^3*b
 + a^2*b^2)*d + 2*(3*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^5 + 2*(a^4 -
 2*a^3*b - 3*a^2*b^2)*d*cosh(d*x + c)^3 - (a^4 - 2*a^3*b - 3*a^2*b^2)*d*cos
h(d*x + c))*sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(csch(c + d*x)**2/(a + b*tanh(c + d*x)**2)**2, x)
```

Giac [B] time = 1.65343, size = 306, normalized size = 3.73

$$-\frac{\frac{3\,b\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\sqrt{ab}a^2}+\frac{2\left(2\,a^2e^{(4\,dx+4\,c)}+3\,abe^{(4\,dx+4\,c)}+3\,b^2e^{(4\,dx+4\,c)}+4\,a^2e^{(2\,dx+2\,c)}-6\,b^2e^{(2\,dx+2\,c)}+2\,a^2+5\,ab+3\,b^2\right)}{\left(a^3+a^2b\right)\left(ae^{(6\,dx+6\,c)}+be^{(6\,dx+6\,c)}+ae^{(4\,dx+4\,c)}-3\,be^{(4\,dx+4\,c)}-ae^{(2\,dx+2\,c)}+3\,be^{(2\,dx+2\,c)}-a-b\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*b*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a
*b))/(sqrt(a*b)*a^2) + 2*(2*a^2*e^(4*d*x + 4*c) + 3*a*b*e^(4*d*x + 4*c) + 3
*b^2*e^(4*d*x + 4*c) + 4*a^2*e^(2*d*x + 2*c) - 6*b^2*e^(2*d*x + 2*c) + 2*a^
2 + 5*a*b + 3*b^2)/((a^3 + a^2*b)*(a*e^(6*d*x + 6*c) + b*e^(6*d*x + 6*c) +
a*e^(4*d*x + 4*c) - 3*b*e^(4*d*x + 4*c) - a*e^(2*d*x + 2*c) + 3*b*e^(2*d*x
+ 2*c) - a - b)))/d
```

3.39 $\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=141

$$-\frac{b\operatorname{sech}(c+dx)}{a^2d\left(a-b\operatorname{sech}^2(c+dx)+b\right)}+\frac{(a+4b)\tanh^{-1}(\cosh(c+dx))}{2a^3d}-\frac{\sqrt{b}(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{2a^3d\sqrt{a+b}}-\frac{\coth(c+dx)\operatorname{cs}}{2ad\left(a-b\operatorname{sech}^2(\right.}$$

```
[Out] ((a + 4*b)*ArcTanh[Cosh[c + d*x]])/(2*a^3*d) - (Sqrt[b]*(3*a + 4*b)*ArcTanh
[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(2*a^3*Sqrt[a + b]*d) - (Coth[c + d*
x]*Csch[c + d*x])/(2*a*d*(a + b - b*Sech[c + d*x]^2)) - (b*Sech[c + d*x])/(
a^2*d*(a + b - b*Sech[c + d*x]^2))
```

Rubi [A] time = 0.209026, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3664, 471, 527, 522, 207, 208}

$$-\frac{b\operatorname{sech}(c+dx)}{a^2d\left(a-b\operatorname{sech}^2(c+dx)+b\right)}+\frac{(a+4b)\tanh^{-1}(\cosh(c+dx))}{2a^3d}-\frac{\sqrt{b}(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{2a^3d\sqrt{a+b}}-\frac{\coth(c+dx)\operatorname{cs}}{2ad\left(a-b\operatorname{sech}^2(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((a + 4*b)*ArcTanh[Cosh[c + d*x]])/(2*a^3*d) - (Sqrt[b]*(3*a + 4*b)*ArcTanh
[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(2*a^3*Sqrt[a + b]*d) - (Coth[c + d*
x]*Csch[c + d*x])/(2*a*d*(a + b - b*Sech[c + d*x]^2)) - (b*Sech[c + d*x])/(
a^2*d*(a + b - b*Sech[c + d*x]^2))
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{x^2}{\left(-1+x^2\right)^2\left(a+b-bx^2\right)^2}\,dx, x, \operatorname{sech}(c+dx)\right)}{d}$$

$$= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a+b-b\operatorname{sech}^2(c+dx)\right)} - \frac{\operatorname{Subst}\left(\int \frac{a+b+3bx^2}{\left(-1+x^2\right)\left(a+b-bx^2\right)^2}\,dx, x, \operatorname{sech}(c+dx)\right)}{2ad}$$

$$= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a+b-b\operatorname{sech}^2(c+dx)\right)} - \frac{b\operatorname{sech}(c+dx)}{a^2d\left(a+b-b\operatorname{sech}^2(c+dx)\right)} - \frac{\operatorname{Subst}\left(\int \frac{2(a+b)(a+2b}{\left(-1+x^2\right)(}\right.}{4}$$

$$= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a+b-b\operatorname{sech}^2(c+dx)\right)} - \frac{b\operatorname{sech}(c+dx)}{a^2d\left(a+b-b\operatorname{sech}^2(c+dx)\right)} - \frac{(a+4b)\operatorname{Subst}\left(\int \right.}{}$$

$$= \frac{(a+4b)\tanh^{-1}(\cosh(c+dx))}{2a^3d} - \frac{\sqrt{b}(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{2a^3\sqrt{a+b}\,d} - \frac{\coth(c+dx)}{2ad\left(a+b-b\right.}$$

Mathematica [C] time = 4.08918, size = 203, normalized size = 1.44

$$-\frac{\frac{8ab\cosh(c+dx)}{(a+b)\cosh(2(c+dx))+a-b} + 4(a+4b)\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) + \frac{4i\sqrt{b}(3a+4b)\tan^{-1}\left(\frac{-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)-i\sqrt{a+b}}{\sqrt{b}}\right)}{\sqrt{a+b}} + \frac{4i\sqrt{b}(3a+4b)\tan^{-1}\left(\frac{\sqrt{a}\tanh}{}\right.}{\sqrt{a+b}}}{8a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -((((4*I)*Sqrt[b]*(3*a + 4*b)*ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d
*x)/2])/Sqrt[b]])/Sqrt[a + b] + ((4*I)*Sqrt[b]*(3*a + 4*b)*ArcTan[((-I)*Sqr
t[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]])/Sqrt[a + b] + (8*a*b*Cosh[c
 + d*x])/(a - b + (a + b)*Cosh[2*(c + d*x)]) + a*Csch[(c + d*x)/2]^2 + 4*(a
```

```
 + 4*b)*Log[Tanh[(c + d*x)/2]] + a*Sech[(c + d*x)/2]^2)/(8*a^3*d)
```

Maple [B] time = 0.108, size = 367, normalized size = 2.6

$$\frac{1}{8\,da^2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-\frac{b}{da^2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4a+2\,(\tanh(1/2\,dx+c/2))^2\,a+4\,(\tanh(1/2\,dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] 1/8/d*tanh(1/2*d*x+1/2*c)^2/a^2-1/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1
/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*tanh(1/2*d*x+1/2*c)^2-2/d/a^
3*b^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2
*c)^2*b+a)*tanh(1/2*d*x+1/2*c)^2-1/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(
1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)-3/2/d/a^2*b/(a*b+b^2)^(1/2)
*arctanh(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2))-2/d/a^3*b
^2/(a*b+b^2)^(1/2)*arctanh(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2
)^(1/2))-1/8/d/a^2/tanh(1/2*d*x+1/2*c)^2-1/2/d/a^2*ln(tanh(1/2*d*x+1/2*c))-
2/d/a^3*ln(tanh(1/2*d*x+1/2*c))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(ae^{(7\,c)}+2\,be^{(7\,c)}\right)e^{(7\,dx)}+\left(3\,ae^{(5\,c)}-2\,be^{(5\,c)}\right)e^{(5\,dx)}+\left(3\,ae^{(3\,c)}-2\,be^{(3\,c)}\right)e^{(3\,dx)}+\left(ae^{c}+2\,be^{c}\right)e^{(dx)}}{4\,a^2bde^{(6\,dx+6\,c)}+4\,a^2bde^{(2\,dx+2\,c)}-a^3d-a^2bd-\left(a^3de^{(8\,c)}+a^2bde^{(8\,c)}\right)e^{(8\,dx)}+2\left(a^3de^{(4\,c)}-3\,a^2bde^{(4\,c)}\right)e^{(4\,dx)}}+\frac{(a+4\,b)\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] ((a*e^(7*c) + 2*b*e^(7*c))*e^(7*d*x) + (3*a*e^(5*c) - 2*b*e^(5*c))*e^(5*d*x
) + (3*a*e^(3*c) - 2*b*e^(3*c))*e^(3*d*x) + (a*e^c + 2*b*e^c)*e^(d*x))/(4*a
^2*b*d*e^(6*d*x + 6*c) + 4*a^2*b*d*e^(2*d*x + 2*c) - a^3*d - a^2*b*d - (a^3
*d*e^(8*c) + a^2*b*d*e^(8*c))*e^(8*d*x) + 2*(a^3*d*e^(4*c) - 3*a^2*b*d*e^(4
*c))*e^(4*d*x)) + 1/2*(a + 4*b)*log((e^(d*x + c) + 1)*e^(-c))/(a^3*d) - 1/2
*(a + 4*b)*log((e^(d*x + c) - 1)*e^(-c))/(a^3*d) + 8*integrate(1/8*((3*a*b*
e^(3*c) + 4*b^2*e^(3*c))*e^(3*d*x) - (3*a*b*e^c + 4*b^2*e^c)*e^(d*x))/(a^4
+ a^3*b + (a^4*e^(4*c) + a^3*b*e^(4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) - a^3*b*
e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 3.30417, size = 15776, normalized size = 111.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^2 + 2*a*b)*cosh(d*x + c)^7 + 28*(a^2 + 2*a*b)*cosh(d*x + c)*sinh
(d*x + c)^6 + 4*(a^2 + 2*a*b)*sinh(d*x + c)^7 + 4*(3*a^2 - 2*a*b)*cosh(d*x
+ c)^5 + 4*(21*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 3*a^2 - 2*a*b)*sinh(d*x + c)
```

```
^5 + 20*(7*(a^2 + 2*a*b)*cosh(d*x + c)^3 + (3*a^2 - 2*a*b)*cosh(d*x + c))*s
inh(d*x + c)^4 + 4*(3*a^2 - 2*a*b)*cosh(d*x + c)^3 + 4*(35*(a^2 + 2*a*b)*co
sh(d*x + c)^4 + 10*(3*a^2 - 2*a*b)*cosh(d*x + c)^2 + 3*a^2 - 2*a*b)*sinh(d*
x + c)^3 + 4*(21*(a^2 + 2*a*b)*cosh(d*x + c)^5 + 10*(3*a^2 - 2*a*b)*cosh(d*
x + c)^3 + 3*(3*a^2 - 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^2 - ((3*a^2 + 7*a
*b + 4*b^2)*cosh(d*x + c)^8 + 8*(3*a^2 + 7*a*b + 4*b^2)*cosh(d*x + c)*sinh(
d*x + c)^7 + (3*a^2 + 7*a*b + 4*b^2)*sinh(d*x + c)^8 - 4*(3*a*b + 4*b^2)*co
sh(d*x + c)^6 + 4*(7*(3*a^2 + 7*a*b + 4*b^2)*cosh(d*x + c)^2 - 3*a*b - 4*b^
2)*sinh(d*x + c)^6 + 8*(7*(3*a^2 + 7*a*b + 4*b^2)*cosh(d*x + c)^3 - 3*(3*a*
b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(3*a^2 - 5*a*b - 12*b^2)*cosh
(d*x + c)^4 + 2*(35*(3*a^2 + 7*a*b + 4*b^2)*cosh(d*x + c)^4 - 30*(3*a*b + 4
*b^2)*cosh(d*x + c)^2 - 3*a^2 + 5*a*b + 12*b^2)*sinh(d*x + c)^4 + 8*(7*(3*a
^2 + 7*a*b + 4*b^2)*cosh(d*x + c)^5 - 10*(3*a*b + 4*b^2)*cosh(d*x + c)^3 -
(3*a^2 - 5*a*b - 12*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(3*a*b + 4*b^2)
*cosh(d*x + c)^2 + 4*(7*(3*a^2 + 7*a*b + 4*b^2)*cosh(d*x + c)^6 - 15*(3*a*b
 + 4*b^2)*cosh(d*x + c)^4 - 3*(3*a^2 - 5*a*b - 12*b^2)*cosh(d*x + c)^2 - 3*
a*b - 4*b^2)*sinh(d*x + c)^2 + 3*a^2 + 7*a*b + 4*b^2 + 8*((3*a^2 + 7*a*b +
4*b^2)*cosh(d*x + c)^7 - 3*(3*a*b + 4*b^2)*cosh(d*x + c)^5 - (3*a^2 - 5*a*b
 - 12*b^2)*cosh(d*x + c)^3 - (3*a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c))*
sqrt(b/(a + b))*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh
(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*
(a + b)*cosh(d*x + c)^2 + a + 3*b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x +
c)^3 + (a + 3*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^3
+ 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a +
b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))*sqrt(
b/(a + b)) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh
(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a
 + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3
 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 4*(a^2 + 2*a*b)*cosh(d*
x + c) - 2*((a^2 + 5*a*b + 4*b^2)*cosh(d*x + c)^8 + 8*(a^2 + 5*a*b + 4*b^2)
*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 5*a*b + 4*b^2)*sinh(d*x + c)^8 - 4*
(a*b + 4*b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 + 5*a*b + 4*b^2)*cosh(d*x + c)^2
- a*b - 4*b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 + 5*a*b + 4*b^2)*cosh(d*x + c)^3
 - 3*(a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(a^2 + a*b - 12*b^2)*
cosh(d*x + c)^4 + 2*(35*(a^2 + 5*a*b + 4*b^2)*cosh(d*x + c)^4 - 30*(a*b + 4
*b^2)*cosh(d*x + c)^2 - a^2 - a*b + 12*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 5
*a*b + 4*b^2)*cosh(d*x + c)^5 - 10*(a*b + 4*b^2)*cosh(d*x + c)^3 - (a^2 + a
*b - 12*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(a*b + 4*b^2)*cosh(d*x + c)
^2 + 4*(7*(a^2 + 5*a*b + 4*b^2)*cosh(d*x + c)^6 - 15*(a*b + 4*b^2)*cosh(d*x
 + c)^4 - 3*(a^2 + a*b - 12*b^2)*cosh(d*x + c)^2 - a*b - 4*b^2)*sinh(d*x +
c)^2 + a^2 + 5*a*b + 4*b^2 + 8*((a^2 + 5*a*b + 4*b^2)*cosh(d*x + c)^7 - 3*(
a*b + 4*b^2)*cosh(d*x + c)^5 - (a^2 + a*b - 12*b^2)*cosh(d*x + c)^3 - (a*b
+ 4*b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) +
1) + 2*((a^2 + 5*a*b + 4*b^2)*cosh(d*x + c)^8 + 8*(a^2 + 5*a*b + 4*b^2)*cos
h(d*x + c)*sinh(d*x + c)^7 + (a^2 + 5*a*b + 4*b^2)*sinh(d*x + c)^8 - 4*(a*b
 + 4*b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 + 5*a*b + 4*b^2)*cosh(d*x + c)^2 - a*
b - 4*b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 + 5*a*b + 4*b^2)*cosh(d*x + c)^3 - 3
*(a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(a^2 + a*b - 12*b^2)*cosh
(d*x + c)^4 + 2*(35*(a^2 + 5*a*b + 4*b^2)*cosh(d*x + c)^4 - 30*(a*b + 4*b^2
)*cosh(d*x + c)^2 - a^2 - a*b + 12*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 5*a*b
 + 4*b^2)*cosh(d*x + c)^5 - 10*(a*b + 4*b^2)*cosh(d*x + c)^3 - (a^2 + a*b -
 12*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(a*b + 4*b^2)*cosh(d*x + c)^2 +
 4*(7*(a^2 + 5*a*b + 4*b^2)*cosh(d*x + c)^6 - 15*(a*b + 4*b^2)*cosh(d*x + c
)^4 - 3*(a^2 + a*b - 12*b^2)*cosh(d*x + c)^2 - a*b - 4*b^2)*sinh(d*x + c)^2
 + a^2 + 5*a*b + 4*b^2 + 8*((a^2 + 5*a*b + 4*b^2)*cosh(d*x + c)^7 - 3*(a*b
+ 4*b^2)*cosh(d*x + c)^5 - (a^2 + a*b - 12*b^2)*cosh(d*x + c)^3 - (a*b + 4*
b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) +
 4*(7*(a^2 + 2*a*b)*cosh(d*x + c)^6 + 5*(3*a^2 - 2*a*b)*cosh(d*x + c)^4 + 3
*(3*a^2 - 2*a*b)*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c))/(4*a^3*b*d*c
```

osh(d*x + c)^6 - (a^4 + a^3*b)*d*cosh(d*x + c)^8 - 8*(a^4 + a^3*b)*d*cosh(d
*x + c)*sinh(d*x + c)^7 - (a^4 + a^3*b)*d*sinh(d*x + c)^8 + 4*a^3*b*d*cosh(
d*x + c)^2 + 4*(a^3*b*d - 7*(a^4 + a^3*b)*d*cosh(d*x + c)^2)*sinh(d*x + c)^
6 + 2*(a^4 - 3*a^3*b)*d*cosh(d*x + c)^4 + 8*(3*a^3*b*d*cosh(d*x + c) - 7*(a
^4 + a^3*b)*d*cosh(d*x + c)^3)*sinh(d*x + c)^5 + 2*(30*a^3*b*d*cosh(d*x + c
)^2 - 35*(a^4 + a^3*b)*d*cosh(d*x + c)^4 + (a^4 - 3*a^3*b)*d)*sinh(d*x + c)
^4 + 8*(10*a^3*b*d*cosh(d*x + c)^3 - 7*(a^4 + a^3*b)*d*cosh(d*x + c)^5 + (a
^4 - 3*a^3*b)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(15*a^3*b*d*cosh(d*x + c
)^4 - 7*(a^4 + a^3*b)*d*cosh(d*x + c)^6 + a^3*b*d + 3*(a^4 - 3*a^3*b)*d*cos
h(d*x + c)^2)*sinh(d*x + c)^2 - (a^4 + a^3*b)*d + 8*(3*a^3*b*d*cosh(d*x + c
)^5 - (a^4 + a^3*b)*d*cosh(d*x + c)^7 + a^3*b*d*cosh(d*x + c) + (a^4 - 3*a^
3*b)*d*cosh(d*x + c)^3)*sinh(d*x + c)), 1/2*(2*(a^2 + 2*a*b)*cosh(d*x + c)^
7 + 14*(a^2 + 2*a*b)*cosh(d*x + c)*sinh(d*x + c)^6 + 2*(a^2 + 2*a*b)*sinh(d
x + c)^7 + 2(3*a^2 - 2*a*b)*cosh(d*x + c)^5 + 2*(21*(a^2 + 2*a*b)*cosh(d*
x + c)^2 + 3*a^2 - 2*a*b)*sinh(d*x + c)^5 + 10*(7*(a^2 + 2*a*b)*cosh(d*x +
c)^3 + (3*a^2 - 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^4 + 2*(3*a^2 - 2*a*b)*c
osh(d*x + c)^3 + 2*(35*(a^2 + 2*a*b)*cosh(d*x + c)^4 + 10*(3*a^2 - 2*a*b)*c
osh(d*x + c)^2 + 3*a^2 - 2*a*b)*sinh(d*x + c)^3 + 2*(21*(a^2 + 2*a*b)*cosh(
d*x + c)^5 + 10*(3*a^2 - 2*a*b)*cosh(d*x + c)^3 + 3*(3*a^2 - 2*a*b)*cosh(d*
x + c))*sinh(d*x + c)^2 + ((3*a^2 + 7*a*b + 4*b^2)*cosh(d*x + c)^8 + 8*(3*a
^2 + 7*a*b + 4*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^2 + 7*a*b + 4*b^2)
*sinh(d*x + c)^8 - 4*(3*a*b + 4*b^2)*cosh(d*x + c)^6 + 4*(7*(3*a^2 + 7*a*b
+ 4*b^2)*cosh(d*x + c)^2 - 3*a*b - 4*b^2)*sinh(d*x + c)^6 + 8*(7*(3*a^2 + 7
*a*b + 4*b^2)*cosh(d*x + c)^3 - 3*(3*a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x +
c)^5 - 2*(3*a^2 - 5*a*b - 12*b^2)*cosh(d*x + c)^4 + 2*(35*(3*a^2 + 7*a*b +
4*b^2)*cosh(d*x + c)^4 - 30*(3*a*b + 4*b^2)*cosh(d*x + c)^2 - 3*a^2 + 5*a*
b + 12*b^2)*sinh(d*x + c)^4 + 8*(7*(3*a^2 + 7*a*b + 4*b^2)*cosh(d*x + c)^5
- 10*(3*a*b + 4*b^2)*cosh(d*x + c)^3 - (3*a^2 - 5*a*b - 12*b^2)*cosh(d*x +
c))*sinh(d*x + c)^3 - 4*(3*a*b + 4*b^2)*cosh(d*x + c)^2 + 4*(7*(3*a^2 + 7*a
*b + 4*b^2)*cosh(d*x + c)^6 - 15*(3*a*b + 4*b^2)*cosh(d*x + c)^4 - 3*(3*a^2
- 5*a*b - 12*b^2)*cosh(d*x + c)^2 - 3*a*b - 4*b^2)*sinh(d*x + c)^2 + 3*a^2
+ 7*a*b + 4*b^2 + 8*((3*a^2 + 7*a*b + 4*b^2)*cosh(d*x + c)^7 - 3*(3*a*b +
4*b^2)*cosh(d*x + c)^5 - (3*a^2 - 5*a*b - 12*b^2)*cosh(d*x + c)^3 - (3*a*b
+ 4*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2*((a + b)
*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d
*x + c)^3 + (a - 3*b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a - 3*b)
*sinh(d*x + c))*sqrt(-b/(a + b))/b) - ((3*a^2 + 7*a*b + 4*b^2)*cosh(d*x + c
)^8 + 8*(3*a^2 + 7*a*b + 4*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^2 + 7*
a*b + 4*b^2)*sinh(d*x + c)^8 - 4*(3*a*b + 4*b^2)*cosh(d*x + c)^6 + 4*(7*(3*
a^2 + 7*a*b + 4*b^2)*cosh(d*x + c)^2 - 3*a*b - 4*b^2)*sinh(d*x + c)^6 + 8*(
7*(3*a^2 + 7*a*b + 4*b^2)*cosh(d*x + c)^3 - 3*(3*a*b + 4*b^2)*cosh(d*x + c)
)*sinh(d*x + c)^5 - 2*(3*a^2 - 5*a*b - 12*b^2)*cosh(d*x + c)^4 + 2*(35*(3*a
^2 + 7*a*b + 4*b^2)*cosh(d*x + c)^4 - 30*(3*a*b + 4*b^2)*cosh(d*x + c)^2 -
3*a^2 + 5*a*b + 12*b^2)*sinh(d*x + c)^4 + 8*(7*(3*a^2 + 7*a*b + 4*b^2)*cosh
(d*x + c)^5 - 10*(3*a*b + 4*b^2)*cosh(d*x + c)^3 - (3*a^2 - 5*a*b - 12*b^2)
*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(3*a*b + 4*b^2)*cosh(d*x + c)^2 + 4*(7*
(3*a^2 + 7*a*b + 4*b^2)*cosh(d*x + c)^6 - 15*(3*a*b + 4*b^2)*cosh(d*x + c)^
4 - 3*(3*a^2 - 5*a*b - 12*b^2)*cosh(d*x + c)^2 - 3*a*b - 4*b^2)*sinh(d*x +
c)^2 + 3*a^2 + 7*a*b + 4*b^2 + 8*((3*a^2 + 7*a*b + 4*b^2)*cosh(d*x + c)^7 -
3*(3*a*b + 4*b^2)*cosh(d*x + c)^5 - (3*a^2 - 5*a*b - 12*b^2)*cosh(d*x + c)
^3 - (3*a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(
1/2*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x + c))*sqrt(-b/(a + b))/b) + 2
*(a^2 + 2*a*b)*cosh(d*x + c) - ((a^2 + 5*a*b + 4*b^2)*cosh(d*x + c)^8 + 8*(
a^2 + 5*a*b + 4*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 5*a*b + 4*b^2)*
sinh(d*x + c)^8 - 4*(a*b + 4*b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 + 5*a*b + 4*b
^2)*cosh(d*x + c)^2 - a*b - 4*b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 + 5*a*b + 4*
b^2)*cosh(d*x + c)^3 - 3*(a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(
a^2 + a*b - 12*b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 + 5*a*b + 4*b^2)*cosh(d*x
+ c)^4 - 30*(a*b + 4*b^2)*cosh(d*x + c)^2 - a^2 - a*b + 12*b^2)*sinh(d*x +

```
c)^4 + 8*(7*(a^2 + 5*a*b + 4*b^2)*cosh(d*x + c)^5 - 10*(a*b + 4*b^2)*cosh(d
*x + c)^3 - (a^2 + a*b - 12*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(a*b +
4*b^2)*cosh(d*x + c)^2 + 4*(7*(a^2 + 5*a*b + 4*b^2)*cosh(d*x + c)^6 - 15*(a
*b + 4*b^2)*cosh(d*x + c)^4 - 3*(a^2 + a*b - 12*b^2)*cosh(d*x + c)^2 - a*b
- 4*b^2)*sinh(d*x + c)^2 + a^2 + 5*a*b + 4*b^2 + 8*((a^2 + 5*a*b + 4*b^2)*c
osh(d*x + c)^7 - 3*(a*b + 4*b^2)*cosh(d*x + c)^5 - (a^2 + a*b - 12*b^2)*cos
h(d*x + c)^3 - (a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c
) + sinh(d*x + c) + 1) + ((a^2 + 5*a*b + 4*b^2)*cosh(d*x + c)^8 + 8*(a^2 +
5*a*b + 4*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 5*a*b + 4*b^2)*sinh(d
*x + c)^8 - 4*(a*b + 4*b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 + 5*a*b + 4*b^2)*co
sh(d*x + c)^2 - a*b - 4*b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 + 5*a*b + 4*b^2)*c
osh(d*x + c)^3 - 3*(a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(a^2 +
a*b - 12*b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 + 5*a*b + 4*b^2)*cosh(d*x + c)^4
 - 30*(a*b + 4*b^2)*cosh(d*x + c)^2 - a^2 - a*b + 12*b^2)*sinh(d*x + c)^4 +
 8*(7*(a^2 + 5*a*b + 4*b^2)*cosh(d*x + c)^5 - 10*(a*b + 4*b^2)*cosh(d*x + c
)^3 - (a^2 + a*b - 12*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(a*b + 4*b^2)
*cosh(d*x + c)^2 + 4*(7*(a^2 + 5*a*b + 4*b^2)*cosh(d*x + c)^6 - 15*(a*b + 4
*b^2)*cosh(d*x + c)^4 - 3*(a^2 + a*b - 12*b^2)*cosh(d*x + c)^2 - a*b - 4*b^
2)*sinh(d*x + c)^2 + a^2 + 5*a*b + 4*b^2 + 8*((a^2 + 5*a*b + 4*b^2)*cosh(d*
x + c)^7 - 3*(a*b + 4*b^2)*cosh(d*x + c)^5 - (a^2 + a*b - 12*b^2)*cosh(d*x
+ c)^3 - (a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + si
nh(d*x + c) - 1) + 2*(7*(a^2 + 2*a*b)*cosh(d*x + c)^6 + 5*(3*a^2 - 2*a*b)*c
osh(d*x + c)^4 + 3*(3*a^2 - 2*a*b)*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x
+ c))/(4*a^3*b*d*cosh(d*x + c)^6 - (a^4 + a^3*b)*d*cosh(d*x + c)^8 - 8*(a^4
 + a^3*b)*d*cosh(d*x + c)*sinh(d*x + c)^7 - (a^4 + a^3*b)*d*sinh(d*x + c)^8
 + 4*a^3*b*d*cosh(d*x + c)^2 + 4*(a^3*b*d - 7*(a^4 + a^3*b)*d*cosh(d*x + c)
^2)*sinh(d*x + c)^6 + 2*(a^4 - 3*a^3*b)*d*cosh(d*x + c)^4 + 8*(3*a^3*b*d*co
sh(d*x + c) - 7*(a^4 + a^3*b)*d*cosh(d*x + c)^3)*sinh(d*x + c)^5 + 2*(30*a^
3*b*d*cosh(d*x + c)^2 - 35*(a^4 + a^3*b)*d*cosh(d*x + c)^4 + (a^4 - 3*a^3*b
)*d)*sinh(d*x + c)^4 + 8*(10*a^3*b*d*cosh(d*x + c)^3 - 7*(a^4 + a^3*b)*d*co
sh(d*x + c)^5 + (a^4 - 3*a^3*b)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(15*a^
3*b*d*cosh(d*x + c)^4 - 7*(a^4 + a^3*b)*d*cosh(d*x + c)^6 + a^3*b*d + 3*(a^
4 - 3*a^3*b)*d*cosh(d*x + c)^2)*sinh(d*x + c)^2 - (a^4 + a^3*b)*d + 8*(3*a^
3*b*d*cosh(d*x + c)^5 - (a^4 + a^3*b)*d*cosh(d*x + c)^7 + a^3*b*d*cosh(d*x
+ c) + (a^4 - 3*a^3*b)*d*cosh(d*x + c)^3)*sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(csch(c + d*x)**3/(a + b*tanh(c + d*x)**2)**2, x)
```

Giac [C] time = 1.97318, size = 5650, normalized size = 40.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*(2*(3*(3*a^5*b + 7*a^4*b^2 + 4*a^3*b^3)*cos(1/2*real_part(arccos(-a/(a
+ b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3
*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) - (3*a^5*b + 7*a^4*b^2
+ 4*a^3*b^3)*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*
real_part(arccos(-a/(a + b) + b/(a + b))))^3 - 9*(3*a^5*b + 7*a^4*b^2 + 4*a
^3*b^3)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_
part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b)
 + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(3*
a^5*b + 7*a^4*b^2 + 4*a^3*b^3)*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a
+ b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*ima
g_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(3*a^5*b + 7*a^4*b^2 + 4*a^3*b^
3)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(
arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a
 + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(3*a^5*b
 + 7*a^4*b^2 + 4*a^3*b^3)*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))
))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(
arccos(-a/(a + b) + b/(a + b))))^2 - 3*(3*a^5*b + 7*a^4*b^2 + 4*a^3*b^3)*co
s(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos
(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)
)))^3 + (3*a^5*b + 7*a^4*b^2 + 4*a^3*b^3)*sin(1/2*real_part(arccos(-a/(a +
b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 -
 (3*a^5*b + 7*a^4*b^2 + 4*a^3*b^3)*cosh(1/2*imag_part(arccos(-a/(a + b) + b
/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) + (3*a^5*b +
 7*a^4*b^2 + 4*a^3*b^3)*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*
sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*arctan((((a^4 + a^3*b)
/(a^4*e^(4*c) + a^3*b*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b))) + e
^(d*x))/(((a^4 + a^3*b)/(a^4*e^(4*c) + a^3*b*e^(4*c)))^(1/4)*sin(1/2*arccos
(-(a - b)/(a + b)))))/(2*a^7*b - (a^4 - a^3*b)*sqrt(-a*b)*a^2*abs(a)) + 2*(
3*(3*a^5*b + 7*a^4*b^2 + 4*a^3*b^3)*cos(1/2*real_part(arccos(-a/(a + b) + b
/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2
*real_part(arccos(-a/(a + b) + b/(a + b)))) - (3*a^5*b + 7*a^4*b^2 + 4*a^3*
b^3)*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_par
t(arccos(-a/(a + b) + b/(a + b))))^3 - 9*(3*a^5*b + 7*a^4*b^2 + 4*a^3*b^3)*
cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arc
cos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a
+ b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(3*a^5*b +
7*a^4*b^2 + 4*a^3*b^3)*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^
2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(a
rccos(-a/(a + b) + b/(a + b)))) + 9*(3*a^5*b + 7*a^4*b^2 + 4*a^3*b^3)*cos(1
/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-
a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))
*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(3*a^5*b + 7*a^4
*b^2 + 4*a^3*b^3)*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1
/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-
a/(a + b) + b/(a + b))))^2 - 3*(3*a^5*b + 7*a^4*b^2 + 4*a^3*b^3)*cos(1/2*re
al_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a +
 b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 +
(3*a^5*b + 7*a^4*b^2 + 4*a^3*b^3)*sin(1/2*real_part(arccos(-a/(a + b) + b/(
a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 - (3*a^5*
b + 7*a^4*b^2 + 4*a^3*b^3)*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)
)))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) + (3*a^5*b + 7*a^4*b
^2 + 4*a^3*b^3)*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2
*imag_part(arccos(-a/(a + b) + b/(a + b)))))*arctan(-(((a^4 + a^3*b)/(a^4*e
^(4*c) + a^3*b*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b))) - e^(d*x))
/(((a^4 + a^3*b)/(a^4*e^(4*c) + a^3*b*e^(4*c)))^(1/4)*sin(1/2*arccos(-(a -
b)/(a + b)))))/(2*a^7*b - (a^4 - a^3*b)*sqrt(-a*b)*a^2*abs(a)) + ((3*a^5*b
+ 7*a^4*b^2 + 4*a^3*b^3)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))
^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 - 3*(3*a^5*b + 7*a
^4*b^2 + 4*a^3*b^3)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh
```

```
(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(
-a/(a + b) + b/(a + b))))^2 - 3*(3*a^5*b + 7*a^4*b^2 + 4*a^3*b^3)*cos(1/2*r
eal_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a
 + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))
+ 9*(3*a^5*b + 7*a^4*b^2 + 4*a^3*b^3)*cos(1/2*real_part(arccos(-a/(a + b) +
 b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2
*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/
(a + b) + b/(a + b)))) + 3*(3*a^5*b + 7*a^4*b^2 + 4*a^3*b^3)*cos(1/2*real_p
art(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b)
 + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 9*(
3*a^5*b + 7*a^4*b^2 + 4*a^3*b^3)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a
 + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_p
art(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b)
 + b/(a + b))))^2 - (3*a^5*b + 7*a^4*b^2 + 4*a^3*b^3)*cos(1/2*real_part(arc
cos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a
 + b))))^3 + 3*(3*a^5*b + 7*a^4*b^2 + 4*a^3*b^3)*cos(1/2*real_part(arccos(-
a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))
^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 - (3*a^5*b + 7*a^4
*b^2 + 4*a^3*b^3)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1
/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + (3*a^5*b + 7*a^4*b^2 + 4*a^
3*b^3)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_par
t(arccos(-a/(a + b) + b/(a + b)))))*log(2*((a^4 + a^3*b)/(a^4*e^(4*c) + a^3
*b*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b)))*e^(d*x) + sqrt((a^4 +
a^3*b)/(a^4*e^(4*c) + a^3*b*e^(4*c))) + e^(2*d*x))/(2*a^7*b - (a^4 - a^3*b)
*sqrt(-a*b)*a^2*abs(a)) - ((3*a^5*b + 7*a^4*b^2 + 4*a^3*b^3)*cos(1/2*real_p
art(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b)
 + b/(a + b))))^3 - 3*(3*a^5*b + 7*a^4*b^2 + 4*a^3*b^3)*cos(1/2*real_part(a
rccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a
 + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(3*a^5*
b + 7*a^4*b^2 + 4*a^3*b^3)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))
))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_pa
rt(arccos(-a/(a + b) + b/(a + b)))) + 9*(3*a^5*b + 7*a^4*b^2 + 4*a^3*b^3)*c
os(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos
(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b
))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(3*a^5*b + 7
*a^4*b^2 + 4*a^3*b^3)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*
cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arcc
os(-a/(a + b) + b/(a + b))))^2 - 9*(3*a^5*b + 7*a^4*b^2 + 4*a^3*b^3)*cos(1/
2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(
a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*
sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - (3*a^5*b + 7*a^4*b^
2 + 4*a^3*b^3)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/
2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + 3*(3*a^5*b + 7*a^4*b^2 + 4
*a^3*b^3)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_p
art(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b)
 + b/(a + b))))^3 - (3*a^5*b + 7*a^4*b^2 + 4*a^3*b^3)*cos(1/2*real_part(arc
cos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a +
 b)))) + (3*a^5*b + 7*a^4*b^2 + 4*a^3*b^3)*cos(1/2*real_part(arccos(-a/(a +
 b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*log
(-2*((a^4 + a^3*b)/(a^4*e^(4*c) + a^3*b*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a
- b)/(a + b)))*e^(d*x) + sqrt((a^4 + a^3*b)/(a^4*e^(4*c) + a^3*b*e^(4*c)))
+ e^(2*d*x))/(2*a^7*b - (a^4 - a^3*b)*sqrt(-a*b)*a^2*abs(a)) + 4*(a*e^c + 4
*b*e^c)*e^(-c)*log(e^(d*x + c) + 1)/a^3 - 4*(a*e^c + 4*b*e^c)*e^(-c)*log(ab
s(e^(d*x + c) - 1))/a^3 - 8*(b*e^(3*d*x + 3*c) + b*e^(d*x + c))/((a*e^(4*d*
x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) +
a + b)*a^2) - 8*(e^(3*d*x + 3*c) + e^(d*x + c))/(a^2*(e^(2*d*x + 2*c) - 1)^
2))/d
```

3.40 $\int \frac{\text{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=113

$$\frac{b(a+b)\tanh(c+dx)}{2a^3d\left(a+b\tanh^2(c+dx)\right)}+\frac{(a+2b)\coth(c+dx)}{a^3d}+\frac{\sqrt{b}(3a+5b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}d}-\frac{\coth^3(c+dx)}{3a^2d}$$

```
[Out] (Sqrt[b]*(3*a + 5*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(7/2)*d)
 + ((a + 2*b)*Coth[c + d*x])/(a^3*d) - Coth[c + d*x]^3/(3*a^2*d) + (b*(a +
b)*Tanh[c + d*x])/(2*a^3*d*(a + b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.153873, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3663, 456, 1261, 205}

$$\frac{b(a+b)\tanh(c+dx)}{2a^3d\left(a+b\tanh^2(c+dx)\right)}+\frac{(a+2b)\coth(c+dx)}{a^3d}+\frac{\sqrt{b}(3a+5b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}d}-\frac{\coth^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (Sqrt[b]*(3*a + 5*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(7/2)*d)
 + ((a + 2*b)*Coth[c + d*x])/(a^3*d) - Coth[c + d*x]^3/(3*a^2*d) + (b*(a +
b)*Tanh[c + d*x])/(2*a^3*d*(a + b*Tanh[c + d*x]^2))
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{x^4\left(a+bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b(a+b)\tanh(c+dx)}{2a^3d\left(a+b\tanh^2(c+dx)\right)} - \frac{b\operatorname{Subst}\left(\int \frac{-\frac{2}{ab}+\frac{2(a+b)x^2}{a^2b}-\frac{(a+b)x^4}{a^3}}{x^4\left(a+bx^2\right)}\,dx, x, \tanh(c+dx)\right)}{2d}$$

$$= \frac{b(a+b)\tanh(c+dx)}{2a^3d\left(a+b\tanh^2(c+dx)\right)} - \frac{b\operatorname{Subst}\left(\int \left(-\frac{2}{a^2bx^4}+\frac{2(a+2b)}{a^3bx^2}+\frac{-3a-5b}{a^3\left(a+bx^2\right)}\right) dx, x, \tanh(c\cdot\right.}{2d}$$

$$= \frac{(a+2b)\coth(c+dx)}{a^3d} - \frac{\coth^3(c+dx)}{3a^2d} + \frac{b(a+b)\tanh(c+dx)}{2a^3d\left(a+b\tanh^2(c+dx)\right)} + \frac{(b(3a+5b))\,\mathrm{S}}{}$$

$$= \frac{\sqrt{b}(3a+5b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}d} + \frac{(a+2b)\coth(c+dx)}{a^3d} - \frac{\coth^3(c+dx)}{3a^2d} + \frac{b(a\cdot}{2a^3d\,(a}$$

Mathematica [A] time = 0.774897, size = 114, normalized size = 1.01

$$\frac{3\sqrt{b}(3a+5b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)+\frac{3\sqrt{a}b(a+b)\sinh(2(c+dx))}{(a+b)\cosh(2(c+dx))+a-b}+2\sqrt{a}\coth(c+dx)\left(-a\operatorname{csch}^2(c+dx)+2a+6b\right)}{6a^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (3*Sqrt[b]*(3*a + 5*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] + 2*Sqrt[a]*
Coth[c + d*x]*(2*a + 6*b - a*Csch[c + d*x]^2) + (3*Sqrt[a]*b*(a + b)*Sinh[2
*(c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)]))/(6*a^(7/2)*d)
```

Maple [B] time = 0.119, size = 1012, normalized size = 9.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] -1/24/d/a^2*tanh(1/2*d*x+1/2*c)^3+3/8/d/a^2*tanh(1/2*d*x+1/2*c)+1/d/a^3*tan
h(1/2*d*x+1/2*c)*b+1/d*b/a^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)
^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*tanh(1/2*d*x+1/2*c)^3+1/d/a^3*b^2/(tanh(1
/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*ta
nh(1/2*d*x+1/2*c)^3+1/d*b/a^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c
)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*tanh(1/2*d*x+1/2*c)+1/d/a^3*b^2/(tanh(1/
2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*tan
h(1/2*d*x+1/2*c)-3/2/d/(b*(a+b))^(1/2)/a/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2
```

```
)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))*b+3/2/
d*b/a^2/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/(
(2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-4/d*b^2/a^2/(b*(a+b))^(1/2)/((2*(b*(a+b
))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-
a-2*b)*a)^(1/2))-3/2/d/(b*(a+b))^(1/2)/a/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2
)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))*b-3/2/d
*b/a^2/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2
*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-4/d*b^2/a^2/(b*(a+b))^(1/2)/((2*(b*(a+b))
^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2
*b)*a)^(1/2))+5/2/d/a^3*b^2/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*t
anh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-5/2/d/a^3*b^3/(b*(a
+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c
)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-5/2/d/a^3*b^2/((2*(b*(a+b))^(1/2)+a+
2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1
/2))-5/2/d/a^3*b^3/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arct
an(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/24/d/a^2/ta
nh(1/2*d*x+1/2*c)^3+3/8/d/a^2/tanh(1/2*d*x+1/2*c)+1/d/a^3/tanh(1/2*d*x+1/2*
c)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 2.78602, size = 12585, normalized size = 111.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [1/12*(12*(3*a*b + 5*b^2)*cosh(d*x + c)^8 + 96*(3*a*b + 5*b^2)*cosh(d*x + c
)*sinh(d*x + c)^7 + 12*(3*a*b + 5*b^2)*sinh(d*x + c)^8 - 24*(2*a^2 + a*b +
10*b^2)*cosh(d*x + c)^6 + 24*(14*(3*a*b + 5*b^2)*cosh(d*x + c)^2 - 2*a^2 -
a*b - 10*b^2)*sinh(d*x + c)^6 + 48*(14*(3*a*b + 5*b^2)*cosh(d*x + c)^3 - 3*
(2*a^2 + a*b + 10*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 8*(10*a^2 - 2*a*b -
 45*b^2)*cosh(d*x + c)^4 + 8*(105*(3*a*b + 5*b^2)*cosh(d*x + c)^4 - 45*(2*a
^2 + a*b + 10*b^2)*cosh(d*x + c)^2 - 10*a^2 + 2*a*b + 45*b^2)*sinh(d*x + c)
^4 + 32*(21*(3*a*b + 5*b^2)*cosh(d*x + c)^5 - 15*(2*a^2 + a*b + 10*b^2)*cos
h(d*x + c)^3 - (10*a^2 - 2*a*b - 45*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 8
*(2*a^2 + 13*a*b + 30*b^2)*cosh(d*x + c)^2 + 8*(42*(3*a*b + 5*b^2)*cosh(d*x
 + c)^6 - 45*(2*a^2 + a*b + 10*b^2)*cosh(d*x + c)^4 - 6*(10*a^2 - 2*a*b - 4
5*b^2)*cosh(d*x + c)^2 - 2*a^2 - 13*a*b - 30*b^2)*sinh(d*x + c)^2 + 3*((3*a
^2 + 8*a*b + 5*b^2)*cosh(d*x + c)^10 + 10*(3*a^2 + 8*a*b + 5*b^2)*cosh(d*x
+ c)*sinh(d*x + c)^9 + (3*a^2 + 8*a*b + 5*b^2)*sinh(d*x + c)^10 - (3*a^2 +
20*a*b + 25*b^2)*cosh(d*x + c)^8 + (45*(3*a^2 + 8*a*b + 5*b^2)*cosh(d*x + c
)^2 - 3*a^2 - 20*a*b - 25*b^2)*sinh(d*x + c)^8 + 8*(15*(3*a^2 + 8*a*b + 5*b
^2)*cosh(d*x + c)^3 - (3*a^2 + 20*a*b + 25*b^2)*cosh(d*x + c))*sinh(d*x + c
)^7 - 2*(3*a^2 - 10*a*b - 25*b^2)*cosh(d*x + c)^6 + 2*(105*(3*a^2 + 8*a*b +
```

```
5*b^2)*cosh(d*x + c)^4 - 14*(3*a^2 + 20*a*b + 25*b^2)*cosh(d*x + c)^2 - 3*
a^2 + 10*a*b + 25*b^2)*sinh(d*x + c)^6 + 4*(63*(3*a^2 + 8*a*b + 5*b^2)*cosh
(d*x + c)^5 - 14*(3*a^2 + 20*a*b + 25*b^2)*cosh(d*x + c)^3 - 3*(3*a^2 - 10*
a*b - 25*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^2 - 10*a*b - 25*b^2)*
cosh(d*x + c)^4 + 2*(105*(3*a^2 + 8*a*b + 5*b^2)*cosh(d*x + c)^6 - 35*(3*a^
2 + 20*a*b + 25*b^2)*cosh(d*x + c)^4 - 15*(3*a^2 - 10*a*b - 25*b^2)*cosh(d*
x + c)^2 + 3*a^2 - 10*a*b - 25*b^2)*sinh(d*x + c)^4 + 8*(15*(3*a^2 + 8*a*b
+ 5*b^2)*cosh(d*x + c)^7 - 7*(3*a^2 + 20*a*b + 25*b^2)*cosh(d*x + c)^5 - 5*
(3*a^2 - 10*a*b - 25*b^2)*cosh(d*x + c)^3 + (3*a^2 - 10*a*b - 25*b^2)*cosh(
d*x + c))*sinh(d*x + c)^3 + (3*a^2 + 20*a*b + 25*b^2)*cosh(d*x + c)^2 + (45
*(3*a^2 + 8*a*b + 5*b^2)*cosh(d*x + c)^8 - 28*(3*a^2 + 20*a*b + 25*b^2)*cos
h(d*x + c)^6 - 30*(3*a^2 - 10*a*b - 25*b^2)*cosh(d*x + c)^4 + 12*(3*a^2 - 1
0*a*b - 25*b^2)*cosh(d*x + c)^2 + 3*a^2 + 20*a*b + 25*b^2)*sinh(d*x + c)^2
- 3*a^2 - 8*a*b - 5*b^2 + 2*(5*(3*a^2 + 8*a*b + 5*b^2)*cosh(d*x + c)^9 - 4*
(3*a^2 + 20*a*b + 25*b^2)*cosh(d*x + c)^7 - 6*(3*a^2 - 10*a*b - 25*b^2)*cos
h(d*x + c)^5 + 4*(3*a^2 - 10*a*b - 25*b^2)*cosh(d*x + c)^3 + (3*a^2 + 20*a*
b + 25*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/a)*log(((a^2 + 2*a*b + b^
2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 +
(a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*
(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*
a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x +
 c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*
x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))
/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a +
b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)
^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x
 + c))*sinh(d*x + c) + a + b)) + 16*a^2 + 76*a*b + 60*b^2 + 16*(6*(3*a*b +
5*b^2)*cosh(d*x + c)^7 - 9*(2*a^2 + a*b + 10*b^2)*cosh(d*x + c)^5 - 2*(10*a
^2 - 2*a*b - 45*b^2)*cosh(d*x + c)^3 - (2*a^2 + 13*a*b + 30*b^2)*cosh(d*x +
 c))*sinh(d*x + c))/((a^4 + a^3*b)*d*cosh(d*x + c)^10 + 10*(a^4 + a^3*b)*d*
cosh(d*x + c)*sinh(d*x + c)^9 + (a^4 + a^3*b)*d*sinh(d*x + c)^10 - (a^4 + 5
*a^3*b)*d*cosh(d*x + c)^8 + (45*(a^4 + a^3*b)*d*cosh(d*x + c)^2 - (a^4 + 5*
a^3*b)*d)*sinh(d*x + c)^8 - 2*(a^4 - 5*a^3*b)*d*cosh(d*x + c)^6 + 8*(15*(a^
4 + a^3*b)*d*cosh(d*x + c)^3 - (a^4 + 5*a^3*b)*d*cosh(d*x + c))*sinh(d*x +
c)^7 + 2*(105*(a^4 + a^3*b)*d*cosh(d*x + c)^4 - 14*(a^4 + 5*a^3*b)*d*cosh(d
*x + c)^2 - (a^4 - 5*a^3*b)*d)*sinh(d*x + c)^6 + 2*(a^4 - 5*a^3*b)*d*cosh(d
*x + c)^4 + 4*(63*(a^4 + a^3*b)*d*cosh(d*x + c)^5 - 14*(a^4 + 5*a^3*b)*d*co
sh(d*x + c)^3 - 3*(a^4 - 5*a^3*b)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105
*(a^4 + a^3*b)*d*cosh(d*x + c)^6 - 35*(a^4 + 5*a^3*b)*d*cosh(d*x + c)^4 - 1
5*(a^4 - 5*a^3*b)*d*cosh(d*x + c)^2 + (a^4 - 5*a^3*b)*d)*sinh(d*x + c)^4 +
(a^4 + 5*a^3*b)*d*cosh(d*x + c)^2 + 8*(15*(a^4 + a^3*b)*d*cosh(d*x + c)^7 -
 7*(a^4 + 5*a^3*b)*d*cosh(d*x + c)^5 - 5*(a^4 - 5*a^3*b)*d*cosh(d*x + c)^3
+ (a^4 - 5*a^3*b)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (45*(a^4 + a^3*b)*d*co
sh(d*x + c)^8 - 28*(a^4 + 5*a^3*b)*d*cosh(d*x + c)^6 - 30*(a^4 - 5*a^3*b)*d
*cosh(d*x + c)^4 + 12*(a^4 - 5*a^3*b)*d*cosh(d*x + c)^2 + (a^4 + 5*a^3*b)*d
)*sinh(d*x + c)^2 - (a^4 + a^3*b)*d + 2*(5*(a^4 + a^3*b)*d*cosh(d*x + c)^9
- 4*(a^4 + 5*a^3*b)*d*cosh(d*x + c)^7 - 6*(a^4 - 5*a^3*b)*d*cosh(d*x + c)^5
 + 4*(a^4 - 5*a^3*b)*d*cosh(d*x + c)^3 + (a^4 + 5*a^3*b)*d*cosh(d*x + c))*s
inh(d*x + c)), 1/6*(6*(3*a*b + 5*b^2)*cosh(d*x + c)^8 + 48*(3*a*b + 5*b^2)*
cosh(d*x + c)*sinh(d*x + c)^7 + 6*(3*a*b + 5*b^2)*sinh(d*x + c)^8 - 12*(2*a
^2 + a*b + 10*b^2)*cosh(d*x + c)^6 + 12*(14*(3*a*b + 5*b^2)*cosh(d*x + c)^2
 - 2*a^2 - a*b - 10*b^2)*sinh(d*x + c)^6 + 24*(14*(3*a*b + 5*b^2)*cosh(d*x
+ c)^3 - 3*(2*a^2 + a*b + 10*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 4*(10*a^
2 - 2*a*b - 45*b^2)*cosh(d*x + c)^4 + 4*(105*(3*a*b + 5*b^2)*cosh(d*x + c)^
4 - 45*(2*a^2 + a*b + 10*b^2)*cosh(d*x + c)^2 - 10*a^2 + 2*a*b + 45*b^2)*si
nh(d*x + c)^4 + 16*(21*(3*a*b + 5*b^2)*cosh(d*x + c)^5 - 15*(2*a^2 + a*b +
10*b^2)*cosh(d*x + c)^3 - (10*a^2 - 2*a*b - 45*b^2)*cosh(d*x + c))*sinh(d*x
 + c)^3 - 4*(2*a^2 + 13*a*b + 30*b^2)*cosh(d*x + c)^2 + 4*(42*(3*a*b + 5*b^
2)*cosh(d*x + c)^6 - 45*(2*a^2 + a*b + 10*b^2)*cosh(d*x + c)^4 - 6*(10*a^2
```

```
- 2*a*b - 45*b^2)*cosh(d*x + c)^2 - 2*a^2 - 13*a*b - 30*b^2)*sinh(d*x + c)^
2 + 3*((3*a^2 + 8*a*b + 5*b^2)*cosh(d*x + c)^10 + 10*(3*a^2 + 8*a*b + 5*b^2
)*cosh(d*x + c)*sinh(d*x + c)^9 + (3*a^2 + 8*a*b + 5*b^2)*sinh(d*x + c)^10
- (3*a^2 + 20*a*b + 25*b^2)*cosh(d*x + c)^8 + (45*(3*a^2 + 8*a*b + 5*b^2)*c
osh(d*x + c)^2 - 3*a^2 - 20*a*b - 25*b^2)*sinh(d*x + c)^8 + 8*(15*(3*a^2 +
8*a*b + 5*b^2)*cosh(d*x + c)^3 - (3*a^2 + 20*a*b + 25*b^2)*cosh(d*x + c))*s
inh(d*x + c)^7 - 2*(3*a^2 - 10*a*b - 25*b^2)*cosh(d*x + c)^6 + 2*(105*(3*a^
2 + 8*a*b + 5*b^2)*cosh(d*x + c)^4 - 14*(3*a^2 + 20*a*b + 25*b^2)*cosh(d*x
+ c)^2 - 3*a^2 + 10*a*b + 25*b^2)*sinh(d*x + c)^6 + 4*(63*(3*a^2 + 8*a*b +
5*b^2)*cosh(d*x + c)^5 - 14*(3*a^2 + 20*a*b + 25*b^2)*cosh(d*x + c)^3 - 3*(
3*a^2 - 10*a*b - 25*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^2 - 10*a*b
 - 25*b^2)*cosh(d*x + c)^4 + 2*(105*(3*a^2 + 8*a*b + 5*b^2)*cosh(d*x + c)^6
 - 35*(3*a^2 + 20*a*b + 25*b^2)*cosh(d*x + c)^4 - 15*(3*a^2 - 10*a*b - 25*b
^2)*cosh(d*x + c)^2 + 3*a^2 - 10*a*b - 25*b^2)*sinh(d*x + c)^4 + 8*(15*(3*a
^2 + 8*a*b + 5*b^2)*cosh(d*x + c)^7 - 7*(3*a^2 + 20*a*b + 25*b^2)*cosh(d*x
+ c)^5 - 5*(3*a^2 - 10*a*b - 25*b^2)*cosh(d*x + c)^3 + (3*a^2 - 10*a*b - 25
*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + (3*a^2 + 20*a*b + 25*b^2)*cosh(d*x +
 c)^2 + (45*(3*a^2 + 8*a*b + 5*b^2)*cosh(d*x + c)^8 - 28*(3*a^2 + 20*a*b +
25*b^2)*cosh(d*x + c)^6 - 30*(3*a^2 - 10*a*b - 25*b^2)*cosh(d*x + c)^4 + 12
*(3*a^2 - 10*a*b - 25*b^2)*cosh(d*x + c)^2 + 3*a^2 + 20*a*b + 25*b^2)*sinh(
d*x + c)^2 - 3*a^2 - 8*a*b - 5*b^2 + 2*(5*(3*a^2 + 8*a*b + 5*b^2)*cosh(d*x
+ c)^9 - 4*(3*a^2 + 20*a*b + 25*b^2)*cosh(d*x + c)^7 - 6*(3*a^2 - 10*a*b -
25*b^2)*cosh(d*x + c)^5 + 4*(3*a^2 - 10*a*b - 25*b^2)*cosh(d*x + c)^3 + (3*
a^2 + 20*a*b + 25*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*(
(a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*s
inh(d*x + c)^2 + a - b)*sqrt(b/a)/b) + 8*a^2 + 38*a*b + 30*b^2 + 8*(6*(3*a*
b + 5*b^2)*cosh(d*x + c)^7 - 9*(2*a^2 + a*b + 10*b^2)*cosh(d*x + c)^5 - 2*(
10*a^2 - 2*a*b - 45*b^2)*cosh(d*x + c)^3 - (2*a^2 + 13*a*b + 30*b^2)*cosh(d
*x + c))*sinh(d*x + c))/((a^4 + a^3*b)*d*cosh(d*x + c)^10 + 10*(a^4 + a^3*b
)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^4 + a^3*b)*d*sinh(d*x + c)^10 - (a^4
 + 5*a^3*b)*d*cosh(d*x + c)^8 + (45*(a^4 + a^3*b)*d*cosh(d*x + c)^2 - (a^4
+ 5*a^3*b)*d)*sinh(d*x + c)^8 - 2*(a^4 - 5*a^3*b)*d*cosh(d*x + c)^6 + 8*(15
*(a^4 + a^3*b)*d*cosh(d*x + c)^3 - (a^4 + 5*a^3*b)*d*cosh(d*x + c))*sinh(d*
x + c)^7 + 2*(105*(a^4 + a^3*b)*d*cosh(d*x + c)^4 - 14*(a^4 + 5*a^3*b)*d*co
sh(d*x + c)^2 - (a^4 - 5*a^3*b)*d)*sinh(d*x + c)^6 + 2*(a^4 - 5*a^3*b)*d*co
sh(d*x + c)^4 + 4*(63*(a^4 + a^3*b)*d*cosh(d*x + c)^5 - 14*(a^4 + 5*a^3*b)*
d*cosh(d*x + c)^3 - 3*(a^4 - 5*a^3*b)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*
(105*(a^4 + a^3*b)*d*cosh(d*x + c)^6 - 35*(a^4 + 5*a^3*b)*d*cosh(d*x + c)^4
 - 15*(a^4 - 5*a^3*b)*d*cosh(d*x + c)^2 + (a^4 - 5*a^3*b)*d)*sinh(d*x + c)^
4 + (a^4 + 5*a^3*b)*d*cosh(d*x + c)^2 + 8*(15*(a^4 + a^3*b)*d*cosh(d*x + c)
^7 - 7*(a^4 + 5*a^3*b)*d*cosh(d*x + c)^5 - 5*(a^4 - 5*a^3*b)*d*cosh(d*x + c
)^3 + (a^4 - 5*a^3*b)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (45*(a^4 + a^3*b)*
d*cosh(d*x + c)^8 - 28*(a^4 + 5*a^3*b)*d*cosh(d*x + c)^6 - 30*(a^4 - 5*a^3*
b)*d*cosh(d*x + c)^4 + 12*(a^4 - 5*a^3*b)*d*cosh(d*x + c)^2 + (a^4 + 5*a^3*
b)*d)*sinh(d*x + c)^2 - (a^4 + a^3*b)*d + 2*(5*(a^4 + a^3*b)*d*cosh(d*x + c
)^9 - 4*(a^4 + 5*a^3*b)*d*cosh(d*x + c)^7 - 6*(a^4 - 5*a^3*b)*d*cosh(d*x +
c)^5 + 4*(a^4 - 5*a^3*b)*d*cosh(d*x + c)^3 + (a^4 + 5*a^3*b)*d*cosh(d*x + c
))*sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(csch(c + d*x)**4/(a + b*tanh(c + d*x)**2)**2, x)
```

Giac [B] time = 1.61048, size = 298, normalized size = 2.64

$$\frac{\frac{3\left(3\,abe^{(2\,c)}+5\,b^{2}e^{(2\,c)}\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)e^{(-2\,c)}}{\sqrt{ab}a^{3}}-\frac{6\left(abe^{(2\,dx+2\,c)}-b^{2}e^{(2\,dx+2\,c)}+ab+b^{2}\right)}{\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)a^{3}}+\frac{8\left(3\,be^{(4\,dx+4\,c)}-3\,ae^{(2\,dx+2\,c)}-6\,be^{(2\,dx+2\,c)}+a+3\,b\right)}{a^{3}\left(e^{(2\,dx+2\,c)}-1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*(3*a*b*e^(2*c) + 5*b^2*e^(2*c))*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^
(2*d*x + 2*c) + a - b)/sqrt(a*b))*e^(-2*c)/(sqrt(a*b)*a^3) - 6*(a*b*e^(2*d*
x + 2*c) - b^2*e^(2*d*x + 2*c) + a*b + b^2)/((a*e^(4*d*x + 4*c) + b*e^(4*d*
x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)*a^3) + 8*(3*b
*e^(4*d*x + 4*c) - 3*a*e^(2*d*x + 2*c) - 6*b*e^(2*d*x + 2*c) + a + 3*b)/(a^
3*(e^(2*d*x + 2*c) - 1)^3))/d
```

3.41 $\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=240

$$\frac{3\sqrt{b}\left(5a^2-10ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}d(a+b)^5}+\frac{3x\left(a^2-10ab+5b^2\right)}{8(a+b)^5}+\frac{3b(a-b)\tanh(c+dx)}{2d(a+b)^4\left(a+b\tanh^2(c+dx)\right)}+\frac{b(7a-5b}{8d(a+b)^3\left(a+\right.}$$

```
[Out] (3*(a^2 - 10*a*b + 5*b^2)*x)/(8*(a + b)^5) + (3*Sqrt[b]*(5*a^2 - 10*a*b + b
^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(8*Sqrt[a]*(a + b)^5*d) - ((5*
a - 3*b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*(a + b)^2*d*(a + b*Tanh[c + d*x]^2
)^2) + (Cosh[c + d*x]^3*Sinh[c + d*x])/(4*(a + b)*d*(a + b*Tanh[c + d*x]^2)
^2) + ((7*a - 5*b)*b*Tanh[c + d*x])/(8*(a + b)^3*d*(a + b*Tanh[c + d*x]^2)^
2) + (3*(a - b)*b*Tanh[c + d*x])/(2*(a + b)^4*d*(a + b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.345065, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3663, 470, 527, 522, 206, 205}

$$\frac{3\sqrt{b}\left(5a^2-10ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}d(a+b)^5}+\frac{3x\left(a^2-10ab+5b^2\right)}{8(a+b)^5}+\frac{3b(a-b)\tanh(c+dx)}{2d(a+b)^4\left(a+b\tanh^2(c+dx)\right)}+\frac{b(7a-5b}{8d(a+b)^3\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (3*(a^2 - 10*a*b + 5*b^2)*x)/(8*(a + b)^5) + (3*Sqrt[b]*(5*a^2 - 10*a*b + b
^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(8*Sqrt[a]*(a + b)^5*d) - ((5*
a - 3*b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*(a + b)^2*d*(a + b*Tanh[c + d*x]^2
)^2) + (Cosh[c + d*x]^3*Sinh[c + d*x])/(4*(a + b)*d*(a + b*Tanh[c + d*x]^2)
^2) + ((7*a - 5*b)*b*Tanh[c + d*x])/(8*(a + b)^3*d*(a + b*Tanh[c + d*x]^2)^
2) + (3*(a - b)*b*Tanh[c + d*x])/(2*(a + b)^4*d*(a + b*Tanh[c + d*x]^2))
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)^3\left(a+bx^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{a+(4a-3b)x^2}{\left(1-x^2\right)^2\left(a+bx^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{4(a+b)d}$$

$$= -\frac{(5a-3b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{(5a-3b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{(7a-}{8(a+b)^3}$$

$$= -\frac{(5a-3b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{(7a-}{8(a+b)^3}$$

$$= -\frac{(5a-3b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{(7a-}{8(a+b)^3}$$

$$= \frac{3\left(a^2-10ab+5b^2\right)x}{8(a+b)^5} + \frac{3\sqrt{b}\left(5a^2-10ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}(a+b)^5d} - \frac{(5a-3b)\cosh}{8(a+b)^2d\left(a\right.}$$

Mathematica [A] time = 0.7631, size = 184, normalized size = 0.77

$$\frac{12\left(a^2-10ab+5b^2\right)(c+dx)+\frac{12\sqrt{b}\left(5a^2-10ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{16ab^2(a+b)\sinh(2(c+dx))}{((a+b)\cosh(2(c+dx))+a-b)^2}-8(a-2b)(a+b)\sinh(2(c+d}{32d(a+b)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (12*(a^2 - 10*a*b + 5*b^2)*(c + d*x) + (12*Sqrt[b]*(5*a^2 - 10*a*b + b^2)*A
rcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/Sqrt[a] - 8*(a - 2*b)*(a + b)*Sinh[
2*(c + d*x)] + (16*a*b^2*(a + b)*Sinh[2*(c + d*x)])/(a - b + (a + b)*Cosh[2
*(c + d*x)])^2 + (4*(9*a - 5*b)*b*(a + b)*Sinh[2*(c + d*x)])/(a - b + (a +
b)*Cosh[2*(c + d*x)]) + (a + b)^2*Sinh[4*(c + d*x)])/(32*(a + b)^5*d)
```

Maple [B] time = 0.128, size = 2366, normalized size = 9.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 15/8/d*b^2/(a+b)^5/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arct
anh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))*a^2+27/8/d*b
^3/(a+b)^5*a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*
tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+15/8/d*b^2/(a+b)^5
/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+
1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))*a^2+27/8/d*b^3/(a+b)^5*a/(b*(a+
b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/
((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-15/8/d*b/(a+b)^5/(b*(a+b))^(1/2)/((2*(
b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(
1/2)+a+2*b)*a)^(1/2))*a^3-15/8/d*b/(a+b)^5/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1
/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b
)*a)^(1/2))*a^3+15/4/d/(a+b)^5*ln(tanh(1/2*d*x+1/2*c)-1)*a*b-3/8/d*b^3/(a+b
)^5/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b
*(a+b))^(1/2)+a+2*b)*a)^(1/2))-5/d*b^4/(a+b)^5/(tanh(1/2*d*x+1/2*c)^4*a+2*t
anh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5
-5/d*b^4/(a+b)^5/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(
1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^3+23/2/d*b^2/(a+b)^5/(tanh(1/2*
d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tan
h(1/2*d*x+1/2*c)^5*a^2-3/4/d*b^3/(a+b)^5/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/
2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)*a-3/8/d
*b^4/(a+b)^5/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*
tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-3/8/d*b^4/(a+b)^5/
(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1
/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+15/4/d*b^2/(a+b)^5*a/((2*(b*(a+b
))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a
+2*b)*a)^(1/2))+3/2/d*b^2/(a+b)^5/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1
/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^7*a^2-3/4/d*b^
3/(a+b)^5/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x
+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^7*a-15/4/d*b^2/(a+b)^5*a/((2*(b*(a+b))
^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-
2*b)*a)^(1/2))+9/4/d*b/(a+b)^5/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*
c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^7*a^3+27/4/d*b/(a
```

```
+b)^5/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2
*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5*a^3+27/4/d*b/(a+b)^5/(tanh(1/2*d*x+1/2*c
)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x
+1/2*c)^3*a^3+9/4/d*b/(a+b)^5/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c
)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)*a^3+15/8/d*b/(a+b)
^5*a^2/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((
2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-15/8/d*b/(a+b)^5*a^2/((2*(b*(a+b))^(1/2)
+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)
^(1/2))-1/4/d*b^3/(a+b)^5/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*
a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5*a+23/2/d*b^2/(a+b)^5
/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2
*b+a)^2*tanh(1/2*d*x+1/2*c)^3*a^2-1/4/d*b^3/(a+b)^5/(tanh(1/2*d*x+1/2*c)^4*
a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2
*c)^3*a+3/2/d*b^2/(a+b)^5/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*
a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)*a^2+3/8/d*b^3/(a+b)^5/
((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a
+b))^(1/2)-a-2*b)*a)^(1/2))+1/2/d/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)^3-1/4/d/(
a+b)^3/(tanh(1/2*d*x+1/2*c)+1)^4+1/2/d/(a+b)^3/(tanh(1/2*d*x+1/2*c)+1)^3+1/
4/d/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)^4-1/8/d/(a+b)^4/(tanh(1/2*d*x+1/2*c)-1)
^2*a+11/8/d/(a+b)^4/(tanh(1/2*d*x+1/2*c)-1)^2*b-3/8/d/(a+b)^4/(tanh(1/2*d*x
+1/2*c)-1)*a+9/8/d/(a+b)^4/(tanh(1/2*d*x+1/2*c)-1)*b-3/8/d/(a+b)^5*ln(tanh(
1/2*d*x+1/2*c)-1)*a^2-15/8/d/(a+b)^5*ln(tanh(1/2*d*x+1/2*c)-1)*b^2+1/8/d/(a
+b)^4/(tanh(1/2*d*x+1/2*c)+1)^2*a-11/8/d/(a+b)^4/(tanh(1/2*d*x+1/2*c)+1)^2*
b-3/8/d/(a+b)^4/(tanh(1/2*d*x+1/2*c)+1)*a+9/8/d/(a+b)^4/(tanh(1/2*d*x+1/2*c
)+1)*b+3/8/d/(a+b)^5*ln(tanh(1/2*d*x+1/2*c)+1)*a^2+15/8/d/(a+b)^5*ln(tanh(1
/2*d*x+1/2*c)+1)*b^2-15/4/d/(a+b)^5*ln(tanh(1/2*d*x+1/2*c)+1)*a*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 4.6622, size = 1246, normalized size = 5.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/64*(24*(a^2 - 10*a*b + 5*b^2)*d*x/(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^
3 + 5*a*b^4 + b^5) + 24*(5*a^2*b*e^(2*c) - 10*a*b^2*e^(2*c) + b^3*e^(2*c))*
arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))*e^(-2
*c)/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*sqrt(a*b)) +
 (a^3*e^(4*d*x + 36*c) + 3*a^2*b*e^(4*d*x + 36*c) + 3*a*b^2*e^(4*d*x + 36*c
) + b^3*e^(4*d*x + 36*c) - 8*a^3*e^(2*d*x + 34*c) + 24*a*b^2*e^(2*d*x + 34*
c) + 16*b^3*e^(2*d*x + 34*c))/(a^6*e^(32*c) + 6*a^5*b*e^(32*c) + 15*a^4*b^2
*e^(32*c) + 20*a^3*b^3*e^(32*c) + 15*a^2*b^4*e^(32*c) + 6*a*b^5*e^(32*c) +
b^6*e^(32*c)) - (6*a^4*e^(12*d*x + 12*c) - 48*a^3*b*e^(12*d*x + 12*c) - 84*
a^2*b^2*e^(12*d*x + 12*c) + 30*b^4*e^(12*d*x + 12*c) + 16*a^4*e^(10*d*x + 1
0*c) - 104*a^3*b*e^(10*d*x + 10*c) - 24*a^2*b^2*e^(10*d*x + 10*c) + 72*a*b^
3*e^(10*d*x + 10*c) - 24*b^4*e^(10*d*x + 10*c) + 5*a^4*e^(8*d*x + 8*c) + 84
*a^3*b*e^(8*d*x + 8*c) + 30*a^2*b^2*e^(8*d*x + 8*c) + 84*a*b^3*e^(8*d*x + 8
*c) - 123*b^4*e^(8*d*x + 8*c) - 20*a^4*e^(6*d*x + 6*c) + 280*a^3*b*e^(6*d*x
 + 6*c) - 64*a^2*b^2*e^(6*d*x + 6*c) - 152*a*b^3*e^(6*d*x + 6*c) + 212*b^4*
e^(6*d*x + 6*c) - 20*a^4*e^(4*d*x + 4*c) + 136*a^3*b*e^(4*d*x + 4*c) + 224*
a^2*b^2*e^(4*d*x + 4*c) - 40*a*b^3*e^(4*d*x + 4*c) - 108*b^4*e^(4*d*x + 4*c
) - 4*a^4*e^(2*d*x + 2*c) + 24*a^2*b^2*e^(2*d*x + 2*c) + 32*a*b^3*e^(2*d*x
+ 2*c) + 12*b^4*e^(2*d*x + 2*c) + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4
)/((a^5*e^(4*c) + 5*a^4*b*e^(4*c) + 10*a^3*b^2*e^(4*c) + 10*a^2*b^3*e^(4*c)
 + 5*a*b^4*e^(4*c) + b^5*e^(4*c))*(a*e^(2*d*x) + b*e^(2*d*x) + a*e^(6*d*x +
 4*c) + b*e^(6*d*x + 4*c) + 2*a*e^(4*d*x + 2*c) - 2*b*e^(4*d*x + 2*c))^2))/
d
```

3.42 $\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=166

$$\frac{\cosh^3(c+dx)}{3d(a+b)^3}-\frac{(a-2b)\cosh(c+dx)}{d(a+b)^4}+\frac{b(7a-4b)\text{sech}(c+dx)}{8d(a+b)^4\left(a-b\text{sech}^2(c+dx)+b\right)}+\frac{ab\text{sech}(c+dx)}{4d(a+b)^3\left(a-b\text{sech}^2(c+dx)+b\right)^2}$$

```
[Out] (5*(3*a - 4*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(8*(a
+ b)^(9/2)*d) - ((a - 2*b)*Cosh[c + d*x])/((a + b)^4*d) + Cosh[c + d*x]^3/(
3*(a + b)^3*d) + (a*b*Sech[c + d*x])/(4*(a + b)^3*d*(a + b - b*Sech[c + d*x
]^2)^2) + ((7*a - 4*b)*b*Sech[c + d*x])/(8*(a + b)^4*d*(a + b - b*Sech[c +
d*x]^2))
```

Rubi [A] time = 0.286174, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3664, 456, 1259, 1261, 208}

$$\frac{\cosh^3(c+dx)}{3d(a+b)^3}-\frac{(a-2b)\cosh(c+dx)}{d(a+b)^4}+\frac{b(7a-4b)\text{sech}(c+dx)}{8d(a+b)^4\left(a-b\text{sech}^2(c+dx)+b\right)}+\frac{ab\text{sech}(c+dx)}{4d(a+b)^3\left(a-b\text{sech}^2(c+dx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```


```
[Out] (5*(3*a - 4*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(8*(a
+ b)^(9/2)*d) - ((a - 2*b)*Cosh[c + d*x])/((a + b)^4*d) + Cosh[c + d*x]^3/(
3*(a + b)^3*d) + (a*b*Sech[c + d*x])/(4*(a + b)^3*d*(a + b - b*Sech[c + d*x
]^2)^2) + ((7*a - 4*b)*b*Sech[c + d*x])/(8*(a + b)^4*d*(a + b - b*Sech[c +
d*x]^2))
```


Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
```

```
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4\left(a+b-bx^2\right)^3}\,dx, x, \text{sech}(c+dx)\right)}{d}$$

$$= \frac{ab\,\text{sech}(c+dx)}{4(a+b)^3d\left(a+b-b\,\text{sech}^2(c+dx)\right)^2} + \frac{b\,\text{Subst}\left(\int \frac{-\frac{4}{b(a+b)}+\frac{4ax^2}{b(a+b)^2}+\frac{3ax^4}{(a+b)^3}}{x^4\left(a+b-bx^2\right)^2}\,dx, x, \text{sech}(c+dx\right.}{4d}$$

$$= \frac{ab\,\text{sech}(c+dx)}{4(a+b)^3d\left(a+b-b\,\text{sech}^2(c+dx)\right)^2} + \frac{(7a-4b)b\,\text{sech}(c+dx)}{8(a+b)^4d\left(a+b-b\,\text{sech}^2(c+dx)\right)} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{ab\,\text{sech}(c+dx)}{4(a+b)^3d\left(a+b-b\,\text{sech}^2(c+dx)\right)^2} + \frac{(7a-4b)b\,\text{sech}(c+dx)}{8(a+b)^4d\left(a+b-b\,\text{sech}^2(c+dx)\right)} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{(a-2b)\cosh(c+dx)}{(a+b)^4d} + \frac{\cosh^3(c+dx)}{3(a+b)^3d} + \frac{ab\,\text{sech}(c+dx)}{4(a+b)^3d\left(a+b-b\,\text{sech}^2(c+dx)\right)^2} + \frac{}{8(a+}$$

$$= \frac{5(3a-4b)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{8(a+b)^{9/2}d} - \frac{(a-2b)\cosh(c+dx)}{(a+b)^4d} + \frac{\cosh^3(c+dx)}{3(a+b)^3d} + \frac{}{4(a+b)^3}$$

Mathematica [C] time = 1.81168, size = 227, normalized size = 1.37

$$\frac{-\frac{6\cosh(c+dx)\left(\left(-27a^2b+6a^3-11ab^2+22b^3\right)\cosh(2(c+dx))-24a^2b+3a^3+30ab^2+3(a-3b)(a+b)^2\cosh^2(2(c+dx))-13b^3\right)}{(a+b)^4((a+b)\cosh(2(c+dx))+a-b)^2}+\frac{2\cosh(3(c+dx))}{(a+b)^3}+\frac{15i\sqrt{b}(3a-4b)\left(\text{ta}\right.}{}}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (((15*I)*(3*a - 4*b)*Sqrt[b]*(ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c +
d*x)/2])/Sqrt[b]] + ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/S
qrt[b]]))/(a + b)^(9/2) - (6*Cosh[c + d*x]*(3*a^3 - 24*a^2*b + 30*a*b^2 - 1
3*b^3 + (6*a^3 - 27*a^2*b - 11*a*b^2 + 22*b^3)*Cosh[2*(c + d*x)] + 3*(a - 3
*b)*(a + b)^2*Cosh[2*(c + d*x)]^2))/((a + b)^4*(a - b + (a + b)*Cosh[2*(c +
 d*x)])^2) + (2*Cosh[3*(c + d*x)])/(a + b)^3)/(24*d)
```

Maple [B] time = 0.109, size = 341, normalized size = 2.1

$$\frac{1}{d}\left(\frac{1}{3\,(a+b)^3}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3}-\frac{1}{2\,(a+b)^3}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}-\frac{a-5\,b}{2\,(a+b)^4}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}-2\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(1/3/(a+b)^3/(tanh(1/2*d*x+1/2*c)+1)^3-1/2/(a+b)^3/(tanh(1/2*d*x+1/2*c)
+1)^2-1/2*(a-5*b)/(a+b)^4/(tanh(1/2*d*x+1/2*c)+1)-2*b/(a+b)^4*((-1/8*(9*a+2
0*b)*a*tanh(1/2*d*x+1/2*c)^6-1/8*(27*a^3+66*a^2*b+56*a*b^2-16*b^3)/a*tanh(1
/2*d*x+1/2*c)^4+(-27/8*a^2-11/2*a*b+2*b^2)*tanh(1/2*d*x+1/2*c)^2-9/8*a^2+1/
4*a*b)/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/
2*c)^2*b+a)^2-5/16*(3*a-4*b)/(a*b+b^2)^(1/2)*arctanh(1/4*(2*tanh(1/2*d*x+1/
2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2)))-1/3/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)^3-1
/2/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)^2-1/2/(a+b)^4*(-a+5*b)/(tanh(1/2*d*x+1/2
*c)-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 3.14298, size = 8107, normalized size = 48.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/96*(30*(3*(6*a^2*b^2 - 8*a*b^3 - (3*a^2*b - 7*a*b^2 + 4*b^3)*sqrt(-a*b))
*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(ar
ccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a
 + b)))) - (6*a^2*b^2 - 8*a*b^3 - (3*a^2*b - 7*a*b^2 + 4*b^3)*sqrt(-a*b))*c
osh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arcc
os(-a/(a + b) + b/(a + b))))^3 - 9*(6*a^2*b^2 - 8*a*b^3 - (3*a^2*b - 7*a*b^
2 + 4*b^3)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2
*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(ar
ccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a
+ b)))) + 3*(6*a^2*b^2 - 8*a*b^3 - (3*a^2*b - 7*a*b^2 + 4*b^3)*sqrt(-a*b))*
cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arc
cos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a
 + b)))) + 9*(6*a^2*b^2 - 8*a*b^3 - (3*a^2*b - 7*a*b^2 + 4*b^3)*sqrt(-a*b))
*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(ar
ccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a +
 b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(6*a^2*b^2
 - 8*a*b^3 - (3*a^2*b - 7*a*b^2 + 4*b^3)*sqrt(-a*b))*cosh(1/2*imag_part(arc
cos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a +
b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(6*a^2*b^
2 - 8*a*b^3 - (3*a^2*b - 7*a*b^2 + 4*b^3)*sqrt(-a*b))*cos(1/2*real_part(arc
cos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a
+ b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (6*a^2*b^2
 - 8*a*b^3 - (3*a^2*b - 7*a*b^2 + 4*b^3)*sqrt(-a*b))*sin(1/2*real_part(arcco
s(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a +
 b))))^3 - (6*a^2*b^2 - 8*a*b^3 - (3*a^2*b - 7*a*b^2 + 4*b^3)*sqrt(-a*b))*c
osh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos
(-a/(a + b) + b/(a + b)))) + (6*a^2*b^2 - 8*a*b^3 - (3*a^2*b - 7*a*b^2 + 4*
b^3)*sqrt(-a*b))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/
2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*arctan((((a^5 + 5*a^4*b + 10*
a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)/(a^5*e^(4*c) + 5*a^4*b*e^(4*c) + 10*a
^3*b^2*e^(4*c) + 10*a^2*b^3*e^(4*c) + 5*a*b^4*e^(4*c) + b^5*e^(4*c)))^(1/4)
*cos(1/2*arccos(-(a - b)/(a + b))) + e^(d*x))/(((a^5 + 5*a^4*b + 10*a^3*b^2
 + 10*a^2*b^3 + 5*a*b^4 + b^5)/(a^5*e^(4*c) + 5*a^4*b*e^(4*c) + 10*a^3*b^2*
e^(4*c) + 10*a^2*b^3*e^(4*c) + 5*a*b^4*e^(4*c) + b^5*e^(4*c)))^(1/4)*sin(1/
2*arccos(-(a - b)/(a + b)))))/(a^6*b + 5*a^5*b^2 + 10*a^4*b^3 + 10*a^3*b^4
+ 5*a^2*b^5 + a*b^6) + 30*(3*(6*a^2*b^2 - 8*a*b^3 - (3*a^2*b - 7*a*b^2 + 4*
b^3)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(
1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-
a/(a + b) + b/(a + b)))) - (6*a^2*b^2 - 8*a*b^3 - (3*a^2*b - 7*a*b^2 + 4*b^
3)*sqrt(-a*b))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/
2*real_part(arccos(-a/(a + b) + b/(a + b))))^3 - 9*(6*a^2*b^2 - 8*a*b^3 - (
3*a^2*b - 7*a*b^2 + 4*b^3)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b)
+ b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(
```

```
1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a
/(a + b) + b/(a + b)))) + 3*(6*a^2*b^2 - 8*a*b^3 - (3*a^2*b - 7*a*b^2 + 4*b
^3)*sqrt(-a*b))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1
/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-
a/(a + b) + b/(a + b)))) + 9*(6*a^2*b^2 - 8*a*b^3 - (3*a^2*b - 7*a*b^2 + 4*
b^3)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(
1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/
(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^
2 - 3*(6*a^2*b^2 - 8*a*b^3 - (3*a^2*b - 7*a*b^2 + 4*b^3)*sqrt(-a*b))*cosh(1
/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(
a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))
^2 - 3*(6*a^2*b^2 - 8*a*b^3 - (3*a^2*b - 7*a*b^2 + 4*b^3)*sqrt(-a*b))*cos(1
/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a
/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))
^3 + (6*a^2*b^2 - 8*a*b^3 - (3*a^2*b - 7*a*b^2 + 4*b^3)*sqrt(-a*b))*sin(1/2
*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/
(a + b) + b/(a + b))))^3 - (6*a^2*b^2 - 8*a*b^3 - (3*a^2*b - 7*a*b^2 + 4*b^
3)*sqrt(-a*b))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*
real_part(arccos(-a/(a + b) + b/(a + b)))) + (6*a^2*b^2 - 8*a*b^3 - (3*a^2*
b - 7*a*b^2 + 4*b^3)*sqrt(-a*b))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a
 + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*arctan(-(((a^
5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)/(a^5*e^(4*c) + 5*a^4
*b*e^(4*c) + 10*a^3*b^2*e^(4*c) + 10*a^2*b^3*e^(4*c) + 5*a*b^4*e^(4*c) + b^
5*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b))) - e^(d*x))/(((a^5 + 5*a
^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)/(a^5*e^(4*c) + 5*a^4*b*e^(4
*c) + 10*a^3*b^2*e^(4*c) + 10*a^2*b^3*e^(4*c) + 5*a*b^4*e^(4*c) + b^5*e^(4*
c)))^(1/4)*sin(1/2*arccos(-(a - b)/(a + b)))))/(a^6*b + 5*a^5*b^2 + 10*a^4*
b^3 + 10*a^3*b^4 + 5*a^2*b^5 + a*b^6) + 4*(9*a*e^(2*d*x + 2*c) - 27*b*e^(2*
d*x + 2*c) - a - b)*e^(-3*d*x)/(a^4*e^(3*c) + 4*a^3*b*e^(3*c) + 6*a^2*b^2*e
^(3*c) + 4*a*b^3*e^(3*c) + b^4*e^(3*c)) + 15*((6*a^2*b^2 - 8*a*b^3 - (3*a^2
*b - 7*a*b^2 + 4*b^3)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(
a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 - 3*(6*a^
2*b^2 - 8*a*b^3 - (3*a^2*b - 7*a*b^2 + 4*b^3)*sqrt(-a*b))*cos(1/2*real_part
(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/
(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(6*a^
2*b^2 - 8*a*b^3 - (3*a^2*b - 7*a*b^2 + 4*b^3)*sqrt(-a*b))*cos(1/2*real_part
(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) +
b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(6*a
^2*b^2 - 8*a*b^3 - (3*a^2*b - 7*a*b^2 + 4*b^3)*sqrt(-a*b))*cos(1/2*real_par
t(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b
/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2
*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(6*a^2*b^2 - 8*a*b^3 - (3*a
^2*b - 7*a*b^2 + 4*b^3)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b
/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*
imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 9*(6*a^2*b^2 - 8*a*b^3 - (3*
a^2*b - 7*a*b^2 + 4*b^3)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) +
b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*re
al_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a
+ b) + b/(a + b))))^2 - (6*a^2*b^2 - 8*a*b^3 - (3*a^2*b - 7*a*b^2 + 4*b^3)*
sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*i
mag_part(arccos(-a/(a + b) + b/(a + b))))^3 + 3*(6*a^2*b^2 - 8*a*b^3 - (3*a
^2*b - 7*a*b^2 + 4*b^3)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b
/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*i
mag_part(arccos(-a/(a + b) + b/(a + b))))^3 - (6*a^2*b^2 - 8*a*b^3 - (3*a^2
*b - 7*a*b^2 + 4*b^3)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(
a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + (6*a^2*b^2
- 8*a*b^3 - (3*a^2*b - 7*a*b^2 + 4*b^3)*sqrt(-a*b))*cos(1/2*real_part(arcco
s(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b
)))))*log(2*((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)/(a^5
```

```
*e^(4*c) + 5*a^4*b*e^(4*c) + 10*a^3*b^2*e^(4*c) + 10*a^2*b^3*e^(4*c) + 5*a*
b^4*e^(4*c) + b^5*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b)))*e^(d*x)
 + sqrt((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)/(a^5*e^(4
*c) + 5*a^4*b*e^(4*c) + 10*a^3*b^2*e^(4*c) + 10*a^2*b^3*e^(4*c) + 5*a*b^4*e
^(4*c) + b^5*e^(4*c))) + e^(2*d*x))/(a^6*b + 5*a^5*b^2 + 10*a^4*b^3 + 10*a^
3*b^4 + 5*a^2*b^5 + a*b^6) - 15*((6*a^2*b^2 - 8*a*b^3 - (3*a^2*b - 7*a*b^2
+ 4*b^3)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*c
osh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 - 3*(6*a^2*b^2 - 8*a*b
^3 - (3*a^2*b - 7*a*b^2 + 4*b^3)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a
 + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*
sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(6*a^2*b^2 - 8*a*b
^3 - (3*a^2*b - 7*a*b^2 + 4*b^3)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a
 + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^
2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(6*a^2*b^2 - 8*a*
b^3 - (3*a^2*b - 7*a*b^2 + 4*b^3)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(
a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2
*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(ar
ccos(-a/(a + b) + b/(a + b)))) + 3*(6*a^2*b^2 - 8*a*b^3 - (3*a^2*b - 7*a*b^
2 + 4*b^3)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3
*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arc
cos(-a/(a + b) + b/(a + b))))^2 - 9*(6*a^2*b^2 - 8*a*b^3 - (3*a^2*b - 7*a*b
^2 + 4*b^3)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*
cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arcco
s(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a +
 b))))^2 - (6*a^2*b^2 - 8*a*b^3 - (3*a^2*b - 7*a*b^2 + 4*b^3)*sqrt(-a*b))*c
os(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arcc
os(-a/(a + b) + b/(a + b))))^3 + 3*(6*a^2*b^2 - 8*a*b^3 - (3*a^2*b - 7*a*b^
2 + 4*b^3)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*s
in(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arcc
os(-a/(a + b) + b/(a + b))))^3 - (6*a^2*b^2 - 8*a*b^3 - (3*a^2*b - 7*a*b^2
+ 4*b^3)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cos
h(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + (6*a^2*b^2 - 8*a*b^3 - (
3*a^2*b - 7*a*b^2 + 4*b^3)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b)
+ b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*log(-2*
((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)/(a^5*e^(4*c) + 5
*a^4*b*e^(4*c) + 10*a^3*b^2*e^(4*c) + 10*a^2*b^3*e^(4*c) + 5*a*b^4*e^(4*c)
+ b^5*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b)))*e^(d*x) + sqrt((a^5
 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)/(a^5*e^(4*c) + 5*a^4*
b*e^(4*c) + 10*a^3*b^2*e^(4*c) + 10*a^2*b^3*e^(4*c) + 5*a*b^4*e^(4*c) + b^5
*e^(4*c))) + e^(2*d*x))/(a^6*b + 5*a^5*b^2 + 10*a^4*b^3 + 10*a^3*b^4 + 5*a^
2*b^5 + a*b^6) - 4*(a^6*e^(3*d*x + 48*c) + 6*a^5*b*e^(3*d*x + 48*c) + 15*a^
4*b^2*e^(3*d*x + 48*c) + 20*a^3*b^3*e^(3*d*x + 48*c) + 15*a^2*b^4*e^(3*d*x
+ 48*c) + 6*a*b^5*e^(3*d*x + 48*c) + b^6*e^(3*d*x + 48*c) - 9*a^6*e^(d*x +
46*c) - 18*a^5*b*e^(d*x + 46*c) + 45*a^4*b^2*e^(d*x + 46*c) + 180*a^3*b^3*e
^(d*x + 46*c) + 225*a^2*b^4*e^(d*x + 46*c) + 126*a*b^5*e^(d*x + 46*c) + 27*
b^6*e^(d*x + 46*c))/(a^9*e^(45*c) + 9*a^8*b*e^(45*c) + 36*a^7*b^2*e^(45*c)
+ 84*a^6*b^3*e^(45*c) + 126*a^5*b^4*e^(45*c) + 126*a^4*b^5*e^(45*c) + 84*a^
3*b^6*e^(45*c) + 36*a^2*b^7*e^(45*c) + 9*a*b^8*e^(45*c) + b^9*e^(45*c)) - 2
4*(9*a^2*b*e^(7*d*x + 7*c) + 5*a*b^2*e^(7*d*x + 7*c) - 4*b^3*e^(7*d*x + 7*c
) + 27*a^2*b*e^(5*d*x + 5*c) - 13*a*b^2*e^(5*d*x + 5*c) + 4*b^3*e^(5*d*x +
5*c) + 27*a^2*b*e^(3*d*x + 3*c) - 13*a*b^2*e^(3*d*x + 3*c) + 4*b^3*e^(3*d*x
 + 3*c) + 9*a^2*b*e^(d*x + c) + 5*a*b^2*e^(d*x + c) - 4*b^3*e^(d*x + c))/((
a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x
+ 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2))/d
```

3.43 $\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=185

$$-\frac{\sqrt{b}\left(15a^2-10ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}d(a+b)^4}-\frac{b(11a-b)\tanh(c+dx)}{8ad(a+b)^3\left(a+b\tanh^2(c+dx)\right)}-\frac{3b\tanh(c+dx)}{4d(a+b)^2\left(a+b\tanh^2(c+dx)\right)^2}$$

```
[Out] -((a - 5*b)*x)/(2*(a + b)^4) - (Sqrt[b]*(15*a^2 - 10*a*b - b^2)*ArcTan[(Sqr
t[b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(3/2)*(a + b)^4*d) + (Cosh[c + d*x]*Sinh
[c + d*x])/(2*(a + b)*d*(a + b*Tanh[c + d*x]^2)^2) - (3*b*Tanh[c + d*x])/(4
*(a + b)^2*d*(a + b*Tanh[c + d*x]^2)^2) - ((11*a - b)*b*Tanh[c + d*x])/(8*a
*(a + b)^3*d*(a + b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.250835, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3663, 471, 527, 522, 206, 205}

$$-\frac{\sqrt{b}\left(15a^2-10ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}d(a+b)^4}-\frac{b(11a-b)\tanh(c+dx)}{8ad(a+b)^3\left(a+b\tanh^2(c+dx)\right)}-\frac{3b\tanh(c+dx)}{4d(a+b)^2\left(a+b\tanh^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```


```
[Out] -((a - 5*b)*x)/(2*(a + b)^4) - (Sqrt[b]*(15*a^2 - 10*a*b - b^2)*ArcTan[(Sqr
t[b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(3/2)*(a + b)^4*d) + (Cosh[c + d*x]*Sinh
[c + d*x])/(2*(a + b)*d*(a + b*Tanh[c + d*x]^2)^2) - (3*b*Tanh[c + d*x])/(4
*(a + b)^2*d*(a + b*Tanh[c + d*x]^2)^2) - ((11*a - b)*b*Tanh[c + d*x])/(8*a
*(a + b)^3*d*(a + b*Tanh[c + d*x]^2))
```


Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)^2\left(a+bx^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{a-5bx^2}{\left(1-x^2\right)\left(a+bx^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{2(a+b)d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{3b\tanh(c+dx)}{4(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{-2a(2a-}{(1-x^2)}\right.}{}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{3b\tanh(c+dx)}{4(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(11a-b)b\,\mathrm{t}}{8a(a+b)^3d\left(a+\right.}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{3b\tanh(c+dx)}{4(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(11a-b)b\,\mathrm{t}}{8a(a+b)^3d\left(a+\right.}$$

$$= -\frac{(a-5b)x}{2(a+b)^4} - \frac{\sqrt{b}\left(15a^2-10ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}(a+b)^4d} + \frac{\cosh(c+dx)\sinh(c+dx}{2(a+b)d\left(a+b\tanh^2(c+d\right.}$$

Mathematica [A] time = 1.20868, size = 158, normalized size = 0.85

$$\frac{\frac{\sqrt{b}\left(-15a^2+10ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}} - \frac{4b^2(a+b)\sinh(2(c+dx))}{((a+b)\cosh(2(c+dx))+a-b)^2} - 4(a-5b)(c+dx) + 2(a+b)\sinh(2(c+dx)) - \frac{b(9a-b)(a+b)\sinh}{a((a+b)\cosh(2(c+}}{8d(a+b)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (-4*(a - 5*b)*(c + d*x) + (Sqrt[b]*(-15*a^2 + 10*a*b + b^2)*ArcTan[(Sqrt[b]
*Tanh[c + d*x])/Sqrt[a]])/a^(3/2) + 2*(a + b)*Sinh[2*(c + d*x)] - (4*b^2*(a
 + b)*Sinh[2*(c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)])^2 - ((9*a - b)
*b*(a + b)*Sinh[2*(c + d*x)])/(a*(a - b + (a + b)*Cosh[2*(c + d*x)])))/(8*(
a + b)^4*d)
```

Maple [B] time = 0.115, size = 2110, normalized size = 11.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 5/8/d*b^2/(a+b)^4*a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arc
tanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+5/8/d*b^2/(
a+b)^4*a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(
1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+15/8/d*b/(a+b)^4*a^2/(b
*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/
2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-1/8/d*b^4/(a+b)^4/(b*(a+b))^(1/2)
/a/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*
(a+b))^(1/2)+a+2*b)*a)^(1/2))+15/8/d*b/(a+b)^4*a^2/(b*(a+b))^(1/2)/((2*(b*(
a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2
)+a+2*b)*a)^(1/2))-1/8/d*b^4/(a+b)^4/(b*(a+b))^(1/2)/a/((2*(b*(a+b))^(1/2)-
a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)
^(1/2))-9/4/d*b/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+
4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^7*a^2+1/d*b^4/(a+b)^4/(t
anh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+
a)^2/a*tanh(1/2*d*x+1/2*c)^5-5/4/d*b^2/(a+b)^4/((2*(b*(a+b))^(1/2)+a+2*b)*a
)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1
/4/d*b^3/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(
1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^7-27/4/d*b^3/(a+b)^4/(tanh(1/2*
d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tan
h(1/2*d*x+1/2*c)^5-27/4/d*b^3/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d
*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^3-1/4/d*b^
3/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x
+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)-1/8/d*b^3/(a+b)^4/a/((2*(b*(a+b))^(1/2
)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a
)^(1/2))-29/2/d*b^2/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^
2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a*tanh(1/2*d*x+1/2*c)^3+1/d*b^4/(a+b)^4/
(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*
b+a)^2/a*tanh(1/2*d*x+1/2*c)^3-5/2/d*b^2/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+2
*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)
*a-27/4/d*b/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*ta
nh(1/2*d*x+1/2*c)^2*b+a)^2*a^2*tanh(1/2*d*x+1/2*c)^3-27/4/d*b/(a+b)^4/(tanh
(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^
2*a^2*tanh(1/2*d*x+1/2*c)^5-9/4/d*b/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh
(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)*a^2-
1/2/d/(a+b)^4*ln(tanh(1/2*d*x+1/2*c)+1)*a+5/2/d/(a+b)^4*ln(tanh(1/2*d*x+1/2
*c)+1)*b+1/2/d/(a+b)^4*ln(tanh(1/2*d*x+1/2*c)-1)*a-5/2/d/(a+b)^4*ln(tanh(1/
2*d*x+1/2*c)-1)*b-1/2/d/(a+b)^3/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/d/(a+b)^3/(ta
nh(1/2*d*x+1/2*c)+1)+1/2/d/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/(a+b)^3/
(tanh(1/2*d*x+1/2*c)-1)-15/8/d*b/(a+b)^4*a/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1
/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-11/8
/d*b^3/(a+b)^4/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(
```

```
a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-11/8/d*b^3/(a+b)
^4/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*
x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/8/d*b^3/(a+b)^4/a/((2*(b*(a
+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2
)-a-2*b)*a)^(1/2))+15/8/d*b/(a+b)^4*a/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*a
rctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+5/4/d*b^2/
(a+b)^4/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/(
(2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-5/2/d*b^2/(a+b)^4/(tanh(1/2*d*x+1/2*c)^
4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1
/2*c)^7*a-29/2/d*b^2/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)
^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a*tanh(1/2*d*x+1/2*c)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.84381, size = 29831, normalized size = 161.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(d*x + c)^12 + 24*(a^4 + 3
*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(d*x + c)*sinh(d*x + c)^11 + 2*(a^4 + 3*a^3
*b + 3*a^2*b^2 + a*b^3)*sinh(d*x + c)^12 + 8*(a^4 + a^3*b - a^2*b^2 - a*b^3
 - (a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d*x)*cosh(d*x + c)^10 + 4*(2*a^4 +
 2*a^3*b - 2*a^2*b^2 - 2*a*b^3 - 2*(a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d*
x + 33*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^1
0 + 40*(11*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(d*x + c)^3 + 2*(a^4 + a
^3*b - a^2*b^2 - a*b^3 - (a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d*x)*cosh(d*
x + c))*sinh(d*x + c)^9 + 2*(5*a^4 + 17*a^3*b - 11*a^2*b^2 - 21*a*b^3 + 2*b
^4 - 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x)*cosh(d*x + c)^8 + 2*(495*(
a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(d*x + c)^4 + 5*a^4 + 17*a^3*b - 11*
a^2*b^2 - 21*a*b^3 + 2*b^4 - 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x + 1
80*(a^4 + a^3*b - a^2*b^2 - a*b^3 - (a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d
*x)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 16*(99*(a^4 + 3*a^3*b + 3*a^2*b^2 +
a*b^3)*cosh(d*x + c)^5 + 60*(a^4 + a^3*b - a^2*b^2 - a*b^3 - (a^4 - 3*a^3*b
 - 9*a^2*b^2 - 5*a*b^3)*d*x)*cosh(d*x + c)^3 + (5*a^4 + 17*a^3*b - 11*a^2*b
^2 - 21*a*b^3 + 2*b^4 - 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x)*cosh(d*
x + c))*sinh(d*x + c)^7 + 4*(27*a^3*b - 21*a^2*b^2 + 29*a*b^3 - 3*b^4 - 4*(
3*a^4 - 17*a^3*b + 13*a^2*b^2 - 15*a*b^3)*d*x)*cosh(d*x + c)^6 + 4*(462*(a^
4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(d*x + c)^6 + 420*(a^4 + a^3*b - a^2*b
^2 - a*b^3 - (a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d*x)*cosh(d*x + c)^4 + 2
7*a^3*b - 21*a^2*b^2 + 29*a*b^3 - 3*b^4 - 4*(3*a^4 - 17*a^3*b + 13*a^2*b^2
- 15*a*b^3)*d*x + 14*(5*a^4 + 17*a^3*b - 11*a^2*b^2 - 21*a*b^3 + 2*b^4 - 16
*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6
+ 8*(198*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(d*x + c)^7 + 252*(a^4 + a
```

```
^3*b - a^2*b^2 - a*b^3 - (a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d*x)*cosh(d*
x + c)^5 + 14*(5*a^4 + 17*a^3*b - 11*a^2*b^2 - 21*a*b^3 + 2*b^4 - 16*(a^4 -
 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x)*cosh(d*x + c)^3 + 3*(27*a^3*b - 21*a^2*b
^2 + 29*a*b^3 - 3*b^4 - 4*(3*a^4 - 17*a^3*b + 13*a^2*b^2 - 15*a*b^3)*d*x)*c
osh(d*x + c))*sinh(d*x + c)^5 - 2*(5*a^4 - 55*a^3*b - 3*a^2*b^2 + 51*a*b^3
- 6*b^4 + 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x)*cosh(d*x + c)^4 + 2*(
495*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(d*x + c)^8 + 840*(a^4 + a^3*b
- a^2*b^2 - a*b^3 - (a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d*x)*cosh(d*x + c
)^6 + 70*(5*a^4 + 17*a^3*b - 11*a^2*b^2 - 21*a*b^3 + 2*b^4 - 16*(a^4 - 5*a^
3*b - a^2*b^2 + 5*a*b^3)*d*x)*cosh(d*x + c)^4 - 5*a^4 + 55*a^3*b + 3*a^2*b^
2 - 51*a*b^3 + 6*b^4 - 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x + 30*(27*
a^3*b - 21*a^2*b^2 + 29*a*b^3 - 3*b^4 - 4*(3*a^4 - 17*a^3*b + 13*a^2*b^2 -
15*a*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 2*a^4 - 6*a^3*b - 6*a^2*b
^2 - 2*a*b^3 + 8*(55*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(d*x + c)^9 +
120*(a^4 + a^3*b - a^2*b^2 - a*b^3 - (a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*
d*x)*cosh(d*x + c)^7 + 14*(5*a^4 + 17*a^3*b - 11*a^2*b^2 - 21*a*b^3 + 2*b^4
 - 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x)*cosh(d*x + c)^5 + 10*(27*a^3
*b - 21*a^2*b^2 + 29*a*b^3 - 3*b^4 - 4*(3*a^4 - 17*a^3*b + 13*a^2*b^2 - 15*
a*b^3)*d*x)*cosh(d*x + c)^3 - (5*a^4 - 55*a^3*b - 3*a^2*b^2 + 51*a*b^3 - 6*
b^4 + 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x)*cosh(d*x + c))*sinh(d*x +
 c)^3 - 4*(2*a^4 - 7*a^3*b - 19*a^2*b^2 - 9*a*b^3 + b^4 + 2*(a^4 - 3*a^3*b
- 9*a^2*b^2 - 5*a*b^3)*d*x)*cosh(d*x + c)^2 + 4*(33*(a^4 + 3*a^3*b + 3*a^2*
b^2 + a*b^3)*cosh(d*x + c)^10 + 90*(a^4 + a^3*b - a^2*b^2 - a*b^3 - (a^4 -
3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d*x)*cosh(d*x + c)^8 + 14*(5*a^4 + 17*a^3*b
- 11*a^2*b^2 - 21*a*b^3 + 2*b^4 - 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*
x)*cosh(d*x + c)^6 + 15*(27*a^3*b - 21*a^2*b^2 + 29*a*b^3 - 3*b^4 - 4*(3*a^
4 - 17*a^3*b + 13*a^2*b^2 - 15*a*b^3)*d*x)*cosh(d*x + c)^4 - 2*a^4 + 7*a^3*
b + 19*a^2*b^2 + 9*a*b^3 - b^4 - 2*(a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d*
x - 3*(5*a^4 - 55*a^3*b - 3*a^2*b^2 + 51*a*b^3 - 6*b^4 + 16*(a^4 - 5*a^3*b
- a^2*b^2 + 5*a*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((15*a^4 + 20*
a^3*b - 6*a^2*b^2 - 12*a*b^3 - b^4)*cosh(d*x + c)^10 + 10*(15*a^4 + 20*a^3*
b - 6*a^2*b^2 - 12*a*b^3 - b^4)*cosh(d*x + c)*sinh(d*x + c)^9 + (15*a^4 + 2
0*a^3*b - 6*a^2*b^2 - 12*a*b^3 - b^4)*sinh(d*x + c)^10 + 4*(15*a^4 - 10*a^3
*b - 16*a^2*b^2 + 10*a*b^3 + b^4)*cosh(d*x + c)^8 + (60*a^4 - 40*a^3*b - 64
*a^2*b^2 + 40*a*b^3 + 4*b^4 + 45*(15*a^4 + 20*a^3*b - 6*a^2*b^2 - 12*a*b^3
- b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(15*a^4 + 20*a^3*b - 6*a^2*
b^2 - 12*a*b^3 - b^4)*cosh(d*x + c)^3 + 4*(15*a^4 - 10*a^3*b - 16*a^2*b^2 +
 10*a*b^3 + b^4)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(45*a^4 - 60*a^3*b + 62
*a^2*b^2 - 28*a*b^3 - 3*b^4)*cosh(d*x + c)^6 + 2*(105*(15*a^4 + 20*a^3*b -
6*a^2*b^2 - 12*a*b^3 - b^4)*cosh(d*x + c)^4 + 45*a^4 - 60*a^3*b + 62*a^2*b^
2 - 28*a*b^3 - 3*b^4 + 56*(15*a^4 - 10*a^3*b - 16*a^2*b^2 + 10*a*b^3 + b^4)
*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(15*a^4 + 20*a^3*b - 6*a^2*b^2 -
12*a*b^3 - b^4)*cosh(d*x + c)^5 + 56*(15*a^4 - 10*a^3*b - 16*a^2*b^2 + 10*a
*b^3 + b^4)*cosh(d*x + c)^3 + 3*(45*a^4 - 60*a^3*b + 62*a^2*b^2 - 28*a*b^3
- 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*(15*a^4 - 10*a^3*b - 16*a^2*b^2
 + 10*a*b^3 + b^4)*cosh(d*x + c)^4 + 2*(105*(15*a^4 + 20*a^3*b - 6*a^2*b^2
- 12*a*b^3 - b^4)*cosh(d*x + c)^6 + 140*(15*a^4 - 10*a^3*b - 16*a^2*b^2 + 1
0*a*b^3 + b^4)*cosh(d*x + c)^4 + 30*a^4 - 20*a^3*b - 32*a^2*b^2 + 20*a*b^3
+ 2*b^4 + 15*(45*a^4 - 60*a^3*b + 62*a^2*b^2 - 28*a*b^3 - 3*b^4)*cosh(d*x +
 c)^2)*sinh(d*x + c)^4 + 8*(15*(15*a^4 + 20*a^3*b - 6*a^2*b^2 - 12*a*b^3 -
b^4)*cosh(d*x + c)^7 + 28*(15*a^4 - 10*a^3*b - 16*a^2*b^2 + 10*a*b^3 + b^4)
*cosh(d*x + c)^5 + 5*(45*a^4 - 60*a^3*b + 62*a^2*b^2 - 28*a*b^3 - 3*b^4)*co
sh(d*x + c)^3 + 2*(15*a^4 - 10*a^3*b - 16*a^2*b^2 + 10*a*b^3 + b^4)*cosh(d*
x + c))*sinh(d*x + c)^3 + (15*a^4 + 20*a^3*b - 6*a^2*b^2 - 12*a*b^3 - b^4)*
cosh(d*x + c)^2 + (45*(15*a^4 + 20*a^3*b - 6*a^2*b^2 - 12*a*b^3 - b^4)*cosh
(d*x + c)^8 + 112*(15*a^4 - 10*a^3*b - 16*a^2*b^2 + 10*a*b^3 + b^4)*cosh(d*
x + c)^6 + 30*(45*a^4 - 60*a^3*b + 62*a^2*b^2 - 28*a*b^3 - 3*b^4)*cosh(d*x
+ c)^4 + 15*a^4 + 20*a^3*b - 6*a^2*b^2 - 12*a*b^3 - b^4 + 24*(15*a^4 - 10*a
^3*b - 16*a^2*b^2 + 10*a*b^3 + b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(5
```

```
*(15*a^4 + 20*a^3*b - 6*a^2*b^2 - 12*a*b^3 - b^4)*cosh(d*x + c)^9 + 16*(15*
a^4 - 10*a^3*b - 16*a^2*b^2 + 10*a*b^3 + b^4)*cosh(d*x + c)^7 + 6*(45*a^4 -
 60*a^3*b + 62*a^2*b^2 - 28*a*b^3 - 3*b^4)*cosh(d*x + c)^5 + 8*(15*a^4 - 10
*a^3*b - 16*a^2*b^2 + 10*a*b^3 + b^4)*cosh(d*x + c)^3 + (15*a^4 + 20*a^3*b
- 6*a^2*b^2 - 12*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/a)*log(
((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*
sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(
d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x
 + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2
 - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(
a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2
- a*b)*sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(
d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a
+ b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3
+ (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 8*(3*(a^4 + 3*a^3*b + 3*
a^2*b^2 + a*b^3)*cosh(d*x + c)^11 + 10*(a^4 + a^3*b - a^2*b^2 - a*b^3 - (a^
4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d*x)*cosh(d*x + c)^9 + 2*(5*a^4 + 17*a^3
*b - 11*a^2*b^2 - 21*a*b^3 + 2*b^4 - 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)
*d*x)*cosh(d*x + c)^7 + 3*(27*a^3*b - 21*a^2*b^2 + 29*a*b^3 - 3*b^4 - 4*(3*
a^4 - 17*a^3*b + 13*a^2*b^2 - 15*a*b^3)*d*x)*cosh(d*x + c)^5 - (5*a^4 - 55*
a^3*b - 3*a^2*b^2 + 51*a*b^3 - 6*b^4 + 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^
3)*d*x)*cosh(d*x + c)^3 - (2*a^4 - 7*a^3*b - 19*a^2*b^2 - 9*a*b^3 + b^4 + 2
*(a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(
(a^7 + 6*a^6*b + 15*a^5*b^2 + 20*a^4*b^3 + 15*a^3*b^4 + 6*a^2*b^5 + a*b^6)*
d*cosh(d*x + c)^10 + 10*(a^7 + 6*a^6*b + 15*a^5*b^2 + 20*a^4*b^3 + 15*a^3*b
^4 + 6*a^2*b^5 + a*b^6)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^7 + 6*a^6*b +
15*a^5*b^2 + 20*a^4*b^3 + 15*a^3*b^4 + 6*a^2*b^5 + a*b^6)*d*sinh(d*x + c)^1
0 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*d*cosh(d*
x + c)^8 + (45*(a^7 + 6*a^6*b + 15*a^5*b^2 + 20*a^4*b^3 + 15*a^3*b^4 + 6*a^
2*b^5 + a*b^6)*d*cosh(d*x + c)^2 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4
 - 4*a^2*b^5 - a*b^6)*d)*sinh(d*x + c)^8 + 2*(3*a^7 + 10*a^6*b + 13*a^5*b^2
 + 12*a^4*b^3 + 13*a^3*b^4 + 10*a^2*b^5 + 3*a*b^6)*d*cosh(d*x + c)^6 + 8*(1
5*(a^7 + 6*a^6*b + 15*a^5*b^2 + 20*a^4*b^3 + 15*a^3*b^4 + 6*a^2*b^5 + a*b^6
)*d*cosh(d*x + c)^3 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5
- a*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^7 + 6*a^6*b + 15*a^5*
b^2 + 20*a^4*b^3 + 15*a^3*b^4 + 6*a^2*b^5 + a*b^6)*d*cosh(d*x + c)^4 + 56*(
a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*d*cosh(d*x + c)^
2 + (3*a^7 + 10*a^6*b + 13*a^5*b^2 + 12*a^4*b^3 + 13*a^3*b^4 + 10*a^2*b^5 +
 3*a*b^6)*d)*sinh(d*x + c)^6 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4
*a^2*b^5 - a*b^6)*d*cosh(d*x + c)^4 + 4*(63*(a^7 + 6*a^6*b + 15*a^5*b^2 + 2
0*a^4*b^3 + 15*a^3*b^4 + 6*a^2*b^5 + a*b^6)*d*cosh(d*x + c)^5 + 56*(a^7 + 4
*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*d*cosh(d*x + c)^3 + 3*(
3*a^7 + 10*a^6*b + 13*a^5*b^2 + 12*a^4*b^3 + 13*a^3*b^4 + 10*a^2*b^5 + 3*a*
b^6)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^7 + 6*a^6*b + 15*a^5*b^2
+ 20*a^4*b^3 + 15*a^3*b^4 + 6*a^2*b^5 + a*b^6)*d*cosh(d*x + c)^6 + 140*(a^7
 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*d*cosh(d*x + c)^4 +
 15*(3*a^7 + 10*a^6*b + 13*a^5*b^2 + 12*a^4*b^3 + 13*a^3*b^4 + 10*a^2*b^5 +
 3*a*b^6)*d*cosh(d*x + c)^2 + 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*
a^2*b^5 - a*b^6)*d)*sinh(d*x + c)^4 + (a^7 + 6*a^6*b + 15*a^5*b^2 + 20*a^4*
b^3 + 15*a^3*b^4 + 6*a^2*b^5 + a*b^6)*d*cosh(d*x + c)^2 + 8*(15*(a^7 + 6*a^
6*b + 15*a^5*b^2 + 20*a^4*b^3 + 15*a^3*b^4 + 6*a^2*b^5 + a*b^6)*d*cosh(d*x
+ c)^7 + 28*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*d*c
osh(d*x + c)^5 + 5*(3*a^7 + 10*a^6*b + 13*a^5*b^2 + 12*a^4*b^3 + 13*a^3*b^4
 + 10*a^2*b^5 + 3*a*b^6)*d*cosh(d*x + c)^3 + 2*(a^7 + 4*a^6*b + 5*a^5*b^2 -
 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (45*(a^7
 + 6*a^6*b + 15*a^5*b^2 + 20*a^4*b^3 + 15*a^3*b^4 + 6*a^2*b^5 + a*b^6)*d*co
sh(d*x + c)^8 + 112*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*
b^6)*d*cosh(d*x + c)^6 + 30*(3*a^7 + 10*a^6*b + 13*a^5*b^2 + 12*a^4*b^3 + 1
3*a^3*b^4 + 10*a^2*b^5 + 3*a*b^6)*d*cosh(d*x + c)^4 + 24*(a^7 + 4*a^6*b + 5
```

*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*d*cosh(d*x + c)^2 + (a^7 + 6*a^6*
b + 15*a^5*b^2 + 20*a^4*b^3 + 15*a^3*b^4 + 6*a^2*b^5 + a*b^6)*d)*sinh(d*x +
 c)^2 + 2*(5*(a^7 + 6*a^6*b + 15*a^5*b^2 + 20*a^4*b^3 + 15*a^3*b^4 + 6*a^2*
b^5 + a*b^6)*d*cosh(d*x + c)^9 + 16*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4
- 4*a^2*b^5 - a*b^6)*d*cosh(d*x + c)^7 + 6*(3*a^7 + 10*a^6*b + 13*a^5*b^2 +
 12*a^4*b^3 + 13*a^3*b^4 + 10*a^2*b^5 + 3*a*b^6)*d*cosh(d*x + c)^5 + 8*(a^7
 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*d*cosh(d*x + c)^3 +
 (a^7 + 6*a^6*b + 15*a^5*b^2 + 20*a^4*b^3 + 15*a^3*b^4 + 6*a^2*b^5 + a*b^6)
*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*
cosh(d*x + c)^12 + 12*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(d*x + c)*sin
h(d*x + c)^11 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sinh(d*x + c)^12 + 4*(a
^4 + a^3*b - a^2*b^2 - a*b^3 - (a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d*x)*c
osh(d*x + c)^10 + 2*(2*a^4 + 2*a^3*b - 2*a^2*b^2 - 2*a*b^3 - 2*(a^4 - 3*a^3
*b - 9*a^2*b^2 - 5*a*b^3)*d*x + 33*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh
(d*x + c)^2)*sinh(d*x + c)^10 + 20*(11*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*
cosh(d*x + c)^3 + 2*(a^4 + a^3*b - a^2*b^2 - a*b^3 - (a^4 - 3*a^3*b - 9*a^2
*b^2 - 5*a*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^9 + (5*a^4 + 17*a^3*b - 1
1*a^2*b^2 - 21*a*b^3 + 2*b^4 - 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x)*
cosh(d*x + c)^8 + (495*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(d*x + c)^4
+ 5*a^4 + 17*a^3*b - 11*a^2*b^2 - 21*a*b^3 + 2*b^4 - 16*(a^4 - 5*a^3*b - a^
2*b^2 + 5*a*b^3)*d*x + 180*(a^4 + a^3*b - a^2*b^2 - a*b^3 - (a^4 - 3*a^3*b
- 9*a^2*b^2 - 5*a*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(99*(a^4 +
 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(d*x + c)^5 + 60*(a^4 + a^3*b - a^2*b^2 -
 a*b^3 - (a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d*x)*cosh(d*x + c)^3 + (5*a^
4 + 17*a^3*b - 11*a^2*b^2 - 21*a*b^3 + 2*b^4 - 16*(a^4 - 5*a^3*b - a^2*b^2
+ 5*a*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(27*a^3*b - 21*a^2*b^2 +
 29*a*b^3 - 3*b^4 - 4*(3*a^4 - 17*a^3*b + 13*a^2*b^2 - 15*a*b^3)*d*x)*cosh(
d*x + c)^6 + 2*(462*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(d*x + c)^6 + 4
20*(a^4 + a^3*b - a^2*b^2 - a*b^3 - (a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d
*x)*cosh(d*x + c)^4 + 27*a^3*b - 21*a^2*b^2 + 29*a*b^3 - 3*b^4 - 4*(3*a^4 -
 17*a^3*b + 13*a^2*b^2 - 15*a*b^3)*d*x + 14*(5*a^4 + 17*a^3*b - 11*a^2*b^2
- 21*a*b^3 + 2*b^4 - 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x)*cosh(d*x +
 c)^2)*sinh(d*x + c)^6 + 4*(198*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(d*
x + c)^7 + 252*(a^4 + a^3*b - a^2*b^2 - a*b^3 - (a^4 - 3*a^3*b - 9*a^2*b^2
- 5*a*b^3)*d*x)*cosh(d*x + c)^5 + 14*(5*a^4 + 17*a^3*b - 11*a^2*b^2 - 21*a*
b^3 + 2*b^4 - 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x)*cosh(d*x + c)^3 +
 3*(27*a^3*b - 21*a^2*b^2 + 29*a*b^3 - 3*b^4 - 4*(3*a^4 - 17*a^3*b + 13*a^2
*b^2 - 15*a*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - (5*a^4 - 55*a^3*b -
3*a^2*b^2 + 51*a*b^3 - 6*b^4 + 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x)*
cosh(d*x + c)^4 + (495*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(d*x + c)^8
+ 840*(a^4 + a^3*b - a^2*b^2 - a*b^3 - (a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3
)*d*x)*cosh(d*x + c)^6 + 70*(5*a^4 + 17*a^3*b - 11*a^2*b^2 - 21*a*b^3 + 2*b
^4 - 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x)*cosh(d*x + c)^4 - 5*a^4 +
55*a^3*b + 3*a^2*b^2 - 51*a*b^3 + 6*b^4 - 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a
*b^3)*d*x + 30*(27*a^3*b - 21*a^2*b^2 + 29*a*b^3 - 3*b^4 - 4*(3*a^4 - 17*a^
3*b + 13*a^2*b^2 - 15*a*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - a^4 -
3*a^3*b - 3*a^2*b^2 - a*b^3 + 4*(55*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cos
h(d*x + c)^9 + 120*(a^4 + a^3*b - a^2*b^2 - a*b^3 - (a^4 - 3*a^3*b - 9*a^2*
b^2 - 5*a*b^3)*d*x)*cosh(d*x + c)^7 + 14*(5*a^4 + 17*a^3*b - 11*a^2*b^2 - 2
1*a*b^3 + 2*b^4 - 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x)*cosh(d*x + c)
^5 + 10*(27*a^3*b - 21*a^2*b^2 + 29*a*b^3 - 3*b^4 - 4*(3*a^4 - 17*a^3*b + 1
3*a^2*b^2 - 15*a*b^3)*d*x)*cosh(d*x + c)^3 - (5*a^4 - 55*a^3*b - 3*a^2*b^2
+ 51*a*b^3 - 6*b^4 + 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x)*cosh(d*x +
 c))*sinh(d*x + c)^3 - 2*(2*a^4 - 7*a^3*b - 19*a^2*b^2 - 9*a*b^3 + b^4 + 2*
(a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d*x)*cosh(d*x + c)^2 + 2*(33*(a^4 + 3
*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(d*x + c)^10 + 90*(a^4 + a^3*b - a^2*b^2 -
a*b^3 - (a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d*x)*cosh(d*x + c)^8 + 14*(5*
a^4 + 17*a^3*b - 11*a^2*b^2 - 21*a*b^3 + 2*b^4 - 16*(a^4 - 5*a^3*b - a^2*b^
2 + 5*a*b^3)*d*x)*cosh(d*x + c)^6 + 15*(27*a^3*b - 21*a^2*b^2 + 29*a*b^3 -

3*b^4 - 4*(3*a^4 - 17*a^3*b + 13*a^2*b^2 - 15*a*b^3)*d*x)*cosh(d*x + c)^4 -
2*a^4 + 7*a^3*b + 19*a^2*b^2 + 9*a*b^3 - b^4 - 2*(a^4 - 3*a^3*b - 9*a^2*b^
2 - 5*a*b^3)*d*x - 3*(5*a^4 - 55*a^3*b - 3*a^2*b^2 + 51*a*b^3 - 6*b^4 + 16*
(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 -
((15*a^4 + 20*a^3*b - 6*a^2*b^2 - 12*a*b^3 - b^4)*cosh(d*x + c)^10 + 10*(1
5*a^4 + 20*a^3*b - 6*a^2*b^2 - 12*a*b^3 - b^4)*cosh(d*x + c)*sinh(d*x + c)^
9 + (15*a^4 + 20*a^3*b - 6*a^2*b^2 - 12*a*b^3 - b^4)*sinh(d*x + c)^10 + 4*(
15*a^4 - 10*a^3*b - 16*a^2*b^2 + 10*a*b^3 + b^4)*cosh(d*x + c)^8 + (60*a^4
- 40*a^3*b - 64*a^2*b^2 + 40*a*b^3 + 4*b^4 + 45*(15*a^4 + 20*a^3*b - 6*a^2*
b^2 - 12*a*b^3 - b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(15*a^4 + 20
*a^3*b - 6*a^2*b^2 - 12*a*b^3 - b^4)*cosh(d*x + c)^3 + 4*(15*a^4 - 10*a^3*b
- 16*a^2*b^2 + 10*a*b^3 + b^4)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(45*a^4
- 60*a^3*b + 62*a^2*b^2 - 28*a*b^3 - 3*b^4)*cosh(d*x + c)^6 + 2*(105*(15*a^
4 + 20*a^3*b - 6*a^2*b^2 - 12*a*b^3 - b^4)*cosh(d*x + c)^4 + 45*a^4 - 60*a^
3*b + 62*a^2*b^2 - 28*a*b^3 - 3*b^4 + 56*(15*a^4 - 10*a^3*b - 16*a^2*b^2 +
10*a*b^3 + b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(15*a^4 + 20*a^3*b
- 6*a^2*b^2 - 12*a*b^3 - b^4)*cosh(d*x + c)^5 + 56*(15*a^4 - 10*a^3*b - 16
*a^2*b^2 + 10*a*b^3 + b^4)*cosh(d*x + c)^3 + 3*(45*a^4 - 60*a^3*b + 62*a^2*
b^2 - 28*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*(15*a^4 - 10*a^3
*b - 16*a^2*b^2 + 10*a*b^3 + b^4)*cosh(d*x + c)^4 + 2*(105*(15*a^4 + 20*a^3
*b - 6*a^2*b^2 - 12*a*b^3 - b^4)*cosh(d*x + c)^6 + 140*(15*a^4 - 10*a^3*b -
16*a^2*b^2 + 10*a*b^3 + b^4)*cosh(d*x + c)^4 + 30*a^4 - 20*a^3*b - 32*a^2*
b^2 + 20*a*b^3 + 2*b^4 + 15*(45*a^4 - 60*a^3*b + 62*a^2*b^2 - 28*a*b^3 - 3*
b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(15*(15*a^4 + 20*a^3*b - 6*a^2*b^
2 - 12*a*b^3 - b^4)*cosh(d*x + c)^7 + 28*(15*a^4 - 10*a^3*b - 16*a^2*b^2 +
10*a*b^3 + b^4)*cosh(d*x + c)^5 + 5*(45*a^4 - 60*a^3*b + 62*a^2*b^2 - 28*a*
b^3 - 3*b^4)*cosh(d*x + c)^3 + 2*(15*a^4 - 10*a^3*b - 16*a^2*b^2 + 10*a*b^3
+ b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + (15*a^4 + 20*a^3*b - 6*a^2*b^2 - 1
2*a*b^3 - b^4)*cosh(d*x + c)^2 + (45*(15*a^4 + 20*a^3*b - 6*a^2*b^2 - 12*a*
b^3 - b^4)*cosh(d*x + c)^8 + 112*(15*a^4 - 10*a^3*b - 16*a^2*b^2 + 10*a*b^3
+ b^4)*cosh(d*x + c)^6 + 30*(45*a^4 - 60*a^3*b + 62*a^2*b^2 - 28*a*b^3 - 3
*b^4)*cosh(d*x + c)^4 + 15*a^4 + 20*a^3*b - 6*a^2*b^2 - 12*a*b^3 - b^4 + 24
*(15*a^4 - 10*a^3*b - 16*a^2*b^2 + 10*a*b^3 + b^4)*cosh(d*x + c)^2)*sinh(d*
x + c)^2 + 2*(5*(15*a^4 + 20*a^3*b - 6*a^2*b^2 - 12*a*b^3 - b^4)*cosh(d*x +
c)^9 + 16*(15*a^4 - 10*a^3*b - 16*a^2*b^2 + 10*a*b^3 + b^4)*cosh(d*x + c)^
7 + 6*(45*a^4 - 60*a^3*b + 62*a^2*b^2 - 28*a*b^3 - 3*b^4)*cosh(d*x + c)^5 +
8*(15*a^4 - 10*a^3*b - 16*a^2*b^2 + 10*a*b^3 + b^4)*cosh(d*x + c)^3 + (15*
a^4 + 20*a^3*b - 6*a^2*b^2 - 12*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c))*
sqrt(b/a)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sin
h(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(b/a)/b) + 4*(3*(a^4 + 3*
a^3*b + 3*a^2*b^2 + a*b^3)*cosh(d*x + c)^11 + 10*(a^4 + a^3*b - a^2*b^2 - a
*b^3 - (a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d*x)*cosh(d*x + c)^9 + 2*(5*a^
4 + 17*a^3*b - 11*a^2*b^2 - 21*a*b^3 + 2*b^4 - 16*(a^4 - 5*a^3*b - a^2*b^2
+ 5*a*b^3)*d*x)*cosh(d*x + c)^7 + 3*(27*a^3*b - 21*a^2*b^2 + 29*a*b^3 - 3*b
^4 - 4*(3*a^4 - 17*a^3*b + 13*a^2*b^2 - 15*a*b^3)*d*x)*cosh(d*x + c)^5 - (5
*a^4 - 55*a^3*b - 3*a^2*b^2 + 51*a*b^3 - 6*b^4 + 16*(a^4 - 5*a^3*b - a^2*b^
2 + 5*a*b^3)*d*x)*cosh(d*x + c)^3 - (2*a^4 - 7*a^3*b - 19*a^2*b^2 - 9*a*b^3
+ b^4 + 2*(a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d*x)*cosh(d*x + c))*sinh(d
*x + c))/((a^7 + 6*a^6*b + 15*a^5*b^2 + 20*a^4*b^3 + 15*a^3*b^4 + 6*a^2*b^5
+ a*b^6)*d*cosh(d*x + c)^10 + 10*(a^7 + 6*a^6*b + 15*a^5*b^2 + 20*a^4*b^3
+ 15*a^3*b^4 + 6*a^2*b^5 + a*b^6)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^7 +
6*a^6*b + 15*a^5*b^2 + 20*a^4*b^3 + 15*a^3*b^4 + 6*a^2*b^5 + a*b^6)*d*sinh(
d*x + c)^10 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)
*d*cosh(d*x + c)^8 + (45*(a^7 + 6*a^6*b + 15*a^5*b^2 + 20*a^4*b^3 + 15*a^3*
b^4 + 6*a^2*b^5 + a*b^6)*d*cosh(d*x + c)^2 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 -
5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*d)*sinh(d*x + c)^8 + 2*(3*a^7 + 10*a^6*b +
13*a^5*b^2 + 12*a^4*b^3 + 13*a^3*b^4 + 10*a^2*b^5 + 3*a*b^6)*d*cosh(d*x + c
)^6 + 8*(15*(a^7 + 6*a^6*b + 15*a^5*b^2 + 20*a^4*b^3 + 15*a^3*b^4 + 6*a^2*b
^5 + a*b^6)*d*cosh(d*x + c)^3 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 -

```
4*a^2*b^5 - a*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^7 + 6*a^6*b
 + 15*a^5*b^2 + 20*a^4*b^3 + 15*a^3*b^4 + 6*a^2*b^5 + a*b^6)*d*cosh(d*x + c
)^4 + 56*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*d*cosh
(d*x + c)^2 + (3*a^7 + 10*a^6*b + 13*a^5*b^2 + 12*a^4*b^3 + 13*a^3*b^4 + 10
*a^2*b^5 + 3*a*b^6)*d)*sinh(d*x + c)^6 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a
^3*b^4 - 4*a^2*b^5 - a*b^6)*d*cosh(d*x + c)^4 + 4*(63*(a^7 + 6*a^6*b + 15*a
^5*b^2 + 20*a^4*b^3 + 15*a^3*b^4 + 6*a^2*b^5 + a*b^6)*d*cosh(d*x + c)^5 + 5
6*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*d*cosh(d*x +
c)^3 + 3*(3*a^7 + 10*a^6*b + 13*a^5*b^2 + 12*a^4*b^3 + 13*a^3*b^4 + 10*a^2*
b^5 + 3*a*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^7 + 6*a^6*b + 1
5*a^5*b^2 + 20*a^4*b^3 + 15*a^3*b^4 + 6*a^2*b^5 + a*b^6)*d*cosh(d*x + c)^6
+ 140*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*d*cosh(d*
x + c)^4 + 15*(3*a^7 + 10*a^6*b + 13*a^5*b^2 + 12*a^4*b^3 + 13*a^3*b^4 + 10
*a^2*b^5 + 3*a*b^6)*d*cosh(d*x + c)^2 + 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^
3*b^4 - 4*a^2*b^5 - a*b^6)*d)*sinh(d*x + c)^4 + (a^7 + 6*a^6*b + 15*a^5*b^2
 + 20*a^4*b^3 + 15*a^3*b^4 + 6*a^2*b^5 + a*b^6)*d*cosh(d*x + c)^2 + 8*(15*(
a^7 + 6*a^6*b + 15*a^5*b^2 + 20*a^4*b^3 + 15*a^3*b^4 + 6*a^2*b^5 + a*b^6)*d
*cosh(d*x + c)^7 + 28*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 -
a*b^6)*d*cosh(d*x + c)^5 + 5*(3*a^7 + 10*a^6*b + 13*a^5*b^2 + 12*a^4*b^3 +
13*a^3*b^4 + 10*a^2*b^5 + 3*a*b^6)*d*cosh(d*x + c)^3 + 2*(a^7 + 4*a^6*b + 5
*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^3
+ (45*(a^7 + 6*a^6*b + 15*a^5*b^2 + 20*a^4*b^3 + 15*a^3*b^4 + 6*a^2*b^5 + a
*b^6)*d*cosh(d*x + c)^8 + 112*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^
2*b^5 - a*b^6)*d*cosh(d*x + c)^6 + 30*(3*a^7 + 10*a^6*b + 13*a^5*b^2 + 12*a
^4*b^3 + 13*a^3*b^4 + 10*a^2*b^5 + 3*a*b^6)*d*cosh(d*x + c)^4 + 24*(a^7 + 4
*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*d*cosh(d*x + c)^2 + (a^
7 + 6*a^6*b + 15*a^5*b^2 + 20*a^4*b^3 + 15*a^3*b^4 + 6*a^2*b^5 + a*b^6)*d)*
sinh(d*x + c)^2 + 2*(5*(a^7 + 6*a^6*b + 15*a^5*b^2 + 20*a^4*b^3 + 15*a^3*b^
4 + 6*a^2*b^5 + a*b^6)*d*cosh(d*x + c)^9 + 16*(a^7 + 4*a^6*b + 5*a^5*b^2 -
5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*d*cosh(d*x + c)^7 + 6*(3*a^7 + 10*a^6*b + 13
*a^5*b^2 + 12*a^4*b^3 + 13*a^3*b^4 + 10*a^2*b^5 + 3*a*b^6)*d*cosh(d*x + c)^
5 + 8*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*d*cosh(d*
x + c)^3 + (a^7 + 6*a^6*b + 15*a^5*b^2 + 20*a^4*b^3 + 15*a^3*b^4 + 6*a^2*b^
5 + a*b^6)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.10308, size = 788, normalized size = 4.26

$$-\frac{\frac{4\,(a-5\,b)dx}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{\left(15\,a^2be^{(2\,c)}-10\,ab^2e^{(2\,c)}-b^3e^{(2\,c)}\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)e^{(-2\,c)}}{\left(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4\right)\sqrt{ab}}-\frac{\left(2\,ae^{(2\,dx+2\,c)}-10\,be^{(2\,dx+2\,c)}-a-b\right)e^{(-2\,d}}{a^4e^{(2\,c)}+4\,a^3be^{(2\,c)}+6\,a^2b^2e^{(2\,c)}+4\,ab^3e^{(2\,c)}+b}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(4*(a - 5*b)*d*x/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + (15*a^2
*b*e^(2*c) - 10*a*b^2*e^(2*c) - b^3*e^(2*c))*arctan(1/2*(a*e^(2*d*x + 2*c)
+ b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))*e^(-2*c)/((a^5 + 4*a^4*b + 6*a^3*b^
2 + 4*a^2*b^3 + a*b^4)*sqrt(a*b)) - (2*a*e^(2*d*x + 2*c) - 10*b*e^(2*d*x +
2*c) - a - b)*e^(-2*d*x)/(a^4*e^(2*c) + 4*a^3*b*e^(2*c) + 6*a^2*b^2*e^(2*c)
 + 4*a*b^3*e^(2*c) + b^4*e^(2*c)) - e^(2*d*x + 16*c)/(a^3*e^(14*c) + 3*a^2*
b*e^(14*c) + 3*a*b^2*e^(14*c) + b^3*e^(14*c)) - 2*(9*a^3*b*e^(6*d*x + 6*c)
- 5*a^2*b^2*e^(6*d*x + 6*c) - 13*a*b^3*e^(6*d*x + 6*c) + b^4*e^(6*d*x + 6*c
) + 27*a^3*b*e^(4*d*x + 4*c) - 21*a^2*b^2*e^(4*d*x + 4*c) + 29*a*b^3*e^(4*d
*x + 4*c) - 3*b^4*e^(4*d*x + 4*c) + 27*a^3*b*e^(2*d*x + 2*c) + a^2*b^2*e^(2
*d*x + 2*c) - 23*a*b^3*e^(2*d*x + 2*c) + 3*b^4*e^(2*d*x + 2*c) + 9*a^3*b +
17*a^2*b^2 + 7*a*b^3 - b^4)/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4
)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d
*x + 2*c) + a + b)^2))/d
```

3.44 $\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=126

$$\frac{15\cosh(c+dx)}{8d(a+b)^3}-\frac{5\cosh(c+dx)}{8d(a+b)^2\left(a-b\text{sech}^2(c+dx)+b\right)}-\frac{\cosh(c+dx)}{4d(a+b)\left(a-b\text{sech}^2(c+dx)+b\right)^2}-\frac{15\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\text{sech}(}{\sqrt{a+}}\right.}{8d(a+b)^{7/2}}$$

```
[Out] (-15*Sqrt[b]*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(8*(a + b)^(7/2)
*d) + (15*Cosh[c + d*x])/(8*(a + b)^3*d) - Cosh[c + d*x]/(4*(a + b)*d*(a +
b - b*Sech[c + d*x]^2)^2) - (5*Cosh[c + d*x])/(8*(a + b)^2*d*(a + b - b*Sec
h[c + d*x]^2))
```

Rubi [A] time = 0.0966862, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3664, 290, 325, 208}

$$\frac{15\cosh(c+dx)}{8d(a+b)^3}-\frac{5\cosh(c+dx)}{8d(a+b)^2\left(a-b\text{sech}^2(c+dx)+b\right)}-\frac{\cosh(c+dx)}{4d(a+b)\left(a-b\text{sech}^2(c+dx)+b\right)^2}-\frac{15\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\text{sech}(}{\sqrt{a+}}\right.}{8d(a+b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```


```
[Out] (-15*Sqrt[b]*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(8*(a + b)^(7/2)
*d) + (15*Cosh[c + d*x])/(8*(a + b)^3*d) - Cosh[c + d*x]/(4*(a + b)*d*(a +
b - b*Sech[c + d*x]^2)^2) - (5*Cosh[c + d*x])/(8*(a + b)^2*d*(a + b - b*Sec
h[c + d*x]^2))
```


Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+b-bx^2\right)^3}\,dx, x, \text{sech}(c+dx)\right)}{d}$$

$$= -\frac{\cosh(c+dx)}{4(a+b)d\left(a+b-b\text{sech}^2(c+dx)\right)^2} - \frac{5\,\text{Subst}\left(\int \frac{1}{x^2\left(a+b-bx^2\right)^2}\,dx, x, \text{sech}(c+dx)\right)}{4(a+b)d}$$

$$= -\frac{\cosh(c+dx)}{4(a+b)d\left(a+b-b\text{sech}^2(c+dx)\right)^2} - \frac{5\cosh(c+dx)}{8(a+b)^2d\left(a+b-b\text{sech}^2(c+dx)\right)} - \frac{15\,\text{Subst}}{}$$

$$= \frac{15\cosh(c+dx)}{8(a+b)^3d} - \frac{\cosh(c+dx)}{4(a+b)d\left(a+b-b\text{sech}^2(c+dx)\right)^2} - \frac{5\cosh(c+dx)}{8(a+b)^2d\left(a+b-b\text{sech}^2(c+\right.}$$

$$= -\frac{15\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{8(a+b)^{7/2}d} + \frac{15\cosh(c+dx)}{8(a+b)^3d} - \frac{\cosh(c+dx)}{4(a+b)d\left(a+b-b\text{sech}^2(c+dx)\right)^2} -$$

Mathematica [C] time = 1.73717, size = 157, normalized size = 1.25

$$\frac{\frac{2\cosh(c+dx)\left(-\frac{4b^2}{((a+b)\cosh(2(c+dx))+a-b)^2}-\frac{9b}{(a+b)\cosh(2(c+dx))+a-b}+4\right)}{(a+b)^3}-\frac{15i\sqrt{b}\left(\tan^{-1}\left(\frac{-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)-i\sqrt{a+b}}{\sqrt{b}}\right)+\tan^{-1}\left(\frac{\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)-i\sqrt{a+b}}{\sqrt{b}}\right)\right)}{(a+b)^{7/2}}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (((-15*I)*Sqrt[b]*(ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2])/Sq
rt[b]] + ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]]))/(
a + b)^(7/2) + (2*Cosh[c + d*x]*(4 - (4*b^2)/(a - b + (a + b)*Cosh[2*(c + d
*x)])^2 - (9*b)/(a - b + (a + b)*Cosh[2*(c + d*x)])))/(a + b)^3)/(8*d)
```

Maple [B] time = 0.089, size = 252, normalized size = 2.

$$\frac{1}{d}\left(\frac{1}{(a+b)^3}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}+2\frac{b}{(a+b)^3}\left(\frac{1}{\left((\tanh(1/2\,dx+c/2))^4a+2\,(\tanh(1/2\,dx+c/2))^2a+4\,(\tanh(1/2\,dx\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(1/(a+b)^3/(tanh(1/2*d*x+1/2*c)+1)+2*b/(a+b)^3*((-1/8*(9*a^2+24*a*b+8*b
^2)/a*tanh(1/2*d*x+1/2*c)^6-1/8/a^2*(27*a^3+78*a^2*b+88*a*b^2+16*b^3)*tanh(
```

```
1/2*d*x+1/2*c)^4-1/8*(27*a^2+56*a*b+8*b^2)/a*tanh(1/2*d*x+1/2*c)^2-9/8*a-1/
4*b)/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*
c)^2*b+a)^2-15/16/(a*b+b^2)^(1/2)*arctanh(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*
a+4*b)/(a*b+b^2)^(1/2)))-1/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*a^2 + 4*a*b + 2*b^2 + 2*(a^2*e^(10*c) + 2*a*b*e^(10*c) + b^2*e^(10*c
))*e^(10*d*x) + 5*(2*a^2*e^(8*c) - a*b*e^(8*c) - 3*b^2*e^(8*c))*e^(8*d*x) +
 5*(4*a^2*e^(6*c) - 7*a*b*e^(6*c) + b^2*e^(6*c))*e^(6*d*x) + 5*(4*a^2*e^(4*
c) - 7*a*b*e^(4*c) + b^2*e^(4*c))*e^(4*d*x) + 5*(2*a^2*e^(2*c) - a*b*e^(2*c
) - 3*b^2*e^(2*c))*e^(2*d*x))/((a^5*d*e^(9*c) + 5*a^4*b*d*e^(9*c) + 10*a^3*
b^2*d*e^(9*c) + 10*a^2*b^3*d*e^(9*c) + 5*a*b^4*d*e^(9*c) + b^5*d*e^(9*c))*e
^(9*d*x) + 4*(a^5*d*e^(7*c) + 3*a^4*b*d*e^(7*c) + 2*a^3*b^2*d*e^(7*c) - 2*a
^2*b^3*d*e^(7*c) - 3*a*b^4*d*e^(7*c) - b^5*d*e^(7*c))*e^(7*d*x) + 2*(3*a^5*
d*e^(5*c) + 7*a^4*b*d*e^(5*c) + 6*a^3*b^2*d*e^(5*c) + 6*a^2*b^3*d*e^(5*c) +
 7*a*b^4*d*e^(5*c) + 3*b^5*d*e^(5*c))*e^(5*d*x) + 4*(a^5*d*e^(3*c) + 3*a^4*
b*d*e^(3*c) + 2*a^3*b^2*d*e^(3*c) - 2*a^2*b^3*d*e^(3*c) - 3*a*b^4*d*e^(3*c)
 - b^5*d*e^(3*c))*e^(3*d*x) + (a^5*d*e^c + 5*a^4*b*d*e^c + 10*a^3*b^2*d*e^c
 + 10*a^2*b^3*d*e^c + 5*a*b^4*d*e^c + b^5*d*e^c)*e^(d*x)) + 1/2*integrate(1
5/2*(b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^
3 + b^4 + (a^4*e^(4*c) + 4*a^3*b*e^(4*c) + 6*a^2*b^2*e^(4*c) + 4*a*b^3*e^(4
*c) + b^4*e^(4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) + 2*a^3*b*e^(2*c) - 2*a*b^3*e
^(2*c) - b^4*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 3.08356, size = 17194, normalized size = 136.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^10 + 80*(a^2 + 2*a*b + b^2)*cosh
(d*x + c)*sinh(d*x + c)^9 + 8*(a^2 + 2*a*b + b^2)*sinh(d*x + c)^10 + 20*(2*
a^2 - a*b - 3*b^2)*cosh(d*x + c)^8 + 20*(18*(a^2 + 2*a*b + b^2)*cosh(d*x +
c)^2 + 2*a^2 - a*b - 3*b^2)*sinh(d*x + c)^8 + 160*(6*(a^2 + 2*a*b + b^2)*co
sh(d*x + c)^3 + (2*a^2 - a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 20*(
4*a^2 - 7*a*b + b^2)*cosh(d*x + c)^6 + 20*(84*(a^2 + 2*a*b + b^2)*cosh(d*x
+ c)^4 + 28*(2*a^2 - a*b - 3*b^2)*cosh(d*x + c)^2 + 4*a^2 - 7*a*b + b^2)*si
nh(d*x + c)^6 + 8*(252*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 140*(2*a^2 - a
*b - 3*b^2)*cosh(d*x + c)^3 + 15*(4*a^2 - 7*a*b + b^2)*cosh(d*x + c))*sinh(
d*x + c)^5 + 20*(4*a^2 - 7*a*b + b^2)*cosh(d*x + c)^4 + 20*(84*(a^2 + 2*a*b
 + b^2)*cosh(d*x + c)^6 + 70*(2*a^2 - a*b - 3*b^2)*cosh(d*x + c)^4 + 15*(4*
a^2 - 7*a*b + b^2)*cosh(d*x + c)^2 + 4*a^2 - 7*a*b + b^2)*sinh(d*x + c)^4 +
 80*(12*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 14*(2*a^2 - a*b - 3*b^2)*cosh
(d*x + c)^5 + 5*(4*a^2 - 7*a*b + b^2)*cosh(d*x + c)^3 + (4*a^2 - 7*a*b + b^
2)*cosh(d*x + c))*sinh(d*x + c)^3 + 20*(2*a^2 - a*b - 3*b^2)*cosh(d*x + c)^
2 + 20*(18*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 28*(2*a^2 - a*b - 3*b^2)*c
```

```
osh(d*x + c)^6 + 15*(4*a^2 - 7*a*b + b^2)*cosh(d*x + c)^4 + 6*(4*a^2 - 7*a*
b + b^2)*cosh(d*x + c)^2 + 2*a^2 - a*b - 3*b^2)*sinh(d*x + c)^2 + 15*((a^2
+ 2*a*b + b^2)*cosh(d*x + c)^9 + 9*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d
*x + c)^8 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^9 + 4*(a^2 - b^2)*cosh(d*x +
c)^7 + 4*(9*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^
7 + 28*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*
sinh(d*x + c)^6 + 2*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^5 + 2*(63*(a^2 +
2*a*b + b^2)*cosh(d*x + c)^4 + 42*(a^2 - b^2)*cosh(d*x + c)^2 + 3*a^2 - 2*a
*b + 3*b^2)*sinh(d*x + c)^5 + 2*(63*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 7
0*(a^2 - b^2)*cosh(d*x + c)^3 + 5*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c))*si
nh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d*x + c)^3 + 4*(21*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)^6 + 35*(a^2 - b^2)*cosh(d*x + c)^4 + 5*(3*a^2 - 2*a*b + 3*b^2)
*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^3 + 4*(9*(a^2 + 2*a*b + b^2)*co
sh(d*x + c)^7 + 21*(a^2 - b^2)*cosh(d*x + c)^5 + 5*(3*a^2 - 2*a*b + 3*b^2)*
cosh(d*x + c)^3 + 3*(a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + (a^2 + 2*a
*b + b^2)*cosh(d*x + c) + (9*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 28*(a^2
- b^2)*cosh(d*x + c)^6 + 10*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^4 + 12*(a
^2 - b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c))*sqrt(b/(a + b
))*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 +
 (a + b)*sinh(d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(
d*x + c)^2 + a + 3*b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + 3
*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*c
osh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a + b)*cosh(d*x +
 c) + (3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))*sqrt(b/(a + b)) +
a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 +
 (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*
x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*co
sh(d*x + c))*sinh(d*x + c) + a + b)) + 8*a^2 + 16*a*b + 8*b^2 + 40*(2*(a^2
+ 2*a*b + b^2)*cosh(d*x + c)^9 + 4*(2*a^2 - a*b - 3*b^2)*cosh(d*x + c)^7 +
3*(4*a^2 - 7*a*b + b^2)*cosh(d*x + c)^5 + 2*(4*a^2 - 7*a*b + b^2)*cosh(d*x
+ c)^3 + (2*a^2 - a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^5 + 5*a^4*
b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^9 + 9*(a^5 + 5
*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)*sinh(d*x
+ c)^8 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*sinh(d
*x + c)^9 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cos
h(d*x + c)^7 + 4*(9*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^
5)*d*cosh(d*x + c)^2 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b
^5)*d)*sinh(d*x + c)^7 + 2*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b
^4 + 3*b^5)*d*cosh(d*x + c)^5 + 28*(3*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*
b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^3 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2
*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c))*sinh(d*x + c)^6 + 2*(63*(a^5 + 5*a^4
*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^4 + 42*(a^5 +
 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^2 + (3*a^
5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d)*sinh(d*x + c)^5 +
 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^
3 + 2*(63*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(
d*x + c)^5 + 70*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*c
osh(d*x + c)^3 + 5*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b
^5)*d*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(21*(a^5 + 5*a^4*b + 10*a^3*b^2 +
10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^6 + 35*(a^5 + 3*a^4*b + 2*a^3*b
^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^4 + 5*(3*a^5 + 7*a^4*b + 6*
a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d*cosh(d*x + c)^2 + (a^5 + 3*a^4*b +
 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d)*sinh(d*x + c)^3 + (a^5 + 5*a^4*b
 + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c) + 4*(9*(a^5 + 5
*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^7 + 21*(a
^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^5 + 5
*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d*cosh(d*x + c
)^3 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x
+ c))*sinh(d*x + c)^2 + (9*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b
```

```
^4 + b^5)*d*cosh(d*x + c)^8 + 28*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3
*a*b^4 - b^5)*d*cosh(d*x + c)^6 + 10*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b
^3 + 7*a*b^4 + 3*b^5)*d*cosh(d*x + c)^4 + 12*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2
*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^2 + (a^5 + 5*a^4*b + 10*a^3*b^2 +
 10*a^2*b^3 + 5*a*b^4 + b^5)*d)*sinh(d*x + c)), 1/8*(4*(a^2 + 2*a*b + b^2)*
cosh(d*x + c)^10 + 40*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^9 + 4
*(a^2 + 2*a*b + b^2)*sinh(d*x + c)^10 + 10*(2*a^2 - a*b - 3*b^2)*cosh(d*x +
 c)^8 + 10*(18*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 2*a^2 - a*b - 3*b^2)*s
inh(d*x + c)^8 + 80*(6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (2*a^2 - a*b -
 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 10*(4*a^2 - 7*a*b + b^2)*cosh(d*x
+ c)^6 + 10*(84*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 28*(2*a^2 - a*b - 3*b
^2)*cosh(d*x + c)^2 + 4*a^2 - 7*a*b + b^2)*sinh(d*x + c)^6 + 4*(252*(a^2 +
2*a*b + b^2)*cosh(d*x + c)^5 + 140*(2*a^2 - a*b - 3*b^2)*cosh(d*x + c)^3 +
15*(4*a^2 - 7*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 10*(4*a^2 - 7*a*b
 + b^2)*cosh(d*x + c)^4 + 10*(84*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 70*(
2*a^2 - a*b - 3*b^2)*cosh(d*x + c)^4 + 15*(4*a^2 - 7*a*b + b^2)*cosh(d*x +
c)^2 + 4*a^2 - 7*a*b + b^2)*sinh(d*x + c)^4 + 40*(12*(a^2 + 2*a*b + b^2)*co
sh(d*x + c)^7 + 14*(2*a^2 - a*b - 3*b^2)*cosh(d*x + c)^5 + 5*(4*a^2 - 7*a*b
 + b^2)*cosh(d*x + c)^3 + (4*a^2 - 7*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c
)^3 + 10*(2*a^2 - a*b - 3*b^2)*cosh(d*x + c)^2 + 10*(18*(a^2 + 2*a*b + b^2)
*cosh(d*x + c)^8 + 28*(2*a^2 - a*b - 3*b^2)*cosh(d*x + c)^6 + 15*(4*a^2 - 7
*a*b + b^2)*cosh(d*x + c)^4 + 6*(4*a^2 - 7*a*b + b^2)*cosh(d*x + c)^2 + 2*a
^2 - a*b - 3*b^2)*sinh(d*x + c)^2 - 15*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^9
 + 9*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^8 + (a^2 + 2*a*b + b^2
)*sinh(d*x + c)^9 + 4*(a^2 - b^2)*cosh(d*x + c)^7 + 4*(9*(a^2 + 2*a*b + b^2
)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^7 + 28*(3*(a^2 + 2*a*b + b^2)*
cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c)^6 + 2*(3*a^2 - 2
*a*b + 3*b^2)*cosh(d*x + c)^5 + 2*(63*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 +
 42*(a^2 - b^2)*cosh(d*x + c)^2 + 3*a^2 - 2*a*b + 3*b^2)*sinh(d*x + c)^5 +
2*(63*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 70*(a^2 - b^2)*cosh(d*x + c)^3
+ 5*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(a^2 - b^2)*
cosh(d*x + c)^3 + 4*(21*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 35*(a^2 - b^2
)*cosh(d*x + c)^4 + 5*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^2 + a^2 - b^2)*
sinh(d*x + c)^3 + 4*(9*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 21*(a^2 - b^2)
*cosh(d*x + c)^5 + 5*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^3 + 3*(a^2 - b^2
)*cosh(d*x + c))*sinh(d*x + c)^2 + (a^2 + 2*a*b + b^2)*cosh(d*x + c) + (9*(
a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 28*(a^2 - b^2)*cosh(d*x + c)^6 + 10*(3
*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^4 + 12*(a^2 - b^2)*cosh(d*x + c)^2 + a^
2 + 2*a*b + b^2)*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2*((a + b)*cosh(d
*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)
^3 + (a - 3*b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a - 3*b)*sinh(d
*x + c))*sqrt(-b/(a + b))/b) + 15*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^9 + 9*
(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^8 + (a^2 + 2*a*b + b^2)*sin
h(d*x + c)^9 + 4*(a^2 - b^2)*cosh(d*x + c)^7 + 4*(9*(a^2 + 2*a*b + b^2)*cos
h(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^7 + 28*(3*(a^2 + 2*a*b + b^2)*cosh(
d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c)^6 + 2*(3*a^2 - 2*a*b
+ 3*b^2)*cosh(d*x + c)^5 + 2*(63*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 42*(
a^2 - b^2)*cosh(d*x + c)^2 + 3*a^2 - 2*a*b + 3*b^2)*sinh(d*x + c)^5 + 2*(63
*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 70*(a^2 - b^2)*cosh(d*x + c)^3 + 5*(
3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(
d*x + c)^3 + 4*(21*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 35*(a^2 - b^2)*cos
h(d*x + c)^4 + 5*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(
d*x + c)^3 + 4*(9*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 21*(a^2 - b^2)*cosh
(d*x + c)^5 + 5*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^3 + 3*(a^2 - b^2)*cos
h(d*x + c))*sinh(d*x + c)^2 + (a^2 + 2*a*b + b^2)*cosh(d*x + c) + (9*(a^2 +
 2*a*b + b^2)*cosh(d*x + c)^8 + 28*(a^2 - b^2)*cosh(d*x + c)^6 + 10*(3*a^2
- 2*a*b + 3*b^2)*cosh(d*x + c)^4 + 12*(a^2 - b^2)*cosh(d*x + c)^2 + a^2 + 2
*a*b + b^2)*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2*((a + b)*cosh(d*x +
c) + (a + b)*sinh(d*x + c))*sqrt(-b/(a + b))/b) + 4*a^2 + 8*a*b + 4*b^2 + 2
```

```
0*(2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^9 + 4*(2*a^2 - a*b - 3*b^2)*cosh(d*x
 + c)^7 + 3*(4*a^2 - 7*a*b + b^2)*cosh(d*x + c)^5 + 2*(4*a^2 - 7*a*b + b^2)
*cosh(d*x + c)^3 + (2*a^2 - a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^
5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^9 +
9*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)
*sinh(d*x + c)^8 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5
)*d*sinh(d*x + c)^9 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 -
b^5)*d*cosh(d*x + c)^7 + 4*(9*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*
a*b^4 + b^5)*d*cosh(d*x + c)^2 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3
*a*b^4 - b^5)*d)*sinh(d*x + c)^7 + 2*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b
^3 + 7*a*b^4 + 3*b^5)*d*cosh(d*x + c)^5 + 28*(3*(a^5 + 5*a^4*b + 10*a^3*b^2
 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^3 + (a^5 + 3*a^4*b + 2*a^3*b
^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c))*sinh(d*x + c)^6 + 2*(63*(a
^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^4 +
 42*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)
^2 + (3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d)*sinh(d*
x + c)^5 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh
(d*x + c)^3 + 2*(63*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^
5)*d*cosh(d*x + c)^5 + 70*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4
- b^5)*d*cosh(d*x + c)^3 + 5*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a
*b^4 + 3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(21*(a^5 + 5*a^4*b + 10*
a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^6 + 35*(a^5 + 3*a^4*b
 + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^4 + 5*(3*a^5 + 7*
a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d*cosh(d*x + c)^2 + (a^5 +
 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d)*sinh(d*x + c)^3 + (a^5
 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c) + 4*(
9*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)
^7 + 21*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x
+ c)^5 + 5*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d*co
sh(d*x + c)^3 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d
*cosh(d*x + c))*sinh(d*x + c)^2 + (9*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b
^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^8 + 28*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a
^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^6 + 10*(3*a^5 + 7*a^4*b + 6*a^3*b^2
 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d*cosh(d*x + c)^4 + 12*(a^5 + 3*a^4*b + 2*a
^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^2 + (a^5 + 5*a^4*b + 10
*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d)*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 2.0028, size = 8412, normalized size = 66.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/32*(30*(3*(a^4*b*e^(4*c) + 4*a^3*b^2*e^(4*c) + 6*a^2*b^3*e^(4*c) + 4*a*b^
4*e^(4*c) + b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))
^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(
arccos(-a/(a + b) + b/(a + b)))) - (a^4*b*e^(4*c) + 4*a^3*b^2*e^(4*c) + 6*a
^2*b^3*e^(4*c) + 4*a*b^4*e^(4*c) + b^5*e^(4*c))*cosh(1/2*imag_part(arccos(-
a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))
))^3 - 9*(a^4*b*e^(4*c) + 4*a^3*b^2*e^(4*c) + 6*a^2*b^3*e^(4*c) + 4*a*b^4*e
^(4*c) + b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*
cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arc
cos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a +
 b)))) + 3*(a^4*b*e^(4*c) + 4*a^3*b^2*e^(4*c) + 6*a^2*b^3*e^(4*c) + 4*a*b^4
*e^(4*c) + b^5*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))
^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(
arccos(-a/(a + b) + b/(a + b)))) + 9*(a^4*b*e^(4*c) + 4*a^3*b^2*e^(4*c) + 6
*a^2*b^3*e^(4*c) + 4*a*b^4*e^(4*c) + b^5*e^(4*c))*cos(1/2*real_part(arccos(
-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b
))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(
arccos(-a/(a + b) + b/(a + b))))^2 - 3*(a^4*b*e^(4*c) + 4*a^3*b^2*e^(4*c) +
 6*a^2*b^3*e^(4*c) + 4*a*b^4*e^(4*c) + b^5*e^(4*c))*cosh(1/2*imag_part(arcc
os(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b
))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(a^4*b*e^(
4*c) + 4*a^3*b^2*e^(4*c) + 6*a^2*b^3*e^(4*c) + 4*a*b^4*e^(4*c) + b^5*e^(4*c
))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(a
rccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a
 + b))))^3 + (a^4*b*e^(4*c) + 4*a^3*b^2*e^(4*c) + 6*a^2*b^3*e^(4*c) + 4*a*b
^4*e^(4*c) + b^5*e^(4*c))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))
)^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 - (a^4*b*e^(4*c)
+ 4*a^3*b^2*e^(4*c) + 6*a^2*b^3*e^(4*c) + 4*a*b^4*e^(4*c) + b^5*e^(4*c))*co
sh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(
-a/(a + b) + b/(a + b)))) + (a^4*b*e^(4*c) + 4*a^3*b^2*e^(4*c) + 6*a^2*b^3*
e^(4*c) + 4*a*b^4*e^(4*c) + b^5*e^(4*c))*sin(1/2*real_part(arccos(-a/(a + b
) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*arcta
n((((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)/(a^4*e^(4*c) + 4*a^3*b*e^(4
*c) + 6*a^2*b^2*e^(4*c) + 4*a*b^3*e^(4*c) + b^4*e^(4*c)))^(1/4)*cos(1/2*arc
cos(-(a - b)/(a + b))) + e^(d*x))/(((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 +
b^4)/(a^4*e^(4*c) + 4*a^3*b*e^(4*c) + 6*a^2*b^2*e^(4*c) + 4*a*b^3*e^(4*c) +
 b^4*e^(4*c)))^(1/4)*sin(1/2*arccos(-(a - b)/(a + b)))))/(2*(a^3*e^(2*c) +
3*a^2*b*e^(2*c) + 3*a*b^2*e^(2*c) + b^3*e^(2*c))^2*a*b + (a^4*e^(2*c) + 2*a
^3*b*e^(2*c) - 2*a*b^3*e^(2*c) - b^4*e^(2*c))*sqrt(-a*b)*abs(-a^3*e^(2*c) -
 3*a^2*b*e^(2*c) - 3*a*b^2*e^(2*c) - b^3*e^(2*c))) + 30*(3*(a^4*b*e^(4*c) +
 4*a^3*b^2*e^(4*c) + 6*a^2*b^3*e^(4*c) + 4*a*b^4*e^(4*c) + b^5*e^(4*c))*cos
(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos
(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b
)))) - (a^4*b*e^(4*c) + 4*a^3*b^2*e^(4*c) + 6*a^2*b^3*e^(4*c) + 4*a*b^4*e^(
4*c) + b^5*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*s
in(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3 - 9*(a^4*b*e^(4*c) + 4*
a^3*b^2*e^(4*c) + 6*a^2*b^3*e^(4*c) + 4*a*b^4*e^(4*c) + b^5*e^(4*c))*cos(1/
2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a
/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))
)*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(a^4*b*e^(4*c) +
4*a^3*b^2*e^(4*c) + 6*a^2*b^3*e^(4*c) + 4*a*b^4*e^(4*c) + b^5*e^(4*c))*cosh
(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(
-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b
)))) + 9*(a^4*b*e^(4*c) + 4*a^3*b^2*e^(4*c) + 6*a^2*b^3*e^(4*c) + 4*a*b^4*e
^(4*c) + b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*
cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arcco
s(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b
))))^2 - 3*(a^4*b*e^(4*c) + 4*a^3*b^2*e^(4*c) + 6*a^2*b^3*e^(4*c) + 4*a*b^4
*e^(4*c) + b^5*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))
```

```
*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(ar
ccos(-a/(a + b) + b/(a + b))))^2 - 3*(a^4*b*e^(4*c) + 4*a^3*b^2*e^(4*c) + 6
*a^2*b^3*e^(4*c) + 4*a*b^4*e^(4*c) + b^5*e^(4*c))*cos(1/2*real_part(arccos(
-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)
)))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (a^4*b*e^(4*c)
+ 4*a^3*b^2*e^(4*c) + 6*a^2*b^3*e^(4*c) + 4*a*b^4*e^(4*c) + b^5*e^(4*c))*si
n(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arcco
s(-a/(a + b) + b/(a + b))))^3 - (a^4*b*e^(4*c) + 4*a^3*b^2*e^(4*c) + 6*a^2*
b^3*e^(4*c) + 4*a*b^4*e^(4*c) + b^5*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(
a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) +
(a^4*b*e^(4*c) + 4*a^3*b^2*e^(4*c) + 6*a^2*b^3*e^(4*c) + 4*a*b^4*e^(4*c) +
b^5*e^(4*c))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*im
ag_part(arccos(-a/(a + b) + b/(a + b)))))*arctan(-(((a^4 + 4*a^3*b + 6*a^2*
b^2 + 4*a*b^3 + b^4)/(a^4*e^(4*c) + 4*a^3*b*e^(4*c) + 6*a^2*b^2*e^(4*c) + 4
*a*b^3*e^(4*c) + b^4*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b))) - e^
(d*x))/(((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)/(a^4*e^(4*c) + 4*a^3*b
*e^(4*c) + 6*a^2*b^2*e^(4*c) + 4*a*b^3*e^(4*c) + b^4*e^(4*c)))^(1/4)*sin(1/
2*arccos(-(a - b)/(a + b)))))/(2*(a^3*e^(2*c) + 3*a^2*b*e^(2*c) + 3*a*b^2*e
^(2*c) + b^3*e^(2*c))^2*a*b + (a^4*e^(2*c) + 2*a^3*b*e^(2*c) - 2*a*b^3*e^(2
*c) - b^4*e^(2*c))*sqrt(-a*b)*abs(-a^3*e^(2*c) - 3*a^2*b*e^(2*c) - 3*a*b^2*
e^(2*c) - b^3*e^(2*c))) + 15*((a^4*b*e^(4*c) + 4*a^3*b^2*e^(4*c) + 6*a^2*b^
3*e^(4*c) + 4*a*b^4*e^(4*c) + b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a +
 b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3
- 3*(a^4*b*e^(4*c) + 4*a^3*b^2*e^(4*c) + 6*a^2*b^3*e^(4*c) + 4*a*b^4*e^(4*c
) + b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/
2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/
(a + b) + b/(a + b))))^2 - 3*(a^4*b*e^(4*c) + 4*a^3*b^2*e^(4*c) + 6*a^2*b^3
*e^(4*c) + 4*a*b^4*e^(4*c) + b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a +
b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*s
inh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(a^4*b*e^(4*c) + 4*a
^3*b^2*e^(4*c) + 6*a^2*b^3*e^(4*c) + 4*a*b^4*e^(4*c) + b^5*e^(4*c))*cos(1/2
*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a
 + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2
*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(a^4*b*e^(4*c) + 4
*a^3*b^2*e^(4*c) + 6*a^2*b^3*e^(4*c) + 4*a*b^4*e^(4*c) + b^5*e^(4*c))*cos(1
/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-
a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))
)^2 - 9*(a^4*b*e^(4*c) + 4*a^3*b^2*e^(4*c) + 6*a^2*b^3*e^(4*c) + 4*a*b^4*e^
(4*c) + b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cos
h(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-
a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)
)))^2 - (a^4*b*e^(4*c) + 4*a^3*b^2*e^(4*c) + 6*a^2*b^3*e^(4*c) + 4*a*b^4*e^
(4*c) + b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*s
inh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + 3*(a^4*b*e^(4*c) + 4
*a^3*b^2*e^(4*c) + 6*a^2*b^3*e^(4*c) + 4*a*b^4*e^(4*c) + b^5*e^(4*c))*cos(1
/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(
a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))
^3 - (a^4*b*e^(4*c) + 4*a^3*b^2*e^(4*c) + 6*a^2*b^3*e^(4*c) + 4*a*b^4*e^(4*
c) + b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1
/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + (a^4*b*e^(4*c) + 4*a^3*b^2*
e^(4*c) + 6*a^2*b^3*e^(4*c) + 4*a*b^4*e^(4*c) + b^5*e^(4*c))*cos(1/2*real_p
art(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) +
 b/(a + b)))))*log(2*((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)/(a^4*e^(4
*c) + 4*a^3*b*e^(4*c) + 6*a^2*b^2*e^(4*c) + 4*a*b^3*e^(4*c) + b^4*e^(4*c)))
^(1/4)*cos(1/2*arccos(-(a - b)/(a + b)))*e^(d*x) + sqrt((a^4 + 4*a^3*b + 6*
a^2*b^2 + 4*a*b^3 + b^4)/(a^4*e^(4*c) + 4*a^3*b*e^(4*c) + 6*a^2*b^2*e^(4*c)
 + 4*a*b^3*e^(4*c) + b^4*e^(4*c))) + e^(2*d*x))/(2*(a^3*e^(2*c) + 3*a^2*b*e
^(2*c) + 3*a*b^2*e^(2*c) + b^3*e^(2*c))^2*a*b + (a^4*e^(2*c) + 2*a^3*b*e^(2
*c) - 2*a*b^3*e^(2*c) - b^4*e^(2*c))*sqrt(-a*b)*abs(-a^3*e^(2*c) - 3*a^2*b*
```

```
e^(2*c) - 3*a*b^2*e^(2*c) - b^3*e^(2*c))) - 15*((a^4*b*e^(4*c) + 4*a^3*b^2*
e^(4*c) + 6*a^2*b^3*e^(4*c) + 4*a*b^4*e^(4*c) + b^5*e^(4*c))*cos(1/2*real_p
art(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b)
 + b/(a + b))))^3 - 3*(a^4*b*e^(4*c) + 4*a^3*b^2*e^(4*c) + 6*a^2*b^3*e^(4*c
) + 4*a*b^4*e^(4*c) + b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/
(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*re
al_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(a^4*b*e^(4*c) + 4*a^3*b^2*e
^(4*c) + 6*a^2*b^3*e^(4*c) + 4*a*b^4*e^(4*c) + b^5*e^(4*c))*cos(1/2*real_pa
rt(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b)
+ b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(a
^4*b*e^(4*c) + 4*a^3*b^2*e^(4*c) + 6*a^2*b^3*e^(4*c) + 4*a*b^4*e^(4*c) + b^
5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag
_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b
) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*
(a^4*b*e^(4*c) + 4*a^3*b^2*e^(4*c) + 6*a^2*b^3*e^(4*c) + 4*a*b^4*e^(4*c) +
b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*
imag_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a
+ b) + b/(a + b))))^2 - 9*(a^4*b*e^(4*c) + 4*a^3*b^2*e^(4*c) + 6*a^2*b^3*e^
(4*c) + 4*a*b^4*e^(4*c) + b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b)
+ b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*
real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(
a + b) + b/(a + b))))^2 - (a^4*b*e^(4*c) + 4*a^3*b^2*e^(4*c) + 6*a^2*b^3*e^
(4*c) + 4*a*b^4*e^(4*c) + b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b)
+ b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + 3*
(a^4*b*e^(4*c) + 4*a^3*b^2*e^(4*c) + 6*a^2*b^3*e^(4*c) + 4*a*b^4*e^(4*c) +
b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*rea
l_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a +
 b) + b/(a + b))))^3 - (a^4*b*e^(4*c) + 4*a^3*b^2*e^(4*c) + 6*a^2*b^3*e^(4*
c) + 4*a*b^4*e^(4*c) + b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b
/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + (a^4*b*e^
(4*c) + 4*a^3*b^2*e^(4*c) + 6*a^2*b^3*e^(4*c) + 4*a*b^4*e^(4*c) + b^5*e^(4*
c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(a
rccos(-a/(a + b) + b/(a + b)))))*log(-2*((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b
^3 + b^4)/(a^4*e^(4*c) + 4*a^3*b*e^(4*c) + 6*a^2*b^2*e^(4*c) + 4*a*b^3*e^(4
*c) + b^4*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b)))*e^(d*x) + sqrt(
(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)/(a^4*e^(4*c) + 4*a^3*b*e^(4*c)
+ 6*a^2*b^2*e^(4*c) + 4*a*b^3*e^(4*c) + b^4*e^(4*c))) + e^(2*d*x))/(2*(a^3*
e^(2*c) + 3*a^2*b*e^(2*c) + 3*a*b^2*e^(2*c) + b^3*e^(2*c))^2*a*b + (a^4*e^(
2*c) + 2*a^3*b*e^(2*c) - 2*a*b^3*e^(2*c) - b^4*e^(2*c))*sqrt(-a*b)*abs(-a^3
*e^(2*c) - 3*a^2*b*e^(2*c) - 3*a*b^2*e^(2*c) - b^3*e^(2*c))) + 16*e^(d*x +
14*c)/(a^3*e^(13*c) + 3*a^2*b*e^(13*c) + 3*a*b^2*e^(13*c) + b^3*e^(13*c)) +
 16*e^(-d*x)/(a^3*e^c + 3*a^2*b*e^c + 3*a*b^2*e^c + b^3*e^c) - 8*(9*a*b*e^(
7*d*x + 7*c) + 9*b^2*e^(7*d*x + 7*c) + 27*a*b*e^(5*d*x + 5*c) - b^2*e^(5*d*
x + 5*c) + 27*a*b*e^(3*d*x + 3*c) - b^2*e^(3*d*x + 3*c) + 9*a*b*e^(d*x + c)
 + 9*b^2*e^(d*x + c))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(a*e^(4*d*x + 4*c) +
 b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2))
/d
```

3.45 $\int \frac{\text{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3} dx$

Optimal. Leaf size=156

$$\frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{8a^3d(a+b)^{5/2}}+\frac{b(7a+4b)\text{sech}(c+dx)}{8a^2d(a+b)^2\left(a-b\text{sech}^2(c+dx)+b\right)}-\frac{\tanh^{-1}(\cosh(c+dx))}{a^3d}+\frac{}{4ad(a}$$

```
[Out] -(ArcTanh[Cosh[c + d*x]]/(a^3*d)) + (Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*ArcT
anh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(8*a^3*(a + b)^(5/2)*d) + (b*Sech
[c + d*x])/(4*a*(a + b)*d*(a + b - b*Sech[c + d*x]^2)^2) + (b*(7*a + 4*b)*S
ech[c + d*x])/(8*a^2*(a + b)^2*d*(a + b - b*Sech[c + d*x]^2))
```

Rubi [A] time = 0.25385, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3664, 414, 527, 522, 207, 208}

$$\frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{8a^3d(a+b)^{5/2}}+\frac{b(7a+4b)\text{sech}(c+dx)}{8a^2d(a+b)^2\left(a-b\text{sech}^2(c+dx)+b\right)}-\frac{\tanh^{-1}(\cosh(c+dx))}{a^3d}+\frac{}{4ad(a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```


```
[Out] -(ArcTanh[Cosh[c + d*x]]/(a^3*d)) + (Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*ArcT
anh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(8*a^3*(a + b)^(5/2)*d) + (b*Sech
[c + d*x])/(4*a*(a + b)*d*(a + b - b*Sech[c + d*x]^2)^2) + (b*(7*a + 4*b)*S
ech[c + d*x])/(8*a^2*(a + b)^2*d*(a + b - b*Sech[c + d*x]^2))
```


Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
```

```
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(-1+x^2)\left(a+b-bx^2\right)^3}\,dx, x, \operatorname{sech}(c+dx)\right)}{d}$$

$$= \frac{b\operatorname{sech}(c+dx)}{4a(a+b)d\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} + \frac{\operatorname{Subst}\left(\int \frac{4a+b+3bx^2}{(-1+x^2)\left(a+b-bx^2\right)^2}\,dx, x, \operatorname{sech}(c+dx)\right)}{4a(a+b)d}$$

$$= \frac{b\operatorname{sech}(c+dx)}{4a(a+b)d\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} + \frac{b(7a+4b)\operatorname{sech}(c+dx)}{8a^2(a+b)^2d\left(a+b-b\operatorname{sech}^2(c+dx)\right)} + \frac{\operatorname{Sub}}{}$$

$$= \frac{b\operatorname{sech}(c+dx)}{4a(a+b)d\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} + \frac{b(7a+4b)\operatorname{sech}(c+dx)}{8a^2(a+b)^2d\left(a+b-b\operatorname{sech}^2(c+dx)\right)} + \frac{\operatorname{Sub}}{}$$

$$= -\frac{\tanh^{-1}(\cosh(c+dx))}{a^3d} + \frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{5/2}d} + \frac{}{4a(a+b)}$$

Mathematica [C] time = 1.28795, size = 236, normalized size = 1.51

$$\frac{\frac{8a^2b^2\cosh(c+dx)}{(a+b)^2((a+b)\cosh(2(c+dx))+a-b)^2} + \frac{i\sqrt{b}\left(15a^2+20ab+8b^2\right)\tan^{-1}\left(\frac{-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)-i\sqrt{a+b}}{\sqrt{b}}\right)}{(a+b)^{5/2}} + \frac{i\sqrt{b}\left(15a^2+20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)-i\sqrt{a+b}}{\sqrt{b}}\right)}{(a+b)^{5/2}} + }{8a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((I*Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Ta
nh[(c + d*x)/2])/Sqrt[b]])/(a + b)^(5/2) + (I*Sqrt[b]*(15*a^2 + 20*a*b + 8*
b^2)*ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]])/(a + b
```

```
)^(5/2) + (8*a^2*b^2*Cosh[c + d*x])/((a + b)^2*(a - b + (a + b)*Cosh[2*(c +
 d*x)])^2) + (2*a*b*(9*a + 4*b)*Cosh[c + d*x])/((a + b)^2*(a - b + (a + b)*
Cosh[2*(c + d*x)])) + 8*Log[Tanh[(c + d*x)/2]])/(8*a^3*d)
```

Maple [B] time = 0.104, size = 1132, normalized size = 7.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 9/4/d*b/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1
/2*c)^2*b+a)^2/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6+7/d/a*b^2/(tanh(1/2*d*
x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2+
2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6+4/d/a^2*b^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tan
h(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2+2*a*b+b^2)*tanh(1/
2*d*x+1/2*c)^6+27/4/d*b/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+
4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4+45/2/d
/a*b^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/
2*c)^2*b+a)^2/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4+30/d/a^2*b^3/(tanh(1/2*
d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^
2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4+12/d/a^3*b^4/(tanh(1/2*d*x+1/2*c)^4*a+2*
tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2+2*a*b+b^2)*tanh
(1/2*d*x+1/2*c)^4+27/4/d*b/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2
*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2+17/
d/a*b^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1
/2*c)^2*b+a)^2/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2+8/d/a^2*b^3/(tanh(1/2*
d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^
2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2+9/4/d*b/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(
1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2+2*a*b+b^2)+3/2/d/a*b
^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)
^2*b+a)^2/(a^2+2*a*b+b^2)+15/8/d/a*b/(a^2+2*a*b+b^2)/(a*b+b^2)^(1/2)*arctan
h(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2))+5/2/d/a^2*b^2/(a
^2+2*a*b+b^2)/(a*b+b^2)^(1/2)*arctanh(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*
b)/(a*b+b^2)^(1/2))+1/d/a^3*b^3/(a^2+2*a*b+b^2)/(a*b+b^2)^(1/2)*arctanh(1/4
*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2))+1/d/a^3*ln(tanh(1/2*d
*x+1/2*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((9*a^2*b*e^(7*c) + 13*a*b^2*e^(7*c) + 4*b^3*e^(7*c))*e^(7*d*x) + (27*a
^2*b*e^(5*c) + 11*a*b^2*e^(5*c) - 4*b^3*e^(5*c))*e^(5*d*x) + (27*a^2*b*e^(3
*c) + 11*a*b^2*e^(3*c) - 4*b^3*e^(3*c))*e^(3*d*x) + (9*a^2*b*e^c + 13*a*b^2
*e^c + 4*b^3*e^c)*e^(d*x))/(a^6*d + 4*a^5*b*d + 6*a^4*b^2*d + 4*a^3*b^3*d +
 a^2*b^4*d + (a^6*d*e^(8*c) + 4*a^5*b*d*e^(8*c) + 6*a^4*b^2*d*e^(8*c) + 4*a
^3*b^3*d*e^(8*c) + a^2*b^4*d*e^(8*c))*e^(8*d*x) + 4*(a^6*d*e^(6*c) + 2*a^5*
b*d*e^(6*c) - 2*a^3*b^3*d*e^(6*c) - a^2*b^4*d*e^(6*c))*e^(6*d*x) + 2*(3*a^6
```

```
*d*e^(4*c) + 4*a^5*b*d*e^(4*c) + 2*a^4*b^2*d*e^(4*c) + 4*a^3*b^3*d*e^(4*c)
+ 3*a^2*b^4*d*e^(4*c))*e^(4*d*x) + 4*(a^6*d*e^(2*c) + 2*a^5*b*d*e^(2*c) - 2
*a^3*b^3*d*e^(2*c) - a^2*b^4*d*e^(2*c))*e^(2*d*x)) - log((e^(d*x + c) + 1)*
e^(-c))/(a^3*d) + log((e^(d*x + c) - 1)*e^(-c))/(a^3*d) - 2*integrate(1/8*(
(15*a^2*b*e^(3*c) + 20*a*b^2*e^(3*c) + 8*b^3*e^(3*c))*e^(3*d*x) - (15*a^2*b
*e^c + 20*a*b^2*e^c + 8*b^3*e^c)*e^(d*x))/(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*
b^3 + (a^6*e^(4*c) + 3*a^5*b*e^(4*c) + 3*a^4*b^2*e^(4*c) + a^3*b^3*e^(4*c))
*e^(4*d*x) + 2*(a^6*e^(2*c) + a^5*b*e^(2*c) - a^4*b^2*e^(2*c) - a^3*b^3*e^(
2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 4.35789, size = 25141, normalized size = 161.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(9*a^3*b + 13*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^7 + 28*(9*a^3*b + 1
3*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + 4*(9*a^3*b + 13*a^2*b^
2 + 4*a*b^3)*sinh(d*x + c)^7 + 4*(27*a^3*b + 11*a^2*b^2 - 4*a*b^3)*cosh(d*x
 + c)^5 + 4*(27*a^3*b + 11*a^2*b^2 - 4*a*b^3 + 21*(9*a^3*b + 13*a^2*b^2 + 4
*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(7*(9*a^3*b + 13*a^2*b^2 + 4*
a*b^3)*cosh(d*x + c)^3 + (27*a^3*b + 11*a^2*b^2 - 4*a*b^3)*cosh(d*x + c))*s
inh(d*x + c)^4 + 4*(27*a^3*b + 11*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^3 + 4*(3
5*(9*a^3*b + 13*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^4 + 27*a^3*b + 11*a^2*b^2
- 4*a*b^3 + 10*(27*a^3*b + 11*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^2)*sinh(d*x
+ c)^3 + 4*(21*(9*a^3*b + 13*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^5 + 10*(27*a^
3*b + 11*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^3 + 3*(27*a^3*b + 11*a^2*b^2 - 4*
a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + ((15*a^4 + 50*a^3*b + 63*a^2*b^2 +
36*a*b^3 + 8*b^4)*cosh(d*x + c)^8 + 8*(15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*
a*b^3 + 8*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (15*a^4 + 50*a^3*b + 63*a^2*
b^2 + 36*a*b^3 + 8*b^4)*sinh(d*x + c)^8 + 4*(15*a^4 + 20*a^3*b - 7*a^2*b^2
- 20*a*b^3 - 8*b^4)*cosh(d*x + c)^6 + 4*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20
*a*b^3 - 8*b^4 + 7*(15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*cosh
(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*
b^3 + 8*b^4)*cosh(d*x + c)^3 + 3*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3
- 8*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(45*a^4 + 30*a^3*b + 29*a^2*b^2
 + 44*a*b^3 + 24*b^4)*cosh(d*x + c)^4 + 2*(35*(15*a^4 + 50*a^3*b + 63*a^2*b
^2 + 36*a*b^3 + 8*b^4)*cosh(d*x + c)^4 + 45*a^4 + 30*a^3*b + 29*a^2*b^2 + 4
4*a*b^3 + 24*b^4 + 30*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*co
sh(d*x + c)^2)*sinh(d*x + c)^4 + 15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3
+ 8*b^4 + 8*(7*(15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*cosh(d*x
 + c)^5 + 10*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*cosh(d*x +
c)^3 + (45*a^4 + 30*a^3*b + 29*a^2*b^2 + 44*a*b^3 + 24*b^4)*cosh(d*x + c))*
sinh(d*x + c)^3 + 4*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*cosh
(d*x + c)^2 + 4*(7*(15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*cosh
(d*x + c)^6 + 15*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*cosh(d*
x + c)^4 + 15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4 + 3*(45*a^4 + 3
0*a^3*b + 29*a^2*b^2 + 44*a*b^3 + 24*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2
+ 8*((15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*cosh(d*x + c)^7 +
3*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*cosh(d*x + c)^5 + (45*
a^4 + 30*a^3*b + 29*a^2*b^2 + 44*a*b^3 + 24*b^4)*cosh(d*x + c)^3 + (15*a^4
+ 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*cosh(d*x + c))*sinh(d*x + c))*sq
rt(b/(a + b))*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d
*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(a
 + b)*cosh(d*x + c)^2 + a + 3*b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)
```

```
^3 + (a + 3*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a + b)*cosh(d*x + c)^3 +
3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a + b)
*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))*sqrt(b/
(a + b)) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d
*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a +
 b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 +
 (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 4*(9*a^3*b + 13*a^2*b^2 +
 4*a*b^3)*cosh(d*x + c) - 16*((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*c
osh(d*x + c)^8 + 8*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c
)*sinh(d*x + c)^7 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sinh(d*x +
c)^8 + 4*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c)^6 + 4*(a^4 + 2*a^3*b
 - 2*a*b^3 - b^4 + 7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x +
 c)^2)*sinh(d*x + c)^6 + 8*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*c
osh(d*x + c)^3 + 3*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x
+ c)^5 + 2*(3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^4
+ 2*(35*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^4 + 3*a^4
 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4 + 30*(a^4 + 2*a^3*b - 2*a*b^3 - b^
4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 +
 b^4 + 8*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^5 + 1
0*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c)^3 + (3*a^4 + 4*a^3*b + 2*a^
2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^4 + 2*a^3*b
 - 2*a*b^3 - b^4)*cosh(d*x + c)^2 + 4*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^
3 + b^4)*cosh(d*x + c)^6 + 15*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c)
^4 + a^4 + 2*a^3*b - 2*a*b^3 - b^4 + 3*(3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b
^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^4 + 4*a^3*b + 6*a^2*b^
2 + 4*a*b^3 + b^4)*cosh(d*x + c)^7 + 3*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh
(d*x + c)^5 + (3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)
^3 + (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c))*log(cosh
(d*x + c) + sinh(d*x + c) + 1) + 16*((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 +
 b^4)*cosh(d*x + c)^8 + 8*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(
d*x + c)*sinh(d*x + c)^7 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sinh
(d*x + c)^8 + 4*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c)^6 + 4*(a^4 +
2*a^3*b - 2*a*b^3 - b^4 + 7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cos
h(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 +
 b^4)*cosh(d*x + c)^3 + 3*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c))*si
nh(d*x + c)^5 + 2*(3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x
+ c)^4 + 2*(35*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^4
+ 3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4 + 30*(a^4 + 2*a^3*b - 2*a*b
^3 - b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*
a*b^3 + b^4 + 8*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c
)^5 + 10*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c)^3 + (3*a^4 + 4*a^3*b
 + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^4 + 2
*a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c)^2 + 4*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 +
 4*a*b^3 + b^4)*cosh(d*x + c)^6 + 15*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d
*x + c)^4 + a^4 + 2*a^3*b - 2*a*b^3 - b^4 + 3*(3*a^4 + 4*a^3*b + 2*a^2*b^2
+ 4*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^4 + 4*a^3*b + 6
*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^7 + 3*(a^4 + 2*a^3*b - 2*a*b^3 - b^
4)*cosh(d*x + c)^5 + (3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d
*x + c)^3 + (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c))*l
og(cosh(d*x + c) + sinh(d*x + c) - 1) + 4*(7*(9*a^3*b + 13*a^2*b^2 + 4*a*b^
3)*cosh(d*x + c)^6 + 5*(27*a^3*b + 11*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^4 +
9*a^3*b + 13*a^2*b^2 + 4*a*b^3 + 3*(27*a^3*b + 11*a^2*b^2 - 4*a*b^3)*cosh(d
*x + c)^2)*sinh(d*x + c))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4
)*d*cosh(d*x + c)^8 + 8*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d
*cosh(d*x + c)*sinh(d*x + c)^7 + (a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a
^3*b^4)*d*sinh(d*x + c)^8 + 4*(a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*cosh(
d*x + c)^6 + 4*(7*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*cosh(
d*x + c)^2 + (a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d)*sinh(d*x + c)^6 + 2*(
3*a^7 + 4*a^6*b + 2*a^5*b^2 + 4*a^4*b^3 + 3*a^3*b^4)*d*cosh(d*x + c)^4 + 8*
```

```
(7*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^3 + 3*
(a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*
(35*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^4 + 3
0*(a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^2 + (3*a^7 + 4*a^6*
b + 2*a^5*b^2 + 4*a^4*b^3 + 3*a^3*b^4)*d)*sinh(d*x + c)^4 + 4*(a^7 + 2*a^6*
b - 2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^2 + 8*(7*(a^7 + 4*a^6*b + 6*a^5*b^
2 + 4*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^5 + 10*(a^7 + 2*a^6*b - 2*a^4*b^3
- a^3*b^4)*d*cosh(d*x + c)^3 + (3*a^7 + 4*a^6*b + 2*a^5*b^2 + 4*a^4*b^3 + 3
*a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^7 + 4*a^6*b + 6*a^5*b^
2 + 4*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^6 + 15*(a^7 + 2*a^6*b - 2*a^4*b^3
- a^3*b^4)*d*cosh(d*x + c)^4 + 3*(3*a^7 + 4*a^6*b + 2*a^5*b^2 + 4*a^4*b^3 +
 3*a^3*b^4)*d*cosh(d*x + c)^2 + (a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d)*si
nh(d*x + c)^2 + (a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d + 8*((a
^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^7 + 3*(a^7
+ 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^5 + (3*a^7 + 4*a^6*b + 2*a
^5*b^2 + 4*a^4*b^3 + 3*a^3*b^4)*d*cosh(d*x + c)^3 + (a^7 + 2*a^6*b - 2*a^4*
b^3 - a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*(2*(9*a^3*b + 13*a^2*b^
2 + 4*a*b^3)*cosh(d*x + c)^7 + 14*(9*a^3*b + 13*a^2*b^2 + 4*a*b^3)*cosh(d*x
 + c)*sinh(d*x + c)^6 + 2*(9*a^3*b + 13*a^2*b^2 + 4*a*b^3)*sinh(d*x + c)^7
+ 2*(27*a^3*b + 11*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^5 + 2*(27*a^3*b + 11*a^
2*b^2 - 4*a*b^3 + 21*(9*a^3*b + 13*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^2)*sinh
(d*x + c)^5 + 10*(7*(9*a^3*b + 13*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^3 + (27*
a^3*b + 11*a^2*b^2 - 4*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 2*(27*a^3*b
+ 11*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^3 + 2*(35*(9*a^3*b + 13*a^2*b^2 + 4*a
*b^3)*cosh(d*x + c)^4 + 27*a^3*b + 11*a^2*b^2 - 4*a*b^3 + 10*(27*a^3*b + 11
*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(21*(9*a^3*b + 13*
a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^5 + 10*(27*a^3*b + 11*a^2*b^2 - 4*a*b^3)*c
osh(d*x + c)^3 + 3*(27*a^3*b + 11*a^2*b^2 - 4*a*b^3)*cosh(d*x + c))*sinh(d*
x + c)^2 + ((15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*cosh(d*x +
c)^8 + 8*(15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*cosh(d*x + c)*
sinh(d*x + c)^7 + (15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*sinh(
d*x + c)^8 + 4*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*cosh(d*x
+ c)^6 + 4*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4 + 7*(15*a^4 +
50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6
+ 8*(7*(15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*cosh(d*x + c)^3
+ 3*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*cosh(d*x + c))*sinh(
d*x + c)^5 + 2*(45*a^4 + 30*a^3*b + 29*a^2*b^2 + 44*a*b^3 + 24*b^4)*cosh(d*
x + c)^4 + 2*(35*(15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*cosh(d
*x + c)^4 + 45*a^4 + 30*a^3*b + 29*a^2*b^2 + 44*a*b^3 + 24*b^4 + 30*(15*a^4
 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^
4 + 15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4 + 8*(7*(15*a^4 + 50*a
^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*cosh(d*x + c)^5 + 10*(15*a^4 + 20*a^3
*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*cosh(d*x + c)^3 + (45*a^4 + 30*a^3*b + 2
9*a^2*b^2 + 44*a*b^3 + 24*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(15*a^4 +
 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*cosh(d*x + c)^2 + 4*(7*(15*a^4 +
50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*cosh(d*x + c)^6 + 15*(15*a^4 + 20
*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*cosh(d*x + c)^4 + 15*a^4 + 20*a^3*b
- 7*a^2*b^2 - 20*a*b^3 - 8*b^4 + 3*(45*a^4 + 30*a^3*b + 29*a^2*b^2 + 44*a*b
^3 + 24*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((15*a^4 + 50*a^3*b + 63*
a^2*b^2 + 36*a*b^3 + 8*b^4)*cosh(d*x + c)^7 + 3*(15*a^4 + 20*a^3*b - 7*a^2*
b^2 - 20*a*b^3 - 8*b^4)*cosh(d*x + c)^5 + (45*a^4 + 30*a^3*b + 29*a^2*b^2 +
 44*a*b^3 + 24*b^4)*cosh(d*x + c)^3 + (15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a
*b^3 - 8*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2*((a
 + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*s
inh(d*x + c)^3 + (a - 3*b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a -
 3*b)*sinh(d*x + c))*sqrt(-b/(a + b))/b) - ((15*a^4 + 50*a^3*b + 63*a^2*b^2
 + 36*a*b^3 + 8*b^4)*cosh(d*x + c)^8 + 8*(15*a^4 + 50*a^3*b + 63*a^2*b^2 +
36*a*b^3 + 8*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (15*a^4 + 50*a^3*b + 63*a
^2*b^2 + 36*a*b^3 + 8*b^4)*sinh(d*x + c)^8 + 4*(15*a^4 + 20*a^3*b - 7*a^2*b
```

```
^2 - 20*a*b^3 - 8*b^4)*cosh(d*x + c)^6 + 4*(15*a^4 + 20*a^3*b - 7*a^2*b^2 -
 20*a*b^3 - 8*b^4 + 7*(15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*c
osh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36
*a*b^3 + 8*b^4)*cosh(d*x + c)^3 + 3*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b
^3 - 8*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(45*a^4 + 30*a^3*b + 29*a^2*
b^2 + 44*a*b^3 + 24*b^4)*cosh(d*x + c)^4 + 2*(35*(15*a^4 + 50*a^3*b + 63*a^
2*b^2 + 36*a*b^3 + 8*b^4)*cosh(d*x + c)^4 + 45*a^4 + 30*a^3*b + 29*a^2*b^2
+ 44*a*b^3 + 24*b^4 + 30*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)
*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b
^3 + 8*b^4 + 8*(7*(15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*cosh(
d*x + c)^5 + 10*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*cosh(d*x
 + c)^3 + (45*a^4 + 30*a^3*b + 29*a^2*b^2 + 44*a*b^3 + 24*b^4)*cosh(d*x + c
))*sinh(d*x + c)^3 + 4*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*c
osh(d*x + c)^2 + 4*(7*(15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*c
osh(d*x + c)^6 + 15*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*cosh
(d*x + c)^4 + 15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4 + 3*(45*a^4
+ 30*a^3*b + 29*a^2*b^2 + 44*a*b^3 + 24*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)
^2 + 8*((15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*cosh(d*x + c)^7
 + 3*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*cosh(d*x + c)^5 + (
45*a^4 + 30*a^3*b + 29*a^2*b^2 + 44*a*b^3 + 24*b^4)*cosh(d*x + c)^3 + (15*a
^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*cosh(d*x + c))*sinh(d*x + c))
*sqrt(-b/(a + b))*arctan(1/2*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x + c)
)*sqrt(-b/(a + b))/b) + 2*(9*a^3*b + 13*a^2*b^2 + 4*a*b^3)*cosh(d*x + c) -
8*((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^8 + 8*(a^4 + 4
*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^4 +
4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sinh(d*x + c)^8 + 4*(a^4 + 2*a^3*b - 2
*a*b^3 - b^4)*cosh(d*x + c)^6 + 4*(a^4 + 2*a^3*b - 2*a*b^3 - b^4 + 7*(a^4 +
 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*
(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^3 + 3*(a^4 + 2
*a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^4 + 4*a^3*b
 + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 2*(35*(a^4 + 4*a^3*b + 6*
a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^4 + 3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*
a*b^3 + 3*b^4 + 30*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c)^2)*sinh(d*
x + c)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 + 8*(7*(a^4 + 4*a^3*b
+ 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^5 + 10*(a^4 + 2*a^3*b - 2*a*b^3
- b^4)*cosh(d*x + c)^3 + (3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*co
sh(d*x + c))*sinh(d*x + c)^3 + 4*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x +
 c)^2 + 4*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^6 +
15*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c)^4 + a^4 + 2*a^3*b - 2*a*b^
3 - b^4 + 3*(3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^2
)*sinh(d*x + c)^2 + 8*((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x
 + c)^7 + 3*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c)^5 + (3*a^4 + 4*a^
3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + (a^4 + 2*a^3*b - 2*a*b
^3 - b^4)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) +
 1) + 8*((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^8 + 8*(a
^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (
a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sinh(d*x + c)^8 + 4*(a^4 + 2*a^3
*b - 2*a*b^3 - b^4)*cosh(d*x + c)^6 + 4*(a^4 + 2*a^3*b - 2*a*b^3 - b^4 + 7*
(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^
6 + 8*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^3 + 3*(a
^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^4 + 4
*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 2*(35*(a^4 + 4*a^3*
b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^4 + 3*a^4 + 4*a^3*b + 2*a^2*b^
2 + 4*a*b^3 + 3*b^4 + 30*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c)^2)*s
inh(d*x + c)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 + 8*(7*(a^4 + 4*
a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^5 + 10*(a^4 + 2*a^3*b - 2*
a*b^3 - b^4)*cosh(d*x + c)^3 + (3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b
^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh
(d*x + c)^2 + 4*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c
```

```
)^6 + 15*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c)^4 + a^4 + 2*a^3*b -
2*a*b^3 - b^4 + 3*(3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x
+ c)^2)*sinh(d*x + c)^2 + 8*((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*co
sh(d*x + c)^7 + 3*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c)^5 + (3*a^4
+ 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + (a^4 + 2*a^3*b -
 2*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x
+ c) - 1) + 2*(7*(9*a^3*b + 13*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^6 + 5*(27*a
^3*b + 11*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^4 + 9*a^3*b + 13*a^2*b^2 + 4*a*b
^3 + 3*(27*a^3*b + 11*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/((
a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^8 + 8*(a^7
 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)*sinh(d*x + c)
^7 + (a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*sinh(d*x + c)^8 +
4*(a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^6 + 4*(7*(a^7 + 4*a
^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^2 + (a^7 + 2*a^6*b
- 2*a^4*b^3 - a^3*b^4)*d)*sinh(d*x + c)^6 + 2*(3*a^7 + 4*a^6*b + 2*a^5*b^2
+ 4*a^4*b^3 + 3*a^3*b^4)*d*cosh(d*x + c)^4 + 8*(7*(a^7 + 4*a^6*b + 6*a^5*b^
2 + 4*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^3 + 3*(a^7 + 2*a^6*b - 2*a^4*b^3 -
 a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^7 + 4*a^6*b + 6*a^5*b
^2 + 4*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^4 + 30*(a^7 + 2*a^6*b - 2*a^4*b^3
 - a^3*b^4)*d*cosh(d*x + c)^2 + (3*a^7 + 4*a^6*b + 2*a^5*b^2 + 4*a^4*b^3 +
3*a^3*b^4)*d)*sinh(d*x + c)^4 + 4*(a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*c
osh(d*x + c)^2 + 8*(7*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*c
osh(d*x + c)^5 + 10*(a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^3
 + (3*a^7 + 4*a^6*b + 2*a^5*b^2 + 4*a^4*b^3 + 3*a^3*b^4)*d*cosh(d*x + c))*s
inh(d*x + c)^3 + 4*(7*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*c
osh(d*x + c)^6 + 15*(a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^4
 + 3*(3*a^7 + 4*a^6*b + 2*a^5*b^2 + 4*a^4*b^3 + 3*a^3*b^4)*d*cosh(d*x + c)^
2 + (a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d)*sinh(d*x + c)^2 + (a^7 + 4*a^6
*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d + 8*((a^7 + 4*a^6*b + 6*a^5*b^2 + 4
*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^7 + 3*(a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*
b^4)*d*cosh(d*x + c)^5 + (3*a^7 + 4*a^6*b + 2*a^5*b^2 + 4*a^4*b^3 + 3*a^3*b
^4)*d*cosh(d*x + c)^3 + (a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*cosh(d*x +
c))*sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral(csch(c + d*x)/(a + b*tanh(c + d*x)**2)**3, x)
```

Giac [C] time = 1.98066, size = 9281, normalized size = 59.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/32*(2*(3*(15*a^8*b*e^(4*c) + 65*a^7*b^2*e^(4*c) + 113*a^6*b^3*e^(4*c) +
99*a^5*b^4*e^(4*c) + 44*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c))*cos(1/2*real_p
```

```
art(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b)
 + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) - (15*
a^8*b*e^(4*c) + 65*a^7*b^2*e^(4*c) + 113*a^6*b^3*e^(4*c) + 99*a^5*b^4*e^(4*
c) + 44*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(
a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^
3 - 9*(15*a^8*b*e^(4*c) + 65*a^7*b^2*e^(4*c) + 113*a^6*b^3*e^(4*c) + 99*a^5
*b^4*e^(4*c) + 44*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c))*cos(1/2*real_part(ar
ccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(
a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*ima
g_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(15*a^8*b*e^(4*c) + 65*a^7*b^2*
e^(4*c) + 113*a^6*b^3*e^(4*c) + 99*a^5*b^4*e^(4*c) + 44*a^4*b^5*e^(4*c) + 8
*a^3*b^6*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin
(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos
(-a/(a + b) + b/(a + b)))) + 9*(15*a^8*b*e^(4*c) + 65*a^7*b^2*e^(4*c) + 113
*a^6*b^3*e^(4*c) + 99*a^5*b^4*e^(4*c) + 44*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4
*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_par
t(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/
(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(15*a^
8*b*e^(4*c) + 65*a^7*b^2*e^(4*c) + 113*a^6*b^3*e^(4*c) + 99*a^5*b^4*e^(4*c)
 + 44*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a
+ b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*si
nh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(15*a^8*b*e^(4*c) +
 65*a^7*b^2*e^(4*c) + 113*a^6*b^3*e^(4*c) + 99*a^5*b^4*e^(4*c) + 44*a^4*b^5
*e^(4*c) + 8*a^3*b^6*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a +
b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_pa
rt(arccos(-a/(a + b) + b/(a + b))))^3 + (15*a^8*b*e^(4*c) + 65*a^7*b^2*e^(4
*c) + 113*a^6*b^3*e^(4*c) + 99*a^5*b^4*e^(4*c) + 44*a^4*b^5*e^(4*c) + 8*a^3
*b^6*e^(4*c))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2
*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 - (15*a^8*b*e^(4*c) + 65*a^7*
b^2*e^(4*c) + 113*a^6*b^3*e^(4*c) + 99*a^5*b^4*e^(4*c) + 44*a^4*b^5*e^(4*c)
 + 8*a^3*b^6*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*s
in(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) + (15*a^8*b*e^(4*c) + 65*
a^7*b^2*e^(4*c) + 113*a^6*b^3*e^(4*c) + 99*a^5*b^4*e^(4*c) + 44*a^4*b^5*e^(
4*c) + 8*a^3*b^6*e^(4*c))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))
)*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*arctan((((a^6 + 3*a^
5*b + 3*a^4*b^2 + a^3*b^3)/(a^6*e^(4*c) + 3*a^5*b*e^(4*c) + 3*a^4*b^2*e^(4*
c) + a^3*b^3*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b))) + e^(d*x))/(
((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)/(a^6*e^(4*c) + 3*a^5*b*e^(4*c) + 3*a
^4*b^2*e^(4*c) + a^3*b^3*e^(4*c)))^(1/4)*sin(1/2*arccos(-(a - b)/(a + b))))
)/(2*(a^5*e^(2*c) + 2*a^4*b*e^(2*c) + a^3*b^2*e^(2*c))^2*a*b + (a^6*e^(2*c)
 + a^5*b*e^(2*c) - a^4*b^2*e^(2*c) - a^3*b^3*e^(2*c))*sqrt(-a*b)*abs(a^5*e^
(2*c) + 2*a^4*b*e^(2*c) + a^3*b^2*e^(2*c))) + 2*(3*(15*a^8*b*e^(4*c) + 65*a
^7*b^2*e^(4*c) + 113*a^6*b^3*e^(4*c) + 99*a^5*b^4*e^(4*c) + 44*a^4*b^5*e^(4
*c) + 8*a^3*b^6*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))
^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(
arccos(-a/(a + b) + b/(a + b)))) - (15*a^8*b*e^(4*c) + 65*a^7*b^2*e^(4*c) +
 113*a^6*b^3*e^(4*c) + 99*a^5*b^4*e^(4*c) + 44*a^4*b^5*e^(4*c) + 8*a^3*b^6*
e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real
_part(arccos(-a/(a + b) + b/(a + b))))^3 - 9*(15*a^8*b*e^(4*c) + 65*a^7*b^2
*e^(4*c) + 113*a^6*b^3*e^(4*c) + 99*a^5*b^4*e^(4*c) + 44*a^4*b^5*e^(4*c) +
8*a^3*b^6*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cos
h(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos
(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)
))) + 3*(15*a^8*b*e^(4*c) + 65*a^7*b^2*e^(4*c) + 113*a^6*b^3*e^(4*c) + 99*a
^5*b^4*e^(4*c) + 44*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c))*cosh(1/2*imag_part
(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b
/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(15*a
^8*b*e^(4*c) + 65*a^7*b^2*e^(4*c) + 113*a^6*b^3*e^(4*c) + 99*a^5*b^4*e^(4*c
) + 44*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c))*cos(1/2*real_part(arccos(-a/(a
```

```
+ b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*s
in(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos
(-a/(a + b) + b/(a + b))))^2 - 3*(15*a^8*b*e^(4*c) + 65*a^7*b^2*e^(4*c) + 1
13*a^6*b^3*e^(4*c) + 99*a^5*b^4*e^(4*c) + 44*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^
(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_par
t(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) +
 b/(a + b))))^2 - 3*(15*a^8*b*e^(4*c) + 65*a^7*b^2*e^(4*c) + 113*a^6*b^3*e^
(4*c) + 99*a^5*b^4*e^(4*c) + 44*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c))*cos(1/
2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/
(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^
3 + (15*a^8*b*e^(4*c) + 65*a^7*b^2*e^(4*c) + 113*a^6*b^3*e^(4*c) + 99*a^5*b
^4*e^(4*c) + 44*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c))*sin(1/2*real_part(arcc
os(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a
+ b))))^3 - (15*a^8*b*e^(4*c) + 65*a^7*b^2*e^(4*c) + 113*a^6*b^3*e^(4*c) +
99*a^5*b^4*e^(4*c) + 44*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c))*cosh(1/2*imag_
part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) +
 b/(a + b)))) + (15*a^8*b*e^(4*c) + 65*a^7*b^2*e^(4*c) + 113*a^6*b^3*e^(4*c
) + 99*a^5*b^4*e^(4*c) + 44*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c))*sin(1/2*re
al_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a +
b) + b/(a + b)))))*arctan(-(((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)/(a^6*e^(
4*c) + 3*a^5*b*e^(4*c) + 3*a^4*b^2*e^(4*c) + a^3*b^3*e^(4*c)))^(1/4)*cos(1/
2*arccos(-(a - b)/(a + b))) - e^(d*x))/(((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b
^3)/(a^6*e^(4*c) + 3*a^5*b*e^(4*c) + 3*a^4*b^2*e^(4*c) + a^3*b^3*e^(4*c)))^
(1/4)*sin(1/2*arccos(-(a - b)/(a + b)))))/(2*(a^5*e^(2*c) + 2*a^4*b*e^(2*c)
 + a^3*b^2*e^(2*c))^2*a*b + (a^6*e^(2*c) + a^5*b*e^(2*c) - a^4*b^2*e^(2*c)
- a^3*b^3*e^(2*c))*sqrt(-a*b)*abs(a^5*e^(2*c) + 2*a^4*b*e^(2*c) + a^3*b^2*e
^(2*c))) + ((15*a^8*b*e^(4*c) + 65*a^7*b^2*e^(4*c) + 113*a^6*b^3*e^(4*c) +
99*a^5*b^4*e^(4*c) + 44*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c))*cos(1/2*real_p
art(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b)
 + b/(a + b))))^3 - 3*(15*a^8*b*e^(4*c) + 65*a^7*b^2*e^(4*c) + 113*a^6*b^3*
e^(4*c) + 99*a^5*b^4*e^(4*c) + 44*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c))*cos(
1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a
/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))
)^2 - 3*(15*a^8*b*e^(4*c) + 65*a^7*b^2*e^(4*c) + 113*a^6*b^3*e^(4*c) + 99*a
^5*b^4*e^(4*c) + 44*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c))*cos(1/2*real_part(
arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b
/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(15*a
^8*b*e^(4*c) + 65*a^7*b^2*e^(4*c) + 113*a^6*b^3*e^(4*c) + 99*a^5*b^4*e^(4*c
) + 44*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c))*cos(1/2*real_part(arccos(-a/(a
+ b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*s
in(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arcc
os(-a/(a + b) + b/(a + b)))) + 3*(15*a^8*b*e^(4*c) + 65*a^7*b^2*e^(4*c) + 1
13*a^6*b^3*e^(4*c) + 99*a^5*b^4*e^(4*c) + 44*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^
(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_p
art(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) +
 b/(a + b))))^2 - 9*(15*a^8*b*e^(4*c) + 65*a^7*b^2*e^(4*c) + 113*a^6*b^3*e^
(4*c) + 99*a^5*b^4*e^(4*c) + 44*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c))*cos(1/
2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(
a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*
sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - (15*a^8*b*e^(4*c) +
 65*a^7*b^2*e^(4*c) + 113*a^6*b^3*e^(4*c) + 99*a^5*b^4*e^(4*c) + 44*a^4*b^5
*e^(4*c) + 8*a^3*b^6*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a +
b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + 3*(15*a^8*b
*e^(4*c) + 65*a^7*b^2*e^(4*c) + 113*a^6*b^3*e^(4*c) + 99*a^5*b^4*e^(4*c) +
44*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b)
 + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1
/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 - (15*a^8*b*e^(4*c) + 65*a^
7*b^2*e^(4*c) + 113*a^6*b^3*e^(4*c) + 99*a^5*b^4*e^(4*c) + 44*a^4*b^5*e^(4*
c) + 8*a^3*b^6*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*
```

```
cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + (15*a^8*b*e^(4*c) + 6
5*a^7*b^2*e^(4*c) + 113*a^6*b^3*e^(4*c) + 99*a^5*b^4*e^(4*c) + 44*a^4*b^5*e
^(4*c) + 8*a^3*b^6*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b)
)))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*log(2*((a^6 + 3*a^
5*b + 3*a^4*b^2 + a^3*b^3)/(a^6*e^(4*c) + 3*a^5*b*e^(4*c) + 3*a^4*b^2*e^(4*
c) + a^3*b^3*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b)))*e^(d*x) + sq
rt((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)/(a^6*e^(4*c) + 3*a^5*b*e^(4*c) + 3
*a^4*b^2*e^(4*c) + a^3*b^3*e^(4*c))) + e^(2*d*x))/(2*(a^5*e^(2*c) + 2*a^4*b
*e^(2*c) + a^3*b^2*e^(2*c))^2*a*b + (a^6*e^(2*c) + a^5*b*e^(2*c) - a^4*b^2*
e^(2*c) - a^3*b^3*e^(2*c))*sqrt(-a*b)*abs(a^5*e^(2*c) + 2*a^4*b*e^(2*c) + a
^3*b^2*e^(2*c))) - ((15*a^8*b*e^(4*c) + 65*a^7*b^2*e^(4*c) + 113*a^6*b^3*e^
(4*c) + 99*a^5*b^4*e^(4*c) + 44*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c))*cos(1/
2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a
/(a + b) + b/(a + b))))^3 - 3*(15*a^8*b*e^(4*c) + 65*a^7*b^2*e^(4*c) + 113*
a^6*b^3*e^(4*c) + 99*a^5*b^4*e^(4*c) + 44*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*
c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(a
rccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(
a + b))))^2 - 3*(15*a^8*b*e^(4*c) + 65*a^7*b^2*e^(4*c) + 113*a^6*b^3*e^(4*c
) + 99*a^5*b^4*e^(4*c) + 44*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c))*cos(1/2*re
al_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a
+ b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) +
 9*(15*a^8*b*e^(4*c) + 65*a^7*b^2*e^(4*c) + 113*a^6*b^3*e^(4*c) + 99*a^5*b^
4*e^(4*c) + 44*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c))*cos(1/2*real_part(arcco
s(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b
))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_p
art(arccos(-a/(a + b) + b/(a + b)))) + 3*(15*a^8*b*e^(4*c) + 65*a^7*b^2*e^(
4*c) + 113*a^6*b^3*e^(4*c) + 99*a^5*b^4*e^(4*c) + 44*a^4*b^5*e^(4*c) + 8*a^
3*b^6*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/
2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(
a + b) + b/(a + b))))^2 - 9*(15*a^8*b*e^(4*c) + 65*a^7*b^2*e^(4*c) + 113*a^
6*b^3*e^(4*c) + 99*a^5*b^4*e^(4*c) + 44*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c)
)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arc
cos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a +
b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - (15*a^8*b*e
^(4*c) + 65*a^7*b^2*e^(4*c) + 113*a^6*b^3*e^(4*c) + 99*a^5*b^4*e^(4*c) + 44
*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) +
 b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + 3*(
15*a^8*b*e^(4*c) + 65*a^7*b^2*e^(4*c) + 113*a^6*b^3*e^(4*c) + 99*a^5*b^4*e^
(4*c) + 44*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c))*cos(1/2*real_part(arccos(-a
/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^
2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 - (15*a^8*b*e^(4*c)
 + 65*a^7*b^2*e^(4*c) + 113*a^6*b^3*e^(4*c) + 99*a^5*b^4*e^(4*c) + 44*a^4*b
^5*e^(4*c) + 8*a^3*b^6*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a
+ b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + (15*a^8*b*e^(
4*c) + 65*a^7*b^2*e^(4*c) + 113*a^6*b^3*e^(4*c) + 99*a^5*b^4*e^(4*c) + 44*a
^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b
/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*log(-2*((a
^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)/(a^6*e^(4*c) + 3*a^5*b*e^(4*c) + 3*a^4*
b^2*e^(4*c) + a^3*b^3*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b)))*e^(
d*x) + sqrt((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)/(a^6*e^(4*c) + 3*a^5*b*e^
(4*c) + 3*a^4*b^2*e^(4*c) + a^3*b^3*e^(4*c))) + e^(2*d*x))/(2*(a^5*e^(2*c)
+ 2*a^4*b*e^(2*c) + a^3*b^2*e^(2*c))^2*a*b + (a^6*e^(2*c) + a^5*b*e^(2*c) -
 a^4*b^2*e^(2*c) - a^3*b^3*e^(2*c))*sqrt(-a*b)*abs(a^5*e^(2*c) + 2*a^4*b*e^
(2*c) + a^3*b^2*e^(2*c))) - 8*(9*a^2*b*e^(7*d*x + 7*c) + 13*a*b^2*e^(7*d*x
+ 7*c) + 4*b^3*e^(7*d*x + 7*c) + 27*a^2*b*e^(5*d*x + 5*c) + 11*a*b^2*e^(5*d
*x + 5*c) - 4*b^3*e^(5*d*x + 5*c) + 27*a^2*b*e^(3*d*x + 3*c) + 11*a*b^2*e^(
3*d*x + 3*c) - 4*b^3*e^(3*d*x + 3*c) + 9*a^2*b*e^(d*x + c) + 13*a*b^2*e^(d*
x + c) + 4*b^3*e^(d*x + c))/((a^4 + 2*a^3*b + a^2*b^2)*(a*e^(4*d*x + 4*c) +
 b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2)
```

```
+ 32*log(e^(d*x + c) + 1)/a^3 - 32*log(abs(e^(d*x + c) - 1))/a^3)/d
```

3.46 $\int \frac{\text{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=112

$$-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}d}+\frac{5\coth(c+dx)}{8a^2d\left(a+b\tanh^2(c+dx)\right)}-\frac{15\coth(c+dx)}{8a^3d}+\frac{\coth(c+dx)}{4ad\left(a+b\tanh^2(c+dx)\right)^2}$$

```
[Out] (-15*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(7/2)*d) - (15*C
oth[c + d*x])/(8*a^3*d) + Coth[c + d*x]/(4*a*d*(a + b*Tanh[c + d*x]^2)^2) +
 (5*Coth[c + d*x])/(8*a^2*d*(a + b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.086946, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3663, 290, 325, 205}

$$-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}d}+\frac{5\coth(c+dx)}{8a^2d\left(a+b\tanh^2(c+dx)\right)}-\frac{15\coth(c+dx)}{8a^3d}+\frac{\coth(c+dx)}{4ad\left(a+b\tanh^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```


```
[Out] (-15*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(7/2)*d) - (15*C
oth[c + d*x])/(8*a^3*d) + Coth[c + d*x]/(4*a*d*(a + b*Tanh[c + d*x]^2)^2) +
 (5*Coth[c + d*x])/(8*a^2*d*(a + b*Tanh[c + d*x]^2))
```


Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\coth(c+dx)}{4ad\left(a+b\tanh^2(c+dx)\right)^2} + \frac{5\operatorname{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{4ad}$$

$$= \frac{\coth(c+dx)}{4ad\left(a+b\tanh^2(c+dx)\right)^2} + \frac{5\coth(c+dx)}{8a^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{15\operatorname{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)}\,dx,\right.}{8a^2d}$$

$$= -\frac{15\coth(c+dx)}{8a^3d} + \frac{\coth(c+dx)}{4ad\left(a+b\tanh^2(c+dx)\right)^2} + \frac{5\coth(c+dx)}{8a^2d\left(a+b\tanh^2(c+dx)\right)} - \frac{(15b}{}$$

$$= -\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}d} - \frac{15\coth(c+dx)}{8a^3d} + \frac{\coth(c+dx)}{4ad\left(a+b\tanh^2(c+dx)\right)^2} + \frac{}{8a^2d}$$

Mathematica [A] time = 0.881923, size = 109, normalized size = 0.97

$$\frac{-15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right) - \frac{\sqrt{a}b\sinh(2(c+dx))((9a+7b)\cosh(2(c+dx))+9a-7b)}{((a+b)\cosh(2(c+dx))+a-b)^2} - 8\sqrt{a}\coth(c+dx)}{8a^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (-15*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] - 8*Sqrt[a]*Coth[c + d
*x] - (Sqrt[a]*b*(9*a - 7*b + (9*a + 7*b)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*
x)])/(a - b + (a + b)*Cosh[2*(c + d*x)])^2)/(8*a^(7/2)*d)
```

Maple [B] time = 0.116, size = 816, normalized size = 7.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] -1/2/d/a^3*tanh(1/2*d*x+1/2*c)-9/4/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(
1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^7-27/
4/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x
+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5-7/d/a^3*b^2/(tanh(1/2*d*x+1/2*c)^4*a
+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*
c)^5-27/4/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh
(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^3-7/d/a^3*b^2/(tanh(1/2*d*x+1/
```

```
2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*
d*x+1/2*c)^3-9/4/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a
+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)+15/8/d/a^2*b/(b*(a+b))^
(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2
*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-15/8/d/a^3*b/((2*(b*(a+b))^(1/2)-a-2*b)*a
)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+
15/8/d/a^3*b^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(
a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+15/8/d/a^2*b/(b*
(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*
c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+15/8/d/a^3*b/((2*(b*(a+b))^(1/2)+a+
2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1
/2))+15/8/d/a^3*b^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arc
tan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/2/d/a^3/ta
nh(1/2*d*x+1/2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 3.14165, size = 19301, normalized size = 172.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [-1/16*(4*(8*a^4 + 23*a^3*b + 45*a^2*b^2 + 45*a*b^3 + 15*b^4)*cosh(d*x + c)
^8 + 32*(8*a^4 + 23*a^3*b + 45*a^2*b^2 + 45*a*b^3 + 15*b^4)*cosh(d*x + c)*s
inh(d*x + c)^7 + 4*(8*a^4 + 23*a^3*b + 45*a^2*b^2 + 45*a*b^3 + 15*b^4)*sinh
(d*x + c)^8 + 8*(16*a^4 + 23*a^3*b - 45*a*b^3 - 30*b^4)*cosh(d*x + c)^6 + 8
*(16*a^4 + 23*a^3*b - 45*a*b^3 - 30*b^4 + 14*(8*a^4 + 23*a^3*b + 45*a^2*b^2
 + 45*a*b^3 + 15*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 16*(14*(8*a^4 + 23
*a^3*b + 45*a^2*b^2 + 45*a*b^3 + 15*b^4)*cosh(d*x + c)^3 + 3*(16*a^4 + 23*a
^3*b - 45*a*b^3 - 30*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 8*(24*a^4 + 32*a
^3*b + 5*a^2*b^2 + 50*a*b^3 + 45*b^4)*cosh(d*x + c)^4 + 8*(35*(8*a^4 + 23*a
^3*b + 45*a^2*b^2 + 45*a*b^3 + 15*b^4)*cosh(d*x + c)^4 + 24*a^4 + 32*a^3*b
+ 5*a^2*b^2 + 50*a*b^3 + 45*b^4 + 15*(16*a^4 + 23*a^3*b - 45*a*b^3 - 30*b^4
)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 32*a^4 + 164*a^3*b + 292*a^2*b^2 + 220
*a*b^3 + 60*b^4 + 32*(7*(8*a^4 + 23*a^3*b + 45*a^2*b^2 + 45*a*b^3 + 15*b^4)
*cosh(d*x + c)^5 + 5*(16*a^4 + 23*a^3*b - 45*a*b^3 - 30*b^4)*cosh(d*x + c)^
3 + (24*a^4 + 32*a^3*b + 5*a^2*b^2 + 50*a*b^3 + 45*b^4)*cosh(d*x + c))*sinh
(d*x + c)^3 + 8*(16*a^4 + 41*a^3*b - 55*a*b^3 - 30*b^4)*cosh(d*x + c)^2 + 8
*(14*(8*a^4 + 23*a^3*b + 45*a^2*b^2 + 45*a*b^3 + 15*b^4)*cosh(d*x + c)^6 +
15*(16*a^4 + 23*a^3*b - 45*a*b^3 - 30*b^4)*cosh(d*x + c)^4 + 16*a^4 + 41*a^
3*b - 55*a*b^3 - 30*b^4 + 6*(24*a^4 + 32*a^3*b + 5*a^2*b^2 + 50*a*b^3 + 45*
b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 15*((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*
a*b^3 + b^4)*cosh(d*x + c)^10 + 10*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b
^4)*cosh(d*x + c)*sinh(d*x + c)^9 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 +
```

```
b^4)*sinh(d*x + c)^10 + (3*a^4 + 4*a^3*b - 6*a^2*b^2 - 12*a*b^3 - 5*b^4)*co
sh(d*x + c)^8 + (3*a^4 + 4*a^3*b - 6*a^2*b^2 - 12*a*b^3 - 5*b^4 + 45*(a^4 +
 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*
(15*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^3 + (3*a^4 +
4*a^3*b - 6*a^2*b^2 - 12*a*b^3 - 5*b^4)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*
(a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*cosh(d*x + c)^6 + 2*(105*(a^4 + 4*a^3*b
 + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^4 + a^4 + 2*a^2*b^2 + 8*a*b^3 +
 5*b^4 + 14*(3*a^4 + 4*a^3*b - 6*a^2*b^2 - 12*a*b^3 - 5*b^4)*cosh(d*x + c)^
2)*sinh(d*x + c)^6 + 4*(63*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh
(d*x + c)^5 + 14*(3*a^4 + 4*a^3*b - 6*a^2*b^2 - 12*a*b^3 - 5*b^4)*cosh(d*x
+ c)^3 + 3*(a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*cosh(d*x + c))*sinh(d*x + c)
^5 - 2*(a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*cosh(d*x + c)^4 + 2*(105*(a^4 +
4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^6 + 35*(3*a^4 + 4*a^3*b
- 6*a^2*b^2 - 12*a*b^3 - 5*b^4)*cosh(d*x + c)^4 - a^4 - 2*a^2*b^2 - 8*a*b^3
 - 5*b^4 + 15*(a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*cosh(d*x + c)^2)*sinh(d*x
 + c)^4 - a^4 - 4*a^3*b - 6*a^2*b^2 - 4*a*b^3 - b^4 + 8*(15*(a^4 + 4*a^3*b
+ 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^7 + 7*(3*a^4 + 4*a^3*b - 6*a^2*b
^2 - 12*a*b^3 - 5*b^4)*cosh(d*x + c)^5 + 5*(a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b
^4)*cosh(d*x + c)^3 - (a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*cosh(d*x + c))*si
nh(d*x + c)^3 - (3*a^4 + 4*a^3*b - 6*a^2*b^2 - 12*a*b^3 - 5*b^4)*cosh(d*x +
 c)^2 + (45*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^8 + 2
8*(3*a^4 + 4*a^3*b - 6*a^2*b^2 - 12*a*b^3 - 5*b^4)*cosh(d*x + c)^6 + 30*(a^
4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*cosh(d*x + c)^4 - 3*a^4 - 4*a^3*b + 6*a^2*
b^2 + 12*a*b^3 + 5*b^4 - 12*(a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*cosh(d*x +
c)^2)*sinh(d*x + c)^2 + 2*(5*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*co
sh(d*x + c)^9 + 4*(3*a^4 + 4*a^3*b - 6*a^2*b^2 - 12*a*b^3 - 5*b^4)*cosh(d*x
 + c)^7 + 6*(a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*cosh(d*x + c)^5 - 4*(a^4 +
2*a^2*b^2 + 8*a*b^3 + 5*b^4)*cosh(d*x + c)^3 - (3*a^4 + 4*a^3*b - 6*a^2*b^2
 - 12*a*b^3 - 5*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/a)*log(((a^2 + 2
*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x
+ c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^
2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 +
 a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*c
osh(d*x + c))*sinh(d*x + c) - 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b
)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sq
rt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^
3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh
(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)
*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 16*(2*(8*a^4 + 23*a^3*b + 45*a^2*
b^2 + 45*a*b^3 + 15*b^4)*cosh(d*x + c)^7 + 3*(16*a^4 + 23*a^3*b - 45*a*b^3
- 30*b^4)*cosh(d*x + c)^5 + 2*(24*a^4 + 32*a^3*b + 5*a^2*b^2 + 50*a*b^3 + 4
5*b^4)*cosh(d*x + c)^3 + (16*a^4 + 41*a^3*b - 55*a*b^3 - 30*b^4)*cosh(d*x +
 c))*sinh(d*x + c))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*co
sh(d*x + c)^10 + 10*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*cos
h(d*x + c)*sinh(d*x + c)^9 + (a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b
^4)*d*sinh(d*x + c)^10 + (3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b^3 - 5*a^3*
b^4)*d*cosh(d*x + c)^8 + (45*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b
^4)*d*cosh(d*x + c)^2 + (3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b^3 - 5*a^3*b
^4)*d)*sinh(d*x + c)^8 + 2*(a^7 + 2*a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*d*cosh
(d*x + c)^6 + 8*(15*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*cos
h(d*x + c)^3 + (3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b^3 - 5*a^3*b^4)*d*cos
h(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3
 + a^3*b^4)*d*cosh(d*x + c)^4 + 14*(3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b^
3 - 5*a^3*b^4)*d*cosh(d*x + c)^2 + (a^7 + 2*a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4
)*d)*sinh(d*x + c)^6 - 2*(a^7 + 2*a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*d*cosh(d
*x + c)^4 + 4*(63*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*cosh(
d*x + c)^5 + 14*(3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b^3 - 5*a^3*b^4)*d*co
sh(d*x + c)^3 + 3*(a^7 + 2*a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*d*cosh(d*x + c)
)*sinh(d*x + c)^5 + 2*(105*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4
```

)*d*cosh(d*x + c)^6 + 35*(3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b^3 - 5*a^3*
b^4)*d*cosh(d*x + c)^4 + 15*(a^7 + 2*a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*d*cos
h(d*x + c)^2 - (a^7 + 2*a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*d)*sinh(d*x + c)^4
 - (3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b^3 - 5*a^3*b^4)*d*cosh(d*x + c)^2
 + 8*(15*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^
7 + 7*(3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b^3 - 5*a^3*b^4)*d*cosh(d*x + c
)^5 + 5*(a^7 + 2*a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*d*cosh(d*x + c)^3 - (a^7
+ 2*a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (45
*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^8 + 28*(
3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b^3 - 5*a^3*b^4)*d*cosh(d*x + c)^6 + 3
0*(a^7 + 2*a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*d*cosh(d*x + c)^4 - 12*(a^7 + 2
*a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*d*cosh(d*x + c)^2 - (3*a^7 + 4*a^6*b - 6*
a^5*b^2 - 12*a^4*b^3 - 5*a^3*b^4)*d)*sinh(d*x + c)^2 - (a^7 + 4*a^6*b + 6*a
^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d + 2*(5*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b
^3 + a^3*b^4)*d*cosh(d*x + c)^9 + 4*(3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b
^3 - 5*a^3*b^4)*d*cosh(d*x + c)^7 + 6*(a^7 + 2*a^5*b^2 + 8*a^4*b^3 + 5*a^3*
b^4)*d*cosh(d*x + c)^5 - 4*(a^7 + 2*a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*d*cosh
(d*x + c)^3 - (3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b^3 - 5*a^3*b^4)*d*cosh
(d*x + c))*sinh(d*x + c)), -1/8*(2*(8*a^4 + 23*a^3*b + 45*a^2*b^2 + 45*a*b^
3 + 15*b^4)*cosh(d*x + c)^8 + 16*(8*a^4 + 23*a^3*b + 45*a^2*b^2 + 45*a*b^3
+ 15*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + 2*(8*a^4 + 23*a^3*b + 45*a^2*b^2
+ 45*a*b^3 + 15*b^4)*sinh(d*x + c)^8 + 4*(16*a^4 + 23*a^3*b - 45*a*b^3 - 30
*b^4)*cosh(d*x + c)^6 + 4*(16*a^4 + 23*a^3*b - 45*a*b^3 - 30*b^4 + 14*(8*a^
4 + 23*a^3*b + 45*a^2*b^2 + 45*a*b^3 + 15*b^4)*cosh(d*x + c)^2)*sinh(d*x +
c)^6 + 8*(14*(8*a^4 + 23*a^3*b + 45*a^2*b^2 + 45*a*b^3 + 15*b^4)*cosh(d*x +
 c)^3 + 3*(16*a^4 + 23*a^3*b - 45*a*b^3 - 30*b^4)*cosh(d*x + c))*sinh(d*x +
 c)^5 + 4*(24*a^4 + 32*a^3*b + 5*a^2*b^2 + 50*a*b^3 + 45*b^4)*cosh(d*x + c)
^4 + 4*(35*(8*a^4 + 23*a^3*b + 45*a^2*b^2 + 45*a*b^3 + 15*b^4)*cosh(d*x + c
)^4 + 24*a^4 + 32*a^3*b + 5*a^2*b^2 + 50*a*b^3 + 45*b^4 + 15*(16*a^4 + 23*a
^3*b - 45*a*b^3 - 30*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*a^4 + 82*a^
3*b + 146*a^2*b^2 + 110*a*b^3 + 30*b^4 + 16*(7*(8*a^4 + 23*a^3*b + 45*a^2*b
^2 + 45*a*b^3 + 15*b^4)*cosh(d*x + c)^5 + 5*(16*a^4 + 23*a^3*b - 45*a*b^3 -
 30*b^4)*cosh(d*x + c)^3 + (24*a^4 + 32*a^3*b + 5*a^2*b^2 + 50*a*b^3 + 45*b
^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(16*a^4 + 41*a^3*b - 55*a*b^3 - 30*b
^4)*cosh(d*x + c)^2 + 4*(14*(8*a^4 + 23*a^3*b + 45*a^2*b^2 + 45*a*b^3 + 15*
b^4)*cosh(d*x + c)^6 + 15*(16*a^4 + 23*a^3*b - 45*a*b^3 - 30*b^4)*cosh(d*x
+ c)^4 + 16*a^4 + 41*a^3*b - 55*a*b^3 - 30*b^4 + 6*(24*a^4 + 32*a^3*b + 5*a
^2*b^2 + 50*a*b^3 + 45*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 15*((a^4 + 4
*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^10 + 10*(a^4 + 4*a^3*b +
6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)*sinh(d*x + c)^9 + (a^4 + 4*a^3*b +
 6*a^2*b^2 + 4*a*b^3 + b^4)*sinh(d*x + c)^10 + (3*a^4 + 4*a^3*b - 6*a^2*b^2
 - 12*a*b^3 - 5*b^4)*cosh(d*x + c)^8 + (3*a^4 + 4*a^3*b - 6*a^2*b^2 - 12*a*
b^3 - 5*b^4 + 45*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^
2)*sinh(d*x + c)^8 + 8*(15*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh
(d*x + c)^3 + (3*a^4 + 4*a^3*b - 6*a^2*b^2 - 12*a*b^3 - 5*b^4)*cosh(d*x + c
))*sinh(d*x + c)^7 + 2*(a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*cosh(d*x + c)^6
+ 2*(105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^4 + a^4
+ 2*a^2*b^2 + 8*a*b^3 + 5*b^4 + 14*(3*a^4 + 4*a^3*b - 6*a^2*b^2 - 12*a*b^3
- 5*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(a^4 + 4*a^3*b + 6*a^2*b^
2 + 4*a*b^3 + b^4)*cosh(d*x + c)^5 + 14*(3*a^4 + 4*a^3*b - 6*a^2*b^2 - 12*a
*b^3 - 5*b^4)*cosh(d*x + c)^3 + 3*(a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*cosh(
d*x + c))*sinh(d*x + c)^5 - 2*(a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*cosh(d*x
+ c)^4 + 2*(105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^6
 + 35*(3*a^4 + 4*a^3*b - 6*a^2*b^2 - 12*a*b^3 - 5*b^4)*cosh(d*x + c)^4 - a^
4 - 2*a^2*b^2 - 8*a*b^3 - 5*b^4 + 15*(a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*co
sh(d*x + c)^2)*sinh(d*x + c)^4 - a^4 - 4*a^3*b - 6*a^2*b^2 - 4*a*b^3 - b^4
+ 8*(15*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^7 + 7*(3*
a^4 + 4*a^3*b - 6*a^2*b^2 - 12*a*b^3 - 5*b^4)*cosh(d*x + c)^5 + 5*(a^4 + 2*
a^2*b^2 + 8*a*b^3 + 5*b^4)*cosh(d*x + c)^3 - (a^4 + 2*a^2*b^2 + 8*a*b^3 + 5

```
*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 - (3*a^4 + 4*a^3*b - 6*a^2*b^2 - 12*a*
b^3 - 5*b^4)*cosh(d*x + c)^2 + (45*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b
^4)*cosh(d*x + c)^8 + 28*(3*a^4 + 4*a^3*b - 6*a^2*b^2 - 12*a*b^3 - 5*b^4)*c
osh(d*x + c)^6 + 30*(a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*cosh(d*x + c)^4 - 3
*a^4 - 4*a^3*b + 6*a^2*b^2 + 12*a*b^3 + 5*b^4 - 12*(a^4 + 2*a^2*b^2 + 8*a*b
^3 + 5*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(5*(a^4 + 4*a^3*b + 6*a^2*
b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^9 + 4*(3*a^4 + 4*a^3*b - 6*a^2*b^2 - 12*
a*b^3 - 5*b^4)*cosh(d*x + c)^7 + 6*(a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*cosh
(d*x + c)^5 - 4*(a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*cosh(d*x + c)^3 - (3*a^
4 + 4*a^3*b - 6*a^2*b^2 - 12*a*b^3 - 5*b^4)*cosh(d*x + c))*sinh(d*x + c))*s
qrt(b/a)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh
(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(b/a)/b) + 8*(2*(8*a^4 + 2
3*a^3*b + 45*a^2*b^2 + 45*a*b^3 + 15*b^4)*cosh(d*x + c)^7 + 3*(16*a^4 + 23*
a^3*b - 45*a*b^3 - 30*b^4)*cosh(d*x + c)^5 + 2*(24*a^4 + 32*a^3*b + 5*a^2*b
^2 + 50*a*b^3 + 45*b^4)*cosh(d*x + c)^3 + (16*a^4 + 41*a^3*b - 55*a*b^3 - 3
0*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^
3 + a^3*b^4)*d*cosh(d*x + c)^10 + 10*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3
 + a^3*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^7 + 4*a^6*b + 6*a^5*b^2 +
4*a^4*b^3 + a^3*b^4)*d*sinh(d*x + c)^10 + (3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12
*a^4*b^3 - 5*a^3*b^4)*d*cosh(d*x + c)^8 + (45*(a^7 + 4*a^6*b + 6*a^5*b^2 +
4*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^2 + (3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*
a^4*b^3 - 5*a^3*b^4)*d)*sinh(d*x + c)^8 + 2*(a^7 + 2*a^5*b^2 + 8*a^4*b^3 +
5*a^3*b^4)*d*cosh(d*x + c)^6 + 8*(15*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3
 + a^3*b^4)*d*cosh(d*x + c)^3 + (3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b^3 -
 5*a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^7 + 4*a^6*b + 6*a^
5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^4 + 14*(3*a^7 + 4*a^6*b - 6*a^
5*b^2 - 12*a^4*b^3 - 5*a^3*b^4)*d*cosh(d*x + c)^2 + (a^7 + 2*a^5*b^2 + 8*a^
4*b^3 + 5*a^3*b^4)*d)*sinh(d*x + c)^6 - 2*(a^7 + 2*a^5*b^2 + 8*a^4*b^3 + 5*
a^3*b^4)*d*cosh(d*x + c)^4 + 4*(63*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 +
 a^3*b^4)*d*cosh(d*x + c)^5 + 14*(3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b^3
- 5*a^3*b^4)*d*cosh(d*x + c)^3 + 3*(a^7 + 2*a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4
)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*
a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^6 + 35*(3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12
*a^4*b^3 - 5*a^3*b^4)*d*cosh(d*x + c)^4 + 15*(a^7 + 2*a^5*b^2 + 8*a^4*b^3 +
 5*a^3*b^4)*d*cosh(d*x + c)^2 - (a^7 + 2*a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*d
)*sinh(d*x + c)^4 - (3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b^3 - 5*a^3*b^4)*
d*cosh(d*x + c)^2 + 8*(15*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)
*d*cosh(d*x + c)^7 + 7*(3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b^3 - 5*a^3*b^
4)*d*cosh(d*x + c)^5 + 5*(a^7 + 2*a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*d*cosh(d
*x + c)^3 - (a^7 + 2*a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*d*cosh(d*x + c))*sinh
(d*x + c)^3 + (45*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*cosh(
d*x + c)^8 + 28*(3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b^3 - 5*a^3*b^4)*d*co
sh(d*x + c)^6 + 30*(a^7 + 2*a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*d*cosh(d*x + c
)^4 - 12*(a^7 + 2*a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*d*cosh(d*x + c)^2 - (3*a
^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b^3 - 5*a^3*b^4)*d)*sinh(d*x + c)^2 - (a^
7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d + 2*(5*(a^7 + 4*a^6*b + 6*
a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^9 + 4*(3*a^7 + 4*a^6*b - 6*a
^5*b^2 - 12*a^4*b^3 - 5*a^3*b^4)*d*cosh(d*x + c)^7 + 6*(a^7 + 2*a^5*b^2 + 8
*a^4*b^3 + 5*a^3*b^4)*d*cosh(d*x + c)^5 - 4*(a^7 + 2*a^5*b^2 + 8*a^4*b^3 +
5*a^3*b^4)*d*cosh(d*x + c)^3 - (3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b^3 -
5*a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral(csch(c + d*x)**2/(a + b*tanh(c + d*x)**2)**3, x)
```

Giac [B] time = 1.91584, size = 474, normalized size = 4.23

$$-\frac{\frac{15\,b\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\sqrt{ab}a^3}-\frac{2\left(9\,a^3be^{(6\,dx+6\,c)}+3\,a^2b^2e^{(6\,dx+6\,c)}-13\,ab^3e^{(6\,dx+6\,c)}-7\,b^4e^{(6\,dx+6\,c)}+27\,a^3be^{(4\,dx+4\,c)}+3\,a^2b^2e^{(4\,dx+4\,c)}+13\,ab^3e^{(4\,dx+4}\right.}{\left(a^5+2\,a^4b+a^3b^2\right)\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+}\right.}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(15*b*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(
a*b))/(sqrt(a*b)*a^3) - 2*(9*a^3*b*e^(6*d*x + 6*c) + 3*a^2*b^2*e^(6*d*x + 6
*c) - 13*a*b^3*e^(6*d*x + 6*c) - 7*b^4*e^(6*d*x + 6*c) + 27*a^3*b*e^(4*d*x
+ 4*c) + 3*a^2*b^2*e^(4*d*x + 4*c) + 13*a*b^3*e^(4*d*x + 4*c) + 21*b^4*e^(4
*d*x + 4*c) + 27*a^3*b*e^(2*d*x + 2*c) + 25*a^2*b^2*e^(2*d*x + 2*c) - 23*a*
b^3*e^(2*d*x + 2*c) - 21*b^4*e^(2*d*x + 2*c) + 9*a^3*b + 25*a^2*b^2 + 23*a*
b^3 + 7*b^4)/((a^5 + 2*a^4*b + a^3*b^2)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4
*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2) + 16/(a^3*(e^(2
*d*x + 2*c) - 1)))/d
```

3.47 $\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3} dx$

Optimal. Leaf size=196

$$-\frac{\sqrt{b}\left(15a^2+40ab+24b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{8a^4d(a+b)^{3/2}}-\frac{b(11a+12b)\operatorname{sech}(c+dx)}{8a^3d(a+b)\left(a-b\operatorname{sech}^2(c+dx)+b\right)}-\frac{3b\operatorname{sech}(c+dx)}{4a^2d\left(a-b\operatorname{sech}^2(c+dx)+\right.}$$

```
[Out] ((a + 6*b)*ArcTanh[Cosh[c + d*x]])/(2*a^4*d) - (Sqrt[b]*(15*a^2 + 40*a*b +
24*b^2)*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(8*a^4*(a + b)^(3/2)*
d) - (Coth[c + d*x]*Csch[c + d*x])/(2*a*d*(a + b - b*Sech[c + d*x]^2)^2) -
(3*b*Sech[c + d*x])/(4*a^2*d*(a + b - b*Sech[c + d*x]^2)^2) - (b*(11*a + 12
*b)*Sech[c + d*x])/(8*a^3*(a + b)*d*(a + b - b*Sech[c + d*x]^2))
```

Rubi [A] time = 0.334025, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3664, 471, 527, 522, 207, 208}

$$-\frac{\sqrt{b}\left(15a^2+40ab+24b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{8a^4d(a+b)^{3/2}}-\frac{b(11a+12b)\operatorname{sech}(c+dx)}{8a^3d(a+b)\left(a-b\operatorname{sech}^2(c+dx)+b\right)}-\frac{3b\operatorname{sech}(c+dx)}{4a^2d\left(a-b\operatorname{sech}^2(c+dx)+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```


```
[Out] ((a + 6*b)*ArcTanh[Cosh[c + d*x]])/(2*a^4*d) - (Sqrt[b]*(15*a^2 + 40*a*b +
24*b^2)*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(8*a^4*(a + b)^(3/2)*
d) - (Coth[c + d*x]*Csch[c + d*x])/(2*a*d*(a + b - b*Sech[c + d*x]^2)^2) -
(3*b*Sech[c + d*x])/(4*a^2*d*(a + b - b*Sech[c + d*x]^2)^2) - (b*(11*a + 12
*b)*Sech[c + d*x])/(8*a^3*(a + b)*d*(a + b - b*Sech[c + d*x]^2))
```


Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{x^2}{\left(-1+x^2\right)^2\left(a+b-bx^2\right)^3}\,dx, x, \operatorname{sech}(c+dx)\right)}{d}$$

$$= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} - \frac{\operatorname{Subst}\left(\int \frac{a+b+5bx^2}{\left(-1+x^2\right)\left(a+b-bx^2\right)^3}\,dx, x, \operatorname{sech}(c+dx)\right)}{2ad}$$

$$= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} - \frac{3b\operatorname{sech}(c+dx)}{4a^2d\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} - \frac{\operatorname{Subst}\left(\int \frac{2(a+b)(2a+3}{\left(-1+x^2\right)(}\right.}{}$$

$$= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} - \frac{3b\operatorname{sech}(c+dx)}{4a^2d\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} - \frac{b(11a+12b)\operatorname{se}}{8a^3(a+b)d\left(a+b-\right.}$$

$$= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} - \frac{3b\operatorname{sech}(c+dx)}{4a^2d\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} - \frac{b(11a+12b)\operatorname{se}}{8a^3(a+b)d\left(a+b-\right.}$$

$$= \frac{(a+6b)\tanh^{-1}(\cosh(c+dx))}{2a^4d} - \frac{\sqrt{b}\left(15a^2+40ab+24b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{8a^4(a+b)^{3/2}d} - \frac{\operatorname{cot}}{2ad\left(\right.}$$

Mathematica [C] time = 4.09531, size = 269, normalized size = 1.37

$$-\frac{\frac{8a^2b^2\cosh(c+dx)}{(a+b)((a+b)\cosh(2(c+dx))+a-b)^2} + \frac{i\sqrt{b}\left(15a^2+40ab+24b^2\right)\tan^{-1}\left(\frac{-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)-i\sqrt{a+b}}{\sqrt{b}}\right)}{(a+b)^{3/2}} + \frac{i\sqrt{b}\left(15a^2+40ab+24b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)-i\sqrt{a+b}}{\sqrt{b}}\right)}{(a+b)^{3/2}} + }{8a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -((I*Sqrt[b]*(15*a^2 + 40*a*b + 24*b^2)*ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*
Tanh[(c + d*x)/2])/Sqrt[b]])/(a + b)^(3/2) + (I*Sqrt[b]*(15*a^2 + 40*a*b +
24*b^2)*ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]])/(a
+ b)^(3/2) + (8*a^2*b^2*Cosh[c + d*x])/((a + b)*(a - b + (a + b)*Cosh[2*(c
+ d*x)])^2) + (2*a*b*(9*a + 8*b)*Cosh[c + d*x])/((a + b)*(a - b + (a + b)*C
osh[2*(c + d*x)])) + a*Csch[(c + d*x)/2]^2 + 4*(a + 6*b)*Log[Tanh[(c + d*x)
/2]] + a*Sech[(c + d*x)/2]^2)/(8*a^4*d)
```

Maple [B] time = 0.118, size = 1083, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 1/8/d*tanh(1/2*d*x+1/2*c)^2/a^3-9/4/d*b/a/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1
/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a+b)*tanh(1/2*d*x+1/2*c)^
6-8/d*b^2/a^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2
*d*x+1/2*c)^2*b+a)^2/(a+b)*tanh(1/2*d*x+1/2*c)^6-6/d*b^3/a^3/(tanh(1/2*d*x+
1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a+b)*t
anh(1/2*d*x+1/2*c)^6-27/4/d*b/a/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2
*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a+b)*tanh(1/2*d*x+1/2*c)^4-51/2/d*b
^2/a^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/
2*c)^2*b+a)^2/(a+b)*tanh(1/2*d*x+1/2*c)^4-38/d*b^3/a^3/(tanh(1/2*d*x+1/2*c)
^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a+b)*tanh(1/
2*d*x+1/2*c)^4-20/d*b^4/a^4/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^
2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a+b)*tanh(1/2*d*x+1/2*c)^4-27/4/d*b/a/(
tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b
+a)^2/(a+b)*tanh(1/2*d*x+1/2*c)^2-20/d*b^2/a^2/(tanh(1/2*d*x+1/2*c)^4*a+2*t
anh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a+b)*tanh(1/2*d*x+1/
2*c)^2-14/d*b^3/a^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*ta
nh(1/2*d*x+1/2*c)^2*b+a)^2/(a+b)*tanh(1/2*d*x+1/2*c)^2-9/4/d*b/a/(tanh(1/2*
d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a+
b)-5/2/d*b^2/a^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(
1/2*d*x+1/2*c)^2*b+a)^2/(a+b)-15/8/d*b/a^2/(a+b)/(a*b+b^2)^(1/2)*arctanh(1/
4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2))-5/d*b^2/a^3/(a+b)/(a
*b+b^2)^(1/2)*arctanh(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/
2))-3/d*b^3/a^4/(a+b)/(a*b+b^2)^(1/2)*arctanh(1/4*(2*tanh(1/2*d*x+1/2*c)^2*
a+2*a+4*b)/(a*b+b^2)^(1/2))-1/8/d/a^3/tanh(1/2*d*x+1/2*c)^2-1/2/d/a^3*ln(ta
nh(1/2*d*x+1/2*c))-3/d/a^4*ln(tanh(1/2*d*x+1/2*c))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*((4*a^3*e^(11*c) + 21*a^2*b*e^(11*c) + 29*a*b^2*e^(11*c) + 12*b^3*e^(1
1*c))*e^(11*d*x) + (20*a^3*e^(9*c) + 37*a^2*b*e^(9*c) - 15*a*b^2*e^(9*c) -
```

```
36*b^3*e^(9*c))*e^(9*d*x) + 2*(20*a^3*e^(7*c) + 3*a^2*b*e^(7*c) - 7*a*b^2*e
^(7*c) + 12*b^3*e^(7*c))*e^(7*d*x) + 2*(20*a^3*e^(5*c) + 3*a^2*b*e^(5*c) -
7*a*b^2*e^(5*c) + 12*b^3*e^(5*c))*e^(5*d*x) + (20*a^3*e^(3*c) + 37*a^2*b*e^
(3*c) - 15*a*b^2*e^(3*c) - 36*b^3*e^(3*c))*e^(3*d*x) + (4*a^3*e^c + 21*a^2*
b*e^c + 29*a*b^2*e^c + 12*b^3*e^c)*e^(d*x))/(a^6*d + 3*a^5*b*d + 3*a^4*b^2*
d + a^3*b^3*d + (a^6*d*e^(12*c) + 3*a^5*b*d*e^(12*c) + 3*a^4*b^2*d*e^(12*c)
 + a^3*b^3*d*e^(12*c))*e^(12*d*x) + 2*(a^6*d*e^(10*c) - a^5*b*d*e^(10*c) -
5*a^4*b^2*d*e^(10*c) - 3*a^3*b^3*d*e^(10*c))*e^(10*d*x) - (a^6*d*e^(8*c) +
3*a^5*b*d*e^(8*c) - 13*a^4*b^2*d*e^(8*c) - 15*a^3*b^3*d*e^(8*c))*e^(8*d*x)
- 4*(a^6*d*e^(6*c) - a^5*b*d*e^(6*c) + 3*a^4*b^2*d*e^(6*c) + 5*a^3*b^3*d*e^
(6*c))*e^(6*d*x) - (a^6*d*e^(4*c) + 3*a^5*b*d*e^(4*c) - 13*a^4*b^2*d*e^(4*c
) - 15*a^3*b^3*d*e^(4*c))*e^(4*d*x) + 2*(a^6*d*e^(2*c) - a^5*b*d*e^(2*c) -
5*a^4*b^2*d*e^(2*c) - 3*a^3*b^3*d*e^(2*c))*e^(2*d*x)) + 1/2*(a + 6*b)*log((
e^(d*x + c) + 1)*e^(-c))/(a^4*d) - 1/2*(a + 6*b)*log((e^(d*x + c) - 1)*e^(-
c))/(a^4*d) + 8*integrate(1/32*((15*a^2*b*e^(3*c) + 40*a*b^2*e^(3*c) + 24*b
^3*e^(3*c))*e^(3*d*x) - (15*a^2*b*e^c + 40*a*b^2*e^c + 24*b^3*e^c)*e^(d*x))
/(a^6 + 2*a^5*b + a^4*b^2 + (a^6*e^(4*c) + 2*a^5*b*e^(4*c) + a^4*b^2*e^(4*c
))*e^(4*d*x) + 2*(a^6*e^(2*c) - a^4*b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral(csch(c + d*x)**3/(a + b*tanh(c + d*x)**2)**3, x)
```

Giac [C] time = 2.44519, size = 8479, normalized size = 43.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/32*(2*(3*(15*a^8*b*e^(4*c) + 70*a^7*b^2*e^(4*c) + 119*a^6*b^3*e^(4*c) + 8
8*a^5*b^4*e^(4*c) + 24*a^4*b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b)
 + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin
```

```
(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) - (15*a^8*b*e^(4*c) + 70*a^
7*b^2*e^(4*c) + 119*a^6*b^3*e^(4*c) + 88*a^5*b^4*e^(4*c) + 24*a^4*b^5*e^(4*
c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part
(arccos(-a/(a + b) + b/(a + b))))^3 - 9*(15*a^8*b*e^(4*c) + 70*a^7*b^2*e^(4
*c) + 119*a^6*b^3*e^(4*c) + 88*a^5*b^4*e^(4*c) + 24*a^4*b^5*e^(4*c))*cos(1/
2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a
/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))
)*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(15*a^8*b*e^(4*c)
 + 70*a^7*b^2*e^(4*c) + 119*a^6*b^3*e^(4*c) + 88*a^5*b^4*e^(4*c) + 24*a^4*b
^5*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*r
eal_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a
 + b) + b/(a + b)))) + 9*(15*a^8*b*e^(4*c) + 70*a^7*b^2*e^(4*c) + 119*a^6*b
^3*e^(4*c) + 88*a^5*b^4*e^(4*c) + 24*a^4*b^5*e^(4*c))*cos(1/2*real_part(arc
cos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a
 + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_p
art(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(15*a^8*b*e^(4*c) + 70*a^7*b^2*e
^(4*c) + 119*a^6*b^3*e^(4*c) + 88*a^5*b^4*e^(4*c) + 24*a^4*b^5*e^(4*c))*cos
h(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-
a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)
)))^2 - 3*(15*a^8*b*e^(4*c) + 70*a^7*b^2*e^(4*c) + 119*a^6*b^3*e^(4*c) + 88
*a^5*b^4*e^(4*c) + 24*a^4*b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b)
+ b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/
2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (15*a^8*b*e^(4*c) + 70*a^7
*b^2*e^(4*c) + 119*a^6*b^3*e^(4*c) + 88*a^5*b^4*e^(4*c) + 24*a^4*b^5*e^(4*c
))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(
arccos(-a/(a + b) + b/(a + b))))^3 - (15*a^8*b*e^(4*c) + 70*a^7*b^2*e^(4*c)
 + 119*a^6*b^3*e^(4*c) + 88*a^5*b^4*e^(4*c) + 24*a^4*b^5*e^(4*c))*cosh(1/2*
imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a +
 b) + b/(a + b)))) + (15*a^8*b*e^(4*c) + 70*a^7*b^2*e^(4*c) + 119*a^6*b^3*e
^(4*c) + 88*a^5*b^4*e^(4*c) + 24*a^4*b^5*e^(4*c))*sin(1/2*real_part(arccos(
-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))
)))*arctan((((a^6 + 2*a^5*b + a^4*b^2)/(a^6*e^(4*c) + 2*a^5*b*e^(4*c) + a^4
*b^2*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b))) + e^(d*x))/(((a^6 +
2*a^5*b + a^4*b^2)/(a^6*e^(4*c) + 2*a^5*b*e^(4*c) + a^4*b^2*e^(4*c)))^(1/4)
*sin(1/2*arccos(-(a - b)/(a + b)))))/(2*(a^5*e^(2*c) + a^4*b*e^(2*c))^2*a*b
 + (a^6*e^(2*c) - a^4*b^2*e^(2*c))*sqrt(-a*b)*abs(-a^5*e^(2*c) - a^4*b*e^(2
*c))) + 2*(3*(15*a^8*b*e^(4*c) + 70*a^7*b^2*e^(4*c) + 119*a^6*b^3*e^(4*c) +
 88*a^5*b^4*e^(4*c) + 24*a^4*b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a +
b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*s
in(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) - (15*a^8*b*e^(4*c) + 70*
a^7*b^2*e^(4*c) + 119*a^6*b^3*e^(4*c) + 88*a^5*b^4*e^(4*c) + 24*a^4*b^5*e^(
4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_pa
rt(arccos(-a/(a + b) + b/(a + b))))^3 - 9*(15*a^8*b*e^(4*c) + 70*a^7*b^2*e^
(4*c) + 119*a^6*b^3*e^(4*c) + 88*a^5*b^4*e^(4*c) + 24*a^4*b^5*e^(4*c))*cos(
1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(
-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)
)))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(15*a^8*b*e^(4*
c) + 70*a^7*b^2*e^(4*c) + 119*a^6*b^3*e^(4*c) + 88*a^5*b^4*e^(4*c) + 24*a^4
*b^5*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2
*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/
(a + b) + b/(a + b)))) + 9*(15*a^8*b*e^(4*c) + 70*a^7*b^2*e^(4*c) + 119*a^6
*b^3*e^(4*c) + 88*a^5*b^4*e^(4*c) + 24*a^4*b^5*e^(4*c))*cos(1/2*real_part(a
rccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/
(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag
_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(15*a^8*b*e^(4*c) + 70*a^7*b^2
*e^(4*c) + 119*a^6*b^3*e^(4*c) + 88*a^5*b^4*e^(4*c) + 24*a^4*b^5*e^(4*c))*c
osh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos
(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a +
b))))^2 - 3*(15*a^8*b*e^(4*c) + 70*a^7*b^2*e^(4*c) + 119*a^6*b^3*e^(4*c) +
```

```
88*a^5*b^4*e^(4*c) + 24*a^4*b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b
) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(
1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (15*a^8*b*e^(4*c) + 70*a
^7*b^2*e^(4*c) + 119*a^6*b^3*e^(4*c) + 88*a^5*b^4*e^(4*c) + 24*a^4*b^5*e^(4
*c))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_par
t(arccos(-a/(a + b) + b/(a + b))))^3 - (15*a^8*b*e^(4*c) + 70*a^7*b^2*e^(4*
c) + 119*a^6*b^3*e^(4*c) + 88*a^5*b^4*e^(4*c) + 24*a^4*b^5*e^(4*c))*cosh(1/
2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a
 + b) + b/(a + b)))) + (15*a^8*b*e^(4*c) + 70*a^7*b^2*e^(4*c) + 119*a^6*b^3
*e^(4*c) + 88*a^5*b^4*e^(4*c) + 24*a^4*b^5*e^(4*c))*sin(1/2*real_part(arcco
s(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b
)))))*arctan(-(((a^6 + 2*a^5*b + a^4*b^2)/(a^6*e^(4*c) + 2*a^5*b*e^(4*c) +
a^4*b^2*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b))) - e^(d*x))/(((a^6
 + 2*a^5*b + a^4*b^2)/(a^6*e^(4*c) + 2*a^5*b*e^(4*c) + a^4*b^2*e^(4*c)))^(1
/4)*sin(1/2*arccos(-(a - b)/(a + b)))))/(2*(a^5*e^(2*c) + a^4*b*e^(2*c))^2*
a*b + (a^6*e^(2*c) - a^4*b^2*e^(2*c))*sqrt(-a*b)*abs(-a^5*e^(2*c) - a^4*b*e
^(2*c))) + ((15*a^8*b*e^(4*c) + 70*a^7*b^2*e^(4*c) + 119*a^6*b^3*e^(4*c) +
88*a^5*b^4*e^(4*c) + 24*a^4*b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b
) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 -
3*(15*a^8*b*e^(4*c) + 70*a^7*b^2*e^(4*c) + 119*a^6*b^3*e^(4*c) + 88*a^5*b^4
*e^(4*c) + 24*a^4*b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a +
 b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_p
art(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(15*a^8*b*e^(4*c) + 70*a^7*b^2*e
^(4*c) + 119*a^6*b^3*e^(4*c) + 88*a^5*b^4*e^(4*c) + 24*a^4*b^5*e^(4*c))*cos
(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos
(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a +
b)))) + 9*(15*a^8*b*e^(4*c) + 70*a^7*b^2*e^(4*c) + 119*a^6*b^3*e^(4*c) + 88
*a^5*b^4*e^(4*c) + 24*a^4*b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b)
+ b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/
2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a
/(a + b) + b/(a + b)))) + 3*(15*a^8*b*e^(4*c) + 70*a^7*b^2*e^(4*c) + 119*a^
6*b^3*e^(4*c) + 88*a^5*b^4*e^(4*c) + 24*a^4*b^5*e^(4*c))*cos(1/2*real_part(
arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b
/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 9*(15*a
^8*b*e^(4*c) + 70*a^7*b^2*e^(4*c) + 119*a^6*b^3*e^(4*c) + 88*a^5*b^4*e^(4*c
) + 24*a^4*b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*
cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arcco
s(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a +
 b))))^2 - (15*a^8*b*e^(4*c) + 70*a^7*b^2*e^(4*c) + 119*a^6*b^3*e^(4*c) + 8
8*a^5*b^4*e^(4*c) + 24*a^4*b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b)
 + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + 3
*(15*a^8*b*e^(4*c) + 70*a^7*b^2*e^(4*c) + 119*a^6*b^3*e^(4*c) + 88*a^5*b^4*
e^(4*c) + 24*a^4*b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a +
b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_pa
rt(arccos(-a/(a + b) + b/(a + b))))^3 - (15*a^8*b*e^(4*c) + 70*a^7*b^2*e^(4
*c) + 119*a^6*b^3*e^(4*c) + 88*a^5*b^4*e^(4*c) + 24*a^4*b^5*e^(4*c))*cos(1/
2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(
a + b) + b/(a + b)))) + (15*a^8*b*e^(4*c) + 70*a^7*b^2*e^(4*c) + 119*a^6*b^
3*e^(4*c) + 88*a^5*b^4*e^(4*c) + 24*a^4*b^5*e^(4*c))*cos(1/2*real_part(arcc
os(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a +
b)))))*log(2*((a^6 + 2*a^5*b + a^4*b^2)/(a^6*e^(4*c) + 2*a^5*b*e^(4*c) + a^
4*b^2*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b)))*e^(d*x) + sqrt((a^6
 + 2*a^5*b + a^4*b^2)/(a^6*e^(4*c) + 2*a^5*b*e^(4*c) + a^4*b^2*e^(4*c))) +
e^(2*d*x))/(2*(a^5*e^(2*c) + a^4*b*e^(2*c))^2*a*b + (a^6*e^(2*c) - a^4*b^2*
e^(2*c))*sqrt(-a*b)*abs(-a^5*e^(2*c) - a^4*b*e^(2*c))) - ((15*a^8*b*e^(4*c)
 + 70*a^7*b^2*e^(4*c) + 119*a^6*b^3*e^(4*c) + 88*a^5*b^4*e^(4*c) + 24*a^4*b
^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*i
mag_part(arccos(-a/(a + b) + b/(a + b))))^3 - 3*(15*a^8*b*e^(4*c) + 70*a^7*
b^2*e^(4*c) + 119*a^6*b^3*e^(4*c) + 88*a^5*b^4*e^(4*c) + 24*a^4*b^5*e^(4*c)
```

```
)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arc
cos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a
+ b))))^2 - 3*(15*a^8*b*e^(4*c) + 70*a^7*b^2*e^(4*c) + 119*a^6*b^3*e^(4*c)
+ 88*a^5*b^4*e^(4*c) + 24*a^4*b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a +
 b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*
sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(15*a^8*b*e^(4*c) +
 70*a^7*b^2*e^(4*c) + 119*a^6*b^3*e^(4*c) + 88*a^5*b^4*e^(4*c) + 24*a^4*b^5
*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_
part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b)
 + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(
15*a^8*b*e^(4*c) + 70*a^7*b^2*e^(4*c) + 119*a^6*b^3*e^(4*c) + 88*a^5*b^4*e^
(4*c) + 24*a^4*b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b)
)))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_par
t(arccos(-a/(a + b) + b/(a + b))))^2 - 9*(15*a^8*b*e^(4*c) + 70*a^7*b^2*e^(
4*c) + 119*a^6*b^3*e^(4*c) + 88*a^5*b^4*e^(4*c) + 24*a^4*b^5*e^(4*c))*cos(1
/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/
(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2
*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - (15*a^8*b*e^(4*c)
+ 70*a^7*b^2*e^(4*c) + 119*a^6*b^3*e^(4*c) + 88*a^5*b^4*e^(4*c) + 24*a^4*b^
5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*im
ag_part(arccos(-a/(a + b) + b/(a + b))))^3 + 3*(15*a^8*b*e^(4*c) + 70*a^7*b
^2*e^(4*c) + 119*a^6*b^3*e^(4*c) + 88*a^5*b^4*e^(4*c) + 24*a^4*b^5*e^(4*c))
*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arcco
s(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a +
 b))))^3 - (15*a^8*b*e^(4*c) + 70*a^7*b^2*e^(4*c) + 119*a^6*b^3*e^(4*c) + 8
8*a^5*b^4*e^(4*c) + 24*a^4*b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b)
 + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + (15*a
^8*b*e^(4*c) + 70*a^7*b^2*e^(4*c) + 119*a^6*b^3*e^(4*c) + 88*a^5*b^4*e^(4*c
) + 24*a^4*b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*
sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*log(-2*((a^6 + 2*a^5*b
 + a^4*b^2)/(a^6*e^(4*c) + 2*a^5*b*e^(4*c) + a^4*b^2*e^(4*c)))^(1/4)*cos(1/
2*arccos(-(a - b)/(a + b)))*e^(d*x) + sqrt((a^6 + 2*a^5*b + a^4*b^2)/(a^6*e
^(4*c) + 2*a^5*b*e^(4*c) + a^4*b^2*e^(4*c))) + e^(2*d*x))/(2*(a^5*e^(2*c) +
 a^4*b*e^(2*c))^2*a*b + (a^6*e^(2*c) - a^4*b^2*e^(2*c))*sqrt(-a*b)*abs(-a^5
*e^(2*c) - a^4*b*e^(2*c))) + 16*(a*e^c + 6*b*e^c)*e^(-c)*log(e^(d*x + c) +
1)/a^4 - 16*(a*e^c + 6*b*e^c)*e^(-c)*log(abs(-e^(d*x + c) + 1))/a^4 - 8*(4*
a^3*e^(11*d*x + 11*c) + 21*a^2*b*e^(11*d*x + 11*c) + 29*a*b^2*e^(11*d*x + 1
1*c) + 12*b^3*e^(11*d*x + 11*c) + 20*a^3*e^(9*d*x + 9*c) + 37*a^2*b*e^(9*d*
x + 9*c) - 15*a*b^2*e^(9*d*x + 9*c) - 36*b^3*e^(9*d*x + 9*c) + 40*a^3*e^(7*
d*x + 7*c) + 6*a^2*b*e^(7*d*x + 7*c) - 14*a*b^2*e^(7*d*x + 7*c) + 24*b^3*e^
(7*d*x + 7*c) + 40*a^3*e^(5*d*x + 5*c) + 6*a^2*b*e^(5*d*x + 5*c) - 14*a*b^2
*e^(5*d*x + 5*c) + 24*b^3*e^(5*d*x + 5*c) + 20*a^3*e^(3*d*x + 3*c) + 37*a^2
*b*e^(3*d*x + 3*c) - 15*a*b^2*e^(3*d*x + 3*c) - 36*b^3*e^(3*d*x + 3*c) + 4*
a^3*e^(d*x + c) + 21*a^2*b*e^(d*x + c) + 29*a*b^2*e^(d*x + c) + 12*b^3*e^(d
*x + c))/((a^4 + a^3*b)*(a*e^(6*d*x + 6*c) + b*e^(6*d*x + 6*c) + a*e^(4*d*x
 + 4*c) - 3*b*e^(4*d*x + 4*c) - a*e^(2*d*x + 2*c) + 3*b*e^(2*d*x + 2*c) - a
 - b)^2))/d
```

3.48 $\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=151

$$\frac{b(7a+11b)\tanh(c+dx)}{8a^4d\left(a+b\tanh^2(c+dx)\right)}+\frac{b(a+b)\tanh(c+dx)}{4a^3d\left(a+b\tanh^2(c+dx)\right)^2}+\frac{(a+3b)\coth(c+dx)}{a^4d}+\frac{5\sqrt{b}(3a+7b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}d}$$

```
[Out] (5*Sqrt[b]*(3*a + 7*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(9/2)*
d) + ((a + 3*b)*Coth[c + d*x])/(a^4*d) - Coth[c + d*x]^3/(3*a^3*d) + (b*(a
+ b)*Tanh[c + d*x])/(4*a^3*d*(a + b*Tanh[c + d*x]^2)^2) + (b*(7*a + 11*b)*T
anh[c + d*x])/(8*a^4*d*(a + b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.200554, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3663, 456, 1259, 1261, 205}

$$\frac{b(7a+11b)\tanh(c+dx)}{8a^4d\left(a+b\tanh^2(c+dx)\right)}+\frac{b(a+b)\tanh(c+dx)}{4a^3d\left(a+b\tanh^2(c+dx)\right)^2}+\frac{(a+3b)\coth(c+dx)}{a^4d}+\frac{5\sqrt{b}(3a+7b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (5*Sqrt[b]*(3*a + 7*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(9/2)*
d) + ((a + 3*b)*Coth[c + d*x])/(a^4*d) - Coth[c + d*x]^3/(3*a^3*d) + (b*(a
+ b)*Tanh[c + d*x])/(4*a^3*d*(a + b*Tanh[c + d*x]^2)^2) + (b*(7*a + 11*b)*T
anh[c + d*x])/(8*a^4*d*(a + b*Tanh[c + d*x]^2))
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
```

```
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{x^4\left(a+bx^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b(a+b)\tanh(c+dx)}{4a^3d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{b\operatorname{Subst}\left(\int \frac{-\frac{4}{ab}+\frac{4(a+b)x^2}{a^2b}-\frac{3(a+b)x^4}{a^3}}{x^4\left(a+bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{4d}$$

$$= \frac{b(a+b)\tanh(c+dx)}{4a^3d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(7a+11b)\tanh(c+dx)}{8a^4d\left(a+b\tanh^2(c+dx)\right)} - \frac{\operatorname{Subst}\left(\int \frac{-8ab+8b(a+2b)x^2 \cdots}{x^4(a+b \cdots}\right.}{\cdots}$$

$$= \frac{b(a+b)\tanh(c+dx)}{4a^3d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(7a+11b)\tanh(c+dx)}{8a^4d\left(a+b\tanh^2(c+dx)\right)} - \frac{\operatorname{Subst}\left(\int\left(-\frac{8b}{x^4}+\frac{8b(a+3b \cdots}{ax^2}\right.\right.}{\cdots}$$

$$= \frac{(a+3b)\coth(c+dx)}{a^4d} - \frac{\coth^3(c+dx)}{3a^3d} + \frac{b(a+b)\tanh(c+dx)}{4a^3d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(7a+11b) \cdots}{8a^4d\left(a+b\,\mathrm{t} \cdots\right.}$$

$$= \frac{5\sqrt{b}(3a+7b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}d} + \frac{(a+3b)\coth(c+dx)}{a^4d} - \frac{\coth^3(c+dx)}{3a^3d} + \frac{b(\cdots}{4a^3d\left(\cdots\right.}$$

Mathematica [A] time = 1.24437, size = 149, normalized size = 0.99

$$\frac{\frac{3\sqrt{ab}\sinh(2(c+dx))\left(\left(9a^2+20ab+11b^2\right)\cosh(2(c+dx))+9a^2+6ab-11b^2\right)}{((a+b)\cosh(2(c+dx))+a-b)^2} + 15\sqrt{b}(3a+7b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right) - 8\sqrt{a}\coth(c+dx)\left(a\mathrm{cs} \cdots\right.}{24a^{9/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (15*Sqrt[b]*(3*a + 7*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] - 8*Sqrt[a]
*Coth[c + d*x]*(-2*a - 9*b + a*Csch[c + d*x]^2) + (3*Sqrt[a]*b*(9*a^2 + 6*a
*b - 11*b^2 + (9*a^2 + 20*a*b + 11*b^2)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)
```

```
])/(a - b + (a + b)*Cosh[2*(c + d*x)])^2)/(24*a^(9/2)*d)
```

Maple [B] time = 0.135, size = 1416, normalized size = 9.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] -15/8/d/a^2*b/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a
*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+3/2/d/a^4*tanh(1/
2*d*x+1/2*c)*b+3/2/d/a^4/tanh(1/2*d*x+1/2*c)*b-35/8/d*b^3/a^4/(b*(a+b))^(1/
2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b
*(a+b))^(1/2)-a-2*b)*a)^(1/2))-35/8/d*b^3/a^4/(b*(a+b))^(1/2)/((2*(b*(a+b))
^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2
*b)*a)^(1/2))+9/4/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*
a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^7-25/4/d/a^3*b^2/(b*(a
+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)
/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+3/8/d/a^3*tanh(1/2*d*x+1/2*c)+3/8/d/a
^3/tanh(1/2*d*x+1/2*c)-25/4/d/a^3*b^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a
-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^
(1/2))-15/8/d/a^2*b/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arc
tan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+27/4/d/a^2*b
/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2
*b+a)^2*tanh(1/2*d*x+1/2*c)^3+67/4/d/a^3*b^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tan
h(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^3+9
/4/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*
x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)+15/8/d/a^3*b/((2*(b*(a+b))^(1/2)-a-2*
b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/
2))-15/8/d/a^3*b/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+
1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-35/8/d*b^2/a^4/((2*(b*(a+b))^(1
/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)
*a)^(1/2))+11/d*b^3/a^4/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+
4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5+11/d*b^3/a^4/(tanh(1/2
*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*ta
nh(1/2*d*x+1/2*c)^3+13/4/d*b^2/a^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+
1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)+13/4/d*b^2/a^
3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^
2*b+a)^2*tanh(1/2*d*x+1/2*c)^7+27/4/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh
(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5+67
/4/d/a^3*b^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*
d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5-1/24/d/a^3*tanh(1/2*d*x+1/2*c)^3-
1/24/d/a^3/tanh(1/2*d*x+1/2*c)^3+35/8/d*b^2/a^4/((2*(b*(a+b))^(1/2)-a-2*b)*
a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral(csch(c + d*x)**4/(a + b*tanh(c + d*x)**2)**3, x)
```

Giac [B] time = 1.91782, size = 549, normalized size = 3.64

$$\frac{\frac{15\left(3\,abe^{(2\,c)}+7\,b^2e^{(2\,c)}\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)e^{(-2\,c)}}{\sqrt{ab}a^4}-\frac{6\left(9\,a^3be^{(6\,dx+6\,c)}+7\,a^2b^2e^{(6\,dx+6\,c)}-13\,ab^3e^{(6\,dx+6\,c)}-11\,b^4e^{(6\,dx+6\,c)}+27\,a^3be^{(4\,dx+4\,c)}+15}{\left(a^5+\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/24*(15*(3*a*b*e^(2*c) + 7*b^2*e^(2*c))*arctan(1/2*(a*e^(2*d*x + 2*c) + b*
e^(2*d*x + 2*c) + a - b)/sqrt(a*b))*e^(-2*c)/(sqrt(a*b)*a^4) - 6*(9*a^3*b*e
^(6*d*x + 6*c) + 7*a^2*b^2*e^(6*d*x + 6*c) - 13*a*b^3*e^(6*d*x + 6*c) - 11*
b^4*e^(6*d*x + 6*c) + 27*a^3*b*e^(4*d*x + 4*c) + 15*a^2*b^2*e^(4*d*x + 4*c)
 + 5*a*b^3*e^(4*d*x + 4*c) + 33*b^4*e^(4*d*x + 4*c) + 27*a^3*b*e^(2*d*x + 2
*c) + 37*a^2*b^2*e^(2*d*x + 2*c) - 23*a*b^3*e^(2*d*x + 2*c) - 33*b^4*e^(2*d
*x + 2*c) + 9*a^3*b + 29*a^2*b^2 + 31*a*b^3 + 11*b^4)/((a^5 + a^4*b)*(a*e^(
4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c
) + a + b)^2) + 16*(9*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*c) - 18*b*e^(2*d
*x + 2*c) + 2*a + 9*b)/(a^4*(e^(2*d*x + 2*c) - 1)^3))/d
```

3.49 $\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)dx$

Optimal. Leaf size=132

$$-\frac{3(a+8b)\log(1-\tanh(c+dx))}{16d}+\frac{3(a-8b)\log(\tanh(c+dx)+1)}{16d}+\frac{\sinh^4(c+dx)(a\tanh(c+dx)+b)}{4d}-\frac{\sinh^2(c+dx}{}$$

```
[Out] (-3*(a + 8*b)*Log[1 - Tanh[c + d*x]])/(16*d) + (3*(a - 8*b)*Log[1 + Tanh[c
+ d*x]])/(16*d) - (3*a*Tanh[c + d*x])/(8*d) - (3*b*Tanh[c + d*x]^2)/(2*d) +
 (Sinh[c + d*x]^4*(b + a*Tanh[c + d*x]))/(4*d) - (Sinh[c + d*x]^2*Tanh[c +
d*x]*(a + 8*b*Tanh[c + d*x]))/(8*d)
```

Rubi [A] time = 0.172493, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3663, 1804, 801, 633, 31}

$$-\frac{3(a+8b)\log(1-\tanh(c+dx))}{16d}+\frac{3(a-8b)\log(\tanh(c+dx)+1)}{16d}+\frac{\sinh^4(c+dx)(a\tanh(c+dx)+b)}{4d}-\frac{\sinh^2(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^3),x]
```



```
[Out] (-3*(a + 8*b)*Log[1 - Tanh[c + d*x]])/(16*d) + (3*(a - 8*b)*Log[1 + Tanh[c
+ d*x]])/(16*d) - (3*a*Tanh[c + d*x])/(8*d) - (3*b*Tanh[c + d*x]^2)/(2*d) +
 (Sinh[c + d*x]^4*(b + a*Tanh[c + d*x]))/(4*d) - (Sinh[c + d*x]^2*Tanh[c +
d*x]*(a + 8*b*Tanh[c + d*x]))/(8*d)
```


Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
```

```
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+bx^3\right)}{\left(1-x^2\right)^3}dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\sinh^4(c+dx)(b+a\tanh(c+dx))}{4d} + \frac{\text{Subst}\left(\int \frac{x^3\left(-4b-ax-4bx^2\right)}{\left(1-x^2\right)^2}dx, x, \tanh\right.}{4d}$$

$$= \frac{\sinh^4(c+dx)(b+a\tanh(c+dx))}{4d} - \frac{\sinh^2(c+dx)\tanh(c+dx)(a+8b}{8d}$$

$$= \frac{\sinh^4(c+dx)(b+a\tanh(c+dx))}{4d} - \frac{\sinh^2(c+dx)\tanh(c+dx)(a+8b}{8d}$$

$$= -\frac{3a\tanh(c+dx)}{8d} - \frac{3b\tanh^2(c+dx)}{2d} + \frac{\sinh^4(c+dx)(b+a\tanh(c+d}{4d}$$

$$= -\frac{3a\tanh(c+dx)}{8d} - \frac{3b\tanh^2(c+dx)}{2d} + \frac{\sinh^4(c+dx)(b+a\tanh(c+d}{4d}$$

$$= -\frac{3(a+8b)\log(1-\tanh(c+dx))}{16d} + \frac{3(a-8b)\log(1+\tanh(c+dx))}{16d} - 3$$

Mathematica [A] time = 0.190275, size = 92, normalized size = 0.7

$$\frac{3a(c+dx)}{8d} - \frac{a\sinh(2(c+dx))}{4d} + \frac{a\sinh(4(c+dx))}{32d} + \frac{b\left(\sinh^4(c+dx) - 4\sinh^2(c+dx) + 2\text{sech}^2(c+dx) + 12\log(c\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (3*a*(c + d*x))/(8*d) + (b*(12*Log[Cosh[c + d*x]] + 2*Sech[c + d*x]^2 - 4*S
inh[c + d*x]^2 + Sinh[c + d*x]^4))/(4*d) - (a*Sinh[2*(c + d*x)])/(4*d) + (a
*Sinh[4*(c + d*x)])/(32*d)
```

Maple [A] time = 0.047, size = 122, normalized size = 0.9

$$\frac{a\cosh(dx+c)(\sinh(dx+c))^3}{4d} - \frac{3a\cosh(dx+c)\sinh(dx+c)}{8d} + \frac{3ax}{8} + \frac{3ac}{8d} + \frac{b(\sinh(dx+c))^6}{4d(\cosh(dx+c))^2} - \frac{3b(\sinh(dx}{4d(\cosh(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3),x)
```

```
[Out] 1/4/d*a*cosh(d*x+c)*sinh(d*x+c)^3-3/8/d*a*cosh(d*x+c)*sinh(d*x+c)+3/8*a*x+3
/8/d*a*c+1/4/d*b*sinh(d*x+c)^6/cosh(d*x+c)^2-3/4/d*b*sinh(d*x+c)^4/cosh(d*x
```

```
+c)^2+3*b*ln(cosh(d*x+c))/d-3/2*b*tanh(d*x+c)^2/d
```

Maxima [A] time = 1.60844, size = 262, normalized size = 1.98

$$\frac{1}{64}a\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)+\frac{1}{64}b\left(\frac{192(dx+c)}{d}-\frac{20e^{(-2dx-2c)}-e^{(-4dx-4c)}}{d}+\frac{192\,\mathrm{lo}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/64*a*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)
/d - e^(-4*d*x - 4*c)/d) + 1/64*b*(192*(d*x + c)/d - (20*e^(-2*d*x - 2*c) -
 e^(-4*d*x - 4*c))/d + 192*log(e^(-2*d*x - 2*c) + 1)/d - (18*e^(-2*d*x - 2*
c) + 39*e^(-4*d*x - 4*c) - 108*e^(-6*d*x - 6*c) - 1)/(d*(e^(-4*d*x - 4*c) +
 2*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c))))
```

Fricas [B] time = 2.49029, size = 4209, normalized size = 31.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/64*((a + b)*cosh(d*x + c)^12 + 12*(a + b)*cosh(d*x + c)*sinh(d*x + c)^11
+ (a + b)*sinh(d*x + c)^12 - 6*(a + 3*b)*cosh(d*x + c)^10 + 6*(11*(a + b)*c
osh(d*x + c)^2 - a - 3*b)*sinh(d*x + c)^10 + 20*(11*(a + b)*cosh(d*x + c)^3
 - 3*(a + 3*b)*cosh(d*x + c))*sinh(d*x + c)^9 + 3*(8*(a - 8*b)*d*x - 5*a -
13*b)*cosh(d*x + c)^8 + 3*(165*(a + b)*cosh(d*x + c)^4 + 8*(a - 8*b)*d*x -
90*(a + 3*b)*cosh(d*x + c)^2 - 5*a - 13*b)*sinh(d*x + c)^8 + 24*(33*(a + b)
*cosh(d*x + c)^5 - 30*(a + 3*b)*cosh(d*x + c)^3 + (8*(a - 8*b)*d*x - 5*a -
13*b)*cosh(d*x + c))*sinh(d*x + c)^7 + 8*(6*(a - 8*b)*d*x + 11*b)*cosh(d*x
+ c)^6 + 4*(231*(a + b)*cosh(d*x + c)^6 - 315*(a + 3*b)*cosh(d*x + c)^4 + 1
2*(a - 8*b)*d*x + 21*(8*(a - 8*b)*d*x - 5*a - 13*b)*cosh(d*x + c)^2 + 22*b)
*sinh(d*x + c)^6 + 24*(33*(a + b)*cosh(d*x + c)^7 - 63*(a + 3*b)*cosh(d*x +
 c)^5 + 7*(8*(a - 8*b)*d*x - 5*a - 13*b)*cosh(d*x + c)^3 + 2*(6*(a - 8*b)*d
*x + 11*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 3*(8*(a - 8*b)*d*x + 5*a - 13*b
)*cosh(d*x + c)^4 + 3*(165*(a + b)*cosh(d*x + c)^8 - 420*(a + 3*b)*cosh(d*x
 + c)^6 + 70*(8*(a - 8*b)*d*x - 5*a - 13*b)*cosh(d*x + c)^4 + 8*(a - 8*b)*d
*x + 40*(6*(a - 8*b)*d*x + 11*b)*cosh(d*x + c)^2 + 5*a - 13*b)*sinh(d*x + c
)^4 + 4*(55*(a + b)*cosh(d*x + c)^9 - 180*(a + 3*b)*cosh(d*x + c)^7 + 42*(8
*(a - 8*b)*d*x - 5*a - 13*b)*cosh(d*x + c)^5 + 40*(6*(a - 8*b)*d*x + 11*b)*
cosh(d*x + c)^3 + 3*(8*(a - 8*b)*d*x + 5*a - 13*b)*cosh(d*x + c))*sinh(d*x
+ c)^3 + 6*(a - 3*b)*cosh(d*x + c)^2 + 6*(11*(a + b)*cosh(d*x + c)^10 - 45*
(a + 3*b)*cosh(d*x + c)^8 + 14*(8*(a - 8*b)*d*x - 5*a - 13*b)*cosh(d*x + c)
^6 + 20*(6*(a - 8*b)*d*x + 11*b)*cosh(d*x + c)^4 + 3*(8*(a - 8*b)*d*x + 5*a
 - 13*b)*cosh(d*x + c)^2 + a - 3*b)*sinh(d*x + c)^2 + 192*(b*cosh(d*x + c)^
8 + 8*b*cosh(d*x + c)*sinh(d*x + c)^7 + b*sinh(d*x + c)^8 + 2*b*cosh(d*x +
c)^6 + 2*(14*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^6 + 4*(14*b*cosh(d*x + c)
^3 + 3*b*cosh(d*x + c))*sinh(d*x + c)^5 + b*cosh(d*x + c)^4 + (70*b*cosh(d*
x + c)^4 + 30*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^4 + 4*(14*b*cosh(d*x + c
)^5 + 10*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(14*b*cos
h(d*x + c)^6 + 15*b*cosh(d*x + c)^4 + 3*b*cosh(d*x + c)^2)*sinh(d*x + c)^2
```

```
+ 4*(2*b*cosh(d*x + c)^7 + 3*b*cosh(d*x + c)^5 + b*cosh(d*x + c)^3)*sinh(d*
x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 12*((a + b)*
cosh(d*x + c)^11 - 5*(a + 3*b)*cosh(d*x + c)^9 + 2*(8*(a - 8*b)*d*x - 5*a -
 13*b)*cosh(d*x + c)^7 + 4*(6*(a - 8*b)*d*x + 11*b)*cosh(d*x + c)^5 + (8*(a
 - 8*b)*d*x + 5*a - 13*b)*cosh(d*x + c)^3 + (a - 3*b)*cosh(d*x + c))*sinh(d
*x + c) - a + b)/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d
*sinh(d*x + c)^8 + 2*d*cosh(d*x + c)^6 + 2*(14*d*cosh(d*x + c)^2 + d)*sinh(
d*x + c)^6 + 4*(14*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 +
 d*cosh(d*x + c)^4 + (70*d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 + d)*sinh
(d*x + c)^4 + 4*(14*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + d*cosh(d*x +
 c))*sinh(d*x + c)^3 + 2*(14*d*cosh(d*x + c)^6 + 15*d*cosh(d*x + c)^4 + 3*d
*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(2*d*cosh(d*x + c)^7 + 3*d*cosh(d*x +
 c)^5 + d*cosh(d*x + c)^3)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4*(a+b*tanh(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37402, size = 278, normalized size = 2.11

$$\frac{24\,(a-8\,b)dx+\left(ae^{(4\,dx+24\,c)}+be^{(4\,dx+24\,c)}-8\,ae^{(2\,dx+22\,c)}-20\,be^{(2\,dx+22\,c)}\right)e^{(-20\,c)}+192\,b\log\left(e^{(2\,dx+2\,c)}+1\right)-\frac{\left(9\,ae^{(8\,dx+8}\right.}{}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/64*(24*(a - 8*b)*d*x + (a*e^(4*d*x + 24*c) + b*e^(4*d*x + 24*c) - 8*a*e^(
2*d*x + 22*c) - 20*b*e^(2*d*x + 22*c))*e^(-20*c) + 192*b*log(e^(2*d*x + 2*c
) + 1) - (9*a*e^(8*d*x + 8*c) + 72*b*e^(8*d*x + 8*c) + 10*a*e^(6*d*x + 6*c)
 + 36*b*e^(6*d*x + 6*c) - 6*a*e^(4*d*x + 4*c) + 111*b*e^(4*d*x + 4*c) - 6*a
*e^(2*d*x + 2*c) + 18*b*e^(2*d*x + 2*c) + a - b)*e^(-4*c)/(e^(2*d*x) + e^(4
*d*x + 2*c))^2)/d
```

3.50 $\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)dx$

Optimal. Leaf size=98

$$\frac{a\cosh^3(c+dx)}{3d}-\frac{a\cosh(c+dx)}{d}+\frac{5b\sinh^3(c+dx)}{6d}-\frac{5b\sinh(c+dx)}{2d}-\frac{b\sinh^3(c+dx)\tanh^2(c+dx)}{2d}+\frac{5b\tan^{-1}(\sin}{2d}$$

```
[Out] (5*b*ArcTan[Sinh[c + d*x]])/(2*d) - (a*Cosh[c + d*x])/d + (a*Cosh[c + d*x]^
3)/(3*d) - (5*b*Sinh[c + d*x])/(2*d) + (5*b*Sinh[c + d*x]^3)/(6*d) - (b*Sin
h[c + d*x]^3*Tanh[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.119697, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3666, 2633, 2592, 288, 302, 203}

$$\frac{a\cosh^3(c+dx)}{3d}-\frac{a\cosh(c+dx)}{d}+\frac{5b\sinh^3(c+dx)}{6d}-\frac{5b\sinh(c+dx)}{2d}-\frac{b\sinh^3(c+dx)\tanh^2(c+dx)}{2d}+\frac{5b\tan^{-1}(\sin}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^3),x]
```


```
[Out] (5*b*ArcTan[Sinh[c + d*x]])/(2*d) - (a*Cosh[c + d*x])/d + (a*Cosh[c + d*x]^
3)/(3*d) - (5*b*Sinh[c + d*x])/(2*d) + (5*b*Sinh[c + d*x]^3)/(6*d) - (b*Sin
h[c + d*x]^3*Tanh[c + d*x]^2)/(2*d)
```


Rule 3666

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)dx = i\int \left(-ia\sinh^3(c+dx)-ib\sinh^3(c+dx)\tanh^3(c+dx)\right)dx$$

$$= a\int \sinh^3(c+dx)\,dx + b\int \sinh^3(c+dx)\tanh^3(c+dx)\,dx$$

$$= -\frac{a\,\text{Subst}\left(\int \left(1-x^2\right)dx, x, \cosh(c+dx)\right)}{d} + \frac{b\,\text{Subst}\left(\int \frac{x^6}{\left(1+x^2\right)^2}dx, x, \sin\right.}{d}$$

$$= -\frac{a\cosh(c+dx)}{d} + \frac{a\cosh^3(c+dx)}{3d} - \frac{b\sinh^3(c+dx)\tanh^2(c+dx)}{2d} + \frac{(5}{}$$

$$= -\frac{a\cosh(c+dx)}{d} + \frac{a\cosh^3(c+dx)}{3d} - \frac{b\sinh^3(c+dx)\tanh^2(c+dx)}{2d} + \frac{(5}{}$$

$$= -\frac{a\cosh(c+dx)}{d} + \frac{a\cosh^3(c+dx)}{3d} - \frac{5b\sinh(c+dx)}{2d} + \frac{5b\sinh^3(c+dx}{6d}$$

$$= \frac{5b\tan^{-1}(\sinh(c+dx))}{2d} - \frac{a\cosh(c+dx)}{d} + \frac{a\cosh^3(c+dx)}{3d} - \frac{5b\sinh(c}{2d}$$

Mathematica [A] time = 0.257416, size = 104, normalized size = 1.06

$$-\frac{3a\cosh(c+dx)}{4d} + \frac{a\cosh(3(c+dx))}{12d} + \frac{b\sinh^3(c+dx)\tanh^2(c+dx)}{3d} - \frac{5b\left(2\sinh(c+dx)\tanh^2(c+dx)-3\left(\tan^{-}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (-3*a*Cosh[c + d*x])/(4*d) + (a*Cosh[3*(c + d*x)])/(12*d) + (b*Sinh[c + d*x
]^3*Tanh[c + d*x]^2)/(3*d) - (5*b*(2*Sinh[c + d*x]*Tanh[c + d*x]^2 - 3*(Arc
Tan[Sinh[c + d*x]] - Sech[c + d*x]*Tanh[c + d*x])))/(6*d)
```

Maple [A] time = 0.043, size = 129, normalized size = 1.3

$$-\frac{2a\cosh(dx+c)}{3d} + \frac{a\cosh(dx+c)(\sinh(dx+c))^2}{3d} + \frac{b(\sinh(dx+c))^5}{3d(\cosh(dx+c))^2} - \frac{5b(\sinh(dx+c))^3}{3d(\cosh(dx+c))^2} - 5\frac{b\sinh(dx+c}{d(\cosh(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3),x)
```

```
[Out] -2/3*a*cosh(d*x+c)/d+1/3/d*a*cosh(d*x+c)*sinh(d*x+c)^2+1/3/d*b*sinh(d*x+c)^
5/cosh(d*x+c)^2-5/3/d*b*sinh(d*x+c)^3/cosh(d*x+c)^2-5/d*b*sinh(d*x+c)/cosh(
```

```
d*x+c)^2+5/2*b*sech(d*x+c)*tanh(d*x+c)/d+5/d*b*arctan(exp(d*x+c))
```

Maxima [A] time = 1.5411, size = 235, normalized size = 2.4

$$\frac{1}{24}b\left(\frac{27e^{(-dx-c)}-e^{(-3dx-3c)}}{d}-\frac{120\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{25e^{(-2dx-2c)}+77e^{(-4dx-4c)}+3e^{(-6dx-6c)}-1}{d\left(e^{(-3dx-3c)}+2e^{(-5dx-5c)}+e^{(-7dx-7c)}\right)}\right)+\frac{1}{24}a\left(\frac{e^{(3dx+3c)}}{d}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/24*b*((27*e^(-d*x - c) - e^(-3*d*x - 3*c))/d - 120*arctan(e^(-d*x - c))/d
 - (25*e^(-2*d*x - 2*c) + 77*e^(-4*d*x - 4*c) + 3*e^(-6*d*x - 6*c) - 1)/(d*
(e^(-3*d*x - 3*c) + 2*e^(-5*d*x - 5*c) + e^(-7*d*x - 7*c)))) + 1/24*a*(e^(3
*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

Fricas [B] time = 2.33449, size = 2942, normalized size = 30.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/24*((a + b)*cosh(d*x + c)^10 + 10*(a + b)*cosh(d*x + c)*sinh(d*x + c)^9 +
 (a + b)*sinh(d*x + c)^10 - (7*a + 25*b)*cosh(d*x + c)^8 + (45*(a + b)*cosh
(d*x + c)^2 - 7*a - 25*b)*sinh(d*x + c)^8 + 8*(15*(a + b)*cosh(d*x + c)^3 -
 (7*a + 25*b)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(13*a + 25*b)*cosh(d*x + c
)^6 + 2*(105*(a + b)*cosh(d*x + c)^4 - 14*(7*a + 25*b)*cosh(d*x + c)^2 - 13
*a - 25*b)*sinh(d*x + c)^6 + 4*(63*(a + b)*cosh(d*x + c)^5 - 14*(7*a + 25*b
)*cosh(d*x + c)^3 - 3*(13*a + 25*b)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(13*
a - 25*b)*cosh(d*x + c)^4 + 2*(105*(a + b)*cosh(d*x + c)^6 - 35*(7*a + 25*b
)*cosh(d*x + c)^4 - 15*(13*a + 25*b)*cosh(d*x + c)^2 - 13*a + 25*b)*sinh(d*
x + c)^4 + 8*(15*(a + b)*cosh(d*x + c)^7 - 7*(7*a + 25*b)*cosh(d*x + c)^5 -
 5*(13*a + 25*b)*cosh(d*x + c)^3 - (13*a - 25*b)*cosh(d*x + c))*sinh(d*x +
c)^3 - (7*a - 25*b)*cosh(d*x + c)^2 + (45*(a + b)*cosh(d*x + c)^8 - 28*(7*a
 + 25*b)*cosh(d*x + c)^6 - 30*(13*a + 25*b)*cosh(d*x + c)^4 - 12*(13*a - 25
*b)*cosh(d*x + c)^2 - 7*a + 25*b)*sinh(d*x + c)^2 + 120*(b*cosh(d*x + c)^7
+ 7*b*cosh(d*x + c)*sinh(d*x + c)^6 + b*sinh(d*x + c)^7 + 2*b*cosh(d*x + c)
^5 + (21*b*cosh(d*x + c)^2 + 2*b)*sinh(d*x + c)^5 + 5*(7*b*cosh(d*x + c)^3
+ 2*b*cosh(d*x + c))*sinh(d*x + c)^4 + b*cosh(d*x + c)^3 + (35*b*cosh(d*x +
 c)^4 + 20*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^3 + (21*b*cosh(d*x + c)^5 +
 20*b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c))*sinh(d*x + c)^2 + (7*b*cosh(d*x
+ c)^6 + 10*b*cosh(d*x + c)^4 + 3*b*cosh(d*x + c)^2)*sinh(d*x + c))*arctan(
cosh(d*x + c) + sinh(d*x + c)) + 2*(5*(a + b)*cosh(d*x + c)^9 - 4*(7*a + 25
*b)*cosh(d*x + c)^7 - 6*(13*a + 25*b)*cosh(d*x + c)^5 - 4*(13*a - 25*b)*cos
h(d*x + c)^3 - (7*a - 25*b)*cosh(d*x + c))*sinh(d*x + c) + a - b)/(d*cosh(d
*x + c)^7 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x + c)^7 + 2*d*cos
h(d*x + c)^5 + (21*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^5 + 5*(7*d*cosh(d
*x + c)^3 + 2*d*cosh(d*x + c))*sinh(d*x + c)^4 + d*cosh(d*x + c)^3 + (35*d*
cosh(d*x + c)^4 + 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^3 + (21*d*cosh(d*
x + c)^5 + 20*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + (7*d
*cosh(d*x + c)^6 + 10*d*cosh(d*x + c)^4 + 3*d*cosh(d*x + c)^2)*sinh(d*x + c
))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3*(a+b*tanh(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29955, size = 192, normalized size = 1.96

$$\frac{120\,b\arctan\left(e^{(dx+c)}\right) - \left(9\,ae^{(2\,dx+2\,c)} - 27\,be^{(2\,dx+2\,c)} - a + b\right)e^{(-3\,dx-3\,c)} + \left(ae^{(3\,dx+30\,c)} + be^{(3\,dx+30\,c)} - 9\,ae^{(dx+28\,c)} - 27\,b$$
$$24\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/24*(120*b*arctan(e^(d*x + c)) - (9*a*e^(2*d*x + 2*c) - 27*b*e^(2*d*x + 2*
c) - a + b)*e^(-3*d*x - 3*c) + (a*e^(3*d*x + 30*c) + b*e^(3*d*x + 30*c) - 9
*a*e^(d*x + 28*c) - 27*b*e^(d*x + 28*c))*e^(-27*c) - 24*(b*e^(3*d*x + 3*c)
- b*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^2)/d
```

3.51 $\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)dx$

Optimal. Leaf size=100

$$\frac{(a+4b)\log(1-\tanh(c+dx))}{4d}-\frac{(a-4b)\log(\tanh(c+dx)+1)}{4d}+\frac{\sinh^2(c+dx)(a\tanh(c+dx)+b)}{2d}+\frac{a\tanh(c+dx)}{2d}$$

```
[Out] ((a + 4*b)*Log[1 - Tanh[c + d*x]])/(4*d) - ((a - 4*b)*Log[1 + Tanh[c + d*x]
])/(4*d) + (a*Tanh[c + d*x])/(2*d) + (b*Tanh[c + d*x]^2)/(2*d) + (Sinh[c +
d*x]^2*(b + a*Tanh[c + d*x]))/(2*d)
```

Rubi [A] time = 0.117426, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3663, 1804, 1802, 633, 31}

$$\frac{(a+4b)\log(1-\tanh(c+dx))}{4d}-\frac{(a-4b)\log(\tanh(c+dx)+1)}{4d}+\frac{\sinh^2(c+dx)(a\tanh(c+dx)+b)}{2d}+\frac{a\tanh(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] ((a + 4*b)*Log[1 - Tanh[c + d*x]])/(4*d) - ((a - 4*b)*Log[1 + Tanh[c + d*x]
])/(4*d) + (a*Tanh[c + d*x])/(2*d) + (b*Tanh[c + d*x]^2)/(2*d) + (Sinh[c +
d*x]^2*(b + a*Tanh[c + d*x]))/(2*d)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx^3)}{(1-x^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\sinh^2(c+dx)(b+a\tanh(c+dx))}{2d} + \frac{\text{Subst}\left(\int \frac{x(-2b-ax-2bx^2)}{1-x^2}\,dx, x, \tanh\right.}{2d}$$

$$= \frac{\sinh^2(c+dx)(b+a\tanh(c+dx))}{2d} + \frac{\text{Subst}\left(\int \left(a+2bx-\frac{a+4bx}{1-x^2}\right)dx, x, \text{t}\right.}{2d}$$

$$= \frac{a\tanh(c+dx)}{2d} + \frac{b\tanh^2(c+dx)}{2d} + \frac{\sinh^2(c+dx)(b+a\tanh(c+dx))}{2d}$$

$$= \frac{a\tanh(c+dx)}{2d} + \frac{b\tanh^2(c+dx)}{2d} + \frac{\sinh^2(c+dx)(b+a\tanh(c+dx))}{2d}$$

$$= \frac{(a+4b)\log(1-\tanh(c+dx))}{4d} - \frac{(a-4b)\log(1+\tanh(c+dx))}{4d} + \frac{a\tan}{}$$

Mathematica [A] time = 0.106663, size = 69, normalized size = 0.69

$$\frac{a(-c-dx)}{2d} + \frac{a\sinh(2(c+dx))}{4d} - \frac{b\left(-\sinh^2(c+dx)+\text{sech}^2(c+dx)+4\log(\cosh(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (a*(-c - d*x))/(2*d) - (b*(4*Log[Cosh[c + d*x]] + Sech[c + d*x]^2 - Sinh[c
+ d*x]^2))/(2*d) + (a*Sinh[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.041, size = 79, normalized size = 0.8

$$\frac{a\cosh(dx+c)\sinh(dx+c)}{2d} - \frac{ax}{2} - \frac{ac}{2d} + \frac{b(\sinh(dx+c))^4}{2d(\cosh(dx+c))^2} - 2\frac{b\ln(\cosh(dx+c))}{d} + \frac{b(\tanh(dx+c))^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3),x)
```

```
[Out] 1/2/d*a*cosh(d*x+c)*sinh(d*x+c)-1/2*a*x-1/2/d*a*c+1/2/d*b*sinh(d*x+c)^4/cos
h(d*x+c)^2-2*b*ln(cosh(d*x+c))/d+b*tanh(d*x+c)^2/d
```

Maxima [A] time = 1.58441, size = 190, normalized size = 1.9

$$-\frac{1}{8}a\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)-\frac{1}{8}b\left(\frac{16(dx+c)}{d}-\frac{e^{(-2dx-2c)}}{d}+\frac{16\log\left(e^{(-2dx-2c)}+1\right)}{d}-\frac{2e^{(-2dx-2c)}-15e^{(-}}{d\left(e^{(-2dx-2c)}+2e^{(-4dx-4}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -1/8*a*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/8*b*(16*(d*x + c)
/d - e^(-2*d*x - 2*c)/d + 16*log(e^(-2*d*x - 2*c) + 1)/d - (2*e^(-2*d*x - 2
*c) - 15*e^(-4*d*x - 4*c) + 1)/(d*(e^(-2*d*x - 2*c) + 2*e^(-4*d*x - 4*c) +
e^(-6*d*x - 6*c))))
```

Fricas [B] time = 2.35088, size = 2453, normalized size = 24.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/8*((a + b)*cosh(d*x + c)^8 + 8*(a + b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a
 + b)*sinh(d*x + c)^8 - 2*(2*(a - 4*b)*d*x - a - b)*cosh(d*x + c)^6 - 2*(2*
(a - 4*b)*d*x - 14*(a + b)*cosh(d*x + c)^2 - a - b)*sinh(d*x + c)^6 + 4*(14
*(a + b)*cosh(d*x + c)^3 - 3*(2*(a - 4*b)*d*x - a - b)*cosh(d*x + c))*sinh(
d*x + c)^5 - 2*(4*(a - 4*b)*d*x + 7*b)*cosh(d*x + c)^4 + 2*(35*(a + b)*cosh
(d*x + c)^4 - 4*(a - 4*b)*d*x - 15*(2*(a - 4*b)*d*x - a - b)*cosh(d*x + c)^
2 - 7*b)*sinh(d*x + c)^4 + 8*(7*(a + b)*cosh(d*x + c)^5 - 5*(2*(a - 4*b)*d*
x - a - b)*cosh(d*x + c)^3 - (4*(a - 4*b)*d*x + 7*b)*cosh(d*x + c))*sinh(d*
x + c)^3 - 2*(2*(a - 4*b)*d*x + a - b)*cosh(d*x + c)^2 + 2*(14*(a + b)*cosh
(d*x + c)^6 - 15*(2*(a - 4*b)*d*x - a - b)*cosh(d*x + c)^4 - 2*(a - 4*b)*d*
x - 6*(4*(a - 4*b)*d*x + 7*b)*cosh(d*x + c)^2 - a + b)*sinh(d*x + c)^2 - 16
*(b*cosh(d*x + c)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b*sinh(d*x + c)^6
 + 2*b*cosh(d*x + c)^4 + (15*b*cosh(d*x + c)^2 + 2*b)*sinh(d*x + c)^4 + 4*(
5*b*cosh(d*x + c)^3 + 2*b*cosh(d*x + c))*sinh(d*x + c)^3 + b*cosh(d*x + c)^
2 + (15*b*cosh(d*x + c)^4 + 12*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 2*(
3*b*cosh(d*x + c)^5 + 4*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c))
*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(2*(a + b)*cosh(d
*x + c)^7 - 3*(2*(a - 4*b)*d*x - a - b)*cosh(d*x + c)^5 - 2*(4*(a - 4*b)*d*
x + 7*b)*cosh(d*x + c)^3 - (2*(a - 4*b)*d*x + a - b)*cosh(d*x + c))*sinh(d*
x + c) - a + b)/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*
sinh(d*x + c)^6 + 2*d*cosh(d*x + c)^4 + (15*d*cosh(d*x + c)^2 + 2*d)*sinh(d
*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 + 2*d*cosh(d*x + c))*sinh(d*x + c)^3 + d
*cosh(d*x + c)^2 + (15*d*cosh(d*x + c)^4 + 12*d*cosh(d*x + c)^2 + d)*sinh(d
*x + c)^2 + 2*(3*d*cosh(d*x + c)^5 + 4*d*cosh(d*x + c)^3 + d*cosh(d*x + c))
*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b \tanh^3(c + dx)\right) \sinh^2(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*tanh(d*x+c)**3),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)*sinh(c + d*x)**2, x)
```

Giac [A] time = 1.26631, size = 192, normalized size = 1.92

$$-\frac{4\,(a-4\,b)dx-\left(2\,ae^{(2\,dx+2\,c)}-8\,be^{(2\,dx+2\,c)}-a+b\right)e^{(-2\,dx-2\,c)}-\left(ae^{(2\,dx+10\,c)}+be^{(2\,dx+10\,c)}\right)e^{(-8\,c)}+16\,b\log\left(e^{(2\,dx+2\,c)}+\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -1/8*(4*(a - 4*b)*d*x - (2*a*e^(2*d*x + 2*c) - 8*b*e^(2*d*x + 2*c) - a + b)
*e^(-2*d*x - 2*c) - (a*e^(2*d*x + 10*c) + b*e^(2*d*x + 10*c))*e^(-8*c) + 16
*b*log(e^(2*d*x + 2*c) + 1) - 8*(3*b*e^(4*d*x + 4*c) + 4*b*e^(2*d*x + 2*c)
+ 3*b)/(e^(2*d*x + 2*c) + 1)^2)/d
```

3.52 $\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$

Optimal. Leaf size=63

$$\frac{a\cosh(c+dx)}{d}+\frac{3b\sinh(c+dx)}{2d}-\frac{3b\tan^{-1}(\sinh(c+dx))}{2d}-\frac{b\sinh(c+dx)\tanh^2(c+dx)}{2d}$$

```
[Out] (-3*b*ArcTan[Sinh[c + d*x]])/(2*d) + (a*Cosh[c + d*x])/d + (3*b*Sinh[c + d*
x])/(2*d) - (b*Sinh[c + d*x]*Tanh[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.0761299, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3666, 2638, 2592, 288, 321, 203}

$$\frac{a\cosh(c+dx)}{d}+\frac{3b\sinh(c+dx)}{2d}-\frac{3b\tan^{-1}(\sinh(c+dx))}{2d}-\frac{b\sinh(c+dx)\tanh^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (-3*b*ArcTan[Sinh[c + d*x]])/(2*d) + (a*Cosh[c + d*x])/d + (3*b*Sinh[c + d*
x])/(2*d) - (b*Sinh[c + d*x]*Tanh[c + d*x]^2)/(2*d)
```

Rule 3666

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)dx &= -\left(i\int\left(ia\sinh(c+dx)+ib\sinh(c+dx)\tanh^3(c+dx)\right)dx\right)\\ &= a\int\sinh(c+dx)\,dx+b\int\sinh(c+dx)\tanh^3(c+dx)\,dx\\ &= \frac{a\cosh(c+dx)}{d}+\frac{b\,\mathrm{Subst}\left(\int\frac{x^4}{\left(1+x^2\right)^2}dx,x,\sinh(c+dx)\right)}{d}\\ &= \frac{a\cosh(c+dx)}{d}-\frac{b\sinh(c+dx)\tanh^2(c+dx)}{2d}+\frac{(3b)\,\mathrm{Subst}\left(\int\frac{x^2}{1+x^2}dx,x\right.}{2d}\\ &= \frac{a\cosh(c+dx)}{d}+\frac{3b\sinh(c+dx)}{2d}-\frac{b\sinh(c+dx)\tanh^2(c+dx)}{2d}-\frac{(3b)\,\mathrm{S}}{}\\ &= -\frac{3b\tan^{-1}(\sinh(c+dx))}{2d}+\frac{a\cosh(c+dx)}{d}+\frac{3b\sinh(c+dx)}{2d}-\frac{b\sinh(c}{}\end{aligned}$$

Mathematica [A] time = 0.120791, size = 72, normalized size = 1.14

$$\frac{a\sinh(c)\sinh(dx)}{d}+\frac{a\cosh(c)\cosh(dx)}{d}+\frac{b\sinh(c+dx)\tanh^2(c+dx)}{d}-\frac{3b\left(\tan^{-1}(\sinh(c+dx))-\tanh(c+dx)\mathrm{se}\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (a*Cosh[c]*Cosh[d*x])/d + (a*Sinh[c]*Sinh[d*x])/d + (b*Sinh[c + d*x]*Tanh[c
 + d*x]^2)/d - (3*b*(ArcTan[Sinh[c + d*x]] - Sech[c + d*x]*Tanh[c + d*x]))/
(2*d)
```

Maple [A] time = 0.04, size = 85, normalized size = 1.4

$$\frac{a\cosh(dx+c)}{d}+\frac{b(\sinh(dx+c))^3}{d(\cosh(dx+c))^2}+3\frac{b\sinh(dx+c)}{d(\cosh(dx+c))^2}-\frac{3b\,\mathrm{sech}(dx+c)\tanh(dx+c)}{2d}-3\frac{b\arctan\left(e^{dx+c}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*(a+b*tanh(d*x+c)^3),x)
```

```
[Out] a*cosh(d*x+c)/d+1/d*b*sinh(d*x+c)^3/cosh(d*x+c)^2+3/d*b*sinh(d*x+c)/cosh(d*
x+c)^2-3/2*b*sech(d*x+c)*tanh(d*x+c)/d-3/d*b*arctan(exp(d*x+c))
```

Maxima [A] time = 1.61063, size = 142, normalized size = 2.25

$$\frac{1}{2}\,b\left(\frac{6\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}}{d}+\frac{4\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}+1}{d\left(e^{(-dx-c)}+2\,e^{(-3\,dx-3\,c)}+e^{(-5\,dx-5\,c)}\right)}\right)+\frac{a\cosh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/2*b*(6*arctan(e^(-d*x - c))/d - e^(-d*x - c)/d + (4*e^(-2*d*x - 2*c) - e^
(-4*d*x - 4*c) + 1)/(d*(e^(-d*x - c) + 2*e^(-3*d*x - 3*c) + e^(-5*d*x - 5*c
)))) + a*cosh(d*x + c)/d
```

Fricas [B] time = 2.36054, size = 1473, normalized size = 23.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/2*((a + b)*cosh(d*x + c)^6 + 6*(a + b)*cosh(d*x + c)*sinh(d*x + c)^5 + (a
 + b)*sinh(d*x + c)^6 + 3*(a + b)*cosh(d*x + c)^4 + 3*(5*(a + b)*cosh(d*x +
 c)^2 + a + b)*sinh(d*x + c)^4 + 4*(5*(a + b)*cosh(d*x + c)^3 + 3*(a + b)*c
osh(d*x + c))*sinh(d*x + c)^3 + 3*(a - b)*cosh(d*x + c)^2 + 3*(5*(a + b)*co
sh(d*x + c)^4 + 6*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 - 6*(b*c
osh(d*x + c)^5 + 5*b*cosh(d*x + c)*sinh(d*x + c)^4 + b*sinh(d*x + c)^5 + 2*
b*cosh(d*x + c)^3 + 2*(5*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^3 + 2*(5*b*co
sh(d*x + c)^3 + 3*b*cosh(d*x + c))*sinh(d*x + c)^2 + b*cosh(d*x + c) + (5*b
*cosh(d*x + c)^4 + 6*b*cosh(d*x + c)^2 + b)*sinh(d*x + c))*arctan(cosh(d*x
+ c) + sinh(d*x + c)) + 6*((a + b)*cosh(d*x + c)^5 + 2*(a + b)*cosh(d*x + c
)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a - b)/(d*cosh(d*x + c)^5 + 5*
d*cosh(d*x + c)*sinh(d*x + c)^4 + d*sinh(d*x + c)^5 + 2*d*cosh(d*x + c)^3 +
 2*(5*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^3 + 2*(5*d*cosh(d*x + c)^3 + 3*d
*cosh(d*x + c))*sinh(d*x + c)^2 + d*cosh(d*x + c) + (5*d*cosh(d*x + c)^4 +
6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tanh^{3}\left(c+dx\right)\right)\sinh\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)**3),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)*sinh(c + d*x), x)
```

Giac [A] time = 1.21657, size = 128, normalized size = 2.03

$$-\frac{6\,b\arctan\left(e^{(dx+c)}\right)-(a-b)e^{(-dx-c)}-\left(ae^{(dx+8\,c)}+be^{(dx+8\,c)}\right)e^{(-7\,c)}-\frac{2\left(be^{(3\,dx+3\,c)}-be^{(dx+c)}\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -1/2*(6*b*arctan(e^(d*x + c)) - (a - b)*e^(-d*x - c) - (a*e^(d*x + 8*c) + b
*e^(d*x + 8*c))*e^(-7*c) - 2*(b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(e^(2*d*x
+ 2*c) + 1)^2)/d
```

3.53 $\int \mathbf{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)dx$

Optimal. Leaf size=49

$$-\frac{a\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b\tan^{-1}(\sinh(c+dx))}{2d}-\frac{b\tanh(c+dx)\mathbf{sech}(c+dx)}{2d}$$

```
[Out] (b*ArcTan[Sinh[c + d*x]])/(2*d) - (a*ArcTanh[Cosh[c + d*x]])/d - (b*Sech[c
+ d*x]*Tanh[c + d*x])/(2*d)
```

Rubi [A] time = 0.0743924, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3666, 3770, 2611}

$$-\frac{a\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b\tan^{-1}(\sinh(c+dx))}{2d}-\frac{b\tanh(c+dx)\mathbf{sech}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (b*ArcTan[Sinh[c + d*x]])/(2*d) - (a*ArcTanh[Cosh[c + d*x]])/d - (b*Sech[c
+ d*x]*Tanh[c + d*x])/(2*d)
```

Rule 3666

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\int \text{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)dx &= i\int \left(-ia\text{csch}(c+dx)-ib\mathbf{sech}(c+dx)\tanh^2(c+dx)\right)dx\\ &= a\int \text{csch}(c+dx)\,dx+b\int \mathbf{sech}(c+dx)\tanh^2(c+dx)\,dx\\ &= -\frac{a\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b\mathbf{sech}(c+dx)\tanh(c+dx)}{2d}+\frac{1}{2}b\int \mathbf{sech}(c+d\\ &= \frac{b\tan^{-1}(\sinh(c+dx))}{2d}-\frac{a\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b\mathbf{sech}(c+dx)\tanh(c+\end{aligned}$$

Mathematica [A] time = 0.0311553, size = 75, normalized size = 1.53

$$\frac{a\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{a\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{b\tan^{-1}(\sinh(c+dx))}{2d}-\frac{b\tanh(c+dx)\text{sech}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (b*ArcTan[Sinh[c + d*x]])/(2*d) - (a*Log[Cosh[c/2 + (d*x)/2]])/d + (a*Log[S
inh[c/2 + (d*x)/2]])/d - (b*Sech[c + d*x]*Tanh[c + d*x])/(2*d)
```

Maple [A] time = 0.049, size = 65, normalized size = 1.3

$$-2\,{\frac{a{\rm Artanh}\left({{\rm e}^{dx+c}}\right)}{d}}-{\frac{b\sinh\left(dx+c\right)}{d\left(\cosh\left(dx+c\right)\right)^{2}}}+{\frac{b{\rm sech}\left(dx+c\right)\tanh\left(dx+c\right)}{2\,d}}+{\frac{b\arctan\left({{\rm e}^{dx+c}}\right)}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*tanh(d*x+c)^3),x)
```

```
[Out] -2/d*a*arctanh(exp(d*x+c))-1/d*b*sinh(d*x+c)/cosh(d*x+c)^2+1/2*b*sech(d*x+c
)*tanh(d*x+c)/d+1/d*b*arctan(exp(d*x+c))
```

Maxima [A] time = 1.51615, size = 112, normalized size = 2.29

$$-b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1\right)}\right)+\frac{a\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -b*(arctan(e^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*
d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a*log(tanh(1/2*d*x + 1/2*c))/d
```

Fricas [B] time = 2.63424, size = 1453, normalized size = 29.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3
 - (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)
^4 + 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 4*
(b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) + b)*arctan(cosh(d*x +
c) + sinh(d*x + c)) - b*cosh(d*x + c) + (a*cosh(d*x + c)^4 + 4*a*cosh(d*x +
 c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*a*cosh(d*x + c)^2 + 2*(3*a*cosh
```

```
(d*x + c)^2 + a)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + a*cosh(d*x + c))*
sinh(d*x + c) + a)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (a*cosh(d*x + c
)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*a*cosh(d*x
+ c)^2 + 2*(3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3
 + a*cosh(d*x + c))*sinh(d*x + c) + a)*log(cosh(d*x + c) + sinh(d*x + c) -
1) + (3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh
(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*
d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x
+ c))*sinh(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tanh^3\left(c + dx\right)\right)\operatorname{csch}\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)**3),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)*csch(c + d*x), x)
```

Giac [A] time = 1.21699, size = 100, normalized size = 2.04

$$\frac{b\arctan\left(e^{(dx+c)}\right) - a\log\left(e^{(dx+c)} + 1\right) + a\log\left(\left|e^{(dx+c)} - 1\right|\right) - \frac{be^{(3\,dx+3\,c)} - be^{(dx+c)}}{\left(e^{(2\,dx+2\,c)} + 1\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] (b*arctan(e^(d*x + c)) - a*log(e^(d*x + c) + 1) + a*log(abs(e^(d*x + c) - 1
)) - (b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^2)/d
```

3.54 $\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$

Optimal. Leaf size=29

$$\frac{b\tanh^2(c+dx)}{2d}-\frac{a\coth(c+dx)}{d}$$

```
[Out] -((a*Coth[c + d*x])/d) + (b*Tanh[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.0341582, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3663, 14}

$$\frac{b\tanh^2(c+dx)}{2d}-\frac{a\coth(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] -((a*Coth[c + d*x])/d) + (b*Tanh[c + d*x]^2)/(2*d)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)dx &= \frac{\operatorname{Subst}\left(\int \frac{a+bx^3}{x^2}\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \left(\frac{a}{x^2}+bx\right)dx,x,\tanh(c+dx)\right)}{d}\\ &= -\frac{a\coth(c+dx)}{d}+\frac{b\tanh^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.0245542, size = 29, normalized size = 1.

$$-\frac{a\coth(c+dx)}{d}-\frac{b\operatorname{sech}^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^3),x]
```

[Out] -((a*Coth[c + d*x])/d) - (b*Sech[c + d*x]^2)/(2*d)

Maple [A] time = 0.05, size = 34, normalized size = 1.2

$$\frac{1}{d}\left(-\coth\left(dx+c\right)a+\frac{b\left(\sinh\left(dx+c\right)\right)^{2}}{2\left(\cosh\left(dx+c\right)\right)^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3),x)

[Out] 1/d*(-coth(d*x+c)*a+1/2*b*sinh(d*x+c)^2/cosh(d*x+c)^2)

Maxima [A] time = 1.04275, size = 59, normalized size = 2.03

$$\frac{2a}{d\left(e^{(-2dx-2c)}-1\right)}-\frac{2b}{d\left(e^{(dx+c)}+e^{(-dx-c)}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")

[Out] 2*a/(d*(e^(-2*d*x - 2*c) - 1)) - 2*b/(d*(e^(d*x + c) + e^(-d*x - c))^2)

Fricas [B] time = 2.44496, size = 375, normalized size = 12.93

$$-\frac{2\left((2a+b)\cosh\left(dx+c\right)^{2}+2b\cosh\left(dx+c\right)\sinh\left(dx+c\right)+(2a+b)\sinh\left(dx+c\right)^{2}+\right.}{d\cosh\left(dx+c\right)^{4}+6d\cosh\left(dx+c\right)^{2}\sinh\left(dx+c\right)^{2}+4d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}+d\sinh\left(dx+c\right)^{4}+4\left(d\cosh\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")

[Out] -2*((2*a + b)*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + (2*a + b)
*sinh(d*x + c)^2 + 2*a - b)/(d*cosh(d*x + c)^4 + 6*d*cosh(d*x + c)^2*sinh(d
*x + c)^2 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 + 4*(d*co
sh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) - d)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tanh^{3}\left(c+dx\right)\right)\operatorname{csch}^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**2*(a+b*tanh(d*x+c)**3),x)

[Out] Integral((a + b*tanh(c + d*x)**3)*csch(c + d*x)**2, x)

Giac [A] time = 1.2117, size = 61, normalized size = 2.1

$$-\frac{2\left(\frac{a}{e^{(2\,dx+2\,c)}-1}+\frac{be^{(2\,dx+2\,c)}}{\left(e^{(2\,dx+2\,c)}+1\right)^{2}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -2*(a/(e^(2*d*x + 2*c) - 1) + b*e^(2*d*x + 2*c)/(e^(2*d*x + 2*c) + 1)^2)/d
```

3.55 $\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)dx$

Optimal. Leaf size=71

$$\frac{a\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+\frac{b\tan^{-1}(\sinh(c+dx))}{2d}+\frac{b\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}$$

```
[Out] (b*ArcTan[Sinh[c + d*x]])/(2*d) + (a*ArcTanh[Cosh[c + d*x]])/(2*d) - (a*Cot
h[c + d*x]*Csch[c + d*x])/(2*d) + (b*Sech[c + d*x]*Tanh[c + d*x])/(2*d)
```

Rubi [A] time = 0.0904974, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3666, 3768, 3770}

$$\frac{a\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+\frac{b\tan^{-1}(\sinh(c+dx))}{2d}+\frac{b\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (b*ArcTan[Sinh[c + d*x]])/(2*d) + (a*ArcTanh[Cosh[c + d*x]])/(2*d) - (a*Cot
h[c + d*x]*Csch[c + d*x])/(2*d) + (b*Sech[c + d*x]*Tanh[c + d*x])/(2*d)
```

Rule 3666

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)dx &= -\left(i\int\left(ia\operatorname{csch}^3(c+dx)+ib\operatorname{sech}^3(c+dx)\right)dx\right)\\ &= a\int \operatorname{csch}^3(c+dx)\,dx+b\int \operatorname{sech}^3(c+dx)\,dx\\ &= -\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+\frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}-\frac{1}{2}a\int \operatorname{csch}(c\\ &= \frac{b\tan^{-1}(\sinh(c+dx))}{2d}+\frac{a\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a\coth(c+dx)\operatorname{csch}(c+\end{aligned}$$

Mathematica [A] time = 0.0259812, size = 95, normalized size = 1.34

$$-\frac{a\text{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a\text{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{b\tan^{-1}(\sinh(c+dx))}{2d}+\frac{b\tanh(c+dx)\text{sec}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (b*ArcTan[Sinh[c + d*x]])/(2*d) - (a*Csch[(c + d*x)/2]^2)/(8*d) - (a*Log[Ta
nh[(c + d*x)/2]])/(2*d) - (a*Sech[(c + d*x)/2]^2)/(8*d) + (b*Sech[c + d*x]*
Tanh[c + d*x])/(2*d)
```

Maple [A] time = 0.059, size = 62, normalized size = 0.9

$$-\frac{\coth(dx+c)\,a\text{csch}\,(dx+c)}{2\,d}+\frac{a\text{Artanh}\left(e^{dx+c}\right)}{d}+\frac{b\text{sech}\,(dx+c)\tanh(dx+c)}{2\,d}+\frac{b\arctan\left(e^{dx+c}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3),x)
```

```
[Out] -1/2*a*coth(d*x+c)*csch(d*x+c)/d+1/d*a*arctanh(exp(d*x+c))+1/2*b*sech(d*x+c
)*tanh(d*x+c)/d+1/d*b*arctan(exp(d*x+c))
```

Maxima [B] time = 1.53103, size = 211, normalized size = 2.97

$$-b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1\right)}\right)+\frac{1}{2}\,a\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+\right.}{d\left(2\,e^{(-2\,dx-2\,c)}-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -b*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*
d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + 1/2*a*(log(e^(-d*x - c) + 1)/d - log
(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x
- 2*c) - e^(-4*d*x - 4*c) - 1)))
```

Fricas [B] time = 2.79467, size = 3216, normalized size = 45.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a - b)*cosh(d*x + c)^7 + 14*(a - b)*cosh(d*x + c)*sinh(d*x + c)^6
+ 2*(a - b)*sinh(d*x + c)^7 + 6*(a + b)*cosh(d*x + c)^5 + 6*(7*(a - b)*cosh
(d*x + c)^2 + a + b)*sinh(d*x + c)^5 + 10*(7*(a - b)*cosh(d*x + c)^3 + 3*(a
```

```
 + b)*cosh(d*x + c))*sinh(d*x + c)^4 + 6*(a - b)*cosh(d*x + c)^3 + 2*(35*(a
 - b)*cosh(d*x + c)^4 + 30*(a + b)*cosh(d*x + c)^2 + 3*a - 3*b)*sinh(d*x +
c)^3 + 6*(7*(a - b)*cosh(d*x + c)^5 + 10*(a + b)*cosh(d*x + c)^3 + 3*(a - b
)*cosh(d*x + c))*sinh(d*x + c)^2 - 2*(b*cosh(d*x + c)^8 + 56*b*cosh(d*x + c
)^3*sinh(d*x + c)^5 + 28*b*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*b*cosh(d*x +
 c)*sinh(d*x + c)^7 + b*sinh(d*x + c)^8 - 2*b*cosh(d*x + c)^4 + 2*(35*b*cos
h(d*x + c)^4 - b)*sinh(d*x + c)^4 + 8*(7*b*cosh(d*x + c)^5 - b*cosh(d*x + c
))*sinh(d*x + c)^3 + 4*(7*b*cosh(d*x + c)^6 - 3*b*cosh(d*x + c)^2)*sinh(d*x
 + c)^2 + 8*(b*cosh(d*x + c)^7 - b*cosh(d*x + c)^3)*sinh(d*x + c) + b)*arct
an(cosh(d*x + c) + sinh(d*x + c)) + 2*(a + b)*cosh(d*x + c) - (a*cosh(d*x +
 c)^8 + 56*a*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*a*cosh(d*x + c)^2*sinh(d*
x + c)^6 + 8*a*cosh(d*x + c)*sinh(d*x + c)^7 + a*sinh(d*x + c)^8 - 2*a*cosh
(d*x + c)^4 + 2*(35*a*cosh(d*x + c)^4 - a)*sinh(d*x + c)^4 + 8*(7*a*cosh(d*
x + c)^5 - a*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a*cosh(d*x + c)^6 - 3*a*
cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(a*cosh(d*x + c)^7 - a*cosh(d*x + c)^3
)*sinh(d*x + c) + a)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + (a*cosh(d*x +
 c)^8 + 56*a*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*a*cosh(d*x + c)^2*sinh(d*
x + c)^6 + 8*a*cosh(d*x + c)*sinh(d*x + c)^7 + a*sinh(d*x + c)^8 - 2*a*cosh
(d*x + c)^4 + 2*(35*a*cosh(d*x + c)^4 - a)*sinh(d*x + c)^4 + 8*(7*a*cosh(d*
x + c)^5 - a*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a*cosh(d*x + c)^6 - 3*a*
cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(a*cosh(d*x + c)^7 - a*cosh(d*x + c)^3
)*sinh(d*x + c) + a)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(7*(a - b)*
cosh(d*x + c)^6 + 15*(a + b)*cosh(d*x + c)^4 + 9*(a - b)*cosh(d*x + c)^2 +
a + b)*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 56*d*cosh(d*x + c)^3*sinh(d*x +
c)^5 + 28*d*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*d*cosh(d*x + c)*sinh(d*x +
c)^7 + d*sinh(d*x + c)^8 - 2*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 -
d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 - d*cosh(d*x + c))*sinh(d*x + c
)^3 + 4*(7*d*cosh(d*x + c)^6 - 3*d*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(d*
cosh(d*x + c)^7 - d*cosh(d*x + c)^3)*sinh(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tanh^3(c + dx)\right)\operatorname{csch}^3(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*tanh(d*x+c)**3),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)*csch(c + d*x)**3, x)
```

Giac [B] time = 1.22179, size = 193, normalized size = 2.72

$$\frac{2\,b\arctan\left(e^{(dx+c)}\right) + a\log\left(e^{(dx+c)} + 1\right) - a\log\left(\left|e^{(dx+c)} - 1\right|\right) - \frac{2\left(ae^{(7\,dx+7\,c)} - be^{(7\,dx+7\,c)} + 3\,ae^{(5\,dx+5\,c)} + 3\,be^{(5\,dx+5\,c)} + 3\,ae^{(3\,dx+3\,c)} - 3\,be^{(3\,dx+3}}{\left(e^{(4\,dx+4\,c)} - 1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/2*(2*b*arctan(e^(d*x + c)) + a*log(e^(d*x + c) + 1) - a*log(abs(e^(d*x +
c) - 1)) - 2*(a*e^(7*d*x + 7*c) - b*e^(7*d*x + 7*c) + 3*a*e^(5*d*x + 5*c) +
 3*b*e^(5*d*x + 5*c) + 3*a*e^(3*d*x + 3*c) - 3*b*e^(3*d*x + 3*c) + a*e^(d*x
 + c) + b*e^(d*x + c))/(e^(4*d*x + 4*c) - 1)^2)/d
```

3.56 $\int \mathrm{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$

Optimal. Leaf size=56

$$-\frac{a\coth^3(c+dx)}{3d}+\frac{a\coth(c+dx)}{d}-\frac{b\tanh^2(c+dx)}{2d}+\frac{b\log(\tanh(c+dx))}{d}$$

```
[Out] (a*Coth[c + d*x])/d - (a*Coth[c + d*x]^3)/(3*d) + (b*Log[Tanh[c + d*x]])/d
- (b*Tanh[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.0582295, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3663, 1802}

$$-\frac{a\coth^3(c+dx)}{3d}+\frac{a\coth(c+dx)}{d}-\frac{b\tanh^2(c+dx)}{2d}+\frac{b\log(\tanh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (a*Coth[c + d*x])/d - (a*Coth[c + d*x]^3)/(3*d) + (b*Log[Tanh[c + d*x]])/d
- (b*Tanh[c + d*x]^2)/(2*d)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \mathrm{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx&=\frac{\mathrm{Subst}\left(\int\frac{(1-x^2)(a+bx^3)}{x^4}\,dx,x,\tanh(c+dx)\right)}{d}\\&=\frac{\mathrm{Subst}\left(\int\left(\frac{a}{x^4}-\frac{a}{x^2}+\frac{b}{x}-bx\right)dx,x,\tanh(c+dx)\right)}{d}\\&=\frac{a\coth(c+dx)}{d}-\frac{a\coth^3(c+dx)}{3d}+\frac{b\log(\tanh(c+dx))}{d}-\frac{b\tanh^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.196557, size = 74, normalized size = 1.32

$$\frac{2a\coth(c+dx)}{3d}-\frac{a\coth(c+dx)\mathrm{csch}^2(c+dx)}{3d}-\frac{b\left(-\mathrm{sech}^2(c+dx)-2\log(\sinh(c+dx))+2\log(\cosh(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (2*a*Coth[c + d*x])/(3*d) - (a*Coth[c + d*x]*Csch[c + d*x]^2)/(3*d) - (b*(2
*Log[Cosh[c + d*x]] - 2*Log[Sinh[c + d*x]] - Sech[c + d*x]^2))/(2*d)
```

Maple [A] time = 0.061, size = 60, normalized size = 1.1

$$\frac{2\coth(dx+c)\,a}{3d}-\frac{\coth(dx+c)\,a\,(\mathrm{csch}(dx+c))^2}{3d}+\frac{b}{2d\,(\cosh(dx+c))^2}+\frac{b\ln(\tanh(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3),x)
```

```
[Out] 2/3*a*coth(d*x+c)/d-1/3/d*a*coth(d*x+c)*csch(d*x+c)^2+1/2/d*b/cosh(d*x+c)^2
+b*ln(tanh(d*x+c))/d
```

Maxima [B] time = 1.5771, size = 248, normalized size = 4.43

$$b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1\right)}\right)+\frac{4}{3}\,a\left(\frac{3}{d\left(3\,e^{(-2\,dx-2\,c)}-3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] b*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x - 2*c)
 + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1)
)) + 4/3*a*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c)
+ e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) +
e^(-6*d*x - 6*c) - 1)))
```

Fricas [B] time = 2.45432, size = 4702, normalized size = 83.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/3*(6*b*cosh(d*x + c)^8 + 48*b*cosh(d*x + c)*sinh(d*x + c)^7 + 6*b*sinh(d*
x + c)^8 - 6*(2*a + 3*b)*cosh(d*x + c)^6 + 6*(28*b*cosh(d*x + c)^2 - 2*a -
3*b)*sinh(d*x + c)^6 + 12*(28*b*cosh(d*x + c)^3 - 3*(2*a + 3*b)*cosh(d*x +
c))*sinh(d*x + c)^5 - 2*(10*a - 9*b)*cosh(d*x + c)^4 + 2*(210*b*cosh(d*x +
c)^4 - 45*(2*a + 3*b)*cosh(d*x + c)^2 - 10*a + 9*b)*sinh(d*x + c)^4 + 8*(42
*b*cosh(d*x + c)^5 - 15*(2*a + 3*b)*cosh(d*x + c)^3 - (10*a - 9*b)*cosh(d*x
 + c))*sinh(d*x + c)^3 - 2*(2*a + 3*b)*cosh(d*x + c)^2 + 2*(84*b*cosh(d*x +
 c)^6 - 45*(2*a + 3*b)*cosh(d*x + c)^4 - 6*(10*a - 9*b)*cosh(d*x + c)^2 - 2
*a - 3*b)*sinh(d*x + c)^2 - 3*(b*cosh(d*x + c)^10 + 10*b*cosh(d*x + c)*sinh
```

```
(d*x + c)^9 + b*sinh(d*x + c)^10 - b*cosh(d*x + c)^8 + (45*b*cosh(d*x + c)^
2 - b)*sinh(d*x + c)^8 + 8*(15*b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*
x + c)^7 - 2*b*cosh(d*x + c)^6 + 2*(105*b*cosh(d*x + c)^4 - 14*b*cosh(d*x +
 c)^2 - b)*sinh(d*x + c)^6 + 4*(63*b*cosh(d*x + c)^5 - 14*b*cosh(d*x + c)^3
 - 3*b*cosh(d*x + c))*sinh(d*x + c)^5 + 2*b*cosh(d*x + c)^4 + 2*(105*b*cosh
(d*x + c)^6 - 35*b*cosh(d*x + c)^4 - 15*b*cosh(d*x + c)^2 + b)*sinh(d*x + c
)^4 + 8*(15*b*cosh(d*x + c)^7 - 7*b*cosh(d*x + c)^5 - 5*b*cosh(d*x + c)^3 +
 b*cosh(d*x + c))*sinh(d*x + c)^3 + b*cosh(d*x + c)^2 + (45*b*cosh(d*x + c)
^8 - 28*b*cosh(d*x + c)^6 - 30*b*cosh(d*x + c)^4 + 12*b*cosh(d*x + c)^2 + b
)*sinh(d*x + c)^2 + 2*(5*b*cosh(d*x + c)^9 - 4*b*cosh(d*x + c)^7 - 6*b*cosh
(d*x + c)^5 + 4*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) - b)*log
(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 3*(b*cosh(d*x + c)^10 +
 10*b*cosh(d*x + c)*sinh(d*x + c)^9 + b*sinh(d*x + c)^10 - b*cosh(d*x + c)^
8 + (45*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^8 + 8*(15*b*cosh(d*x + c)^3 -
b*cosh(d*x + c))*sinh(d*x + c)^7 - 2*b*cosh(d*x + c)^6 + 2*(105*b*cosh(d*x
+ c)^4 - 14*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^6 + 4*(63*b*cosh(d*x + c)^
5 - 14*b*cosh(d*x + c)^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^5 + 2*b*cosh(d*
x + c)^4 + 2*(105*b*cosh(d*x + c)^6 - 35*b*cosh(d*x + c)^4 - 15*b*cosh(d*x
+ c)^2 + b)*sinh(d*x + c)^4 + 8*(15*b*cosh(d*x + c)^7 - 7*b*cosh(d*x + c)^5
 - 5*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c)^3 + b*cosh(d*x + c)
^2 + (45*b*cosh(d*x + c)^8 - 28*b*cosh(d*x + c)^6 - 30*b*cosh(d*x + c)^4 +
12*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 2*(5*b*cosh(d*x + c)^9 - 4*b*co
sh(d*x + c)^7 - 6*b*cosh(d*x + c)^5 + 4*b*cosh(d*x + c)^3 + b*cosh(d*x + c)
)*sinh(d*x + c) - b)*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) +
 4*(12*b*cosh(d*x + c)^7 - 9*(2*a + 3*b)*cosh(d*x + c)^5 - 2*(10*a - 9*b)*c
osh(d*x + c)^3 - (2*a + 3*b)*cosh(d*x + c))*sinh(d*x + c) + 4*a)/(d*cosh(d*
x + c)^10 + 10*d*cosh(d*x + c)*sinh(d*x + c)^9 + d*sinh(d*x + c)^10 - d*cos
h(d*x + c)^8 + (45*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^8 + 8*(15*d*cosh(d*
x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c)^7 - 2*d*cosh(d*x + c)^6 + 2*(105*
d*cosh(d*x + c)^4 - 14*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^6 + 4*(63*d*cos
h(d*x + c)^5 - 14*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^5 +
2*d*cosh(d*x + c)^4 + 2*(105*d*cosh(d*x + c)^6 - 35*d*cosh(d*x + c)^4 - 15*
d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 8*(15*d*cosh(d*x + c)^7 - 7*d*cosh
(d*x + c)^5 - 5*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^3 + d*co
sh(d*x + c)^2 + (45*d*cosh(d*x + c)^8 - 28*d*cosh(d*x + c)^6 - 30*d*cosh(d*
x + c)^4 + 12*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 2*(5*d*cosh(d*x + c)
^9 - 4*d*cosh(d*x + c)^7 - 6*d*cosh(d*x + c)^5 + 4*d*cosh(d*x + c)^3 + d*co
sh(d*x + c))*sinh(d*x + c) - d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tanh^3(c + dx)\right)\operatorname{csch}^4(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*tanh(d*x+c)**3),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)*csch(c + d*x)**4, x)
```

Giac [B] time = 1.2359, size = 201, normalized size = 3.59

$$-\frac{6\,b\log\left(e^{(2\,dx+2\,c)}+1\right)-6\,b\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)-\frac{3\left(3\,be^{(4\,dx+4\,c)}+10\,be^{(2\,dx+2\,c)}+3\,b\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^2}+\frac{11\,be^{(6\,dx+6\,c)}-33\,be^{(4\,dx+4\,c)}+24\,ae^{(2\,dx+2\,c)}+33\,be^{(}}{\left(e^{(2\,dx+2\,c)}-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -1/6*(6*b*log(e^(2*d*x + 2*c) + 1) - 6*b*log(abs(e^(2*d*x + 2*c) - 1)) - 3*
(3*b*e^(4*d*x + 4*c) + 10*b*e^(2*d*x + 2*c) + 3*b)/(e^(2*d*x + 2*c) + 1)^2
+ (11*b*e^(6*d*x + 6*c) - 33*b*e^(4*d*x + 4*c) + 24*a*e^(2*d*x + 2*c) + 33*
b*e^(2*d*x + 2*c) - 8*a - 11*b)/(e^(2*d*x + 2*c) - 1)^3)/d
```

3.57 $\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=170

$$\frac{\sinh(c+dx)\cosh^3(c+dx)\left(a^2+2ab\tanh(c+dx)+b^2\right)}{4d}-\frac{\sinh(c+dx)\cosh(c+dx)\left(5a^2+20ab\tanh(c+dx)+17b\right.}{8d}$$

```
[Out] (3*(a^2 + 21*b^2)*x)/8 + (6*a*b*Log[Cosh[c + d*x]])/d - (6*b^2*Tanh[c + d*x
])/d - (a*b*Tanh[c + d*x]^2)/d - (b^2*Tanh[c + d*x]^3)/d - (b^2*Tanh[c + d*
x]^5)/(5*d) + (Cosh[c + d*x]^3*Sinh[c + d*x]*(a^2 + b^2 + 2*a*b*Tanh[c + d*
x]))/(4*d) - (Cosh[c + d*x]*Sinh[c + d*x]*(5*a^2 + 17*b^2 + 20*a*b*Tanh[c +
 d*x]))/(8*d)
```

Rubi [A] time = 0.296606, antiderivative size = 206, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3663, 1804, 1802, 633, 31}

$$-\frac{3\left(a^2+21b^2\right)\tanh(c+dx)}{8d}-\frac{3\left(a^2+16ab+21b^2\right)\log(1-\tanh(c+dx))}{16d}+\frac{3\left(a^2-16ab+21b^2\right)\log(\tanh(c+dx)+}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] (-3*(a^2 + 16*a*b + 21*b^2)*Log[1 - Tanh[c + d*x]])/(16*d) + (3*(a^2 - 16*a
*b + 21*b^2)*Log[1 + Tanh[c + d*x]])/(16*d) - (3*(a^2 + 21*b^2)*Tanh[c + d*
x])/(8*d) - (3*a*b*Tanh[c + d*x]^2)/d - (b^2*Tanh[c + d*x]^3)/d - (b^2*Tanh
[c + d*x]^5)/(5*d) - (Sinh[c + d*x]^2*Tanh[c + d*x]*(a^2 + 13*b^2 + 16*a*b*
Tanh[c + d*x]))/(8*d) + (Sinh[c + d*x]^4*(2*a*b + (a^2 + b^2)*Tanh[c + d*x]
))/(4*d)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+bx^3\right)^2}{\left(1-x^2\right)^3}dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\sinh^4(c+dx)\left(2ab+\left(a^2+b^2\right)\tanh(c+dx)\right)}{4d} + \frac{\text{Subst}\left(\int \frac{x^3\left(-8ab-\left(a^2+5b^2\right)x-\right.}{(1-}\right.}{}$$

$$= -\frac{\sinh^2(c+dx)\tanh(c+dx)\left(a^2+13b^2+16ab\tanh(c+dx)\right)}{8d} + \frac{\sinh^4(c+}{}$$

$$= -\frac{\sinh^2(c+dx)\tanh(c+dx)\left(a^2+13b^2+16ab\tanh(c+dx)\right)}{8d} + \frac{\sinh^4(c+}{}$$

$$= -\frac{3\left(a^2+21b^2\right)\tanh(c+dx)}{8d} - \frac{3ab\tanh^2(c+dx)}{d} - \frac{b^2\tanh^3(c+dx)}{d} - \frac{b^2}{}$$

$$= -\frac{3\left(a^2+21b^2\right)\tanh(c+dx)}{8d} - \frac{3ab\tanh^2(c+dx)}{d} - \frac{b^2\tanh^3(c+dx)}{d} - \frac{b^2}{}$$

$$= -\frac{3\left(a^2+16ab+21b^2\right)\log(1-\tanh(c+dx))}{16d} + \frac{3\left(a^2-16ab+21b^2\right)\log(1}{16d}$$

Mathematica [A] time = 2.80884, size = 156, normalized size = 0.92

$$\frac{60\left(a^2+21b^2\right)(c+dx)-40\left(a^2+4b^2\right)\sinh(2(c+dx))+5\left(a^2+b^2\right)\sinh(4(c+dx))-200ab\cosh(2(c+dx))+10ab\cos}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] (60*(a^2 + 21*b^2)*(c + d*x) - 200*a*b*Cosh[2*(c + d*x)] + 10*a*b*Cosh[4*(c
 + d*x)] + 960*a*b*Log[Cosh[c + d*x]] + 160*a*b*Sech[c + d*x]^2 - 40*(a^2 +
 4*b^2)*Sinh[2*(c + d*x)] + 5*(a^2 + b^2)*Sinh[4*(c + d*x)] - 1152*b^2*Tanh
[c + d*x] + 224*b^2*Sech[c + d*x]^2*Tanh[c + d*x] - 32*b^2*Sech[c + d*x]^4*
Tanh[c + d*x])/(160*d)
```

Maple [A] time = 0.058, size = 243, normalized size = 1.4

$$\frac{a^2\cosh(dx+c)(\sinh(dx+c))^3}{4d} - \frac{3a^2\sinh(dx+c)\cosh(dx+c)}{8d} + \frac{3a^2x}{8} + \frac{3a^2c}{8d} + \frac{ab(\sinh(dx+c))^6}{2d(\cosh(dx+c))^2} - \frac{3ab(\sinh(}{2d(\cosh(d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x)
```

```
[Out] 1/4/d*a^2*cosh(d*x+c)*sinh(d*x+c)^3-3/8/d*a^2*sinh(d*x+c)*cosh(d*x+c)+3/8*a
^2*x+3/8/d*a^2*c+1/2/d*a*b*sinh(d*x+c)^6/cosh(d*x+c)^2-3/2/d*a*b*sinh(d*x+c
)^4/cosh(d*x+c)^2+6*a*b*ln(cosh(d*x+c))/d-3*a*b*tanh(d*x+c)^2/d+1/4/d*b^2*s
inh(d*x+c)^9/cosh(d*x+c)^5-9/8/d*b^2*sinh(d*x+c)^7/cosh(d*x+c)^5+63/8*b^2*x
+63/8/d*c*b^2-63/8*b^2*tanh(d*x+c)/d-21/8*b^2*tanh(d*x+c)^3/d-63/40*b^2*tan
h(d*x+c)^5/d
```

Maxima [B] time = 1.58581, size = 512, normalized size = 3.01

$$\frac{1}{64}a^2\left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d}\right) + \frac{1}{320}b^2\left(\frac{2520(dx+c)}{d} + \frac{5\left(32e^{(-2dx-2c)} - e^{(-4dx-4}\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 1/64*a^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + 1/320*b^2*(2520*(d*x + c)/d + 5*(32*e^(-2*d*x
- 2*c) - e^(-4*d*x - 4*c))/d - (135*e^(-2*d*x - 2*c) + 5358*e^(-4*d*x - 4*c
) + 18190*e^(-6*d*x - 6*c) + 28455*e^(-8*d*x - 8*c) + 19995*e^(-10*d*x - 10
*c) + 6560*e^(-12*d*x - 12*c) - 5)/(d*(e^(-4*d*x - 4*c) + 5*e^(-6*d*x - 6*c
) + 10*e^(-8*d*x - 8*c) + 10*e^(-10*d*x - 10*c) + 5*e^(-12*d*x - 12*c) + e^
(-14*d*x - 14*c)))) + 1/32*a*b*(192*(d*x + c)/d - (20*e^(-2*d*x - 2*c) - e^
(-4*d*x - 4*c))/d + 192*log(e^(-2*d*x - 2*c) + 1)/d - (18*e^(-2*d*x - 2*c)
+ 39*e^(-4*d*x - 4*c) - 108*e^(-6*d*x - 6*c) - 1)/(d*(e^(-4*d*x - 4*c) + 2*
e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c))))
```

Fricas [B] time = 2.9417, size = 13666, normalized size = 80.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/320*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^18 + 90*(a^2 + 2*a*b + b^2)*cosh
(d*x + c)*sinh(d*x + c)^17 + 5*(a^2 + 2*a*b + b^2)*sinh(d*x + c)^18 - 15*(a
^2 + 10*a*b + 9*b^2)*cosh(d*x + c)^16 + 15*(51*(a^2 + 2*a*b + b^2)*cosh(d*x
 + c)^2 - a^2 - 10*a*b - 9*b^2)*sinh(d*x + c)^16 + 240*(17*(a^2 + 2*a*b + b
^2)*cosh(d*x + c)^3 - (a^2 + 10*a*b + 9*b^2)*cosh(d*x + c))*sinh(d*x + c)^1
5 + 30*(4*(a^2 - 16*a*b + 21*b^2)*d*x - 5*a^2 - 30*a*b - 25*b^2)*cosh(d*x +
 c)^14 + 30*(510*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 - 16*a*b + 21
*b^2)*d*x - 60*(a^2 + 10*a*b + 9*b^2)*cosh(d*x + c)^2 - 5*a^2 - 30*a*b - 25
*b^2)*sinh(d*x + c)^14 + 420*(102*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 - 20*
(a^2 + 10*a*b + 9*b^2)*cosh(d*x + c)^3 + (4*(a^2 - 16*a*b + 21*b^2)*d*x - 5
*a^2 - 30*a*b - 25*b^2)*cosh(d*x + c))*sinh(d*x + c)^13 + 10*(60*(a^2 - 16*
a*b + 21*b^2)*d*x - 31*a^2 - 82*a*b + 501*b^2)*cosh(d*x + c)^12 + 10*(9282*
(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 - 2730*(a^2 + 10*a*b + 9*b^2)*cosh(d*x
+ c)^4 + 60*(a^2 - 16*a*b + 21*b^2)*d*x + 273*(4*(a^2 - 16*a*b + 21*b^2)*d*
x - 5*a^2 - 30*a*b - 25*b^2)*cosh(d*x + c)^2 - 31*a^2 - 82*a*b + 501*b^2)*s
inh(d*x + c)^12 + 120*(1326*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 - 546*(a^2
+ 10*a*b + 9*b^2)*cosh(d*x + c)^5 + 91*(4*(a^2 - 16*a*b + 21*b^2)*d*x - 5*a
```

^2 - 30*a*b - 25*b^2)*cosh(d*x + c)^3 + (60*(a^2 - 16*a*b + 21*b^2)*d*x - 3
1*a^2 - 82*a*b + 501*b^2)*cosh(d*x + c))*sinh(d*x + c)^11 + 60*(20*(a^2 - 1
6*a*b + 21*b^2)*d*x - 3*a^2 + 15*a*b + 307*b^2)*cosh(d*x + c)^10 + 30*(7293
*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 - 4004*(a^2 + 10*a*b + 9*b^2)*cosh(d*x
 + c)^6 + 1001*(4*(a^2 - 16*a*b + 21*b^2)*d*x - 5*a^2 - 30*a*b - 25*b^2)*co
sh(d*x + c)^4 + 40*(a^2 - 16*a*b + 21*b^2)*d*x + 22*(60*(a^2 - 16*a*b + 21*
b^2)*d*x - 31*a^2 - 82*a*b + 501*b^2)*cosh(d*x + c)^2 - 6*a^2 + 30*a*b + 61
4*b^2)*sinh(d*x + c)^10 + 20*(12155*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^9 - 8
580*(a^2 + 10*a*b + 9*b^2)*cosh(d*x + c)^7 + 3003*(4*(a^2 - 16*a*b + 21*b^2
)*d*x - 5*a^2 - 30*a*b - 25*b^2)*cosh(d*x + c)^5 + 110*(60*(a^2 - 16*a*b +
21*b^2)*d*x - 31*a^2 - 82*a*b + 501*b^2)*cosh(d*x + c)^3 + 30*(20*(a^2 - 16
*a*b + 21*b^2)*d*x - 3*a^2 + 15*a*b + 307*b^2)*cosh(d*x + c))*sinh(d*x + c)
^9 + 60*(20*(a^2 - 16*a*b + 21*b^2)*d*x + 3*a^2 + 15*a*b + 461*b^2)*cosh(d*
x + c)^8 + 30*(7293*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^10 - 6435*(a^2 + 10*a
*b + 9*b^2)*cosh(d*x + c)^8 + 3003*(4*(a^2 - 16*a*b + 21*b^2)*d*x - 5*a^2 -
 30*a*b - 25*b^2)*cosh(d*x + c)^6 + 165*(60*(a^2 - 16*a*b + 21*b^2)*d*x - 3
1*a^2 - 82*a*b + 501*b^2)*cosh(d*x + c)^4 + 40*(a^2 - 16*a*b + 21*b^2)*d*x
+ 90*(20*(a^2 - 16*a*b + 21*b^2)*d*x - 3*a^2 + 15*a*b + 307*b^2)*cosh(d*x +
 c)^2 + 6*a^2 + 30*a*b + 922*b^2)*sinh(d*x + c)^8 + 240*(663*(a^2 + 2*a*b +
 b^2)*cosh(d*x + c)^11 - 715*(a^2 + 10*a*b + 9*b^2)*cosh(d*x + c)^9 + 429*(
4*(a^2 - 16*a*b + 21*b^2)*d*x - 5*a^2 - 30*a*b - 25*b^2)*cosh(d*x + c)^7 +
33*(60*(a^2 - 16*a*b + 21*b^2)*d*x - 31*a^2 - 82*a*b + 501*b^2)*cosh(d*x +
c)^5 + 30*(20*(a^2 - 16*a*b + 21*b^2)*d*x - 3*a^2 + 15*a*b + 307*b^2)*cosh(
d*x + c)^3 + 2*(20*(a^2 - 16*a*b + 21*b^2)*d*x + 3*a^2 + 15*a*b + 461*b^2)*
cosh(d*x + c))*sinh(d*x + c)^7 + 10*(60*(a^2 - 16*a*b + 21*b^2)*d*x + 31*a^
2 - 82*a*b + 1803*b^2)*cosh(d*x + c)^6 + 10*(9282*(a^2 + 2*a*b + b^2)*cosh(
d*x + c)^12 - 12012*(a^2 + 10*a*b + 9*b^2)*cosh(d*x + c)^10 + 9009*(4*(a^2
- 16*a*b + 21*b^2)*d*x - 5*a^2 - 30*a*b - 25*b^2)*cosh(d*x + c)^8 + 924*(60
*(a^2 - 16*a*b + 21*b^2)*d*x - 31*a^2 - 82*a*b + 501*b^2)*cosh(d*x + c)^6 +
 1260*(20*(a^2 - 16*a*b + 21*b^2)*d*x - 3*a^2 + 15*a*b + 307*b^2)*cosh(d*x
+ c)^4 + 60*(a^2 - 16*a*b + 21*b^2)*d*x + 168*(20*(a^2 - 16*a*b + 21*b^2)*d
*x + 3*a^2 + 15*a*b + 461*b^2)*cosh(d*x + c)^2 + 31*a^2 - 82*a*b + 1803*b^2
)*sinh(d*x + c)^6 + 60*(714*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^13 - 1092*(a^
2 + 10*a*b + 9*b^2)*cosh(d*x + c)^11 + 1001*(4*(a^2 - 16*a*b + 21*b^2)*d*x
- 5*a^2 - 30*a*b - 25*b^2)*cosh(d*x + c)^9 + 132*(60*(a^2 - 16*a*b + 21*b^2
)*d*x - 31*a^2 - 82*a*b + 501*b^2)*cosh(d*x + c)^7 + 252*(20*(a^2 - 16*a*b
+ 21*b^2)*d*x - 3*a^2 + 15*a*b + 307*b^2)*cosh(d*x + c)^5 + 56*(20*(a^2 - 1
6*a*b + 21*b^2)*d*x + 3*a^2 + 15*a*b + 461*b^2)*cosh(d*x + c)^3 + (60*(a^2
- 16*a*b + 21*b^2)*d*x + 31*a^2 - 82*a*b + 1803*b^2)*cosh(d*x + c))*sinh(d*
x + c)^5 + 6*(20*(a^2 - 16*a*b + 21*b^2)*d*x + 25*a^2 - 150*a*b + 893*b^2)*
cosh(d*x + c)^4 + 6*(2550*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^14 - 4550*(a^2
+ 10*a*b + 9*b^2)*cosh(d*x + c)^12 + 5005*(4*(a^2 - 16*a*b + 21*b^2)*d*x -
5*a^2 - 30*a*b - 25*b^2)*cosh(d*x + c)^10 + 825*(60*(a^2 - 16*a*b + 21*b^2)
*d*x - 31*a^2 - 82*a*b + 501*b^2)*cosh(d*x + c)^8 + 2100*(20*(a^2 - 16*a*b
+ 21*b^2)*d*x - 3*a^2 + 15*a*b + 307*b^2)*cosh(d*x + c)^6 + 700*(20*(a^2 -
16*a*b + 21*b^2)*d*x + 3*a^2 + 15*a*b + 461*b^2)*cosh(d*x + c)^4 + 20*(a^2
- 16*a*b + 21*b^2)*d*x + 25*(60*(a^2 - 16*a*b + 21*b^2)*d*x + 31*a^2 - 82*a
*b + 1803*b^2)*cosh(d*x + c)^2 + 25*a^2 - 150*a*b + 893*b^2)*sinh(d*x + c)^
4 + 8*(510*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^15 - 1050*(a^2 + 10*a*b + 9*b^
2)*cosh(d*x + c)^13 + 1365*(4*(a^2 - 16*a*b + 21*b^2)*d*x - 5*a^2 - 30*a*b
- 25*b^2)*cosh(d*x + c)^11 + 275*(60*(a^2 - 16*a*b + 21*b^2)*d*x - 31*a^2 -
 82*a*b + 501*b^2)*cosh(d*x + c)^9 + 900*(20*(a^2 - 16*a*b + 21*b^2)*d*x -
3*a^2 + 15*a*b + 307*b^2)*cosh(d*x + c)^7 + 420*(20*(a^2 - 16*a*b + 21*b^2)
*d*x + 3*a^2 + 15*a*b + 461*b^2)*cosh(d*x + c)^5 + 25*(60*(a^2 - 16*a*b + 2
1*b^2)*d*x + 31*a^2 - 82*a*b + 1803*b^2)*cosh(d*x + c)^3 + 3*(20*(a^2 - 16*
a*b + 21*b^2)*d*x + 25*a^2 - 150*a*b + 893*b^2)*cosh(d*x + c))*sinh(d*x + c
)^3 + 15*(a^2 - 10*a*b + 9*b^2)*cosh(d*x + c)^2 + 3*(255*(a^2 + 2*a*b + b^2
)*cosh(d*x + c)^16 - 600*(a^2 + 10*a*b + 9*b^2)*cosh(d*x + c)^14 + 910*(4*(
a^2 - 16*a*b + 21*b^2)*d*x - 5*a^2 - 30*a*b - 25*b^2)*cosh(d*x + c)^12 + 22

```
0*(60*(a^2 - 16*a*b + 21*b^2)*d*x - 31*a^2 - 82*a*b + 501*b^2)*cosh(d*x + c
)^10 + 900*(20*(a^2 - 16*a*b + 21*b^2)*d*x - 3*a^2 + 15*a*b + 307*b^2)*cosh
(d*x + c)^8 + 560*(20*(a^2 - 16*a*b + 21*b^2)*d*x + 3*a^2 + 15*a*b + 461*b^
2)*cosh(d*x + c)^6 + 50*(60*(a^2 - 16*a*b + 21*b^2)*d*x + 31*a^2 - 82*a*b +
 1803*b^2)*cosh(d*x + c)^4 + 12*(20*(a^2 - 16*a*b + 21*b^2)*d*x + 25*a^2 -
150*a*b + 893*b^2)*cosh(d*x + c)^2 + 5*a^2 - 50*a*b + 45*b^2)*sinh(d*x + c)
^2 - 5*a^2 + 10*a*b - 5*b^2 + 1920*(a*b*cosh(d*x + c)^14 + 14*a*b*cosh(d*x
+ c)*sinh(d*x + c)^13 + a*b*sinh(d*x + c)^14 + 5*a*b*cosh(d*x + c)^12 + (91
*a*b*cosh(d*x + c)^2 + 5*a*b)*sinh(d*x + c)^12 + 10*a*b*cosh(d*x + c)^10 +
4*(91*a*b*cosh(d*x + c)^3 + 15*a*b*cosh(d*x + c))*sinh(d*x + c)^11 + (1001*
a*b*cosh(d*x + c)^4 + 330*a*b*cosh(d*x + c)^2 + 10*a*b)*sinh(d*x + c)^10 +
10*a*b*cosh(d*x + c)^8 + 2*(1001*a*b*cosh(d*x + c)^5 + 550*a*b*cosh(d*x + c
)^3 + 50*a*b*cosh(d*x + c))*sinh(d*x + c)^9 + (3003*a*b*cosh(d*x + c)^6 + 2
475*a*b*cosh(d*x + c)^4 + 450*a*b*cosh(d*x + c)^2 + 10*a*b)*sinh(d*x + c)^8
 + 5*a*b*cosh(d*x + c)^6 + 8*(429*a*b*cosh(d*x + c)^7 + 495*a*b*cosh(d*x +
c)^5 + 150*a*b*cosh(d*x + c)^3 + 10*a*b*cosh(d*x + c))*sinh(d*x + c)^7 + (3
003*a*b*cosh(d*x + c)^8 + 4620*a*b*cosh(d*x + c)^6 + 2100*a*b*cosh(d*x + c)
^4 + 280*a*b*cosh(d*x + c)^2 + 5*a*b)*sinh(d*x + c)^6 + a*b*cosh(d*x + c)^4
 + 2*(1001*a*b*cosh(d*x + c)^9 + 1980*a*b*cosh(d*x + c)^7 + 1260*a*b*cosh(d
*x + c)^5 + 280*a*b*cosh(d*x + c)^3 + 15*a*b*cosh(d*x + c))*sinh(d*x + c)^5
 + (1001*a*b*cosh(d*x + c)^10 + 2475*a*b*cosh(d*x + c)^8 + 2100*a*b*cosh(d*
x + c)^6 + 700*a*b*cosh(d*x + c)^4 + 75*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x
 + c)^4 + 4*(91*a*b*cosh(d*x + c)^11 + 275*a*b*cosh(d*x + c)^9 + 300*a*b*co
sh(d*x + c)^7 + 140*a*b*cosh(d*x + c)^5 + 25*a*b*cosh(d*x + c)^3 + a*b*cosh
(d*x + c))*sinh(d*x + c)^3 + (91*a*b*cosh(d*x + c)^12 + 330*a*b*cosh(d*x +
c)^10 + 450*a*b*cosh(d*x + c)^8 + 280*a*b*cosh(d*x + c)^6 + 75*a*b*cosh(d*x
 + c)^4 + 6*a*b*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(7*a*b*cosh(d*x + c)^1
3 + 30*a*b*cosh(d*x + c)^11 + 50*a*b*cosh(d*x + c)^9 + 40*a*b*cosh(d*x + c)
^7 + 15*a*b*cosh(d*x + c)^5 + 2*a*b*cosh(d*x + c)^3)*sinh(d*x + c))*log(2*c
osh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 6*(15*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)^17 - 40*(a^2 + 10*a*b + 9*b^2)*cosh(d*x + c)^15 + 70*(4*(a^2 -
 16*a*b + 21*b^2)*d*x - 5*a^2 - 30*a*b - 25*b^2)*cosh(d*x + c)^13 + 20*(60*
(a^2 - 16*a*b + 21*b^2)*d*x - 31*a^2 - 82*a*b + 501*b^2)*cosh(d*x + c)^11 +
 100*(20*(a^2 - 16*a*b + 21*b^2)*d*x - 3*a^2 + 15*a*b + 307*b^2)*cosh(d*x +
 c)^9 + 80*(20*(a^2 - 16*a*b + 21*b^2)*d*x + 3*a^2 + 15*a*b + 461*b^2)*cosh
(d*x + c)^7 + 10*(60*(a^2 - 16*a*b + 21*b^2)*d*x + 31*a^2 - 82*a*b + 1803*b
^2)*cosh(d*x + c)^5 + 4*(20*(a^2 - 16*a*b + 21*b^2)*d*x + 25*a^2 - 150*a*b
+ 893*b^2)*cosh(d*x + c)^3 + 5*(a^2 - 10*a*b + 9*b^2)*cosh(d*x + c))*sinh(d
*x + c))/(d*cosh(d*x + c)^14 + 14*d*cosh(d*x + c)*sinh(d*x + c)^13 + d*sinh
(d*x + c)^14 + 5*d*cosh(d*x + c)^12 + (91*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x
 + c)^12 + 4*(91*d*cosh(d*x + c)^3 + 15*d*cosh(d*x + c))*sinh(d*x + c)^11 +
 10*d*cosh(d*x + c)^10 + (1001*d*cosh(d*x + c)^4 + 330*d*cosh(d*x + c)^2 +
10*d)*sinh(d*x + c)^10 + 2*(1001*d*cosh(d*x + c)^5 + 550*d*cosh(d*x + c)^3
+ 50*d*cosh(d*x + c))*sinh(d*x + c)^9 + 10*d*cosh(d*x + c)^8 + (3003*d*cosh
(d*x + c)^6 + 2475*d*cosh(d*x + c)^4 + 450*d*cosh(d*x + c)^2 + 10*d)*sinh(d
*x + c)^8 + 8*(429*d*cosh(d*x + c)^7 + 495*d*cosh(d*x + c)^5 + 150*d*cosh(d
*x + c)^3 + 10*d*cosh(d*x + c))*sinh(d*x + c)^7 + 5*d*cosh(d*x + c)^6 + (30
03*d*cosh(d*x + c)^8 + 4620*d*cosh(d*x + c)^6 + 2100*d*cosh(d*x + c)^4 + 28
0*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^6 + 2*(1001*d*cosh(d*x + c)^9 + 19
80*d*cosh(d*x + c)^7 + 1260*d*cosh(d*x + c)^5 + 280*d*cosh(d*x + c)^3 + 15*
d*cosh(d*x + c))*sinh(d*x + c)^5 + d*cosh(d*x + c)^4 + (1001*d*cosh(d*x + c
)^10 + 2475*d*cosh(d*x + c)^8 + 2100*d*cosh(d*x + c)^6 + 700*d*cosh(d*x + c
)^4 + 75*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 4*(91*d*cosh(d*x + c)^11
+ 275*d*cosh(d*x + c)^9 + 300*d*cosh(d*x + c)^7 + 140*d*cosh(d*x + c)^5 + 2
5*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^3 + (91*d*cosh(d*x + c
)^12 + 330*d*cosh(d*x + c)^10 + 450*d*cosh(d*x + c)^8 + 280*d*cosh(d*x + c)
^6 + 75*d*cosh(d*x + c)^4 + 6*d*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(7*d*c
osh(d*x + c)^13 + 30*d*cosh(d*x + c)^11 + 50*d*cosh(d*x + c)^9 + 40*d*cosh(
d*x + c)^7 + 15*d*cosh(d*x + c)^5 + 2*d*cosh(d*x + c)^3)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4*(a+b*tanh(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.27347, size = 508, normalized size = 2.99

$$120\left(a^2-16\,ab+21\,b^2\right)dx+1920\,ab\log\left(e^{(2\,dx+2\,c)}+1\right)-5\left(18\,a^2e^{(4\,dx+4\,c)}-288\,abe^{(4\,dx+4\,c)}+378\,b^2e^{(4\,dx+4\,c)}-8\,a^2e^{(2\,dx-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/320*(120*(a^2 - 16*a*b + 21*b^2)*d*x + 1920*a*b*log(e^(2*d*x + 2*c) + 1)
- 5*(18*a^2*e^(4*d*x + 4*c) - 288*a*b*e^(4*d*x + 4*c) + 378*b^2*e^(4*d*x +
4*c) - 8*a^2*e^(2*d*x + 2*c) + 40*a*b*e^(2*d*x + 2*c) - 32*b^2*e^(2*d*x + 2
*c) + a^2 - 2*a*b + b^2)*e^(-4*d*x - 4*c) + 5*(a^2*e^(4*d*x + 36*c) + 2*a*b
*e^(4*d*x + 36*c) + b^2*e^(4*d*x + 36*c) - 8*a^2*e^(2*d*x + 34*c) - 40*a*b*
e^(2*d*x + 34*c) - 32*b^2*e^(2*d*x + 34*c))*e^(-32*c) - 32*(137*a*b*e^(10*d
*x + 10*c) + 645*a*b*e^(8*d*x + 8*c) - 200*b^2*e^(8*d*x + 8*c) + 1250*a*b*e
^(6*d*x + 6*c) - 600*b^2*e^(6*d*x + 6*c) + 1250*a*b*e^(4*d*x + 4*c) - 840*b
^2*e^(4*d*x + 4*c) + 645*a*b*e^(2*d*x + 2*c) - 520*b^2*e^(2*d*x + 2*c) + 13
7*a*b - 144*b^2)/(e^(2*d*x + 2*c) + 1)^5)/d
```

3.58 $\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=182

$$\frac{a^2\cosh^3(c+dx)}{3d}-\frac{a^2\cosh(c+dx)}{d}+\frac{5ab\sinh^3(c+dx)}{3d}-\frac{5ab\sinh(c+dx)}{d}-\frac{ab\sinh^3(c+dx)\tanh^2(c+dx)}{d}+\frac{5ab}{}$$

```
[Out] (5*a*b*ArcTan[Sinh[c + d*x]])/d - (a^2*Cosh[c + d*x])/d - (4*b^2*Cosh[c + d
*x])/d + (a^2*Cosh[c + d*x]^3)/(3*d) + (b^2*Cosh[c + d*x]^3)/(3*d) - (6*b^2
*Sech[c + d*x])/d + (4*b^2*Sech[c + d*x]^3)/(3*d) - (b^2*Sech[c + d*x]^5)/(
5*d) - (5*a*b*Sinh[c + d*x])/d + (5*a*b*Sinh[c + d*x]^3)/(3*d) - (a*b*Sinh[
c + d*x]^3*Tanh[c + d*x]^2)/d
```

Rubi [A] time = 0.224828, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3666, 2633, 2592, 288, 302, 203, 2590, 270}

$$\frac{a^2\cosh^3(c+dx)}{3d}-\frac{a^2\cosh(c+dx)}{d}+\frac{5ab\sinh^3(c+dx)}{3d}-\frac{5ab\sinh(c+dx)}{d}-\frac{ab\sinh^3(c+dx)\tanh^2(c+dx)}{d}+\frac{5ab}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] (5*a*b*ArcTan[Sinh[c + d*x]])/d - (a^2*Cosh[c + d*x])/d - (4*b^2*Cosh[c + d
*x])/d + (a^2*Cosh[c + d*x]^3)/(3*d) + (b^2*Cosh[c + d*x]^3)/(3*d) - (6*b^2
*Sech[c + d*x])/d + (4*b^2*Sech[c + d*x]^3)/(3*d) - (b^2*Sech[c + d*x]^5)/(
5*d) - (5*a*b*Sinh[c + d*x])/d + (5*a*b*Sinh[c + d*x]^3)/(3*d) - (a*b*Sinh[
c + d*x]^3*Tanh[c + d*x]^2)/d
```

Rule 3666

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = i\int\left(-ia^2\sinh^3(c+dx)-2iab\sinh^3(c+dx)\tanh^3(c+dx)-ib^2\sinh^3(c+\right.$$

$$= a^2\int\sinh^3(c+dx)\,dx+(2ab)\int\sinh^3(c+dx)\tanh^3(c+dx)\,dx+b^2\int\text{si}$$

$$= -\frac{a^2\,\text{Subst}\left(\int\left(1-x^2\right)dx,x,\cosh(c+dx)\right)}{d}+\frac{(2ab)\,\text{Subst}\left(\int\frac{x^6}{\left(1+x^2\right)^2}dx,x,\right.}{d}$$

$$= -\frac{a^2\cosh(c+dx)}{d}+\frac{a^2\cosh^3(c+dx)}{3d}-\frac{ab\sinh^3(c+dx)\tanh^2(c+dx)}{d}+$$

$$= -\frac{a^2\cosh(c+dx)}{d}-\frac{4b^2\cosh(c+dx)}{d}+\frac{a^2\cosh^3(c+dx)}{3d}+\frac{b^2\cosh^3(c+d}{3d}$$

$$= -\frac{a^2\cosh(c+dx)}{d}-\frac{4b^2\cosh(c+dx)}{d}+\frac{a^2\cosh^3(c+dx)}{3d}+\frac{b^2\cosh^3(c+d}{3d}$$

$$= \frac{5ab\tan^{-1}(\sinh(c+dx))}{d}-\frac{a^2\cosh(c+dx)}{d}-\frac{4b^2\cosh(c+dx)}{d}+\frac{a^2\cosh^3}{3}$$

Mathematica [A] time = 0.720434, size = 121, normalized size = 0.66

$$\frac{-45\left(a^2+5b^2\right)\cosh(c+dx)+5\left(a^2+b^2\right)\cosh(3(c+dx))-2b\left(30\text{sech}(c+dx)(a\tanh(c+dx)+6b)-5a\left(-27\sinh(c+\right.\right.}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] (-45*(a^2 + 5*b^2)*Cosh[c + d*x] + 5*(a^2 + b^2)*Cosh[3*(c + d*x)] - 2*b*(-
40*b*Sech[c + d*x]^3 + 6*b*Sech[c + d*x]^5 - 5*a*(60*ArcTan[Tanh[(c + d*x)/
2]] - 27*Sinh[c + d*x] + Sinh[3*(c + d*x)]) + 30*Sech[c + d*x]*(6*b + a*Tan
h[c + d*x])))/(60*d)
```

Maple [A] time = 0.058, size = 296, normalized size = 1.6

$$-\frac{2\,a^2\cosh(dx+c)}{3\,d}+\frac{a^2\cosh(dx+c)(\sinh(dx+c))^2}{3\,d}+\frac{2\,ab(\sinh(dx+c))^5}{3\,d(\cosh(dx+c))^2}-\frac{10\,ab(\sinh(dx+c))^3}{3\,d(\cosh(dx+c))^2}-10\,\frac{ab\sinh(}{d(\cosh(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x)
```

```
[Out] -2/3*a^2*cosh(d*x+c)/d+1/3/d*a^2*cosh(d*x+c)*sinh(d*x+c)^2+2/3/d*a*b*sinh(d
*x+c)^5/cosh(d*x+c)^2-10/3/d*a*b*sinh(d*x+c)^3/cosh(d*x+c)^2-10/d*a*b*sinh(
d*x+c)/cosh(d*x+c)^2+5/d*a*b*sech(d*x+c)*tanh(d*x+c)+10/d*a*b*arctan(exp(d*
x+c))+1/3/d*b^2*sinh(d*x+c)^8/cosh(d*x+c)^5-8/3/d*b^2*sinh(d*x+c)^6/cosh(d*
x+c)^5-16/d*b^2*sinh(d*x+c)^4/cosh(d*x+c)^5-64/5/d*b^2*sinh(d*x+c)^2/cosh(d
*x+c)^5+128/15/d*b^2*sinh(d*x+c)^2/cosh(d*x+c)^3+128/15/d*b^2*sinh(d*x+c)^2
/cosh(d*x+c)-128/15*b^2*cosh(d*x+c)/d
```

Maxima [B] time = 1.55516, size = 470, normalized size = 2.58

$$-\frac{1}{120}\,b^2\left(\frac{5\left(45\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}\right)}{d}+\frac{200\,e^{(-2\,dx-2\,c)}+2515\,e^{(-4\,dx-4\,c)}+6680\,e^{(-6\,dx-6\,c)}+9073\,e^{(-8\,dx-8\,c)}+5600\,e^{(-10}}{d\left(e^{(-3\,dx-3\,c)}+5\,e^{(-5\,dx-5\,c)}+10\,e^{(-7\,dx-7\,c)}+10\,e^{(-9\,dx-9\,c)}+5\,e^{(-11\,dx-}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] -1/120*b^2*(5*(45*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + (200*e^(-2*d*x - 2*c
) + 2515*e^(-4*d*x - 4*c) + 6680*e^(-6*d*x - 6*c) + 9073*e^(-8*d*x - 8*c) +
 5600*e^(-10*d*x - 10*c) + 1665*e^(-12*d*x - 12*c) - 5)/(d*(e^(-3*d*x - 3*c
) + 5*e^(-5*d*x - 5*c) + 10*e^(-7*d*x - 7*c) + 10*e^(-9*d*x - 9*c) + 5*e^(-
11*d*x - 11*c) + e^(-13*d*x - 13*c)))) + 1/12*a*b*((27*e^(-d*x - c) - e^(-3
*d*x - 3*c))/d - 120*arctan(e^(-d*x - c))/d - (25*e^(-2*d*x - 2*c) + 77*e^(
-4*d*x - 4*c) + 3*e^(-6*d*x - 6*c) - 1)/(d*(e^(-3*d*x - 3*c) + 2*e^(-5*d*x
- 5*c) + e^(-7*d*x - 7*c)))) + 1/24*a^2*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/
d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

Fricas [B] time = 2.77703, size = 9211, normalized size = 50.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/120*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^16 + 80*(a^2 + 2*a*b + b^2)*cosh
(d*x + c)*sinh(d*x + c)^15 + 5*(a^2 + 2*a*b + b^2)*sinh(d*x + c)^16 - 20*(a
```

```
^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^14 + 20*(30*(a^2 + 2*a*b + b^2)*cosh(d*
x + c)^2 - a^2 - 11*a*b - 10*b^2)*sinh(d*x + c)^14 + 280*(10*(a^2 + 2*a*b +
 b^2)*cosh(d*x + c)^3 - (a^2 + 11*a*b + 10*b^2)*cosh(d*x + c))*sinh(d*x + c
)^13 - 20*(11*a^2 + 61*a*b + 137*b^2)*cosh(d*x + c)^12 + 20*(455*(a^2 + 2*a
*b + b^2)*cosh(d*x + c)^4 - 91*(a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^2 - 11
*a^2 - 61*a*b - 137*b^2)*sinh(d*x + c)^12 + 80*(273*(a^2 + 2*a*b + b^2)*cos
h(d*x + c)^5 - 91*(a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^3 - 3*(11*a^2 + 61*
a*b + 137*b^2)*cosh(d*x + c))*sinh(d*x + c)^11 - 20*(31*a^2 + 87*a*b + 390*
b^2)*cosh(d*x + c)^10 + 20*(2002*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 - 1001
*(a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^4 - 66*(11*a^2 + 61*a*b + 137*b^2)*c
osh(d*x + c)^2 - 31*a^2 - 87*a*b - 390*b^2)*sinh(d*x + c)^10 + 40*(1430*(a^
2 + 2*a*b + b^2)*cosh(d*x + c)^7 - 1001*(a^2 + 11*a*b + 10*b^2)*cosh(d*x +
c)^5 - 110*(11*a^2 + 61*a*b + 137*b^2)*cosh(d*x + c)^3 - 5*(31*a^2 + 87*a*b
 + 390*b^2)*cosh(d*x + c))*sinh(d*x + c)^9 - 2*(425*a^2 + 5649*b^2)*cosh(d*
x + c)^8 + 2*(32175*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 - 30030*(a^2 + 11*a
*b + 10*b^2)*cosh(d*x + c)^6 - 4950*(11*a^2 + 61*a*b + 137*b^2)*cosh(d*x +
c)^4 - 450*(31*a^2 + 87*a*b + 390*b^2)*cosh(d*x + c)^2 - 425*a^2 - 5649*b^2
)*sinh(d*x + c)^8 + 16*(3575*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^9 - 4290*(a^
2 + 11*a*b + 10*b^2)*cosh(d*x + c)^7 - 990*(11*a^2 + 61*a*b + 137*b^2)*cosh
(d*x + c)^5 - 150*(31*a^2 + 87*a*b + 390*b^2)*cosh(d*x + c)^3 - (425*a^2 +
5649*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 20*(31*a^2 - 87*a*b + 390*b^2)*c
osh(d*x + c)^6 + 4*(10010*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^10 - 15015*(a^2
 + 11*a*b + 10*b^2)*cosh(d*x + c)^8 - 4620*(11*a^2 + 61*a*b + 137*b^2)*cosh
(d*x + c)^6 - 1050*(31*a^2 + 87*a*b + 390*b^2)*cosh(d*x + c)^4 - 14*(425*a^
2 + 5649*b^2)*cosh(d*x + c)^2 - 155*a^2 + 435*a*b - 1950*b^2)*sinh(d*x + c)
^6 + 8*(2730*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^11 - 5005*(a^2 + 11*a*b + 10
*b^2)*cosh(d*x + c)^9 - 1980*(11*a^2 + 61*a*b + 137*b^2)*cosh(d*x + c)^7 -
630*(31*a^2 + 87*a*b + 390*b^2)*cosh(d*x + c)^5 - 14*(425*a^2 + 5649*b^2)*c
osh(d*x + c)^3 - 15*(31*a^2 - 87*a*b + 390*b^2)*cosh(d*x + c))*sinh(d*x + c
)^5 - 20*(11*a^2 - 61*a*b + 137*b^2)*cosh(d*x + c)^4 + 20*(455*(a^2 + 2*a*b
 + b^2)*cosh(d*x + c)^12 - 1001*(a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^10 -
495*(11*a^2 + 61*a*b + 137*b^2)*cosh(d*x + c)^8 - 210*(31*a^2 + 87*a*b + 39
0*b^2)*cosh(d*x + c)^6 - 7*(425*a^2 + 5649*b^2)*cosh(d*x + c)^4 - 15*(31*a^
2 - 87*a*b + 390*b^2)*cosh(d*x + c)^2 - 11*a^2 + 61*a*b - 137*b^2)*sinh(d*x
 + c)^4 + 16*(175*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^13 - 455*(a^2 + 11*a*b
+ 10*b^2)*cosh(d*x + c)^11 - 275*(11*a^2 + 61*a*b + 137*b^2)*cosh(d*x + c)^
9 - 150*(31*a^2 + 87*a*b + 390*b^2)*cosh(d*x + c)^7 - 7*(425*a^2 + 5649*b^2
)*cosh(d*x + c)^5 - 25*(31*a^2 - 87*a*b + 390*b^2)*cosh(d*x + c)^3 - 5*(11*
a^2 - 61*a*b + 137*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 20*(a^2 - 11*a*b +
 10*b^2)*cosh(d*x + c)^2 + 4*(150*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^14 - 45
5*(a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^12 - 330*(11*a^2 + 61*a*b + 137*b^2
)*cosh(d*x + c)^10 - 225*(31*a^2 + 87*a*b + 390*b^2)*cosh(d*x + c)^8 - 14*(
425*a^2 + 5649*b^2)*cosh(d*x + c)^6 - 75*(31*a^2 - 87*a*b + 390*b^2)*cosh(d
*x + c)^4 - 30*(11*a^2 - 61*a*b + 137*b^2)*cosh(d*x + c)^2 - 5*a^2 + 55*a*b
 - 50*b^2)*sinh(d*x + c)^2 + 5*a^2 - 10*a*b + 5*b^2 + 1200*(a*b*cosh(d*x +
c)^13 + 13*a*b*cosh(d*x + c)*sinh(d*x + c)^12 + a*b*sinh(d*x + c)^13 + 5*a*
b*cosh(d*x + c)^11 + (78*a*b*cosh(d*x + c)^2 + 5*a*b)*sinh(d*x + c)^11 + 10
*a*b*cosh(d*x + c)^9 + 11*(26*a*b*cosh(d*x + c)^3 + 5*a*b*cosh(d*x + c))*si
nh(d*x + c)^10 + 5*(143*a*b*cosh(d*x + c)^4 + 55*a*b*cosh(d*x + c)^2 + 2*a*
b)*sinh(d*x + c)^9 + 10*a*b*cosh(d*x + c)^7 + 3*(429*a*b*cosh(d*x + c)^5 +
275*a*b*cosh(d*x + c)^3 + 30*a*b*cosh(d*x + c))*sinh(d*x + c)^8 + 2*(858*a*
b*cosh(d*x + c)^6 + 825*a*b*cosh(d*x + c)^4 + 180*a*b*cosh(d*x + c)^2 + 5*a
*b)*sinh(d*x + c)^7 + 5*a*b*cosh(d*x + c)^5 + 2*(858*a*b*cosh(d*x + c)^7 +
1155*a*b*cosh(d*x + c)^5 + 420*a*b*cosh(d*x + c)^3 + 35*a*b*cosh(d*x + c))*
sinh(d*x + c)^6 + (1287*a*b*cosh(d*x + c)^8 + 2310*a*b*cosh(d*x + c)^6 + 12
60*a*b*cosh(d*x + c)^4 + 210*a*b*cosh(d*x + c)^2 + 5*a*b)*sinh(d*x + c)^5 +
 a*b*cosh(d*x + c)^3 + 5*(143*a*b*cosh(d*x + c)^9 + 330*a*b*cosh(d*x + c)^7
 + 252*a*b*cosh(d*x + c)^5 + 70*a*b*cosh(d*x + c)^3 + 5*a*b*cosh(d*x + c))*
sinh(d*x + c)^4 + (286*a*b*cosh(d*x + c)^10 + 825*a*b*cosh(d*x + c)^8 + 840
```

```
*a*b*cosh(d*x + c)^6 + 350*a*b*cosh(d*x + c)^4 + 50*a*b*cosh(d*x + c)^2 + a
*b)*sinh(d*x + c)^3 + (78*a*b*cosh(d*x + c)^11 + 275*a*b*cosh(d*x + c)^9 +
360*a*b*cosh(d*x + c)^7 + 210*a*b*cosh(d*x + c)^5 + 50*a*b*cosh(d*x + c)^3
+ 3*a*b*cosh(d*x + c))*sinh(d*x + c)^2 + (13*a*b*cosh(d*x + c)^12 + 55*a*b*
cosh(d*x + c)^10 + 90*a*b*cosh(d*x + c)^8 + 70*a*b*cosh(d*x + c)^6 + 25*a*b
*cosh(d*x + c)^4 + 3*a*b*cosh(d*x + c)^2)*sinh(d*x + c))*arctan(cosh(d*x +
c) + sinh(d*x + c)) + 8*(10*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^15 - 35*(a^2
+ 11*a*b + 10*b^2)*cosh(d*x + c)^13 - 30*(11*a^2 + 61*a*b + 137*b^2)*cosh(d
*x + c)^11 - 25*(31*a^2 + 87*a*b + 390*b^2)*cosh(d*x + c)^9 - 2*(425*a^2 +
5649*b^2)*cosh(d*x + c)^7 - 15*(31*a^2 - 87*a*b + 390*b^2)*cosh(d*x + c)^5
- 10*(11*a^2 - 61*a*b + 137*b^2)*cosh(d*x + c)^3 - 5*(a^2 - 11*a*b + 10*b^2
)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^13 + 13*d*cosh(d*x + c)*si
nh(d*x + c)^12 + d*sinh(d*x + c)^13 + 5*d*cosh(d*x + c)^11 + (78*d*cosh(d*x
 + c)^2 + 5*d)*sinh(d*x + c)^11 + 11*(26*d*cosh(d*x + c)^3 + 5*d*cosh(d*x +
 c))*sinh(d*x + c)^10 + 10*d*cosh(d*x + c)^9 + 5*(143*d*cosh(d*x + c)^4 + 5
5*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^9 + 3*(429*d*cosh(d*x + c)^5 + 275
*d*cosh(d*x + c)^3 + 30*d*cosh(d*x + c))*sinh(d*x + c)^8 + 10*d*cosh(d*x +
c)^7 + 2*(858*d*cosh(d*x + c)^6 + 825*d*cosh(d*x + c)^4 + 180*d*cosh(d*x +
c)^2 + 5*d)*sinh(d*x + c)^7 + 2*(858*d*cosh(d*x + c)^7 + 1155*d*cosh(d*x +
c)^5 + 420*d*cosh(d*x + c)^3 + 35*d*cosh(d*x + c))*sinh(d*x + c)^6 + 5*d*co
sh(d*x + c)^5 + (1287*d*cosh(d*x + c)^8 + 2310*d*cosh(d*x + c)^6 + 1260*d*c
osh(d*x + c)^4 + 210*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^5 + 5*(143*d*co
sh(d*x + c)^9 + 330*d*cosh(d*x + c)^7 + 252*d*cosh(d*x + c)^5 + 70*d*cosh(d
*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^4 + d*cosh(d*x + c)^3 + (286*d
*cosh(d*x + c)^10 + 825*d*cosh(d*x + c)^8 + 840*d*cosh(d*x + c)^6 + 350*d*c
osh(d*x + c)^4 + 50*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^3 + (78*d*cosh(d*x
 + c)^11 + 275*d*cosh(d*x + c)^9 + 360*d*cosh(d*x + c)^7 + 210*d*cosh(d*x +
 c)^5 + 50*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + (13*d*c
osh(d*x + c)^12 + 55*d*cosh(d*x + c)^10 + 90*d*cosh(d*x + c)^8 + 70*d*cosh(
d*x + c)^6 + 25*d*cosh(d*x + c)^4 + 3*d*cosh(d*x + c)^2)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3*(a+b*tanh(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.04108, size = 405, normalized size = 2.23

$$1200\,ab\arctan\left(e^{(dx+c)}\right) - 5\left(9\,a^2e^{(2\,dx+2\,c)} - 54\,abe^{(2\,dx+2\,c)} + 45\,b^2e^{(2\,dx+2\,c)} - a^2 + 2\,ab - b^2\right)e^{(-3\,dx-3\,c)} + 5\left(a^2e^{(3\,dx+48\,c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/120*(1200*a*b*arctan(e^(d*x + c)) - 5*(9*a^2*e^(2*d*x + 2*c) - 54*a*b*e^(
2*d*x + 2*c) + 45*b^2*e^(2*d*x + 2*c) - a^2 + 2*a*b - b^2)*e^(-3*d*x - 3*c)
 + 5*(a^2*e^(3*d*x + 48*c) + 2*a*b*e^(3*d*x + 48*c) + b^2*e^(3*d*x + 48*c)
```

```
- 9*a^2*e^(d*x + 46*c) - 54*a*b*e^(d*x + 46*c) - 45*b^2*e^(d*x + 46*c))*e^(
-45*c) - 16*(15*a*b*e^(9*d*x + 9*c) + 90*b^2*e^(9*d*x + 9*c) + 30*a*b*e^(7*
d*x + 7*c) + 280*b^2*e^(7*d*x + 7*c) + 428*b^2*e^(5*d*x + 5*c) - 30*a*b*e^(
3*d*x + 3*c) + 280*b^2*e^(3*d*x + 3*c) - 15*a*b*e^(d*x + c) + 90*b^2*e^(d*x
 + c))/(e^(2*d*x + 2*c) + 1)^5)/d
```

3.59 $\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=129

$$\frac{\sinh(c+dx)\cosh(c+dx)\left(a^2+2ab\tanh(c+dx)+b^2\right)}{2d}-\frac{1}{2}x\left(a^2+7b^2\right)+\frac{ab\tanh^2(c+dx)}{d}-\frac{4ab\log(\cosh(c+dx))}{d}$$

```
[Out] -((a^2 + 7*b^2)*x)/2 - (4*a*b*Log[Cosh[c + d*x]])/d + (3*b^2*Tanh[c + d*x])
/d + (a*b*Tanh[c + d*x]^2)/d + (2*b^2*Tanh[c + d*x]^3)/(3*d) + (b^2*Tanh[c
+ d*x]^5)/(5*d) + (Cosh[c + d*x]*Sinh[c + d*x]*(a^2 + b^2 + 2*a*b*Tanh[c +
d*x]))/(2*d)
```

Rubi [A] time = 0.205109, antiderivative size = 159, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3663, 1804, 1802, 633, 31}

$$\frac{\left(a^2+7b^2\right)\tanh(c+dx)}{2d}+\frac{\sinh^2(c+dx)\left(\left(a^2+b^2\right)\tanh(c+dx)+2ab\right)}{2d}+\frac{ab\tanh^2(c+dx)}{d}+\frac{(a+b)(a+7b)\log(1}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] ((a + b)*(a + 7*b)*Log[1 - Tanh[c + d*x]])/(4*d) - ((a - 7*b)*(a - b)*Log[1
 + Tanh[c + d*x]])/(4*d) + ((a^2 + 7*b^2)*Tanh[c + d*x])/(2*d) + (a*b*Tanh[
c + d*x]^2)/d + (2*b^2*Tanh[c + d*x]^3)/(3*d) + (b^2*Tanh[c + d*x]^5)/(5*d)
 + (Sinh[c + d*x]^2*(2*a*b + (a^2 + b^2)*Tanh[c + d*x]))/(2*d)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
```

```
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+bx^3\right)^2}{\left(1-x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\sinh^2(c+dx)\left(2ab+\left(a^2+b^2\right)\tanh(c+dx)\right)}{2d} + \frac{\text{Subst}\left(\int \frac{x\left(-4ab-\left(a^2+3b^2\right)x-\right.}{1-}\right.}{}$$

$$= \frac{\sinh^2(c+dx)\left(2ab+\left(a^2+b^2\right)\tanh(c+dx)\right)}{2d} + \frac{\text{Subst}\left(\int \left(a^2+7b^2+4ab\right.\right.}{}$$

$$= \frac{\left(a^2+7b^2\right)\tanh(c+dx)}{2d} + \frac{ab\tanh^2(c+dx)}{d} + \frac{2b^2\tanh^3(c+dx)}{3d} + \frac{b^2\tan}{}$$

$$= \frac{\left(a^2+7b^2\right)\tanh(c+dx)}{2d} + \frac{ab\tanh^2(c+dx)}{d} + \frac{2b^2\tanh^3(c+dx)}{3d} + \frac{b^2\tan}{}$$

$$= \frac{(a+b)(a+7b)\log(1-\tanh(c+dx))}{4d} - \frac{(a-7b)(a-b)\log(1+\tanh(c+d}{4d}$$

Mathematica [A] time = 1.54989, size = 137, normalized size = 1.06

$$\frac{15a^2\sinh(2(c+dx)) - 30a^2c - 30a^2dx + 30ab\cosh(2(c+dx)) - 240ab\log(\cosh(c+dx)) - 4b\text{sech}^2(c+dx)(15a+16b\,\text{t}}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] (-30*a^2*c - 210*b^2*c - 30*a^2*d*x - 210*b^2*d*x + 30*a*b*Cosh[2*(c + d*x)
] - 240*a*b*Log[Cosh[c + d*x]] + 15*a^2*Sinh[2*(c + d*x)] + 15*b^2*Sinh[2*(
c + d*x)] + 232*b^2*Tanh[c + d*x] + 12*b^2*Sech[c + d*x]^4*Tanh[c + d*x] -
4*b*Sech[c + d*x]^2*(15*a + 16*b*Tanh[c + d*x]))/(60*d)
```

Maple [A] time = 0.056, size = 173, normalized size = 1.3

$$\frac{a^2\sinh(dx+c)\cosh(dx+c)}{2d} - \frac{a^2x}{2} - \frac{a^2c}{2d} + \frac{ab(\sinh(dx+c))^4}{d(\cosh(dx+c))^2} - 4\frac{ab\ln(\cosh(dx+c))}{d} + 2\frac{ab(\tanh(dx+c))^2}{d} + \frac{b^2(}{2d(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x)
```

```
[Out] 1/2/d*a^2*sinh(d*x+c)*cosh(d*x+c)-1/2*a^2*x-1/2/d*a^2*c+1/d*a*b*sinh(d*x+c)
^4/cosh(d*x+c)^2-4*a*b*ln(cosh(d*x+c))/d+2*a*b*tanh(d*x+c)^2/d+1/2/d*b^2*si
```

```
nh(d*x+c)^7/cosh(d*x+c)^5-7/2*b^2*x-7/2/d*c*b^2+7/2*b^2*tanh(d*x+c)/d+7/6*b
^2*tanh(d*x+c)^3/d+7/10*b^2*tanh(d*x+c)^5/d
```

Maxima [B] time = 1.57803, size = 406, normalized size = 3.15

$$-\frac{1}{8}a^2\left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d}\right) - \frac{1}{120}b^2\left(\frac{420(dx+c)}{d} + \frac{15e^{(-2dx-2c)}}{d} - \frac{1003e^{(-2dx-2c)} + 3350e^{(-4dx-4c)} + 5590e^{\cdots}}{d\left(e^{(-2dx-2c)} + 5e^{(-4dx-4c)} + 10e^{(-6dx-\cdots}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] -1/8*a^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/120*b^2*(420*(d
*x + c)/d + 15*e^(-2*d*x - 2*c)/d - (1003*e^(-2*d*x - 2*c) + 3350*e^(-4*d*x
 - 4*c) + 5590*e^(-6*d*x - 6*c) + 3915*e^(-8*d*x - 8*c) + 1455*e^(-10*d*x -
 10*c) + 15)/(d*(e^(-2*d*x - 2*c) + 5*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c
) + 10*e^(-8*d*x - 8*c) + 5*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c)))) - 1/
4*a*b*(16*(d*x + c)/d - e^(-2*d*x - 2*c)/d + 16*log(e^(-2*d*x - 2*c) + 1)/d
 - (2*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 1)/(d*(e^(-2*d*x - 2*c) + 2*
e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))))
```

Fricas [B] time = 2.87053, size = 9671, normalized size = 74.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/120*(15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^14 + 210*(a^2 + 2*a*b + b^2)*co
sh(d*x + c)*sinh(d*x + c)^13 + 15*(a^2 + 2*a*b + b^2)*sinh(d*x + c)^14 - 15
*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b - 5*b^2)*cosh(d*x + c)^12 -
15*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 91*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 -
5*a^2 - 10*a*b - 5*b^2)*sinh(d*x + c)^12 + 60*(91*(a^2 + 2*a*b + b^2)*cosh(
d*x + c)^3 - 3*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b - 5*b^2)*cosh(
d*x + c))*sinh(d*x + c)^11 - 15*(20*(a^2 - 8*a*b + 7*b^2)*d*x - 9*a^2 + 10*
a*b + 87*b^2)*cosh(d*x + c)^10 + 15*(1001*(a^2 + 2*a*b + b^2)*cosh(d*x + c)
^4 - 20*(a^2 - 8*a*b + 7*b^2)*d*x - 66*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2
 - 10*a*b - 5*b^2)*cosh(d*x + c)^2 + 9*a^2 - 10*a*b - 87*b^2)*sinh(d*x + c)
^10 + 30*(1001*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 - 110*(4*(a^2 - 8*a*b +
7*b^2)*d*x - 5*a^2 - 10*a*b - 5*b^2)*cosh(d*x + c)^3 - 5*(20*(a^2 - 8*a*b +
 7*b^2)*d*x - 9*a^2 + 10*a*b + 87*b^2)*cosh(d*x + c))*sinh(d*x + c)^9 - 15*
(40*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 + 66*a*b + 251*b^2)*cosh(d*x + c)^8 +
 15*(3003*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 - 495*(4*(a^2 - 8*a*b + 7*b^2
)*d*x - 5*a^2 - 10*a*b - 5*b^2)*cosh(d*x + c)^4 - 40*(a^2 - 8*a*b + 7*b^2)*
d*x - 45*(20*(a^2 - 8*a*b + 7*b^2)*d*x - 9*a^2 + 10*a*b + 87*b^2)*cosh(d*x
+ c)^2 + 5*a^2 - 66*a*b - 251*b^2)*sinh(d*x + c)^8 + 120*(429*(a^2 + 2*a*b
+ b^2)*cosh(d*x + c)^7 - 99*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b -
 5*b^2)*cosh(d*x + c)^5 - 15*(20*(a^2 - 8*a*b + 7*b^2)*d*x - 9*a^2 + 10*a*b
 + 87*b^2)*cosh(d*x + c)^3 - (40*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 + 66*a*b
 + 251*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 5*(120*(a^2 - 8*a*b + 7*b^2)*d
*x + 15*a^2 + 198*a*b + 1103*b^2)*cosh(d*x + c)^6 + 5*(9009*(a^2 + 2*a*b +
b^2)*cosh(d*x + c)^8 - 2772*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b -
 5*b^2)*cosh(d*x + c)^6 - 630*(20*(a^2 - 8*a*b + 7*b^2)*d*x - 9*a^2 + 10*a*
```

```
b + 87*b^2)*cosh(d*x + c)^4 - 120*(a^2 - 8*a*b + 7*b^2)*d*x - 84*(40*(a^2 -
 8*a*b + 7*b^2)*d*x - 5*a^2 + 66*a*b + 251*b^2)*cosh(d*x + c)^2 - 15*a^2 -
198*a*b - 1103*b^2)*sinh(d*x + c)^6 + 30*(1001*(a^2 + 2*a*b + b^2)*cosh(d*x
 + c)^9 - 396*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b - 5*b^2)*cosh(d
*x + c)^7 - 126*(20*(a^2 - 8*a*b + 7*b^2)*d*x - 9*a^2 + 10*a*b + 87*b^2)*co
sh(d*x + c)^5 - 28*(40*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 + 66*a*b + 251*b^2
)*cosh(d*x + c)^3 - (120*(a^2 - 8*a*b + 7*b^2)*d*x + 15*a^2 + 198*a*b + 110
3*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 5*(60*(a^2 - 8*a*b + 7*b^2)*d*x + 2
7*a^2 + 30*a*b + 667*b^2)*cosh(d*x + c)^4 + 5*(3003*(a^2 + 2*a*b + b^2)*cos
h(d*x + c)^10 - 1485*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b - 5*b^2)
*cosh(d*x + c)^8 - 630*(20*(a^2 - 8*a*b + 7*b^2)*d*x - 9*a^2 + 10*a*b + 87*
b^2)*cosh(d*x + c)^6 - 210*(40*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 + 66*a*b +
 251*b^2)*cosh(d*x + c)^4 - 60*(a^2 - 8*a*b + 7*b^2)*d*x - 15*(120*(a^2 - 8
*a*b + 7*b^2)*d*x + 15*a^2 + 198*a*b + 1103*b^2)*cosh(d*x + c)^2 - 27*a^2 -
 30*a*b - 667*b^2)*sinh(d*x + c)^4 + 20*(273*(a^2 + 2*a*b + b^2)*cosh(d*x +
 c)^11 - 165*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b - 5*b^2)*cosh(d*
x + c)^9 - 90*(20*(a^2 - 8*a*b + 7*b^2)*d*x - 9*a^2 + 10*a*b + 87*b^2)*cosh
(d*x + c)^7 - 42*(40*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 + 66*a*b + 251*b^2)*
cosh(d*x + c)^5 - 5*(120*(a^2 - 8*a*b + 7*b^2)*d*x + 15*a^2 + 198*a*b + 110
3*b^2)*cosh(d*x + c)^3 - (60*(a^2 - 8*a*b + 7*b^2)*d*x + 27*a^2 + 30*a*b +
667*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - (60*(a^2 - 8*a*b + 7*b^2)*d*x + 7
5*a^2 - 150*a*b + 1003*b^2)*cosh(d*x + c)^2 + (1365*(a^2 + 2*a*b + b^2)*cos
h(d*x + c)^12 - 990*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b - 5*b^2)*
cosh(d*x + c)^10 - 675*(20*(a^2 - 8*a*b + 7*b^2)*d*x - 9*a^2 + 10*a*b + 87*
b^2)*cosh(d*x + c)^8 - 420*(40*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 + 66*a*b +
 251*b^2)*cosh(d*x + c)^6 - 75*(120*(a^2 - 8*a*b + 7*b^2)*d*x + 15*a^2 + 19
8*a*b + 1103*b^2)*cosh(d*x + c)^4 - 60*(a^2 - 8*a*b + 7*b^2)*d*x - 30*(60*(
a^2 - 8*a*b + 7*b^2)*d*x + 27*a^2 + 30*a*b + 667*b^2)*cosh(d*x + c)^2 - 75*
a^2 + 150*a*b - 1003*b^2)*sinh(d*x + c)^2 - 15*a^2 + 30*a*b - 15*b^2 - 480*
(a*b*cosh(d*x + c)^12 + 12*a*b*cosh(d*x + c)*sinh(d*x + c)^11 + a*b*sinh(d*
x + c)^12 + 5*a*b*cosh(d*x + c)^10 + (66*a*b*cosh(d*x + c)^2 + 5*a*b)*sinh(
d*x + c)^10 + 10*a*b*cosh(d*x + c)^8 + 10*(22*a*b*cosh(d*x + c)^3 + 5*a*b*c
osh(d*x + c))*sinh(d*x + c)^9 + 5*(99*a*b*cosh(d*x + c)^4 + 45*a*b*cosh(d*x
 + c)^2 + 2*a*b)*sinh(d*x + c)^8 + 10*a*b*cosh(d*x + c)^6 + 8*(99*a*b*cosh(
d*x + c)^5 + 75*a*b*cosh(d*x + c)^3 + 10*a*b*cosh(d*x + c))*sinh(d*x + c)^7
 + 2*(462*a*b*cosh(d*x + c)^6 + 525*a*b*cosh(d*x + c)^4 + 140*a*b*cosh(d*x
+ c)^2 + 5*a*b)*sinh(d*x + c)^6 + 5*a*b*cosh(d*x + c)^4 + 4*(198*a*b*cosh(d
*x + c)^7 + 315*a*b*cosh(d*x + c)^5 + 140*a*b*cosh(d*x + c)^3 + 15*a*b*cosh
(d*x + c))*sinh(d*x + c)^5 + 5*(99*a*b*cosh(d*x + c)^8 + 210*a*b*cosh(d*x +
 c)^6 + 140*a*b*cosh(d*x + c)^4 + 30*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x +
c)^4 + a*b*cosh(d*x + c)^2 + 20*(11*a*b*cosh(d*x + c)^9 + 30*a*b*cosh(d*x +
 c)^7 + 28*a*b*cosh(d*x + c)^5 + 10*a*b*cosh(d*x + c)^3 + a*b*cosh(d*x + c)
)*sinh(d*x + c)^3 + (66*a*b*cosh(d*x + c)^10 + 225*a*b*cosh(d*x + c)^8 + 28
0*a*b*cosh(d*x + c)^6 + 150*a*b*cosh(d*x + c)^4 + 30*a*b*cosh(d*x + c)^2 +
a*b)*sinh(d*x + c)^2 + 2*(6*a*b*cosh(d*x + c)^11 + 25*a*b*cosh(d*x + c)^9 +
 40*a*b*cosh(d*x + c)^7 + 30*a*b*cosh(d*x + c)^5 + 10*a*b*cosh(d*x + c)^3 +
 a*b*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sin
h(d*x + c))) + 2*(105*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^13 - 90*(4*(a^2 - 8
*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b - 5*b^2)*cosh(d*x + c)^11 - 75*(20*(a^2
- 8*a*b + 7*b^2)*d*x - 9*a^2 + 10*a*b + 87*b^2)*cosh(d*x + c)^9 - 60*(40*(a
^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 + 66*a*b + 251*b^2)*cosh(d*x + c)^7 - 15*(1
20*(a^2 - 8*a*b + 7*b^2)*d*x + 15*a^2 + 198*a*b + 1103*b^2)*cosh(d*x + c)^5
 - 10*(60*(a^2 - 8*a*b + 7*b^2)*d*x + 27*a^2 + 30*a*b + 667*b^2)*cosh(d*x +
 c)^3 - (60*(a^2 - 8*a*b + 7*b^2)*d*x + 75*a^2 - 150*a*b + 1003*b^2)*cosh(d
*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^12 + 12*d*cosh(d*x + c)*sinh(d*x +
 c)^11 + d*sinh(d*x + c)^12 + 5*d*cosh(d*x + c)^10 + (66*d*cosh(d*x + c)^2
+ 5*d)*sinh(d*x + c)^10 + 10*(22*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sin
h(d*x + c)^9 + 10*d*cosh(d*x + c)^8 + 5*(99*d*cosh(d*x + c)^4 + 45*d*cosh(d
*x + c)^2 + 2*d)*sinh(d*x + c)^8 + 8*(99*d*cosh(d*x + c)^5 + 75*d*cosh(d*x
```

```
+ c)^3 + 10*d*cosh(d*x + c))*sinh(d*x + c)^7 + 10*d*cosh(d*x + c)^6 + 2*(46
2*d*cosh(d*x + c)^6 + 525*d*cosh(d*x + c)^4 + 140*d*cosh(d*x + c)^2 + 5*d)*
sinh(d*x + c)^6 + 4*(198*d*cosh(d*x + c)^7 + 315*d*cosh(d*x + c)^5 + 140*d*
cosh(d*x + c)^3 + 15*d*cosh(d*x + c))*sinh(d*x + c)^5 + 5*d*cosh(d*x + c)^4
 + 5*(99*d*cosh(d*x + c)^8 + 210*d*cosh(d*x + c)^6 + 140*d*cosh(d*x + c)^4
+ 30*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 20*(11*d*cosh(d*x + c)^9 + 30
*d*cosh(d*x + c)^7 + 28*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + d*cosh(d
*x + c))*sinh(d*x + c)^3 + d*cosh(d*x + c)^2 + (66*d*cosh(d*x + c)^10 + 225
*d*cosh(d*x + c)^8 + 280*d*cosh(d*x + c)^6 + 150*d*cosh(d*x + c)^4 + 30*d*c
osh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 2*(6*d*cosh(d*x + c)^11 + 25*d*cosh(d
*x + c)^9 + 40*d*cosh(d*x + c)^7 + 30*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c
)^3 + d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*tanh(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.83589, size = 402, normalized size = 3.12

$$-\frac{60\left(a^2-8\,ab+7\,b^2\right)dx+480\,ab\log\left(e^{(2\,dx+2\,c)}+1\right)-15\left(2\,a^2e^{(2\,dx+2\,c)}-16\,abe^{(2\,dx+2\,c)}+14\,b^2e^{(2\,dx+2\,c)}-a^2+2\,ab-\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -1/120*(60*(a^2 - 8*a*b + 7*b^2)*d*x + 480*a*b*log(e^(2*d*x + 2*c) + 1) - 1
5*(2*a^2*e^(2*d*x + 2*c) - 16*a*b*e^(2*d*x + 2*c) + 14*b^2*e^(2*d*x + 2*c)
- a^2 + 2*a*b - b^2)*e^(-2*d*x - 2*c) - 15*(a^2*e^(2*d*x + 16*c) + 2*a*b*e^
(2*d*x + 16*c) + b^2*e^(2*d*x + 16*c))*e^(-14*c) - 8*(137*a*b*e^(10*d*x + 1
0*c) + 625*a*b*e^(8*d*x + 8*c) - 180*b^2*e^(8*d*x + 8*c) + 1190*a*b*e^(6*d*
x + 6*c) - 480*b^2*e^(6*d*x + 6*c) + 1190*a*b*e^(4*d*x + 4*c) - 680*b^2*e^(
4*d*x + 4*c) + 625*a*b*e^(2*d*x + 2*c) - 400*b^2*e^(2*d*x + 2*c) + 137*a*b
- 116*b^2)/(e^(2*d*x + 2*c) + 1)^5)/d
```

3.60 $\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=123

$$\frac{a^2\cosh(c+dx)}{d}+\frac{3ab\sinh(c+dx)}{d}-\frac{3ab\tan^{-1}(\sinh(c+dx))}{d}-\frac{ab\sinh(c+dx)\tanh^2(c+dx)}{d}+\frac{b^2\cosh(c+dx)}{d}+\frac{b^2\text{s}}{}$$

```
[Out] (-3*a*b*ArcTan[Sinh[c + d*x]])/d + (a^2*Cosh[c + d*x])/d + (b^2*Cosh[c + d*
x])/d + (3*b^2*Sech[c + d*x])/d - (b^2*Sech[c + d*x]^3)/d + (b^2*Sech[c + d
*x]^5)/(5*d) + (3*a*b*Sinh[c + d*x])/d - (a*b*Sinh[c + d*x]*Tanh[c + d*x]^2
)/d
```

Rubi [A] time = 0.146609, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3666, 2638, 2592, 288, 321, 203, 2590, 270}

$$\frac{a^2\cosh(c+dx)}{d}+\frac{3ab\sinh(c+dx)}{d}-\frac{3ab\tan^{-1}(\sinh(c+dx))}{d}-\frac{ab\sinh(c+dx)\tanh^2(c+dx)}{d}+\frac{b^2\cosh(c+dx)}{d}+\frac{b^2\text{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^3)^2,x]
```


```
[Out] (-3*a*b*ArcTan[Sinh[c + d*x]])/d + (a^2*Cosh[c + d*x])/d + (b^2*Cosh[c + d*
x])/d + (3*b^2*Sech[c + d*x])/d - (b^2*Sech[c + d*x]^3)/d + (b^2*Sech[c + d
*x]^5)/(5*d) + (3*a*b*Sinh[c + d*x])/d - (a*b*Sinh[c + d*x]*Tanh[c + d*x]^2
)/d
```


Rule 3666

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = -\left(i\int \left(ia^2\sinh(c+dx)+2iab\sinh(c+dx)\tanh^3(c+dx)+ib^2\sinh(c+\right.\right.$$
$$= a^2\int \sinh(c+dx)\,dx+(2ab)\int \sinh(c+dx)\tanh^3(c+dx)\,dx+b^2\int \mathrm{si}$$
$$= \frac{a^2\cosh(c+dx)}{d}+\frac{(2ab)\,\mathrm{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^2}\,dx,x,\sinh(c+dx)\right)}{d}-\frac{b^2\,\mathrm{Subs}}{}$$
$$= \frac{a^2\cosh(c+dx)}{d}-\frac{ab\sinh(c+dx)\tanh^2(c+dx)}{d}+\frac{(3ab)\,\mathrm{Subst}\left(\int \frac{x^2}{1+x^2}\right.}{d}$$
$$= \frac{a^2\cosh(c+dx)}{d}+\frac{b^2\cosh(c+dx)}{d}+\frac{3b^2\mathrm{sech}(c+dx)}{d}-\frac{b^2\mathrm{sech}^3(c+dx}{d}$$
$$= -\frac{3ab\tan^{-1}(\sinh(c+dx))}{d}+\frac{a^2\cosh(c+dx)}{d}+\frac{b^2\cosh(c+dx)}{d}+\frac{3b^2\mathrm{se}}{}$$

Mathematica [A] time = 0.393281, size = 90, normalized size = 0.73

$$\frac{5\left(a^2+b^2\right)\cosh(c+dx)+b\left(5\mathrm{sech}(c+dx)(a\tanh(c+dx)+3b)+10a\left(\sinh(c+dx)-3\tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)\right)\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] (5*(a^2 + b^2)*Cosh[c + d*x] + b*(-5*b*Sech[c + d*x]^3 + b*Sech[c + d*x]^5
+ 10*a*(-3*ArcTan[Tanh[(c + d*x)/2]] + Sinh[c + d*x]) + 5*Sech[c + d*x]*(3*
b + a*Tanh[c + d*x])))/(5*d)
```

Maple [A] time = 0.054, size = 225, normalized size = 1.8

$$\frac{a^2\cosh(dx+c)}{d}+2\frac{ab(\sinh(dx+c))^3}{d(\cosh(dx+c))^2}+6\frac{ab\sinh(dx+c)}{d(\cosh(dx+c))^2}-3\frac{ab\mathrm{sech}(dx+c)\tanh(dx+c)}{d}-6\frac{ab\arctan\left(e^{dx+c}\right)}{d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x)
```

```
[Out] a^2*cosh(d*x+c)/d+2/d*a*b*sinh(d*x+c)^3/cosh(d*x+c)^2+6/d*a*b*sinh(d*x+c)/c
osh(d*x+c)^2-3/d*a*b*sech(d*x+c)*tanh(d*x+c)-6/d*a*b*arctan(exp(d*x+c))+1/d
*b^2*sinh(d*x+c)^6/cosh(d*x+c)^5+6/d*b^2*sinh(d*x+c)^4/cosh(d*x+c)^5+24/5/d
*b^2*sinh(d*x+c)^2/cosh(d*x+c)^5-16/5/d*b^2*sinh(d*x+c)^2/cosh(d*x+c)^3-16/
5/d*b^2*sinh(d*x+c)^2/cosh(d*x+c)+16/5*b^2*cosh(d*x+c)/d
```

Maxima [B] time = 1.5844, size = 342, normalized size = 2.78

$$ab\left(\frac{6\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}}{d}+\frac{4e^{(-2dx-2c)}-e^{(-4dx-4c)}+1}{d\left(e^{(-dx-c)}+2e^{(-3dx-3c)}+e^{(-5dx-5c)}\right)}\right)+\frac{1}{10}b^2\left(\frac{5e^{(-dx-c)}}{d}+\frac{85e^{(-2dx-2c)}+210e^{(-4dx-}}{d\left(e^{(-dx-c)}+5e^{(-3dx-3c)}+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] a*b*(6*arctan(e^(-d*x - c))/d - e^(-d*x - c)/d + (4*e^(-2*d*x - 2*c) - e^(-
4*d*x - 4*c) + 1)/(d*(e^(-d*x - c) + 2*e^(-3*d*x - 3*c) + e^(-5*d*x - 5*c))
)) + 1/10*b^2*(5*e^(-d*x - c)/d + (85*e^(-2*d*x - 2*c) + 210*e^(-4*d*x - 4*
c) + 314*e^(-6*d*x - 6*c) + 185*e^(-8*d*x - 8*c) + 65*e^(-10*d*x - 10*c) +
5)/(d*(e^(-d*x - c) + 5*e^(-3*d*x - 3*c) + 10*e^(-5*d*x - 5*c) + 10*e^(-7*d
*x - 7*c) + 5*e^(-9*d*x - 9*c) + e^(-11*d*x - 11*c)))) + a^2*cosh(d*x + c)/
d
```

Fricas [B] time = 2.51216, size = 5998, normalized size = 48.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/10*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^12 + 60*(a^2 + 2*a*b + b^2)*cosh(
d*x + c)*sinh(d*x + c)^11 + 5*(a^2 + 2*a*b + b^2)*sinh(d*x + c)^12 + 30*(a^
2 + 2*a*b + 3*b^2)*cosh(d*x + c)^10 + 30*(11*(a^2 + 2*a*b + b^2)*cosh(d*x +
 c)^2 + a^2 + 2*a*b + 3*b^2)*sinh(d*x + c)^10 + 100*(11*(a^2 + 2*a*b + b^2)
*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^9 +
 5*(15*a^2 + 18*a*b + 47*b^2)*cosh(d*x + c)^8 + 5*(495*(a^2 + 2*a*b + b^2)*
cosh(d*x + c)^4 + 270*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^2 + 15*a^2 + 18*a
*b + 47*b^2)*sinh(d*x + c)^8 + 40*(99*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 +
 90*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^3 + (15*a^2 + 18*a*b + 47*b^2)*cosh
(d*x + c))*sinh(d*x + c)^7 + 4*(25*a^2 + 91*b^2)*cosh(d*x + c)^6 + 4*(1155*
(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 1575*(a^2 + 2*a*b + 3*b^2)*cosh(d*x +
 c)^4 + 35*(15*a^2 + 18*a*b + 47*b^2)*cosh(d*x + c)^2 + 25*a^2 + 91*b^2)*si
nh(d*x + c)^6 + 8*(495*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 945*(a^2 + 2*a
```

```
*b + 3*b^2)*cosh(d*x + c)^5 + 35*(15*a^2 + 18*a*b + 47*b^2)*cosh(d*x + c)^3
 + 3*(25*a^2 + 91*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 5*(15*a^2 - 18*a*b
+ 47*b^2)*cosh(d*x + c)^4 + 5*(495*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 12
60*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^6 + 70*(15*a^2 + 18*a*b + 47*b^2)*co
sh(d*x + c)^4 + 12*(25*a^2 + 91*b^2)*cosh(d*x + c)^2 + 15*a^2 - 18*a*b + 47
*b^2)*sinh(d*x + c)^4 + 20*(55*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^9 + 180*(a
^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^7 + 14*(15*a^2 + 18*a*b + 47*b^2)*cosh(d*
x + c)^5 + 4*(25*a^2 + 91*b^2)*cosh(d*x + c)^3 + (15*a^2 - 18*a*b + 47*b^2)
*cosh(d*x + c))*sinh(d*x + c)^3 + 30*(a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^2
+ 10*(33*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^10 + 135*(a^2 + 2*a*b + 3*b^2)*c
osh(d*x + c)^8 + 14*(15*a^2 + 18*a*b + 47*b^2)*cosh(d*x + c)^6 + 6*(25*a^2
+ 91*b^2)*cosh(d*x + c)^4 + 3*(15*a^2 - 18*a*b + 47*b^2)*cosh(d*x + c)^2 +
3*a^2 - 6*a*b + 9*b^2)*sinh(d*x + c)^2 + 5*a^2 - 10*a*b + 5*b^2 - 60*(a*b*c
osh(d*x + c)^11 + 11*a*b*cosh(d*x + c)*sinh(d*x + c)^10 + a*b*sinh(d*x + c)
^11 + 5*a*b*cosh(d*x + c)^9 + 5*(11*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c
)^9 + 10*a*b*cosh(d*x + c)^7 + 15*(11*a*b*cosh(d*x + c)^3 + 3*a*b*cosh(d*x
+ c))*sinh(d*x + c)^8 + 10*(33*a*b*cosh(d*x + c)^4 + 18*a*b*cosh(d*x + c)^2
 + a*b)*sinh(d*x + c)^7 + 10*a*b*cosh(d*x + c)^5 + 14*(33*a*b*cosh(d*x + c)
^5 + 30*a*b*cosh(d*x + c)^3 + 5*a*b*cosh(d*x + c))*sinh(d*x + c)^6 + 2*(231
*a*b*cosh(d*x + c)^6 + 315*a*b*cosh(d*x + c)^4 + 105*a*b*cosh(d*x + c)^2 +
5*a*b)*sinh(d*x + c)^5 + 5*a*b*cosh(d*x + c)^3 + 10*(33*a*b*cosh(d*x + c)^7
 + 63*a*b*cosh(d*x + c)^5 + 35*a*b*cosh(d*x + c)^3 + 5*a*b*cosh(d*x + c))*s
inh(d*x + c)^4 + 5*(33*a*b*cosh(d*x + c)^8 + 84*a*b*cosh(d*x + c)^6 + 70*a*
b*cosh(d*x + c)^4 + 20*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^3 + a*b*cos
h(d*x + c) + 5*(11*a*b*cosh(d*x + c)^9 + 36*a*b*cosh(d*x + c)^7 + 42*a*b*co
sh(d*x + c)^5 + 20*a*b*cosh(d*x + c)^3 + 3*a*b*cosh(d*x + c))*sinh(d*x + c)
^2 + (11*a*b*cosh(d*x + c)^10 + 45*a*b*cosh(d*x + c)^8 + 70*a*b*cosh(d*x +
c)^6 + 50*a*b*cosh(d*x + c)^4 + 15*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)
)*arctan(cosh(d*x + c) + sinh(d*x + c)) + 4*(15*(a^2 + 2*a*b + b^2)*cosh(d*
x + c)^11 + 75*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^9 + 10*(15*a^2 + 18*a*b
+ 47*b^2)*cosh(d*x + c)^7 + 6*(25*a^2 + 91*b^2)*cosh(d*x + c)^5 + 5*(15*a^2
 - 18*a*b + 47*b^2)*cosh(d*x + c)^3 + 15*(a^2 - 2*a*b + 3*b^2)*cosh(d*x + c
))*sinh(d*x + c))/(d*cosh(d*x + c)^11 + 11*d*cosh(d*x + c)*sinh(d*x + c)^10
 + d*sinh(d*x + c)^11 + 5*d*cosh(d*x + c)^9 + 5*(11*d*cosh(d*x + c)^2 + d)*
sinh(d*x + c)^9 + 15*(11*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x +
c)^8 + 10*d*cosh(d*x + c)^7 + 10*(33*d*cosh(d*x + c)^4 + 18*d*cosh(d*x + c)
^2 + d)*sinh(d*x + c)^7 + 14*(33*d*cosh(d*x + c)^5 + 30*d*cosh(d*x + c)^3 +
 5*d*cosh(d*x + c))*sinh(d*x + c)^6 + 10*d*cosh(d*x + c)^5 + 2*(231*d*cosh(
d*x + c)^6 + 315*d*cosh(d*x + c)^4 + 105*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x
+ c)^5 + 10*(33*d*cosh(d*x + c)^7 + 63*d*cosh(d*x + c)^5 + 35*d*cosh(d*x +
c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(33*d*c
osh(d*x + c)^8 + 84*d*cosh(d*x + c)^6 + 70*d*cosh(d*x + c)^4 + 20*d*cosh(d*
x + c)^2 + d)*sinh(d*x + c)^3 + 5*(11*d*cosh(d*x + c)^9 + 36*d*cosh(d*x + c
)^7 + 42*d*cosh(d*x + c)^5 + 20*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh
(d*x + c)^2 + d*cosh(d*x + c) + (11*d*cosh(d*x + c)^10 + 45*d*cosh(d*x + c)
^8 + 70*d*cosh(d*x + c)^6 + 50*d*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 + d
)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)**3)**2,x)

[Out] Timed out
```

Giac [A] time = 1.61689, size = 289, normalized size = 2.35

$$-\frac{60\,ab\arctan\left(e^{(dx+c)}\right)-5\left(a^2-2\,ab+b^2\right)e^{(-dx-c)}-5\left(a^2e^{(dx+14\,c)}+2\,abe^{(dx+14\,c)}+b^2e^{(dx+14\,c)}\right)e^{(-13\,c)}-\frac{4\left(5\,abe^{(9\,dx+9\,c)}+15\,b^2\right.}{}}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -1/10*(60*a*b*arctan(e^(d*x + c)) - 5*(a^2 - 2*a*b + b^2)*e^(-d*x - c) - 5*
(a^2*e^(d*x + 14*c) + 2*a*b*e^(d*x + 14*c) + b^2*e^(d*x + 14*c))*e^(-13*c)
- 4*(5*a*b*e^(9*d*x + 9*c) + 15*b^2*e^(9*d*x + 9*c) + 10*a*b*e^(7*d*x + 7*c
) + 40*b^2*e^(7*d*x + 7*c) + 66*b^2*e^(5*d*x + 5*c) - 10*a*b*e^(3*d*x + 3*c
) + 40*b^2*e^(3*d*x + 3*c) - 5*a*b*e^(d*x + c) + 15*b^2*e^(d*x + c))/(e^(2*
d*x + 2*c) + 1)^5)/d
```

3.61 $\int \mathbf{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=98

$$-\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{ab\tan^{-1}(\sinh(c+dx))}{d}-\frac{ab\tanh(c+dx)\text{sech}(c+dx)}{d}-\frac{b^2\text{sech}^5(c+dx)}{5d}+\frac{2b^2\text{sech}^3(c}{3d}$$

```
[Out] (a*b*ArcTan[Sinh[c + d*x]])/d - (a^2*ArcTanh[Cosh[c + d*x]])/d - (b^2*Sech[
c + d*x])/d + (2*b^2*Sech[c + d*x]^3)/(3*d) - (b^2*Sech[c + d*x]^5)/(5*d) -
 (a*b*Sech[c + d*x]*Tanh[c + d*x])/d
```

Rubi [A] time = 0.136202, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3666, 3770, 2611, 2606, 194}

$$-\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{ab\tan^{-1}(\sinh(c+dx))}{d}-\frac{ab\tanh(c+dx)\text{sech}(c+dx)}{d}-\frac{b^2\text{sech}^5(c+dx)}{5d}+\frac{2b^2\text{sech}^3(c}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] (a*b*ArcTan[Sinh[c + d*x]])/d - (a^2*ArcTanh[Cosh[c + d*x]])/d - (b^2*Sech[
c + d*x])/d + (2*b^2*Sech[c + d*x]^3)/(3*d) - (b^2*Sech[c + d*x]^5)/(5*d) -
 (a*b*Sech[c + d*x]*Tanh[c + d*x])/d
```

Rule 3666

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^2 dx = i\int \left(-ia^2\operatorname{csch}(c+dx) - 2iab\operatorname{sech}(c+dx)\tanh^2(c+dx) - ib^2\operatorname{sech}(c+dx)\right.$$

$$= a^2\int \operatorname{csch}(c+dx)\,dx + (2ab)\int \operatorname{sech}(c+dx)\tanh^2(c+dx)\,dx + b^2\int \operatorname{sec}$$

$$= -\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d} - \frac{ab\operatorname{sech}(c+dx)\tanh(c+dx)}{d} + (ab)\int \operatorname{sech}(c$$

$$= \frac{ab\tan^{-1}(\sinh(c+dx))}{d} - \frac{a^2\tanh^{-1}(\cosh(c+dx))}{d} - \frac{ab\operatorname{sech}(c+dx)\tanh(}{d}$$

$$= \frac{ab\tan^{-1}(\sinh(c+dx))}{d} - \frac{a^2\tanh^{-1}(\cosh(c+dx))}{d} - \frac{b^2\operatorname{sech}(c+dx)}{d} + \frac{2b^2}{}$$

Mathematica [A] time = 0.156975, size = 106, normalized size = 1.08

$$\frac{a^2\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{2ab\tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{ab\tanh(c+dx)\operatorname{sech}(c+dx)}{d} - \frac{b^2\operatorname{sech}^5(c+dx)}{5d} + \frac{2b^2\operatorname{sec}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] (2*a*b*ArcTan[Tanh[(c + d*x)/2]])/d + (a^2*Log[Tanh[(c + d*x)/2]])/d - (b^2
*Sech[c + d*x])/d + (2*b^2*Sech[c + d*x]^3)/(3*d) - (b^2*Sech[c + d*x]^5)/(
5*d) - (a*b*Sech[c + d*x]*Tanh[c + d*x])/d
```

Maple [A] time = 0.072, size = 180, normalized size = 1.8

$$-2\frac{a^2\operatorname{Artanh}\left(e^{dx+c}\right)}{d} - 2\frac{ab\sinh(dx+c)}{d(\cosh(dx+c))^2} + \frac{ab\operatorname{sech}(dx+c)\tanh(dx+c)}{d} + 2\frac{ab\arctan\left(e^{dx+c}\right)}{d} - \frac{b^2(\sinh(dx+c))^4}{d(\cosh(dx+c))^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x)
```

```
[Out] -2/d*a^2*arctanh(exp(d*x+c))-2/d*a*b*sinh(d*x+c)/cosh(d*x+c)^2+1/d*a*b*sech
(d*x+c)*tanh(d*x+c)+2/d*a*b*arctan(exp(d*x+c))-1/d*b^2*sinh(d*x+c)^4/cosh(d
*x+c)^5-4/5/d*b^2*sinh(d*x+c)^2/cosh(d*x+c)^5+8/15/d*b^2*sinh(d*x+c)^2/cosh
(d*x+c)^3+8/15/d*b^2*sinh(d*x+c)^2/cosh(d*x+c)-8/15*b^2*cosh(d*x+c)/d
```

Maxima [B] time = 1.53543, size = 603, normalized size = 6.15

$$-2\,ab\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} + \frac{e^{(-dx-c)} - e^{(-3\,dx-3\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)} + e^{(-4\,dx-4\,c)} + 1\right)}\right) - \frac{2}{15}\,b^2\left(\frac{15\,e^{(-dx-c)}}{d\left(5\,e^{(-2\,dx-2\,c)} + 10\,e^{(-4\,dx-4\,c)} + 10\,e^{(-6\,dx-6\,c)} + 5\,e^{(-8d}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] -2*a*b*(arctan(e^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^
(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) - 2/15*b^2*(15*e^(-d*x - c)/(d*(5*
e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x
- 8*c) + e^(-10*d*x - 10*c) + 1)) + 20*e^(-3*d*x - 3*c)/(d*(5*e^(-2*d*x - 2
*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-
10*d*x - 10*c) + 1)) + 58*e^(-5*d*x - 5*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-
4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c
) + 1)) + 20*e^(-7*d*x - 7*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c)
+ 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*
e^(-9*d*x - 9*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*
x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + a^2*log(tanh(1/
2*d*x + 1/2*c))/d
```

Fricas [B] time = 2.67318, size = 6643, normalized size = 67.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] -1/15*(30*(a*b + b^2)*cosh(d*x + c)^9 + 270*(a*b + b^2)*cosh(d*x + c)*sinh(
d*x + c)^8 + 30*(a*b + b^2)*sinh(d*x + c)^9 + 20*(3*a*b + 2*b^2)*cosh(d*x +
 c)^7 + 20*(54*(a*b + b^2)*cosh(d*x + c)^2 + 3*a*b + 2*b^2)*sinh(d*x + c)^7
 + 116*b^2*cosh(d*x + c)^5 + 140*(18*(a*b + b^2)*cosh(d*x + c)^3 + (3*a*b +
 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^6 + 4*(945*(a*b + b^2)*cosh(d*x + c)^4
 + 105*(3*a*b + 2*b^2)*cosh(d*x + c)^2 + 29*b^2)*sinh(d*x + c)^5 + 20*(189*
(a*b + b^2)*cosh(d*x + c)^5 + 35*(3*a*b + 2*b^2)*cosh(d*x + c)^3 + 29*b^2*c
osh(d*x + c))*sinh(d*x + c)^4 - 20*(3*a*b - 2*b^2)*cosh(d*x + c)^3 + 20*(12
6*(a*b + b^2)*cosh(d*x + c)^6 + 35*(3*a*b + 2*b^2)*cosh(d*x + c)^4 + 58*b^2
*cosh(d*x + c)^2 - 3*a*b + 2*b^2)*sinh(d*x + c)^3 + 20*(54*(a*b + b^2)*cosh
(d*x + c)^7 + 21*(3*a*b + 2*b^2)*cosh(d*x + c)^5 + 58*b^2*cosh(d*x + c)^3 -
 3*(3*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 30*(a*b*cosh(d*x + c)^1
0 + 10*a*b*cosh(d*x + c)*sinh(d*x + c)^9 + a*b*sinh(d*x + c)^10 + 5*a*b*cos
h(d*x + c)^8 + 5*(9*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^8 + 10*a*b*cos
h(d*x + c)^6 + 40*(3*a*b*cosh(d*x + c)^3 + a*b*cosh(d*x + c))*sinh(d*x + c)
^7 + 10*(21*a*b*cosh(d*x + c)^4 + 14*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x +
c)^6 + 10*a*b*cosh(d*x + c)^4 + 4*(63*a*b*cosh(d*x + c)^5 + 70*a*b*cosh(d*x
 + c)^3 + 15*a*b*cosh(d*x + c))*sinh(d*x + c)^5 + 10*(21*a*b*cosh(d*x + c)^
6 + 35*a*b*cosh(d*x + c)^4 + 15*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^4
+ 5*a*b*cosh(d*x + c)^2 + 40*(3*a*b*cosh(d*x + c)^7 + 7*a*b*cosh(d*x + c)^5
 + 5*a*b*cosh(d*x + c)^3 + a*b*cosh(d*x + c))*sinh(d*x + c)^3 + 5*(9*a*b*co
sh(d*x + c)^8 + 28*a*b*cosh(d*x + c)^6 + 30*a*b*cosh(d*x + c)^4 + 12*a*b*co
sh(d*x + c)^2 + a*b)*sinh(d*x + c)^2 + a*b + 10*(a*b*cosh(d*x + c)^9 + 4*a*
b*cosh(d*x + c)^7 + 6*a*b*cosh(d*x + c)^5 + 4*a*b*cosh(d*x + c)^3 + a*b*cos
h(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - 30*(a*b
- b^2)*cosh(d*x + c) + 15*(a^2*cosh(d*x + c)^10 + 10*a^2*cosh(d*x + c)*sinh
(d*x + c)^9 + a^2*sinh(d*x + c)^10 + 5*a^2*cosh(d*x + c)^8 + 5*(9*a^2*cosh(
d*x + c)^2 + a^2)*sinh(d*x + c)^8 + 10*a^2*cosh(d*x + c)^6 + 40*(3*a^2*cosh
(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c)^7 + 10*(21*a^2*cosh(d*x + c)
^4 + 14*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^6 + 10*a^2*cosh(d*x + c)^4
 + 4*(63*a^2*cosh(d*x + c)^5 + 70*a^2*cosh(d*x + c)^3 + 15*a^2*cosh(d*x + c
))*sinh(d*x + c)^5 + 10*(21*a^2*cosh(d*x + c)^6 + 35*a^2*cosh(d*x + c)^4 +
15*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^4 + 5*a^2*cosh(d*x + c)^2 + 40*
(3*a^2*cosh(d*x + c)^7 + 7*a^2*cosh(d*x + c)^5 + 5*a^2*cosh(d*x + c)^3 + a^
```

```
2*cosh(d*x + c))*sinh(d*x + c)^3 + 5*(9*a^2*cosh(d*x + c)^8 + 28*a^2*cosh(d
*x + c)^6 + 30*a^2*cosh(d*x + c)^4 + 12*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x
 + c)^2 + a^2 + 10*(a^2*cosh(d*x + c)^9 + 4*a^2*cosh(d*x + c)^7 + 6*a^2*cos
h(d*x + c)^5 + 4*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c))*lo
g(cosh(d*x + c) + sinh(d*x + c) + 1) - 15*(a^2*cosh(d*x + c)^10 + 10*a^2*co
sh(d*x + c)*sinh(d*x + c)^9 + a^2*sinh(d*x + c)^10 + 5*a^2*cosh(d*x + c)^8
+ 5*(9*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^8 + 10*a^2*cosh(d*x + c)^6
+ 40*(3*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c)^7 + 10*(21*a
^2*cosh(d*x + c)^4 + 14*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^6 + 10*a^2
*cosh(d*x + c)^4 + 4*(63*a^2*cosh(d*x + c)^5 + 70*a^2*cosh(d*x + c)^3 + 15*
a^2*cosh(d*x + c))*sinh(d*x + c)^5 + 10*(21*a^2*cosh(d*x + c)^6 + 35*a^2*co
sh(d*x + c)^4 + 15*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^4 + 5*a^2*cosh(
d*x + c)^2 + 40*(3*a^2*cosh(d*x + c)^7 + 7*a^2*cosh(d*x + c)^5 + 5*a^2*cosh
(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c)^3 + 5*(9*a^2*cosh(d*x + c)^8
 + 28*a^2*cosh(d*x + c)^6 + 30*a^2*cosh(d*x + c)^4 + 12*a^2*cosh(d*x + c)^2
 + a^2)*sinh(d*x + c)^2 + a^2 + 10*(a^2*cosh(d*x + c)^9 + 4*a^2*cosh(d*x +
c)^7 + 6*a^2*cosh(d*x + c)^5 + 4*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*s
inh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 10*(27*(a*b + b^2)*c
osh(d*x + c)^8 + 14*(3*a*b + 2*b^2)*cosh(d*x + c)^6 + 58*b^2*cosh(d*x + c)^
4 - 6*(3*a*b - 2*b^2)*cosh(d*x + c)^2 - 3*a*b + 3*b^2)*sinh(d*x + c))/(d*co
sh(d*x + c)^10 + 10*d*cosh(d*x + c)*sinh(d*x + c)^9 + d*sinh(d*x + c)^10 +
5*d*cosh(d*x + c)^8 + 5*(9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 40*(3*d
*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^7 + 10*d*cosh(d*x + c)^6
+ 10*(21*d*cosh(d*x + c)^4 + 14*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 4*
(63*d*cosh(d*x + c)^5 + 70*d*cosh(d*x + c)^3 + 15*d*cosh(d*x + c))*sinh(d*x
 + c)^5 + 10*d*cosh(d*x + c)^4 + 10*(21*d*cosh(d*x + c)^6 + 35*d*cosh(d*x +
 c)^4 + 15*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 40*(3*d*cosh(d*x + c)^7
 + 7*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x +
c)^3 + 5*d*cosh(d*x + c)^2 + 5*(9*d*cosh(d*x + c)^8 + 28*d*cosh(d*x + c)^6
+ 30*d*cosh(d*x + c)^4 + 12*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 10*(d*
cosh(d*x + c)^9 + 4*d*cosh(d*x + c)^7 + 6*d*cosh(d*x + c)^5 + 4*d*cosh(d*x
+ c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tanh^3(c + dx)\right)^2 \operatorname{csch}(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)**3)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)**2*csch(c + d*x), x)
```

Giac [A] time = 1.51089, size = 240, normalized size = 2.45

$$\frac{30\,ab\arctan\left(e^{(dx+c)}\right) - 15\,a^2\log\left(e^{(dx+c)}+1\right) + 15\,a^2\log\left(\left|e^{(dx+c)}-1\right|\right) - \frac{2\left(15\,abe^{(9\,dx+9\,c)}+15\,b^2e^{(9\,dx+9\,c)}+30\,abe^{(7\,dx+7\,c)}+20\,b^2e^{(7\,dx+7\,c}\right.}{(}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/15*(30*a*b*arctan(e^(d*x + c)) - 15*a^2*log(e^(d*x + c) + 1) + 15*a^2*log
(abs(e^(d*x + c) - 1)) - 2*(15*a*b*e^(9*d*x + 9*c) + 15*b^2*e^(9*d*x + 9*c)
```

```
 + 30*a*b*e^(7*d*x + 7*c) + 20*b^2*e^(7*d*x + 7*c) + 58*b^2*e^(5*d*x + 5*c)
 - 30*a*b*e^(3*d*x + 3*c) + 20*b^2*e^(3*d*x + 3*c) - 15*a*b*e^(d*x + c) + 1
5*b^2*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^5)/d
```

3.62 $\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=47

$$-\frac{a^2\coth(c+dx)}{d}+\frac{ab\tanh^2(c+dx)}{d}+\frac{b^2\tanh^5(c+dx)}{5d}$$

```
[Out] -((a^2*Coth[c + d*x])/d) + (a*b*Tanh[c + d*x]^2)/d + (b^2*Tanh[c + d*x]^5)/
(5*d)
```

Rubi [A] time = 0.0581107, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3663, 270}

$$-\frac{a^2\coth(c+dx)}{d}+\frac{ab\tanh^2(c+dx)}{d}+\frac{b^2\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] -((a^2*Coth[c + d*x])/d) + (a*b*Tanh[c + d*x]^2)/d + (b^2*Tanh[c + d*x]^5)/
(5*d)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\left(a+bx^3\right)^2}{x^2}\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \left(\frac{a^2}{x^2}+2abx+b^2x^4\right)dx,x,\tanh(c+dx)\right)}{d}\\ &= -\frac{a^2\coth(c+dx)}{d}+\frac{ab\tanh^2(c+dx)}{d}+\frac{b^2\tanh^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.220764, size = 94, normalized size = 2.

$$-\frac{a^2\coth(c+dx)}{d}-\frac{ab\operatorname{sech}^2(c+dx)}{d}+\frac{b^2\tanh(c+dx)}{5d}+\frac{b^2\tanh(c+dx)\operatorname{sech}^4(c+dx)}{5d}-\frac{2b^2\tanh(c+dx)\operatorname{sech}^2(c+d}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] -((a^2*Coth[c + d*x])/d) - (a*b*Sech[c + d*x]^2)/d + (b^2*Tanh[c + d*x])/(5
*d) - (2*b^2*Sech[c + d*x]^2*Tanh[c + d*x])/(5*d) + (b^2*Sech[c + d*x]^4*Ta
nh[c + d*x])/(5*d)
```

Maple [B] time = 0.067, size = 105, normalized size = 2.2

$$\frac{1}{d}\left(-a^2\coth(dx+c)+\frac{ab(\sinh(dx+c))^2}{(\cosh(dx+c))^2}+b^2\left(-\frac{(\sinh(dx+c))^3}{2(\cosh(dx+c))^5}-\frac{3\sinh(dx+c)}{8(\cosh(dx+c))^5}+\frac{3\tanh(dx+c)}{8}\left(\frac{8}{15}+\cdots\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x)
```

```
[Out] 1/d*(-a^2*coth(d*x+c)+a*b*sinh(d*x+c)^2/cosh(d*x+c)^2+b^2*(-1/2*sinh(d*x+c)
^3/cosh(d*x+c)^5-3/8*sinh(d*x+c)/cosh(d*x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+
4/15*sech(d*x+c)^2)*tanh(d*x+c)))
```

Maxima [B] time = 1.05468, size = 346, normalized size = 7.36

$$\frac{2}{5}b^2\left(\frac{10\,e^{(-4\,dx-4\,c)}}{d\left(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1\right)}+\frac{\cdots}{d\left(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 2/5*b^2*(10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) +
 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 5*e^
(-8*d*x - 8*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
- 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x
- 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e
^(-10*d*x - 10*c) + 1))) + 2*a^2/(d*(e^(-2*d*x - 2*c) - 1)) - 4*a*b/(d*(e^(
d*x + c) + e^(-d*x - c))^2)
```

Fricas [B] time = 2.17751, size = 1334, normalized size = 28.38

$$-\frac{4\left(\left(5\,a^2+5\,ab+2\,b^2\right)\cosh(dx+c)^5+5\left(5\,a^2+5\,ab+2\,b^2\right)\cosh(dx+c)\sinh(dx+c)^4+\left(5\,ab+3\,b^2\right)\cdots\right.}{5\left(d\cosh(dx+c)^7+7\,d\cosh(dx+c)\sinh(dx+c)^6+d\sinh(dx+c)^7+3\,d\cosh(dx+c)^5+\left(21\,d\cosh(dx+c)^2+\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] -4/5*((5*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^5 + 5*(5*a^2 + 5*a*b + 2*b^2)*c
osh(d*x + c)*sinh(d*x + c)^4 + (5*a*b + 3*b^2)*sinh(d*x + c)^5 + (25*a^2 +
5*a*b - 2*b^2)*cosh(d*x + c)^3 + (10*(5*a*b + 3*b^2)*cosh(d*x + c)^2 + 15*a
```

```
*b - 3*b^2)*sinh(d*x + c)^3 + (10*(5*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^3 +
 3*(25*a^2 + 5*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(5*a^2 - a*
b)*cosh(d*x + c) + (5*(5*a*b + 3*b^2)*cosh(d*x + c)^4 + 9*(5*a*b - b^2)*cos
h(d*x + c)^2 + 10*a*b + 10*b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*cos
h(d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x + c)^7 + 3*d*cosh(d*x + c)^5 + (21*
d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^5 + 5*(7*d*cosh(d*x + c)^3 + 3*d*cos
h(d*x + c))*sinh(d*x + c)^4 + d*cosh(d*x + c)^3 + (35*d*cosh(d*x + c)^4 + 5
0*d*cosh(d*x + c)^2 + 9*d)*sinh(d*x + c)^3 + 3*(7*d*cosh(d*x + c)^5 + 10*d*
cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^2 - 5*d*cosh(d*x + c) + (7
*d*cosh(d*x + c)^6 + 25*d*cosh(d*x + c)^4 + 27*d*cosh(d*x + c)^2 + 5*d)*sin
h(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tanh^3(c + dx)\right)^2 \operatorname{csch}^2(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*tanh(d*x+c)**3)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)**2*csch(c + d*x)**2, x)
```

Giac [B] time = 1.61257, size = 165, normalized size = 3.51

$$-\frac{2\left(\frac{5\,a^2}{e^{(2\,dx+2\,c)}-1} + \frac{10\,abe^{(8\,dx+8\,c)}+5\,b^2e^{(8\,dx+8\,c)}+30\,abe^{(6\,dx+6\,c)}+30\,abe^{(4\,dx+4\,c)}+10\,b^2e^{(4\,dx+4\,c)}+10\,abe^{(2\,dx+2\,c)}+b^2}{\left(e^{(2\,dx+2\,c)}+1\right)^5}\right)}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -2/5*(5*a^2/(e^(2*d*x + 2*c) - 1) + (10*a*b*e^(8*d*x + 8*c) + 5*b^2*e^(8*d*
x + 8*c) + 30*a*b*e^(6*d*x + 6*c) + 30*a*b*e^(4*d*x + 4*c) + 10*b^2*e^(4*d*
x + 4*c) + 10*a*b*e^(2*d*x + 2*c) + b^2)/(e^(2*d*x + 2*c) + 1)^5)/d
```

3.63 $\int \mathbf{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=107

$$\frac{a^2\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^2\coth(c+dx)\mathbf{csch}(c+dx)}{2d}+\frac{ab\tan^{-1}(\sinh(c+dx))}{d}+\frac{ab\tanh(c+dx)\mathbf{sech}(c+dx)}{d}+$$

```
[Out] (a*b*ArcTan[Sinh[c + d*x]])/d + (a^2*ArcTanh[Cosh[c + d*x]])/(2*d) - (a^2*C
oth[c + d*x]*Csch[c + d*x])/(2*d) - (b^2*Sech[c + d*x]^3)/(3*d) + (b^2*Sech
[c + d*x]^5)/(5*d) + (a*b*Sech[c + d*x]*Tanh[c + d*x])/d
```

Rubi [A] time = 0.159902, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3666, 3768, 3770, 2606, 14}

$$\frac{a^2\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^2\coth(c+dx)\mathbf{csch}(c+dx)}{2d}+\frac{ab\tan^{-1}(\sinh(c+dx))}{d}+\frac{ab\tanh(c+dx)\mathbf{sech}(c+dx)}{d}+$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] (a*b*ArcTan[Sinh[c + d*x]])/d + (a^2*ArcTanh[Cosh[c + d*x]])/(2*d) - (a^2*C
oth[c + d*x]*Csch[c + d*x])/(2*d) - (b^2*Sech[c + d*x]^3)/(3*d) + (b^2*Sech
[c + d*x]^5)/(5*d) + (a*b*Sech[c + d*x]*Tanh[c + d*x])/d
```

Rule 3666

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \text{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2 dx = -\left(i\int \left(ia^2\text{csch}^3(c+dx)+2iab\,\text{sech}^3(c+dx)+ib^2\text{sech}^3(c+dx)\tanh^3(c\right.\right.$$
$$= a^2\int \text{csch}^3(c+dx)\,dx+(2ab)\int \text{sech}^3(c+dx)\,dx+b^2\int \text{sech}^3(c+dx)\,\text{t}$$
$$= -\frac{a^2\coth(c+dx)\text{csch}(c+dx)}{2d}+\frac{ab\,\text{sech}(c+dx)\tanh(c+dx)}{d}-\frac{1}{2}a^2\int \text{cs}$$
$$= \frac{ab\tan^{-1}(\sinh(c+dx))}{d}+\frac{a^2\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^2\coth(c+dx)\text{csch}}{2d}$$
$$= \frac{ab\tan^{-1}(\sinh(c+dx))}{d}+\frac{a^2\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^2\coth(c+dx)\text{csch}}{2d}$$

Mathematica [A] time = 0.150823, size = 138, normalized size = 1.29

$$-\frac{a^2\text{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a^2\text{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a^2\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{2ab\tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{ab\tanh(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] (2*a*b*ArcTan[Tanh[(c + d*x)/2]])/d - (a^2*Csch[(c + d*x)/2]^2)/(8*d) - (a^
2*Log[Tanh[(c + d*x)/2]])/(2*d) - (a^2*Sech[(c + d*x)/2]^2)/(8*d) - (b^2*Se
ch[c + d*x]^3)/(3*d) + (b^2*Sech[c + d*x]^5)/(5*d) + (a*b*Sech[c + d*x]*Tan
h[c + d*x])/d
```

Maple [A] time = 0.085, size = 154, normalized size = 1.4

$$-\frac{a^2\coth(dx+c)\,\text{csch}(dx+c)}{2d}+\frac{a^2\text{Artanh}\left(e^{dx+c}\right)}{d}+\frac{ab\,\text{sech}(dx+c)\tanh(dx+c)}{d}+2\frac{ab\arctan\left(e^{dx+c}\right)}{d}-\frac{b^2(\sinh(}{5d(\cosh(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x)
```

```
[Out] -1/2*a^2*coth(d*x+c)*csch(d*x+c)/d+1/d*a^2*arctanh(exp(d*x+c))+1/d*a*b*sech
(d*x+c)*tanh(d*x+c)+2/d*a*b*arctan(exp(d*x+c))-1/5/d*b^2*sinh(d*x+c)^2/cosh
(d*x+c)^5+2/15/d*b^2*sinh(d*x+c)^2/cosh(d*x+c)^3+2/15/d*b^2*sinh(d*x+c)^2/c
osh(d*x+c)-2/15*b^2*cosh(d*x+c)/d
```

Maxima [B] time = 1.5607, size = 510, normalized size = 4.77

$$-2ab\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-e^{(-3dx-3c)}}{d\left(2e^{(-2dx-2c)}+e^{(-4dx-4c)}+1\right)}\right)+\frac{1}{2}a^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}\right.}{d\left(2e^{(-2dx-2c)}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] -2*a*b*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^
(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + 1/2*a^2*(log(e^(-d*x - c) + 1)/d
 - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-
2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - 8/15*b^2*(5*e^(-3*d*x - 3*c)/(d*(5
*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x
 - 8*c) + e^(-10*d*x - 10*c) + 1)) - 2*e^(-5*d*x - 5*c)/(d*(5*e^(-2*d*x - 2
*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-
10*d*x - 10*c) + 1)) + 5*e^(-7*d*x - 7*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4
*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c)
 + 1)))
```

Fricas [B] time = 2.99178, size = 12523, normalized size = 117.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] -1/30*(30*(a^2 - 2*a*b)*cosh(d*x + c)^13 + 390*(a^2 - 2*a*b)*cosh(d*x + c)*
sinh(d*x + c)^12 + 30*(a^2 - 2*a*b)*sinh(d*x + c)^13 + 20*(9*a^2 + 4*b^2)*c
osh(d*x + c)^11 + 20*(117*(a^2 - 2*a*b)*cosh(d*x + c)^2 + 9*a^2 + 4*b^2)*si
nh(d*x + c)^11 + 220*(39*(a^2 - 2*a*b)*cosh(d*x + c)^3 + (9*a^2 + 4*b^2)*co
sh(d*x + c))*sinh(d*x + c)^10 + 6*(75*a^2 + 30*a*b - 32*b^2)*cosh(d*x + c)^
9 + 2*(10725*(a^2 - 2*a*b)*cosh(d*x + c)^4 + 550*(9*a^2 + 4*b^2)*cosh(d*x +
 c)^2 + 225*a^2 + 90*a*b - 96*b^2)*sinh(d*x + c)^9 + 6*(6435*(a^2 - 2*a*b)*
cosh(d*x + c)^5 + 550*(9*a^2 + 4*b^2)*cosh(d*x + c)^3 + 9*(75*a^2 + 30*a*b
- 32*b^2)*cosh(d*x + c))*sinh(d*x + c)^8 + 8*(75*a^2 + 28*b^2)*cosh(d*x + c
)^7 + 8*(6435*(a^2 - 2*a*b)*cosh(d*x + c)^6 + 825*(9*a^2 + 4*b^2)*cosh(d*x
+ c)^4 + 27*(75*a^2 + 30*a*b - 32*b^2)*cosh(d*x + c)^2 + 75*a^2 + 28*b^2)*s
inh(d*x + c)^7 + 8*(6435*(a^2 - 2*a*b)*cosh(d*x + c)^7 + 1155*(9*a^2 + 4*b^
2)*cosh(d*x + c)^5 + 63*(75*a^2 + 30*a*b - 32*b^2)*cosh(d*x + c)^3 + 7*(75*
a^2 + 28*b^2)*cosh(d*x + c))*sinh(d*x + c)^6 + 6*(75*a^2 - 30*a*b - 32*b^2)
*cosh(d*x + c)^5 + 6*(6435*(a^2 - 2*a*b)*cosh(d*x + c)^8 + 1540*(9*a^2 + 4*
b^2)*cosh(d*x + c)^6 + 126*(75*a^2 + 30*a*b - 32*b^2)*cosh(d*x + c)^4 + 28*
(75*a^2 + 28*b^2)*cosh(d*x + c)^2 + 75*a^2 - 30*a*b - 32*b^2)*sinh(d*x + c)
^5 + 2*(10725*(a^2 - 2*a*b)*cosh(d*x + c)^9 + 3300*(9*a^2 + 4*b^2)*cosh(d*x
 + c)^7 + 378*(75*a^2 + 30*a*b - 32*b^2)*cosh(d*x + c)^5 + 140*(75*a^2 + 28
*b^2)*cosh(d*x + c)^3 + 15*(75*a^2 - 30*a*b - 32*b^2)*cosh(d*x + c))*sinh(d
*x + c)^4 + 20*(9*a^2 + 4*b^2)*cosh(d*x + c)^3 + 4*(2145*(a^2 - 2*a*b)*cosh
(d*x + c)^10 + 825*(9*a^2 + 4*b^2)*cosh(d*x + c)^8 + 126*(75*a^2 + 30*a*b -
 32*b^2)*cosh(d*x + c)^6 + 70*(75*a^2 + 28*b^2)*cosh(d*x + c)^4 + 15*(75*a^
2 - 30*a*b - 32*b^2)*cosh(d*x + c)^2 + 45*a^2 + 20*b^2)*sinh(d*x + c)^3 + 4
*(585*(a^2 - 2*a*b)*cosh(d*x + c)^11 + 275*(9*a^2 + 4*b^2)*cosh(d*x + c)^9
+ 54*(75*a^2 + 30*a*b - 32*b^2)*cosh(d*x + c)^7 + 42*(75*a^2 + 28*b^2)*cosh
(d*x + c)^5 + 15*(75*a^2 - 30*a*b - 32*b^2)*cosh(d*x + c)^3 + 15*(9*a^2 + 4
*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 60*(a*b*cosh(d*x + c)^14 + 14*a*b*co
sh(d*x + c)*sinh(d*x + c)^13 + a*b*sinh(d*x + c)^14 + 3*a*b*cosh(d*x + c)^1
2 + (91*a*b*cosh(d*x + c)^2 + 3*a*b)*sinh(d*x + c)^12 + a*b*cosh(d*x + c)^1
0 + 4*(91*a*b*cosh(d*x + c)^3 + 9*a*b*cosh(d*x + c))*sinh(d*x + c)^11 + (10
01*a*b*cosh(d*x + c)^4 + 198*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^10 -
5*a*b*cosh(d*x + c)^8 + 2*(1001*a*b*cosh(d*x + c)^5 + 330*a*b*cosh(d*x + c)
^3 + 5*a*b*cosh(d*x + c))*sinh(d*x + c)^9 + (3003*a*b*cosh(d*x + c)^6 + 148
5*a*b*cosh(d*x + c)^4 + 45*a*b*cosh(d*x + c)^2 - 5*a*b)*sinh(d*x + c)^8 - 5
*a*b*cosh(d*x + c)^6 + 8*(429*a*b*cosh(d*x + c)^7 + 297*a*b*cosh(d*x + c)^5
```

+ 15*a*b*cosh(d*x + c)^3 - 5*a*b*cosh(d*x + c))*sinh(d*x + c)^7 + (3003*a*
b*cosh(d*x + c)^8 + 2772*a*b*cosh(d*x + c)^6 + 210*a*b*cosh(d*x + c)^4 - 14
0*a*b*cosh(d*x + c)^2 - 5*a*b)*sinh(d*x + c)^6 + a*b*cosh(d*x + c)^4 + 2*(1
001*a*b*cosh(d*x + c)^9 + 1188*a*b*cosh(d*x + c)^7 + 126*a*b*cosh(d*x + c)^
5 - 140*a*b*cosh(d*x + c)^3 - 15*a*b*cosh(d*x + c))*sinh(d*x + c)^5 + (1001
*a*b*cosh(d*x + c)^10 + 1485*a*b*cosh(d*x + c)^8 + 210*a*b*cosh(d*x + c)^6
- 350*a*b*cosh(d*x + c)^4 - 75*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^4 +
3*a*b*cosh(d*x + c)^2 + 4*(91*a*b*cosh(d*x + c)^11 + 165*a*b*cosh(d*x + c)
^9 + 30*a*b*cosh(d*x + c)^7 - 70*a*b*cosh(d*x + c)^5 - 25*a*b*cosh(d*x + c)
^3 + a*b*cosh(d*x + c))*sinh(d*x + c)^3 + (91*a*b*cosh(d*x + c)^12 + 198*a*
b*cosh(d*x + c)^10 + 45*a*b*cosh(d*x + c)^8 - 140*a*b*cosh(d*x + c)^6 - 75*
a*b*cosh(d*x + c)^4 + 6*a*b*cosh(d*x + c)^2 + 3*a*b)*sinh(d*x + c)^2 + a*b
+ 2*(7*a*b*cosh(d*x + c)^13 + 18*a*b*cosh(d*x + c)^11 + 5*a*b*cosh(d*x + c)
^9 - 20*a*b*cosh(d*x + c)^7 - 15*a*b*cosh(d*x + c)^5 + 2*a*b*cosh(d*x + c)^
3 + 3*a*b*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c
)) + 30*(a^2 + 2*a*b)*cosh(d*x + c) - 15*(a^2*cosh(d*x + c)^14 + 14*a^2*cos
h(d*x + c)*sinh(d*x + c)^13 + a^2*sinh(d*x + c)^14 + 3*a^2*cosh(d*x + c)^12
+ (91*a^2*cosh(d*x + c)^2 + 3*a^2)*sinh(d*x + c)^12 + a^2*cosh(d*x + c)^10
+ 4*(91*a^2*cosh(d*x + c)^3 + 9*a^2*cosh(d*x + c))*sinh(d*x + c)^11 + (100
1*a^2*cosh(d*x + c)^4 + 198*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^10 - 5
*a^2*cosh(d*x + c)^8 + 2*(1001*a^2*cosh(d*x + c)^5 + 330*a^2*cosh(d*x + c)^
3 + 5*a^2*cosh(d*x + c))*sinh(d*x + c)^9 + (3003*a^2*cosh(d*x + c)^6 + 1485
*a^2*cosh(d*x + c)^4 + 45*a^2*cosh(d*x + c)^2 - 5*a^2)*sinh(d*x + c)^8 - 5*
a^2*cosh(d*x + c)^6 + 8*(429*a^2*cosh(d*x + c)^7 + 297*a^2*cosh(d*x + c)^5
+ 15*a^2*cosh(d*x + c)^3 - 5*a^2*cosh(d*x + c))*sinh(d*x + c)^7 + (3003*a^2
*cosh(d*x + c)^8 + 2772*a^2*cosh(d*x + c)^6 + 210*a^2*cosh(d*x + c)^4 - 140
*a^2*cosh(d*x + c)^2 - 5*a^2)*sinh(d*x + c)^6 + a^2*cosh(d*x + c)^4 + 2*(10
01*a^2*cosh(d*x + c)^9 + 1188*a^2*cosh(d*x + c)^7 + 126*a^2*cosh(d*x + c)^5
- 140*a^2*cosh(d*x + c)^3 - 15*a^2*cosh(d*x + c))*sinh(d*x + c)^5 + (1001*
a^2*cosh(d*x + c)^10 + 1485*a^2*cosh(d*x + c)^8 + 210*a^2*cosh(d*x + c)^6 -
350*a^2*cosh(d*x + c)^4 - 75*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^4 +
3*a^2*cosh(d*x + c)^2 + 4*(91*a^2*cosh(d*x + c)^11 + 165*a^2*cosh(d*x + c)^
9 + 30*a^2*cosh(d*x + c)^7 - 70*a^2*cosh(d*x + c)^5 - 25*a^2*cosh(d*x + c)^
3 + a^2*cosh(d*x + c))*sinh(d*x + c)^3 + (91*a^2*cosh(d*x + c)^12 + 198*a^2
*cosh(d*x + c)^10 + 45*a^2*cosh(d*x + c)^8 - 140*a^2*cosh(d*x + c)^6 - 75*a
^2*cosh(d*x + c)^4 + 6*a^2*cosh(d*x + c)^2 + 3*a^2)*sinh(d*x + c)^2 + a^2 +
2*(7*a^2*cosh(d*x + c)^13 + 18*a^2*cosh(d*x + c)^11 + 5*a^2*cosh(d*x + c)^
9 - 20*a^2*cosh(d*x + c)^7 - 15*a^2*cosh(d*x + c)^5 + 2*a^2*cosh(d*x + c)^3
+ 3*a^2*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) +
1) + 15*(a^2*cosh(d*x + c)^14 + 14*a^2*cosh(d*x + c)*sinh(d*x + c)^13 + a^2
*sinh(d*x + c)^14 + 3*a^2*cosh(d*x + c)^12 + (91*a^2*cosh(d*x + c)^2 + 3*a^
2)*sinh(d*x + c)^12 + a^2*cosh(d*x + c)^10 + 4*(91*a^2*cosh(d*x + c)^3 + 9*
a^2*cosh(d*x + c))*sinh(d*x + c)^11 + (1001*a^2*cosh(d*x + c)^4 + 198*a^2*c
osh(d*x + c)^2 + a^2)*sinh(d*x + c)^10 - 5*a^2*cosh(d*x + c)^8 + 2*(1001*a^
2*cosh(d*x + c)^5 + 330*a^2*cosh(d*x + c)^3 + 5*a^2*cosh(d*x + c))*sinh(d*x
+ c)^9 + (3003*a^2*cosh(d*x + c)^6 + 1485*a^2*cosh(d*x + c)^4 + 45*a^2*cos
h(d*x + c)^2 - 5*a^2)*sinh(d*x + c)^8 - 5*a^2*cosh(d*x + c)^6 + 8*(429*a^2*
cosh(d*x + c)^7 + 297*a^2*cosh(d*x + c)^5 + 15*a^2*cosh(d*x + c)^3 - 5*a^2*
cosh(d*x + c))*sinh(d*x + c)^7 + (3003*a^2*cosh(d*x + c)^8 + 2772*a^2*cosh(
d*x + c)^6 + 210*a^2*cosh(d*x + c)^4 - 140*a^2*cosh(d*x + c)^2 - 5*a^2)*sin
h(d*x + c)^6 + a^2*cosh(d*x + c)^4 + 2*(1001*a^2*cosh(d*x + c)^9 + 1188*a^2
*cosh(d*x + c)^7 + 126*a^2*cosh(d*x + c)^5 - 140*a^2*cosh(d*x + c)^3 - 15*a
^2*cosh(d*x + c))*sinh(d*x + c)^5 + (1001*a^2*cosh(d*x + c)^10 + 1485*a^2*c
osh(d*x + c)^8 + 210*a^2*cosh(d*x + c)^6 - 350*a^2*cosh(d*x + c)^4 - 75*a^2
*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^4 + 3*a^2*cosh(d*x + c)^2 + 4*(91*a^2
*cosh(d*x + c)^11 + 165*a^2*cosh(d*x + c)^9 + 30*a^2*cosh(d*x + c)^7 - 70*a
^2*cosh(d*x + c)^5 - 25*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x +
c)^3 + (91*a^2*cosh(d*x + c)^12 + 198*a^2*cosh(d*x + c)^10 + 45*a^2*cosh(d
*x + c)^8 - 140*a^2*cosh(d*x + c)^6 - 75*a^2*cosh(d*x + c)^4 + 6*a^2*cosh(d

```
*x + c)^2 + 3*a^2)*sinh(d*x + c)^2 + a^2 + 2*(7*a^2*cosh(d*x + c)^13 + 18*a
^2*cosh(d*x + c)^11 + 5*a^2*cosh(d*x + c)^9 - 20*a^2*cosh(d*x + c)^7 - 15*a
^2*cosh(d*x + c)^5 + 2*a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x
+ c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(195*(a^2 - 2*a*b)*cosh(d*
x + c)^12 + 110*(9*a^2 + 4*b^2)*cosh(d*x + c)^10 + 27*(75*a^2 + 30*a*b - 32
*b^2)*cosh(d*x + c)^8 + 28*(75*a^2 + 28*b^2)*cosh(d*x + c)^6 + 15*(75*a^2 -
 30*a*b - 32*b^2)*cosh(d*x + c)^4 + 30*(9*a^2 + 4*b^2)*cosh(d*x + c)^2 + 15
*a^2 + 30*a*b)*sinh(d*x + c))/(d*cosh(d*x + c)^14 + 14*d*cosh(d*x + c)*sinh
(d*x + c)^13 + d*sinh(d*x + c)^14 + 3*d*cosh(d*x + c)^12 + (91*d*cosh(d*x +
 c)^2 + 3*d)*sinh(d*x + c)^12 + 4*(91*d*cosh(d*x + c)^3 + 9*d*cosh(d*x + c)
)*sinh(d*x + c)^11 + d*cosh(d*x + c)^10 + (1001*d*cosh(d*x + c)^4 + 198*d*c
osh(d*x + c)^2 + d)*sinh(d*x + c)^10 + 2*(1001*d*cosh(d*x + c)^5 + 330*d*co
sh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^9 - 5*d*cosh(d*x + c)^8 +
(3003*d*cosh(d*x + c)^6 + 1485*d*cosh(d*x + c)^4 + 45*d*cosh(d*x + c)^2 - 5
*d)*sinh(d*x + c)^8 + 8*(429*d*cosh(d*x + c)^7 + 297*d*cosh(d*x + c)^5 + 15
*d*cosh(d*x + c)^3 - 5*d*cosh(d*x + c))*sinh(d*x + c)^7 - 5*d*cosh(d*x + c)
^6 + (3003*d*cosh(d*x + c)^8 + 2772*d*cosh(d*x + c)^6 + 210*d*cosh(d*x + c)
^4 - 140*d*cosh(d*x + c)^2 - 5*d)*sinh(d*x + c)^6 + 2*(1001*d*cosh(d*x + c)
^9 + 1188*d*cosh(d*x + c)^7 + 126*d*cosh(d*x + c)^5 - 140*d*cosh(d*x + c)^3
 - 15*d*cosh(d*x + c))*sinh(d*x + c)^5 + d*cosh(d*x + c)^4 + (1001*d*cosh(d
*x + c)^10 + 1485*d*cosh(d*x + c)^8 + 210*d*cosh(d*x + c)^6 - 350*d*cosh(d*
x + c)^4 - 75*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 4*(91*d*cosh(d*x + c
)^11 + 165*d*cosh(d*x + c)^9 + 30*d*cosh(d*x + c)^7 - 70*d*cosh(d*x + c)^5
- 25*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*d*cosh(d*x +
c)^2 + (91*d*cosh(d*x + c)^12 + 198*d*cosh(d*x + c)^10 + 45*d*cosh(d*x + c)
^8 - 140*d*cosh(d*x + c)^6 - 75*d*cosh(d*x + c)^4 + 6*d*cosh(d*x + c)^2 + 3
*d)*sinh(d*x + c)^2 + 2*(7*d*cosh(d*x + c)^13 + 18*d*cosh(d*x + c)^11 + 5*d
*cosh(d*x + c)^9 - 20*d*cosh(d*x + c)^7 - 15*d*cosh(d*x + c)^5 + 2*d*cosh(d
*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tanh^3(c + dx)\right)^2 \operatorname{csch}^3(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*tanh(d*x+c)**3)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)**2*csch(c + d*x)**3, x)
```

Giac [A] time = 1.5929, size = 259, normalized size = 2.42

$$\frac{60\,ab\arctan\left(e^{(dx+c)}\right)+15\,a^2\log\left(e^{(dx+c)}+1\right)-15\,a^2\log\left(\left|e^{(dx+c)}-1\right|\right)-\frac{30\left(a^2e^{(3\,dx+3\,c)}+a^2e^{(dx+c)}\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^2}+\frac{4\left(15\,abe^{(9\,dx+9\,c)}+30\,abe^{(7\,dx}}{}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/30*(60*a*b*arctan(e^(d*x + c)) + 15*a^2*log(e^(d*x + c) + 1) - 15*a^2*log
(abs(e^(d*x + c) - 1)) - 30*(a^2*e^(3*d*x + 3*c) + a^2*e^(d*x + c))/(e^(2*d
*x + 2*c) - 1)^2 + 4*(15*a*b*e^(9*d*x + 9*c) + 30*a*b*e^(7*d*x + 7*c) - 20*
b^2*e^(7*d*x + 7*c) + 8*b^2*e^(5*d*x + 5*c) - 30*a*b*e^(3*d*x + 3*c) - 20*b
```

```
^2*e^(3*d*x + 3*c) - 15*a*b*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^5)/d
```

3.64 $\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=97

$$-\frac{a^2\coth^3(c+dx)}{3d}+\frac{a^2\coth(c+dx)}{d}-\frac{ab\tanh^2(c+dx)}{d}+\frac{2ab\log(\tanh(c+dx))}{d}-\frac{b^2\tanh^5(c+dx)}{5d}+\frac{b^2\tanh^3(c+}{3d}$$

```
[Out] (a^2*Coth[c + d*x])/d - (a^2*Coth[c + d*x]^3)/(3*d) + (2*a*b*Log[Tanh[c + d
*x]])/d - (a*b*Tanh[c + d*x]^2)/d + (b^2*Tanh[c + d*x]^3)/(3*d) - (b^2*Tanh
[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0980367, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3663, 1802}

$$-\frac{a^2\coth^3(c+dx)}{3d}+\frac{a^2\coth(c+dx)}{d}-\frac{ab\tanh^2(c+dx)}{d}+\frac{2ab\log(\tanh(c+dx))}{d}-\frac{b^2\tanh^5(c+dx)}{5d}+\frac{b^2\tanh^3(c+}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] (a^2*Coth[c + d*x])/d - (a^2*Coth[c + d*x]^3)/(3*d) + (2*a*b*Log[Tanh[c + d
*x]])/d - (a*b*Tanh[c + d*x]^2)/d + (b^2*Tanh[c + d*x]^3)/(3*d) - (b^2*Tanh
[c + d*x]^5)/(5*d)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)(a+bx^3)^2}{x^4}\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \left(\frac{a^2}{x^4}-\frac{a^2}{x^2}+\frac{2ab}{x}-2abx+b^2x^2-b^2x^4\right)dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{a^2\coth(c+dx)}{d}-\frac{a^2\coth^3(c+dx)}{3d}+\frac{2ab\log(\tanh(c+dx))}{d}-\frac{ab\tanh}{}\end{aligned}$$

Mathematica [A] time = 0.185879, size = 147, normalized size = 1.52

$$\frac{2a^2\coth(c+dx)}{3d}-\frac{a^2\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}+\frac{ab\operatorname{sech}^2(c+dx)}{d}+\frac{2ab\log(\sinh(c+dx))}{d}-\frac{2ab\log(\cosh(c+dx}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] (2*a^2*Coth[c + d*x])/(3*d) - (a^2*Coth[c + d*x]*Csch[c + d*x]^2)/(3*d) - (
2*a*b*Log[Cosh[c + d*x]])/d + (2*a*b*Log[Sinh[c + d*x]])/d + (a*b*Sech[c +
d*x]^2)/d + (2*b^2*Tanh[c + d*x])/(15*d) + (b^2*Sech[c + d*x]^2*Tanh[c + d*
x])/(15*d) - (b^2*Sech[c + d*x]^4*Tanh[c + d*x])/(5*d)
```

Maple [A] time = 0.083, size = 146, normalized size = 1.5

$$\frac{2\,a^{2}\coth\left(dx+c\right)}{3\,d}-\frac{a^{2}\coth\left(dx+c\right)\left(\operatorname{csch}\left(dx+c\right)\right)^{2}}{3\,d}+\frac{ab}{d\left(\cosh\left(dx+c\right)\right)^{2}}+2\,\frac{ab\ln\left(\tanh\left(dx+c\right)\right)}{d}-\frac{b^{2}\sinh\left(dx+c\right)}{4\,d\left(\cosh\left(dx+c\right)\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x)
```

```
[Out] 2/3*a^2*coth(d*x+c)/d-1/3/d*a^2*coth(d*x+c)*csch(d*x+c)^2+1/d*a*b/cosh(d*x+
c)^2+2*a*b*ln(tanh(d*x+c))/d-1/4/d*b^2*sinh(d*x+c)/cosh(d*x+c)^5+2/15*b^2*t
anh(d*x+c)/d+1/20/d*b^2*tanh(d*x+c)*sech(d*x+c)^4+1/15/d*b^2*tanh(d*x+c)*se
ch(d*x+c)^2
```

Maxima [B] time = 1.58208, size = 632, normalized size = 6.52

$$2\,ab\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1\right)}\right)+\frac{4}{15}\,b^{2}\left(\frac{}{d\left(5\,e^{(-2\,dx-2}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 2*a*b*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x -
2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c)
+ 1))) + 4/15*b^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x
 - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1
)) - 5*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e
^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*e^(-6*
d*x - 6*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*
c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*
c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-1
0*d*x - 10*c) + 1))) + 4/3*a^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) -
 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3
*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

Fricas [B] time = 2.81461, size = 11420, normalized size = 117.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 2/15*(30*a*b*cosh(d*x + c)^14 + 420*a*b*cosh(d*x + c)*sinh(d*x + c)^13 + 30
*a*b*sinh(d*x + c)^14 - 30*(a^2 + b^2)*cosh(d*x + c)^12 + 30*(91*a*b*cosh(d
*x + c)^2 - a^2 - b^2)*sinh(d*x + c)^12 + 120*(91*a*b*cosh(d*x + c)^3 - 3*(
a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c)^11 - 10*(14*a^2 + 9*a*b - 10*b^2)*c
osh(d*x + c)^10 + 10*(3003*a*b*cosh(d*x + c)^4 - 198*(a^2 + b^2)*cosh(d*x +
 c)^2 - 14*a^2 - 9*a*b + 10*b^2)*sinh(d*x + c)^10 + 20*(3003*a*b*cosh(d*x +
 c)^5 - 330*(a^2 + b^2)*cosh(d*x + c)^3 - 5*(14*a^2 + 9*a*b - 10*b^2)*cosh(
d*x + c))*sinh(d*x + c)^9 - 10*(25*a^2 + 13*b^2)*cosh(d*x + c)^8 + 10*(9009
*a*b*cosh(d*x + c)^6 - 1485*(a^2 + b^2)*cosh(d*x + c)^4 - 45*(14*a^2 + 9*a*
b - 10*b^2)*cosh(d*x + c)^2 - 25*a^2 - 13*b^2)*sinh(d*x + c)^8 + 80*(1287*a
*b*cosh(d*x + c)^7 - 297*(a^2 + b^2)*cosh(d*x + c)^5 - 15*(14*a^2 + 9*a*b -
 10*b^2)*cosh(d*x + c)^3 - (25*a^2 + 13*b^2)*cosh(d*x + c))*sinh(d*x + c)^7
 - 2*(100*a^2 - 45*a*b - 44*b^2)*cosh(d*x + c)^6 + 2*(45045*a*b*cosh(d*x +
c)^8 - 13860*(a^2 + b^2)*cosh(d*x + c)^6 - 1050*(14*a^2 + 9*a*b - 10*b^2)*c
osh(d*x + c)^4 - 140*(25*a^2 + 13*b^2)*cosh(d*x + c)^2 - 100*a^2 + 45*a*b +
 44*b^2)*sinh(d*x + c)^6 + 4*(15015*a*b*cosh(d*x + c)^9 - 5940*(a^2 + b^2)*
cosh(d*x + c)^7 - 630*(14*a^2 + 9*a*b - 10*b^2)*cosh(d*x + c)^5 - 140*(25*a
^2 + 13*b^2)*cosh(d*x + c)^3 - 3*(100*a^2 - 45*a*b - 44*b^2)*cosh(d*x + c))
*sinh(d*x + c)^5 - 2*(25*a^2 + 17*b^2)*cosh(d*x + c)^4 + 2*(15015*a*b*cosh(
d*x + c)^10 - 7425*(a^2 + b^2)*cosh(d*x + c)^8 - 1050*(14*a^2 + 9*a*b - 10*
b^2)*cosh(d*x + c)^6 - 350*(25*a^2 + 13*b^2)*cosh(d*x + c)^4 - 15*(100*a^2
- 45*a*b - 44*b^2)*cosh(d*x + c)^2 - 25*a^2 - 17*b^2)*sinh(d*x + c)^4 + 8*(
1365*a*b*cosh(d*x + c)^11 - 825*(a^2 + b^2)*cosh(d*x + c)^9 - 150*(14*a^2 +
 9*a*b - 10*b^2)*cosh(d*x + c)^7 - 70*(25*a^2 + 13*b^2)*cosh(d*x + c)^5 - 5
*(100*a^2 - 45*a*b - 44*b^2)*cosh(d*x + c)^3 - (25*a^2 + 17*b^2)*cosh(d*x +
 c))*sinh(d*x + c)^3 + 2*(10*a^2 - 15*a*b + 2*b^2)*cosh(d*x + c)^2 + 2*(136
5*a*b*cosh(d*x + c)^12 - 990*(a^2 + b^2)*cosh(d*x + c)^10 - 225*(14*a^2 + 9
*a*b - 10*b^2)*cosh(d*x + c)^8 - 140*(25*a^2 + 13*b^2)*cosh(d*x + c)^6 - 15
*(100*a^2 - 45*a*b - 44*b^2)*cosh(d*x + c)^4 - 6*(25*a^2 + 17*b^2)*cosh(d*x
 + c)^2 + 10*a^2 - 15*a*b + 2*b^2)*sinh(d*x + c)^2 + 10*a^2 + 2*b^2 - 15*(a
*b*cosh(d*x + c)^16 + 16*a*b*cosh(d*x + c)*sinh(d*x + c)^15 + a*b*sinh(d*x
+ c)^16 + 2*a*b*cosh(d*x + c)^14 + 2*(60*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*
x + c)^14 - 2*a*b*cosh(d*x + c)^12 + 28*(20*a*b*cosh(d*x + c)^3 + a*b*cosh(
d*x + c))*sinh(d*x + c)^13 + 2*(910*a*b*cosh(d*x + c)^4 + 91*a*b*cosh(d*x +
 c)^2 - a*b)*sinh(d*x + c)^12 - 6*a*b*cosh(d*x + c)^10 + 8*(546*a*b*cosh(d*
x + c)^5 + 91*a*b*cosh(d*x + c)^3 - 3*a*b*cosh(d*x + c))*sinh(d*x + c)^11 +
 2*(4004*a*b*cosh(d*x + c)^6 + 1001*a*b*cosh(d*x + c)^4 - 66*a*b*cosh(d*x +
 c)^2 - 3*a*b)*sinh(d*x + c)^10 + 4*(2860*a*b*cosh(d*x + c)^7 + 1001*a*b*co
sh(d*x + c)^5 - 110*a*b*cosh(d*x + c)^3 - 15*a*b*cosh(d*x + c))*sinh(d*x +
c)^9 + 6*(2145*a*b*cosh(d*x + c)^8 + 1001*a*b*cosh(d*x + c)^6 - 165*a*b*cos
h(d*x + c)^4 - 45*a*b*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 6*a*b*cosh(d*x + c
)^6 + 16*(715*a*b*cosh(d*x + c)^9 + 429*a*b*cosh(d*x + c)^7 - 99*a*b*cosh(d
*x + c)^5 - 45*a*b*cosh(d*x + c)^3)*sinh(d*x + c)^7 + 2*(4004*a*b*cosh(d*x
+ c)^10 + 3003*a*b*cosh(d*x + c)^8 - 924*a*b*cosh(d*x + c)^6 - 630*a*b*cosh
(d*x + c)^4 + 3*a*b)*sinh(d*x + c)^6 + 2*a*b*cosh(d*x + c)^4 + 4*(1092*a*b*
cosh(d*x + c)^11 + 1001*a*b*cosh(d*x + c)^9 - 396*a*b*cosh(d*x + c)^7 - 378
*a*b*cosh(d*x + c)^5 + 9*a*b*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(910*a*b*co
sh(d*x + c)^12 + 1001*a*b*cosh(d*x + c)^10 - 495*a*b*cosh(d*x + c)^8 - 630*
a*b*cosh(d*x + c)^6 + 45*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^4 - 2*a*b
*cosh(d*x + c)^2 + 8*(70*a*b*cosh(d*x + c)^13 + 91*a*b*cosh(d*x + c)^11 - 5
5*a*b*cosh(d*x + c)^9 - 90*a*b*cosh(d*x + c)^7 + 15*a*b*cosh(d*x + c)^3 + a
*b*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(60*a*b*cosh(d*x + c)^14 + 91*a*b*cos
h(d*x + c)^12 - 66*a*b*cosh(d*x + c)^10 - 135*a*b*cosh(d*x + c)^8 + 45*a*b*
cosh(d*x + c)^4 + 6*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x + c)^2 - a*b + 4*(4
*a*b*cosh(d*x + c)^15 + 7*a*b*cosh(d*x + c)^13 - 6*a*b*cosh(d*x + c)^11 - 1
5*a*b*cosh(d*x + c)^9 + 9*a*b*cosh(d*x + c)^5 + 2*a*b*cosh(d*x + c)^3 - a*b
*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*
```

```
x + c))) + 15*(a*b*cosh(d*x + c)^16 + 16*a*b*cosh(d*x + c)*sinh(d*x + c)^15
 + a*b*sinh(d*x + c)^16 + 2*a*b*cosh(d*x + c)^14 + 2*(60*a*b*cosh(d*x + c)^
2 + a*b)*sinh(d*x + c)^14 - 2*a*b*cosh(d*x + c)^12 + 28*(20*a*b*cosh(d*x +
c)^3 + a*b*cosh(d*x + c))*sinh(d*x + c)^13 + 2*(910*a*b*cosh(d*x + c)^4 + 9
1*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x + c)^12 - 6*a*b*cosh(d*x + c)^10 + 8*
(546*a*b*cosh(d*x + c)^5 + 91*a*b*cosh(d*x + c)^3 - 3*a*b*cosh(d*x + c))*si
nh(d*x + c)^11 + 2*(4004*a*b*cosh(d*x + c)^6 + 1001*a*b*cosh(d*x + c)^4 - 6
6*a*b*cosh(d*x + c)^2 - 3*a*b)*sinh(d*x + c)^10 + 4*(2860*a*b*cosh(d*x + c)
^7 + 1001*a*b*cosh(d*x + c)^5 - 110*a*b*cosh(d*x + c)^3 - 15*a*b*cosh(d*x +
 c))*sinh(d*x + c)^9 + 6*(2145*a*b*cosh(d*x + c)^8 + 1001*a*b*cosh(d*x + c)
^6 - 165*a*b*cosh(d*x + c)^4 - 45*a*b*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 6*
a*b*cosh(d*x + c)^6 + 16*(715*a*b*cosh(d*x + c)^9 + 429*a*b*cosh(d*x + c)^7
 - 99*a*b*cosh(d*x + c)^5 - 45*a*b*cosh(d*x + c)^3)*sinh(d*x + c)^7 + 2*(40
04*a*b*cosh(d*x + c)^10 + 3003*a*b*cosh(d*x + c)^8 - 924*a*b*cosh(d*x + c)^
6 - 630*a*b*cosh(d*x + c)^4 + 3*a*b)*sinh(d*x + c)^6 + 2*a*b*cosh(d*x + c)^
4 + 4*(1092*a*b*cosh(d*x + c)^11 + 1001*a*b*cosh(d*x + c)^9 - 396*a*b*cosh(
d*x + c)^7 - 378*a*b*cosh(d*x + c)^5 + 9*a*b*cosh(d*x + c))*sinh(d*x + c)^5
 + 2*(910*a*b*cosh(d*x + c)^12 + 1001*a*b*cosh(d*x + c)^10 - 495*a*b*cosh(d
*x + c)^8 - 630*a*b*cosh(d*x + c)^6 + 45*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*
x + c)^4 - 2*a*b*cosh(d*x + c)^2 + 8*(70*a*b*cosh(d*x + c)^13 + 91*a*b*cosh
(d*x + c)^11 - 55*a*b*cosh(d*x + c)^9 - 90*a*b*cosh(d*x + c)^7 + 15*a*b*cos
h(d*x + c)^3 + a*b*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(60*a*b*cosh(d*x + c)
^14 + 91*a*b*cosh(d*x + c)^12 - 66*a*b*cosh(d*x + c)^10 - 135*a*b*cosh(d*x
+ c)^8 + 45*a*b*cosh(d*x + c)^4 + 6*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x + c
)^2 - a*b + 4*(4*a*b*cosh(d*x + c)^15 + 7*a*b*cosh(d*x + c)^13 - 6*a*b*cosh
(d*x + c)^11 - 15*a*b*cosh(d*x + c)^9 + 9*a*b*cosh(d*x + c)^5 + 2*a*b*cosh(
d*x + c)^3 - a*b*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*
x + c) - sinh(d*x + c))) + 4*(105*a*b*cosh(d*x + c)^13 - 90*(a^2 + b^2)*cos
h(d*x + c)^11 - 25*(14*a^2 + 9*a*b - 10*b^2)*cosh(d*x + c)^9 - 20*(25*a^2 +
 13*b^2)*cosh(d*x + c)^7 - 3*(100*a^2 - 45*a*b - 44*b^2)*cosh(d*x + c)^5 -
2*(25*a^2 + 17*b^2)*cosh(d*x + c)^3 + (10*a^2 - 15*a*b + 2*b^2)*cosh(d*x +
c))*sinh(d*x + c))/(d*cosh(d*x + c)^16 + 16*d*cosh(d*x + c)*sinh(d*x + c)^1
5 + d*sinh(d*x + c)^16 + 2*d*cosh(d*x + c)^14 + 2*(60*d*cosh(d*x + c)^2 + d
)*sinh(d*x + c)^14 + 28*(20*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x +
 c)^13 - 2*d*cosh(d*x + c)^12 + 2*(910*d*cosh(d*x + c)^4 + 91*d*cosh(d*x +
c)^2 - d)*sinh(d*x + c)^12 + 8*(546*d*cosh(d*x + c)^5 + 91*d*cosh(d*x + c)^
3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^11 - 6*d*cosh(d*x + c)^10 + 2*(4004*d*
cosh(d*x + c)^6 + 1001*d*cosh(d*x + c)^4 - 66*d*cosh(d*x + c)^2 - 3*d)*sinh
(d*x + c)^10 + 4*(2860*d*cosh(d*x + c)^7 + 1001*d*cosh(d*x + c)^5 - 110*d*c
osh(d*x + c)^3 - 15*d*cosh(d*x + c))*sinh(d*x + c)^9 + 6*(2145*d*cosh(d*x +
 c)^8 + 1001*d*cosh(d*x + c)^6 - 165*d*cosh(d*x + c)^4 - 45*d*cosh(d*x + c)
^2)*sinh(d*x + c)^8 + 16*(715*d*cosh(d*x + c)^9 + 429*d*cosh(d*x + c)^7 - 9
9*d*cosh(d*x + c)^5 - 45*d*cosh(d*x + c)^3)*sinh(d*x + c)^7 + 6*d*cosh(d*x
+ c)^6 + 2*(4004*d*cosh(d*x + c)^10 + 3003*d*cosh(d*x + c)^8 - 924*d*cosh(d
*x + c)^6 - 630*d*cosh(d*x + c)^4 + 3*d)*sinh(d*x + c)^6 + 4*(1092*d*cosh(d
*x + c)^11 + 1001*d*cosh(d*x + c)^9 - 396*d*cosh(d*x + c)^7 - 378*d*cosh(d*
x + c)^5 + 9*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*d*cosh(d*x + c)^4 + 2*(91
0*d*cosh(d*x + c)^12 + 1001*d*cosh(d*x + c)^10 - 495*d*cosh(d*x + c)^8 - 63
0*d*cosh(d*x + c)^6 + 45*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 8*(70*d*c
osh(d*x + c)^13 + 91*d*cosh(d*x + c)^11 - 55*d*cosh(d*x + c)^9 - 90*d*cosh(
d*x + c)^7 + 15*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^3 - 2*d*
cosh(d*x + c)^2 + 2*(60*d*cosh(d*x + c)^14 + 91*d*cosh(d*x + c)^12 - 66*d*c
osh(d*x + c)^10 - 135*d*cosh(d*x + c)^8 + 45*d*cosh(d*x + c)^4 + 6*d*cosh(d
*x + c)^2 - d)*sinh(d*x + c)^2 + 4*(4*d*cosh(d*x + c)^15 + 7*d*cosh(d*x + c
)^13 - 6*d*cosh(d*x + c)^11 - 15*d*cosh(d*x + c)^9 + 9*d*cosh(d*x + c)^5 +
2*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) - d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tanh^3\left(c + dx\right)\right)^2 \operatorname{csch}^4\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*tanh(d*x+c)**3)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)**2*csch(c + d*x)**4, x)
```

Giac [B] time = 1.69259, size = 336, normalized size = 3.46

$$-\frac{60\,ab\log\left(e^{(2\,dx+2\,c)}+1\right) - 60\,ab\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right) + \frac{10\left(11\,abe^{(6\,dx+6\,c)} - 33\,abe^{(4\,dx+4\,c)} + 12\,a^2e^{(2\,dx+2\,c)} + 33\,abe^{(2\,dx+2\,c)} - 4\,a^2 - 11\,ab\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^3} - \cdots}{30\,\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -1/30*(60*a*b*log(e^(2*d*x + 2*c) + 1) - 60*a*b*log(abs(e^(2*d*x + 2*c) - 1
)) + 10*(11*a*b*e^(6*d*x + 6*c) - 33*a*b*e^(4*d*x + 4*c) + 12*a^2*e^(2*d*x
+ 2*c) + 33*a*b*e^(2*d*x + 2*c) - 4*a^2 - 11*a*b)/(e^(2*d*x + 2*c) - 1)^3 -
 (137*a*b*e^(10*d*x + 10*c) + 805*a*b*e^(8*d*x + 8*c) + 1730*a*b*e^(6*d*x +
 6*c) - 120*b^2*e^(6*d*x + 6*c) + 1730*a*b*e^(4*d*x + 4*c) + 40*b^2*e^(4*d*
x + 4*c) + 805*a*b*e^(2*d*x + 2*c) - 40*b^2*e^(2*d*x + 2*c) + 137*a*b - 8*b
^2)/(e^(2*d*x + 2*c) + 1)^5)/d
```

3.65 $\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=275

$$-\frac{b\left(3a^2+10b^2\right)\tanh^2(c+dx)}{2d}+\frac{3b\left(3a^2+5b^2\right)\log(\cosh(c+dx))}{d}+\frac{\sinh(c+dx)\cosh^3(c+dx)\left(b\left(3a^2+b^2\right)\tanh(c+\right.}{4d}$$

```
[Out] (3*a*(a^2 + 63*b^2)*x)/8 + (3*b*(3*a^2 + 5*b^2)*Log[Cosh[c + d*x]])/d - (18
*a*b^2*Tanh[c + d*x])/d - (b*(3*a^2 + 10*b^2)*Tanh[c + d*x]^2)/(2*d) - (3*a
*b^2*Tanh[c + d*x]^3)/d - (3*b^3*Tanh[c + d*x]^4)/(2*d) - (3*a*b^2*Tanh[c +
 d*x]^5)/(5*d) - (b^3*Tanh[c + d*x]^6)/(2*d) - (b^3*Tanh[c + d*x]^8)/(8*d)
+ (Cosh[c + d*x]^3*Sinh[c + d*x]*(a*(a^2 + 3*b^2) + b*(3*a^2 + b^2)*Tanh[c
+ d*x]))/(4*d) - (Cosh[c + d*x]*Sinh[c + d*x]*(a*(5*a^2 + 51*b^2) + 2*b*(15
*a^2 + 11*b^2)*Tanh[c + d*x]))/(8*d)
```

Rubi [A] time = 0.493673, antiderivative size = 306, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3663, 1804, 1802, 633, 31}

$$-\frac{3b\left(3a^2+5b^2\right)\tanh^2(c+dx)}{2d}-\frac{3a\left(a^2+63b^2\right)\tanh(c+dx)}{8d}-\frac{3(a+b)\left(a^2+23ab+40b^2\right)\log(1-\tanh(c+dx))}{16d}+\frac{3(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] (-3*(a + b)*(a^2 + 23*a*b + 40*b^2)*Log[1 - Tanh[c + d*x]])/(16*d) + (3*(a
- b)*(a^2 - 23*a*b + 40*b^2)*Log[1 + Tanh[c + d*x]])/(16*d) - (3*a*(a^2 + 6
3*b^2)*Tanh[c + d*x])/(8*d) - (3*b*(3*a^2 + 5*b^2)*Tanh[c + d*x]^2)/(2*d) -
 (3*a*b^2*Tanh[c + d*x]^3)/d - (3*b^3*Tanh[c + d*x]^4)/(2*d) - (3*a*b^2*Tan
h[c + d*x]^5)/(5*d) - (b^3*Tanh[c + d*x]^6)/(2*d) - (b^3*Tanh[c + d*x]^8)/(
8*d) + (Sinh[c + d*x]^4*(b*(3*a^2 + b^2) + a*(a^2 + 3*b^2)*Tanh[c + d*x]))/
(4*d) - (Sinh[c + d*x]^2*Tanh[c + d*x]*(a*(a^2 + 39*b^2) + 4*b*(6*a^2 + 5*b
^2)*Tanh[c + d*x]))/(8*d)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+bx^3\right)^3}{\left(1-x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\sinh^4(c+dx)\left(b\left(3a^2+b^2\right)+a\left(a^2+3b^2\right)\tanh(c+dx)\right)}{4d} + \frac{\text{Subst}\left(\int \frac{x^3}{-}\right.}{}$$

$$= \frac{\sinh^4(c+dx)\left(b\left(3a^2+b^2\right)+a\left(a^2+3b^2\right)\tanh(c+dx)\right)}{4d} - \frac{\sinh^2(c+d}{}$$

$$= \frac{\sinh^4(c+dx)\left(b\left(3a^2+b^2\right)+a\left(a^2+3b^2\right)\tanh(c+dx)\right)}{4d} - \frac{\sinh^2(c+d}{}$$

$$= -\frac{3a\left(a^2+63b^2\right)\tanh(c+dx)}{8d} - \frac{3b\left(3a^2+5b^2\right)\tanh^2(c+dx)}{2d} - \frac{3ab^2\,\text{t}}{}$$

$$= -\frac{3a\left(a^2+63b^2\right)\tanh(c+dx)}{8d} - \frac{3b\left(3a^2+5b^2\right)\tanh^2(c+dx)}{2d} - \frac{3ab^2\,\text{t}}{}$$

$$= -\frac{3(a+b)\left(a^2+23ab+40b^2\right)\log(1-\tanh(c+dx))}{16d} + \frac{3(a-b)\left(a^2-23\right.}{}$$

Mathematica [A] time = 6.23935, size = 294, normalized size = 1.07

$$\frac{3a\left(a^2+63b^2\right)(c+dx)}{8d} - \frac{a\left(a^2+12b^2\right)\sinh(2(c+dx))}{4d} + \frac{a\left(a^2+3b^2\right)\sinh(4(c+dx))}{32d} - \frac{b\left(15a^2+11b^2\right)\cosh(2(c+}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] (3*a*(a^2 + 63*b^2)*(c + d*x))/(8*d) - (b*(15*a^2 + 11*b^2)*Cosh[2*(c + d*x
)])/(8*d) + (b*(3*a^2 + b^2)*Cosh[4*(c + d*x)])/(32*d) + (3*(3*a^2*b + 5*b^
3)*Log[Cosh[c + d*x]])/d + (b*(3*a^2 + 20*b^2)*Sech[c + d*x]^2)/(2*d) - (15
*b^3*Sech[c + d*x]^4)/(4*d) + (b^3*Sech[c + d*x]^6)/d - (b^3*Sech[c + d*x]^
8)/(8*d) - (a*(a^2 + 12*b^2)*Sinh[2*(c + d*x)])/(4*d) + (a*(a^2 + 3*b^2)*Si
nh[4*(c + d*x)])/(32*d) - (108*a*b^2*Tanh[c + d*x])/(5*d) + (21*a*b^2*Sech[
c + d*x]^2*Tanh[c + d*x])/(5*d) - (3*a*b^2*Sech[c + d*x]^4*Tanh[c + d*x])/(
5*d)
```

Maple [A] time = 0.076, size = 385, normalized size = 1.4

$$\frac{a^3\cosh(dx+c)(\sinh(dx+c))^3}{4d} - \frac{3a^3\cosh(dx+c)\sinh(dx+c)}{8d} + \frac{3a^3x}{8} + \frac{3a^3c}{8d} + \frac{3a^2b(\sinh(dx+c))^6}{4d(\cosh(dx+c))^2} - \frac{9a^2b(\sinh}{4d(\cosh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x)
```

```
[Out] 1/4/d*a^3*cosh(d*x+c)*sinh(d*x+c)^3-3/8/d*a^3*cosh(d*x+c)*sinh(d*x+c)+3/8*a
^3*x+3/8/d*a^3*c+3/4/d*a^2*b*sinh(d*x+c)^6/cosh(d*x+c)^2-9/4/d*a^2*b*sinh(d
*x+c)^4/cosh(d*x+c)^2+9/d*a^2*b*ln(cosh(d*x+c))-9/2*a^2*b*tanh(d*x+c)^2/d+3
/4/d*a*b^2*sinh(d*x+c)^9/cosh(d*x+c)^5-27/8/d*a*b^2*sinh(d*x+c)^7/cosh(d*x+
c)^5+189/8*a*b^2*x+189/8/d*a*b^2*c-189/8*a*b^2*tanh(d*x+c)/d-63/8*a*b^2*tan
h(d*x+c)^3/d-189/40*a*b^2*tanh(d*x+c)^5/d+1/4/d*b^3*sinh(d*x+c)^12/cosh(d*x
+c)^8-3/2/d*b^3*sinh(d*x+c)^10/cosh(d*x+c)^8+15/d*b^3*ln(cosh(d*x+c))-15/2/
d*b^3*tanh(d*x+c)^2-15/4*b^3*tanh(d*x+c)^4/d-5/2*b^3*tanh(d*x+c)^6/d-15/8*b
^3*tanh(d*x+c)^8/d
```

Maxima [B] time = 1.80729, size = 873, normalized size = 3.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] 1/64*a^3*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + 3/320*a*b^2*(2520*(d*x + c)/d + 5*(32*e^(-2*d*
x - 2*c) - e^(-4*d*x - 4*c))/d - (135*e^(-2*d*x - 2*c) + 5358*e^(-4*d*x - 4
*c) + 18190*e^(-6*d*x - 6*c) + 28455*e^(-8*d*x - 8*c) + 19995*e^(-10*d*x -
10*c) + 6560*e^(-12*d*x - 12*c) - 5)/(d*(e^(-4*d*x - 4*c) + 5*e^(-6*d*x - 6
*c) + 10*e^(-8*d*x - 8*c) + 10*e^(-10*d*x - 10*c) + 5*e^(-12*d*x - 12*c) +
e^(-14*d*x - 14*c)))) + 1/64*b^3*(960*(d*x + c)/d - (44*e^(-2*d*x - 2*c) -
e^(-4*d*x - 4*c))/d + 960*log(e^(-2*d*x - 2*c) + 1)/d - (36*e^(-2*d*x - 2*c
) + 324*e^(-4*d*x - 4*c) - 1384*e^(-6*d*x - 6*c) - 9126*e^(-8*d*x - 8*c) -
24112*e^(-10*d*x - 10*c) - 31868*e^(-12*d*x - 12*c) - 25912*e^(-14*d*x - 14
*c) - 11169*e^(-16*d*x - 16*c) - 2516*e^(-18*d*x - 18*c) - 1)/(d*(e^(-4*d*x
 - 4*c) + 8*e^(-6*d*x - 6*c) + 28*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c)
+ 70*e^(-12*d*x - 12*c) + 56*e^(-14*d*x - 14*c) + 28*e^(-16*d*x - 16*c) + 8
*e^(-18*d*x - 18*c) + e^(-20*d*x - 20*c)))) + 3/64*a^2*b*(192*(d*x + c)/d -
 (20*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c))/d + 192*log(e^(-2*d*x - 2*c) + 1)
/d - (18*e^(-2*d*x - 2*c) + 39*e^(-4*d*x - 4*c) - 108*e^(-6*d*x - 6*c) - 1)
/(d*(e^(-4*d*x - 4*c) + 2*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c))))
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4*(a+b*tanh(d*x+c)**3)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 4.87444, size = 956, normalized size = 3.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 1/2240*(840*(a^3 - 24*a^2*b + 63*a*b^2 - 40*b^3)*d*x + 6720*(3*a^2*b*e^(2*c
) + 5*b^3*e^(2*c))*e^(-2*c)*log(e^(2*d*x + 2*c) + 1) - 35*(18*a^3*e^(4*d*x
+ 4*c) - 432*a^2*b*e^(4*d*x + 4*c) + 1134*a*b^2*e^(4*d*x + 4*c) - 720*b^3*e
^(4*d*x + 4*c) - 8*a^3*e^(2*d*x + 2*c) + 60*a^2*b*e^(2*d*x + 2*c) - 96*a*b^
2*e^(2*d*x + 2*c) + 44*b^3*e^(2*d*x + 2*c) + a^3 - 3*a^2*b + 3*a*b^2 - b^3)
*e^(-4*d*x - 4*c) + 35*(a^3*e^(4*d*x + 48*c) + 3*a^2*b*e^(4*d*x + 48*c) + 3
*a*b^2*e^(4*d*x + 48*c) + b^3*e^(4*d*x + 48*c) - 8*a^3*e^(2*d*x + 46*c) - 6
0*a^2*b*e^(2*d*x + 46*c) - 96*a*b^2*e^(2*d*x + 46*c) - 44*b^3*e^(2*d*x + 46
*c))*e^(-44*c) - 8*(6849*a^2*b*e^(16*d*x + 16*c) + 11415*b^3*e^(16*d*x + 16
*c) + 53112*a^2*b*e^(14*d*x + 14*c) - 16800*a*b^2*e^(14*d*x + 14*c) + 80120
*b^3*e^(14*d*x + 14*c) + 181692*a^2*b*e^(12*d*x + 12*c) - 100800*a*b^2*e^(1
2*d*x + 12*c) + 269220*b^3*e^(12*d*x + 12*c) + 358344*a^2*b*e^(10*d*x + 10*
c) - 272160*a*b^2*e^(10*d*x + 10*c) + 520520*b^3*e^(10*d*x + 10*c) + 445830
*a^2*b*e^(8*d*x + 8*c) - 423360*a*b^2*e^(8*d*x + 8*c) + 648970*b^3*e^(8*d*x
 + 8*c) + 358344*a^2*b*e^(6*d*x + 6*c) - 405216*a*b^2*e^(6*d*x + 6*c) + 520
520*b^3*e^(6*d*x + 6*c) + 181692*a^2*b*e^(4*d*x + 4*c) - 237888*a*b^2*e^(4*
d*x + 4*c) + 269220*b^3*e^(4*d*x + 4*c) + 53112*a^2*b*e^(2*d*x + 2*c) - 799
68*a*b^2*e^(2*d*x + 2*c) + 80120*b^3*e^(2*d*x + 2*c) + 6849*a^2*b - 12096*a
*b^2 + 11415*b^3)/(e^(2*d*x + 2*c) + 1)^8)/d
```

3.66 $\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=351

$$\frac{5a^2b\sinh^3(c+dx)}{2d}-\frac{15a^2b\sinh(c+dx)}{2d}-\frac{3a^2b\sinh^3(c+dx)\tanh^2(c+dx)}{2d}+\frac{15a^2b\tan^{-1}(\sinh(c+dx))}{2d}+\frac{a^3\cosh^3(c}{3d}$$

```
[Out] (15*a^2*b*ArcTan[Sinh[c + d*x]])/(2*d) + (1155*b^3*ArcTan[Sinh[c + d*x]])/(
128*d) - (a^3*Cosh[c + d*x])/d - (12*a*b^2*Cosh[c + d*x])/d + (a^3*Cosh[c +
 d*x]^3)/(3*d) + (a*b^2*Cosh[c + d*x]^3)/d - (18*a*b^2*Sech[c + d*x])/d + (
4*a*b^2*Sech[c + d*x]^3)/d - (3*a*b^2*Sech[c + d*x]^5)/(5*d) - (15*a^2*b*Si
nh[c + d*x])/(2*d) - (1155*b^3*Sinh[c + d*x])/(128*d) + (5*a^2*b*Sinh[c + d
*x]^3)/(2*d) + (385*b^3*Sinh[c + d*x]^3)/(128*d) - (3*a^2*b*Sinh[c + d*x]^3
*Tanh[c + d*x]^2)/(2*d) - (231*b^3*Sinh[c + d*x]^3*Tanh[c + d*x]^2)/(128*d)
 - (33*b^3*Sinh[c + d*x]^3*Tanh[c + d*x]^4)/(64*d) - (11*b^3*Sinh[c + d*x]^
3*Tanh[c + d*x]^6)/(48*d) - (b^3*Sinh[c + d*x]^3*Tanh[c + d*x]^8)/(8*d)
```

Rubi [A] time = 0.3499, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3666, 2633, 2592, 288, 302, 203, 2590, 270}

$$\frac{5a^2b\sinh^3(c+dx)}{2d}-\frac{15a^2b\sinh(c+dx)}{2d}-\frac{3a^2b\sinh^3(c+dx)\tanh^2(c+dx)}{2d}+\frac{15a^2b\tan^{-1}(\sinh(c+dx))}{2d}+\frac{a^3\cosh^3(c}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^3,x]
```


```
[Out] (15*a^2*b*ArcTan[Sinh[c + d*x]])/(2*d) + (1155*b^3*ArcTan[Sinh[c + d*x]])/(
128*d) - (a^3*Cosh[c + d*x])/d - (12*a*b^2*Cosh[c + d*x])/d + (a^3*Cosh[c +
 d*x]^3)/(3*d) + (a*b^2*Cosh[c + d*x]^3)/d - (18*a*b^2*Sech[c + d*x])/d + (
4*a*b^2*Sech[c + d*x]^3)/d - (3*a*b^2*Sech[c + d*x]^5)/(5*d) - (15*a^2*b*Si
nh[c + d*x])/(2*d) - (1155*b^3*Sinh[c + d*x])/(128*d) + (5*a^2*b*Sinh[c + d
*x]^3)/(2*d) + (385*b^3*Sinh[c + d*x]^3)/(128*d) - (3*a^2*b*Sinh[c + d*x]^3
*Tanh[c + d*x]^2)/(2*d) - (231*b^3*Sinh[c + d*x]^3*Tanh[c + d*x]^2)/(128*d)
 - (33*b^3*Sinh[c + d*x]^3*Tanh[c + d*x]^4)/(64*d) - (11*b^3*Sinh[c + d*x]^
3*Tanh[c + d*x]^6)/(48*d) - (b^3*Sinh[c + d*x]^3*Tanh[c + d*x]^8)/(8*d)
```


Rule 3666

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
```

```
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = i\int\left(-ia^3\sinh^3(c+dx)-3ia^2b\sinh^3(c+dx)\tanh^3(c+dx)-3iab^2\sinh^3(\right.$$

$$= a^3\int\sinh^3(c+dx)\,dx+\left(3a^2b\right)\int\sinh^3(c+dx)\tanh^3(c+dx)\,dx+\left(3ab^2\right)$$

$$= -\frac{a^3\,\text{Subst}\left(\int\left(1-x^2\right)dx,x,\cosh(c+dx)\right)}{d}+\frac{\left(3a^2b\right)\text{Subst}\left(\int\frac{x^6}{\left(1+x^2\right)^2}\,dx,x\right.}{d}$$

$$= -\frac{a^3\cosh(c+dx)}{d}+\frac{a^3\cosh^3(c+dx)}{3d}-\frac{3a^2b\sinh^3(c+dx)\tanh^2(c+dx)}{2d}$$

$$= -\frac{a^3\cosh(c+dx)}{d}-\frac{12ab^2\cosh(c+dx)}{d}+\frac{a^3\cosh^3(c+dx)}{3d}+\frac{ab^2\cosh^3(c}{d}$$

$$= -\frac{a^3\cosh(c+dx)}{d}-\frac{12ab^2\cosh(c+dx)}{d}+\frac{a^3\cosh^3(c+dx)}{3d}+\frac{ab^2\cosh^3(c}{d}$$

$$= \frac{15a^2b\tan^{-1}(\sinh(c+dx))}{2d}-\frac{a^3\cosh(c+dx)}{d}-\frac{12ab^2\cosh(c+dx)}{d}+\frac{a^3\text{c}}{}$$

$$= \frac{15a^2b\tan^{-1}(\sinh(c+dx))}{2d}-\frac{a^3\cosh(c+dx)}{d}-\frac{12ab^2\cosh(c+dx)}{d}+\frac{a^3\text{c}}{}$$

$$= \frac{15a^2b\tan^{-1}(\sinh(c+dx))}{2d}-\frac{a^3\cosh(c+dx)}{d}-\frac{12ab^2\cosh(c+dx)}{d}+\frac{a^3\text{c}}{}$$

$$= \frac{15a^2b\tan^{-1}(\sinh(c+dx))}{2d}+\frac{1155b^3\tan^{-1}(\sinh(c+dx))}{128d}-\frac{a^3\cosh(c+dx}{d}$$

Mathematica [A] time = 6.54458, size = 291, normalized size = 0.83

$$-\frac{3b\left(9a^2+7b^2\right)\sinh(c+dx)}{4d}+\frac{b\left(3a^2+b^2\right)\sinh(3(c+dx))}{12d}-\frac{3a\left(a^2+15b^2\right)\cosh(c+dx)}{4d}+\frac{a\left(a^2+3b^2\right)\cosh(3(c+dx}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] (15*b*(64*a^2 + 77*b^2)*ArcTan[Tanh[(c + d*x)/2]])/(64*d) - (3*a*(a^2 + 15*
b^2)*Cosh[c + d*x])/(4*d) + (a*(a^2 + 3*b^2)*Cosh[3*(c + d*x)])/(12*d) - (1
8*a*b^2*Sech[c + d*x])/d + (4*a*b^2*Sech[c + d*x]^3)/d - (3*a*b^2*Sech[c +
d*x]^5)/(5*d) - (3*b*(9*a^2 + 7*b^2)*Sinh[c + d*x])/(4*d) - (3*Sech[c + d*x
]^2*(64*a^2*b*Sinh[c + d*x] + 255*b^3*Sinh[c + d*x]))/(128*d) + (b*(3*a^2 +
 b^2)*Sinh[3*(c + d*x)])/(12*d) + (515*b^3*Sech[c + d*x]^3*Tanh[c + d*x])/(
192*d) - (41*b^3*Sech[c + d*x]^5*Tanh[c + d*x])/(48*d) + (b^3*Sech[c + d*x]
^7*Tanh[c + d*x])/(8*d)
```

Maple [A] time = 0.13, size = 554, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x)
```

```
[Out] 77/16/d*b^3*tanh(d*x+c)*sech(d*x+c)^5+1/3/d*a^3*cosh(d*x+c)*sinh(d*x+c)^2+1
5/d*a^2*b*arctan(exp(d*x+c))+1/3/d*b^3*sinh(d*x+c)^11/cosh(d*x+c)^8-11/3/d*
b^3*sinh(d*x+c)^9/cosh(d*x+c)^8-33/d*b^3*sinh(d*x+c)^7/cosh(d*x+c)^8-77/d*b
^3*sinh(d*x+c)^5/cosh(d*x+c)^8-77/d*b^3*sinh(d*x+c)^3/cosh(d*x+c)^8-33/d*b^
3*sinh(d*x+c)/cosh(d*x+c)^8+33/8/d*b^3*tanh(d*x+c)*sech(d*x+c)^7-8/d*a*b^2*
sinh(d*x+c)^6/cosh(d*x+c)^5+1155/128*b^3*sech(d*x+c)*tanh(d*x+c)/d+385/64*b
^3*sech(d*x+c)^3*tanh(d*x+c)/d-128/5*a*b^2*cosh(d*x+c)/d-2/3*a^3*cosh(d*x+c
)/d-48/d*a*b^2*sinh(d*x+c)^4/cosh(d*x+c)^5-192/5/d*a*b^2*sinh(d*x+c)^2/cosh
(d*x+c)^5+128/5/d*a*b^2*sinh(d*x+c)^2/cosh(d*x+c)^3+128/5/d*a*b^2*sinh(d*x+
c)^2/cosh(d*x+c)+1/d*a^2*b*sinh(d*x+c)^5/cosh(d*x+c)^2-5/d*a^2*b*sinh(d*x+c
)^3/cosh(d*x+c)^2-15/d*a^2*b*sinh(d*x+c)/cosh(d*x+c)^2+1/d*a*b^2*sinh(d*x+c
)^8/cosh(d*x+c)^5+15/2*a^2*b*sech(d*x+c)*tanh(d*x+c)/d+1155/64/d*b^3*arctan
(exp(d*x+c))
```

Maxima [A] time = 1.78683, size = 815, normalized size = 2.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] 1/192*b^3*(8*(63*e^(-d*x - c) - e^(-3*d*x - 3*c))/d - 3465*arctan(e^(-d*x -
 c))/d - (440*e^(-2*d*x - 2*c) + 6103*e^(-4*d*x - 4*c) + 21019*e^(-6*d*x -
6*c) + 41207*e^(-8*d*x - 8*c) + 40243*e^(-10*d*x - 10*c) + 22589*e^(-12*d*x
 - 12*c) + 505*e^(-14*d*x - 14*c) - 3331*e^(-16*d*x - 16*c) - 1791*e^(-18*d
*x - 18*c) - 8)/(d*(e^(-3*d*x - 3*c) + 8*e^(-5*d*x - 5*c) + 28*e^(-7*d*x -
7*c) + 56*e^(-9*d*x - 9*c) + 70*e^(-11*d*x - 11*c) + 56*e^(-13*d*x - 13*c)
+ 28*e^(-15*d*x - 15*c) + 8*e^(-17*d*x - 17*c) + e^(-19*d*x - 19*c)))) - 1/
40*a*b^2*(5*(45*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + (200*e^(-2*d*x - 2*c)
+ 2515*e^(-4*d*x - 4*c) + 6680*e^(-6*d*x - 6*c) + 9073*e^(-8*d*x - 8*c) + 5
600*e^(-10*d*x - 10*c) + 1665*e^(-12*d*x - 12*c) - 5)/(d*(e^(-3*d*x - 3*c)
+ 5*e^(-5*d*x - 5*c) + 10*e^(-7*d*x - 7*c) + 10*e^(-9*d*x - 9*c) + 5*e^(-11
*d*x - 11*c) + e^(-13*d*x - 13*c)))) + 1/8*a^2*b*((27*e^(-d*x - c) - e^(-3*
d*x - 3*c))/d - 120*arctan(e^(-d*x - c))/d - (25*e^(-2*d*x - 2*c) + 77*e^(-
4*d*x - 4*c) + 3*e^(-6*d*x - 6*c) - 1)/(d*(e^(-3*d*x - 3*c) + 2*e^(-5*d*x -
 5*c) + e^(-7*d*x - 7*c)))) + 1/24*a^3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d
 - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

Fricas [B] time = 3.60726, size = 23883, normalized size = 68.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/960*(40*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^22 + 880*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^21 + 40*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*sinh(d*x + c)^22 - 40*(a^3 + 57*a^2*b + 111*a*b^2 + 55*b^3)*
cosh(d*x + c)^20 - 40*(a^3 + 57*a^2*b + 111*a*b^2 + 55*b^3 - 231*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^20 + 800*(77*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 - (a^3 + 57*a^2*b + 111*a*b^2 + 55*b
^3)*cosh(d*x + c))*sinh(d*x + c)^19 - 5*(424*a^3 + 4440*a^2*b + 15960*a*b^2
 + 5599*b^3)*cosh(d*x + c)^18 + 5*(58520*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*co
```

sh(d*x + c)^4 - 424*a^3 - 4440*a^2*b - 15960*a*b^2 - 5599*b^3 - 1520*(a^3 + 57*a^2*b + 111*a*b^2 + 55*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^18 + 30*(351 12*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 - 1520*(a^3 + 57*a^2*b + 111*a*b^2 + 55*b^3)*cosh(d*x + c)^3 - 3*(424*a^3 + 4440*a^2*b + 15960*a*b^ 2 + 5599*b^3)*cosh(d*x + c))*sinh(d*x + c)^17 - 15*(712*a^3 + 4840*a^2*b + 26584*a*b^2 + 5665*b^3)*cosh(d*x + c)^16 + 15*(198968*(a^3 + 3*a^2*b + 3*a* b^2 + b^3)*cosh(d*x + c)^6 - 12920*(a^3 + 57*a^2*b + 111*a*b^2 + 55*b^3)*co sh(d*x + c)^4 - 712*a^3 - 4840*a^2*b - 26584*a*b^2 - 5665*b^3 - 51*(424*a^3 + 4440*a^2*b + 15960*a*b^2 + 5599*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^16 + 240*(28424*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 - 2584*(a^3 + 5 7*a^2*b + 111*a*b^2 + 55*b^3)*cosh(d*x + c)^5 - 17*(424*a^3 + 4440*a^2*b + 15960*a*b^2 + 5599*b^3)*cosh(d*x + c)^3 - (712*a^3 + 4840*a^2*b + 26584*a*b ^2 + 5665*b^3)*cosh(d*x + c))*sinh(d*x + c)^15 - 3*(9040*a^3 + 36400*a^2*b + 344944*a*b^2 + 45265*b^3)*cosh(d*x + c)^14 + 3*(4263600*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 - 516800*(a^3 + 57*a^2*b + 111*a*b^2 + 55*b^ 3)*cosh(d*x + c)^6 - 5100*(424*a^3 + 4440*a^2*b + 15960*a*b^2 + 5599*b^3)*c osh(d*x + c)^4 - 9040*a^3 - 36400*a^2*b - 344944*a*b^2 - 45265*b^3 - 600*(7 12*a^3 + 4840*a^2*b + 26584*a*b^2 + 5665*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^14 + 2*(9948400*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^9 - 1550400 *(a^3 + 57*a^2*b + 111*a*b^2 + 55*b^3)*cosh(d*x + c)^7 - 21420*(424*a^3 + 4 440*a^2*b + 15960*a*b^2 + 5599*b^3)*cosh(d*x + c)^5 - 4200*(712*a^3 + 4840* a^2*b + 26584*a*b^2 + 5665*b^3)*cosh(d*x + c)^3 - 21*(9040*a^3 + 36400*a^2* b + 344944*a*b^2 + 45265*b^3)*cosh(d*x + c))*sinh(d*x + c)^13 - 3*(14000*a^ 3 + 18800*a^2*b + 542672*a*b^2 + 20405*b^3)*cosh(d*x + c)^12 + (25865840*(a ^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^10 - 5038800*(a^3 + 57*a^2*b + 111*a*b^2 + 55*b^3)*cosh(d*x + c)^8 - 92820*(424*a^3 + 4440*a^2*b + 15960*a *b^2 + 5599*b^3)*cosh(d*x + c)^6 - 27300*(712*a^3 + 4840*a^2*b + 26584*a*b^ 2 + 5665*b^3)*cosh(d*x + c)^4 - 42000*a^3 - 56400*a^2*b - 1628016*a*b^2 - 6 1215*b^3 - 273*(9040*a^3 + 36400*a^2*b + 344944*a*b^2 + 45265*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^12 + 4*(7054320*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh (d*x + c)^11 - 1679600*(a^3 + 57*a^2*b + 111*a*b^2 + 55*b^3)*cosh(d*x + c)^ 9 - 39780*(424*a^3 + 4440*a^2*b + 15960*a*b^2 + 5599*b^3)*cosh(d*x + c)^7 - 16380*(712*a^3 + 4840*a^2*b + 26584*a*b^2 + 5665*b^3)*cosh(d*x + c)^5 - 27 3*(9040*a^3 + 36400*a^2*b + 344944*a*b^2 + 45265*b^3)*cosh(d*x + c)^3 - 9*(14000*a^3 + 18800*a^2*b + 542672*a*b^2 + 20405*b^3)*cosh(d*x + c))*sinh(d*x + c)^11 - 3*(14000*a^3 - 18800*a^2*b + 542672*a*b^2 - 20405*b^3)*cosh(d*x + c)^10 + (25865840*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^12 - 7390 240*(a^3 + 57*a^2*b + 111*a*b^2 + 55*b^3)*cosh(d*x + c)^10 - 218790*(424*a^ 3 + 4440*a^2*b + 15960*a*b^2 + 5599*b^3)*cosh(d*x + c)^8 - 120120*(712*a^3 + 4840*a^2*b + 26584*a*b^2 + 5665*b^3)*cosh(d*x + c)^6 - 3003*(9040*a^3 + 3 6400*a^2*b + 344944*a*b^2 + 45265*b^3)*cosh(d*x + c)^4 - 42000*a^3 + 56400* a^2*b - 1628016*a*b^2 + 61215*b^3 - 198*(14000*a^3 + 18800*a^2*b + 542672*a *b^2 + 20405*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 2*(9948400*(a^3 + 3*a ^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^13 - 3359200*(a^3 + 57*a^2*b + 111*a*b^ 2 + 55*b^3)*cosh(d*x + c)^11 - 121550*(424*a^3 + 4440*a^2*b + 15960*a*b^2 + 5599*b^3)*cosh(d*x + c)^9 - 85800*(712*a^3 + 4840*a^2*b + 26584*a*b^2 + 56 65*b^3)*cosh(d*x + c)^7 - 3003*(9040*a^3 + 36400*a^2*b + 344944*a*b^2 + 452 65*b^3)*cosh(d*x + c)^5 - 330*(14000*a^3 + 18800*a^2*b + 542672*a*b^2 + 204 05*b^3)*cosh(d*x + c)^3 - 15*(14000*a^3 - 18800*a^2*b + 542672*a*b^2 - 2040 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 - 3*(9040*a^3 - 36400*a^2*b + 344944* a*b^2 - 45265*b^3)*cosh(d*x + c)^8 + 3*(4263600*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^14 - 1679600*(a^3 + 57*a^2*b + 111*a*b^2 + 55*b^3)*cosh(d*x + c)^12 - 72930*(424*a^3 + 4440*a^2*b + 15960*a*b^2 + 5599*b^3)*cosh(d* x + c)^10 - 64350*(712*a^3 + 4840*a^2*b + 26584*a*b^2 + 5665*b^3)*cosh(d*x + c)^8 - 3003*(9040*a^3 + 36400*a^2*b + 344944*a*b^2 + 45265*b^3)*cosh(d*x + c)^6 - 495*(14000*a^3 + 18800*a^2*b + 542672*a*b^2 + 20405*b^3)*cosh(d*x + c)^4 - 9040*a^3 + 36400*a^2*b - 344944*a*b^2 + 45265*b^3 - 45*(14000*a^3 - 18800*a^2*b + 542672*a*b^2 - 20405*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 24*(284240*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^15 - 129200*(a^3

+ 57*a^2*b + 111*a*b^2 + 55*b^3)*cosh(d*x + c)^13 - 6630*(424*a^3 + 4440*a
^2*b + 15960*a*b^2 + 5599*b^3)*cosh(d*x + c)^11 - 7150*(712*a^3 + 4840*a^2*
b + 26584*a*b^2 + 5665*b^3)*cosh(d*x + c)^9 - 429*(9040*a^3 + 36400*a^2*b +
344944*a*b^2 + 45265*b^3)*cosh(d*x + c)^7 - 99*(14000*a^3 + 18800*a^2*b +
542672*a*b^2 + 20405*b^3)*cosh(d*x + c)^5 - 15*(14000*a^3 - 18800*a^2*b + 5
42672*a*b^2 - 20405*b^3)*cosh(d*x + c)^3 - (9040*a^3 - 36400*a^2*b + 344944
*a*b^2 - 45265*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 - 15*(712*a^3 - 4840*a^2
*b + 26584*a*b^2 - 5665*b^3)*cosh(d*x + c)^6 + 3*(994840*(a^3 + 3*a^2*b + 3
*a*b^2 + b^3)*cosh(d*x + c)^16 - 516800*(a^3 + 57*a^2*b + 111*a*b^2 + 55*b^
3)*cosh(d*x + c)^14 - 30940*(424*a^3 + 4440*a^2*b + 15960*a*b^2 + 5599*b^3)
*cosh(d*x + c)^12 - 40040*(712*a^3 + 4840*a^2*b + 26584*a*b^2 + 5665*b^3)*c
osh(d*x + c)^10 - 3003*(9040*a^3 + 36400*a^2*b + 344944*a*b^2 + 45265*b^3)*
cosh(d*x + c)^8 - 924*(14000*a^3 + 18800*a^2*b + 542672*a*b^2 + 20405*b^3)*
cosh(d*x + c)^6 - 210*(14000*a^3 - 18800*a^2*b + 542672*a*b^2 - 20405*b^3)*
cosh(d*x + c)^4 - 3560*a^3 + 24200*a^2*b - 132920*a*b^2 + 28325*b^3 - 28*(9
040*a^3 - 36400*a^2*b + 344944*a*b^2 - 45265*b^3)*cosh(d*x + c)^2)*sinh(d*x
+ c)^6 + 6*(175560*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^17 - 1033
60*(a^3 + 57*a^2*b + 111*a*b^2 + 55*b^3)*cosh(d*x + c)^15 - 7140*(424*a^3 +
4440*a^2*b + 15960*a*b^2 + 5599*b^3)*cosh(d*x + c)^13 - 10920*(712*a^3 + 4
840*a^2*b + 26584*a*b^2 + 5665*b^3)*cosh(d*x + c)^11 - 1001*(9040*a^3 + 364
00*a^2*b + 344944*a*b^2 + 45265*b^3)*cosh(d*x + c)^9 - 396*(14000*a^3 + 188
00*a^2*b + 542672*a*b^2 + 20405*b^3)*cosh(d*x + c)^7 - 126*(14000*a^3 - 188
00*a^2*b + 542672*a*b^2 - 20405*b^3)*cosh(d*x + c)^5 - 28*(9040*a^3 - 36400
*a^2*b + 344944*a*b^2 - 45265*b^3)*cosh(d*x + c)^3 - 15*(712*a^3 - 4840*a^2
*b + 26584*a*b^2 - 5665*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 5*(424*a^3 -
4440*a^2*b + 15960*a*b^2 - 5599*b^3)*cosh(d*x + c)^4 + (292600*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*cosh(d*x + c)^18 - 193800*(a^3 + 57*a^2*b + 111*a*b^2 +
55*b^3)*cosh(d*x + c)^16 - 15300*(424*a^3 + 4440*a^2*b + 15960*a*b^2 + 559
9*b^3)*cosh(d*x + c)^14 - 27300*(712*a^3 + 4840*a^2*b + 26584*a*b^2 + 5665*
b^3)*cosh(d*x + c)^12 - 3003*(9040*a^3 + 36400*a^2*b + 344944*a*b^2 + 45265
*b^3)*cosh(d*x + c)^10 - 1485*(14000*a^3 + 18800*a^2*b + 542672*a*b^2 + 204
05*b^3)*cosh(d*x + c)^8 - 630*(14000*a^3 - 18800*a^2*b + 542672*a*b^2 - 204
05*b^3)*cosh(d*x + c)^6 - 210*(9040*a^3 - 36400*a^2*b + 344944*a*b^2 - 4526
5*b^3)*cosh(d*x + c)^4 - 2120*a^3 + 22200*a^2*b - 79800*a*b^2 + 27995*b^3 -
225*(712*a^3 - 4840*a^2*b + 26584*a*b^2 - 5665*b^3)*cosh(d*x + c)^2)*sinh(
d*x + c)^4 + 4*(15400*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^19 - 11
400*(a^3 + 57*a^2*b + 111*a*b^2 + 55*b^3)*cosh(d*x + c)^17 - 1020*(424*a^3
+ 4440*a^2*b + 15960*a*b^2 + 5599*b^3)*cosh(d*x + c)^15 - 2100*(712*a^3 + 4
840*a^2*b + 26584*a*b^2 + 5665*b^3)*cosh(d*x + c)^13 - 273*(9040*a^3 + 3640
0*a^2*b + 344944*a*b^2 + 45265*b^3)*cosh(d*x + c)^11 - 165*(14000*a^3 + 188
00*a^2*b + 542672*a*b^2 + 20405*b^3)*cosh(d*x + c)^9 - 90*(14000*a^3 - 1880
0*a^2*b + 542672*a*b^2 - 20405*b^3)*cosh(d*x + c)^7 - 42*(9040*a^3 - 36400*
a^2*b + 344944*a*b^2 - 45265*b^3)*cosh(d*x + c)^5 - 75*(712*a^3 - 4840*a^2*
b + 26584*a*b^2 - 5665*b^3)*cosh(d*x + c)^3 - 5*(424*a^3 - 4440*a^2*b + 159
60*a*b^2 - 5599*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 40*a^3 - 120*a^2*b +
120*a*b^2 - 40*b^3 - 40*(a^3 - 57*a^2*b + 111*a*b^2 - 55*b^3)*cosh(d*x + c)
^2 + (9240*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^20 - 7600*(a^3 + 5
7*a^2*b + 111*a*b^2 + 55*b^3)*cosh(d*x + c)^18 - 765*(424*a^3 + 4440*a^2*b
+ 15960*a*b^2 + 5599*b^3)*cosh(d*x + c)^16 - 1800*(712*a^3 + 4840*a^2*b + 2
6584*a*b^2 + 5665*b^3)*cosh(d*x + c)^14 - 273*(9040*a^3 + 36400*a^2*b + 344
944*a*b^2 + 45265*b^3)*cosh(d*x + c)^12 - 198*(14000*a^3 + 18800*a^2*b + 54
2672*a*b^2 + 20405*b^3)*cosh(d*x + c)^10 - 135*(14000*a^3 - 18800*a^2*b + 5
42672*a*b^2 - 20405*b^3)*cosh(d*x + c)^8 - 84*(9040*a^3 - 36400*a^2*b + 344
944*a*b^2 - 45265*b^3)*cosh(d*x + c)^6 - 225*(712*a^3 - 4840*a^2*b + 26584*
a*b^2 - 5665*b^3)*cosh(d*x + c)^4 - 40*a^3 + 2280*a^2*b - 4440*a*b^2 + 2200
b^3 - 30(424*a^3 - 4440*a^2*b + 15960*a*b^2 - 5599*b^3)*cosh(d*x + c)^2)*
sinh(d*x + c)^2 + 225*((64*a^2*b + 77*b^3)*cosh(d*x + c)^19 + 19*(64*a^2*b
+ 77*b^3)*cosh(d*x + c)*sinh(d*x + c)^18 + (64*a^2*b + 77*b^3)*sinh(d*x + c
)^19 + 8*(64*a^2*b + 77*b^3)*cosh(d*x + c)^17 + (512*a^2*b + 616*b^3 + 171*

```
(64*a^2*b + 77*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^17 + 17*(57*(64*a^2*b +
77*b^3)*cosh(d*x + c)^3 + 8*(64*a^2*b + 77*b^3)*cosh(d*x + c))*sinh(d*x + c
)^16 + 28*(64*a^2*b + 77*b^3)*cosh(d*x + c)^15 + 4*(969*(64*a^2*b + 77*b^3)
*cosh(d*x + c)^4 + 448*a^2*b + 539*b^3 + 272*(64*a^2*b + 77*b^3)*cosh(d*x +
 c)^2)*sinh(d*x + c)^15 + 4*(2907*(64*a^2*b + 77*b^3)*cosh(d*x + c)^5 + 136
0*(64*a^2*b + 77*b^3)*cosh(d*x + c)^3 + 105*(64*a^2*b + 77*b^3)*cosh(d*x +
c))*sinh(d*x + c)^14 + 56*(64*a^2*b + 77*b^3)*cosh(d*x + c)^13 + 28*(969*(6
4*a^2*b + 77*b^3)*cosh(d*x + c)^6 + 680*(64*a^2*b + 77*b^3)*cosh(d*x + c)^4
 + 128*a^2*b + 154*b^3 + 105*(64*a^2*b + 77*b^3)*cosh(d*x + c)^2)*sinh(d*x
+ c)^13 + 52*(969*(64*a^2*b + 77*b^3)*cosh(d*x + c)^7 + 952*(64*a^2*b + 77*
b^3)*cosh(d*x + c)^5 + 245*(64*a^2*b + 77*b^3)*cosh(d*x + c)^3 + 14*(64*a^2
*b + 77*b^3)*cosh(d*x + c))*sinh(d*x + c)^12 + 70*(64*a^2*b + 77*b^3)*cosh(
d*x + c)^11 + 2*(37791*(64*a^2*b + 77*b^3)*cosh(d*x + c)^8 + 49504*(64*a^2*
b + 77*b^3)*cosh(d*x + c)^6 + 19110*(64*a^2*b + 77*b^3)*cosh(d*x + c)^4 + 2
240*a^2*b + 2695*b^3 + 2184*(64*a^2*b + 77*b^3)*cosh(d*x + c)^2)*sinh(d*x +
 c)^11 + 22*(4199*(64*a^2*b + 77*b^3)*cosh(d*x + c)^9 + 7072*(64*a^2*b + 77
*b^3)*cosh(d*x + c)^7 + 3822*(64*a^2*b + 77*b^3)*cosh(d*x + c)^5 + 728*(64*
a^2*b + 77*b^3)*cosh(d*x + c)^3 + 35*(64*a^2*b + 77*b^3)*cosh(d*x + c))*sin
h(d*x + c)^10 + 56*(64*a^2*b + 77*b^3)*cosh(d*x + c)^9 + 2*(46189*(64*a^2*b
 + 77*b^3)*cosh(d*x + c)^10 + 97240*(64*a^2*b + 77*b^3)*cosh(d*x + c)^8 + 7
0070*(64*a^2*b + 77*b^3)*cosh(d*x + c)^6 + 20020*(64*a^2*b + 77*b^3)*cosh(d
*x + c)^4 + 1792*a^2*b + 2156*b^3 + 1925*(64*a^2*b + 77*b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^9 + 2*(37791*(64*a^2*b + 77*b^3)*cosh(d*x + c)^11 + 97240*
(64*a^2*b + 77*b^3)*cosh(d*x + c)^9 + 90090*(64*a^2*b + 77*b^3)*cosh(d*x +
c)^7 + 36036*(64*a^2*b + 77*b^3)*cosh(d*x + c)^5 + 5775*(64*a^2*b + 77*b^3)
*cosh(d*x + c)^3 + 252*(64*a^2*b + 77*b^3)*cosh(d*x + c))*sinh(d*x + c)^8 +
 28*(64*a^2*b + 77*b^3)*cosh(d*x + c)^7 + 4*(12597*(64*a^2*b + 77*b^3)*cosh
(d*x + c)^12 + 38896*(64*a^2*b + 77*b^3)*cosh(d*x + c)^10 + 45045*(64*a^2*b
 + 77*b^3)*cosh(d*x + c)^8 + 24024*(64*a^2*b + 77*b^3)*cosh(d*x + c)^6 + 57
75*(64*a^2*b + 77*b^3)*cosh(d*x + c)^4 + 448*a^2*b + 539*b^3 + 504*(64*a^2*
b + 77*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 28*(969*(64*a^2*b + 77*b^3)*
cosh(d*x + c)^13 + 3536*(64*a^2*b + 77*b^3)*cosh(d*x + c)^11 + 5005*(64*a^2
*b + 77*b^3)*cosh(d*x + c)^9 + 3432*(64*a^2*b + 77*b^3)*cosh(d*x + c)^7 + 1
155*(64*a^2*b + 77*b^3)*cosh(d*x + c)^5 + 168*(64*a^2*b + 77*b^3)*cosh(d*x
+ c)^3 + 7*(64*a^2*b + 77*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 8*(64*a^2*b
 + 77*b^3)*cosh(d*x + c)^5 + 4*(2907*(64*a^2*b + 77*b^3)*cosh(d*x + c)^14 +
 12376*(64*a^2*b + 77*b^3)*cosh(d*x + c)^12 + 21021*(64*a^2*b + 77*b^3)*cos
h(d*x + c)^10 + 18018*(64*a^2*b + 77*b^3)*cosh(d*x + c)^8 + 8085*(64*a^2*b
+ 77*b^3)*cosh(d*x + c)^6 + 1764*(64*a^2*b + 77*b^3)*cosh(d*x + c)^4 + 128*
a^2*b + 154*b^3 + 147*(64*a^2*b + 77*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5
+ 4*(969*(64*a^2*b + 77*b^3)*cosh(d*x + c)^15 + 4760*(64*a^2*b + 77*b^3)*co
sh(d*x + c)^13 + 9555*(64*a^2*b + 77*b^3)*cosh(d*x + c)^11 + 10010*(64*a^2*
b + 77*b^3)*cosh(d*x + c)^9 + 5775*(64*a^2*b + 77*b^3)*cosh(d*x + c)^7 + 17
64*(64*a^2*b + 77*b^3)*cosh(d*x + c)^5 + 245*(64*a^2*b + 77*b^3)*cosh(d*x +
 c)^3 + 10*(64*a^2*b + 77*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + (64*a^2*b +
 77*b^3)*cosh(d*x + c)^3 + (969*(64*a^2*b + 77*b^3)*cosh(d*x + c)^16 + 5440
*(64*a^2*b + 77*b^3)*cosh(d*x + c)^14 + 12740*(64*a^2*b + 77*b^3)*cosh(d*x
+ c)^12 + 16016*(64*a^2*b + 77*b^3)*cosh(d*x + c)^10 + 11550*(64*a^2*b + 77
*b^3)*cosh(d*x + c)^8 + 4704*(64*a^2*b + 77*b^3)*cosh(d*x + c)^6 + 980*(64*
a^2*b + 77*b^3)*cosh(d*x + c)^4 + 64*a^2*b + 77*b^3 + 80*(64*a^2*b + 77*b^3
)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + (171*(64*a^2*b + 77*b^3)*cosh(d*x + c)
^17 + 1088*(64*a^2*b + 77*b^3)*cosh(d*x + c)^15 + 2940*(64*a^2*b + 77*b^3)*
cosh(d*x + c)^13 + 4368*(64*a^2*b + 77*b^3)*cosh(d*x + c)^11 + 3850*(64*a^2
*b + 77*b^3)*cosh(d*x + c)^9 + 2016*(64*a^2*b + 77*b^3)*cosh(d*x + c)^7 + 5
88*(64*a^2*b + 77*b^3)*cosh(d*x + c)^5 + 80*(64*a^2*b + 77*b^3)*cosh(d*x +
c)^3 + 3*(64*a^2*b + 77*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + (19*(64*a^2*b
 + 77*b^3)*cosh(d*x + c)^18 + 136*(64*a^2*b + 77*b^3)*cosh(d*x + c)^16 + 42
0*(64*a^2*b + 77*b^3)*cosh(d*x + c)^14 + 728*(64*a^2*b + 77*b^3)*cosh(d*x +
 c)^12 + 770*(64*a^2*b + 77*b^3)*cosh(d*x + c)^10 + 504*(64*a^2*b + 77*b^3)
```

```
*cosh(d*x + c)^8 + 196*(64*a^2*b + 77*b^3)*cosh(d*x + c)^6 + 40*(64*a^2*b +
 77*b^3)*cosh(d*x + c)^4 + 3*(64*a^2*b + 77*b^3)*cosh(d*x + c)^2)*sinh(d*x
+ c))*arctan(cosh(d*x + c) + sinh(d*x + c)) + 2*(440*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*cosh(d*x + c)^21 - 400*(a^3 + 57*a^2*b + 111*a*b^2 + 55*b^3)*cosh
(d*x + c)^19 - 45*(424*a^3 + 4440*a^2*b + 15960*a*b^2 + 5599*b^3)*cosh(d*x
+ c)^17 - 120*(712*a^3 + 4840*a^2*b + 26584*a*b^2 + 5665*b^3)*cosh(d*x + c)
^15 - 21*(9040*a^3 + 36400*a^2*b + 344944*a*b^2 + 45265*b^3)*cosh(d*x + c)^
13 - 18*(14000*a^3 + 18800*a^2*b + 542672*a*b^2 + 20405*b^3)*cosh(d*x + c)^
11 - 15*(14000*a^3 - 18800*a^2*b + 542672*a*b^2 - 20405*b^3)*cosh(d*x + c)^
9 - 12*(9040*a^3 - 36400*a^2*b + 344944*a*b^2 - 45265*b^3)*cosh(d*x + c)^7
- 45*(712*a^3 - 4840*a^2*b + 26584*a*b^2 - 5665*b^3)*cosh(d*x + c)^5 - 10*(
424*a^3 - 4440*a^2*b + 15960*a*b^2 - 5599*b^3)*cosh(d*x + c)^3 - 40*(a^3 -
57*a^2*b + 111*a*b^2 - 55*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x +
c)^19 + 19*d*cosh(d*x + c)*sinh(d*x + c)^18 + d*sinh(d*x + c)^19 + 8*d*cosh
(d*x + c)^17 + (171*d*cosh(d*x + c)^2 + 8*d)*sinh(d*x + c)^17 + 17*(57*d*co
sh(d*x + c)^3 + 8*d*cosh(d*x + c))*sinh(d*x + c)^16 + 28*d*cosh(d*x + c)^15
 + 4*(969*d*cosh(d*x + c)^4 + 272*d*cosh(d*x + c)^2 + 7*d)*sinh(d*x + c)^15
 + 4*(2907*d*cosh(d*x + c)^5 + 1360*d*cosh(d*x + c)^3 + 105*d*cosh(d*x + c)
)*sinh(d*x + c)^14 + 56*d*cosh(d*x + c)^13 + 28*(969*d*cosh(d*x + c)^6 + 68
0*d*cosh(d*x + c)^4 + 105*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^13 + 52*(9
69*d*cosh(d*x + c)^7 + 952*d*cosh(d*x + c)^5 + 245*d*cosh(d*x + c)^3 + 14*d
*cosh(d*x + c))*sinh(d*x + c)^12 + 70*d*cosh(d*x + c)^11 + 2*(37791*d*cosh(
d*x + c)^8 + 49504*d*cosh(d*x + c)^6 + 19110*d*cosh(d*x + c)^4 + 2184*d*cos
h(d*x + c)^2 + 35*d)*sinh(d*x + c)^11 + 22*(4199*d*cosh(d*x + c)^9 + 7072*d
*cosh(d*x + c)^7 + 3822*d*cosh(d*x + c)^5 + 728*d*cosh(d*x + c)^3 + 35*d*co
sh(d*x + c))*sinh(d*x + c)^10 + 56*d*cosh(d*x + c)^9 + 2*(46189*d*cosh(d*x
+ c)^10 + 97240*d*cosh(d*x + c)^8 + 70070*d*cosh(d*x + c)^6 + 20020*d*cosh(
d*x + c)^4 + 1925*d*cosh(d*x + c)^2 + 28*d)*sinh(d*x + c)^9 + 2*(37791*d*co
sh(d*x + c)^11 + 97240*d*cosh(d*x + c)^9 + 90090*d*cosh(d*x + c)^7 + 36036*
d*cosh(d*x + c)^5 + 5775*d*cosh(d*x + c)^3 + 252*d*cosh(d*x + c))*sinh(d*x
+ c)^8 + 28*d*cosh(d*x + c)^7 + 4*(12597*d*cosh(d*x + c)^12 + 38896*d*cosh(
d*x + c)^10 + 45045*d*cosh(d*x + c)^8 + 24024*d*cosh(d*x + c)^6 + 5775*d*co
sh(d*x + c)^4 + 504*d*cosh(d*x + c)^2 + 7*d)*sinh(d*x + c)^7 + 28*(969*d*co
sh(d*x + c)^13 + 3536*d*cosh(d*x + c)^11 + 5005*d*cosh(d*x + c)^9 + 3432*d*
cosh(d*x + c)^7 + 1155*d*cosh(d*x + c)^5 + 168*d*cosh(d*x + c)^3 + 7*d*cosh
(d*x + c))*sinh(d*x + c)^6 + 8*d*cosh(d*x + c)^5 + 4*(2907*d*cosh(d*x + c)^
14 + 12376*d*cosh(d*x + c)^12 + 21021*d*cosh(d*x + c)^10 + 18018*d*cosh(d*x
 + c)^8 + 8085*d*cosh(d*x + c)^6 + 1764*d*cosh(d*x + c)^4 + 147*d*cosh(d*x
+ c)^2 + 2*d)*sinh(d*x + c)^5 + 4*(969*d*cosh(d*x + c)^15 + 4760*d*cosh(d*x
 + c)^13 + 9555*d*cosh(d*x + c)^11 + 10010*d*cosh(d*x + c)^9 + 5775*d*cosh(
d*x + c)^7 + 1764*d*cosh(d*x + c)^5 + 245*d*cosh(d*x + c)^3 + 10*d*cosh(d*x
 + c))*sinh(d*x + c)^4 + d*cosh(d*x + c)^3 + (969*d*cosh(d*x + c)^16 + 5440
*d*cosh(d*x + c)^14 + 12740*d*cosh(d*x + c)^12 + 16016*d*cosh(d*x + c)^10 +
 11550*d*cosh(d*x + c)^8 + 4704*d*cosh(d*x + c)^6 + 980*d*cosh(d*x + c)^4 +
 80*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^3 + (171*d*cosh(d*x + c)^17 + 1088
*d*cosh(d*x + c)^15 + 2940*d*cosh(d*x + c)^13 + 4368*d*cosh(d*x + c)^11 + 3
850*d*cosh(d*x + c)^9 + 2016*d*cosh(d*x + c)^7 + 588*d*cosh(d*x + c)^5 + 80
*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + (19*d*cosh(d*x +
c)^18 + 136*d*cosh(d*x + c)^16 + 420*d*cosh(d*x + c)^14 + 728*d*cosh(d*x +
c)^12 + 770*d*cosh(d*x + c)^10 + 504*d*cosh(d*x + c)^8 + 196*d*cosh(d*x + c
)^6 + 40*d*cosh(d*x + c)^4 + 3*d*cosh(d*x + c)^2)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3*(a+b*tanh(d*x+c)**3)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 3.95178, size = 811, normalized size = 2.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 1/960*(225*(64*a^2*b*e^c + 77*b^3*e^c)*arctan(e^(d*x + c))*e^(-c) - 40*(9*a
^3*e^(2*d*x + 2*c) - 81*a^2*b*e^(2*d*x + 2*c) + 135*a*b^2*e^(2*d*x + 2*c) -
 63*b^3*e^(2*d*x + 2*c) - a^3 + 3*a^2*b - 3*a*b^2 + b^3)*e^(-3*d*x - 3*c) +
 40*(a^3*e^(3*d*x + 66*c) + 3*a^2*b*e^(3*d*x + 66*c) + 3*a*b^2*e^(3*d*x + 6
6*c) + b^3*e^(3*d*x + 66*c) - 9*a^3*e^(d*x + 64*c) - 81*a^2*b*e^(d*x + 64*c
) - 135*a*b^2*e^(d*x + 64*c) - 63*b^3*e^(d*x + 64*c))*e^(-63*c) - (2880*a^2
*b*e^(15*d*x + 15*c) + 34560*a*b^2*e^(15*d*x + 15*c) + 11475*b^3*e^(15*d*x
+ 15*c) + 14400*a^2*b*e^(13*d*x + 13*c) + 211200*a*b^2*e^(13*d*x + 13*c) +
36775*b^3*e^(13*d*x + 13*c) + 25920*a^2*b*e^(11*d*x + 11*c) + 590592*a*b^2*
e^(11*d*x + 11*c) + 67715*b^3*e^(11*d*x + 11*c) + 14400*a^2*b*e^(9*d*x + 9*
c) + 957696*a*b^2*e^(9*d*x + 9*c) + 27055*b^3*e^(9*d*x + 9*c) - 14400*a^2*b
*e^(7*d*x + 7*c) + 957696*a*b^2*e^(7*d*x + 7*c) - 27055*b^3*e^(7*d*x + 7*c)
 - 25920*a^2*b*e^(5*d*x + 5*c) + 590592*a*b^2*e^(5*d*x + 5*c) - 67715*b^3*e
^(5*d*x + 5*c) - 14400*a^2*b*e^(3*d*x + 3*c) + 211200*a*b^2*e^(3*d*x + 3*c)
 - 36775*b^3*e^(3*d*x + 3*c) - 2880*a^2*b*e^(d*x + c) + 34560*a*b^2*e^(d*x
+ c) - 11475*b^3*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^8)/d
```

3.67 $\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=220

$$\frac{b\left(3a^2+4b^2\right)\tanh^2(c+dx)}{2d}-\frac{b\left(6a^2+5b^2\right)\log(\cosh(c+dx))}{d}+\frac{\sinh(c+dx)\cosh(c+dx)\left(b\left(3a^2+b^2\right)\tanh(c+dx\right.}{2d}$$

```
[Out] -(a*(a^2 + 21*b^2)*x)/2 - (b*(6*a^2 + 5*b^2)*Log[Cosh[c + d*x]])/d + (9*a*b
^2*Tanh[c + d*x])/d + (b*(3*a^2 + 4*b^2)*Tanh[c + d*x]^2)/(2*d) + (2*a*b^2*
Tanh[c + d*x]^3)/d + (3*b^3*Tanh[c + d*x]^4)/(4*d) + (3*a*b^2*Tanh[c + d*x]
^5)/(5*d) + (b^3*Tanh[c + d*x]^6)/(3*d) + (b^3*Tanh[c + d*x]^8)/(8*d) + (Co
sh[c + d*x]*Sinh[c + d*x]*(a*(a^2 + 3*b^2) + b*(3*a^2 + b^2)*Tanh[c + d*x])
)/(2*d)
```

Rubi [A] time = 0.314838, antiderivative size = 241, normalized size of antiderivative = 1.1, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3663, 1804, 1802, 633, 31}

$$\frac{b\left(3a^2+4b^2\right)\tanh^2(c+dx)}{2d}+\frac{a\left(a^2+21b^2\right)\tanh(c+dx)}{2d}+\frac{\sinh^2(c+dx)\left(a\left(a^2+3b^2\right)\tanh(c+dx)+b\left(3a^2+b^2\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] ((a + b)^2*(a + 10*b)*Log[1 - Tanh[c + d*x]])/(4*d) - ((a - 10*b)*(a - b)^2
*Log[1 + Tanh[c + d*x]])/(4*d) + (a*(a^2 + 21*b^2)*Tanh[c + d*x])/(2*d) + (
b*(3*a^2 + 4*b^2)*Tanh[c + d*x]^2)/(2*d) + (2*a*b^2*Tanh[c + d*x]^3)/d + (3
*b^3*Tanh[c + d*x]^4)/(4*d) + (3*a*b^2*Tanh[c + d*x]^5)/(5*d) + (b^3*Tanh[c
 + d*x]^6)/(3*d) + (b^3*Tanh[c + d*x]^8)/(8*d) + (Sinh[c + d*x]^2*(b*(3*a^2
 + b^2) + a*(a^2 + 3*b^2)*Tanh[c + d*x]))/(2*d)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+bx^3\right)^3}{\left(1-x^2\right)^2}dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\sinh^2(c+dx)\left(b\left(3a^2+b^2\right)+a\left(a^2+3b^2\right)\tanh(c+dx)\right)}{2d} + \frac{\text{Subst}\left(\int \frac{x(-2b}{}\right.}{}$$

$$= \frac{\sinh^2(c+dx)\left(b\left(3a^2+b^2\right)+a\left(a^2+3b^2\right)\tanh(c+dx)\right)}{2d} + \frac{\text{Subst}\left(\int \left(a^3 +\right.\right.}{}$$

$$= \frac{a\left(a^2+21b^2\right)\tanh(c+dx)}{2d} + \frac{b\left(3a^2+4b^2\right)\tanh^2(c+dx)}{2d} + \frac{2ab^2\tanh^3(c}{d}$$

$$= \frac{a\left(a^2+21b^2\right)\tanh(c+dx)}{2d} + \frac{b\left(3a^2+4b^2\right)\tanh^2(c+dx)}{2d} + \frac{2ab^2\tanh^3(c}{d}$$

$$= \frac{(a+b)^2(a+10b)\log(1-\tanh(c+dx))}{4d} - \frac{(a-10b)(a-b)^2\log(1+\tanh(c}{4d}$$

Mathematica [A] time = 6.2196, size = 244, normalized size = 1.11

$$-\frac{a\left(a^2+21b^2\right)(c+dx)}{2d} + \frac{a\left(a^2+3b^2\right)\sinh(2(c+dx))}{4d} + \frac{b\left(3a^2+b^2\right)\cosh(2(c+dx))}{4d} - \frac{b\left(3a^2+10b^2\right)\text{sech}^2(c+dx)}{2d} +$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] -(a*(a^2 + 21*b^2)*(c + d*x))/(2*d) + (b*(3*a^2 + b^2)*Cosh[2*(c + d*x)])/(
4*d) + ((-6*a^2*b - 5*b^3)*Log[Cosh[c + d*x]])/d - (b*(3*a^2 + 10*b^2)*Sech
[c + d*x]^2)/(2*d) + (5*b^3*Sech[c + d*x]^4)/(2*d) - (5*b^3*Sech[c + d*x]^6
)/(6*d) + (b^3*Sech[c + d*x]^8)/(8*d) + (a*(a^2 + 3*b^2)*Sinh[2*(c + d*x)])
/(4*d) + (58*a*b^2*Tanh[c + d*x])/(5*d) - (16*a*b^2*Sech[c + d*x]^2*Tanh[c
+ d*x])/(5*d) + (3*a*b^2*Sech[c + d*x]^4*Tanh[c + d*x])/(5*d)
```

Maple [A] time = 0.073, size = 289, normalized size = 1.3

$$\frac{a^3\cosh(dx+c)\sinh(dx+c)}{2d} - \frac{a^3x}{2} - \frac{a^3c}{2d} + \frac{3a^2b(\sinh(dx+c))^4}{2d(\cosh(dx+c))^2} - 6\frac{a^2b\ln(\cosh(dx+c))}{d} + 3\frac{a^2b(\tanh(dx+c))^2}{d} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x)
```

```
[Out] 1/2/d*a^3*cosh(d*x+c)*sinh(d*x+c)-1/2*a^3*x-1/2/d*a^3*c+3/2/d*a^2*b*sinh(d*
x+c)^4/cosh(d*x+c)^2-6/d*a^2*b*ln(cosh(d*x+c))+3*a^2*b*tanh(d*x+c)^2/d+3/2/
d*a*b^2*sinh(d*x+c)^7/cosh(d*x+c)^5-21/2*a*b^2*x-21/2/d*a*b^2*c+21/2*a*b^2*
tanh(d*x+c)/d+7/2*a*b^2*tanh(d*x+c)^3/d+21/10*a*b^2*tanh(d*x+c)^5/d+1/2/d*b
^3*sinh(d*x+c)^10/cosh(d*x+c)^8-5/d*b^3*ln(cosh(d*x+c))+5/2/d*b^3*tanh(d*x+
c)^2+5/4*b^3*tanh(d*x+c)^4/d+5/6*b^3*tanh(d*x+c)^6/d+5/8*b^3*tanh(d*x+c)^8/
d
```

Maxima [B] time = 1.75078, size = 734, normalized size = 3.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] -1/8*a^3*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/40*a*b^2*(420*(
d*x + c)/d + 15*e^(-2*d*x - 2*c)/d - (1003*e^(-2*d*x - 2*c) + 3350*e^(-4*d*
x - 4*c) + 5590*e^(-6*d*x - 6*c) + 3915*e^(-8*d*x - 8*c) + 1455*e^(-10*d*x
- 10*c) + 15)/(d*(e^(-2*d*x - 2*c) + 5*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*
c) + 10*e^(-8*d*x - 8*c) + 5*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c)))) - 1
/24*b^3*(120*(d*x + c)/d - 3*e^(-2*d*x - 2*c)/d + 120*log(e^(-2*d*x - 2*c)
+ 1)/d - (24*e^(-2*d*x - 2*c) - 396*e^(-4*d*x - 4*c) - 1752*e^(-6*d*x - 6*c
) - 4430*e^(-8*d*x - 8*c) - 5464*e^(-10*d*x - 10*c) - 4556*e^(-12*d*x - 12*
c) - 1896*e^(-14*d*x - 14*c) - 477*e^(-16*d*x - 16*c) + 3)/(d*(e^(-2*d*x -
2*c) + 8*e^(-4*d*x - 4*c) + 28*e^(-6*d*x - 6*c) + 56*e^(-8*d*x - 8*c) + 70*
e^(-10*d*x - 10*c) + 56*e^(-12*d*x - 12*c) + 28*e^(-14*d*x - 14*c) + 8*e^(-
16*d*x - 16*c) + e^(-18*d*x - 18*c)))) - 3/8*a^2*b*(16*(d*x + c)/d - e^(-2*
d*x - 2*c)/d + 16*log(e^(-2*d*x - 2*c) + 1)/d - (2*e^(-2*d*x - 2*c) - 15*e^
(-4*d*x - 4*c) + 1)/(d*(e^(-2*d*x - 2*c) + 2*e^(-4*d*x - 4*c) + e^(-6*d*x -
 6*c))))
```

Fricas [B] time = 4.05151, size = 25608, normalized size = 116.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/120*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^20 + 300*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^19 + 15*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*sinh(d*x + c)^20 + 60*(2*a^3 + 6*a^2*b + 6*a*b^2 + 2*b^3 - (
a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^18 + 30*(4*a^3 + 12*
a^2*b + 12*a*b^2 + 4*b^3 - 2*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x + 95*
(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^18 + 180*(95
*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + 6*(2*a^3 + 6*a^2*b + 6*a
*b^2 + 2*b^3 - (a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c))*sin
h(d*x + c)^17 + 15*(27*a^3 + 39*a^2*b - 207*a*b^2 - 131*b^3 - 32*(a^3 - 12*
a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^16 + 15*(4845*(a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 27*a^3 + 39*a^2*b - 207*a*b^2 - 131*b^3
- 32*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x + 612*(2*a^3 + 6*a^2*b + 6*a*
```

b^2 + 2*b^3 - (a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^2)*si
nh(d*x + c)^16 + 240*(969*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 +
 204*(2*a^3 + 6*a^2*b + 6*a*b^2 + 2*b^3 - (a^3 - 12*a^2*b + 21*a*b^2 - 10*b
^3)*d*x)*cosh(d*x + c)^3 + (27*a^3 + 39*a^2*b - 207*a*b^2 - 131*b^3 - 32*(a
^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^15 + 2
40*(3*a^3 - 6*a^2*b - 93*a*b^2 - 36*b^3 - 7*(a^3 - 12*a^2*b + 21*a*b^2 - 10
*b^3)*d*x)*cosh(d*x + c)^14 + 120*(4845*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos
h(d*x + c)^6 + 1530*(2*a^3 + 6*a^2*b + 6*a*b^2 + 2*b^3 - (a^3 - 12*a^2*b +
21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^4 + 6*a^3 - 12*a^2*b - 186*a*b^2 - 72
b^3 - 14(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x + 15*(27*a^3 + 39*a^2*b
- 207*a*b^2 - 131*b^3 - 32*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d
*x + c)^2)*sinh(d*x + c)^14 + 240*(4845*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos
h(d*x + c)^7 + 2142*(2*a^3 + 6*a^2*b + 6*a*b^2 + 2*b^3 - (a^3 - 12*a^2*b +
21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^5 + 35*(27*a^3 + 39*a^2*b - 207*a*b^2
 - 131*b^3 - 32*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^3 +
 14*(3*a^3 - 6*a^2*b - 93*a*b^2 - 36*b^3 - 7*(a^3 - 12*a^2*b + 21*a*b^2 - 1
0*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^13 + 10*(63*a^3 - 639*a^2*b - 6195
*a*b^2 - 2173*b^3 - 336*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x
+ c)^12 + 10*(188955*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 1113
84*(2*a^3 + 6*a^2*b + 6*a*b^2 + 2*b^3 - (a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3
)*d*x)*cosh(d*x + c)^6 + 2730*(27*a^3 + 39*a^2*b - 207*a*b^2 - 131*b^3 - 32
*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^4 + 63*a^3 - 639*a
^2*b - 6195*a*b^2 - 2173*b^3 - 336*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x
 + 2184*(3*a^3 - 6*a^2*b - 93*a*b^2 - 36*b^3 - 7*(a^3 - 12*a^2*b + 21*a*b^2
 - 10*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^12 + 120*(20995*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*cosh(d*x + c)^9 + 15912*(2*a^3 + 6*a^2*b + 6*a*b^2 + 2*
b^3 - (a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^7 + 546*(27*a
^3 + 39*a^2*b - 207*a*b^2 - 131*b^3 - 32*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^
3)*d*x)*cosh(d*x + c)^5 + 728*(3*a^3 - 6*a^2*b - 93*a*b^2 - 36*b^3 - 7*(a^3
 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^3 + (63*a^3 - 639*a^2*b
 - 6195*a*b^2 - 2173*b^3 - 336*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*co
sh(d*x + c))*sinh(d*x + c)^11 - 40*(234*a^2*b + 2436*a*b^2 + 662*b^3 + 105*
(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^10 + 20*(138567*(a^
3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^10 + 131274*(2*a^3 + 6*a^2*b + 6
*a*b^2 + 2*b^3 - (a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^8
+ 6006*(27*a^3 + 39*a^2*b - 207*a*b^2 - 131*b^3 - 32*(a^3 - 12*a^2*b + 21*a
*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^6 + 12012*(3*a^3 - 6*a^2*b - 93*a*b^2 - 3
6*b^3 - 7*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^4 - 468*a
^2*b - 4872*a*b^2 - 1324*b^3 - 210*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x
 + 33*(63*a^3 - 639*a^2*b - 6195*a*b^2 - 2173*b^3 - 336*(a^3 - 12*a^2*b + 2
1*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 40*(62985*(a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^11 + 72930*(2*a^3 + 6*a^2*b + 6*a*b
^2 + 2*b^3 - (a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^9 + 42
90*(27*a^3 + 39*a^2*b - 207*a*b^2 - 131*b^3 - 32*(a^3 - 12*a^2*b + 21*a*b^2
 - 10*b^3)*d*x)*cosh(d*x + c)^7 + 12012*(3*a^3 - 6*a^2*b - 93*a*b^2 - 36*b^
3 - 7*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^5 + 55*(63*a^
3 - 639*a^2*b - 6195*a*b^2 - 2173*b^3 - 336*(a^3 - 12*a^2*b + 21*a*b^2 - 10
*b^3)*d*x)*cosh(d*x + c)^3 - 10*(234*a^2*b + 2436*a*b^2 + 662*b^3 + 105*(a^
3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^9 - 2*(
315*a^3 + 3195*a^2*b + 46977*a*b^2 + 10865*b^3 + 1680*(a^3 - 12*a^2*b + 21*
a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^8 + 2*(944775*(a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*cosh(d*x + c)^12 + 1312740*(2*a^3 + 6*a^2*b + 6*a*b^2 + 2*b^3 - (a^3
- 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^10 + 96525*(27*a^3 + 39*
a^2*b - 207*a*b^2 - 131*b^3 - 32*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*
cosh(d*x + c)^8 + 360360*(3*a^3 - 6*a^2*b - 93*a*b^2 - 36*b^3 - 7*(a^3 - 12
*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^6 + 2475*(63*a^3 - 639*a^2*b
 - 6195*a*b^2 - 2173*b^3 - 336*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*co
sh(d*x + c)^4 - 315*a^3 - 3195*a^2*b - 46977*a*b^2 - 10865*b^3 - 1680*(a^3
- 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x - 900*(234*a^2*b + 2436*a*b^2 + 662*b^3

+ 105*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x
+ c)^8 + 16*(72675*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^13 + 11934
0*(2*a^3 + 6*a^2*b + 6*a*b^2 + 2*b^3 - (a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)
*d*x)*cosh(d*x + c)^11 + 10725*(27*a^3 + 39*a^2*b - 207*a*b^2 - 131*b^3 - 3
2*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^9 + 51480*(3*a^3
- 6*a^2*b - 93*a*b^2 - 36*b^3 - 7*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)
*cosh(d*x + c)^7 + 495*(63*a^3 - 639*a^2*b - 6195*a*b^2 - 2173*b^3 - 336*(a
^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^5 - 300*(234*a^2*b +
2436*a*b^2 + 662*b^3 + 105*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d
*x + c)^3 - (315*a^3 + 3195*a^2*b + 46977*a*b^2 + 10865*b^3 + 1680*(a^3 - 1
2*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 - 48*(15*a
^3 + 30*a^2*b + 1159*a*b^2 + 180*b^3 + 35*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b
^3)*d*x)*cosh(d*x + c)^6 + 8*(72675*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*
x + c)^14 + 139230*(2*a^3 + 6*a^2*b + 6*a*b^2 + 2*b^3 - (a^3 - 12*a^2*b + 2
1*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^12 + 15015*(27*a^3 + 39*a^2*b - 207*a*
b^2 - 131*b^3 - 32*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^
10 + 90090*(3*a^3 - 6*a^2*b - 93*a*b^2 - 36*b^3 - 7*(a^3 - 12*a^2*b + 21*a*
b^2 - 10*b^3)*d*x)*cosh(d*x + c)^8 + 1155*(63*a^3 - 639*a^2*b - 6195*a*b^2
- 2173*b^3 - 336*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^6
- 1050*(234*a^2*b + 2436*a*b^2 + 662*b^3 + 105*(a^3 - 12*a^2*b + 21*a*b^2 -
10*b^3)*d*x)*cosh(d*x + c)^4 - 90*a^3 - 180*a^2*b - 6954*a*b^2 - 1080*b^3
- 210*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x - 7*(315*a^3 + 3195*a^2*b +
46977*a*b^2 + 10865*b^3 + 1680*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*co
sh(d*x + c)^2)*sinh(d*x + c)^6 + 16*(14535*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*
cosh(d*x + c)^15 + 32130*(2*a^3 + 6*a^2*b + 6*a*b^2 + 2*b^3 - (a^3 - 12*a^2
*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^13 + 4095*(27*a^3 + 39*a^2*b - 2
07*a*b^2 - 131*b^3 - 32*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x
+ c)^11 + 30030*(3*a^3 - 6*a^2*b - 93*a*b^2 - 36*b^3 - 7*(a^3 - 12*a^2*b +
21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^9 + 495*(63*a^3 - 639*a^2*b - 6195*a*
b^2 - 2173*b^3 - 336*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c
)^7 - 630*(234*a^2*b + 2436*a*b^2 + 662*b^3 + 105*(a^3 - 12*a^2*b + 21*a*b^
2 - 10*b^3)*d*x)*cosh(d*x + c)^5 - 7*(315*a^3 + 3195*a^2*b + 46977*a*b^2 +
10865*b^3 + 1680*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^3
- 18*(15*a^3 + 30*a^2*b + 1159*a*b^2 + 180*b^3 + 35*(a^3 - 12*a^2*b + 21*a*
b^2 - 10*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 3*(135*a^3 - 195*a^2*b
+ 6389*a*b^2 + 655*b^3 + 160*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh
(d*x + c)^4 + (72675*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^16 + 183
600*(2*a^3 + 6*a^2*b + 6*a*b^2 + 2*b^3 - (a^3 - 12*a^2*b + 21*a*b^2 - 10*b^
3)*d*x)*cosh(d*x + c)^14 + 27300*(27*a^3 + 39*a^2*b - 207*a*b^2 - 131*b^3 -
32*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^12 + 240240*(3*
a^3 - 6*a^2*b - 93*a*b^2 - 36*b^3 - 7*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*
d*x)*cosh(d*x + c)^10 + 4950*(63*a^3 - 639*a^2*b - 6195*a*b^2 - 2173*b^3 -
336*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^8 - 8400*(234*a
^2*b + 2436*a*b^2 + 662*b^3 + 105*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)
*cosh(d*x + c)^6 - 140*(315*a^3 + 3195*a^2*b + 46977*a*b^2 + 10865*b^3 + 16
80*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^4 - 405*a^3 + 58
5*a^2*b - 19167*a*b^2 - 1965*b^3 - 480*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)
*d*x - 720*(15*a^3 + 30*a^2*b + 1159*a*b^2 + 180*b^3 + 35*(a^3 - 12*a^2*b +
21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(4275*(a^3 +
3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^17 + 12240*(2*a^3 + 6*a^2*b + 6*a*b^
2 + 2*b^3 - (a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^15 + 21
00*(27*a^3 + 39*a^2*b - 207*a*b^2 - 131*b^3 - 32*(a^3 - 12*a^2*b + 21*a*b^2
- 10*b^3)*d*x)*cosh(d*x + c)^13 + 21840*(3*a^3 - 6*a^2*b - 93*a*b^2 - 36*b
^3 - 7*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^11 + 550*(63
*a^3 - 639*a^2*b - 6195*a*b^2 - 2173*b^3 - 336*(a^3 - 12*a^2*b + 21*a*b^2 -
10*b^3)*d*x)*cosh(d*x + c)^9 - 1200*(234*a^2*b + 2436*a*b^2 + 662*b^3 + 10
5*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^7 - 28*(315*a^3 +
3195*a^2*b + 46977*a*b^2 + 10865*b^3 + 1680*(a^3 - 12*a^2*b + 21*a*b^2 - 1
0*b^3)*d*x)*cosh(d*x + c)^5 - 240*(15*a^3 + 30*a^2*b + 1159*a*b^2 + 180*b^3

+ 35*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^3 - 3*(135*a^
3 - 195*a^2*b + 6389*a*b^2 + 655*b^3 + 160*(a^3 - 12*a^2*b + 21*a*b^2 - 10*
b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 15*a^3 + 45*a^2*b - 45*a*b^2 + 1
5*b^3 - 12*(10*a^3 - 30*a^2*b + 262*a*b^2 - 10*b^3 + 5*(a^3 - 12*a^2*b + 21
*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^2 + 2*(1425*(a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*cosh(d*x + c)^18 + 4590*(2*a^3 + 6*a^2*b + 6*a*b^2 + 2*b^3 - (a^3 - 12
*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^16 + 900*(27*a^3 + 39*a^2*b
- 207*a*b^2 - 131*b^3 - 32*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d
x + c)^14 + 10920(3*a^3 - 6*a^2*b - 93*a*b^2 - 36*b^3 - 7*(a^3 - 12*a^2*b
+ 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^12 + 330*(63*a^3 - 639*a^2*b - 619
5*a*b^2 - 2173*b^3 - 336*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x
+ c)^10 - 900*(234*a^2*b + 2436*a*b^2 + 662*b^3 + 105*(a^3 - 12*a^2*b + 21
*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^8 - 28*(315*a^3 + 3195*a^2*b + 46977*a*
b^2 + 10865*b^3 + 1680*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x +
c)^6 - 360*(15*a^3 + 30*a^2*b + 1159*a*b^2 + 180*b^3 + 35*(a^3 - 12*a^2*b
+ 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^4 - 60*a^3 + 180*a^2*b - 1572*a*b^2
+ 60*b^3 - 30*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x - 9*(135*a^3 - 195*
a^2*b + 6389*a*b^2 + 655*b^3 + 160*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x
)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 120*((6*a^2*b + 5*b^3)*cosh(d*x + c)^1
8 + 18*(6*a^2*b + 5*b^3)*cosh(d*x + c)*sinh(d*x + c)^17 + (6*a^2*b + 5*b^3)
*sinh(d*x + c)^18 + 8*(6*a^2*b + 5*b^3)*cosh(d*x + c)^16 + (48*a^2*b + 40*b
^3 + 153*(6*a^2*b + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^16 + 16*(51*(6*a^
2*b + 5*b^3)*cosh(d*x + c)^3 + 8*(6*a^2*b + 5*b^3)*cosh(d*x + c))*sinh(d*x
+ c)^15 + 28*(6*a^2*b + 5*b^3)*cosh(d*x + c)^14 + 4*(765*(6*a^2*b + 5*b^3)*
cosh(d*x + c)^4 + 42*a^2*b + 35*b^3 + 240*(6*a^2*b + 5*b^3)*cosh(d*x + c)^2
)*sinh(d*x + c)^14 + 56*(153*(6*a^2*b + 5*b^3)*cosh(d*x + c)^5 + 80*(6*a^2*
b + 5*b^3)*cosh(d*x + c)^3 + 7*(6*a^2*b + 5*b^3)*cosh(d*x + c))*sinh(d*x +
c)^13 + 56*(6*a^2*b + 5*b^3)*cosh(d*x + c)^12 + 28*(663*(6*a^2*b + 5*b^3)*c
osh(d*x + c)^6 + 520*(6*a^2*b + 5*b^3)*cosh(d*x + c)^4 + 12*a^2*b + 10*b^3
+ 91*(6*a^2*b + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^12 + 16*(1989*(6*a^2*
b + 5*b^3)*cosh(d*x + c)^7 + 2184*(6*a^2*b + 5*b^3)*cosh(d*x + c)^5 + 637*(
6*a^2*b + 5*b^3)*cosh(d*x + c)^3 + 42*(6*a^2*b + 5*b^3)*cosh(d*x + c))*sinh
(d*x + c)^11 + 70*(6*a^2*b + 5*b^3)*cosh(d*x + c)^10 + 2*(21879*(6*a^2*b +
5*b^3)*cosh(d*x + c)^8 + 32032*(6*a^2*b + 5*b^3)*cosh(d*x + c)^6 + 14014*(6
*a^2*b + 5*b^3)*cosh(d*x + c)^4 + 210*a^2*b + 175*b^3 + 1848*(6*a^2*b + 5*b
^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 4*(12155*(6*a^2*b + 5*b^3)*cosh(d*x
+ c)^9 + 22880*(6*a^2*b + 5*b^3)*cosh(d*x + c)^7 + 14014*(6*a^2*b + 5*b^3)
*cosh(d*x + c)^5 + 3080*(6*a^2*b + 5*b^3)*cosh(d*x + c)^3 + 175*(6*a^2*b +
5*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 + 56*(6*a^2*b + 5*b^3)*cosh(d*x + c)^
8 + 2*(21879*(6*a^2*b + 5*b^3)*cosh(d*x + c)^10 + 51480*(6*a^2*b + 5*b^3)*c
osh(d*x + c)^8 + 42042*(6*a^2*b + 5*b^3)*cosh(d*x + c)^6 + 13860*(6*a^2*b +
5*b^3)*cosh(d*x + c)^4 + 168*a^2*b + 140*b^3 + 1575*(6*a^2*b + 5*b^3)*cosh
(d*x + c)^2)*sinh(d*x + c)^8 + 16*(1989*(6*a^2*b + 5*b^3)*cosh(d*x + c)^11
+ 5720*(6*a^2*b + 5*b^3)*cosh(d*x + c)^9 + 6006*(6*a^2*b + 5*b^3)*cosh(d*x
+ c)^7 + 2772*(6*a^2*b + 5*b^3)*cosh(d*x + c)^5 + 525*(6*a^2*b + 5*b^3)*cos
h(d*x + c)^3 + 28*(6*a^2*b + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 28*(6*
a^2*b + 5*b^3)*cosh(d*x + c)^6 + 28*(663*(6*a^2*b + 5*b^3)*cosh(d*x + c)^12
+ 2288*(6*a^2*b + 5*b^3)*cosh(d*x + c)^10 + 3003*(6*a^2*b + 5*b^3)*cosh(d*
x + c)^8 + 1848*(6*a^2*b + 5*b^3)*cosh(d*x + c)^6 + 525*(6*a^2*b + 5*b^3)*c
osh(d*x + c)^4 + 6*a^2*b + 5*b^3 + 56*(6*a^2*b + 5*b^3)*cosh(d*x + c)^2)*si
nh(d*x + c)^6 + 56*(153*(6*a^2*b + 5*b^3)*cosh(d*x + c)^13 + 624*(6*a^2*b +
5*b^3)*cosh(d*x + c)^11 + 1001*(6*a^2*b + 5*b^3)*cosh(d*x + c)^9 + 792*(6*
a^2*b + 5*b^3)*cosh(d*x + c)^7 + 315*(6*a^2*b + 5*b^3)*cosh(d*x + c)^5 + 56
*(6*a^2*b + 5*b^3)*cosh(d*x + c)^3 + 3*(6*a^2*b + 5*b^3)*cosh(d*x + c))*sin
h(d*x + c)^5 + 8*(6*a^2*b + 5*b^3)*cosh(d*x + c)^4 + 4*(765*(6*a^2*b + 5*b^
3)*cosh(d*x + c)^14 + 3640*(6*a^2*b + 5*b^3)*cosh(d*x + c)^12 + 7007*(6*a^2
*b + 5*b^3)*cosh(d*x + c)^10 + 6930*(6*a^2*b + 5*b^3)*cosh(d*x + c)^8 + 367
5*(6*a^2*b + 5*b^3)*cosh(d*x + c)^6 + 980*(6*a^2*b + 5*b^3)*cosh(d*x + c)^4
+ 12*a^2*b + 10*b^3 + 105*(6*a^2*b + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)

```
^4 + 16*(51*(6*a^2*b + 5*b^3)*cosh(d*x + c)^15 + 280*(6*a^2*b + 5*b^3)*cosh
(d*x + c)^13 + 637*(6*a^2*b + 5*b^3)*cosh(d*x + c)^11 + 770*(6*a^2*b + 5*b^
3)*cosh(d*x + c)^9 + 525*(6*a^2*b + 5*b^3)*cosh(d*x + c)^7 + 196*(6*a^2*b +
 5*b^3)*cosh(d*x + c)^5 + 35*(6*a^2*b + 5*b^3)*cosh(d*x + c)^3 + 2*(6*a^2*b
 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + (6*a^2*b + 5*b^3)*cosh(d*x + c)^
2 + (153*(6*a^2*b + 5*b^3)*cosh(d*x + c)^16 + 960*(6*a^2*b + 5*b^3)*cosh(d*
x + c)^14 + 2548*(6*a^2*b + 5*b^3)*cosh(d*x + c)^12 + 3696*(6*a^2*b + 5*b^3
)*cosh(d*x + c)^10 + 3150*(6*a^2*b + 5*b^3)*cosh(d*x + c)^8 + 1568*(6*a^2*b
 + 5*b^3)*cosh(d*x + c)^6 + 420*(6*a^2*b + 5*b^3)*cosh(d*x + c)^4 + 6*a^2*b
 + 5*b^3 + 48*(6*a^2*b + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(9*(6*
a^2*b + 5*b^3)*cosh(d*x + c)^17 + 64*(6*a^2*b + 5*b^3)*cosh(d*x + c)^15 + 1
96*(6*a^2*b + 5*b^3)*cosh(d*x + c)^13 + 336*(6*a^2*b + 5*b^3)*cosh(d*x + c)
^11 + 350*(6*a^2*b + 5*b^3)*cosh(d*x + c)^9 + 224*(6*a^2*b + 5*b^3)*cosh(d*
x + c)^7 + 84*(6*a^2*b + 5*b^3)*cosh(d*x + c)^5 + 16*(6*a^2*b + 5*b^3)*cosh
(d*x + c)^3 + (6*a^2*b + 5*b^3)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*
x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(75*(a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*cosh(d*x + c)^19 + 270*(2*a^3 + 6*a^2*b + 6*a*b^2 + 2*b^3 - (a^3 - 12*
a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^17 + 60*(27*a^3 + 39*a^2*b -
207*a*b^2 - 131*b^3 - 32*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x
 + c)^15 + 840*(3*a^3 - 6*a^2*b - 93*a*b^2 - 36*b^3 - 7*(a^3 - 12*a^2*b + 2
1*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^13 + 30*(63*a^3 - 639*a^2*b - 6195*a*b
^2 - 2173*b^3 - 336*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)
^11 - 100*(234*a^2*b + 2436*a*b^2 + 662*b^3 + 105*(a^3 - 12*a^2*b + 21*a*b^
2 - 10*b^3)*d*x)*cosh(d*x + c)^9 - 4*(315*a^3 + 3195*a^2*b + 46977*a*b^2 +
10865*b^3 + 1680*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^7
- 72*(15*a^3 + 30*a^2*b + 1159*a*b^2 + 180*b^3 + 35*(a^3 - 12*a^2*b + 21*a*
b^2 - 10*b^3)*d*x)*cosh(d*x + c)^5 - 3*(135*a^3 - 195*a^2*b + 6389*a*b^2 +
655*b^3 + 160*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^3 - 6
*(10*a^3 - 30*a^2*b + 262*a*b^2 - 10*b^3 + 5*(a^3 - 12*a^2*b + 21*a*b^2 - 1
0*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^18 + 18*d*cosh(d
*x + c)*sinh(d*x + c)^17 + d*sinh(d*x + c)^18 + 8*d*cosh(d*x + c)^16 + (153
*d*cosh(d*x + c)^2 + 8*d)*sinh(d*x + c)^16 + 16*(51*d*cosh(d*x + c)^3 + 8*d
*cosh(d*x + c))*sinh(d*x + c)^15 + 28*d*cosh(d*x + c)^14 + 4*(765*d*cosh(d*
x + c)^4 + 240*d*cosh(d*x + c)^2 + 7*d)*sinh(d*x + c)^14 + 56*(153*d*cosh(d
*x + c)^5 + 80*d*cosh(d*x + c)^3 + 7*d*cosh(d*x + c))*sinh(d*x + c)^13 + 56
*d*cosh(d*x + c)^12 + 28*(663*d*cosh(d*x + c)^6 + 520*d*cosh(d*x + c)^4 + 9
1*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^12 + 16*(1989*d*cosh(d*x + c)^7 +
2184*d*cosh(d*x + c)^5 + 637*d*cosh(d*x + c)^3 + 42*d*cosh(d*x + c))*sinh(d
*x + c)^11 + 70*d*cosh(d*x + c)^10 + 2*(21879*d*cosh(d*x + c)^8 + 32032*d*c
osh(d*x + c)^6 + 14014*d*cosh(d*x + c)^4 + 1848*d*cosh(d*x + c)^2 + 35*d)*s
inh(d*x + c)^10 + 4*(12155*d*cosh(d*x + c)^9 + 22880*d*cosh(d*x + c)^7 + 14
014*d*cosh(d*x + c)^5 + 3080*d*cosh(d*x + c)^3 + 175*d*cosh(d*x + c))*sinh(
d*x + c)^9 + 56*d*cosh(d*x + c)^8 + 2*(21879*d*cosh(d*x + c)^10 + 51480*d*c
osh(d*x + c)^8 + 42042*d*cosh(d*x + c)^6 + 13860*d*cosh(d*x + c)^4 + 1575*d
*cosh(d*x + c)^2 + 28*d)*sinh(d*x + c)^8 + 16*(1989*d*cosh(d*x + c)^11 + 57
20*d*cosh(d*x + c)^9 + 6006*d*cosh(d*x + c)^7 + 2772*d*cosh(d*x + c)^5 + 52
5*d*cosh(d*x + c)^3 + 28*d*cosh(d*x + c))*sinh(d*x + c)^7 + 28*d*cosh(d*x +
 c)^6 + 28*(663*d*cosh(d*x + c)^12 + 2288*d*cosh(d*x + c)^10 + 3003*d*cosh(
d*x + c)^8 + 1848*d*cosh(d*x + c)^6 + 525*d*cosh(d*x + c)^4 + 56*d*cosh(d*x
 + c)^2 + d)*sinh(d*x + c)^6 + 56*(153*d*cosh(d*x + c)^13 + 624*d*cosh(d*x
+ c)^11 + 1001*d*cosh(d*x + c)^9 + 792*d*cosh(d*x + c)^7 + 315*d*cosh(d*x +
 c)^5 + 56*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 8*d*cos
h(d*x + c)^4 + 4*(765*d*cosh(d*x + c)^14 + 3640*d*cosh(d*x + c)^12 + 7007*d
*cosh(d*x + c)^10 + 6930*d*cosh(d*x + c)^8 + 3675*d*cosh(d*x + c)^6 + 980*d
*cosh(d*x + c)^4 + 105*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^4 + 16*(51*d*
cosh(d*x + c)^15 + 280*d*cosh(d*x + c)^13 + 637*d*cosh(d*x + c)^11 + 770*d*
cosh(d*x + c)^9 + 525*d*cosh(d*x + c)^7 + 196*d*cosh(d*x + c)^5 + 35*d*cosh
(d*x + c)^3 + 2*d*cosh(d*x + c))*sinh(d*x + c)^3 + d*cosh(d*x + c)^2 + (153
*d*cosh(d*x + c)^16 + 960*d*cosh(d*x + c)^14 + 2548*d*cosh(d*x + c)^12 + 36
```

```
96*d*cosh(d*x + c)^10 + 3150*d*cosh(d*x + c)^8 + 1568*d*cosh(d*x + c)^6 + 4
20*d*cosh(d*x + c)^4 + 48*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 2*(9*d*c
osh(d*x + c)^17 + 64*d*cosh(d*x + c)^15 + 196*d*cosh(d*x + c)^13 + 336*d*co
sh(d*x + c)^11 + 350*d*cosh(d*x + c)^9 + 224*d*cosh(d*x + c)^7 + 84*d*cosh(
d*x + c)^5 + 16*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*tanh(d*x+c)**3)**3,x)

[Out] Timed out
```

Giac [B] time = 3.47288, size = 799, normalized size = 3.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")

[Out] -1/840*(420*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x + 840*(6*a^2*b*e^(2*c)
 + 5*b^3*e^(2*c))*e^(-2*c)*log(e^(2*d*x + 2*c) + 1) - 105*(2*a^3*e^(2*d*x +
 2*c) - 24*a^2*b*e^(2*d*x + 2*c) + 42*a*b^2*e^(2*d*x + 2*c) - 20*b^3*e^(2*d
*x + 2*c) - a^3 + 3*a^2*b - 3*a*b^2 + b^3)*e^(-2*d*x - 2*c) - 105*(a^3*e^(2
*d*x + 22*c) + 3*a^2*b*e^(2*d*x + 22*c) + 3*a*b^2*e^(2*d*x + 22*c) + b^3*e^
(2*d*x + 22*c))*e^(-20*c) - (13698*a^2*b*e^(16*d*x + 16*c) + 11415*b^3*e^(1
6*d*x + 16*c) + 104544*a^2*b*e^(14*d*x + 14*c) - 30240*a*b^2*e^(14*d*x + 14
*c) + 74520*b^3*e^(14*d*x + 14*c) + 353304*a^2*b*e^(12*d*x + 12*c) - 171360
*a*b^2*e^(12*d*x + 12*c) + 252420*b^3*e^(12*d*x + 12*c) + 691488*a^2*b*e^(1
0*d*x + 10*c) - 446880*a*b^2*e^(10*d*x + 10*c) + 476840*b^3*e^(10*d*x + 10*
c) + 858060*a^2*b*e^(8*d*x + 8*c) - 682080*a*b^2*e^(8*d*x + 8*c) + 601930*b
^3*e^(8*d*x + 8*c) + 691488*a^2*b*e^(6*d*x + 6*c) - 644448*a*b^2*e^(6*d*x +
 6*c) + 476840*b^3*e^(6*d*x + 6*c) + 353304*a^2*b*e^(4*d*x + 4*c) - 374304*
a*b^2*e^(4*d*x + 4*c) + 252420*b^3*e^(4*d*x + 4*c) + 104544*a^2*b*e^(2*d*x
+ 2*c) - 125664*a*b^2*e^(2*d*x + 2*c) + 74520*b^3*e^(2*d*x + 2*c) + 13698*a
^2*b - 19488*a*b^2 + 11415*b^3)/(e^(2*d*x + 2*c) + 1)^8)/d
```

3.68 $\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=269

$$\frac{9a^2b\sinh(c+dx)}{2d}-\frac{9a^2b\tan^{-1}(\sinh(c+dx))}{2d}-\frac{3a^2b\sinh(c+dx)\tanh^2(c+dx)}{2d}+\frac{a^3\cosh(c+dx)}{d}+\frac{3ab^2\cosh(c+$$

```
[Out] (-9*a^2*b*ArcTan[Sinh[c + d*x]])/(2*d) - (315*b^3*ArcTan[Sinh[c + d*x]])/(1
28*d) + (a^3*Cosh[c + d*x])/d + (3*a*b^2*Cosh[c + d*x])/d + (9*a*b^2*Sech[c
 + d*x])/d - (3*a*b^2*Sech[c + d*x]^3)/d + (3*a*b^2*Sech[c + d*x]^5)/(5*d)
+ (9*a^2*b*Sinh[c + d*x])/(2*d) + (315*b^3*Sinh[c + d*x])/(128*d) - (3*a^2*
b*Sinh[c + d*x]*Tanh[c + d*x]^2)/(2*d) - (105*b^3*Sinh[c + d*x]*Tanh[c + d*
x]^2)/(128*d) - (21*b^3*Sinh[c + d*x]*Tanh[c + d*x]^4)/(64*d) - (3*b^3*Sinh
[c + d*x]*Tanh[c + d*x]^6)/(16*d) - (b^3*Sinh[c + d*x]*Tanh[c + d*x]^8)/(8*
d)
```

Rubi [A] time = 0.24916, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3666, 2638, 2592, 288, 321, 203, 2590, 270}

$$\frac{9a^2b\sinh(c+dx)}{2d}-\frac{9a^2b\tan^{-1}(\sinh(c+dx))}{2d}-\frac{3a^2b\sinh(c+dx)\tanh^2(c+dx)}{2d}+\frac{a^3\cosh(c+dx)}{d}+\frac{3ab^2\cosh(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^3)^3,x]
```



```
[Out] (-9*a^2*b*ArcTan[Sinh[c + d*x]])/(2*d) - (315*b^3*ArcTan[Sinh[c + d*x]])/(1
28*d) + (a^3*Cosh[c + d*x])/d + (3*a*b^2*Cosh[c + d*x])/d + (9*a*b^2*Sech[c
 + d*x])/d - (3*a*b^2*Sech[c + d*x]^3)/d + (3*a*b^2*Sech[c + d*x]^5)/(5*d)
+ (9*a^2*b*Sinh[c + d*x])/(2*d) + (315*b^3*Sinh[c + d*x])/(128*d) - (3*a^2*
b*Sinh[c + d*x]*Tanh[c + d*x]^2)/(2*d) - (105*b^3*Sinh[c + d*x]*Tanh[c + d*
x]^2)/(128*d) - (21*b^3*Sinh[c + d*x]*Tanh[c + d*x]^4)/(64*d) - (3*b^3*Sinh
[c + d*x]*Tanh[c + d*x]^6)/(16*d) - (b^3*Sinh[c + d*x]*Tanh[c + d*x]^8)/(8*
d)
```


Rule 3666

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = -\left(i\int\left(ia^3\sinh(c+dx)+3ia^2b\sinh(c+dx)\tanh^3(c+dx)+3iab^2\sinh\right.\right.$$

$$= a^3\int\sinh(c+dx)\,dx+\left(3a^2b\right)\int\sinh(c+dx)\tanh^3(c+dx)\,dx+\left(3ab^2\right.$$

$$= \frac{a^3\cosh(c+dx)}{d}+\frac{\left(3a^2b\right)\mathrm{Subst}\left(\int\frac{x^4}{(1+x^2)^2}\,dx,x,\sinh(c+dx)\right)}{d}-\frac{\left(3ab^2\right)}{}$$

$$= \frac{a^3\cosh(c+dx)}{d}-\frac{3a^2b\sinh(c+dx)\tanh^2(c+dx)}{2d}-\frac{b^3\sinh(c+dx)\tan}{8d}$$

$$= \frac{a^3\cosh(c+dx)}{d}+\frac{3ab^2\cosh(c+dx)}{d}+\frac{9ab^2\mathrm{sech}(c+dx)}{d}-\frac{3ab^2\mathrm{sech}^3(}{d}$$

$$= -\frac{9a^2b\tan^{-1}(\sinh(c+dx))}{2d}+\frac{a^3\cosh(c+dx)}{d}+\frac{3ab^2\cosh(c+dx)}{d}+\frac{9a}{}$$

$$= -\frac{9a^2b\tan^{-1}(\sinh(c+dx))}{2d}+\frac{a^3\cosh(c+dx)}{d}+\frac{3ab^2\cosh(c+dx)}{d}+\frac{9a}{}$$

$$= -\frac{9a^2b\tan^{-1}(\sinh(c+dx))}{2d}+\frac{a^3\cosh(c+dx)}{d}+\frac{3ab^2\cosh(c+dx)}{d}+\frac{9a}{}$$

$$= -\frac{9a^2b\tan^{-1}(\sinh(c+dx))}{2d}-\frac{315b^3\tan^{-1}(\sinh(c+dx))}{128d}+\frac{a^3\cosh(c+}{d}$$

Mathematica [A] time = 6.28942, size = 233, normalized size = 0.87

$$\frac{b\left(3a^2+b^2\right)\sinh(c+dx)}{d}+\frac{a\left(a^2+3b^2\right)\cosh(c+dx)}{d}-\frac{9b\left(64a^2+35b^2\right)\tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{64d}+\frac{\mathrm{sech}^2(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] (-9*b*(64*a^2 + 35*b^2)*ArcTan[Tanh[(c + d*x)/2]])/(64*d) + (a*(a^2 + 3*b^2
)*Cosh[c + d*x])/d + (9*a*b^2*Sech[c + d*x])/d - (3*a*b^2*Sech[c + d*x]^3)/
d + (3*a*b^2*Sech[c + d*x]^5)/(5*d) + (b*(3*a^2 + b^2)*Sinh[c + d*x])/d + (
Sech[c + d*x]^2*(192*a^2*b*Sinh[c + d*x] + 325*b^3*Sinh[c + d*x]))/(128*d)
- (105*b^3*Sech[c + d*x]^3*Tanh[c + d*x])/(64*d) + (11*b^3*Sech[c + d*x]^5*
Tanh[c + d*x])/(16*d) - (b^3*Sech[c + d*x]^7*Tanh[c + d*x])/(8*d)
```

Maple [A] time = 0.067, size = 458, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x)
```

```
[Out] a^3*cosh(d*x+c)/d+3/d*a^2*b*sinh(d*x+c)^3/cosh(d*x+c)^2+9/d*a^2*b*sinh(d*x+
c)/cosh(d*x+c)^2-9/2*a^2*b*sech(d*x+c)*tanh(d*x+c)/d-9/d*a^2*b*arctan(exp(d
*x+c))+3/d*a*b^2*sinh(d*x+c)^6/cosh(d*x+c)^5+18/d*a*b^2*sinh(d*x+c)^4/cosh(
d*x+c)^5+72/5/d*a*b^2*sinh(d*x+c)^2/cosh(d*x+c)^5-48/5/d*a*b^2*sinh(d*x+c)^
```

```
2/cosh(d*x+c)^3-48/5/d*a*b^2*sinh(d*x+c)^2/cosh(d*x+c)+48/5*a*b^2*cosh(d*x+
c)/d+1/d*b^3*sinh(d*x+c)^9/cosh(d*x+c)^8+9/d*b^3*sinh(d*x+c)^7/cosh(d*x+c)^
8+21/d*b^3*sinh(d*x+c)^5/cosh(d*x+c)^8+21/d*b^3*sinh(d*x+c)^3/cosh(d*x+c)^8
+9/d*b^3*sinh(d*x+c)/cosh(d*x+c)^8-9/8/d*b^3*tanh(d*x+c)*sech(d*x+c)^7-21/1
6/d*b^3*tanh(d*x+c)*sech(d*x+c)^5-105/64*b^3*sech(d*x+c)^3*tanh(d*x+c)/d-31
5/128*b^3*sech(d*x+c)*tanh(d*x+c)/d-315/64/d*b^3*arctan(exp(d*x+c))
```

Maxima [A] time = 1.74748, size = 653, normalized size = 2.43

$$\frac{1}{64}b^3\left(\frac{315\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{32\,e^{(-dx-c)}}{d}+\frac{581\,e^{(-2\,dx-2\,c)}+1681\,e^{(-4\,dx-4\,c)}+3605\,e^{(-6\,dx-6\,c)}+2569\,e^{(-8\,dx-8\,c)}+1463\,e^{(-}}{d\left(e^{(-dx-c)}+8\,e^{(-3\,dx-3\,c)}+28\,e^{(-5\,dx-5\,c)}+56\,e^{(-7\,dx-7\,c)}+70\,e^{(-9\,dx-9\,c)}+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] 1/64*b^3*(315*arctan(e^(-d*x - c))/d - 32*e^(-d*x - c)/d + (581*e^(-2*d*x -
 2*c) + 1681*e^(-4*d*x - 4*c) + 3605*e^(-6*d*x - 6*c) + 2569*e^(-8*d*x - 8*
c) + 1463*e^(-10*d*x - 10*c) - 917*e^(-12*d*x - 12*c) - 529*e^(-14*d*x - 14
*c) - 293*e^(-16*d*x - 16*c) + 32)/(d*(e^(-d*x - c) + 8*e^(-3*d*x - 3*c) +
28*e^(-5*d*x - 5*c) + 56*e^(-7*d*x - 7*c) + 70*e^(-9*d*x - 9*c) + 56*e^(-11
*d*x - 11*c) + 28*e^(-13*d*x - 13*c) + 8*e^(-15*d*x - 15*c) + e^(-17*d*x -
17*c)))) + 3/2*a^2*b*(6*arctan(e^(-d*x - c))/d - e^(-d*x - c)/d + (4*e^(-2*
d*x - 2*c) - e^(-4*d*x - 4*c) + 1)/(d*(e^(-d*x - c) + 2*e^(-3*d*x - 3*c) +
e^(-5*d*x - 5*c)))) + 3/10*a*b^2*(5*e^(-d*x - c)/d + (85*e^(-2*d*x - 2*c) +
 210*e^(-4*d*x - 4*c) + 314*e^(-6*d*x - 6*c) + 185*e^(-8*d*x - 8*c) + 65*e^
(-10*d*x - 10*c) + 5)/(d*(e^(-d*x - c) + 5*e^(-3*d*x - 3*c) + 10*e^(-5*d*x
- 5*c) + 10*e^(-7*d*x - 7*c) + 5*e^(-9*d*x - 9*c) + e^(-11*d*x - 11*c)))) +
 a^3*cosh(d*x + c)/d
```

Fricas [B] time = 3.98452, size = 17484, normalized size = 65.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/320*(160*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^18 + 2880*(a^3 + 3
*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^17 + 160*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*sinh(d*x + c)^18 + 45*(32*a^3 + 96*a^2*b + 224*a*b^2 + 61
*b^3)*cosh(d*x + c)^16 + 45*(32*a^3 + 96*a^2*b + 224*a*b^2 + 61*b^3 + 544*(
a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^16 + 240*(544
*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + 3*(32*a^3 + 96*a^2*b + 2
24*a*b^2 + 61*b^3)*cosh(d*x + c))*sinh(d*x + c)^15 + 15*(384*a^3 + 960*a^2*
b + 3328*a*b^2 + 475*b^3)*cosh(d*x + c)^14 + 15*(32640*(a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*cosh(d*x + c)^4 + 384*a^3 + 960*a^2*b + 3328*a*b^2 + 475*b^3 +
360*(32*a^3 + 96*a^2*b + 224*a*b^2 + 61*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)
^14 + 210*(6528*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 120*(32*a
^3 + 96*a^2*b + 224*a*b^2 + 61*b^3)*cosh(d*x + c)^3 + (384*a^3 + 960*a^2*b
+ 3328*a*b^2 + 475*b^3)*cosh(d*x + c))*sinh(d*x + c)^13 + 3*(4480*a^3 + 736
0*a^2*b + 43008*a*b^2 + 4515*b^3)*cosh(d*x + c)^12 + 3*(990080*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 27300*(32*a^3 + 96*a^2*b + 224*a*b^2
+ 61*b^3)*cosh(d*x + c)^4 + 4480*a^3 + 7360*a^2*b + 43008*a*b^2 + 4515*b^3
```

+ 455*(384*a^3 + 960*a^2*b + 3328*a*b^2 + 475*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^12 + 12*(424320*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 16380*(32*a^3 + 96*a^2*b + 224*a*b^2 + 61*b^3)*cosh(d*x + c)^5 + 455*(384*a^3 + 960*a^2*b + 3328*a*b^2 + 475*b^3)*cosh(d*x + c)^3 + 3*(4480*a^3 + 7360*a^2*b + 43008*a*b^2 + 4515*b^3)*cosh(d*x + c))*sinh(d*x + c)^11 + 3*(6720*a^3 + 3840*a^2*b + 67904*a*b^2 + 1295*b^3)*cosh(d*x + c)^10 + 3*(2333760*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 120120*(32*a^3 + 96*a^2*b + 224*a*b^2 + 61*b^3)*cosh(d*x + c)^6 + 5005*(384*a^3 + 960*a^2*b + 3328*a*b^2 + 475*b^3)*cosh(d*x + c)^4 + 6720*a^3 + 3840*a^2*b + 67904*a*b^2 + 1295*b^3 + 66*(4480*a^3 + 7360*a^2*b + 43008*a*b^2 + 4515*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 10*(777920*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^9 + 51480*(32*a^3 + 96*a^2*b + 224*a*b^2 + 61*b^3)*cosh(d*x + c)^7 + 3003*(384*a^3 + 960*a^2*b + 3328*a*b^2 + 475*b^3)*cosh(d*x + c)^5 + 66*(4480*a^3 + 7360*a^2*b + 43008*a*b^2 + 4515*b^3)*cosh(d*x + c)^3 + 3*(6720*a^3 + 3840*a^2*b + 67904*a*b^2 + 1295*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 + 3*(6720*a^3 - 3840*a^2*b + 67904*a*b^2 - 1295*b^3)*cosh(d*x + c)^8 + 3*(2333760*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^10 + 193050*(32*a^3 + 96*a^2*b + 224*a*b^2 + 61*b^3)*cosh(d*x + c)^8 + 15015*(384*a^3 + 960*a^2*b + 3328*a*b^2 + 475*b^3)*cosh(d*x + c)^6 + 495*(4480*a^3 + 7360*a^2*b + 43008*a*b^2 + 4515*b^3)*cosh(d*x + c)^4 + 6720*a^3 - 3840*a^2*b + 67904*a*b^2 - 1295*b^3 + 45*(6720*a^3 + 3840*a^2*b + 67904*a*b^2 + 1295*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 24*(212160*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^11 + 21450*(32*a^3 + 96*a^2*b + 224*a*b^2 + 61*b^3)*cosh(d*x + c)^9 + 2145*(384*a^3 + 960*a^2*b + 3328*a*b^2 + 475*b^3)*cosh(d*x + c)^7 + 99*(4480*a^3 + 7360*a^2*b + 43008*a*b^2 + 4515*b^3)*cosh(d*x + c)^5 + 15*(6720*a^3 + 3840*a^2*b + 67904*a*b^2 + 1295*b^3)*cosh(d*x + c)^3 + (6720*a^3 - 3840*a^2*b + 67904*a*b^2 - 1295*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 3*(4480*a^3 - 7360*a^2*b + 43008*a*b^2 - 4515*b^3)*cosh(d*x + c)^6 + 3*(990080*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^12 + 120120*(32*a^3 + 96*a^2*b + 224*a*b^2 + 61*b^3)*cosh(d*x + c)^10 + 15015*(384*a^3 + 960*a^2*b + 3328*a*b^2 + 475*b^3)*cosh(d*x + c)^8 + 924*(4480*a^3 + 7360*a^2*b + 43008*a*b^2 + 4515*b^3)*cosh(d*x + c)^6 + 210*(6720*a^3 + 3840*a^2*b + 67904*a*b^2 + 1295*b^3)*cosh(d*x + c)^4 + 4480*a^3 - 7360*a^2*b + 43008*a*b^2 - 4515*b^3 + 28*(6720*a^3 - 3840*a^2*b + 67904*a*b^2 - 1295*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 6*(228480*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^13 + 32760*(32*a^3 + 96*a^2*b + 224*a*b^2 + 61*b^3)*cosh(d*x + c)^11 + 5005*(384*a^3 + 960*a^2*b + 3328*a*b^2 + 475*b^3)*cosh(d*x + c)^9 + 396*(4480*a^3 + 7360*a^2*b + 43008*a*b^2 + 4515*b^3)*cosh(d*x + c)^7 + 126*(6720*a^3 + 3840*a^2*b + 67904*a*b^2 + 1295*b^3)*cosh(d*x + c)^5 + 28*(6720*a^3 - 3840*a^2*b + 67904*a*b^2 - 1295*b^3)*cosh(d*x + c)^3 + 3*(4480*a^3 - 7360*a^2*b + 43008*a*b^2 - 4515*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 15*(384*a^3 - 960*a^2*b + 3328*a*b^2 - 475*b^3)*cosh(d*x + c)^4 + 15*(32640*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^14 + 5460*(32*a^3 + 96*a^2*b + 224*a*b^2 + 61*b^3)*cosh(d*x + c)^12 + 1001*(384*a^3 + 960*a^2*b + 3328*a*b^2 + 475*b^3)*cosh(d*x + c)^10 + 99*(4480*a^3 + 7360*a^2*b + 43008*a*b^2 + 4515*b^3)*cosh(d*x + c)^8 + 42*(6720*a^3 + 3840*a^2*b + 67904*a*b^2 + 1295*b^3)*cosh(d*x + c)^6 + 14*(6720*a^3 - 3840*a^2*b + 67904*a*b^2 - 1295*b^3)*cosh(d*x + c)^4 + 384*a^3 - 960*a^2*b + 3328*a*b^2 - 475*b^3 + 3*(4480*a^3 - 7360*a^2*b + 43008*a*b^2 - 4515*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 12*(10880*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^15 + 2100*(32*a^3 + 96*a^2*b + 224*a*b^2 + 61*b^3)*cosh(d*x + c)^13 + 455*(384*a^3 + 960*a^2*b + 3328*a*b^2 + 475*b^3)*cosh(d*x + c)^11 + 55*(4480*a^3 + 7360*a^2*b + 43008*a*b^2 + 4515*b^3)*cosh(d*x + c)^9 + 30*(6720*a^3 + 3840*a^2*b + 67904*a*b^2 + 1295*b^3)*cosh(d*x + c)^7 + 14*(6720*a^3 - 3840*a^2*b + 67904*a*b^2 - 1295*b^3)*cosh(d*x + c)^5 + 5*(4480*a^3 - 7360*a^2*b + 43008*a*b^2 - 4515*b^3)*cosh(d*x + c)^3 + 5*(384*a^3 - 960*a^2*b + 3328*a*b^2 - 475*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 160*a^3 - 480*a^2*b + 480*a*b^2 - 160*b^3 + 45*(32*a^3 - 96*a^2*b + 224*a*b^2 - 61*b^3)*cosh(d*x + c)^2 + 3*(8160*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^16 + 1800*(32*a^3 + 96*a^2*b + 224*a*b^2 + 61*b^3)*cosh(d*x + c)^14 +

```
455*(384*a^3 + 960*a^2*b + 3328*a*b^2 + 475*b^3)*cosh(d*x + c)^12 + 66*(448
0*a^3 + 7360*a^2*b + 43008*a*b^2 + 4515*b^3)*cosh(d*x + c)^10 + 45*(6720*a^
3 + 3840*a^2*b + 67904*a*b^2 + 1295*b^3)*cosh(d*x + c)^8 + 28*(6720*a^3 - 3
840*a^2*b + 67904*a*b^2 - 1295*b^3)*cosh(d*x + c)^6 + 15*(4480*a^3 - 7360*a
^2*b + 43008*a*b^2 - 4515*b^3)*cosh(d*x + c)^4 + 480*a^3 - 1440*a^2*b + 336
0*a*b^2 - 915*b^3 + 30*(384*a^3 - 960*a^2*b + 3328*a*b^2 - 475*b^3)*cosh(d*
x + c)^2)*sinh(d*x + c)^2 - 45*((64*a^2*b + 35*b^3)*cosh(d*x + c)^17 + 17*(
64*a^2*b + 35*b^3)*cosh(d*x + c)*sinh(d*x + c)^16 + (64*a^2*b + 35*b^3)*sin
h(d*x + c)^17 + 8*(64*a^2*b + 35*b^3)*cosh(d*x + c)^15 + 8*(64*a^2*b + 35*b
^3 + 17*(64*a^2*b + 35*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^15 + 40*(17*(64*
a^2*b + 35*b^3)*cosh(d*x + c)^3 + 3*(64*a^2*b + 35*b^3)*cosh(d*x + c))*sinh
(d*x + c)^14 + 28*(64*a^2*b + 35*b^3)*cosh(d*x + c)^13 + 28*(85*(64*a^2*b +
 35*b^3)*cosh(d*x + c)^4 + 64*a^2*b + 35*b^3 + 30*(64*a^2*b + 35*b^3)*cosh(
d*x + c)^2)*sinh(d*x + c)^13 + 364*(17*(64*a^2*b + 35*b^3)*cosh(d*x + c)^5
+ 10*(64*a^2*b + 35*b^3)*cosh(d*x + c)^3 + (64*a^2*b + 35*b^3)*cosh(d*x + c
))*sinh(d*x + c)^12 + 56*(64*a^2*b + 35*b^3)*cosh(d*x + c)^11 + 56*(221*(64
*a^2*b + 35*b^3)*cosh(d*x + c)^6 + 195*(64*a^2*b + 35*b^3)*cosh(d*x + c)^4
+ 64*a^2*b + 35*b^3 + 39*(64*a^2*b + 35*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)
^11 + 88*(221*(64*a^2*b + 35*b^3)*cosh(d*x + c)^7 + 273*(64*a^2*b + 35*b^3)
*cosh(d*x + c)^5 + 91*(64*a^2*b + 35*b^3)*cosh(d*x + c)^3 + 7*(64*a^2*b + 3
5*b^3)*cosh(d*x + c))*sinh(d*x + c)^10 + 70*(64*a^2*b + 35*b^3)*cosh(d*x +
c)^9 + 10*(2431*(64*a^2*b + 35*b^3)*cosh(d*x + c)^8 + 4004*(64*a^2*b + 35*b
^3)*cosh(d*x + c)^6 + 2002*(64*a^2*b + 35*b^3)*cosh(d*x + c)^4 + 448*a^2*b
+ 245*b^3 + 308*(64*a^2*b + 35*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^9 + 2*(1
2155*(64*a^2*b + 35*b^3)*cosh(d*x + c)^9 + 25740*(64*a^2*b + 35*b^3)*cosh(d
*x + c)^7 + 18018*(64*a^2*b + 35*b^3)*cosh(d*x + c)^5 + 4620*(64*a^2*b + 35
*b^3)*cosh(d*x + c)^3 + 315*(64*a^2*b + 35*b^3)*cosh(d*x + c))*sinh(d*x + c
)^8 + 56*(64*a^2*b + 35*b^3)*cosh(d*x + c)^7 + 8*(2431*(64*a^2*b + 35*b^3)*
cosh(d*x + c)^10 + 6435*(64*a^2*b + 35*b^3)*cosh(d*x + c)^8 + 6006*(64*a^2*
b + 35*b^3)*cosh(d*x + c)^6 + 2310*(64*a^2*b + 35*b^3)*cosh(d*x + c)^4 + 44
8*a^2*b + 245*b^3 + 315*(64*a^2*b + 35*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^
7 + 56*(221*(64*a^2*b + 35*b^3)*cosh(d*x + c)^11 + 715*(64*a^2*b + 35*b^3)*
cosh(d*x + c)^9 + 858*(64*a^2*b + 35*b^3)*cosh(d*x + c)^7 + 462*(64*a^2*b +
 35*b^3)*cosh(d*x + c)^5 + 105*(64*a^2*b + 35*b^3)*cosh(d*x + c)^3 + 7*(64*
a^2*b + 35*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 28*(64*a^2*b + 35*b^3)*cos
h(d*x + c)^5 + 28*(221*(64*a^2*b + 35*b^3)*cosh(d*x + c)^12 + 858*(64*a^2*b
 + 35*b^3)*cosh(d*x + c)^10 + 1287*(64*a^2*b + 35*b^3)*cosh(d*x + c)^8 + 92
4*(64*a^2*b + 35*b^3)*cosh(d*x + c)^6 + 315*(64*a^2*b + 35*b^3)*cosh(d*x +
c)^4 + 64*a^2*b + 35*b^3 + 42*(64*a^2*b + 35*b^3)*cosh(d*x + c)^2)*sinh(d*x
 + c)^5 + 140*(17*(64*a^2*b + 35*b^3)*cosh(d*x + c)^13 + 78*(64*a^2*b + 35*
b^3)*cosh(d*x + c)^11 + 143*(64*a^2*b + 35*b^3)*cosh(d*x + c)^9 + 132*(64*a
^2*b + 35*b^3)*cosh(d*x + c)^7 + 63*(64*a^2*b + 35*b^3)*cosh(d*x + c)^5 + 1
4*(64*a^2*b + 35*b^3)*cosh(d*x + c)^3 + (64*a^2*b + 35*b^3)*cosh(d*x + c))*
sinh(d*x + c)^4 + 8*(64*a^2*b + 35*b^3)*cosh(d*x + c)^3 + 8*(85*(64*a^2*b +
 35*b^3)*cosh(d*x + c)^14 + 455*(64*a^2*b + 35*b^3)*cosh(d*x + c)^12 + 1001
*(64*a^2*b + 35*b^3)*cosh(d*x + c)^10 + 1155*(64*a^2*b + 35*b^3)*cosh(d*x +
 c)^8 + 735*(64*a^2*b + 35*b^3)*cosh(d*x + c)^6 + 245*(64*a^2*b + 35*b^3)*c
osh(d*x + c)^4 + 64*a^2*b + 35*b^3 + 35*(64*a^2*b + 35*b^3)*cosh(d*x + c)^2
)*sinh(d*x + c)^3 + 8*(17*(64*a^2*b + 35*b^3)*cosh(d*x + c)^15 + 105*(64*a^
2*b + 35*b^3)*cosh(d*x + c)^13 + 273*(64*a^2*b + 35*b^3)*cosh(d*x + c)^11 +
 385*(64*a^2*b + 35*b^3)*cosh(d*x + c)^9 + 315*(64*a^2*b + 35*b^3)*cosh(d*x
 + c)^7 + 147*(64*a^2*b + 35*b^3)*cosh(d*x + c)^5 + 35*(64*a^2*b + 35*b^3)*
cosh(d*x + c)^3 + 3*(64*a^2*b + 35*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + (6
4*a^2*b + 35*b^3)*cosh(d*x + c) + (17*(64*a^2*b + 35*b^3)*cosh(d*x + c)^16
+ 120*(64*a^2*b + 35*b^3)*cosh(d*x + c)^14 + 364*(64*a^2*b + 35*b^3)*cosh(d
*x + c)^12 + 616*(64*a^2*b + 35*b^3)*cosh(d*x + c)^10 + 630*(64*a^2*b + 35*
b^3)*cosh(d*x + c)^8 + 392*(64*a^2*b + 35*b^3)*cosh(d*x + c)^6 + 140*(64*a^
2*b + 35*b^3)*cosh(d*x + c)^4 + 64*a^2*b + 35*b^3 + 24*(64*a^2*b + 35*b^3)*
cosh(d*x + c)^2)*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) + 6*(
```

```
480*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^17 + 120*(32*a^3 + 96*a^2
*b + 224*a*b^2 + 61*b^3)*cosh(d*x + c)^15 + 35*(384*a^3 + 960*a^2*b + 3328*
a*b^2 + 475*b^3)*cosh(d*x + c)^13 + 6*(4480*a^3 + 7360*a^2*b + 43008*a*b^2
+ 4515*b^3)*cosh(d*x + c)^11 + 5*(6720*a^3 + 3840*a^2*b + 67904*a*b^2 + 129
5*b^3)*cosh(d*x + c)^9 + 4*(6720*a^3 - 3840*a^2*b + 67904*a*b^2 - 1295*b^3)
*cosh(d*x + c)^7 + 3*(4480*a^3 - 7360*a^2*b + 43008*a*b^2 - 4515*b^3)*cosh(
d*x + c)^5 + 10*(384*a^3 - 960*a^2*b + 3328*a*b^2 - 475*b^3)*cosh(d*x + c)^
3 + 15*(32*a^3 - 96*a^2*b + 224*a*b^2 - 61*b^3)*cosh(d*x + c))*sinh(d*x + c
))/(d*cosh(d*x + c)^17 + 17*d*cosh(d*x + c)*sinh(d*x + c)^16 + d*sinh(d*x +
 c)^17 + 8*d*cosh(d*x + c)^15 + 8*(17*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^
15 + 40*(17*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^14 + 28*d*
cosh(d*x + c)^13 + 28*(85*d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 + d)*sin
h(d*x + c)^13 + 364*(17*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + d*cosh(d
*x + c))*sinh(d*x + c)^12 + 56*d*cosh(d*x + c)^11 + 56*(221*d*cosh(d*x + c)
^6 + 195*d*cosh(d*x + c)^4 + 39*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^11 + 8
8*(221*d*cosh(d*x + c)^7 + 273*d*cosh(d*x + c)^5 + 91*d*cosh(d*x + c)^3 + 7
*d*cosh(d*x + c))*sinh(d*x + c)^10 + 70*d*cosh(d*x + c)^9 + 10*(2431*d*cosh
(d*x + c)^8 + 4004*d*cosh(d*x + c)^6 + 2002*d*cosh(d*x + c)^4 + 308*d*cosh(
d*x + c)^2 + 7*d)*sinh(d*x + c)^9 + 2*(12155*d*cosh(d*x + c)^9 + 25740*d*co
sh(d*x + c)^7 + 18018*d*cosh(d*x + c)^5 + 4620*d*cosh(d*x + c)^3 + 315*d*co
sh(d*x + c))*sinh(d*x + c)^8 + 56*d*cosh(d*x + c)^7 + 8*(2431*d*cosh(d*x +
c)^10 + 6435*d*cosh(d*x + c)^8 + 6006*d*cosh(d*x + c)^6 + 2310*d*cosh(d*x +
 c)^4 + 315*d*cosh(d*x + c)^2 + 7*d)*sinh(d*x + c)^7 + 56*(221*d*cosh(d*x +
 c)^11 + 715*d*cosh(d*x + c)^9 + 858*d*cosh(d*x + c)^7 + 462*d*cosh(d*x + c
)^5 + 105*d*cosh(d*x + c)^3 + 7*d*cosh(d*x + c))*sinh(d*x + c)^6 + 28*d*cos
h(d*x + c)^5 + 28*(221*d*cosh(d*x + c)^12 + 858*d*cosh(d*x + c)^10 + 1287*d
*cosh(d*x + c)^8 + 924*d*cosh(d*x + c)^6 + 315*d*cosh(d*x + c)^4 + 42*d*cos
h(d*x + c)^2 + d)*sinh(d*x + c)^5 + 140*(17*d*cosh(d*x + c)^13 + 78*d*cosh(
d*x + c)^11 + 143*d*cosh(d*x + c)^9 + 132*d*cosh(d*x + c)^7 + 63*d*cosh(d*x
 + c)^5 + 14*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^4 + 8*d*cos
h(d*x + c)^3 + 8*(85*d*cosh(d*x + c)^14 + 455*d*cosh(d*x + c)^12 + 1001*d*c
osh(d*x + c)^10 + 1155*d*cosh(d*x + c)^8 + 735*d*cosh(d*x + c)^6 + 245*d*co
sh(d*x + c)^4 + 35*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^3 + 8*(17*d*cosh(d*
x + c)^15 + 105*d*cosh(d*x + c)^13 + 273*d*cosh(d*x + c)^11 + 385*d*cosh(d*
x + c)^9 + 315*d*cosh(d*x + c)^7 + 147*d*cosh(d*x + c)^5 + 35*d*cosh(d*x +
c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + d*cosh(d*x + c) + (17*d*cosh(d*
x + c)^16 + 120*d*cosh(d*x + c)^14 + 364*d*cosh(d*x + c)^12 + 616*d*cosh(d*
x + c)^10 + 630*d*cosh(d*x + c)^8 + 392*d*cosh(d*x + c)^6 + 140*d*cosh(d*x
+ c)^4 + 24*d*cosh(d*x + c)^2 + d)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)**3)**3,x)

[Out] Timed out
```

Giac [A] time = 2.63282, size = 655, normalized size = 2.43

$$-\frac{45\left(64\,a^2be^c+35\,b^3e^c\right)\arctan\left(e^{(dx+c)}\right)e^{(-c)}-160\left(a^3-3\,a^2b+3\,ab^2-b^3\right)e^{(-dx-c)}-160\left(a^3e^{(dx+20\,c)}+3\,a^2be^{(dx+20\,c)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] -1/320*(45*(64*a^2*b*e^c + 35*b^3*e^c)*arctan(e^(d*x + c))*e^(-c) - 160*(a^
3 - 3*a^2*b + 3*a*b^2 - b^3)*e^(-d*x - c) - 160*(a^3*e^(d*x + 20*c) + 3*a^2
*b*e^(d*x + 20*c) + 3*a*b^2*e^(d*x + 20*c) + b^3*e^(d*x + 20*c))*e^(-19*c)
- (960*a^2*b*e^(15*d*x + 15*c) + 5760*a*b^2*e^(15*d*x + 15*c) + 1625*b^3*e^
(15*d*x + 15*c) + 4800*a^2*b*e^(13*d*x + 13*c) + 32640*a*b^2*e^(13*d*x + 13
*c) + 3925*b^3*e^(13*d*x + 13*c) + 8640*a^2*b*e^(11*d*x + 11*c) + 88704*a*b
^2*e^(11*d*x + 11*c) + 9065*b^3*e^(11*d*x + 11*c) + 4800*a^2*b*e^(9*d*x + 9
*c) + 143232*a*b^2*e^(9*d*x + 9*c) + 1645*b^3*e^(9*d*x + 9*c) - 4800*a^2*b*
e^(7*d*x + 7*c) + 143232*a*b^2*e^(7*d*x + 7*c) - 1645*b^3*e^(7*d*x + 7*c) -
 8640*a^2*b*e^(5*d*x + 5*c) + 88704*a*b^2*e^(5*d*x + 5*c) - 9065*b^3*e^(5*d
*x + 5*c) - 4800*a^2*b*e^(3*d*x + 3*c) + 32640*a*b^2*e^(3*d*x + 3*c) - 3925
*b^3*e^(3*d*x + 3*c) - 960*a^2*b*e^(d*x + c) + 5760*a*b^2*e^(d*x + c) - 162
5*b^3*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^8)/d
```

3.69 $\int \mathbf{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=219

$$\frac{3a^2b\tan^{-1}(\sinh(c+dx))}{2d}-\frac{3a^2b\tanh(c+dx)\mathbf{sech}(c+dx)}{2d}-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}-\frac{3ab^2\mathbf{sech}^5(c+dx)}{5d}+\frac{2ab^2\mathrm{s}}{}$$

```
[Out] (3*a^2*b*ArcTan[Sinh[c + d*x]])/(2*d) + (35*b^3*ArcTan[Sinh[c + d*x]])/(128
*d) - (a^3*ArcTanh[Cosh[c + d*x]])/d - (3*a*b^2*Sech[c + d*x])/d + (2*a*b^2
*Sech[c + d*x]^3)/d - (3*a*b^2*Sech[c + d*x]^5)/(5*d) - (3*a^2*b*Sech[c + d
*x]*Tanh[c + d*x])/(2*d) - (35*b^3*Sech[c + d*x]*Tanh[c + d*x])/(128*d) - (
35*b^3*Sech[c + d*x]*Tanh[c + d*x]^3)/(192*d) - (7*b^3*Sech[c + d*x]*Tanh[c
 + d*x]^5)/(48*d) - (b^3*Sech[c + d*x]*Tanh[c + d*x]^7)/(8*d)
```

Rubi [A] time = 0.2635, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3666, 3770, 2611, 2606, 194}

$$\frac{3a^2b\tan^{-1}(\sinh(c+dx))}{2d}-\frac{3a^2b\tanh(c+dx)\mathbf{sech}(c+dx)}{2d}-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}-\frac{3ab^2\mathbf{sech}^5(c+dx)}{5d}+\frac{2ab^2\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] (3*a^2*b*ArcTan[Sinh[c + d*x]])/(2*d) + (35*b^3*ArcTan[Sinh[c + d*x]])/(128
*d) - (a^3*ArcTanh[Cosh[c + d*x]])/d - (3*a*b^2*Sech[c + d*x])/d + (2*a*b^2
*Sech[c + d*x]^3)/d - (3*a*b^2*Sech[c + d*x]^5)/(5*d) - (3*a^2*b*Sech[c + d
*x]*Tanh[c + d*x])/(2*d) - (35*b^3*Sech[c + d*x]*Tanh[c + d*x])/(128*d) - (
35*b^3*Sech[c + d*x]*Tanh[c + d*x]^3)/(192*d) - (7*b^3*Sech[c + d*x]*Tanh[c
 + d*x]^5)/(48*d) - (b^3*Sech[c + d*x]*Tanh[c + d*x]^7)/(8*d)
```

Rule 3666

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^3 dx = i\int \left(-ia^3\operatorname{csch}(c+dx)-3ia^2b\operatorname{sech}(c+dx)\tanh^2(c+dx)-3iab^2\operatorname{sech}(c+
$$
$$= a^3\int \operatorname{csch}(c+dx)\,dx+\left(3a^2b\right)\int \operatorname{sech}(c+dx)\tanh^2(c+dx)\,dx+\left(3ab^2\right)\int$$
$$= -\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}-\frac{3a^2b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}-\frac{b^3\operatorname{sech}(c+}{}$$
$$= \frac{3a^2b\tan^{-1}(\sinh(c+dx))}{2d}-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}-\frac{3a^2b\operatorname{sech}(c+dx)\,\mathrm{ta}}{2d}$$
$$= \frac{3a^2b\tan^{-1}(\sinh(c+dx))}{2d}-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}-\frac{3ab^2\operatorname{sech}(c+dx)}{d}+$$
$$= \frac{3a^2b\tan^{-1}(\sinh(c+dx))}{2d}-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}-\frac{3ab^2\operatorname{sech}(c+dx)}{d}+$$
$$= \frac{3a^2b\tan^{-1}(\sinh(c+dx))}{2d}+\frac{35b^3\tan^{-1}(\sinh(c+dx))}{128d}-\frac{a^3\tanh^{-1}(\cosh(c}{d}$$

Mathematica [A] time = 2.88559, size = 154, normalized size = 0.7

$$\frac{-45b\operatorname{sech}(c+dx)\left(\left(64a^2+31b^2\right)\tanh(c+dx)+128ab\right)+30\left(b\left(192a^2+35b^2\right)\tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+64a^3\log\left(\mathrm{ta}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] (30*(b*(192*a^2 + 35*b^2)*ArcTan[Tanh[(c + d*x)/2]] + 64*a^3*Log[Tanh[(c +
d*x)/2]]) + 240*b^3*Sech[c + d*x]^7*Tanh[c + d*x] - 8*b^2*Sech[c + d*x]^5*(
144*a + 125*b*Tanh[c + d*x]) + 10*b^2*Sech[c + d*x]^3*(384*a + 163*b*Tanh[c
 + d*x]) - 45*b*Sech[c + d*x]*(128*a*b + (64*a^2 + 31*b^2)*Tanh[c + d*x]))/
(1920*d)
```

Maple [A] time = 0.096, size = 387, normalized size = 1.8

$$-2\,\frac{a^3\operatorname{Artanh}\left(e^{dx+c}\right)}{d}-3\,\frac{a^2b\sinh(dx+c)}{d\left(\cosh(dx+c)\right)^2}+\frac{3\,a^2b\operatorname{sech}(dx+c)\tanh(dx+c)}{2\,d}+3\,\frac{a^2b\arctan\left(e^{dx+c}\right)}{d}-3\,\frac{ab^2\left(\sinh(dx\right.}{d\left(\cosh(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x)
```

```
[Out] -2/d*a^3*arctanh(exp(d*x+c))-3/d*a^2*b*sinh(d*x+c)/cosh(d*x+c)^2+3/2*a^2*b*
sech(d*x+c)*tanh(d*x+c)/d+3/d*a^2*b*arctan(exp(d*x+c))-3/d*a*b^2*sinh(d*x+c
)^4/cosh(d*x+c)^5-12/5/d*a*b^2*sinh(d*x+c)^2/cosh(d*x+c)^5+8/5/d*a*b^2*sinh
```

```
(d*x+c)^2/cosh(d*x+c)^3+8/5/d*a*b^2*sinh(d*x+c)^2/cosh(d*x+c)-8/5*a*b^2*cos
h(d*x+c)/d-1/d*b^3*sinh(d*x+c)^7/cosh(d*x+c)^8-7/3/d*b^3*sinh(d*x+c)^5/cosh
(d*x+c)^8-7/3/d*b^3*sinh(d*x+c)^3/cosh(d*x+c)^8-1/d*b^3*sinh(d*x+c)/cosh(d*
x+c)^8+1/8/d*b^3*tanh(d*x+c)*sech(d*x+c)^7+7/48/d*b^3*tanh(d*x+c)*sech(d*x+
c)^5+35/192*b^3*sech(d*x+c)^3*tanh(d*x+c)/d+35/128*b^3*sech(d*x+c)*tanh(d*x
+c)/d+35/64/d*b^3*arctan(exp(d*x+c))
```

Maxima [B] time = 1.72007, size = 883, normalized size = 4.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] -1/192*b^3*(105*arctan(e^(-d*x - c))/d + (279*e^(-d*x - c) + 91*e^(-3*d*x -
 3*c) + 1799*e^(-5*d*x - 5*c) - 1085*e^(-7*d*x - 7*c) + 1085*e^(-9*d*x - 9*
c) - 1799*e^(-11*d*x - 11*c) - 91*e^(-13*d*x - 13*c) - 279*e^(-15*d*x - 15*
c))/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70
*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-1
4*d*x - 14*c) + e^(-16*d*x - 16*c) + 1))) - 3*a^2*b*(arctan(e^(-d*x - c))/d
 + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4
*c) + 1))) - 2/5*a*b^2*(15*e^(-d*x - c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d
*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) +
 1)) + 20*e^(-3*d*x - 3*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 1
0*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 58*e^(
-5*d*x - 5*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x -
 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 20*e^(-7*d*x - 7*c)
/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(
-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*e^(-9*d*x - 9*c)/(d*(5*e^(-2*
d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c)
 + e^(-10*d*x - 10*c) + 1))) + a^3*log(tanh(1/2*d*x + 1/2*c))/d
```

Fricas [B] time = 3.60023, size = 19499, normalized size = 89.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] -1/960*(45*(64*a^2*b + 128*a*b^2 + 31*b^3)*cosh(d*x + c)^15 + 675*(64*a^2*b
 + 128*a*b^2 + 31*b^3)*cosh(d*x + c)*sinh(d*x + c)^14 + 45*(64*a^2*b + 128*
a*b^2 + 31*b^3)*sinh(d*x + c)^15 + 5*(2880*a^2*b + 4992*a*b^2 + 91*b^3)*cos
h(d*x + c)^13 + 5*(2880*a^2*b + 4992*a*b^2 + 91*b^3 + 945*(64*a^2*b + 128*a
*b^2 + 31*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^13 + 65*(315*(64*a^2*b + 128*
a*b^2 + 31*b^3)*cosh(d*x + c)^3 + (2880*a^2*b + 4992*a*b^2 + 91*b^3)*cosh(d
*x + c))*sinh(d*x + c)^12 + (25920*a^2*b + 62592*a*b^2 + 8995*b^3)*cosh(d*x
 + c)^11 + (61425*(64*a^2*b + 128*a*b^2 + 31*b^3)*cosh(d*x + c)^4 + 25920*a
^2*b + 62592*a*b^2 + 8995*b^3 + 390*(2880*a^2*b + 4992*a*b^2 + 91*b^3)*cosh
(d*x + c)^2)*sinh(d*x + c)^11 + 11*(12285*(64*a^2*b + 128*a*b^2 + 31*b^3)*c
osh(d*x + c)^5 + 130*(2880*a^2*b + 4992*a*b^2 + 91*b^3)*cosh(d*x + c)^3 + (
25920*a^2*b + 62592*a*b^2 + 8995*b^3)*cosh(d*x + c))*sinh(d*x + c)^10 + (14
400*a^2*b + 103296*a*b^2 - 5425*b^3)*cosh(d*x + c)^9 + (225225*(64*a^2*b +
128*a*b^2 + 31*b^3)*cosh(d*x + c)^6 + 3575*(2880*a^2*b + 4992*a*b^2 + 91*b^
```

```
3)*cosh(d*x + c)^4 + 14400*a^2*b + 103296*a*b^2 - 5425*b^3 + 55*(25920*a^2*
b + 62592*a*b^2 + 8995*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^9 + 3*(96525*(64
*a^2*b + 128*a*b^2 + 31*b^3)*cosh(d*x + c)^7 + 2145*(2880*a^2*b + 4992*a*b^
2 + 91*b^3)*cosh(d*x + c)^5 + 55*(25920*a^2*b + 62592*a*b^2 + 8995*b^3)*cos
h(d*x + c)^3 + 3*(14400*a^2*b + 103296*a*b^2 - 5425*b^3)*cosh(d*x + c))*sin
h(d*x + c)^8 - (14400*a^2*b - 103296*a*b^2 - 5425*b^3)*cosh(d*x + c)^7 + (2
89575*(64*a^2*b + 128*a*b^2 + 31*b^3)*cosh(d*x + c)^8 + 8580*(2880*a^2*b +
4992*a*b^2 + 91*b^3)*cosh(d*x + c)^6 + 330*(25920*a^2*b + 62592*a*b^2 + 899
5*b^3)*cosh(d*x + c)^4 - 14400*a^2*b + 103296*a*b^2 + 5425*b^3 + 36*(14400*
a^2*b + 103296*a*b^2 - 5425*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + (225225
*(64*a^2*b + 128*a*b^2 + 31*b^3)*cosh(d*x + c)^9 + 8580*(2880*a^2*b + 4992*
a*b^2 + 91*b^3)*cosh(d*x + c)^7 + 462*(25920*a^2*b + 62592*a*b^2 + 8995*b^3
)*cosh(d*x + c)^5 + 84*(14400*a^2*b + 103296*a*b^2 - 5425*b^3)*cosh(d*x + c
)^3 - 7*(14400*a^2*b - 103296*a*b^2 - 5425*b^3)*cosh(d*x + c))*sinh(d*x + c
)^6 - (25920*a^2*b - 62592*a*b^2 + 8995*b^3)*cosh(d*x + c)^5 + (135135*(64*
a^2*b + 128*a*b^2 + 31*b^3)*cosh(d*x + c)^10 + 6435*(2880*a^2*b + 4992*a*b^
2 + 91*b^3)*cosh(d*x + c)^8 + 462*(25920*a^2*b + 62592*a*b^2 + 8995*b^3)*co
sh(d*x + c)^6 + 126*(14400*a^2*b + 103296*a*b^2 - 5425*b^3)*cosh(d*x + c)^4
 - 25920*a^2*b + 62592*a*b^2 - 8995*b^3 - 21*(14400*a^2*b - 103296*a*b^2 -
5425*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + (61425*(64*a^2*b + 128*a*b^2 +
 31*b^3)*cosh(d*x + c)^11 + 3575*(2880*a^2*b + 4992*a*b^2 + 91*b^3)*cosh(d*
x + c)^9 + 330*(25920*a^2*b + 62592*a*b^2 + 8995*b^3)*cosh(d*x + c)^7 + 126
*(14400*a^2*b + 103296*a*b^2 - 5425*b^3)*cosh(d*x + c)^5 - 35*(14400*a^2*b
- 103296*a*b^2 - 5425*b^3)*cosh(d*x + c)^3 - 5*(25920*a^2*b - 62592*a*b^2 +
 8995*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 - 5*(2880*a^2*b - 4992*a*b^2 + 91
*b^3)*cosh(d*x + c)^3 + (20475*(64*a^2*b + 128*a*b^2 + 31*b^3)*cosh(d*x + c
)^12 + 1430*(2880*a^2*b + 4992*a*b^2 + 91*b^3)*cosh(d*x + c)^10 + 165*(2592
0*a^2*b + 62592*a*b^2 + 8995*b^3)*cosh(d*x + c)^8 + 84*(14400*a^2*b + 10329
6*a*b^2 - 5425*b^3)*cosh(d*x + c)^6 - 35*(14400*a^2*b - 103296*a*b^2 - 5425
*b^3)*cosh(d*x + c)^4 - 14400*a^2*b + 24960*a*b^2 - 455*b^3 - 10*(25920*a^2
*b - 62592*a*b^2 + 8995*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + (4725*(64*a
^2*b + 128*a*b^2 + 31*b^3)*cosh(d*x + c)^13 + 390*(2880*a^2*b + 4992*a*b^2
+ 91*b^3)*cosh(d*x + c)^11 + 55*(25920*a^2*b + 62592*a*b^2 + 8995*b^3)*cosh
(d*x + c)^9 + 36*(14400*a^2*b + 103296*a*b^2 - 5425*b^3)*cosh(d*x + c)^7 -
21*(14400*a^2*b - 103296*a*b^2 - 5425*b^3)*cosh(d*x + c)^5 - 10*(25920*a^2*
b - 62592*a*b^2 + 8995*b^3)*cosh(d*x + c)^3 - 15*(2880*a^2*b - 4992*a*b^2 +
 91*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - 15*((192*a^2*b + 35*b^3)*cosh(d*x
 + c)^16 + 16*(192*a^2*b + 35*b^3)*cosh(d*x + c)*sinh(d*x + c)^15 + (192*a^
2*b + 35*b^3)*sinh(d*x + c)^16 + 8*(192*a^2*b + 35*b^3)*cosh(d*x + c)^14 +
8*(192*a^2*b + 35*b^3 + 15*(192*a^2*b + 35*b^3)*cosh(d*x + c)^2)*sinh(d*x +
 c)^14 + 112*(5*(192*a^2*b + 35*b^3)*cosh(d*x + c)^3 + (192*a^2*b + 35*b^3)
*cosh(d*x + c))*sinh(d*x + c)^13 + 28*(192*a^2*b + 35*b^3)*cosh(d*x + c)^12
 + 28*(65*(192*a^2*b + 35*b^3)*cosh(d*x + c)^4 + 192*a^2*b + 35*b^3 + 26*(1
92*a^2*b + 35*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^12 + 112*(39*(192*a^2*b +
 35*b^3)*cosh(d*x + c)^5 + 26*(192*a^2*b + 35*b^3)*cosh(d*x + c)^3 + 3*(192
*a^2*b + 35*b^3)*cosh(d*x + c))*sinh(d*x + c)^11 + 56*(192*a^2*b + 35*b^3)*
cosh(d*x + c)^10 + 56*(143*(192*a^2*b + 35*b^3)*cosh(d*x + c)^6 + 143*(192*
a^2*b + 35*b^3)*cosh(d*x + c)^4 + 192*a^2*b + 35*b^3 + 33*(192*a^2*b + 35*b
^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 16*(715*(192*a^2*b + 35*b^3)*cosh(d
*x + c)^7 + 1001*(192*a^2*b + 35*b^3)*cosh(d*x + c)^5 + 385*(192*a^2*b + 35
*b^3)*cosh(d*x + c)^3 + 35*(192*a^2*b + 35*b^3)*cosh(d*x + c))*sinh(d*x + c
)^9 + 70*(192*a^2*b + 35*b^3)*cosh(d*x + c)^8 + 2*(6435*(192*a^2*b + 35*b^3
)*cosh(d*x + c)^8 + 12012*(192*a^2*b + 35*b^3)*cosh(d*x + c)^6 + 6930*(192*
a^2*b + 35*b^3)*cosh(d*x + c)^4 + 6720*a^2*b + 1225*b^3 + 1260*(192*a^2*b +
 35*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 16*(715*(192*a^2*b + 35*b^3)*co
sh(d*x + c)^9 + 1716*(192*a^2*b + 35*b^3)*cosh(d*x + c)^7 + 1386*(192*a^2*b
 + 35*b^3)*cosh(d*x + c)^5 + 420*(192*a^2*b + 35*b^3)*cosh(d*x + c)^3 + 35*
(192*a^2*b + 35*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 56*(192*a^2*b + 35*b^
3)*cosh(d*x + c)^6 + 56*(143*(192*a^2*b + 35*b^3)*cosh(d*x + c)^10 + 429*(1
```

```
92*a^2*b + 35*b^3)*cosh(d*x + c)^8 + 462*(192*a^2*b + 35*b^3)*cosh(d*x + c)
^6 + 210*(192*a^2*b + 35*b^3)*cosh(d*x + c)^4 + 192*a^2*b + 35*b^3 + 35*(19
2*a^2*b + 35*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 112*(39*(192*a^2*b + 3
5*b^3)*cosh(d*x + c)^11 + 143*(192*a^2*b + 35*b^3)*cosh(d*x + c)^9 + 198*(1
92*a^2*b + 35*b^3)*cosh(d*x + c)^7 + 126*(192*a^2*b + 35*b^3)*cosh(d*x + c)
^5 + 35*(192*a^2*b + 35*b^3)*cosh(d*x + c)^3 + 3*(192*a^2*b + 35*b^3)*cosh(
d*x + c))*sinh(d*x + c)^5 + 28*(192*a^2*b + 35*b^3)*cosh(d*x + c)^4 + 28*(6
5*(192*a^2*b + 35*b^3)*cosh(d*x + c)^12 + 286*(192*a^2*b + 35*b^3)*cosh(d*x
 + c)^10 + 495*(192*a^2*b + 35*b^3)*cosh(d*x + c)^8 + 420*(192*a^2*b + 35*b
^3)*cosh(d*x + c)^6 + 175*(192*a^2*b + 35*b^3)*cosh(d*x + c)^4 + 192*a^2*b
+ 35*b^3 + 30*(192*a^2*b + 35*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 112*(
5*(192*a^2*b + 35*b^3)*cosh(d*x + c)^13 + 26*(192*a^2*b + 35*b^3)*cosh(d*x
+ c)^11 + 55*(192*a^2*b + 35*b^3)*cosh(d*x + c)^9 + 60*(192*a^2*b + 35*b^3)
*cosh(d*x + c)^7 + 35*(192*a^2*b + 35*b^3)*cosh(d*x + c)^5 + 10*(192*a^2*b
+ 35*b^3)*cosh(d*x + c)^3 + (192*a^2*b + 35*b^3)*cosh(d*x + c))*sinh(d*x +
c)^3 + 192*a^2*b + 35*b^3 + 8*(192*a^2*b + 35*b^3)*cosh(d*x + c)^2 + 8*(15*
(192*a^2*b + 35*b^3)*cosh(d*x + c)^14 + 91*(192*a^2*b + 35*b^3)*cosh(d*x +
c)^12 + 231*(192*a^2*b + 35*b^3)*cosh(d*x + c)^10 + 315*(192*a^2*b + 35*b^3
)*cosh(d*x + c)^8 + 245*(192*a^2*b + 35*b^3)*cosh(d*x + c)^6 + 105*(192*a^2
*b + 35*b^3)*cosh(d*x + c)^4 + 192*a^2*b + 35*b^3 + 21*(192*a^2*b + 35*b^3)
*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 16*((192*a^2*b + 35*b^3)*cosh(d*x + c)^
15 + 7*(192*a^2*b + 35*b^3)*cosh(d*x + c)^13 + 21*(192*a^2*b + 35*b^3)*cosh
(d*x + c)^11 + 35*(192*a^2*b + 35*b^3)*cosh(d*x + c)^9 + 35*(192*a^2*b + 35
*b^3)*cosh(d*x + c)^7 + 21*(192*a^2*b + 35*b^3)*cosh(d*x + c)^5 + 7*(192*a^
2*b + 35*b^3)*cosh(d*x + c)^3 + (192*a^2*b + 35*b^3)*cosh(d*x + c))*sinh(d*
x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - 45*(64*a^2*b - 128*a*b^2 +
31*b^3)*cosh(d*x + c) + 960*(a^3*cosh(d*x + c)^16 + 16*a^3*cosh(d*x + c)*si
nh(d*x + c)^15 + a^3*sinh(d*x + c)^16 + 8*a^3*cosh(d*x + c)^14 + 28*a^3*cos
h(d*x + c)^12 + 8*(15*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^14 + 112*(5*
a^3*cosh(d*x + c)^3 + a^3*cosh(d*x + c))*sinh(d*x + c)^13 + 56*a^3*cosh(d*x
 + c)^10 + 28*(65*a^3*cosh(d*x + c)^4 + 26*a^3*cosh(d*x + c)^2 + a^3)*sinh(
d*x + c)^12 + 112*(39*a^3*cosh(d*x + c)^5 + 26*a^3*cosh(d*x + c)^3 + 3*a^3*
cosh(d*x + c))*sinh(d*x + c)^11 + 70*a^3*cosh(d*x + c)^8 + 56*(143*a^3*cosh
(d*x + c)^6 + 143*a^3*cosh(d*x + c)^4 + 33*a^3*cosh(d*x + c)^2 + a^3)*sinh(
d*x + c)^10 + 16*(715*a^3*cosh(d*x + c)^7 + 1001*a^3*cosh(d*x + c)^5 + 385*
a^3*cosh(d*x + c)^3 + 35*a^3*cosh(d*x + c))*sinh(d*x + c)^9 + 56*a^3*cosh(d
*x + c)^6 + 2*(6435*a^3*cosh(d*x + c)^8 + 12012*a^3*cosh(d*x + c)^6 + 6930*
a^3*cosh(d*x + c)^4 + 1260*a^3*cosh(d*x + c)^2 + 35*a^3)*sinh(d*x + c)^8 +
16*(715*a^3*cosh(d*x + c)^9 + 1716*a^3*cosh(d*x + c)^7 + 1386*a^3*cosh(d*x
+ c)^5 + 420*a^3*cosh(d*x + c)^3 + 35*a^3*cosh(d*x + c))*sinh(d*x + c)^7 +
28*a^3*cosh(d*x + c)^4 + 56*(143*a^3*cosh(d*x + c)^10 + 429*a^3*cosh(d*x +
c)^8 + 462*a^3*cosh(d*x + c)^6 + 210*a^3*cosh(d*x + c)^4 + 35*a^3*cosh(d*x
+ c)^2 + a^3)*sinh(d*x + c)^6 + 112*(39*a^3*cosh(d*x + c)^11 + 143*a^3*cosh
(d*x + c)^9 + 198*a^3*cosh(d*x + c)^7 + 126*a^3*cosh(d*x + c)^5 + 35*a^3*co
sh(d*x + c)^3 + 3*a^3*cosh(d*x + c))*sinh(d*x + c)^5 + 8*a^3*cosh(d*x + c)^
2 + 28*(65*a^3*cosh(d*x + c)^12 + 286*a^3*cosh(d*x + c)^10 + 495*a^3*cosh(d
*x + c)^8 + 420*a^3*cosh(d*x + c)^6 + 175*a^3*cosh(d*x + c)^4 + 30*a^3*cosh
(d*x + c)^2 + a^3)*sinh(d*x + c)^4 + 112*(5*a^3*cosh(d*x + c)^13 + 26*a^3*c
osh(d*x + c)^11 + 55*a^3*cosh(d*x + c)^9 + 60*a^3*cosh(d*x + c)^7 + 35*a^3*
cosh(d*x + c)^5 + 10*a^3*cosh(d*x + c)^3 + a^3*cosh(d*x + c))*sinh(d*x + c)
^3 + a^3 + 8*(15*a^3*cosh(d*x + c)^14 + 91*a^3*cosh(d*x + c)^12 + 231*a^3*c
osh(d*x + c)^10 + 315*a^3*cosh(d*x + c)^8 + 245*a^3*cosh(d*x + c)^6 + 105*a
^3*cosh(d*x + c)^4 + 21*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^2 + 16*(a^
3*cosh(d*x + c)^15 + 7*a^3*cosh(d*x + c)^13 + 21*a^3*cosh(d*x + c)^11 + 35*
a^3*cosh(d*x + c)^9 + 35*a^3*cosh(d*x + c)^7 + 21*a^3*cosh(d*x + c)^5 + 7*a
^3*cosh(d*x + c)^3 + a^3*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) +
sinh(d*x + c) + 1) - 960*(a^3*cosh(d*x + c)^16 + 16*a^3*cosh(d*x + c)*sinh(
d*x + c)^15 + a^3*sinh(d*x + c)^16 + 8*a^3*cosh(d*x + c)^14 + 28*a^3*cosh(d
*x + c)^12 + 8*(15*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^14 + 112*(5*a^3
```

*cosh(d*x + c)^3 + a^3*cosh(d*x + c))*sinh(d*x + c)^13 + 56*a^3*cosh(d*x + c)^10 + 28*(65*a^3*cosh(d*x + c)^4 + 26*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^12 + 112*(39*a^3*cosh(d*x + c)^5 + 26*a^3*cosh(d*x + c)^3 + 3*a^3*cosh(d*x + c))*sinh(d*x + c)^11 + 70*a^3*cosh(d*x + c)^8 + 56*(143*a^3*cosh(d*x + c)^6 + 143*a^3*cosh(d*x + c)^4 + 33*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^10 + 16*(715*a^3*cosh(d*x + c)^7 + 1001*a^3*cosh(d*x + c)^5 + 385*a^3*cosh(d*x + c)^3 + 35*a^3*cosh(d*x + c))*sinh(d*x + c)^9 + 56*a^3*cosh(d*x + c)^6 + 2*(6435*a^3*cosh(d*x + c)^8 + 12012*a^3*cosh(d*x + c)^6 + 6930*a^3*cosh(d*x + c)^4 + 1260*a^3*cosh(d*x + c)^2 + 35*a^3)*sinh(d*x + c)^8 + 16*(715*a^3*cosh(d*x + c)^9 + 1716*a^3*cosh(d*x + c)^7 + 1386*a^3*cosh(d*x + c)^5 + 420*a^3*cosh(d*x + c)^3 + 35*a^3*cosh(d*x + c))*sinh(d*x + c)^7 + 28*a^3*cosh(d*x + c)^4 + 56*(143*a^3*cosh(d*x + c)^10 + 429*a^3*cosh(d*x + c)^8 + 462*a^3*cosh(d*x + c)^6 + 210*a^3*cosh(d*x + c)^4 + 35*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^6 + 112*(39*a^3*cosh(d*x + c)^11 + 143*a^3*cosh(d*x + c)^9 + 198*a^3*cosh(d*x + c)^7 + 126*a^3*cosh(d*x + c)^5 + 35*a^3*cosh(d*x + c)^3 + 3*a^3*cosh(d*x + c))*sinh(d*x + c)^5 + 8*a^3*cosh(d*x + c)^2 + 28*(65*a^3*cosh(d*x + c)^12 + 286*a^3*cosh(d*x + c)^10 + 495*a^3*cosh(d*x + c)^8 + 420*a^3*cosh(d*x + c)^6 + 175*a^3*cosh(d*x + c)^4 + 30*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^4 + 112*(5*a^3*cosh(d*x + c)^13 + 26*a^3*cosh(d*x + c)^11 + 55*a^3*cosh(d*x + c)^9 + 60*a^3*cosh(d*x + c)^7 + 35*a^3*cosh(d*x + c)^5 + 10*a^3*cosh(d*x + c)^3 + a^3*cosh(d*x + c))*sinh(d*x + c)^3 + a^3 + 8*(15*a^3*cosh(d*x + c)^14 + 91*a^3*cosh(d*x + c)^12 + 231*a^3*cosh(d*x + c)^10 + 315*a^3*cosh(d*x + c)^8 + 245*a^3*cosh(d*x + c)^6 + 105*a^3*cosh(d*x + c)^4 + 21*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^2 + 16*(a^3*cosh(d*x + c)^15 + 7*a^3*cosh(d*x + c)^13 + 21*a^3*cosh(d*x + c)^11 + 35*a^3*cosh(d*x + c)^9 + 35*a^3*cosh(d*x + c)^7 + 21*a^3*cosh(d*x + c)^5 + 7*a^3*cosh(d*x + c)^3 + a^3*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + (675*(64*a^2*b + 128*a*b^2 + 31*b^3)*cosh(d*x + c)^14 + 65*(2880*a^2*b + 4992*a*b^2 + 91*b^3)*cosh(d*x + c)^12 + 11*(25920*a^2*b + 62592*a*b^2 + 8995*b^3)*cosh(d*x + c)^10 + 9*(14400*a^2*b + 103296*a*b^2 - 5425*b^3)*cosh(d*x + c)^8 - 7*(14400*a^2*b - 103296*a*b^2 - 5425*b^3)*cosh(d*x + c)^6 - 5*(25920*a^2*b - 62592*a*b^2 + 8995*b^3)*cosh(d*x + c)^4 - 2880*a^2*b + 5760*a*b^2 - 1395*b^3 - 15*(2880*a^2*b - 4992*a*b^2 + 91*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^16 + 16*d*cosh(d*x + c)*sinh(d*x + c)^15 + d*sinh(d*x + c)^16 + 8*d*cosh(d*x + c)^14 + 8*(15*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^14 + 112*(5*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^13 + 28*d*cosh(d*x + c)^12 + 28*(65*d*cosh(d*x + c)^4 + 26*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^12 + 112*(39*d*cosh(d*x + c)^5 + 26*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^11 + 56*d*cosh(d*x + c)^10 + 56*(143*d*cosh(d*x + c)^6 + 143*d*cosh(d*x + c)^4 + 33*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^10 + 16*(715*d*cosh(d*x + c)^7 + 1001*d*cosh(d*x + c)^5 + 385*d*cosh(d*x + c)^3 + 35*d*cosh(d*x + c))*sinh(d*x + c)^9 + 70*d*cosh(d*x + c)^8 + 2*(6435*d*cosh(d*x + c)^8 + 12012*d*cosh(d*x + c)^6 + 6930*d*cosh(d*x + c)^4 + 1260*d*cosh(d*x + c)^2 + 35*d)*sinh(d*x + c)^8 + 16*(715*d*cosh(d*x + c)^9 + 1716*d*cosh(d*x + c)^7 + 1386*d*cosh(d*x + c)^5 + 420*d*cosh(d*x + c)^3 + 35*d*cosh(d*x + c))*sinh(d*x + c)^7 + 56*d*cosh(d*x + c)^6 + 56*(143*d*cosh(d*x + c)^10 + 429*d*cosh(d*x + c)^8 + 462*d*cosh(d*x + c)^6 + 210*d*cosh(d*x + c)^4 + 35*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 112*(39*d*cosh(d*x + c)^11 + 143*d*cosh(d*x + c)^9 + 198*d*cosh(d*x + c)^7 + 126*d*cosh(d*x + c)^5 + 35*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 28*d*cosh(d*x + c)^4 + 28*(65*d*cosh(d*x + c)^12 + 286*d*cosh(d*x + c)^10 + 495*d*cosh(d*x + c)^8 + 420*d*cosh(d*x + c)^6 + 175*d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 112*(5*d*cosh(d*x + c)^13 + 26*d*cosh(d*x + c)^11 + 55*d*cosh(d*x + c)^9 + 60*d*cosh(d*x + c)^7 + 35*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^3 + 8*d*cosh(d*x + c)^2 + 8*(15*d*cosh(d*x + c)^14 + 91*d*cosh(d*x + c)^12 + 231*d*cosh(d*x + c)^10 + 315*d*cosh(d*x + c)^8 + 245*d*cosh(d*x + c)^6 + 105*d*cosh(d*x + c)^4 + 21*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 16*(d*cosh(d*x + c)^15 + 7*d*cosh(d*x + c)^13 + 21*d*cosh(d*x + c)^11 + 35*d*cosh

```
(d*x + c)^9 + 35*d*cosh(d*x + c)^7 + 21*d*cosh(d*x + c)^5 + 7*d*cosh(d*x +
c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tanh^3\left(c + dx\right)\right)^3 \operatorname{csch}\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)**3)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)**3*csch(c + d*x), x)
```

Giac [B] time = 2.28561, size = 570, normalized size = 2.6

$$-\frac{960\,a^3\log\left(e^{(dx+c)}+1\right) - 960\,a^3\log\left(\left|e^{(dx+c)}-1\right|\right) - 15\left(192\,a^2be^c + 35\,b^3e^c\right)\arctan\left(e^{(dx+c)}\right)e^{(-c)} + \frac{2880\,a^2be^{(15\,dx+15\,c)}+5\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] -1/960*(960*a^3*log(e^(d*x + c) + 1) - 960*a^3*log(abs(e^(d*x + c) - 1)) -
15*(192*a^2*b*e^c + 35*b^3*e^c)*arctan(e^(d*x + c))*e^(-c) + (2880*a^2*b*e^
(15*d*x + 15*c) + 5760*a*b^2*e^(15*d*x + 15*c) + 1395*b^3*e^(15*d*x + 15*c)
 + 14400*a^2*b*e^(13*d*x + 13*c) + 24960*a*b^2*e^(13*d*x + 13*c) + 455*b^3*
e^(13*d*x + 13*c) + 25920*a^2*b*e^(11*d*x + 11*c) + 62592*a*b^2*e^(11*d*x +
 11*c) + 8995*b^3*e^(11*d*x + 11*c) + 14400*a^2*b*e^(9*d*x + 9*c) + 103296*
a*b^2*e^(9*d*x + 9*c) - 5425*b^3*e^(9*d*x + 9*c) - 14400*a^2*b*e^(7*d*x + 7
*c) + 103296*a*b^2*e^(7*d*x + 7*c) + 5425*b^3*e^(7*d*x + 7*c) - 25920*a^2*b
*e^(5*d*x + 5*c) + 62592*a*b^2*e^(5*d*x + 5*c) - 8995*b^3*e^(5*d*x + 5*c) -
 14400*a^2*b*e^(3*d*x + 3*c) + 24960*a*b^2*e^(3*d*x + 3*c) - 455*b^3*e^(3*d
*x + 3*c) - 2880*a^2*b*e^(d*x + c) + 5760*a*b^2*e^(d*x + c) - 1395*b^3*e^(d
*x + c))/(e^(2*d*x + 2*c) + 1)^8)/d
```

3.70 $\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=71

$$\frac{3a^2b\tanh^2(c+dx)}{2d}-\frac{a^3\coth(c+dx)}{d}+\frac{3ab^2\tanh^5(c+dx)}{5d}+\frac{b^3\tanh^8(c+dx)}{8d}$$

```
[Out] -((a^3*Coth[c + d*x])/d) + (3*a^2*b*Tanh[c + d*x]^2)/(2*d) + (3*a*b^2*Tanh[
c + d*x]^5)/(5*d) + (b^3*Tanh[c + d*x]^8)/(8*d)
```

Rubi [A] time = 0.0645756, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3663, 270}

$$\frac{3a^2b\tanh^2(c+dx)}{2d}-\frac{a^3\coth(c+dx)}{d}+\frac{3ab^2\tanh^5(c+dx)}{5d}+\frac{b^3\tanh^8(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] -((a^3*Coth[c + d*x])/d) + (3*a^2*b*Tanh[c + d*x]^2)/(2*d) + (3*a*b^2*Tanh[
c + d*x]^5)/(5*d) + (b^3*Tanh[c + d*x]^8)/(8*d)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx&=\frac{\operatorname{Subst}\left(\int \frac{\left(a+bx^3\right)^3}{x^2}\,dx,x,\tanh(c+dx)\right)}{d}\\&=\frac{\operatorname{Subst}\left(\int \left(\frac{a^3}{x^2}+3a^2bx+3ab^2x^4+b^3x^7\right)dx,x,\tanh(c+dx)\right)}{d}\\&=-\frac{a^3\coth(c+dx)}{d}+\frac{3a^2b\tanh^2(c+dx)}{2d}+\frac{3ab^2\tanh^5(c+dx)}{5d}+\frac{b^3\tanh^8}{8d}\end{aligned}$$

Mathematica [A] time = 0.687407, size = 113, normalized size = 1.59

$$\frac{b\left(-4\operatorname{sech}^2(c+dx)\left(15a^2+12ab\tanh(c+dx)+5b^2\right)+24ab\tanh(c+dx)+6b\operatorname{sech}^4(c+dx)(4a\tanh(c+dx)+5b)+5}{40d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] (-40*a^3*Coth[c + d*x] + b*(-20*b^2*Sech[c + d*x]^6 + 5*b^2*Sech[c + d*x]^8
 + 24*a*b*Tanh[c + d*x] + 6*b*Sech[c + d*x]^4*(5*b + 4*a*Tanh[c + d*x]) - 4
*Sech[c + d*x]^2*(15*a^2 + 5*b^2 + 12*a*b*Tanh[c + d*x])))/(40*d)
```

Maple [B] time = 0.086, size = 223, normalized size = 3.1

$$\frac{1}{d}\left(-a^3\coth(dx+c)+\frac{3a^2b(\sinh(dx+c))^2}{2(\cosh(dx+c))^2}+3ab^2\left(-1/2\frac{(\sinh(dx+c))^3}{(\cosh(dx+c))^5}-3/8\frac{\sinh(dx+c)}{(\cosh(dx+c))^5}+3/8\left(\frac{8}{15}+1/5\,(\mathrm{s}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x)
```

```
[Out] 1/d*(-a^3*coth(d*x+c)+3/2*a^2*b*sinh(d*x+c)^2/cosh(d*x+c)^2+3*a*b^2*(-1/2*s
inh(d*x+c)^3/cosh(d*x+c)^5-3/8*sinh(d*x+c)/cosh(d*x+c)^5+3/8*(8/15+1/5*sech
(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c))+b^3*(-1/2*sinh(d*x+c)^6/cosh(d*x
+c)^8-3/4*sinh(d*x+c)^4/cosh(d*x+c)^8-3/8*sinh(d*x+c)^2/cosh(d*x+c)^8+1/8*s
inh(d*x+c)^2/cosh(d*x+c)^6+1/8*sinh(d*x+c)^2/cosh(d*x+c)^4+1/8*sinh(d*x+c)^
2/cosh(d*x+c)^2))
```

Maxima [B] time = 1.19483, size = 917, normalized size = 12.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] -2*b^3*(e^(-2*d*x - 2*c)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*
e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*
d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1)) + 7*e^(-6*d*x
 - 6*c)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c)
+ 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e
^(-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1)) + 7*e^(-10*d*x - 10*c)/(d*(8*e
^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*x
- 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*
c) + e^(-16*d*x - 16*c) + 1)) + e^(-14*d*x - 14*c)/(d*(8*e^(-2*d*x - 2*c) +
 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c) + 56*e^(-1
0*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e^(-16*d*x -
 16*c) + 1))) + 6/5*a*b^2*(10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
10*c) + 1)) + 5*e^(-8*d*x - 8*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*
c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) +
1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^
(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 2*a^3/(d*(e^(-2*d*x - 2*c) - 1
)) - 6*a^2*b/(d*(e^(d*x + c) + e^(-d*x - c))^2)
```

Fricas [B] time = 2.23443, size = 3071, normalized size = 43.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] -2/5*((10*a^3 + 15*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^8 + 8*(15*a^2*b
+ 18*a*b^2 + 5*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (10*a^3 + 15*a^2*b + 12
*a*b^2 + 5*b^3)*sinh(d*x + c)^8 + 2*(40*a^3 + 30*a^2*b + 12*a*b^2 - 5*b^3)*
cosh(d*x + c)^6 + 2*(40*a^3 + 30*a^2*b + 12*a*b^2 - 5*b^3 + 14*(10*a^3 + 15
*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(14*(15*a^2
*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 27*(5*a^2*b + 2*a*b^2)*cosh(d*x +
c))*sinh(d*x + c)^5 + 20*(14*a^3 + 3*a^2*b + 2*b^3)*cosh(d*x + c)^4 + 10*(7
*(10*a^3 + 15*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 28*a^3 + 6*a^2*b
+ 4*b^3 + 3*(40*a^3 + 30*a^2*b + 12*a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*
x + c)^4 + 8*(7*(15*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 45*(5*a^2*b
 + 2*a*b^2)*cosh(d*x + c)^3 + 15*(7*a^2*b + 2*a*b^2 + b^3)*cosh(d*x + c))*s
inh(d*x + c)^3 + 350*a^3 - 75*a^2*b - 12*a*b^2 + 35*b^3 + 2*(280*a^3 - 30*a
^2*b - 12*a*b^2 - 35*b^3)*cosh(d*x + c)^2 + 2*(14*(10*a^3 + 15*a^2*b + 12*a
*b^2 + 5*b^3)*cosh(d*x + c)^6 + 15*(40*a^3 + 30*a^2*b + 12*a*b^2 - 5*b^3)*c
osh(d*x + c)^4 + 280*a^3 - 30*a^2*b - 12*a*b^2 - 35*b^3 + 60*(14*a^3 + 3*a^
2*b + 2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(2*(15*a^2*b + 18*a*b^2 +
 5*b^3)*cosh(d*x + c)^7 + 27*(5*a^2*b + 2*a*b^2)*cosh(d*x + c)^5 + 30*(7*a^
2*b + 2*a*b^2 + b^3)*cosh(d*x + c)^3 + 21*(5*a^2*b + 2*a*b^2)*cosh(d*x + c)
)*sinh(d*x + c))/(d*cosh(d*x + c)^10 + 10*d*cosh(d*x + c)*sinh(d*x + c)^9 +
 d*sinh(d*x + c)^10 + 6*d*cosh(d*x + c)^8 + 3*(15*d*cosh(d*x + c)^2 + 2*d)*
sinh(d*x + c)^8 + 8*(15*d*cosh(d*x + c)^3 + 8*d*cosh(d*x + c))*sinh(d*x + c
)^7 + 13*d*cosh(d*x + c)^6 + (210*d*cosh(d*x + c)^4 + 168*d*cosh(d*x + c)^2
 + 13*d)*sinh(d*x + c)^6 + 2*(126*d*cosh(d*x + c)^5 + 224*d*cosh(d*x + c)^3
 + 81*d*cosh(d*x + c))*sinh(d*x + c)^5 + 8*d*cosh(d*x + c)^4 + (210*d*cosh(
d*x + c)^6 + 420*d*cosh(d*x + c)^4 + 195*d*cosh(d*x + c)^2 + 8*d)*sinh(d*x
+ c)^4 + 4*(30*d*cosh(d*x + c)^7 + 112*d*cosh(d*x + c)^5 + 135*d*cosh(d*x +
 c)^3 + 48*d*cosh(d*x + c))*sinh(d*x + c)^3 - 14*d*cosh(d*x + c)^2 + (45*d*
cosh(d*x + c)^8 + 168*d*cosh(d*x + c)^6 + 195*d*cosh(d*x + c)^4 + 48*d*cosh
(d*x + c)^2 - 14*d)*sinh(d*x + c)^2 + 2*(5*d*cosh(d*x + c)^9 + 32*d*cosh(d*
x + c)^7 + 81*d*cosh(d*x + c)^5 + 96*d*cosh(d*x + c)^3 + 42*d*cosh(d*x + c)
)*sinh(d*x + c) - 14*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\tanh^3\left(c+dx\right)\right)^3\operatorname{csch}^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*tanh(d*x+c)**3)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)**3*csch(c + d*x)**2, x)
```

Giac [B] time = 2.49159, size = 420, normalized size = 5.92

$$-\frac{2\left(\frac{5\,a^3}{e^{(2\,dx+2\,c)}-1}+\frac{15\,a^2be^{(14\,dx+14\,c)}+15\,ab^2e^{(14\,dx+14\,c)}+5\,b^3e^{(14\,dx+14\,c)}+90\,a^2be^{(12\,dx+12\,c)}+45\,ab^2e^{(12\,dx+12\,c)}+225\,a^2be^{(10\,dx+10\,c)}+75\,ab^2e^{(10\,dx+10\,c)}+35\,b^3e^{(\cdots}}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] -2/5*(5*a^3/(e^(2*d*x + 2*c) - 1) + (15*a^2*b*e^(14*d*x + 14*c) + 15*a*b^2*
e^(14*d*x + 14*c) + 5*b^3*e^(14*d*x + 14*c) + 90*a^2*b*e^(12*d*x + 12*c) +
45*a*b^2*e^(12*d*x + 12*c) + 225*a^2*b*e^(10*d*x + 10*c) + 75*a*b^2*e^(10*d
*x + 10*c) + 35*b^3*e^(10*d*x + 10*c) + 300*a^2*b*e^(8*d*x + 8*c) + 105*a*b
^2*e^(8*d*x + 8*c) + 225*a^2*b*e^(6*d*x + 6*c) + 93*a*b^2*e^(6*d*x + 6*c) +
 35*b^3*e^(6*d*x + 6*c) + 90*a^2*b*e^(4*d*x + 4*c) + 39*a*b^2*e^(4*d*x + 4*
c) + 15*a^2*b*e^(2*d*x + 2*c) + 9*a*b^2*e^(2*d*x + 2*c) + 5*b^3*e^(2*d*x +
2*c) + 3*a*b^2)/(e^(2*d*x + 2*c) + 1)^8)/d
```

3.71 $\int \mathbf{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=232

$$\frac{3a^2b\tan^{-1}(\sinh(c+dx))}{2d}+\frac{3a^2b\tanh(c+dx)\mathbf{sech}(c+dx)}{2d}+\frac{a^3\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^3\coth(c+dx)\mathbf{csch}(c+dx)}{2d}+$$

```
[Out] (3*a^2*b*ArcTan[Sinh[c + d*x]])/(2*d) + (5*b^3*ArcTan[Sinh[c + d*x]])/(128*
d) + (a^3*ArcTanh[Cosh[c + d*x]])/(2*d) - (a^3*Coth[c + d*x]*Csch[c + d*x])
/(2*d) - (a*b^2*Sech[c + d*x]^3)/d + (3*a*b^2*Sech[c + d*x]^5)/(5*d) + (3*a
^2*b*Sech[c + d*x]*Tanh[c + d*x])/(2*d) + (5*b^3*Sech[c + d*x]*Tanh[c + d*x
])/(128*d) - (5*b^3*Sech[c + d*x]^3*Tanh[c + d*x])/(64*d) - (5*b^3*Sech[c +
 d*x]^3*Tanh[c + d*x]^3)/(48*d) - (b^3*Sech[c + d*x]^3*Tanh[c + d*x]^5)/(8*
d)
```

Rubi [A] time = 0.316071, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3666, 3768, 3770, 2606, 14, 2611}

$$\frac{3a^2b\tan^{-1}(\sinh(c+dx))}{2d}+\frac{3a^2b\tanh(c+dx)\mathbf{sech}(c+dx)}{2d}+\frac{a^3\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^3\coth(c+dx)\mathbf{csch}(c+dx)}{2d}+$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^3,x]
```


```
[Out] (3*a^2*b*ArcTan[Sinh[c + d*x]])/(2*d) + (5*b^3*ArcTan[Sinh[c + d*x]])/(128*
d) + (a^3*ArcTanh[Cosh[c + d*x]])/(2*d) - (a^3*Coth[c + d*x]*Csch[c + d*x])
/(2*d) - (a*b^2*Sech[c + d*x]^3)/d + (3*a*b^2*Sech[c + d*x]^5)/(5*d) + (3*a
^2*b*Sech[c + d*x]*Tanh[c + d*x])/(2*d) + (5*b^3*Sech[c + d*x]*Tanh[c + d*x
])/(128*d) - (5*b^3*Sech[c + d*x]^3*Tanh[c + d*x])/(64*d) - (5*b^3*Sech[c +
 d*x]^3*Tanh[c + d*x]^3)/(48*d) - (b^3*Sech[c + d*x]^3*Tanh[c + d*x]^5)/(8*
d)
```


Rule 3666

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = -\left(i\int\left(ia^3\operatorname{csch}^3(c+dx)+3ia^2b\operatorname{sech}^3(c+dx)+3iab^2\operatorname{sech}^3(c+dx)\operatorname{ta}\right.\right.$$

$$= a^3\int\operatorname{csch}^3(c+dx)\,dx+\left(3a^2b\right)\int\operatorname{sech}^3(c+dx)\,dx+\left(3ab^2\right)\int\operatorname{sech}^3($$

$$= -\frac{a^3\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+\frac{3a^2b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}-\frac{b^3\operatorname{s}}{}$$

$$= \frac{3a^2b\tan^{-1}(\sinh(c+dx))}{2d}+\frac{a^3\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^3\coth(c+dx}{2d}$$

$$= \frac{3a^2b\tan^{-1}(\sinh(c+dx))}{2d}+\frac{a^3\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^3\coth(c+dx}{2d}$$

$$= \frac{3a^2b\tan^{-1}(\sinh(c+dx))}{2d}+\frac{a^3\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^3\coth(c+dx}{2d}$$

$$= \frac{3a^2b\tan^{-1}(\sinh(c+dx))}{2d}+\frac{5b^3\tan^{-1}(\sinh(c+dx))}{128d}+\frac{a^3\tanh^{-1}(\cosh}{2d}$$

Mathematica [A] time = 6.31567, size = 243, normalized size = 1.05

$$\frac{b\left(192a^2+5b^2\right)\tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{64d}+\frac{\operatorname{sech}^2(c+dx)\left(192a^2b\sinh(c+dx)+5b^3\sinh(c+dx)\right)}{128d}-\frac{a^3\operatorname{csch}^2\left(\frac{1}{2}(c\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] (b*(192*a^2 + 5*b^2)*ArcTan[Tanh[(c + d*x)/2]])/(64*d) - (a^3*Csch[(c + d*x
)/2]^2)/(8*d) - (a^3*Log[Tanh[(c + d*x)/2]])/(2*d) - (a^3*Sech[(c + d*x)/2]
^2)/(8*d) - (a*b^2*Sech[c + d*x]^3)/d + (3*a*b^2*Sech[c + d*x]^5)/(5*d) + (
Sech[c + d*x]^2*(192*a^2*b*Sinh[c + d*x] + 5*b^3*Sinh[c + d*x]))/(128*d) -
(59*b^3*Sech[c + d*x]^3*Tanh[c + d*x])/(192*d) + (17*b^3*Sech[c + d*x]^5*Ta
nh[c + d*x])/(48*d) - (b^3*Sech[c + d*x]^7*Tanh[c + d*x])/(8*d)
```

Maple [A] time = 0.099, size = 334, normalized size = 1.4

$$-\frac{a^3\coth(dx+c)\operatorname{csch}(dx+c)}{2d}+\frac{a^3\operatorname{Artanh}\left(e^{dx+c}\right)}{d}+\frac{3a^2b\operatorname{sech}(dx+c)\tanh(dx+c)}{2d}+3\frac{a^2b\arctan\left(e^{dx+c}\right)}{d}-\frac{3ab}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x)
```

```
[Out] -1/2*a^3*coth(d*x+c)*csch(d*x+c)/d+1/d*a^3*arctanh(exp(d*x+c))+3/2*a^2*b*se
ch(d*x+c)*tanh(d*x+c)/d+3/d*a^2*b*arctan(exp(d*x+c))-3/5/d*a*b^2*sinh(d*x+c
)^2/cosh(d*x+c)^5+2/5/d*a*b^2*sinh(d*x+c)^2/cosh(d*x+c)^3+2/5/d*a*b^2*sinh(
d*x+c)^2/cosh(d*x+c)-2/5*a*b^2*cosh(d*x+c)/d-1/3/d*b^3*sinh(d*x+c)^5/cosh(d
*x+c)^8-1/3/d*b^3*sinh(d*x+c)^3/cosh(d*x+c)^8-1/7/d*b^3*sinh(d*x+c)/cosh(d*
x+c)^8+1/56/d*b^3*tanh(d*x+c)*sech(d*x+c)^7+1/48/d*b^3*tanh(d*x+c)*sech(d*x
+c)^5+5/192*b^3*sech(d*x+c)^3*tanh(d*x+c)/d+5/128*b^3*sech(d*x+c)*tanh(d*x+
c)/d+5/64/d*b^3*arctan(exp(d*x+c))
```

Maxima [B] time = 1.77763, size = 791, normalized size = 3.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] -1/192*b^3*(15*arctan(e^(-d*x - c))/d - (15*e^(-d*x - c) - 397*e^(-3*d*x -
3*c) + 895*e^(-5*d*x - 5*c) - 1765*e^(-7*d*x - 7*c) + 1765*e^(-9*d*x - 9*c)
 - 895*e^(-11*d*x - 11*c) + 397*e^(-13*d*x - 13*c) - 15*e^(-15*d*x - 15*c))
/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^
(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d
*x - 14*c) + e^(-16*d*x - 16*c) + 1))) - 3*a^2*b*(arctan(e^(-d*x - c))/d -
(e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c)
 + 1))) + 1/2*a^3*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e
^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) -
 1))) - 8/5*a*b^2*(5*e^(-3*d*x - 3*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x
 - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1
)) - 2*e^(-5*d*x - 5*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e
^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 5*e^(-7*d
*x - 7*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c
) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)))
```

Fricas [B] time = 3.9052, size = 30318, normalized size = 130.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] -1/960*(15*(64*a^3 - 192*a^2*b - 5*b^3)*cosh(d*x + c)^19 + 285*(64*a^3 - 19
2*a^2*b - 5*b^3)*cosh(d*x + c)*sinh(d*x + c)^18 + 15*(64*a^3 - 192*a^2*b -
5*b^3)*sinh(d*x + c)^19 + 5*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3)*
cosh(d*x + c)^17 + 5*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3 + 513*(6
4*a^3 - 192*a^2*b - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^17 + 85*(171*(64*
a^3 - 192*a^2*b - 5*b^3)*cosh(d*x + c)^3 + (1728*a^3 - 1728*a^2*b + 1536*a*
b^2 + 427*b^3)*cosh(d*x + c))*sinh(d*x + c)^16 + 24*(1440*a^3 + 192*a*b^2 -
 355*b^3)*cosh(d*x + c)^15 + 4*(14535*(64*a^3 - 192*a^2*b - 5*b^3)*cosh(d*x
 + c)^4 + 8640*a^3 + 1152*a*b^2 - 2130*b^3 + 170*(1728*a^3 - 1728*a^2*b + 1
```

536*a*b^2 + 427*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^15 + 20*(8721*(64*a^3 - 192*a^2*b - 5*b^3)*cosh(d*x + c)^5 + 170*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3)*cosh(d*x + c)^3 + 18*(1440*a^3 + 192*a*b^2 - 355*b^3)*cosh(d*x + c))*sinh(d*x + c)^14 + 16*(5040*a^3 + 1440*a^2*b - 672*a*b^2 + 1235*b^3)*cosh(d*x + c)^13 + 4*(101745*(64*a^3 - 192*a^2*b - 5*b^3)*cosh(d*x + c)^6 + 2975*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3)*cosh(d*x + c)^4 + 20160*a^3 + 5760*a^2*b - 2688*a*b^2 + 4940*b^3 + 630*(1440*a^3 + 192*a*b^2 - 355*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^13 + 52*(14535*(64*a^3 - 192*a^2*b - 5*b^3)*cosh(d*x + c)^7 + 595*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3)*cosh(d*x + c)^5 + 210*(1440*a^3 + 192*a*b^2 - 355*b^3)*cosh(d*x + c)^3 + 4*(5040*a^3 + 1440*a^2*b - 672*a*b^2 + 1235*b^3)*cosh(d*x + c))*sinh(d*x + c)^12 + 2*(60480*a^3 + 8640*a^2*b - 768*a*b^2 - 15475*b^3)*cosh(d*x + c)^11 + 2*(566865*(64*a^3 - 192*a^2*b - 5*b^3)*cosh(d*x + c)^8 + 30940*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3)*cosh(d*x + c)^6 + 16380*(1440*a^3 + 192*a*b^2 - 355*b^3)*cosh(d*x + c)^4 + 60480*a^3 + 8640*a^2*b - 768*a*b^2 - 15475*b^3 + 624*(5040*a^3 + 1440*a^2*b - 672*a*b^2 + 1235*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^11 + 22*(62985*(64*a^3 - 192*a^2*b - 5*b^3)*cosh(d*x + c)^9 + 4420*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3)*cosh(d*x + c)^7 + 3276*(1440*a^3 + 192*a*b^2 - 355*b^3)*cosh(d*x + c)^5 + 208*(5040*a^3 + 1440*a^2*b - 672*a*b^2 + 1235*b^3)*cosh(d*x + c)^3 + (60480*a^3 + 8640*a^2*b - 768*a*b^2 - 15475*b^3)*cosh(d*x + c))*sinh(d*x + c)^10 + 2*(60480*a^3 - 8640*a^2*b - 768*a*b^2 + 15475*b^3)*cosh(d*x + c)^9 + 2*(692835*(64*a^3 - 192*a^2*b - 5*b^3)*cosh(d*x + c)^10 + 60775*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3)*cosh(d*x + c)^8 + 60060*(1440*a^3 + 192*a*b^2 - 355*b^3)*cosh(d*x + c)^6 + 5720*(5040*a^3 + 1440*a^2*b - 672*a*b^2 + 1235*b^3)*cosh(d*x + c)^4 + 60480*a^3 - 8640*a^2*b - 768*a*b^2 + 15475*b^3 + 55*(60480*a^3 + 8640*a^2*b - 768*a*b^2 - 15475*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^9 + 2*(566865*(64*a^3 - 192*a^2*b - 5*b^3)*cosh(d*x + c)^11 + 60775*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3)*cosh(d*x + c)^9 + 77220*(1440*a^3 + 192*a*b^2 - 355*b^3)*cosh(d*x + c)^7 + 10296*(5040*a^3 + 1440*a^2*b - 672*a*b^2 + 1235*b^3)*cosh(d*x + c)^5 + 165*(60480*a^3 + 8640*a^2*b - 768*a*b^2 - 15475*b^3)*cosh(d*x + c)^3 + 9*(60480*a^3 - 8640*a^2*b - 768*a*b^2 + 15475*b^3)*cosh(d*x + c))*sinh(d*x + c)^8 + 16*(5040*a^3 - 1440*a^2*b - 672*a*b^2 - 1235*b^3)*cosh(d*x + c)^7 + 4*(188955*(64*a^3 - 192*a^2*b - 5*b^3)*cosh(d*x + c)^12 + 24310*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3)*cosh(d*x + c)^10 + 38610*(1440*a^3 + 192*a*b^2 - 355*b^3)*cosh(d*x + c)^8 + 6864*(5040*a^3 + 1440*a^2*b - 672*a*b^2 + 1235*b^3)*cosh(d*x + c)^6 + 165*(60480*a^3 + 8640*a^2*b - 768*a*b^2 - 15475*b^3)*cosh(d*x + c)^4 + 20160*a^3 - 5760*a^2*b - 2688*a*b^2 - 4940*b^3 + 18*(60480*a^3 - 8640*a^2*b - 768*a*b^2 + 15475*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 4*(101745*(64*a^3 - 192*a^2*b - 5*b^3)*cosh(d*x + c)^13 + 15470*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3)*cosh(d*x + c)^11 + 30030*(1440*a^3 + 192*a*b^2 - 355*b^3)*cosh(d*x + c)^9 + 6864*(5040*a^3 + 1440*a^2*b - 672*a*b^2 + 1235*b^3)*cosh(d*x + c)^7 + 231*(60480*a^3 + 8640*a^2*b - 768*a*b^2 - 15475*b^3)*cosh(d*x + c)^5 + 42*(60480*a^3 - 8640*a^2*b - 768*a*b^2 + 15475*b^3)*cosh(d*x + c)^3 + 28*(5040*a^3 - 1440*a^2*b - 672*a*b^2 - 1235*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 24*(1440*a^3 + 192*a*b^2 + 355*b^3)*cosh(d*x + c)^5 + 4*(43605*(64*a^3 - 192*a^2*b - 5*b^3)*cosh(d*x + c)^14 + 7735*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3)*cosh(d*x + c)^12 + 18018*(1440*a^3 + 192*a*b^2 - 355*b^3)*cosh(d*x + c)^10 + 5148*(5040*a^3 + 1440*a^2*b - 672*a*b^2 + 1235*b^3)*cosh(d*x + c)^8 + 231*(60480*a^3 + 8640*a^2*b - 768*a*b^2 - 15475*b^3)*cosh(d*x + c)^6 + 63*(60480*a^3 - 8640*a^2*b - 768*a*b^2 + 15475*b^3)*cosh(d*x + c)^4 + 8640*a^3 + 1152*a*b^2 + 2130*b^3 + 84*(5040*a^3 - 1440*a^2*b - 672*a*b^2 - 1235*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 4*(14535*(64*a^3 - 192*a^2*b - 5*b^3)*cosh(d*x + c)^15 + 2975*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3)*cosh(d*x + c)^13 + 8190*(1440*a^3 + 192*a*b^2 - 355*b^3)*cosh(d*x + c)^11 + 2860*(5040*a^3 + 1440*a^2*b - 672*a*b^2 + 1235*b^3)*cosh(d*x + c)^9 + 165*(60480*a^3 + 8640*a^2*b - 768*a*b^2 - 15475*b^3)*cosh(d*x + c)^7 + 63*(60480*a^3 - 8640*a^2*b - 768*a*b^2 + 15475*b^3)*cosh(d*x + c)^5 + 140*(5040*a^

3 - 1440*a^2*b - 672*a*b^2 - 1235*b^3)*cosh(d*x + c)^3 + 30*(1440*a^3 + 192
*a*b^2 + 355*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 5*(1728*a^3 + 1728*a^2*b
 + 1536*a*b^2 - 427*b^3)*cosh(d*x + c)^3 + (14535*(64*a^3 - 192*a^2*b - 5*b
^3)*cosh(d*x + c)^16 + 3400*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3)*
cosh(d*x + c)^14 + 10920*(1440*a^3 + 192*a*b^2 - 355*b^3)*cosh(d*x + c)^12
 + 4576*(5040*a^3 + 1440*a^2*b - 672*a*b^2 + 1235*b^3)*cosh(d*x + c)^10 + 33
0*(60480*a^3 + 8640*a^2*b - 768*a*b^2 - 15475*b^3)*cosh(d*x + c)^8 + 168*(6
0480*a^3 - 8640*a^2*b - 768*a*b^2 + 15475*b^3)*cosh(d*x + c)^6 + 560*(5040*
a^3 - 1440*a^2*b - 672*a*b^2 - 1235*b^3)*cosh(d*x + c)^4 + 8640*a^3 + 8640*
a^2*b + 7680*a*b^2 - 2135*b^3 + 240*(1440*a^3 + 192*a*b^2 + 355*b^3)*cosh(d
*x + c)^2)*sinh(d*x + c)^3 + (2565*(64*a^3 - 192*a^2*b - 5*b^3)*cosh(d*x +
c)^17 + 680*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3)*cosh(d*x + c)^15
 + 2520*(1440*a^3 + 192*a*b^2 - 355*b^3)*cosh(d*x + c)^13 + 1248*(5040*a^3
 + 1440*a^2*b - 672*a*b^2 + 1235*b^3)*cosh(d*x + c)^11 + 110*(60480*a^3 + 86
40*a^2*b - 768*a*b^2 - 15475*b^3)*cosh(d*x + c)^9 + 72*(60480*a^3 - 8640*a^
2*b - 768*a*b^2 + 15475*b^3)*cosh(d*x + c)^7 + 336*(5040*a^3 - 1440*a^2*b -
 672*a*b^2 - 1235*b^3)*cosh(d*x + c)^5 + 240*(1440*a^3 + 192*a*b^2 + 355*b^
3)*cosh(d*x + c)^3 + 15*(1728*a^3 + 1728*a^2*b + 1536*a*b^2 - 427*b^3)*cosh
(d*x + c))*sinh(d*x + c)^2 - 15*((192*a^2*b + 5*b^3)*cosh(d*x + c)^20 + 20*
(192*a^2*b + 5*b^3)*cosh(d*x + c)*sinh(d*x + c)^19 + (192*a^2*b + 5*b^3)*si
nh(d*x + c)^20 + 6*(192*a^2*b + 5*b^3)*cosh(d*x + c)^18 + 2*(576*a^2*b + 15
b^3 + 95(192*a^2*b + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^18 + 12*(95*(1
92*a^2*b + 5*b^3)*cosh(d*x + c)^3 + 9*(192*a^2*b + 5*b^3)*cosh(d*x + c))*si
nh(d*x + c)^17 + 13*(192*a^2*b + 5*b^3)*cosh(d*x + c)^16 + (4845*(192*a^2*b
 + 5*b^3)*cosh(d*x + c)^4 + 2496*a^2*b + 65*b^3 + 918*(192*a^2*b + 5*b^3)*c
osh(d*x + c)^2)*sinh(d*x + c)^16 + 16*(969*(192*a^2*b + 5*b^3)*cosh(d*x + c
)^5 + 306*(192*a^2*b + 5*b^3)*cosh(d*x + c)^3 + 13*(192*a^2*b + 5*b^3)*cosh
(d*x + c))*sinh(d*x + c)^15 + 8*(192*a^2*b + 5*b^3)*cosh(d*x + c)^14 + 8*(4
845*(192*a^2*b + 5*b^3)*cosh(d*x + c)^6 + 2295*(192*a^2*b + 5*b^3)*cosh(d*x
 + c)^4 + 192*a^2*b + 5*b^3 + 195*(192*a^2*b + 5*b^3)*cosh(d*x + c)^2)*sinh
(d*x + c)^14 + 16*(4845*(192*a^2*b + 5*b^3)*cosh(d*x + c)^7 + 3213*(192*a^2
*b + 5*b^3)*cosh(d*x + c)^5 + 455*(192*a^2*b + 5*b^3)*cosh(d*x + c)^3 + 7*(
192*a^2*b + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^13 - 14*(192*a^2*b + 5*b^3)
*cosh(d*x + c)^12 + 2*(62985*(192*a^2*b + 5*b^3)*cosh(d*x + c)^8 + 55692*(1
92*a^2*b + 5*b^3)*cosh(d*x + c)^6 + 11830*(192*a^2*b + 5*b^3)*cosh(d*x + c)
^4 - 1344*a^2*b - 35*b^3 + 364*(192*a^2*b + 5*b^3)*cosh(d*x + c)^2)*sinh(d*
x + c)^12 + 8*(20995*(192*a^2*b + 5*b^3)*cosh(d*x + c)^9 + 23868*(192*a^2*b
 + 5*b^3)*cosh(d*x + c)^7 + 7098*(192*a^2*b + 5*b^3)*cosh(d*x + c)^5 + 364*
(192*a^2*b + 5*b^3)*cosh(d*x + c)^3 - 21*(192*a^2*b + 5*b^3)*cosh(d*x + c))
*sinh(d*x + c)^11 - 28*(192*a^2*b + 5*b^3)*cosh(d*x + c)^10 + 4*(46189*(192
*a^2*b + 5*b^3)*cosh(d*x + c)^10 + 65637*(192*a^2*b + 5*b^3)*cosh(d*x + c)^
8 + 26026*(192*a^2*b + 5*b^3)*cosh(d*x + c)^6 + 2002*(192*a^2*b + 5*b^3)*co
sh(d*x + c)^4 - 1344*a^2*b - 35*b^3 - 231*(192*a^2*b + 5*b^3)*cosh(d*x + c)
^2)*sinh(d*x + c)^10 + 8*(20995*(192*a^2*b + 5*b^3)*cosh(d*x + c)^11 + 3646
5*(192*a^2*b + 5*b^3)*cosh(d*x + c)^9 + 18590*(192*a^2*b + 5*b^3)*cosh(d*x
+ c)^7 + 2002*(192*a^2*b + 5*b^3)*cosh(d*x + c)^5 - 385*(192*a^2*b + 5*b^3)
*cosh(d*x + c)^3 - 35*(192*a^2*b + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 -
14*(192*a^2*b + 5*b^3)*cosh(d*x + c)^8 + 2*(62985*(192*a^2*b + 5*b^3)*cosh(
d*x + c)^12 + 131274*(192*a^2*b + 5*b^3)*cosh(d*x + c)^10 + 83655*(192*a^2*
b + 5*b^3)*cosh(d*x + c)^8 + 12012*(192*a^2*b + 5*b^3)*cosh(d*x + c)^6 - 34
65*(192*a^2*b + 5*b^3)*cosh(d*x + c)^4 - 1344*a^2*b - 35*b^3 - 630*(192*a^2
*b + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 16*(4845*(192*a^2*b + 5*b^3)
*cosh(d*x + c)^13 + 11934*(192*a^2*b + 5*b^3)*cosh(d*x + c)^11 + 9295*(192*
a^2*b + 5*b^3)*cosh(d*x + c)^9 + 1716*(192*a^2*b + 5*b^3)*cosh(d*x + c)^7 -
 693*(192*a^2*b + 5*b^3)*cosh(d*x + c)^5 - 210*(192*a^2*b + 5*b^3)*cosh(d*x
 + c)^3 - 7*(192*a^2*b + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 8*(192*a^2
*b + 5*b^3)*cosh(d*x + c)^6 + 8*(4845*(192*a^2*b + 5*b^3)*cosh(d*x + c)^14
+ 13923*(192*a^2*b + 5*b^3)*cosh(d*x + c)^12 + 13013*(192*a^2*b + 5*b^3)*co
sh(d*x + c)^10 + 3003*(192*a^2*b + 5*b^3)*cosh(d*x + c)^8 - 1617*(192*a^2*b

+ 5*b^3)*cosh(d*x + c)^6 - 735*(192*a^2*b + 5*b^3)*cosh(d*x + c)^4 + 192*a
^2*b + 5*b^3 - 49*(192*a^2*b + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 16
(969(192*a^2*b + 5*b^3)*cosh(d*x + c)^15 + 3213*(192*a^2*b + 5*b^3)*cosh(
d*x + c)^13 + 3549*(192*a^2*b + 5*b^3)*cosh(d*x + c)^11 + 1001*(192*a^2*b +
5*b^3)*cosh(d*x + c)^9 - 693*(192*a^2*b + 5*b^3)*cosh(d*x + c)^7 - 441*(19
2*a^2*b + 5*b^3)*cosh(d*x + c)^5 - 49*(192*a^2*b + 5*b^3)*cosh(d*x + c)^3 +
3*(192*a^2*b + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 13*(192*a^2*b + 5*b
^3)*cosh(d*x + c)^4 + (4845*(192*a^2*b + 5*b^3)*cosh(d*x + c)^16 + 18360*(1
92*a^2*b + 5*b^3)*cosh(d*x + c)^14 + 23660*(192*a^2*b + 5*b^3)*cosh(d*x + c
)^12 + 8008*(192*a^2*b + 5*b^3)*cosh(d*x + c)^10 - 6930*(192*a^2*b + 5*b^3)
*cosh(d*x + c)^8 - 5880*(192*a^2*b + 5*b^3)*cosh(d*x + c)^6 - 980*(192*a^2*
b + 5*b^3)*cosh(d*x + c)^4 + 2496*a^2*b + 65*b^3 + 120*(192*a^2*b + 5*b^3)*
cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(285*(192*a^2*b + 5*b^3)*cosh(d*x + c)
^17 + 1224*(192*a^2*b + 5*b^3)*cosh(d*x + c)^15 + 1820*(192*a^2*b + 5*b^3)*
cosh(d*x + c)^13 + 728*(192*a^2*b + 5*b^3)*cosh(d*x + c)^11 - 770*(192*a^2*
b + 5*b^3)*cosh(d*x + c)^9 - 840*(192*a^2*b + 5*b^3)*cosh(d*x + c)^7 - 196*
(192*a^2*b + 5*b^3)*cosh(d*x + c)^5 + 40*(192*a^2*b + 5*b^3)*cosh(d*x + c)^
3 + 13*(192*a^2*b + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 192*a^2*b + 5*b
^3 + 6*(192*a^2*b + 5*b^3)*cosh(d*x + c)^2 + 2*(95*(192*a^2*b + 5*b^3)*cosh
(d*x + c)^18 + 459*(192*a^2*b + 5*b^3)*cosh(d*x + c)^16 + 780*(192*a^2*b +
5*b^3)*cosh(d*x + c)^14 + 364*(192*a^2*b + 5*b^3)*cosh(d*x + c)^12 - 462*(1
92*a^2*b + 5*b^3)*cosh(d*x + c)^10 - 630*(192*a^2*b + 5*b^3)*cosh(d*x + c)^
8 - 196*(192*a^2*b + 5*b^3)*cosh(d*x + c)^6 + 60*(192*a^2*b + 5*b^3)*cosh(d
*x + c)^4 + 576*a^2*b + 15*b^3 + 39*(192*a^2*b + 5*b^3)*cosh(d*x + c)^2)*si
nh(d*x + c)^2 + 4*(5*(192*a^2*b + 5*b^3)*cosh(d*x + c)^19 + 27*(192*a^2*b +
5*b^3)*cosh(d*x + c)^17 + 52*(192*a^2*b + 5*b^3)*cosh(d*x + c)^15 + 28*(19
2*a^2*b + 5*b^3)*cosh(d*x + c)^13 - 42*(192*a^2*b + 5*b^3)*cosh(d*x + c)^11
- 70*(192*a^2*b + 5*b^3)*cosh(d*x + c)^9 - 28*(192*a^2*b + 5*b^3)*cosh(d*x
+ c)^7 + 12*(192*a^2*b + 5*b^3)*cosh(d*x + c)^5 + 13*(192*a^2*b + 5*b^3)*c
osh(d*x + c)^3 + 3*(192*a^2*b + 5*b^3)*cosh(d*x + c))*sinh(d*x + c))*arctan
(cosh(d*x + c) + sinh(d*x + c)) + 15*(64*a^3 + 192*a^2*b + 5*b^3)*cosh(d*x
+ c) - 480*(a^3*cosh(d*x + c)^20 + 20*a^3*cosh(d*x + c)*sinh(d*x + c)^19 +
a^3*sinh(d*x + c)^20 + 6*a^3*cosh(d*x + c)^18 + 13*a^3*cosh(d*x + c)^16 + 2
*(95*a^3*cosh(d*x + c)^2 + 3*a^3)*sinh(d*x + c)^18 + 12*(95*a^3*cosh(d*x +
c)^3 + 9*a^3*cosh(d*x + c))*sinh(d*x + c)^17 + 8*a^3*cosh(d*x + c)^14 + (48
45*a^3*cosh(d*x + c)^4 + 918*a^3*cosh(d*x + c)^2 + 13*a^3)*sinh(d*x + c)^16
+ 16*(969*a^3*cosh(d*x + c)^5 + 306*a^3*cosh(d*x + c)^3 + 13*a^3*cosh(d*x
+ c))*sinh(d*x + c)^15 - 14*a^3*cosh(d*x + c)^12 + 8*(4845*a^3*cosh(d*x + c
)^6 + 2295*a^3*cosh(d*x + c)^4 + 195*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x +
c)^14 + 16*(4845*a^3*cosh(d*x + c)^7 + 3213*a^3*cosh(d*x + c)^5 + 455*a^3*c
osh(d*x + c)^3 + 7*a^3*cosh(d*x + c))*sinh(d*x + c)^13 - 28*a^3*cosh(d*x +
c)^10 + 2*(62985*a^3*cosh(d*x + c)^8 + 55692*a^3*cosh(d*x + c)^6 + 11830*a^
3*cosh(d*x + c)^4 + 364*a^3*cosh(d*x + c)^2 - 7*a^3)*sinh(d*x + c)^12 + 8*(
20995*a^3*cosh(d*x + c)^9 + 23868*a^3*cosh(d*x + c)^7 + 7098*a^3*cosh(d*x +
c)^5 + 364*a^3*cosh(d*x + c)^3 - 21*a^3*cosh(d*x + c))*sinh(d*x + c)^11 -
14*a^3*cosh(d*x + c)^8 + 4*(46189*a^3*cosh(d*x + c)^10 + 65637*a^3*cosh(d*x
+ c)^8 + 26026*a^3*cosh(d*x + c)^6 + 2002*a^3*cosh(d*x + c)^4 - 231*a^3*co
sh(d*x + c)^2 - 7*a^3)*sinh(d*x + c)^10 + 8*(20995*a^3*cosh(d*x + c)^11 + 3
6465*a^3*cosh(d*x + c)^9 + 18590*a^3*cosh(d*x + c)^7 + 2002*a^3*cosh(d*x +
c)^5 - 385*a^3*cosh(d*x + c)^3 - 35*a^3*cosh(d*x + c))*sinh(d*x + c)^9 + 8*
a^3*cosh(d*x + c)^6 + 2*(62985*a^3*cosh(d*x + c)^12 + 131274*a^3*cosh(d*x +
c)^10 + 83655*a^3*cosh(d*x + c)^8 + 12012*a^3*cosh(d*x + c)^6 - 3465*a^3*c
osh(d*x + c)^4 - 630*a^3*cosh(d*x + c)^2 - 7*a^3)*sinh(d*x + c)^8 + 16*(484
5*a^3*cosh(d*x + c)^13 + 11934*a^3*cosh(d*x + c)^11 + 9295*a^3*cosh(d*x + c
)^9 + 1716*a^3*cosh(d*x + c)^7 - 693*a^3*cosh(d*x + c)^5 - 210*a^3*cosh(d*x
+ c)^3 - 7*a^3*cosh(d*x + c))*sinh(d*x + c)^7 + 13*a^3*cosh(d*x + c)^4 + 8
*(4845*a^3*cosh(d*x + c)^14 + 13923*a^3*cosh(d*x + c)^12 + 13013*a^3*cosh(d
*x + c)^10 + 3003*a^3*cosh(d*x + c)^8 - 1617*a^3*cosh(d*x + c)^6 - 735*a^3*
cosh(d*x + c)^4 - 49*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^6 + 16*(969*a

^3*cosh(d*x + c)^15 + 3213*a^3*cosh(d*x + c)^13 + 3549*a^3*cosh(d*x + c)^11
 + 1001*a^3*cosh(d*x + c)^9 - 693*a^3*cosh(d*x + c)^7 - 441*a^3*cosh(d*x +
c)^5 - 49*a^3*cosh(d*x + c)^3 + 3*a^3*cosh(d*x + c))*sinh(d*x + c)^5 + 6*a^
3*cosh(d*x + c)^2 + (4845*a^3*cosh(d*x + c)^16 + 18360*a^3*cosh(d*x + c)^14
 + 23660*a^3*cosh(d*x + c)^12 + 8008*a^3*cosh(d*x + c)^10 - 6930*a^3*cosh(d
*x + c)^8 - 5880*a^3*cosh(d*x + c)^6 - 980*a^3*cosh(d*x + c)^4 + 120*a^3*co
sh(d*x + c)^2 + 13*a^3)*sinh(d*x + c)^4 + 4*(285*a^3*cosh(d*x + c)^17 + 122
4*a^3*cosh(d*x + c)^15 + 1820*a^3*cosh(d*x + c)^13 + 728*a^3*cosh(d*x + c)^
11 - 770*a^3*cosh(d*x + c)^9 - 840*a^3*cosh(d*x + c)^7 - 196*a^3*cosh(d*x +
 c)^5 + 40*a^3*cosh(d*x + c)^3 + 13*a^3*cosh(d*x + c))*sinh(d*x + c)^3 + a^
3 + 2*(95*a^3*cosh(d*x + c)^18 + 459*a^3*cosh(d*x + c)^16 + 780*a^3*cosh(d*
x + c)^14 + 364*a^3*cosh(d*x + c)^12 - 462*a^3*cosh(d*x + c)^10 - 630*a^3*c
osh(d*x + c)^8 - 196*a^3*cosh(d*x + c)^6 + 60*a^3*cosh(d*x + c)^4 + 39*a^3*
cosh(d*x + c)^2 + 3*a^3)*sinh(d*x + c)^2 + 4*(5*a^3*cosh(d*x + c)^19 + 27*a
^3*cosh(d*x + c)^17 + 52*a^3*cosh(d*x + c)^15 + 28*a^3*cosh(d*x + c)^13 - 4
2*a^3*cosh(d*x + c)^11 - 70*a^3*cosh(d*x + c)^9 - 28*a^3*cosh(d*x + c)^7 +
12*a^3*cosh(d*x + c)^5 + 13*a^3*cosh(d*x + c)^3 + 3*a^3*cosh(d*x + c))*sinh
(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 480*(a^3*cosh(d*x + c)^
20 + 20*a^3*cosh(d*x + c)*sinh(d*x + c)^19 + a^3*sinh(d*x + c)^20 + 6*a^3*c
osh(d*x + c)^18 + 13*a^3*cosh(d*x + c)^16 + 2*(95*a^3*cosh(d*x + c)^2 + 3*a
^3)*sinh(d*x + c)^18 + 12*(95*a^3*cosh(d*x + c)^3 + 9*a^3*cosh(d*x + c))*si
nh(d*x + c)^17 + 8*a^3*cosh(d*x + c)^14 + (4845*a^3*cosh(d*x + c)^4 + 918*a
^3*cosh(d*x + c)^2 + 13*a^3)*sinh(d*x + c)^16 + 16*(969*a^3*cosh(d*x + c)^5
 + 306*a^3*cosh(d*x + c)^3 + 13*a^3*cosh(d*x + c))*sinh(d*x + c)^15 - 14*a^
3*cosh(d*x + c)^12 + 8*(4845*a^3*cosh(d*x + c)^6 + 2295*a^3*cosh(d*x + c)^4
 + 195*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^14 + 16*(4845*a^3*cosh(d*x
+ c)^7 + 3213*a^3*cosh(d*x + c)^5 + 455*a^3*cosh(d*x + c)^3 + 7*a^3*cosh(d*
x + c))*sinh(d*x + c)^13 - 28*a^3*cosh(d*x + c)^10 + 2*(62985*a^3*cosh(d*x
+ c)^8 + 55692*a^3*cosh(d*x + c)^6 + 11830*a^3*cosh(d*x + c)^4 + 364*a^3*co
sh(d*x + c)^2 - 7*a^3)*sinh(d*x + c)^12 + 8*(20995*a^3*cosh(d*x + c)^9 + 23
868*a^3*cosh(d*x + c)^7 + 7098*a^3*cosh(d*x + c)^5 + 364*a^3*cosh(d*x + c)^
3 - 21*a^3*cosh(d*x + c))*sinh(d*x + c)^11 - 14*a^3*cosh(d*x + c)^8 + 4*(46
189*a^3*cosh(d*x + c)^10 + 65637*a^3*cosh(d*x + c)^8 + 26026*a^3*cosh(d*x +
 c)^6 + 2002*a^3*cosh(d*x + c)^4 - 231*a^3*cosh(d*x + c)^2 - 7*a^3)*sinh(d*
x + c)^10 + 8*(20995*a^3*cosh(d*x + c)^11 + 36465*a^3*cosh(d*x + c)^9 + 185
90*a^3*cosh(d*x + c)^7 + 2002*a^3*cosh(d*x + c)^5 - 385*a^3*cosh(d*x + c)^3
 - 35*a^3*cosh(d*x + c))*sinh(d*x + c)^9 + 8*a^3*cosh(d*x + c)^6 + 2*(62985
*a^3*cosh(d*x + c)^12 + 131274*a^3*cosh(d*x + c)^10 + 83655*a^3*cosh(d*x +
c)^8 + 12012*a^3*cosh(d*x + c)^6 - 3465*a^3*cosh(d*x + c)^4 - 630*a^3*cosh(
d*x + c)^2 - 7*a^3)*sinh(d*x + c)^8 + 16*(4845*a^3*cosh(d*x + c)^13 + 11934
*a^3*cosh(d*x + c)^11 + 9295*a^3*cosh(d*x + c)^9 + 1716*a^3*cosh(d*x + c)^7
 - 693*a^3*cosh(d*x + c)^5 - 210*a^3*cosh(d*x + c)^3 - 7*a^3*cosh(d*x + c))
*sinh(d*x + c)^7 + 13*a^3*cosh(d*x + c)^4 + 8*(4845*a^3*cosh(d*x + c)^14 +
13923*a^3*cosh(d*x + c)^12 + 13013*a^3*cosh(d*x + c)^10 + 3003*a^3*cosh(d*x
 + c)^8 - 1617*a^3*cosh(d*x + c)^6 - 735*a^3*cosh(d*x + c)^4 - 49*a^3*cosh(
d*x + c)^2 + a^3)*sinh(d*x + c)^6 + 16*(969*a^3*cosh(d*x + c)^15 + 3213*a^3
*cosh(d*x + c)^13 + 3549*a^3*cosh(d*x + c)^11 + 1001*a^3*cosh(d*x + c)^9 -
693*a^3*cosh(d*x + c)^7 - 441*a^3*cosh(d*x + c)^5 - 49*a^3*cosh(d*x + c)^3
+ 3*a^3*cosh(d*x + c))*sinh(d*x + c)^5 + 6*a^3*cosh(d*x + c)^2 + (4845*a^3*
cosh(d*x + c)^16 + 18360*a^3*cosh(d*x + c)^14 + 23660*a^3*cosh(d*x + c)^12
+ 8008*a^3*cosh(d*x + c)^10 - 6930*a^3*cosh(d*x + c)^8 - 5880*a^3*cosh(d*x
+ c)^6 - 980*a^3*cosh(d*x + c)^4 + 120*a^3*cosh(d*x + c)^2 + 13*a^3)*sinh(d
x + c)^4 + 4(285*a^3*cosh(d*x + c)^17 + 1224*a^3*cosh(d*x + c)^15 + 1820*
a^3*cosh(d*x + c)^13 + 728*a^3*cosh(d*x + c)^11 - 770*a^3*cosh(d*x + c)^9 -
 840*a^3*cosh(d*x + c)^7 - 196*a^3*cosh(d*x + c)^5 + 40*a^3*cosh(d*x + c)^3
 + 13*a^3*cosh(d*x + c))*sinh(d*x + c)^3 + a^3 + 2*(95*a^3*cosh(d*x + c)^18
 + 459*a^3*cosh(d*x + c)^16 + 780*a^3*cosh(d*x + c)^14 + 364*a^3*cosh(d*x +
 c)^12 - 462*a^3*cosh(d*x + c)^10 - 630*a^3*cosh(d*x + c)^8 - 196*a^3*cosh(
d*x + c)^6 + 60*a^3*cosh(d*x + c)^4 + 39*a^3*cosh(d*x + c)^2 + 3*a^3)*sinh(

```
d*x + c)^2 + 4*(5*a^3*cosh(d*x + c)^19 + 27*a^3*cosh(d*x + c)^17 + 52*a^3*c
osh(d*x + c)^15 + 28*a^3*cosh(d*x + c)^13 - 42*a^3*cosh(d*x + c)^11 - 70*a^
3*cosh(d*x + c)^9 - 28*a^3*cosh(d*x + c)^7 + 12*a^3*cosh(d*x + c)^5 + 13*a^
3*cosh(d*x + c)^3 + 3*a^3*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) +
 sinh(d*x + c) - 1) + (285*(64*a^3 - 192*a^2*b - 5*b^3)*cosh(d*x + c)^18 +
85*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3)*cosh(d*x + c)^16 + 360*(1
440*a^3 + 192*a*b^2 - 355*b^3)*cosh(d*x + c)^14 + 208*(5040*a^3 + 1440*a^2*
b - 672*a*b^2 + 1235*b^3)*cosh(d*x + c)^12 + 22*(60480*a^3 + 8640*a^2*b - 7
68*a*b^2 - 15475*b^3)*cosh(d*x + c)^10 + 18*(60480*a^3 - 8640*a^2*b - 768*a
*b^2 + 15475*b^3)*cosh(d*x + c)^8 + 112*(5040*a^3 - 1440*a^2*b - 672*a*b^2
- 1235*b^3)*cosh(d*x + c)^6 + 120*(1440*a^3 + 192*a*b^2 + 355*b^3)*cosh(d*x
 + c)^4 + 960*a^3 + 2880*a^2*b + 75*b^3 + 15*(1728*a^3 + 1728*a^2*b + 1536*
a*b^2 - 427*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^20 + 20*d
*cosh(d*x + c)*sinh(d*x + c)^19 + d*sinh(d*x + c)^20 + 6*d*cosh(d*x + c)^18
 + 2*(95*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^18 + 12*(95*d*cosh(d*x + c)
^3 + 9*d*cosh(d*x + c))*sinh(d*x + c)^17 + 13*d*cosh(d*x + c)^16 + (4845*d*
cosh(d*x + c)^4 + 918*d*cosh(d*x + c)^2 + 13*d)*sinh(d*x + c)^16 + 16*(969*
d*cosh(d*x + c)^5 + 306*d*cosh(d*x + c)^3 + 13*d*cosh(d*x + c))*sinh(d*x +
c)^15 + 8*d*cosh(d*x + c)^14 + 8*(4845*d*cosh(d*x + c)^6 + 2295*d*cosh(d*x
+ c)^4 + 195*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^14 + 16*(4845*d*cosh(d*x
+ c)^7 + 3213*d*cosh(d*x + c)^5 + 455*d*cosh(d*x + c)^3 + 7*d*cosh(d*x + c)
)*sinh(d*x + c)^13 - 14*d*cosh(d*x + c)^12 + 2*(62985*d*cosh(d*x + c)^8 + 5
5692*d*cosh(d*x + c)^6 + 11830*d*cosh(d*x + c)^4 + 364*d*cosh(d*x + c)^2 -
7*d)*sinh(d*x + c)^12 + 8*(20995*d*cosh(d*x + c)^9 + 23868*d*cosh(d*x + c)^
7 + 7098*d*cosh(d*x + c)^5 + 364*d*cosh(d*x + c)^3 - 21*d*cosh(d*x + c))*si
nh(d*x + c)^11 - 28*d*cosh(d*x + c)^10 + 4*(46189*d*cosh(d*x + c)^10 + 6563
7*d*cosh(d*x + c)^8 + 26026*d*cosh(d*x + c)^6 + 2002*d*cosh(d*x + c)^4 - 23
1*d*cosh(d*x + c)^2 - 7*d)*sinh(d*x + c)^10 + 8*(20995*d*cosh(d*x + c)^11 +
 36465*d*cosh(d*x + c)^9 + 18590*d*cosh(d*x + c)^7 + 2002*d*cosh(d*x + c)^5
 - 385*d*cosh(d*x + c)^3 - 35*d*cosh(d*x + c))*sinh(d*x + c)^9 - 14*d*cosh(
d*x + c)^8 + 2*(62985*d*cosh(d*x + c)^12 + 131274*d*cosh(d*x + c)^10 + 8365
5*d*cosh(d*x + c)^8 + 12012*d*cosh(d*x + c)^6 - 3465*d*cosh(d*x + c)^4 - 63
0*d*cosh(d*x + c)^2 - 7*d)*sinh(d*x + c)^8 + 16*(4845*d*cosh(d*x + c)^13 +
11934*d*cosh(d*x + c)^11 + 9295*d*cosh(d*x + c)^9 + 1716*d*cosh(d*x + c)^7
- 693*d*cosh(d*x + c)^5 - 210*d*cosh(d*x + c)^3 - 7*d*cosh(d*x + c))*sinh(d
*x + c)^7 + 8*d*cosh(d*x + c)^6 + 8*(4845*d*cosh(d*x + c)^14 + 13923*d*cosh
(d*x + c)^12 + 13013*d*cosh(d*x + c)^10 + 3003*d*cosh(d*x + c)^8 - 1617*d*c
osh(d*x + c)^6 - 735*d*cosh(d*x + c)^4 - 49*d*cosh(d*x + c)^2 + d)*sinh(d*x
 + c)^6 + 16*(969*d*cosh(d*x + c)^15 + 3213*d*cosh(d*x + c)^13 + 3549*d*cos
h(d*x + c)^11 + 1001*d*cosh(d*x + c)^9 - 693*d*cosh(d*x + c)^7 - 441*d*cosh
(d*x + c)^5 - 49*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 1
3*d*cosh(d*x + c)^4 + (4845*d*cosh(d*x + c)^16 + 18360*d*cosh(d*x + c)^14 +
 23660*d*cosh(d*x + c)^12 + 8008*d*cosh(d*x + c)^10 - 6930*d*cosh(d*x + c)^
8 - 5880*d*cosh(d*x + c)^6 - 980*d*cosh(d*x + c)^4 + 120*d*cosh(d*x + c)^2
+ 13*d)*sinh(d*x + c)^4 + 4*(285*d*cosh(d*x + c)^17 + 1224*d*cosh(d*x + c)^
15 + 1820*d*cosh(d*x + c)^13 + 728*d*cosh(d*x + c)^11 - 770*d*cosh(d*x + c)
^9 - 840*d*cosh(d*x + c)^7 - 196*d*cosh(d*x + c)^5 + 40*d*cosh(d*x + c)^3 +
 13*d*cosh(d*x + c))*sinh(d*x + c)^3 + 6*d*cosh(d*x + c)^2 + 2*(95*d*cosh(d
*x + c)^18 + 459*d*cosh(d*x + c)^16 + 780*d*cosh(d*x + c)^14 + 364*d*cosh(d
*x + c)^12 - 462*d*cosh(d*x + c)^10 - 630*d*cosh(d*x + c)^8 - 196*d*cosh(d*
x + c)^6 + 60*d*cosh(d*x + c)^4 + 39*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)
^2 + 4*(5*d*cosh(d*x + c)^19 + 27*d*cosh(d*x + c)^17 + 52*d*cosh(d*x + c)^1
5 + 28*d*cosh(d*x + c)^13 - 42*d*cosh(d*x + c)^11 - 70*d*cosh(d*x + c)^9 -
28*d*cosh(d*x + c)^7 + 12*d*cosh(d*x + c)^5 + 13*d*cosh(d*x + c)^3 + 3*d*co
sh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tanh^3(c + dx)\right)^3 \operatorname{csch}^3(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*tanh(d*x+c)**3)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)**3*csch(c + d*x)**3, x)
```

Giac [B] time = 2.56392, size = 586, normalized size = 2.53

$$\frac{480\,a^3\log\left(e^{(dx+c)}+1\right) - 480\,a^3\log\left(\left|e^{(dx+c)}-1\right|\right) + 15\left(192\,a^2be^c + 5\,b^3e^c\right)\arctan\left(e^{(dx+c)}\right)e^{(-c)} - \frac{960\left(a^3e^{(3\,dx+3\,c)}+a^3e^{(dx+c)}\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^2} + \cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 1/960*(480*a^3*log(e^(d*x + c) + 1) - 480*a^3*log(abs(e^(d*x + c) - 1)) + 1
5*(192*a^2*b*e^c + 5*b^3*e^c)*arctan(e^(d*x + c))*e^(-c) - 960*(a^3*e^(3*d*
x + 3*c) + a^3*e^(d*x + c))/(e^(2*d*x + 2*c) - 1)^2 + (2880*a^2*b*e^(15*d*x
 + 15*c) + 75*b^3*e^(15*d*x + 15*c) + 14400*a^2*b*e^(13*d*x + 13*c) - 7680*
a*b^2*e^(13*d*x + 13*c) - 1985*b^3*e^(13*d*x + 13*c) + 25920*a^2*b*e^(11*d*
x + 11*c) - 19968*a*b^2*e^(11*d*x + 11*c) + 4475*b^3*e^(11*d*x + 11*c) + 14
400*a^2*b*e^(9*d*x + 9*c) - 21504*a*b^2*e^(9*d*x + 9*c) - 8825*b^3*e^(9*d*x
 + 9*c) - 14400*a^2*b*e^(7*d*x + 7*c) - 21504*a*b^2*e^(7*d*x + 7*c) + 8825*
b^3*e^(7*d*x + 7*c) - 25920*a^2*b*e^(5*d*x + 5*c) - 19968*a*b^2*e^(5*d*x +
5*c) - 4475*b^3*e^(5*d*x + 5*c) - 14400*a^2*b*e^(3*d*x + 3*c) - 7680*a*b^2*
e^(3*d*x + 3*c) + 1985*b^3*e^(3*d*x + 3*c) - 2880*a^2*b*e^(d*x + c) - 75*b^
3*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^8)/d
```

3.72 $\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=138

$$-\frac{3a^2b\tanh^2(c+dx)}{2d}+\frac{3a^2b\log(\tanh(c+dx))}{d}-\frac{a^3\coth^3(c+dx)}{3d}+\frac{a^3\coth(c+dx)}{d}-\frac{3ab^2\tanh^5(c+dx)}{5d}+\frac{ab^2\tanh$$

```
[Out] (a^3*Coth[c + d*x])/d - (a^3*Coth[c + d*x]^3)/(3*d) + (3*a^2*b*Log[Tanh[c +
 d*x]])/d - (3*a^2*b*Tanh[c + d*x]^2)/(2*d) + (a*b^2*Tanh[c + d*x]^3)/d - (
3*a*b^2*Tanh[c + d*x]^5)/(5*d) + (b^3*Tanh[c + d*x]^6)/(6*d) - (b^3*Tanh[c
+ d*x]^8)/(8*d)
```

Rubi [A] time = 0.115098, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3663, 1802}

$$-\frac{3a^2b\tanh^2(c+dx)}{2d}+\frac{3a^2b\log(\tanh(c+dx))}{d}-\frac{a^3\coth^3(c+dx)}{3d}+\frac{a^3\coth(c+dx)}{d}-\frac{3ab^2\tanh^5(c+dx)}{5d}+\frac{ab^2\tanh$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] (a^3*Coth[c + d*x])/d - (a^3*Coth[c + d*x]^3)/(3*d) + (3*a^2*b*Log[Tanh[c +
 d*x]])/d - (3*a^2*b*Tanh[c + d*x]^2)/(2*d) + (a*b^2*Tanh[c + d*x]^3)/d - (
3*a*b^2*Tanh[c + d*x]^5)/(5*d) + (b^3*Tanh[c + d*x]^6)/(6*d) - (b^3*Tanh[c
+ d*x]^8)/(8*d)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)(a+bx^3)^3}{x^4}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \left(\frac{a^3}{x^4}-\frac{a^3}{x^2}+\frac{3a^2b}{x}-3a^2bx+3ab^2x^2-3ab^2x^4+b^3x^5-b^3x^7\right) dx\right.}{d}\\ &= \frac{a^3\coth(c+dx)}{d}-\frac{a^3\coth^3(c+dx)}{3d}+\frac{3a^2b\log(\tanh(c+dx))}{d}-\frac{3a^2b\,t}{}\end{aligned}$$

Mathematica [A] time = 0.168904, size = 213, normalized size = 1.54

$$\frac{3a^2b\,\mathrm{sech}^2(c+dx)}{2d}+\frac{3a^2b\log(\sinh(c+dx))}{d}-\frac{3a^2b\log(\cosh(c+dx))}{d}+\frac{2a^3\coth(c+dx)}{3d}-\frac{a^3\coth(c+dx)\mathrm{csch}^2(c+a}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] (2*a^3*Coth[c + d*x])/(3*d) - (a^3*Coth[c + d*x]*Csch[c + d*x]^2)/(3*d) - (
3*a^2*b*Log[Cosh[c + d*x]])/d + (3*a^2*b*Log[Sinh[c + d*x]])/d + (3*a^2*b*S
ech[c + d*x]^2)/(2*d) - (b^3*Sech[c + d*x]^4)/(4*d) + (b^3*Sech[c + d*x]^6)
/(3*d) - (b^3*Sech[c + d*x]^8)/(8*d) + (2*a*b^2*Tanh[c + d*x])/(5*d) + (a*b
^2*Sech[c + d*x]^2*Tanh[c + d*x])/(5*d) - (3*a*b^2*Sech[c + d*x]^4*Tanh[c +
 d*x])/(5*d)
```

Maple [B] time = 0.106, size = 275, normalized size = 2.

$$\frac{2\,a^3\coth(dx+c)}{3\,d}-\frac{a^3\coth(dx+c)(\mathrm{csch}(dx+c))^2}{3\,d}+\frac{3\,a^2b}{2\,d\,(\cosh(dx+c))^2}+3\,\frac{a^2b\ln(\tanh(dx+c))}{d}-\frac{3\,ab^2\sinh(dx+}{4\,d\,(\cosh(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x)
```

```
[Out] 2/3*a^3*coth(d*x+c)/d-1/3/d*a^3*coth(d*x+c)*csch(d*x+c)^2+3/2/d*a^2*b/cosh(
d*x+c)^2+3*a^2*b*ln(tanh(d*x+c))/d-3/4/d*a*b^2*sinh(d*x+c)/cosh(d*x+c)^5+2/
5*a*b^2*tanh(d*x+c)/d+3/20/d*a*b^2*tanh(d*x+c)*sech(d*x+c)^4+1/5/d*a*b^2*ta
nh(d*x+c)*sech(d*x+c)^2-1/4/d*b^3*sinh(d*x+c)^4/cosh(d*x+c)^8-1/8/d*b^3*sin
h(d*x+c)^2/cosh(d*x+c)^8+1/24/d*b^3*sinh(d*x+c)^2/cosh(d*x+c)^6+1/24/d*b^3*
sinh(d*x+c)^2/cosh(d*x+c)^4+1/24/d*b^3*sinh(d*x+c)^2/cosh(d*x+c)^2
```

Maxima [B] time = 1.80962, size = 1346, normalized size = 9.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] 3*a^2*b*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x
- 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c
) + 1))) + 4/5*a*b^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*
d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c)
+ 1)) - 5*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 1
0*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*e^(
-6*d*x - 6*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x -
 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x -
 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^
(-10*d*x - 10*c) + 1))) + 4/3*a^3*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c
) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c)
- 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) - 4/3*b^3*(3*e^(-4*d*x - 4*c
)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e
^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*
```

```
d*x - 14*c) + e^(-16*d*x - 16*c) + 1)) - 4*e^(-6*d*x - 6*c)/(d*(8*e^(-2*d*x
 - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c) +
 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e^(
-16*d*x - 16*c) + 1)) + 10*e^(-8*d*x - 8*c)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(
-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x -
 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e^(-16*d*x - 16*c)
+ 1)) - 4*e^(-10*d*x - 10*c)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) +
 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(
-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1)) + 3*e^(-1
2*d*x - 12*c)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x -
 6*c) + 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c)
 + 8*e^(-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1)))
```

Fricas [B] time = 3.63018, size = 25978, normalized size = 188.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(90*a^2*b*cosh(d*x + c)^20 + 1800*a^2*b*cosh(d*x + c)*sinh(d*x + c)^19
 + 90*a^2*b*sinh(d*x + c)^20 - 30*(2*a^3 - 9*a^2*b + 6*a*b^2 + 2*b^3)*cosh(
d*x + c)^18 + 30*(570*a^2*b*cosh(d*x + c)^2 - 2*a^3 + 9*a^2*b - 6*a*b^2 - 2
*b^3)*sinh(d*x + c)^18 + 540*(190*a^2*b*cosh(d*x + c)^3 - (2*a^3 - 9*a^2*b
+ 6*a*b^2 + 2*b^3)*cosh(d*x + c))*sinh(d*x + c)^17 - 20*(23*a^3 - 3*a*b^2 -
 13*b^3)*cosh(d*x + c)^16 + 10*(43605*a^2*b*cosh(d*x + c)^4 - 46*a^3 + 6*a*
b^2 + 26*b^3 - 459*(2*a^3 - 9*a^2*b + 6*a*b^2 + 2*b^3)*cosh(d*x + c)^2)*sin
h(d*x + c)^16 + 160*(8721*a^2*b*cosh(d*x + c)^5 - 153*(2*a^3 - 9*a^2*b + 6*
a*b^2 + 2*b^3)*cosh(d*x + c)^3 - 2*(23*a^3 - 3*a*b^2 - 13*b^3)*cosh(d*x + c
))*sinh(d*x + c)^15 - 20*(76*a^3 + 36*a^2*b - 24*a*b^2 + 31*b^3)*cosh(d*x +
 c)^14 + 20*(174420*a^2*b*cosh(d*x + c)^6 - 4590*(2*a^3 - 9*a^2*b + 6*a*b^2
 + 2*b^3)*cosh(d*x + c)^4 - 76*a^3 - 36*a^2*b + 24*a*b^2 - 31*b^3 - 120*(23
*a^3 - 3*a*b^2 - 13*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^14 + 40*(174420*a^2
*b*cosh(d*x + c)^7 - 6426*(2*a^3 - 9*a^2*b + 6*a*b^2 + 2*b^3)*cosh(d*x + c)
^5 - 280*(23*a^3 - 3*a*b^2 - 13*b^3)*cosh(d*x + c)^3 - 7*(76*a^3 + 36*a^2*b
 - 24*a*b^2 + 31*b^3)*cosh(d*x + c))*sinh(d*x + c)^13 - 4*(700*a^3 + 135*a^
2*b + 48*a*b^2 - 245*b^3)*cosh(d*x + c)^12 + 4*(2834325*a^2*b*cosh(d*x + c)
^8 - 139230*(2*a^3 - 9*a^2*b + 6*a*b^2 + 2*b^3)*cosh(d*x + c)^6 - 9100*(23*
a^3 - 3*a*b^2 - 13*b^3)*cosh(d*x + c)^4 - 700*a^3 - 135*a^2*b - 48*a*b^2 +
245*b^3 - 455*(76*a^3 + 36*a^2*b - 24*a*b^2 + 31*b^3)*cosh(d*x + c)^2)*sinh
(d*x + c)^12 + 16*(944775*a^2*b*cosh(d*x + c)^9 - 59670*(2*a^3 - 9*a^2*b +
6*a*b^2 + 2*b^3)*cosh(d*x + c)^7 - 5460*(23*a^3 - 3*a*b^2 - 13*b^3)*cosh(d*
x + c)^5 - 455*(76*a^3 + 36*a^2*b - 24*a*b^2 + 31*b^3)*cosh(d*x + c)^3 - 3*
(700*a^3 + 135*a^2*b + 48*a*b^2 - 245*b^3)*cosh(d*x + c))*sinh(d*x + c)^11
- 20*(154*a^3 - 27*a^2*b + 18*a*b^2 + 49*b^3)*cosh(d*x + c)^10 + 4*(4157010
*a^2*b*cosh(d*x + c)^10 - 328185*(2*a^3 - 9*a^2*b + 6*a*b^2 + 2*b^3)*cosh(d
*x + c)^8 - 40040*(23*a^3 - 3*a*b^2 - 13*b^3)*cosh(d*x + c)^6 - 5005*(76*a^
3 + 36*a^2*b - 24*a*b^2 + 31*b^3)*cosh(d*x + c)^4 - 770*a^3 + 135*a^2*b - 9
0*a*b^2 - 245*b^3 - 66*(700*a^3 + 135*a^2*b + 48*a*b^2 - 245*b^3)*cosh(d*x
+ c)^2)*sinh(d*x + c)^10 + 40*(377910*a^2*b*cosh(d*x + c)^11 - 36465*(2*a^3
 - 9*a^2*b + 6*a*b^2 + 2*b^3)*cosh(d*x + c)^9 - 5720*(23*a^3 - 3*a*b^2 - 13
*b^3)*cosh(d*x + c)^7 - 1001*(76*a^3 + 36*a^2*b - 24*a*b^2 + 31*b^3)*cosh(d
*x + c)^5 - 22*(700*a^3 + 135*a^2*b + 48*a*b^2 - 245*b^3)*cosh(d*x + c)^3 -
 5*(154*a^3 - 27*a^2*b + 18*a*b^2 + 49*b^3)*cosh(d*x + c))*sinh(d*x + c)^9
- 4*(490*a^3 - 180*a^2*b - 54*a*b^2 - 155*b^3)*cosh(d*x + c)^8 + 4*(2834325
*a^2*b*cosh(d*x + c)^12 - 328185*(2*a^3 - 9*a^2*b + 6*a*b^2 + 2*b^3)*cosh(d
```

x + c)^10 - 64350(23*a^3 - 3*a*b^2 - 13*b^3)*cosh(d*x + c)^8 - 15015*(76*
a^3 + 36*a^2*b - 24*a*b^2 + 31*b^3)*cosh(d*x + c)^6 - 495*(700*a^3 + 135*a^
2*b + 48*a*b^2 - 245*b^3)*cosh(d*x + c)^4 - 490*a^3 + 180*a^2*b + 54*a*b^2
+ 155*b^3 - 225*(154*a^3 - 27*a^2*b + 18*a*b^2 + 49*b^3)*cosh(d*x + c)^2)*s
inh(d*x + c)^8 + 32*(218025*a^2*b*cosh(d*x + c)^13 - 29835*(2*a^3 - 9*a^2*b
 + 6*a*b^2 + 2*b^3)*cosh(d*x + c)^11 - 7150*(23*a^3 - 3*a*b^2 - 13*b^3)*cos
h(d*x + c)^9 - 2145*(76*a^3 + 36*a^2*b - 24*a*b^2 + 31*b^3)*cosh(d*x + c)^7
 - 99*(700*a^3 + 135*a^2*b + 48*a*b^2 - 245*b^3)*cosh(d*x + c)^5 - 75*(154*
a^3 - 27*a^2*b + 18*a*b^2 + 49*b^3)*cosh(d*x + c)^3 - (490*a^3 - 180*a^2*b
 - 54*a*b^2 - 155*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 - 20*(28*a^3 + 13*b^3)
*cosh(d*x + c)^6 + 4*(872100*a^2*b*cosh(d*x + c)^14 - 139230*(2*a^3 - 9*a^2
*b + 6*a*b^2 + 2*b^3)*cosh(d*x + c)^12 - 40040*(23*a^3 - 3*a*b^2 - 13*b^3)*
cosh(d*x + c)^10 - 15015*(76*a^3 + 36*a^2*b - 24*a*b^2 + 31*b^3)*cosh(d*x +
 c)^8 - 924*(700*a^3 + 135*a^2*b + 48*a*b^2 - 245*b^3)*cosh(d*x + c)^6 - 10
50*(154*a^3 - 27*a^2*b + 18*a*b^2 + 49*b^3)*cosh(d*x + c)^4 - 140*a^3 - 65*
b^3 - 28*(490*a^3 - 180*a^2*b - 54*a*b^2 - 155*b^3)*cosh(d*x + c)^2)*sinh(d
x + c)^6 + 8(174420*a^2*b*cosh(d*x + c)^15 - 32130*(2*a^3 - 9*a^2*b + 6*a
*b^2 + 2*b^3)*cosh(d*x + c)^13 - 10920*(23*a^3 - 3*a*b^2 - 13*b^3)*cosh(d*x
 + c)^11 - 5005*(76*a^3 + 36*a^2*b - 24*a*b^2 + 31*b^3)*cosh(d*x + c)^9 - 3
96*(700*a^3 + 135*a^2*b + 48*a*b^2 - 245*b^3)*cosh(d*x + c)^7 - 630*(154*a^
3 - 27*a^2*b + 18*a*b^2 + 49*b^3)*cosh(d*x + c)^5 - 28*(490*a^3 - 180*a^2*b
 - 54*a*b^2 - 155*b^3)*cosh(d*x + c)^3 - 15*(28*a^3 + 13*b^3)*cosh(d*x + c)
)*sinh(d*x + c)^5 + 2*(40*a^3 - 135*a^2*b - 48*a*b^2 + 30*b^3)*cosh(d*x + c
)^4 + 2*(218025*a^2*b*cosh(d*x + c)^16 - 45900*(2*a^3 - 9*a^2*b + 6*a*b^2 +
 2*b^3)*cosh(d*x + c)^14 - 18200*(23*a^3 - 3*a*b^2 - 13*b^3)*cosh(d*x + c)^
12 - 10010*(76*a^3 + 36*a^2*b - 24*a*b^2 + 31*b^3)*cosh(d*x + c)^10 - 990*(
700*a^3 + 135*a^2*b + 48*a*b^2 - 245*b^3)*cosh(d*x + c)^8 - 2100*(154*a^3 -
 27*a^2*b + 18*a*b^2 + 49*b^3)*cosh(d*x + c)^6 - 140*(490*a^3 - 180*a^2*b -
 54*a*b^2 - 155*b^3)*cosh(d*x + c)^4 + 40*a^3 - 135*a^2*b - 48*a*b^2 + 30*b
^3 - 150*(28*a^3 + 13*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(12825*a^2*
b*cosh(d*x + c)^17 - 3060*(2*a^3 - 9*a^2*b + 6*a*b^2 + 2*b^3)*cosh(d*x + c)
^15 - 1400*(23*a^3 - 3*a*b^2 - 13*b^3)*cosh(d*x + c)^13 - 910*(76*a^3 + 36*
a^2*b - 24*a*b^2 + 31*b^3)*cosh(d*x + c)^11 - 110*(700*a^3 + 135*a^2*b + 48
*a*b^2 - 245*b^3)*cosh(d*x + c)^9 - 300*(154*a^3 - 27*a^2*b + 18*a*b^2 + 49
*b^3)*cosh(d*x + c)^7 - 28*(490*a^3 - 180*a^2*b - 54*a*b^2 - 155*b^3)*cosh(
d*x + c)^5 - 50*(28*a^3 + 13*b^3)*cosh(d*x + c)^3 + (40*a^3 - 135*a^2*b - 4
8*a*b^2 + 30*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 20*a^3 + 12*a*b^2 + 10*(
10*a^3 - 9*a^2*b + 6*a*b^2)*cosh(d*x + c)^2 + 2*(8550*a^2*b*cosh(d*x + c)^1
8 - 2295*(2*a^3 - 9*a^2*b + 6*a*b^2 + 2*b^3)*cosh(d*x + c)^16 - 1200*(23*a^
3 - 3*a*b^2 - 13*b^3)*cosh(d*x + c)^14 - 910*(76*a^3 + 36*a^2*b - 24*a*b^2
+ 31*b^3)*cosh(d*x + c)^12 - 132*(700*a^3 + 135*a^2*b + 48*a*b^2 - 245*b^3)
*cosh(d*x + c)^10 - 450*(154*a^3 - 27*a^2*b + 18*a*b^2 + 49*b^3)*cosh(d*x +
 c)^8 - 56*(490*a^3 - 180*a^2*b - 54*a*b^2 - 155*b^3)*cosh(d*x + c)^6 - 150
*(28*a^3 + 13*b^3)*cosh(d*x + c)^4 + 50*a^3 - 45*a^2*b + 30*a*b^2 + 6*(40*a
^3 - 135*a^2*b - 48*a*b^2 + 30*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 45*(
a^2*b*cosh(d*x + c)^22 + 22*a^2*b*cosh(d*x + c)*sinh(d*x + c)^21 + a^2*b*si
nh(d*x + c)^22 + 5*a^2*b*cosh(d*x + c)^20 + 7*a^2*b*cosh(d*x + c)^18 + (231
*a^2*b*cosh(d*x + c)^2 + 5*a^2*b)*sinh(d*x + c)^20 + 20*(77*a^2*b*cosh(d*x
+ c)^3 + 5*a^2*b*cosh(d*x + c))*sinh(d*x + c)^19 - 5*a^2*b*cosh(d*x + c)^16
 + (7315*a^2*b*cosh(d*x + c)^4 + 950*a^2*b*cosh(d*x + c)^2 + 7*a^2*b)*sinh(
d*x + c)^18 + 6*(4389*a^2*b*cosh(d*x + c)^5 + 950*a^2*b*cosh(d*x + c)^3 + 2
1*a^2*b*cosh(d*x + c))*sinh(d*x + c)^17 - 22*a^2*b*cosh(d*x + c)^14 + (7461
3*a^2*b*cosh(d*x + c)^6 + 24225*a^2*b*cosh(d*x + c)^4 + 1071*a^2*b*cosh(d*x
 + c)^2 - 5*a^2*b)*sinh(d*x + c)^16 + 16*(10659*a^2*b*cosh(d*x + c)^7 + 484
5*a^2*b*cosh(d*x + c)^5 + 357*a^2*b*cosh(d*x + c)^3 - 5*a^2*b*cosh(d*x + c)
)*sinh(d*x + c)^15 - 14*a^2*b*cosh(d*x + c)^12 + 2*(159885*a^2*b*cosh(d*x +
 c)^8 + 96900*a^2*b*cosh(d*x + c)^6 + 10710*a^2*b*cosh(d*x + c)^4 - 300*a^2
*b*cosh(d*x + c)^2 - 11*a^2*b)*sinh(d*x + c)^14 + 4*(124355*a^2*b*cosh(d*x
+ c)^9 + 96900*a^2*b*cosh(d*x + c)^7 + 14994*a^2*b*cosh(d*x + c)^5 - 700*a^

2*b*cosh(d*x + c)^3 - 77*a^2*b*cosh(d*x + c))*sinh(d*x + c)^13 + 14*a^2*b*c
osh(d*x + c)^10 + 2*(323323*a^2*b*cosh(d*x + c)^10 + 314925*a^2*b*cosh(d*x
+ c)^8 + 64974*a^2*b*cosh(d*x + c)^6 - 4550*a^2*b*cosh(d*x + c)^4 - 1001*a^
2*b*cosh(d*x + c)^2 - 7*a^2*b)*sinh(d*x + c)^12 + 8*(88179*a^2*b*cosh(d*x +
 c)^11 + 104975*a^2*b*cosh(d*x + c)^9 + 27846*a^2*b*cosh(d*x + c)^7 - 2730*
a^2*b*cosh(d*x + c)^5 - 1001*a^2*b*cosh(d*x + c)^3 - 21*a^2*b*cosh(d*x + c)
)*sinh(d*x + c)^11 + 22*a^2*b*cosh(d*x + c)^8 + 2*(323323*a^2*b*cosh(d*x +
c)^12 + 461890*a^2*b*cosh(d*x + c)^10 + 153153*a^2*b*cosh(d*x + c)^8 - 2002
0*a^2*b*cosh(d*x + c)^6 - 11011*a^2*b*cosh(d*x + c)^4 - 462*a^2*b*cosh(d*x
+ c)^2 + 7*a^2*b)*sinh(d*x + c)^10 + 4*(124355*a^2*b*cosh(d*x + c)^13 + 209
950*a^2*b*cosh(d*x + c)^11 + 85085*a^2*b*cosh(d*x + c)^9 - 14300*a^2*b*cosh
(d*x + c)^7 - 11011*a^2*b*cosh(d*x + c)^5 - 770*a^2*b*cosh(d*x + c)^3 + 35*
a^2*b*cosh(d*x + c))*sinh(d*x + c)^9 + 5*a^2*b*cosh(d*x + c)^6 + 2*(159885*
a^2*b*cosh(d*x + c)^14 + 314925*a^2*b*cosh(d*x + c)^12 + 153153*a^2*b*cosh(
d*x + c)^10 - 32175*a^2*b*cosh(d*x + c)^8 - 33033*a^2*b*cosh(d*x + c)^6 - 3
465*a^2*b*cosh(d*x + c)^4 + 315*a^2*b*cosh(d*x + c)^2 + 11*a^2*b)*sinh(d*x
+ c)^8 + 16*(10659*a^2*b*cosh(d*x + c)^15 + 24225*a^2*b*cosh(d*x + c)^13 +
13923*a^2*b*cosh(d*x + c)^11 - 3575*a^2*b*cosh(d*x + c)^9 - 4719*a^2*b*cosh
(d*x + c)^7 - 693*a^2*b*cosh(d*x + c)^5 + 105*a^2*b*cosh(d*x + c)^3 + 11*a^
2*b*cosh(d*x + c))*sinh(d*x + c)^7 - 7*a^2*b*cosh(d*x + c)^4 + (74613*a^2*b
*cosh(d*x + c)^16 + 193800*a^2*b*cosh(d*x + c)^14 + 129948*a^2*b*cosh(d*x +
 c)^12 - 40040*a^2*b*cosh(d*x + c)^10 - 66066*a^2*b*cosh(d*x + c)^8 - 12936
*a^2*b*cosh(d*x + c)^6 + 2940*a^2*b*cosh(d*x + c)^4 + 616*a^2*b*cosh(d*x +
c)^2 + 5*a^2*b)*sinh(d*x + c)^6 + 2*(13167*a^2*b*cosh(d*x + c)^17 + 38760*a
^2*b*cosh(d*x + c)^15 + 29988*a^2*b*cosh(d*x + c)^13 - 10920*a^2*b*cosh(d*x
 + c)^11 - 22022*a^2*b*cosh(d*x + c)^9 - 5544*a^2*b*cosh(d*x + c)^7 + 1764*
a^2*b*cosh(d*x + c)^5 + 616*a^2*b*cosh(d*x + c)^3 + 15*a^2*b*cosh(d*x + c))
*sinh(d*x + c)^5 - 5*a^2*b*cosh(d*x + c)^2 + (7315*a^2*b*cosh(d*x + c)^18 +
 24225*a^2*b*cosh(d*x + c)^16 + 21420*a^2*b*cosh(d*x + c)^14 - 9100*a^2*b*c
osh(d*x + c)^12 - 22022*a^2*b*cosh(d*x + c)^10 - 6930*a^2*b*cosh(d*x + c)^8
 + 2940*a^2*b*cosh(d*x + c)^6 + 1540*a^2*b*cosh(d*x + c)^4 + 75*a^2*b*cosh(
d*x + c)^2 - 7*a^2*b)*sinh(d*x + c)^4 + 4*(385*a^2*b*cosh(d*x + c)^19 + 142
5*a^2*b*cosh(d*x + c)^17 + 1428*a^2*b*cosh(d*x + c)^15 - 700*a^2*b*cosh(d*x
 + c)^13 - 2002*a^2*b*cosh(d*x + c)^11 - 770*a^2*b*cosh(d*x + c)^9 + 420*a^
2*b*cosh(d*x + c)^7 + 308*a^2*b*cosh(d*x + c)^5 + 25*a^2*b*cosh(d*x + c)^3
- 7*a^2*b*cosh(d*x + c))*sinh(d*x + c)^3 - a^2*b + (231*a^2*b*cosh(d*x + c)
^20 + 950*a^2*b*cosh(d*x + c)^18 + 1071*a^2*b*cosh(d*x + c)^16 - 600*a^2*b*
cosh(d*x + c)^14 - 2002*a^2*b*cosh(d*x + c)^12 - 924*a^2*b*cosh(d*x + c)^10
 + 630*a^2*b*cosh(d*x + c)^8 + 616*a^2*b*cosh(d*x + c)^6 + 75*a^2*b*cosh(d*
x + c)^4 - 42*a^2*b*cosh(d*x + c)^2 - 5*a^2*b)*sinh(d*x + c)^2 + 2*(11*a^2*
b*cosh(d*x + c)^21 + 50*a^2*b*cosh(d*x + c)^19 + 63*a^2*b*cosh(d*x + c)^17
- 40*a^2*b*cosh(d*x + c)^15 - 154*a^2*b*cosh(d*x + c)^13 - 84*a^2*b*cosh(d*
x + c)^11 + 70*a^2*b*cosh(d*x + c)^9 + 88*a^2*b*cosh(d*x + c)^7 + 15*a^2*b*
cosh(d*x + c)^5 - 14*a^2*b*cosh(d*x + c)^3 - 5*a^2*b*cosh(d*x + c))*sinh(d*
x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 45*(a^2*b*co
sh(d*x + c)^22 + 22*a^2*b*cosh(d*x + c)*sinh(d*x + c)^21 + a^2*b*sinh(d*x +
 c)^22 + 5*a^2*b*cosh(d*x + c)^20 + 7*a^2*b*cosh(d*x + c)^18 + (231*a^2*b*c
osh(d*x + c)^2 + 5*a^2*b)*sinh(d*x + c)^20 + 20*(77*a^2*b*cosh(d*x + c)^3 +
 5*a^2*b*cosh(d*x + c))*sinh(d*x + c)^19 - 5*a^2*b*cosh(d*x + c)^16 + (7315
*a^2*b*cosh(d*x + c)^4 + 950*a^2*b*cosh(d*x + c)^2 + 7*a^2*b)*sinh(d*x + c)
^18 + 6*(4389*a^2*b*cosh(d*x + c)^5 + 950*a^2*b*cosh(d*x + c)^3 + 21*a^2*b*
cosh(d*x + c))*sinh(d*x + c)^17 - 22*a^2*b*cosh(d*x + c)^14 + (74613*a^2*b*
cosh(d*x + c)^6 + 24225*a^2*b*cosh(d*x + c)^4 + 1071*a^2*b*cosh(d*x + c)^2
- 5*a^2*b)*sinh(d*x + c)^16 + 16*(10659*a^2*b*cosh(d*x + c)^7 + 4845*a^2*b*
cosh(d*x + c)^5 + 357*a^2*b*cosh(d*x + c)^3 - 5*a^2*b*cosh(d*x + c))*sinh(d
*x + c)^15 - 14*a^2*b*cosh(d*x + c)^12 + 2*(159885*a^2*b*cosh(d*x + c)^8 +
96900*a^2*b*cosh(d*x + c)^6 + 10710*a^2*b*cosh(d*x + c)^4 - 300*a^2*b*cosh(
d*x + c)^2 - 11*a^2*b)*sinh(d*x + c)^14 + 4*(124355*a^2*b*cosh(d*x + c)^9 +
 96900*a^2*b*cosh(d*x + c)^7 + 14994*a^2*b*cosh(d*x + c)^5 - 700*a^2*b*cosh

(d*x + c)^3 - 77*a^2*b*cosh(d*x + c))*sinh(d*x + c)^13 + 14*a^2*b*cosh(d*x + c)^10 + 2*(323323*a^2*b*cosh(d*x + c)^10 + 314925*a^2*b*cosh(d*x + c)^8 + 64974*a^2*b*cosh(d*x + c)^6 - 4550*a^2*b*cosh(d*x + c)^4 - 1001*a^2*b*cosh (d*x + c)^2 - 7*a^2*b)*sinh(d*x + c)^12 + 8*(88179*a^2*b*cosh(d*x + c)^11 + 104975*a^2*b*cosh(d*x + c)^9 + 27846*a^2*b*cosh(d*x + c)^7 - 2730*a^2*b*co sh(d*x + c)^5 - 1001*a^2*b*cosh(d*x + c)^3 - 21*a^2*b*cosh(d*x + c))*sinh(d *x + c)^11 + 22*a^2*b*cosh(d*x + c)^8 + 2*(323323*a^2*b*cosh(d*x + c)^12 + 461890*a^2*b*cosh(d*x + c)^10 + 153153*a^2*b*cosh(d*x + c)^8 - 20020*a^2*b* cosh(d*x + c)^6 - 11011*a^2*b*cosh(d*x + c)^4 - 462*a^2*b*cosh(d*x + c)^2 + 7*a^2*b)*sinh(d*x + c)^10 + 4*(124355*a^2*b*cosh(d*x + c)^13 + 209950*a^2* b*cosh(d*x + c)^11 + 85085*a^2*b*cosh(d*x + c)^9 - 14300*a^2*b*cosh(d*x + c)^7 - 11011*a^2*b*cosh(d*x + c)^5 - 770*a^2*b*cosh(d*x + c)^3 + 35*a^2*b*co sh(d*x + c))*sinh(d*x + c)^9 + 5*a^2*b*cosh(d*x + c)^6 + 2*(159885*a^2*b*co sh(d*x + c)^14 + 314925*a^2*b*cosh(d*x + c)^12 + 153153*a^2*b*cosh(d*x + c) ^10 - 32175*a^2*b*cosh(d*x + c)^8 - 33033*a^2*b*cosh(d*x + c)^6 - 3465*a^2* b*cosh(d*x + c)^4 + 315*a^2*b*cosh(d*x + c)^2 + 11*a^2*b)*sinh(d*x + c)^8 + 16*(10659*a^2*b*cosh(d*x + c)^15 + 24225*a^2*b*cosh(d*x + c)^13 + 13923*a^ 2*b*cosh(d*x + c)^11 - 3575*a^2*b*cosh(d*x + c)^9 - 4719*a^2*b*cosh(d*x + c)^7 - 693*a^2*b*cosh(d*x + c)^5 + 105*a^2*b*cosh(d*x + c)^3 + 11*a^2*b*cosh (d*x + c))*sinh(d*x + c)^7 - 7*a^2*b*cosh(d*x + c)^4 + (74613*a^2*b*cosh(d* x + c)^16 + 193800*a^2*b*cosh(d*x + c)^14 + 129948*a^2*b*cosh(d*x + c)^12 - 40040*a^2*b*cosh(d*x + c)^10 - 66066*a^2*b*cosh(d*x + c)^8 - 12936*a^2*b*c osh(d*x + c)^6 + 2940*a^2*b*cosh(d*x + c)^4 + 616*a^2*b*cosh(d*x + c)^2 + 5 *a^2*b)*sinh(d*x + c)^6 + 2*(13167*a^2*b*cosh(d*x + c)^17 + 38760*a^2*b*cos h(d*x + c)^15 + 29988*a^2*b*cosh(d*x + c)^13 - 10920*a^2*b*cosh(d*x + c)^11 - 22022*a^2*b*cosh(d*x + c)^9 - 5544*a^2*b*cosh(d*x + c)^7 + 1764*a^2*b*co sh(d*x + c)^5 + 616*a^2*b*cosh(d*x + c)^3 + 15*a^2*b*cosh(d*x + c))*sinh(d* x + c)^5 - 5*a^2*b*cosh(d*x + c)^2 + (7315*a^2*b*cosh(d*x + c)^18 + 24225*a ^2*b*cosh(d*x + c)^16 + 21420*a^2*b*cosh(d*x + c)^14 - 9100*a^2*b*cosh(d*x + c)^12 - 22022*a^2*b*cosh(d*x + c)^10 - 6930*a^2*b*cosh(d*x + c)^8 + 2940* a^2*b*cosh(d*x + c)^6 + 1540*a^2*b*cosh(d*x + c)^4 + 75*a^2*b*cosh(d*x + c) ^2 - 7*a^2*b)*sinh(d*x + c)^4 + 4*(385*a^2*b*cosh(d*x + c)^19 + 1425*a^2*b* cosh(d*x + c)^17 + 1428*a^2*b*cosh(d*x + c)^15 - 700*a^2*b*cosh(d*x + c)^13 - 2002*a^2*b*cosh(d*x + c)^11 - 770*a^2*b*cosh(d*x + c)^9 + 420*a^2*b*cosh (d*x + c)^7 + 308*a^2*b*cosh(d*x + c)^5 + 25*a^2*b*cosh(d*x + c)^3 - 7*a^2* b*cosh(d*x + c))*sinh(d*x + c)^3 - a^2*b + (231*a^2*b*cosh(d*x + c)^20 + 95 0*a^2*b*cosh(d*x + c)^18 + 1071*a^2*b*cosh(d*x + c)^16 - 600*a^2*b*cosh(d*x + c)^14 - 2002*a^2*b*cosh(d*x + c)^12 - 924*a^2*b*cosh(d*x + c)^10 + 630*a ^2*b*cosh(d*x + c)^8 + 616*a^2*b*cosh(d*x + c)^6 + 75*a^2*b*cosh(d*x + c)^4 - 42*a^2*b*cosh(d*x + c)^2 - 5*a^2*b)*sinh(d*x + c)^2 + 2*(11*a^2*b*cosh(d *x + c)^21 + 50*a^2*b*cosh(d*x + c)^19 + 63*a^2*b*cosh(d*x + c)^17 - 40*a^2 *b*cosh(d*x + c)^15 - 154*a^2*b*cosh(d*x + c)^13 - 84*a^2*b*cosh(d*x + c)^1 1 + 70*a^2*b*cosh(d*x + c)^9 + 88*a^2*b*cosh(d*x + c)^7 + 15*a^2*b*cosh(d*x + c)^5 - 14*a^2*b*cosh(d*x + c)^3 - 5*a^2*b*cosh(d*x + c))*sinh(d*x + c))* log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(450*a^2*b*cosh(d* x + c)^19 - 135*(2*a^3 - 9*a^2*b + 6*a*b^2 + 2*b^3)*cosh(d*x + c)^17 - 80*(23*a^3 - 3*a*b^2 - 13*b^3)*cosh(d*x + c)^15 - 70*(76*a^3 + 36*a^2*b - 24*a* b^2 + 31*b^3)*cosh(d*x + c)^13 - 12*(700*a^3 + 135*a^2*b + 48*a*b^2 - 245*b ^3)*cosh(d*x + c)^11 - 50*(154*a^3 - 27*a^2*b + 18*a*b^2 + 49*b^3)*cosh(d*x + c)^9 - 8*(490*a^3 - 180*a^2*b - 54*a*b^2 - 155*b^3)*cosh(d*x + c)^7 - 30 *(28*a^3 + 13*b^3)*cosh(d*x + c)^5 + 2*(40*a^3 - 135*a^2*b - 48*a*b^2 + 30* b^3)*cosh(d*x + c)^3 + 5*(10*a^3 - 9*a^2*b + 6*a*b^2)*cosh(d*x + c))*sinh(d *x + c))/(d*cosh(d*x + c)^22 + 22*d*cosh(d*x + c)*sinh(d*x + c)^21 + d*sinh (d*x + c)^22 + 5*d*cosh(d*x + c)^20 + (231*d*cosh(d*x + c)^2 + 5*d)*sinh(d* x + c)^20 + 20*(77*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^19 + 7*d*cosh(d*x + c)^18 + (7315*d*cosh(d*x + c)^4 + 950*d*cosh(d*x + c)^2 + 7*d)*sinh(d*x + c)^18 + 6*(4389*d*cosh(d*x + c)^5 + 950*d*cosh(d*x + c)^3 + 21*d*cosh(d*x + c))*sinh(d*x + c)^17 - 5*d*cosh(d*x + c)^16 + (74613*d*cos h(d*x + c)^6 + 24225*d*cosh(d*x + c)^4 + 1071*d*cosh(d*x + c)^2 - 5*d)*sinh

```
(d*x + c)^16 + 16*(10659*d*cosh(d*x + c)^7 + 4845*d*cosh(d*x + c)^5 + 357*d
*cosh(d*x + c)^3 - 5*d*cosh(d*x + c))*sinh(d*x + c)^15 - 22*d*cosh(d*x + c)
^14 + 2*(159885*d*cosh(d*x + c)^8 + 96900*d*cosh(d*x + c)^6 + 10710*d*cosh(
d*x + c)^4 - 300*d*cosh(d*x + c)^2 - 11*d)*sinh(d*x + c)^14 + 4*(124355*d*c
osh(d*x + c)^9 + 96900*d*cosh(d*x + c)^7 + 14994*d*cosh(d*x + c)^5 - 700*d*
cosh(d*x + c)^3 - 77*d*cosh(d*x + c))*sinh(d*x + c)^13 - 14*d*cosh(d*x + c)
^12 + 2*(323323*d*cosh(d*x + c)^10 + 314925*d*cosh(d*x + c)^8 + 64974*d*cos
h(d*x + c)^6 - 4550*d*cosh(d*x + c)^4 - 1001*d*cosh(d*x + c)^2 - 7*d)*sinh(
d*x + c)^12 + 8*(88179*d*cosh(d*x + c)^11 + 104975*d*cosh(d*x + c)^9 + 2784
6*d*cosh(d*x + c)^7 - 2730*d*cosh(d*x + c)^5 - 1001*d*cosh(d*x + c)^3 - 21*
d*cosh(d*x + c))*sinh(d*x + c)^11 + 14*d*cosh(d*x + c)^10 + 2*(323323*d*cos
h(d*x + c)^12 + 461890*d*cosh(d*x + c)^10 + 153153*d*cosh(d*x + c)^8 - 2002
0*d*cosh(d*x + c)^6 - 11011*d*cosh(d*x + c)^4 - 462*d*cosh(d*x + c)^2 + 7*d
)*sinh(d*x + c)^10 + 4*(124355*d*cosh(d*x + c)^13 + 209950*d*cosh(d*x + c)^
11 + 85085*d*cosh(d*x + c)^9 - 14300*d*cosh(d*x + c)^7 - 11011*d*cosh(d*x +
 c)^5 - 770*d*cosh(d*x + c)^3 + 35*d*cosh(d*x + c))*sinh(d*x + c)^9 + 22*d*
cosh(d*x + c)^8 + 2*(159885*d*cosh(d*x + c)^14 + 314925*d*cosh(d*x + c)^12
+ 153153*d*cosh(d*x + c)^10 - 32175*d*cosh(d*x + c)^8 - 33033*d*cosh(d*x +
c)^6 - 3465*d*cosh(d*x + c)^4 + 315*d*cosh(d*x + c)^2 + 11*d)*sinh(d*x + c)
^8 + 16*(10659*d*cosh(d*x + c)^15 + 24225*d*cosh(d*x + c)^13 + 13923*d*cosh
(d*x + c)^11 - 3575*d*cosh(d*x + c)^9 - 4719*d*cosh(d*x + c)^7 - 693*d*cosh
(d*x + c)^5 + 105*d*cosh(d*x + c)^3 + 11*d*cosh(d*x + c))*sinh(d*x + c)^7 +
 5*d*cosh(d*x + c)^6 + (74613*d*cosh(d*x + c)^16 + 193800*d*cosh(d*x + c)^1
4 + 129948*d*cosh(d*x + c)^12 - 40040*d*cosh(d*x + c)^10 - 66066*d*cosh(d*x
 + c)^8 - 12936*d*cosh(d*x + c)^6 + 2940*d*cosh(d*x + c)^4 + 616*d*cosh(d*x
 + c)^2 + 5*d)*sinh(d*x + c)^6 + 2*(13167*d*cosh(d*x + c)^17 + 38760*d*cosh
(d*x + c)^15 + 29988*d*cosh(d*x + c)^13 - 10920*d*cosh(d*x + c)^11 - 22022*
d*cosh(d*x + c)^9 - 5544*d*cosh(d*x + c)^7 + 1764*d*cosh(d*x + c)^5 + 616*d
*cosh(d*x + c)^3 + 15*d*cosh(d*x + c))*sinh(d*x + c)^5 - 7*d*cosh(d*x + c)^
4 + (7315*d*cosh(d*x + c)^18 + 24225*d*cosh(d*x + c)^16 + 21420*d*cosh(d*x
+ c)^14 - 9100*d*cosh(d*x + c)^12 - 22022*d*cosh(d*x + c)^10 - 6930*d*cosh(
d*x + c)^8 + 2940*d*cosh(d*x + c)^6 + 1540*d*cosh(d*x + c)^4 + 75*d*cosh(d*
x + c)^2 - 7*d)*sinh(d*x + c)^4 + 4*(385*d*cosh(d*x + c)^19 + 1425*d*cosh(d
*x + c)^17 + 1428*d*cosh(d*x + c)^15 - 700*d*cosh(d*x + c)^13 - 2002*d*cosh
(d*x + c)^11 - 770*d*cosh(d*x + c)^9 + 420*d*cosh(d*x + c)^7 + 308*d*cosh(d
*x + c)^5 + 25*d*cosh(d*x + c)^3 - 7*d*cosh(d*x + c))*sinh(d*x + c)^3 - 5*d
*cosh(d*x + c)^2 + (231*d*cosh(d*x + c)^20 + 950*d*cosh(d*x + c)^18 + 1071*
d*cosh(d*x + c)^16 - 600*d*cosh(d*x + c)^14 - 2002*d*cosh(d*x + c)^12 - 924
*d*cosh(d*x + c)^10 + 630*d*cosh(d*x + c)^8 + 616*d*cosh(d*x + c)^6 + 75*d*
cosh(d*x + c)^4 - 42*d*cosh(d*x + c)^2 - 5*d)*sinh(d*x + c)^2 + 2*(11*d*cos
h(d*x + c)^21 + 50*d*cosh(d*x + c)^19 + 63*d*cosh(d*x + c)^17 - 40*d*cosh(d
*x + c)^15 - 154*d*cosh(d*x + c)^13 - 84*d*cosh(d*x + c)^11 + 70*d*cosh(d*x
 + c)^9 + 88*d*cosh(d*x + c)^7 + 15*d*cosh(d*x + c)^5 - 14*d*cosh(d*x + c)^
3 - 5*d*cosh(d*x + c))*sinh(d*x + c) - d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tanh^3\left(c + dx\right)\right)^3 \operatorname{csch}^4\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*tanh(d*x+c)**3)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)**3*csch(c + d*x)**4, x)
```

Giac [B] time = 2.87652, size = 590, normalized size = 4.28

$$-\frac{2520\, a^{2} b \log\left(e^{(2\, dx+2\, c)}+1\right)-2520\, a^{2} b \log\left(\left|e^{(2\, dx+2\, c)}-1\right|\right)+\frac{140\left(33\, a^{2} b e^{(6\, dx+6\, c)}-99\, a^{2} b e^{(4\, dx+4\, c)}+24\, a^{3} e^{(2\, dx+2\, c)}+99\, a^{2} b e^{(2\, dx+2\, c)}-8\, a^{3}-\cdots\right.}{\left(e^{(2\, dx+2\, c)}-1\right)^{3}}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] -1/840*(2520*a^2*b*log(e^(2*d*x + 2*c) + 1) - 2520*a^2*b*log(abs(e^(2*d*x +
 2*c) - 1)) + 140*(33*a^2*b*e^(6*d*x + 6*c) - 99*a^2*b*e^(4*d*x + 4*c) + 24
*a^3*e^(2*d*x + 2*c) + 99*a^2*b*e^(2*d*x + 2*c) - 8*a^3 - 33*a^2*b)/(e^(2*d
*x + 2*c) - 1)^3 - (6849*a^2*b*e^(16*d*x + 16*c) + 59832*a^2*b*e^(14*d*x +
14*c) + 222012*a^2*b*e^(12*d*x + 12*c) - 10080*a*b^2*e^(12*d*x + 12*c) - 33
60*b^3*e^(12*d*x + 12*c) + 459144*a^2*b*e^(10*d*x + 10*c) - 26880*a*b^2*e^(
10*d*x + 10*c) + 4480*b^3*e^(10*d*x + 10*c) + 580230*a^2*b*e^(8*d*x + 8*c)
- 23520*a*b^2*e^(8*d*x + 8*c) - 11200*b^3*e^(8*d*x + 8*c) + 459144*a^2*b*e^
(6*d*x + 6*c) - 10752*a*b^2*e^(6*d*x + 6*c) + 4480*b^3*e^(6*d*x + 6*c) + 22
2012*a^2*b*e^(4*d*x + 4*c) - 8736*a*b^2*e^(4*d*x + 4*c) - 3360*b^3*e^(4*d*x
 + 4*c) + 59832*a^2*b*e^(2*d*x + 2*c) - 5376*a*b^2*e^(2*d*x + 2*c) + 6849*a
^2*b - 672*a*b^2)/(e^(2*d*x + 2*c) + 1)^8)/d
```

3.73 $\int \frac{\sinh^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

Optimal. Leaf size=491

$$-\frac{a^2 b\left(a^2+2b^2\right)\log\left(a+b\tanh^3(c+dx)\right)}{d\left(a^2-b^2\right)^3}+\frac{a^{2/3}\sqrt[3]{b}\left(7a^2b^2+3a^{2/3}b^{4/3}\left(2a^2+b^2\right)+a^4+b^4\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c\right.}{6d\left(a^2-b^2\right)^3}$$

```
[Out] -((a^(2/3)*b^(1/3)*(a^2 + 3*a^(4/3)*b^(2/3) - b^2)*ArcTan[(a^(1/3) - 2*b^(1
/3)*Tanh[c + d*x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*(a^(4/3) + a^(2/3)*b^(2/3)
+ b^(4/3))^3*d)) - (3*a*(a - 5*b)*Log[1 - Tanh[c + d*x]])/(16*(a + b)^3*d)
+ (3*a*(a + 5*b)*Log[1 + Tanh[c + d*x]])/(16*(a - b)^3*d) - (a^(2/3)*b^(1/3
)*(a^4 + 7*a^2*b^2 + b^4 + 3*a^(2/3)*b^(4/3)*(2*a^2 + b^2))*Log[a^(1/3) + b
^(1/3)*Tanh[c + d*x]])/(3*(a^2 - b^2)^3*d) + (a^(2/3)*b^(1/3)*(a^4 + 7*a^2*
b^2 + b^4 + 3*a^(2/3)*b^(4/3)*(2*a^2 + b^2))*Log[a^(2/3) - a^(1/3)*b^(1/3)*
Tanh[c + d*x] + b^(2/3)*Tanh[c + d*x]^2])/(6*(a^2 - b^2)^3*d) - (a^2*b*(a^2
 + 2*b^2)*Log[a + b*Tanh[c + d*x]^3])/((a^2 - b^2)^3*d) + 1/(16*(a + b)*d*(
1 - Tanh[c + d*x])^2) - (5*a - b)/(16*(a + b)^2*d*(1 - Tanh[c + d*x])) - 1/
(16*(a - b)*d*(1 + Tanh[c + d*x])^2) + (5*a + b)/(16*(a - b)^2*d*(1 + Tanh[
c + d*x]))
```

Rubi [A] time = 0.891549, antiderivative size = 491, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {3663, 6725, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{a^2 b\left(a^2+2b^2\right)\log\left(a+b\tanh^3(c+dx)\right)}{d\left(a^2-b^2\right)^3}+\frac{a^{2/3}\sqrt[3]{b}\left(7a^2b^2+3a^{2/3}b^{4/3}\left(2a^2+b^2\right)+a^4+b^4\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c\right.}{6d\left(a^2-b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^3),x]
```


```
[Out] -((a^(2/3)*b^(1/3)*(a^2 + 3*a^(4/3)*b^(2/3) - b^2)*ArcTan[(a^(1/3) - 2*b^(1
/3)*Tanh[c + d*x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*(a^(4/3) + a^(2/3)*b^(2/3)
+ b^(4/3))^3*d)) - (3*a*(a - 5*b)*Log[1 - Tanh[c + d*x]])/(16*(a + b)^3*d)
+ (3*a*(a + 5*b)*Log[1 + Tanh[c + d*x]])/(16*(a - b)^3*d) - (a^(2/3)*b^(1/3
)*(a^4 + 7*a^2*b^2 + b^4 + 3*a^(2/3)*b^(4/3)*(2*a^2 + b^2))*Log[a^(1/3) + b
^(1/3)*Tanh[c + d*x]])/(3*(a^2 - b^2)^3*d) + (a^(2/3)*b^(1/3)*(a^4 + 7*a^2*
b^2 + b^4 + 3*a^(2/3)*b^(4/3)*(2*a^2 + b^2))*Log[a^(2/3) - a^(1/3)*b^(1/3)*
Tanh[c + d*x] + b^(2/3)*Tanh[c + d*x]^2])/(6*(a^2 - b^2)^3*d) - (a^2*b*(a^2
 + 2*b^2)*Log[a + b*Tanh[c + d*x]^3])/((a^2 - b^2)^3*d) + 1/(16*(a + b)*d*(
1 - Tanh[c + d*x])^2) - (5*a - b)/(16*(a + b)^2*d*(1 - Tanh[c + d*x])) - 1/
(16*(a - b)*d*(1 + Tanh[c + d*x])^2) + (5*a + b)/(16*(a - b)^2*d*(1 + Tanh[
c + d*x]))
```


Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^3(a+bx^3)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{8(a+b)(-1+x)^3} + \frac{-5a+b}{16(a+b)^2(-1+x)^2} - \frac{3a(a-5b)}{16(a+b)^3(-1+x)} + \frac{1}{8(a-b)(1+x)^3} + \frac{-5a-b}{16(a-b)^2(1+x)^2} + \ldots\right.\right.}{d}$$

$$= -\frac{3a(a-5b)\log(1-\tanh(c+dx))}{16(a+b)^3d} + \frac{3a(a+5b)\log(1+\tanh(c+dx))}{16(a-b)^3d} + \frac{\ldots}{16(a+b)d(1-\ldots}$$

$$= -\frac{3a(a-5b)\log(1-\tanh(c+dx))}{16(a+b)^3d} + \frac{3a(a+5b)\log(1+\tanh(c+dx))}{16(a-b)^3d} + \frac{\ldots}{16(a+b)d(1-\ldots}$$

$$= -\frac{3a(a-5b)\log(1-\tanh(c+dx))}{16(a+b)^3d} + \frac{3a(a+5b)\log(1+\tanh(c+dx))}{16(a-b)^3d} - \frac{a^2b\left(a^2+2b^2\right)\ldots}{\ldots}$$

$$= -\frac{3a(a-5b)\log(1-\tanh(c+dx))}{16(a+b)^3d} + \frac{3a(a+5b)\log(1+\tanh(c+dx))}{16(a-b)^3d} - \frac{a^{2/3}\sqrt[3]{b}\left(a^4+7\ldots\right.}{\ldots}$$

$$= -\frac{3a(a-5b)\log(1-\tanh(c+dx))}{16(a+b)^3d} + \frac{3a(a+5b)\log(1+\tanh(c+dx))}{16(a-b)^3d} - \frac{a^{2/3}\sqrt[3]{b}\left(a^4+7\ldots\right.}{\ldots}$$

$$= -\frac{a^{2/3}\sqrt[3]{b}\left(a^2+3a^{4/3}b^{2/3}-b^2\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\tanh(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\left(a^{4/3}+a^{2/3}b^{2/3}+b^{4/3}\right)^3 d} - \frac{3a(a-5b)\log(1-\tanh(c+dx))}{16(a+b)^3d}\ldots$$

Mathematica [C] time = 4.37695, size = 645, normalized size = 1.31

$$\frac{3\left(-8a\left(a^2b+a^3+2ab^2+2b^3\right)\sinh(2(c+dx))+a(a-b)\left(12\left(a^2-6ab+5b^2\right)(c+dx)+(a+b)^2\sinh(4(c+dx))\right)+4\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (-32*a*b*RootSum[a - b + 3*a*#1 + 3*b*#1 + 3*a*#1^2 - 3*b*#1^2 + a*#1^3 + b
*#1^3 & , (-6*a^3*c - 12*a*b^2*c - 6*a^3*d*x - 12*a*b^2*d*x + 3*a^3*Log[E^(
2*(c + d*x)) - #1] + 6*a*b^2*Log[E^(2*(c + d*x)) - #1] - 8*a^3*c*#1 + 4*a^2
*b*c*#1 + 8*a*b^2*c*#1 - 4*b^3*c*#1 - 8*a^3*d*x*#1 + 4*a^2*b*d*x*#1 + 8*a*b
^2*d*x*#1 - 4*b^3*d*x*#1 + 4*a^3*Log[E^(2*(c + d*x)) - #1]*#1 - 2*a^2*b*Log
[E^(2*(c + d*x)) - #1]*#1 - 4*a*b^2*Log[E^(2*(c + d*x)) - #1]*#1 + 2*b^3*Lo
g[E^(2*(c + d*x)) - #1]*#1 - 10*a^3*c*#1^2 + 20*a^2*b*c*#1^2 - 20*a*b^2*c*#
1^2 + 4*b^3*c*#1^2 - 10*a^3*d*x*#1^2 + 20*a^2*b*d*x*#1^2 - 20*a*b^2*d*x*#1^
2 + 4*b^3*d*x*#1^2 + 5*a^3*Log[E^(2*(c + d*x)) - #1]*#1^2 - 10*a^2*b*Log[E^
(2*(c + d*x)) - #1]*#1^2 + 10*a*b^2*Log[E^(2*(c + d*x)) - #1]*#1^2 - 2*b^3*
Log[E^(2*(c + d*x)) - #1]*#1^2)/(a - b + 2*a*#1 + 2*b*#1 + a*#1^2 - b*#1^2)
 & ] + 3*(4*b*(5*a^3 + 5*a^2*b + a*b^2 + b^3)*Cosh[2*(c + d*x)] - (a - b)*b
*(a + b)^2*Cosh[4*(c + d*x)] - 8*a*(a^3 + a^2*b + 2*a*b^2 + 2*b^3)*Sinh[2*(
c + d*x)] + a*(a - b)*(12*(a^2 - 6*a*b + 5*b^2)*(c + d*x) + (a + b)^2*Sinh[
```

```
4*(c + d*x)]))))/(96*(a - b)^2*(a + b)^3*d)
```

Maple [C] time = 0.128, size = 603, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^3),x)
```

```
[Out] -8/d/(32*a-32*b)/(tanh(1/2*d*x+1/2*c)+1)^4+32/d/(64*a-64*b)/(tanh(1/2*d*x+1
/2*c)+1)^3+1/8/d/(a-b)^2/(tanh(1/2*d*x+1/2*c)+1)^2*a+5/8/d/(a-b)^2/(tanh(1/
2*d*x+1/2*c)+1)^2*b-3/8/d/(a-b)^2/(tanh(1/2*d*x+1/2*c)+1)*a-3/8/d/(a-b)^2/(
tanh(1/2*d*x+1/2*c)+1)*b+3/8/d*a^2/(a-b)^3*ln(tanh(1/2*d*x+1/2*c)+1)+15/8/d
*a/(a-b)^3*ln(tanh(1/2*d*x+1/2*c)+1)*b+8/d/(32*a+32*b)/(tanh(1/2*d*x+1/2*c)
-1)^4+32/d/(64*a+64*b)/(tanh(1/2*d*x+1/2*c)-1)^3-1/8/d/(a+b)^2/(tanh(1/2*d*
x+1/2*c)-1)^2*a+5/8/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)^2*b-3/8/d/(a+b)^2/(ta
nh(1/2*d*x+1/2*c)-1)*a+3/8/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)*b-3/8/d/(a+b)^
3*ln(tanh(1/2*d*x+1/2*c)-1)*a^2+15/8/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)-1)*a*
b-1/3/d*a*b/(a-b)^3/(a+b)^3*sum((3*a^2*(a^2+2*b^2)*_R^5+3*a*b*(-2*a^2-b^2)*
_R^4+2*(4*a^4+13*a^2*b^2+b^4)*_R^3+12*a*b*(a^2+2*b^2)*_R^2+(a^4-8*a^2*b^2-2
*b^4)*_R+6*a^3*b+3*a*b^3)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1
/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -6*a^4*b*(integrate(((a + b)*e^(4*d*x + 4*c) + 3*(a - b)*e^(2*d*x + 2*c) +
3*a + 3*b)*e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x +
4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4
- b^6) - (d*x + c)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d)) - 12*a^2*b^3*(i
ntegrate(((a + b)*e^(4*d*x + 4*c) + 3*(a - b)*e^(2*d*x + 2*c) + 3*a + 3*b)*
e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a
 + b)*e^(2*d*x + 2*c) + a - b), x)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) - (d
*x + c)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d)) + 10*a^4*b*integrate(e^(4*
d*x + 4*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)
*e^(2*d*x + 2*c) + a - b), x)/(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4
+ b^5) - 20*a^3*b^2*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x + 6*c) + 3*
(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^5 + a^4
*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5) + 20*a^2*b^3*integrate(e^(4*d*x +
 4*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2
*d*x + 2*c) + a - b), x)/(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5
) - 4*a*b^4*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*
e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^5 + a^4*b - 2*a
^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5) + 8*a^4*b*integrate(e^(2*d*x + 2*c)/((a +
 b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c)
 + a - b), x)/(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5) - 4*a^3*b
^2*integrate(e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x
+ 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^5 + a^4*b - 2*a^3*b^2 -
```

```
2*a^2*b^3 + a*b^4 + b^5) - 8*a^2*b^3*integrate(e^(2*d*x + 2*c)/((a + b)*e^(
6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a -
b), x)/(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5) + 4*a*b^4*integr
ate(e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) +
3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3
 + a*b^4 + b^5) - 1/64*(a^4 + 2*a^3*b - 2*a*b^3 - b^4 - 24*(a^4*d*e^(4*c) -
 7*a^3*b*d*e^(4*c) + 11*a^2*b^2*d*e^(4*c) - 5*a*b^3*d*e^(4*c))*x*e^(4*d*x)
- (a^4*e^(8*c) - 2*a^2*b^2*e^(8*c) + b^4*e^(8*c))*e^(8*d*x) + 4*(2*a^4*e^(6
*c) - 3*a^3*b*e^(6*c) - a^2*b^2*e^(6*c) + 3*a*b^3*e^(6*c) - b^4*e^(6*c))*e^
(6*d*x) - 4*(2*a^4*e^(2*c) + 7*a^3*b*e^(2*c) + 9*a^2*b^2*e^(2*c) + 5*a*b^3*
e^(2*c) + b^4*e^(2*c))*e^(2*d*x))*e^(-4*d*x)/(a^5*d*e^(4*c) + a^4*b*d*e^(4*
c) - 2*a^3*b^2*d*e^(4*c) - 2*a^2*b^3*d*e^(4*c) + a*b^4*d*e^(4*c) + b^5*d*e^
(4*c))
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a+b*tanh(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.33021, size = 489, normalized size = 1.

$$\frac{\frac{24\left(a^2+5ab\right)dx}{a^3-3a^2b+3ab^2-b^3}-\frac{\left(18a^2e^{(4dx+4c)}+90abe^{(4dx+4c)}-8a^2e^{(2dx+2c)}+4abe^{(2dx+2c)}+4b^2e^{(2dx+2c)}+a^2-2ab+b^2\right)e^{(-4dx)}}{a^3e^{(4c)}-3a^2be^{(4c)}+3ab^2e^{(4c)}-b^3e^{(4c)}}-\frac{64\left(a^4b+2a^2b^3\right)\log\left(\left|ae^{(6dx+6c)}+\right.\right.}{}}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/64*(24*(a^2 + 5*a*b)*d*x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (18*a^2*e^(4*d
*x + 4*c) + 90*a*b*e^(4*d*x + 4*c) - 8*a^2*e^(2*d*x + 2*c) + 4*a*b*e^(2*d*x
 + 2*c) + 4*b^2*e^(2*d*x + 2*c) + a^2 - 2*a*b + b^2)*e^(-4*d*x)/(a^3*e^(4*c
) - 3*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c) - b^3*e^(4*c)) - 64*(a^4*b + 2*a^2*b^
3)*log(abs(a*e^(6*d*x + 6*c) + b*e^(6*d*x + 6*c) + 3*a*e^(4*d*x + 4*c) - 3*
b*e^(4*d*x + 4*c) + 3*a*e^(2*d*x + 2*c) + 3*b*e^(2*d*x + 2*c) + a - b))/(a^
6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) + (a*e^(4*d*x + 24*c) + b*e^(4*d*x + 24*c)
 - 8*a*e^(2*d*x + 22*c) + 4*b*e^(2*d*x + 22*c))/(a^2*e^(20*c) + 2*a*b*e^(20
*c) + b^2*e^(20*c)))/d
```

3.74 $\int \frac{\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

Optimal. Leaf size=32

$$i\text{Unintegrable}\left(-\frac{i\sinh^3(c+dx)}{a+b\tanh^3(c+dx)},x\right)$$

```
[Out] I*Unintegrable[((-I)*Sinh[c + d*x]^3)/(a + b*Tanh[c + d*x]^3), x]
```

Rubi [A] time = 0.0461342, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] I*Defer[Int][((-I)*Sinh[c + d*x]^3)/(a + b*Tanh[c + d*x]^3), x]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = i\int -\frac{i\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

Mathematica [A] time = 0.483965, size = 826, normalized size = 25.81

$$\cosh(3(c+dx))a^3+27b\sinh(c+dx)a^2-b\sinh(3(c+dx))a^2-9\left(a^2+3b^2\right)\cosh(c+dx)a-b^2\cosh(3(c+dx))a-2b\text{Ro}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (-9*a*(a^2 + 3*b^2)*Cosh[c + d*x] + a^3*Cosh[3*(c + d*x)] - a*b^2*Cosh[3*(c
 + d*x)] - 2*a*b*RootSum[a - b + 3*a*#1^2 + 3*b*#1^2 + 3*a*#1^4 - 3*b*#1^4
+ a*#1^6 + b*#1^6 & , (3*a^2*c + 3*a*b*c + 3*b^2*c + 3*a^2*d*x + 3*a*b*d*x
+ 3*b^2*d*x + 6*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c +
d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 6*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c
 + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 6*b^2*Log[-Cosh
[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2
]*#1] + 2*a^2*c*#1^2 - 2*b^2*c*#1^2 + 2*a^2*d*x*#1^2 - 2*b^2*d*x*#1^2 + 4*a
^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh
[(c + d*x)/2]*#1]*#1^2 - 4*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] +
 Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 3*a^2*c*#1^4 - 3*a*b*c
*#1^4 + 3*b^2*c*#1^4 + 3*a^2*d*x*#1^4 - 3*a*b*d*x*#1^4 + 3*b^2*d*x*#1^4 + 6
*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Si
nh[(c + d*x)/2]*#1]*#1^4 - 6*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2]
```

```
 + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + 6*b^2*Log[-Cosh[(c +
 d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]
*#1^4)/(a*#1 + b*#1 + 2*a*#1^3 - 2*b*#1^3 + a*#1^5 + b*#1^5) & ] + 27*a^2*b
*Sinh[c + d*x] + 9*b^3*Sinh[c + d*x] - a^2*b*Sinh[3*(c + d*x)] + b^3*Sinh[3
*(c + d*x)])/(12*(a - b)^2*(a + b)^2*d)
```

Maple [A] time = 0.11, size = 346, normalized size = 10.8

$$-8\frac{1}{d(16a-16b)(\tanh(1/2dx+c/2)+1)^2}+\frac{16}{3d(16a-16b)}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3}-\frac{a}{2d(a-b)^2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^3),x)
```

```
[Out] -8/d/(16*a-16*b)/(tanh(1/2*d*x+1/2*c)+1)^2+16/3/d/(tanh(1/2*d*x+1/2*c)+1)^3
/(16*a-16*b)-1/2/d/(a-b)^2/(tanh(1/2*d*x+1/2*c)+1)*a-1/d/(a-b)^2/(tanh(1/2*
d*x+1/2*c)+1)*b-16/3/d/(tanh(1/2*d*x+1/2*c)-1)^3/(16*a+16*b)-8/d/(16*a+16*b
)/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)*a-1/d/(a+
b)^2/(tanh(1/2*d*x+1/2*c)-1)*b-1/3/d*a*b/(a+b)^2/(a-b)^2*sum(((2*a^2+b^2)*_
R^4-6*_R^3*a*b+2*(4*a^2+5*b^2)*_R^2-6*a*b*_R+2*a^2+b^2)/(_R^5*a+2*_R^3*a+4*
_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+
3*_Z^2*a+a))
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/24*(a^3 + a^2*b - a*b^2 - b^3 + (a^3*e^(6*c) - a^2*b*e^(6*c) - a*b^2*e^(6
*c) + b^3*e^(6*c))*e^(6*d*x) - 9*(a^3*e^(4*c) - 3*a^2*b*e^(4*c) + 3*a*b^2*e
^(4*c) - b^3*e^(4*c))*e^(4*d*x) - 9*(a^3*e^(2*c) + 3*a^2*b*e^(2*c) + 3*a*b^
2*e^(2*c) + b^3*e^(2*c))*e^(2*d*x))*e^(-3*d*x)/(a^4*d*e^(3*c) - 2*a^2*b^2*d
*e^(3*c) + b^4*d*e^(3*c)) - 1/8*integrate(16*(3*(a^3*b*e^(5*c) - a^2*b^2*e^
(5*c) + a*b^3*e^(5*c))*e^(5*d*x) + 2*(a^3*b*e^(3*c) - a*b^3*e^(3*c))*e^(3*d
*x) + 3*(a^3*b*e^c + a^2*b^2*e^c + a*b^3*e^c)*e^(d*x))/(a^5 - a^4*b - 2*a^3
*b^2 + 2*a^2*b^3 + a*b^4 - b^5 + (a^5*e^(6*c) + a^4*b*e^(6*c) - 2*a^3*b^2*e
^(6*c) - 2*a^2*b^3*e^(6*c) + a*b^4*e^(6*c) + b^5*e^(6*c))*e^(6*d*x) + 3*(a^
5*e^(4*c) - a^4*b*e^(4*c) - 2*a^3*b^2*e^(4*c) + 2*a^2*b^3*e^(4*c) + a*b^4*e
^(4*c) - b^5*e^(4*c))*e^(4*d*x) + 3*(a^5*e^(2*c) + a^4*b*e^(2*c) - 2*a^3*b^
2*e^(2*c) - 2*a^2*b^3*e^(2*c) + a*b^4*e^(2*c) + b^5*e^(2*c))*e^(2*d*x)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a+b*tanh(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.05765, size = 473, normalized size = 14.78

$$-\frac{\frac{\left(9\,ae^{(2\,dx+2\,c)}+9\,be^{(2\,dx+2\,c)}-a+b\right)e^{(-3\,dx)}}{a^2e^{(3\,c)}-2\,abe^{(3\,c)}+b^2e^{(3\,c)}}-\frac{a^2e^{(3\,dx+30\,c)}+2\,abe^{(3\,dx+30\,c)}+b^2e^{(3\,dx+30\,c)}-9\,a^2e^{(dx+28\,c)}+9\,b^2e^{(dx+28\,c)}}{a^3e^{(27\,c)}+3\,a^2be^{(27\,c)}+3\,ab^2e^{(27\,c)}+b^3e^{(27\,c)}}}{24\,d}-\frac{\frac{6\left(a^3be^c+a^2b^2e^c+ab^3e^c\right)dx}{ad-bd}-\frac{\left(a^3be^c+a\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -1/24*((9*a*e^(2*d*x + 2*c) + 9*b*e^(2*d*x + 2*c) - a + b)*e^(-3*d*x)/(a^2*
e^(3*c) - 2*a*b*e^(3*c) + b^2*e^(3*c)) - (a^2*e^(3*d*x + 30*c) + 2*a*b*e^(3
*d*x + 30*c) + b^2*e^(3*d*x + 30*c) - 9*a^2*e^(d*x + 28*c) + 9*b^2*e^(d*x +
 28*c))/(a^3*e^(27*c) + 3*a^2*b*e^(27*c) + 3*a*b^2*e^(27*c) + b^3*e^(27*c))
)/d - (6*(a^3*b*e^c + a^2*b^2*e^c + a*b^3*e^c)*d*x/(a*d - b*d) - (a^3*b*e^c
 + a^2*b^2*e^c + a*b^3*e^c)*log(abs(a*e^(6*d*x + 6*c) + b*e^(6*d*x + 6*c) +
 3*a*e^(4*d*x + 4*c) - 3*b*e^(4*d*x + 4*c) + 3*a*e^(2*d*x + 2*c) + 3*b*e^(2
*d*x + 2*c) + a - b))/(a*d - b*d))/((a^4 - 2*a^2*b^2 + b^4)*d)
```

3.75 $\int \frac{\sinh^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

Optimal. Leaf size=384

$$-\frac{a^{2/3}\sqrt[3]{b}\left(3a^{2/3}b^{4/3}+a^2+2b^2\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+b^{2/3}\tanh^2(c+dx)\right)}{6d\left(a^2-b^2\right)^2}+\frac{b\left(2a^2+b^2\right)\log\left(a+b\tanh^3(c\right.}{3d\left(a^2-b^2\right)^2}$$

```
[Out] (a^(2/3)*b^(1/3)*(a^2 - 3*a^(2/3)*b^(4/3) + 2*b^2)*ArcTan[(a^(1/3) - 2*b^(1
/3)*Tanh[c + d*x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*(a^2 - b^2)^2*d) + ((a - 2*
b)*Log[1 - Tanh[c + d*x]])/(4*(a + b)^2*d) - ((a + 2*b)*Log[1 + Tanh[c + d*
x]])/(4*(a - b)^2*d) + (a^(2/3)*b^(1/3)*(a^2 + 3*a^(2/3)*b^(4/3) + 2*b^2)*L
og[a^(1/3) + b^(1/3)*Tanh[c + d*x]])/(3*(a^2 - b^2)^2*d) - (a^(2/3)*b^(1/3)
*(a^2 + 3*a^(2/3)*b^(4/3) + 2*b^2)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Tanh[c + d
*x] + b^(2/3)*Tanh[c + d*x]^2])/(6*(a^2 - b^2)^2*d) + (b*(2*a^2 + b^2)*Log[
a + b*Tanh[c + d*x]^3])/(3*(a^2 - b^2)^2*d) + 1/(4*(a + b)*d*(1 - Tanh[c +
d*x])) - 1/(4*(a - b)*d*(1 + Tanh[c + d*x]))
```

Rubi [A] time = 0.629447, antiderivative size = 384, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {3663, 6725, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{a^{2/3}\sqrt[3]{b}\left(3a^{2/3}b^{4/3}+a^2+2b^2\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+b^{2/3}\tanh^2(c+dx)\right)}{6d\left(a^2-b^2\right)^2}+\frac{b\left(2a^2+b^2\right)\log\left(a+b\tanh^3(c\right.}{3d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (a^(2/3)*b^(1/3)*(a^2 - 3*a^(2/3)*b^(4/3) + 2*b^2)*ArcTan[(a^(1/3) - 2*b^(1
/3)*Tanh[c + d*x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*(a^2 - b^2)^2*d) + ((a - 2*
b)*Log[1 - Tanh[c + d*x]])/(4*(a + b)^2*d) - ((a + 2*b)*Log[1 + Tanh[c + d*
x]])/(4*(a - b)^2*d) + (a^(2/3)*b^(1/3)*(a^2 + 3*a^(2/3)*b^(4/3) + 2*b^2)*L
og[a^(1/3) + b^(1/3)*Tanh[c + d*x]])/(3*(a^2 - b^2)^2*d) - (a^(2/3)*b^(1/3)
*(a^2 + 3*a^(2/3)*b^(4/3) + 2*b^2)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Tanh[c + d
*x] + b^(2/3)*Tanh[c + d*x]^2])/(6*(a^2 - b^2)^2*d) + (b*(2*a^2 + b^2)*Log[
a + b*Tanh[c + d*x]^3])/(3*(a^2 - b^2)^2*d) + 1/(4*(a + b)*d*(1 - Tanh[c +
d*x])) - 1/(4*(a - b)*d*(1 + Tanh[c + d*x]))
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^2}{(1-x^2)^2(a+bx^3)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{4(a+b)(-1+x)^2} + \frac{a-2b}{4(a+b)^2(-1+x)} + \frac{1}{4(a-b)(1+x)^2} + \frac{-a-2b}{4(a-b)^2(1+x)} + \frac{b\left(3a^2b-a\left(a^2+2b^2\right)x+b\left(2a^2+\right.\right.}{\left(a^2-b^2\right)^2\left(a+bx^3\right)}\right.\right.}{d}$$

$$= \frac{(a-2b)\log(1-\tanh(c+dx))}{4(a+b)^2d} - \frac{(a+2b)\log(1+\tanh(c+dx))}{4(a-b)^2d} + \frac{1}{4(a+b)d(1-\tanh(c}$$

$$= \frac{(a-2b)\log(1-\tanh(c+dx))}{4(a+b)^2d} - \frac{(a+2b)\log(1+\tanh(c+dx))}{4(a-b)^2d} + \frac{1}{4(a+b)d(1-\tanh(c}$$

$$= \frac{(a-2b)\log(1-\tanh(c+dx))}{4(a+b)^2d} - \frac{(a+2b)\log(1+\tanh(c+dx))}{4(a-b)^2d} + \frac{b\left(2a^2+b^2\right)\log\left(a+\right.}{3\left(a^2-b\right.}$$

$$= \frac{(a-2b)\log(1-\tanh(c+dx))}{4(a+b)^2d} - \frac{(a+2b)\log(1+\tanh(c+dx))}{4(a-b)^2d} + \frac{a^{2/3}\sqrt[3]{b}\left(a^2+3a^{2/3}b^{4/3}\right.}{}$$

$$= \frac{(a-2b)\log(1-\tanh(c+dx))}{4(a+b)^2d} - \frac{(a+2b)\log(1+\tanh(c+dx))}{4(a-b)^2d} + \frac{a^{2/3}\sqrt[3]{b}\left(a^2+3a^{2/3}b^{4/3}\right.}{}$$

$$= \frac{a^{2/3}\sqrt[3]{b}\left(a^2-3a^{2/3}b^{4/3}+2b^2\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\tanh(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\left(a^2-b^2\right)^2 d} + \frac{(a-2b)\log(1-\tanh(c+dx))}{4(a+b)^2d} -$$

Mathematica [C] time = 3.39126, size = 423, normalized size = 1.1

$$-\frac{4b\operatorname{RootSum}\left[\#1^3a+3\#1^2a+\#1^3b-3\#1^2b+3\#1a+3\#1b+a-b\&, \frac{-4\#1^2a^2\log\left(e^{2(c+dx)}-\#1\right)+8\#1^2a^2c+8\#1^2a^2dx+4\#1^2ab\log(\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] -(6*(a^2 - 3*a*b + 2*b^2)*(c + d*x) + 3*b*(a + b)*Cosh[2*(c + d*x)] + 4*b*R
ootSum[a - b + 3*a*#1 + 3*b*#1 + 3*a*#1^2 - 3*b*#1^2 + a*#1^3 + b*#1^3 & ,
(4*a^2*c + 2*b^2*c + 4*a^2*d*x + 2*b^2*d*x - 2*a^2*Log[E^(2*(c + d*x)) - #1
] - b^2*Log[E^(2*(c + d*x)) - #1] + 4*a^2*c*#1 - 4*b^2*c*#1 + 4*a^2*d*x*#1
- 4*b^2*d*x*#1 - 2*a^2*Log[E^(2*(c + d*x)) - #1]*#1 + 2*b^2*Log[E^(2*(c + d
*x)) - #1]*#1 + 8*a^2*c*#1^2 - 8*a*b*c*#1^2 + 2*b^2*c*#1^2 + 8*a^2*d*x*#1^2
 - 8*a*b*d*x*#1^2 + 2*b^2*d*x*#1^2 - 4*a^2*Log[E^(2*(c + d*x)) - #1]*#1^2 +
 4*a*b*Log[E^(2*(c + d*x)) - #1]*#1^2 - b^2*Log[E^(2*(c + d*x)) - #1]*#1^2)
/(a - b + 2*a*#1 + 2*b*#1 + a*#1^2 - b*#1^2) & ] - 3*a*(a + b)*Sinh[2*(c +
d*x)])/(12*(a - b)*(a + b)^2*d)
```

Maple [C] time = 0.109, size = 356, normalized size = 0.9

$$-4\,\frac{1}{d\,(8\,a-8\,b)\,(\tanh(1/2\,dx+c/2)+1)^2}+8\,\frac{1}{d\,(16\,a-16\,b)\,(\tanh(1/2\,dx+c/2)+1)}-\frac{a}{2\,d\,(a-b)^2}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^3),x)
```

```
[Out] -4/d/(8*a-8*b)/(tanh(1/2*d*x+1/2*c)+1)^2+8/d/(16*a-16*b)/(tanh(1/2*d*x+1/2*
c)+1)-1/2/d/(a-b)^2*ln(tanh(1/2*d*x+1/2*c)+1)*a-1/d/(a-b)^2*ln(tanh(1/2*d*x
+1/2*c)+1)*b+4/d/(8*a+8*b)/(tanh(1/2*d*x+1/2*c)-1)^2+8/d/(16*a+16*b)/(tanh(
1/2*d*x+1/2*c)-1)+1/2/d/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)-1)*a-1/d/(a+b)^2*ln(
tanh(1/2*d*x+1/2*c)-1)*b+1/3/d*b/(a-b)^2/(a+b)^2*sum((a*(2*a^2+b^2)*_R^5-3*
_R^4*a^2*b+6*a*(a^2+b^2)*_R^3+4*b*(2*a^2+b^2)*_R^2-3*a*b^2*_R+3*a^2*b)/(_R^
5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_
Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$4\,a^2b\left(\frac{-(a-b)\int\frac{1}{\left(ae^{(6\,c)}+be^{(6\,c)}\right)e^{(6\,dx)}+3\left(ae^{(4\,c)}-be^{(4\,c)}\right)e^{(4\,dx)}+3\left(ae^{(2\,c)}+be^{(2\,c)}\right)e^{(2\,dx)}+a-b}\,dx+x}{a^4-2\,a^2b^2+b^4}-\frac{dx+c}{\left(a^4-2\,a^2b^2+b^4\right)d}\right)+2\,b^3\left(\frac{-(a-b)\int}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 4*a^2*b*(integrate(((a + b)*e^(4*d*x + 4*c) + 3*(a - b)*e^(2*d*x + 2*c) + 3
*a + 3*b)*e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4
*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^4 - 2*a^2*b^2 + b^4) - (d*x
 + c)/((a^4 - 2*a^2*b^2 + b^4)*d)) + 2*b^3*(integrate(((a + b)*e^(4*d*x + 4
*c) + 3*(a - b)*e^(2*d*x + 2*c) + 3*a + 3*b)*e^(2*d*x + 2*c)/((a + b)*e^(6*
d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b)
, x)/(a^4 - 2*a^2*b^2 + b^4) - (d*x + c)/((a^4 - 2*a^2*b^2 + b^4)*d)) - 8*a
^2*b*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*
x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^3 + a^2*b - a*b^2 - b^
3) + 8*a*b^2*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)
*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^3 + a^2*b - a*
b^2 - b^3) - 2*b^3*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x + 6*c) + 3*(
a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^3 + a^2*
b - a*b^2 - b^3) - 4*a^2*b*integrate(e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*
c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^
3 + a^2*b - a*b^2 - b^3) + 4*b^3*integrate(e^(2*d*x + 2*c)/((a + b)*e^(6*d*
x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b),
x)/(a^3 + a^2*b - a*b^2 - b^3) - 1/8*(4*(a^2*d*e^(2*c) - 3*a*b*d*e^(2*c) +
2*b^2*d*e^(2*c))*x*e^(2*d*x) + a^2 + 2*a*b + b^2 - (a^2*e^(4*c) - b^2*e^(4*
c))*e^(4*d*x))*e^(-2*d*x)/(a^3*d*e^(2*c) + a^2*b*d*e^(2*c) - a*b^2*d*e^(2*c
) - b^3*d*e^(2*c))
```

Fricas [C] time = 16.0621, size = 22579, normalized size = 58.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/72*(36*(a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3)*d*x*cosh(d*x + c)^2 - 9*(a^3 -
a^2*b - a*b^2 + b^3)*cosh(d*x + c)^4 - 36*(a^3 - a^2*b - a*b^2 + b^3)*cosh(
d*x + c)*sinh(d*x + c)^3 - 9*(a^3 - a^2*b - a*b^2 + b^3)*sinh(d*x + c)^4 +
9*a^3 + 9*a^2*b - 9*a*b^2 - 9*b^3 - 4*((a^4 - 2*a^2*b^2 + b^4)*d*cosh(d*x +
 c)^2 + 2*(a^4 - 2*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d*x + c) + (a^4 - 2*
a^2*b^2 + b^4)*d*sinh(d*x + c)^2)*((b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)
 - (2*a^2*b + b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3) + 1)/(-1/
18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*
b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 +
1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2
 - b^2)^4*d^3))^(1/3) - 9*(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*
d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4
*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) +
1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^(1/3)*(I*sqrt(3) + 1) + 6*(2*
a^2*b + b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))*log(1/18*(a^5 + 2*a^4*b - 2*a^3
*b^2 - 4*a^2*b^3 + a*b^4 + 2*b^5)*((b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)
 - (2*a^2*b + b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3) + 1)/(-1/
18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*
b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 +
1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2
 - b^2)^4*d^3))^(1/3) - 9*(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*
d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4
*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) +
1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^(1/3)*(I*sqrt(3) + 1) + 6*(2*
a^2*b + b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))^2*d^2 + a^3 + 2*a^2*b + 2*a*b^2
 + 4*b^3 - 1/3*(a^4 + 3*a^3*b + 13*a^2*b^2 + 6*a*b^3 + 4*b^4)*((b^2/(a^4*d^
2 - 2*a^2*b^2*d^2 + b^4*d^2) - (2*a^2*b + b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4
*d)^2)*(-I*sqrt(3) + 1)/(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^
2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d
 - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/
54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^(1/3) - 9*(-1/18*(2*a^2*b + b^3
)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) +
 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 -
 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^(
1/3)*(I*sqrt(3) + 1) + 6*(2*a^2*b + b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))*d +
 (a^3 + 8*a*b^2)*cosh(d*x + c)^2 + 2*(a^3 + 8*a*b^2)*cosh(d*x + c)*sinh(d*x
 + c) + (a^3 + 8*a*b^2)*sinh(d*x + c)^2) + 18*(2*(a^3 + 4*a^2*b + 5*a*b^2 +
 2*b^3)*d*x - 3*(a^3 - a^2*b - a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^
2 - 2*(18*(2*a^2*b + b^3)*cosh(d*x + c)^2 + 36*(2*a^2*b + b^3)*cosh(d*x + c
)*sinh(d*x + c) + 18*(2*a^2*b + b^3)*sinh(d*x + c)^2 - ((a^4 - 2*a^2*b^2 +
b^4)*d*cosh(d*x + c)^2 + 2*(a^4 - 2*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d*x
 + c) + (a^4 - 2*a^2*b^2 + b^4)*d*sinh(d*x + c)^2)*((b^2/(a^4*d^2 - 2*a^2*b
^2*d^2 + b^4*d^2) - (2*a^2*b + b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*
sqrt(3) + 1)/(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2
)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^
2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8
*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^(1/3) - 9*(-1/18*(2*a^2*b + b^3)*b^2/((a^4
*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^
2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*
d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^(1/3)*(I*sqr
t(3) + 1) + 6*(2*a^2*b + b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d)) - 3*sqrt(1/3)*
((a^4 - 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^2 + 2*(a^4 - 2*a^2*b^2 + b^4)*d*co
sh(d*x + c)*sinh(d*x + c) + (a^4 - 2*a^2*b^2 + b^4)*d*sinh(d*x + c)^2)*sqrt
((288*a^4*b^2 + 720*a^2*b^4 - 36*b^6 - (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2
*b^6 + b^8)*((b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (2*a^2*b + b^3)^2/(
```

```
a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3) + 1)/(-1/18*(2*a^2*b + b^3)*b^2
/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27
*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^
2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^(1/3)
 - 9*(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d
 - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4
*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2
*b/((a^2 - b^2)^4*d^3))^(1/3)*(I*sqrt(3) + 1) + 6*(2*a^2*b + b^3)/(a^4*d -
2*a^2*b^2*d + b^4*d))^2*d^2 + 12*(2*a^6*b - 3*a^4*b^3 + b^7)*((b^2/(a^4*d^2
 - 2*a^2*b^2*d^2 + b^4*d^2) - (2*a^2*b + b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*
d)^2)*(-I*sqrt(3) + 1)/(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2
 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d
 - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/5
4*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^(1/3) - 9*(-1/18*(2*a^2*b + b^3)
*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) +
1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 -
2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^(1
/3)*(I*sqrt(3) + 1) + 6*(2*a^2*b + b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))*d)/(
(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d^2)))*log(-1/36*(a^6 + 3*a
^5*b - 6*a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 + 2*b^6)*((b^2/(a^4*d^2 - 2*a^2*b^2*
d^2 + b^4*d^2) - (2*a^2*b + b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqr
t(3) + 1)/(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(
a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d
 + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^
2)*a^2*b/((a^2 - b^2)^4*d^3))^(1/3) - 9*(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^
2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b
 + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3
 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^(1/3)*(I*sqrt(3
) + 1) + 6*(2*a^2*b + b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))^2*d^2 + a^4 - 3*a
^3*b + 10*a^2*b^2 - 15*a*b^3 - 2*b^4 + 1/6*(a^5 + 4*a^4*b + 16*a^3*b^2 + 19
*a^2*b^3 + 10*a*b^4 + 4*b^5)*((b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (2
*a^2*b + b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3) + 1)/(-1/18*(2
*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d
 + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*
b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^
2)^4*d^3))^(1/3) - 9*(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 +
 b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d -
2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*
(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^(1/3)*(I*sqrt(3) + 1) + 6*(2*a^2*b
 + b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))*d + (a^4 + a^3*b + 8*a^2*b^2 + 8*a*b
^3)*cosh(d*x + c)^2 + 2*(a^4 + a^3*b + 8*a^2*b^2 + 8*a*b^3)*cosh(d*x + c)*s
inh(d*x + c) + (a^4 + a^3*b + 8*a^2*b^2 + 8*a*b^3)*sinh(d*x + c)^2 + 1/12*s
qrt(1/3)*((a^6 + 3*a^5*b - 6*a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 + 2*b^6)*((b^2/(
a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (2*a^2*b + b^3)^2/(a^4*d - 2*a^2*b^2*d
 + b^4*d)^2)*(-I*sqrt(3) + 1)/(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*
b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/
(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3
) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^(1/3) - 9*(-1/18*(2*a^2*b
 + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4
*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4
*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d
^3))^(1/3)*(I*sqrt(3) + 1) + 6*(2*a^2*b + b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d
))*d^2 + 6*(a^5 - 2*a^4*b - 2*a^3*b^2 + 4*a^2*b^3 + a*b^4 - 2*b^5)*d)*sqrt(
(288*a^4*b^2 + 720*a^2*b^4 - 36*b^6 - (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*
b^6 + b^8)*((b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (2*a^2*b + b^3)^2/(a
^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3) + 1)/(-1/18*(2*a^2*b + b^3)*b^2/
((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*
(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2
*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^(1/3) -
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9*(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d -
 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*
d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*
b/((a^2 - b^2)^4*d^3))^(1/3)*(I*sqrt(3) + 1) + 6*(2*a^2*b + b^3)/(a^4*d - 2
*a^2*b^2*d + b^4*d))^2*d^2 + 12*(2*a^6*b - 3*a^4*b^3 + b^7)*((b^2/(a^4*d^2
- 2*a^2*b^2*d^2 + b^4*d^2) - (2*a^2*b + b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d
)^2)*(-I*sqrt(3) + 1)/(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2
+ b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d -
 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54
*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^(1/3) - 9*(-1/18*(2*a^2*b + b^3)*
b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1
/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2
*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^(1/
3)*(I*sqrt(3) + 1) + 6*(2*a^2*b + b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))*d)/((
a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d^2))) - 2*(18*(2*a^2*b + b^
3)*cosh(d*x + c)^2 + 36*(2*a^2*b + b^3)*cosh(d*x + c)*sinh(d*x + c) + 18*(2
*a^2*b + b^3)*sinh(d*x + c)^2 - ((a^4 - 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^2
+ 2*(a^4 - 2*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d*x + c) + (a^4 - 2*a^2*b^
2 + b^4)*d*sinh(d*x + c)^2)*((b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (2*
a^2*b + b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3) + 1)/(-1/18*(2*
a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d
+ b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b
/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2
)^4*d^3))^(1/3) - 9*(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 +
b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2
*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(
a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^(1/3)*(I*sqrt(3) + 1) + 6*(2*a^2*b
+ b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d)) + 3*sqrt(1/3)*((a^4 - 2*a^2*b^2 + b^4
)*d*cosh(d*x + c)^2 + 2*(a^4 - 2*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d*x +
c) + (a^4 - 2*a^2*b^2 + b^4)*d*sinh(d*x + c)^2)*sqrt((288*a^4*b^2 + 720*a^2
*b^4 - 36*b^6 - (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*((b^2/(a^4*
d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (2*a^2*b + b^3)^2/(a^4*d - 2*a^2*b^2*d + b
^4*d)^2)*(-I*sqrt(3) + 1)/(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*
d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4
*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) +
1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^(1/3) - 9*(-1/18*(2*a^2*b + b
^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d))
 + 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3
 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))
^(1/3)*(I*sqrt(3) + 1) + 6*(2*a^2*b + b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))^2
*d^2 + 12*(2*a^6*b - 3*a^4*b^3 + b^7)*((b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*
d^2) - (2*a^2*b + b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3) + 1)/
(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*
a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^
3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/(
(a^2 - b^2)^4*d^3))^(1/3) - 9*(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*
b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/
(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3
) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^(1/3)*(I*sqrt(3) + 1) + 6
*(2*a^2*b + b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))*d)/((a^8 - 4*a^6*b^2 + 6*a^
4*b^4 - 4*a^2*b^6 + b^8)*d^2)))*log(-1/36*(a^6 + 3*a^5*b - 6*a^3*b^3 - 3*a^
2*b^4 + 3*a*b^5 + 2*b^6)*((b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (2*a^2
*b + b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3) + 1)/(-1/18*(2*a^2
*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b
^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a
^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4
*d^3))^(1/3) - 9*(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4
*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^
2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2
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 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^(1/3)*(I*sqrt(3) + 1) + 6*(2*a^2*b + b
^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))^2*d^2 + a^4 - 3*a^3*b + 10*a^2*b^2 - 15*
a*b^3 - 2*b^4 + 1/6*(a^5 + 4*a^4*b + 16*a^3*b^2 + 19*a^2*b^3 + 10*a*b^4 + 4
*b^5)*((b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (2*a^2*b + b^3)^2/(a^4*d
- 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3) + 1)/(-1/18*(2*a^2*b + b^3)*b^2/((a^4
*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^
2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*
d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^(1/3) - 9*(-
1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^
2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3
+ 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a
^2 - b^2)^4*d^3))^(1/3)*(I*sqrt(3) + 1) + 6*(2*a^2*b + b^3)/(a^4*d - 2*a^2*
b^2*d + b^4*d))*d + (a^4 + a^3*b + 8*a^2*b^2 + 8*a*b^3)*cosh(d*x + c)^2 + 2
*(a^4 + a^3*b + 8*a^2*b^2 + 8*a*b^3)*cosh(d*x + c)*sinh(d*x + c) + (a^4 + a
^3*b + 8*a^2*b^2 + 8*a*b^3)*sinh(d*x + c)^2 - 1/12*sqrt(1/3)*((a^6 + 3*a^5*
b - 6*a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 + 2*b^6)*((b^2/(a^4*d^2 - 2*a^2*b^2*d^2
 + b^4*d^2) - (2*a^2*b + b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3
) + 1)/(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4
*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d +
b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*
a^2*b/((a^2 - b^2)^4*d^3))^(1/3) - 9*(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 -
 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b +
b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 +
b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^(1/3)*(I*sqrt(3) +
 1) + 6*(2*a^2*b + b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))*d^2 + 6*(a^5 - 2*a^4
*b - 2*a^3*b^2 + 4*a^2*b^3 + a*b^4 - 2*b^5)*d)*sqrt((288*a^4*b^2 + 720*a^2*
b^4 - 36*b^6 - (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*((b^2/(a^4*d
^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (2*a^2*b + b^3)^2/(a^4*d - 2*a^2*b^2*d + b^
4*d)^2)*(-I*sqrt(3) + 1)/(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d
^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*
d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1
/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^(1/3) - 9*(-1/18*(2*a^2*b + b^
3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d))
+ 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3
- 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^
(1/3)*(I*sqrt(3) + 1) + 6*(2*a^2*b + b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))^2*
d^2 + 12*(2*a^6*b - 3*a^4*b^3 + b^7)*((b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d
^2) - (2*a^2*b + b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3) + 1)/(
-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a
^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3
 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((
a^2 - b^2)^4*d^3))^(1/3) - 9*(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b
^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(
a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3)
 + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^(1/3)*(I*sqrt(3) + 1) + 6*
(2*a^2*b + b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))*d)/((a^8 - 4*a^6*b^2 + 6*a^4
*b^4 - 4*a^2*b^6 + b^8)*d^2))) + 36*(2*(a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3)*d*
x*cosh(d*x + c) - (a^3 - a^2*b - a*b^2 + b^3)*cosh(d*x + c)^3)*sinh(d*x + c
))/((a^4 - 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^2 + 2*(a^4 - 2*a^2*b^2 + b^4)*d
*cosh(d*x + c)*sinh(d*x + c) + (a^4 - 2*a^2*b^2 + b^4)*d*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+b*tanh(d*x+c)**3),x)
```

[Out] Timed out

Giac [A] time = 1.78404, size = 301, normalized size = 0.78

$$-\frac{\frac{12\,(a+2\,b)dx}{a^2-2\,ab+b^2}-\frac{3\left(2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}-a+b\right)e^{(-2\,dx)}}{a^2e^{(2\,c)}-2\,abe^{(2\,c)}+b^2e^{(2\,c)}}-\frac{8\left(2\,a^2b+b^3\right)\log\left(\left|ae^{(6\,dx+6\,c)}+be^{(6\,dx+6\,c)}+3\,ae^{(4\,dx+4\,c)}-3\,be^{(4\,dx+4\,c)}+3\,ae^{(2\,dx+2\,c)}+3\,be^{(2\,dx+2}\right.\right.}{a^4-2\,a^2b^2+b^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -1/24*(12*(a + 2*b)*d*x/(a^2 - 2*a*b + b^2) - 3*(2*a*e^(2*d*x + 2*c) + 4*b*
e^(2*d*x + 2*c) - a + b)*e^(-2*d*x)/(a^2*e^(2*c) - 2*a*b*e^(2*c) + b^2*e^(2
*c)) - 8*(2*a^2*b + b^3)*log(abs(a*e^(6*d*x + 6*c) + b*e^(6*d*x + 6*c) + 3*
a*e^(4*d*x + 4*c) - 3*b*e^(4*d*x + 4*c) + 3*a*e^(2*d*x + 2*c) + 3*b*e^(2*d*
x + 2*c) + a - b))/(a^4 - 2*a^2*b^2 + b^4) - 3*e^(2*d*x + 10*c)/(a*e^(8*c)
+ b*e^(8*c)))/d
```

3.76 $\int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

Optimal. Leaf size=30

$$-i\text{Unintegrable}\left(\frac{i\sinh(c+dx)}{a+b\tanh^3(c+dx)},x\right)$$

```
[Out] (-I)*Unintegrable[(I*Sinh[c + d*x])/(a + b*Tanh[c + d*x]^3), x]
```

Rubi [A] time = 0.0276825, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (-I)*Defer[Int][(I*Sinh[c + d*x])/(a + b*Tanh[c + d*x]^3), x]
```

Rubi steps

$$\int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx = -\left(i\int \frac{i\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx\right)$$

Mathematica [A] time = 0.226157, size = 409, normalized size = 13.63

$$b\text{RootSum}\left[\#1^6a+3\#1^4a+3\#1^2a+\#1^6b-3\#1^4b+3\#1^2b+a-b\&,\frac{4\#1^4a\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (6*a*Cosh[c + d*x] + b*RootSum[a - b + 3*a*#1^2 + 3*b*#1^2 + 3*a*#1^4 - 3*b
*#1^4 + a*#1^6 + b*#1^6 & , (2*a*c + b*c + 2*a*d*x + b*d*x + 4*a*Log[-Cosh[
(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]
*#1] + 2*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#
1 - Sinh[(c + d*x)/2]*#1] + 2*a*c*#1^4 - b*c*#1^4 + 2*a*d*x*#1^4 - b*d*x*#1
^4 + 4*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1
- Sinh[(c + d*x)/2]*#1]*#1^4 - 2*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/
2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4)/(a*#1 + b*#1 + 2*a*
#1^3 - 2*b*#1^3 + a*#1^5 + b*#1^5) & ] - 6*b*Sinh[c + d*x])/(6*(a - b)*(a +
 b)*d)
```

Maple [A] time = 0.108, size = 164, normalized size = 5.5

$$-4\,\frac{1}{d\,(4\,a+4\,b)\,(\tanh\,(1/2\,dx+c/2)-1)}+\frac{b}{3\,d\,(a-b)\,(a+b)}\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3\,a\_Z^4+8\,b\_Z^3+3\,a\_Z^2+a\right)}\frac{\_R^4a-2\_R^3b+6\ldots}{\_R^5a+2\_R^3a+\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a+b*tanh(d*x+c)^3),x)
```

```
[Out] -4/d/(4*a+4*b)/(tanh(1/2*d*x+1/2*c)-1)+1/3/d*b/(a-b)/(a+b)*sum((_R^4*a-2*_R
^3*b+6*_R^2*a-2*_R*b+a)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2
*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))+4/d/(4*a-4*b)/(tanh
(1/2*d*x+1/2*c)+1)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{\left(\left(ae^{(2\,c)}-be^{(2\,c)}\right)e^{(2\,dx)}+a+b\right)e^{(-dx)}}{2\left(a^2de^c-b^2de^c\right)}+\frac{1}{2}\int\frac{4\left(\left(2\,abe^{(5\,c)}-b^2\ldots\right.\right.}{a^3-a^2b-ab^2+b^3+\left(a^3e^{(6\,c)}+a^2be^{(6\,c)}-ab^2e^{(6\,c)}-b^3e^{(6\,c)}\right)e^{(6\,dx)}+3\left(a^3\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/2*((a*e^(2*c) - b*e^(2*c))*e^(2*d*x) + a + b)*e^(-d*x)/(a^2*d*e^c - b^2*d
*e^c) + 1/2*integrate(4*((2*a*b*e^(5*c) - b^2*e^(5*c))*e^(5*d*x) + (2*a*b*e
^c + b^2*e^c)*e^(d*x))/(a^3 - a^2*b - a*b^2 + b^3 + (a^3*e^(6*c) + a^2*b*e^
(6*c) - a*b^2*e^(6*c) - b^3*e^(6*c))*e^(6*d*x) + 3*(a^3*e^(4*c) - a^2*b*e^(
4*c) - a*b^2*e^(4*c) + b^3*e^(4*c))*e^(4*d*x) + 3*(a^3*e^(2*c) + a^2*b*e^(2
*c) - a*b^2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.62107, size = 263, normalized size = 8.77

$$\frac{\frac{e^{(dx+8c)}}{ae^{(7c)}+be^{(7c)}}+\frac{e^{(-dx)}}{ae^{c}-be^{c}}}{2d}+\frac{\frac{6\left(2abe^{c}+b^{2}e^{c}\right)dx}{ad-bd}-\frac{\left(2abe^{c}+b^{2}e^{c}\right)\log\left(\left|ae^{(6dx+6c)}+be^{(6dx+6c)}+3ae^{(4dx+4c)}-3be^{(4dx+4c)}+3ae^{(2dx+2c)}+3be^{(2dx+2c)}+a-b\right|\right)}{ad-bd}}{3\left(a^{2}-b^{2}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/2*(e^(d*x + 8*c)/(a*e^(7*c) + b*e^(7*c)) + e^(-d*x)/(a*e^c - b*e^c))/d +
1/3*(6*(2*a*b*e^c + b^2*e^c)*d*x/(a*d - b*d) - (2*a*b*e^c + b^2*e^c)*log(ab
s(a*e^(6*d*x + 6*c) + b*e^(6*d*x + 6*c) + 3*a*e^(4*d*x + 4*c) - 3*b*e^(4*d*
x + 4*c) + 3*a*e^(2*d*x + 2*c) + 3*b*e^(2*d*x + 2*c) + a - b))/(a*d - b*d))
/((a^2 - b^2)*d)
```

3.77 $\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

Optimal. Leaf size=30

$$i\text{Unintegrable}\left(-\frac{i\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)},x\right)$$

```
[Out] I*Unintegrable[((-I)*Csch[c + d*x])/(a + b*Tanh[c + d*x]^3), x]
```

Rubi [A] time = 0.0408265, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csch[c + d*x]/(a + b*Tanh[c + d*x]^3),x]

[Out] I*Defer[Int][((-I)*Csch[c + d*x])/(a + b*Tanh[c + d*x]^3), x]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)}\,dx = i\int -\frac{i\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

Mathematica [A] time = 0.158439, size = 319, normalized size = 10.63

$$6\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) - b\text{RootSum}\left[\#1^6a + 3\#1^4a + 3\#1^2a + \#1^6b - 3\#1^4b + 3\#1^2b + a - b\&, \frac{2\#1^4\log\left(-\#1\sinh\left(\frac{1}{2}(c+\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a + b*Tanh[c + d*x]^3),x]

[Out] (6*Log[Tanh[(c + d*x)/2]] - b*RootSum[a - b + 3*a*#1^2 + 3*b*#1^2 + 3*a*#1^
4 - 3*b*#1^4 + a*#1^6 + b*#1^6 & , (c + d*x + 2*Log[-Cosh[(c + d*x)/2] - Si
nh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] - 2*c*#1^2 -
 2*d*x*#1^2 - 4*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)
/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + c*#1^4 + d*x*#1^4 + 2*Log[-Cosh[(c +
d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*
#1^4)/(a*#1 + b*#1 + 2*a*#1^3 - 2*b*#1^3 + a*#1^5 + b*#1^5) & ])/(6*a*d)
```

Maple [A] time = 0.105, size = 98, normalized size = 3.3

$$\frac{1}{da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right) - \frac{4b}{3da}\sum_{\_R=\text{RootOf}(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a)} \frac{\_R^2}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\ln\left(\tanh\left(\frac{dx}{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a+b*tanh(d*x+c)^3),x)
```

```
[Out] 1/d/a*ln(tanh(1/2*d*x+1/2*c))-4/3/d/a*b*sum(_R^2/(_R^5*a+2*_R^3*a+4*_R^2*b+
_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*
a+a))
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(\left(e^{(dx+c)}+1\right)e^{(-c)}\right)}{ad}+\frac{\log\left(\left(e^{(dx+c)}-1\right)e^{(-c)}\right)}{ad}-2\int\frac{be^{(5\,dx+5\,c)}-2\,be^{(3\,dx+3\,c)}+be^{(dx+c)}}{a^2-ab+\left(a^2e^{(6\,c)}+abe^{(6\,c)}\right)e^{(6\,dx)}+3\left(a^2e^{(4\,c)}-abe^{(4\,c)}\right)e^{(4\,dx)}+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -log((e^(d*x + c) + 1)*e^(-c))/(a*d) + log((e^(d*x + c) - 1)*e^(-c))/(a*d)
- 2*integrate((b*e^(5*d*x + 5*c) - 2*b*e^(3*d*x + 3*c) + b*e^(d*x + c))/(a^
2 - a*b + (a^2*e^(6*c) + a*b*e^(6*c))*e^(6*d*x) + 3*(a^2*e^(4*c) - a*b*e^(4
*c))*e^(4*d*x) + 3*(a^2*e^(2*c) + a*b*e^(2*c))*e^(2*d*x)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)**3),x)
```

```
[Out] Integral(csch(c + d*x)/(a + b*tanh(c + d*x)**3), x)
```

Giac [A] time = 1.45323, size = 207, normalized size = 6.9

$$-\frac{\frac{\log\left(e^{(dx+c)}+1\right)}{a}-\frac{\log\left(\left|e^{(dx+c)}-1\right|\right)}{a}}{d}-\frac{\frac{6\,bdxe^{c}}{ad-bd}-\frac{be^{c}\log\left(\left|ae^{(6\,dx+6\,c)}+be^{(6\,dx+6\,c)}+3\,ae^{(4\,dx+4\,c)}-3\,be^{(4\,dx+4\,c)}+3\,ae^{(2\,dx+2\,c)}+3\,be^{(2\,dx+2\,c)}+a-b\right|\right)}{ad-bd}}{3\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -(log(e^(d*x + c) + 1)/a - log(abs(e^(d*x + c) - 1))/a)/d - 1/3*(6*b*d*x*e^
c/(a*d - b*d) - b*e^c*log(abs(a*e^(6*d*x + 6*c) + b*e^(6*d*x + 6*c) + 3*a*e
^(4*d*x + 4*c) - 3*b*e^(4*d*x + 4*c) + 3*a*e^(2*d*x + 2*c) + 3*b*e^(2*d*x +
 2*c) + a - b))/(a*d - b*d))/(a*d)
```

3.78 $\int \frac{\text{csch}^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

Optimal. Leaf size=157

$$-\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+b^{2/3}\tanh^2(c+dx)\right)}{6a^{4/3}d}+\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3a^{4/3}d}+\frac{\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c}{\sqrt{3}\sqrt[3]{a}}\right.}{\sqrt{3}a^{4/3}d}$$

```
[Out] (b^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Tanh[c + d*x])/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*a^(4/3)*d) - Coth[c + d*x]/(a*d) + (b^(1/3)*Log[a^(1/3) + b^(1/3)*Tan
h[c + d*x]])/(3*a^(4/3)*d) - (b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Tanh[c
+ d*x] + b^(2/3)*Tanh[c + d*x]^2])/(6*a^(4/3)*d)
```

Rubi [A] time = 0.140349, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3663, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+b^{2/3}\tanh^2(c+dx)\right)}{6a^{4/3}d}+\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3a^{4/3}d}+\frac{\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c}{\sqrt{3}\sqrt[3]{a}}\right.}{\sqrt{3}a^{4/3}d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (b^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Tanh[c + d*x])/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*a^(4/3)*d) - Coth[c + d*x]/(a*d) + (b^(1/3)*Log[a^(1/3) + b^(1/3)*Tan
h[c + d*x]])/(3*a^(4/3)*d) - (b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Tanh[c
+ d*x] + b^(2/3)*Tanh[c + d*x]^2])/(6*a^(4/3)*d)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{x^2\left(a+bx^3\right)}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$= -\frac{\coth(c+dx)}{ad} - \frac{b\operatorname{Subst}\left(\int \frac{x}{a+bx^3}\,dx,x,\tanh(c+dx)\right)}{ad}$$

$$= -\frac{\coth(c+dx)}{ad} + \frac{b^{2/3}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx,x,\tanh(c+dx)\right)}{3a^{4/3}d} - \frac{b^{2/3}\operatorname{Subst}\left(\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+\cdots}\right.}{3a^{4/3}d}$$

$$= -\frac{\coth(c+dx)}{ad} + \frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3a^{4/3}d} - \frac{\sqrt[3]{b}\operatorname{Subst}\left(\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx,x,\right.}{6a^{4/3}d}$$

$$= -\frac{\coth(c+dx)}{ad} + \frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3a^{4/3}d} - \frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)\cdots\right.}{6a^{4/3}d}$$

$$= \frac{\sqrt[3]{b}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\tanh(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{4/3}d} - \frac{\coth(c+dx)}{ad} + \frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3a^{4/3}d} - \frac{\sqrt[3]{b}\log\left(\cdots\right.}{}$$

Mathematica [C] time = 0.134613, size = 190, normalized size = 1.21

$$-\frac{2b\operatorname{RootSum}\left[\#1^3a+3\#1^2a+\#1^3b-3\#1^2b+3\#1a+3\#1b+a-b\&,\frac{-\log(-\#1\sinh(c+dx)+\#1\cosh(c+dx)-\sinh(c+dx)-\cosh(c+\cdots}{}\right.}{3ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] -(3*Coth[c + d*x] + 2*b*RootSum[a - b + 3*a*#1 + 3*b*#1 + 3*a*#1^2 - 3*b*#1
^2 + a*#1^3 + b*#1^3 & , (-c - d*x - Log[-Cosh[c + d*x] - Sinh[c + d*x] + C
osh[c + d*x]*#1 - Sinh[c + d*x]*#1] + c*#1 + d*x*#1 + Log[-Cosh[c + d*x] -
Sinh[c + d*x] + Cosh[c + d*x]*#1 - Sinh[c + d*x]*#1]*#1)/(a + b + 2*a*#1 -
2*b*#1 + a*#1^2 + b*#1^2) & ])/(3*a*d)
```

Maple [C] time = 0.112, size = 121, normalized size = 0.8

$$-\frac{1}{2da}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1}+\frac{2b}{3da}\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\_R^3-\_R}{\_R^5a+2\_R^3a+4\_R^2b+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a+b*tanh(d*x+c)^3),x)
```

```
[Out] -1/2/d/a*tanh(1/2*d*x+1/2*c)-1/2/d/a/tanh(1/2*d*x+1/2*c)+2/3/d/a*b*sum((_R^
3-_R)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(
_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2}{ade^{(2dx+2c)}-ad}-4\int\frac{be^{(4dx+4c)}-be^{(2dx+2c)}}{a^2-ab+\left(a^2e^{(6c)}+abe^{(6c)}\right)e^{(6dx)}+3\left(a^2e^{(4c)}-abe^{(4c)}\right)e^{(4dx)}+3\left(a^2e^{(2c)}+abe^{(2c)}\right)e^{(2dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -2/(a*d*e^(2*d*x + 2*c) - a*d) - 4*integrate((b*e^(4*d*x + 4*c) - b*e^(2*d*
x + 2*c))/(a^2 - a*b + (a^2*e^(6*c) + a*b*e^(6*c))*e^(6*d*x) + 3*(a^2*e^(4*
c) - a*b*e^(4*c))*e^(4*d*x) + 3*(a^2*e^(2*c) + a*b*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 2.66535, size = 1801, normalized size = 11.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/6*(2*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x + c) +
sqrt(3)*sinh(d*x + c)^2 - sqrt(3))*(b/a)^(1/3)*arctan(-1/3*(sqrt(3)*b*cosh(
d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*b*sinh(d*x +
 c)^2 - (sqrt(3)*a*cosh(d*x + c)^2 + 2*sqrt(3)*a*cosh(d*x + c)*sinh(d*x + c
) + sqrt(3)*a*sinh(d*x + c)^2 + sqrt(3)*a)*(b/a)^(2/3) - (sqrt(3)*b*cosh(d*
x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*b*sinh(d*x + c
```

```
)^2 - sqrt(3)*b)*(b/a)^(1/3))/b) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(
d*x + c) + sinh(d*x + c)^2 - 1)*(b/a)^(1/3)*log((a + b)*cosh(d*x + c)^4 + 4
*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b
)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 +
 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) - 2*(a*c
osh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 - a)*(
b/a)^(2/3) + 2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sin
h(d*x + c)^2 + a)*(b/a)^(1/3) + a + b) - 2*(cosh(d*x + c)^2 + 2*cosh(d*x +
c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(b/a)^(1/3)*log((a + b)*cosh(d*x +
c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + 2*
a*(b/a)^(2/3) - 2*a*(b/a)^(1/3) + a - b) + 12)/(a*d*cosh(d*x + c)^2 + 2*a*d
*cosh(d*x + c)*sinh(d*x + c) + a*d*sinh(d*x + c)^2 - a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a+b*tanh(d*x+c)**3),x)
```

```
[Out] Integral(csch(c + d*x)**2/(a + b*tanh(c + d*x)**3), x)
```

Giac [A] time = 1.33049, size = 28, normalized size = 0.18

$$-\frac{2}{ad\left(e^{(2\,dx+2\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -2/(a*d*(e^(2*d*x + 2*c) - 1))
```

3.79 $\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

Optimal. Leaf size=32

$$-i\text{Unintegrable}\left(\frac{i\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)},x\right)$$

```
[Out] (-I)*Unintegrable[(I*Csch[c + d*x]^3)/(a + b*Tanh[c + d*x]^3), x]
```

Rubi [A] time = 0.0471116, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (-I)*Defer[Int][(I*Csch[c + d*x]^3)/(a + b*Tanh[c + d*x]^3), x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = -\left(i\int \frac{i\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx\right)$$

Mathematica [A] time = 0.361888, size = 201, normalized size = 6.28

$$-\frac{16b\text{RootSum}\left[\#1^6a+3\#1^4a+3\#1^2a+\#1^6b-3\#1^4b+3\#1^2b+a-b\&,\frac{2\#1\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+\right.\right.}{\#1^4a+2\#1^2a+\#1^4b-2\#1^2b+a}\right.}{24ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] -(16*b*RootSum[a - b + 3*a*#1^2 + 3*b*#1^2 + 3*a*#1^4 - 3*b*#1^4 + a*#1^6 +
 b*#1^6 & , (c*#1 + d*x*#1 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] +
 Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1)/(a + b + 2*a*#1^2 - 2*b*#
1^2 + a*#1^4 + b*#1^4) & ] + 3*(Csch[(c + d*x)/2]^2 + 4*Log[Tanh[(c + d*x)/
2]] + Sech[(c + d*x)/2]^2))/(24*a*d)
```

Maple [A] time = 0.125, size = 144, normalized size = 4.5

$$\frac{1}{8da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-\frac{1}{8da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-2}-\frac{1}{2da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{b}{3da}\sum_{\_R=\text{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(a+b*tanh(d*x+c)^3),x)
```

```
[Out] 1/8/d/a*tanh(1/2*d*x+1/2*c)^2-1/8/d/a/tanh(1/2*d*x+1/2*c)^2-1/2/d/a*ln(tanh
(1/2*d*x+1/2*c))-1/3/d/a*b*sum((_R^4-2*_R^2+1)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R
*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+
a))
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-8\,b\int\frac{e^{(3\,dx+3\,c)}}{a^2-ab+\left(a^2e^{(6\,c)}+abe^{(6\,c)}\right)e^{(6\,dx)}+3\left(a^2e^{(4\,c)}-abe^{(4\,c)}\right)e^{(4\,dx)}+3\left(a^2e^{(2\,c)}+abe^{(2\,c)}\right)e^{(2\,dx)}}\,dx-\frac{e^{(3\,dx+3\,c)}}{ade^{(4\,dx+4\,c)}-2\,\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -8*b*integrate(e^(3*d*x + 3*c)/(a^2 - a*b + (a^2*e^(6*c) + a*b*e^(6*c))*e^(
6*d*x) + 3*(a^2*e^(4*c) - a*b*e^(4*c))*e^(4*d*x) + 3*(a^2*e^(2*c) + a*b*e^(
2*c))*e^(2*d*x)), x) - (e^(3*d*x + 3*c) + e^(d*x + c))/(a*d*e^(4*d*x + 4*c)
 - 2*a*d*e^(2*d*x + 2*c) + a*d) + 1/2*log((e^(d*x + c) + 1)*e^(-c))/(a*d) -
 1/2*log((e^(d*x + c) - 1)*e^(-c))/(a*d)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(a+b*tanh(d*x+c)**3),x)
```

```
[Out] Integral(csch(c + d*x)**3/(a + b*tanh(c + d*x)**3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{csch}(dx+c)^3}{b\tanh(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(csch(d*x + c)^3/(b*tanh(d*x + c)^3 + a), x)
```

3.80 $\int \frac{\mathbf{csch}^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

Optimal. Leaf size=215

$$\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+b^{2/3}\tanh^2(c+dx)\right)}{6a^{4/3}d}+\frac{b\log\left(a+b\tanh^3(c+dx)\right)}{3a^2d}-\frac{b\log(\tanh(c+dx))}{a^2d}-\sqrt[3]{b}\,l$$

```
[Out] -((b^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Tanh[c + d*x])/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*a^(4/3)*d)) + Coth[c + d*x]/(a*d) - Coth[c + d*x]^3/(3*a*d) - (b*Lo
g[Tanh[c + d*x]])/(a^2*d) - (b^(1/3)*Log[a^(1/3) + b^(1/3)*Tanh[c + d*x]])/
(3*a^(4/3)*d) + (b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Tanh[c + d*x] + b^(2
/3)*Tanh[c + d*x]^2])/(6*a^(4/3)*d) + (b*Log[a + b*Tanh[c + d*x]^3])/(3*a^2
*d)
```

Rubi [A] time = 0.238659, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3663, 1834, 1871, 12, 292, 31, 634, 617, 204, 628, 260}

$$\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+b^{2/3}\tanh^2(c+dx)\right)}{6a^{4/3}d}+\frac{b\log\left(a+b\tanh^3(c+dx)\right)}{3a^2d}-\frac{b\log(\tanh(c+dx))}{a^2d}-\sqrt[3]{b}\,l$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] -((b^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Tanh[c + d*x])/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*a^(4/3)*d)) + Coth[c + d*x]/(a*d) - Coth[c + d*x]^3/(3*a*d) - (b*Lo
g[Tanh[c + d*x]])/(a^2*d) - (b^(1/3)*Log[a^(1/3) + b^(1/3)*Tanh[c + d*x]])/
(3*a^(4/3)*d) + (b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Tanh[c + d*x] + b^(2
/3)*Tanh[c + d*x]^2])/(6*a^(4/3)*d) + (b*Log[a + b*Tanh[c + d*x]^3])/(3*a^2
*d)
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{x^4(a+bx^3)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^4}-\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{bx(a+bx)}{a^2(a+bx^3)}\right)dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\coth(c+dx)}{ad}-\frac{\coth^3(c+dx)}{3ad}-\frac{b\log(\tanh(c+dx))}{a^2d}+\frac{b\,\text{Subst}\left(\int \frac{x(a+bx)}{a+bx^3}\,dx, x, \tanh(c\right.}{a^2d}$$

$$= \frac{\coth(c+dx)}{ad}-\frac{\coth^3(c+dx)}{3ad}-\frac{b\log(\tanh(c+dx))}{a^2d}+\frac{b\,\text{Subst}\left(\int \frac{ax}{a+bx^3}\,dx, x, \tanh(c+\right.}{a^2d}$$

$$= \frac{\coth(c+dx)}{ad}-\frac{\coth^3(c+dx)}{3ad}-\frac{b\log(\tanh(c+dx))}{a^2d}+\frac{b\log\left(a+b\tanh^3(c+dx)\right)}{3a^2d}+\frac{b}{}$$

$$= \frac{\coth(c+dx)}{ad}-\frac{\coth^3(c+dx)}{3ad}-\frac{b\log(\tanh(c+dx))}{a^2d}+\frac{b\log\left(a+b\tanh^3(c+dx)\right)}{3a^2d}-\frac{b^2}{}$$

$$= \frac{\coth(c+dx)}{ad}-\frac{\coth^3(c+dx)}{3ad}-\frac{b\log(\tanh(c+dx))}{a^2d}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3a^{4/3}d}$$

$$= \frac{\coth(c+dx)}{ad}-\frac{\coth^3(c+dx)}{3ad}-\frac{b\log(\tanh(c+dx))}{a^2d}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3a^{4/3}d}$$

$$= -\frac{\sqrt[3]{b}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\tanh(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{4/3}d}+\frac{\coth(c+dx)}{ad}-\frac{\coth^3(c+dx)}{3ad}-\frac{b\log(\tanh(c+dx))}{a^2d}-\sqrt[3]{}$$

Mathematica [C] time = 3.15715, size = 322, normalized size = 1.5

$$b\text{RootSum}\left[\#1^3a+3\#1^2a+\#1^3b-3\#1^2b+3\#1a+3\#1b+a-b\&, \frac{-\#1^2a\log\left(e^{2(c+dx)}-\#1\right)+2\#1^2ac+2\#1^2adx-\#1^2b\log\left(e^{2(c+dx)}-\#1\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (-(a*Coth[c + d*x]*(-2 + Csch[c + d*x]^2)) + 3*b*(c + d*x - Log[Sinh[c + d*
x]]) + b*RootSum[a - b + 3*a*#1 + 3*b*#1 + 3*a*#1^2 - 3*b*#1^2 + a*#1^3 + b
*#1^3 & , (-2*a*c + 2*b*c - 2*a*d*x + 2*b*d*x + a*Log[E^(2*(c + d*x)) - #1]
 - b*Log[E^(2*(c + d*x)) - #1] - 8*a*c*#1 - 4*b*c*#1 - 8*a*d*x*#1 - 4*b*d*x
*#1 + 4*a*Log[E^(2*(c + d*x)) - #1]*#1 + 2*b*Log[E^(2*(c + d*x)) - #1]*#1 +
 2*a*c*#1^2 + 2*b*c*#1^2 + 2*a*d*x*#1^2 + 2*b*d*x*#1^2 - a*Log[E^(2*(c + d*
x)) - #1]*#1^2 - b*Log[E^(2*(c + d*x)) - #1]*#1^2)/(a - b + 2*a*#1 + 2*b*#1
 + a*#1^2 - b*#1^2) & ])/(3*a^2*d)
```

Maple [C] time = 0.131, size = 187, normalized size = 0.9

$$-\frac{1}{24\,da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{3}{8\,da}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{24\,da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-3}+\frac{3}{8\,da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1}-\frac{b}{da^2}\ln\left(\mathrm{t}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4/(a+b*tanh(d*x+c)^3),x)
```

```
[Out] -1/24/d/a*tanh(1/2*d*x+1/2*c)^3+3/8/d/a*tanh(1/2*d*x+1/2*c)-1/24/d/a/tanh(1
/2*d*x+1/2*c)^3+3/8/d/a/tanh(1/2*d*x+1/2*c)-1/d/a^2*b*ln(tanh(1/2*d*x+1/2*c
))+1/3/d/a^2*b*sum((_R^5*a+4*_R^2*b+3*_R*a)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)
*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$2\,ab\left(\frac{-(a-b)\int\frac{1}{\left(ae^{(6\,c)}+be^{(6\,c)}\right)e^{(6\,dx)}+3\left(ae^{(4\,c)}-be^{(4\,c)}\right)e^{(4\,dx)}+3\left(ae^{(2\,c)}+be^{(2\,c)}\right)e^{(2\,dx)}+a-b}\,dx+x}{a^3-a^2b}-\frac{dx+c}{\left(a^3-a^2b\right)d}\right)-2\,b^2\left(\frac{-(a-b)\int\frac{}{\left(ae^{(6\,c)}+be\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 2*a*b*(integrate(((a + b)*e^(4*d*x + 4*c) + 3*(a - b)*e^(2*d*x + 2*c) + 3*a
 + 3*b)*e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c
) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^3 - a^2*b) - (d*x + c)/((a^3
- a^2*b)*d)) - 2*b^2*(integrate(((a + b)*e^(4*d*x + 4*c) + 3*(a - b)*e^(2*d
*x + 2*c) + 3*a + 3*b)*e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)
*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^3 - a^2*b) - (
d*x + c)/((a^3 - a^2*b)*d)) + 2*b*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d
*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b),
 x)/a + 2*b^2*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b
)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/a^2 - 8*b*integr
ate(e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) +
3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/a - 4*b^2*integrate(e^(2*d*x + 2*c)/
((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x +
 2*c) + a - b), x)/a^2 + 2/3*(3*b*d*x*e^(6*d*x + 6*c) - 9*b*d*x*e^(4*d*x +
4*c) - 3*b*d*x + 3*(3*b*d*x*e^(2*c) - 2*a*e^(2*c))*e^(2*d*x) + 2*a)/(a^2*d*
e^(6*d*x + 6*c) - 3*a^2*d*e^(4*d*x + 4*c) + 3*a^2*d*e^(2*d*x + 2*c) - a^2*d
) - b*log((e^(d*x + c) + 1)*e^(-c))/(a^2*d) - b*log((e^(d*x + c) - 1)*e^(-c
))/(a^2*d)
```

Fricas [C] time = 17.1023, size = 4415, normalized size = 20.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/12*(12*sqrt(1/3)*(a^2*d*e^(6*d*x + 6*c) - 3*a^2*d*e^(4*d*x + 4*c) + 3*a^2
*d*e^(2*d*x + 2*c) - a^2*d)*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3)
 - b^3/(a^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d))^2*a^4*d^2
+ 4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b^
3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d))*a^2*b*d + 4*b^2)/(a^4*d^2))*arctan(-1/8*
sqrt(1/3)*((a^6 + a^5*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(a
^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d))^2*d^3*e^(2*d*x + 2*
c) - 4*(2*a^3*b + a^2*b^2 - a*b^3)*d*e^(2*d*x + 2*c) - 2*((a^5 - a^4*b - 2*
a^3*b^2)*d^2*e^(2*d*x + 2*c) + (a^5 + a^4*b)*d^2)*((1/2)^(1/3)*(I*sqrt(3) +
 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^
```

```
2*d)) - (((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*b
 - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d))^2*a^4*d^3 - 2*(a^3 - 2*a^2*b)*((1/2
)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b^3)/(a^6*d
^3))^(1/3) - 2*b/(a^2*d))*d^2 - 4*(2*a*b - b^2)*d)*sqrt(a^4 + 4*a^3*b + 5*a
^2*b^2 + 2*a*b^3 - 1/2*((a^6 + a^5*b)*d^2*e^(2*d*x + 2*c) - (a^6 + a^5*b)*d
^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b^
3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d))^2 + ((a^5 - a^4*b - 2*a^3*b^2)*d*e^(2*d*
x + 2*c) + (a^5 + 3*a^4*b + 2*a^3*b^2)*d)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(
a^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d)) +
(a^4 + 2*a^3*b + a^2*b^2)*e^(4*d*x + 4*c) + 2*(a^4 + a^3*b - a^2*b^2 - a*b^
3)*e^(2*d*x + 2*c)))*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/
(a^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d))^2*a^4*d^2 + 4*((1
/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b^3)/(a^6
*d^3))^(1/3) - 2*b/(a^2*d))*a^2*b*d + 4*b^2)/(a^4*d^2))/(a^2*b + a*b^2)) -
2*(a^2*d*e^(6*d*x + 6*c) - 3*a^2*d*e^(4*d*x + 4*c) + 3*a^2*d*e^(2*d*x + 2*c
) - a^2*d)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2
*b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d))*log(1/2*((1/2)^(1/3)*(I*sqrt(3) +
 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^
2*d))^2*a^4*d^2 - (a^3 - 2*a^2*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3)
 - b^3/(a^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d))*d + a^2 -
3*a*b + 2*b^2 + (a^2 + a*b)*e^(2*d*x + 2*c)) - 48*a*e^(2*d*x + 2*c) + ((a^2
*d*e^(6*d*x + 6*c) - 3*a^2*d*e^(4*d*x + 4*c) + 3*a^2*d*e^(2*d*x + 2*c) - a^
2*d)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b
^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d)) + 6*b*e^(6*d*x + 6*c) - 18*b*e^(4*d*x +
 4*c) + 18*b*e^(2*d*x + 2*c) - 6*b)*log(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3
 - 1/2*((a^6 + a^5*b)*d^2*e^(2*d*x + 2*c) - (a^6 + a^5*b)*d^2)*((1/2)^(1/3)
*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1
/3) - 2*b/(a^2*d))^2 + ((a^5 - a^4*b - 2*a^3*b^2)*d*e^(2*d*x + 2*c) + (a^5
+ 3*a^4*b + 2*a^3*b^2)*d)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(
a^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d)) + (a^4 + 2*a^3*b +
 a^2*b^2)*e^(4*d*x + 4*c) + 2*(a^4 + a^3*b - a^2*b^2 - a*b^3)*e^(2*d*x + 2*
c)) - 12*(b*e^(6*d*x + 6*c) - 3*b*e^(4*d*x + 4*c) + 3*b*e^(2*d*x + 2*c) - b
)*log(e^(2*d*x + 2*c) - 1) + 16*a)/(a^2*d*e^(6*d*x + 6*c) - 3*a^2*d*e^(4*d*
x + 4*c) + 3*a^2*d*e^(2*d*x + 2*c) - a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4/(a+b*tanh(d*x+c)**3),x)
```

```
[Out] Integral(csch(c + d*x)**4/(a + b*tanh(c + d*x)**3), x)
```

Giac [A] time = 1.42274, size = 243, normalized size = 1.13

$$\frac{\frac{2\,b\log\left(\left|ae^{(6\,dx+6\,c)}+be^{(6\,dx+6\,c)}+3\,ae^{(4\,dx+4\,c)}-3\,be^{(4\,dx+4\,c)}+3\,ae^{(2\,dx+2\,c)}+3\,be^{(2\,dx+2\,c)}+a-b\right|\right)}{a^2}-\frac{6\,b\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)}{a^2}+\frac{11\,be^{(6\,dx+6\,c)}-33\,be^{(4\,dx+4\,c)}-24\,ae^{\cdots}}{a^2\left(e^{(2\,dx+\cdots}\right.}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/6*(2*b*log(abs(a*e^(6*d*x + 6*c) + b*e^(6*d*x + 6*c) + 3*a*e^(4*d*x + 4*c
) - 3*b*e^(4*d*x + 4*c) + 3*a*e^(2*d*x + 2*c) + 3*b*e^(2*d*x + 2*c) + a - b
))/a^2 - 6*b*log(abs(e^(2*d*x + 2*c) - 1))/a^2 + (11*b*e^(6*d*x + 6*c) - 33
*b*e^(4*d*x + 4*c) - 24*a*e^(2*d*x + 2*c) + 33*b*e^(2*d*x + 2*c) + 8*a - 11
*b)/(a^2*(e^(2*d*x + 2*c) - 1)^3))/d
```

3.81 $\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=63

$$\frac{(a+b)\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{(3a-b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{1}{8}x(3a-b)$$

```
[Out] ((3*a - b)*x)/8 + ((3*a - b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + ((a + b)*
Cosh[c + d*x]^3*Sinh[c + d*x])/(4*d)
```

Rubi [A] time = 0.0496557, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3675, 385, 199, 206}

$$\frac{(a+b)\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{(3a-b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{1}{8}x(3a-b)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((3*a - b)*x)/8 + ((3*a - b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + ((a + b)*
Cosh[c + d*x]^3*Sinh[c + d*x])/(4*d)
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)dx = \frac{\text{Subst}\left(\int \frac{a+bx^2}{(1-x^2)^3}dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{(a+b)\cosh^3(c+dx)\sinh(c+dx)}{4d} + \frac{(3a-b)\,\text{Subst}\left(\int \frac{1}{(1-x^2)^2}dx, x, \tanh(c\right.}{4d}$$

$$= \frac{(3a-b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{(a+b)\cosh^3(c+dx)\sinh(c+dx)}{4d} + $$

$$= \frac{1}{8}(3a-b)x + \frac{(3a-b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{(a+b)\cosh^3(c+dx)\sin}{4d}$$

Mathematica [A] time = 0.162677, size = 44, normalized size = 0.7

$$\frac{(a+b)\sinh(4(c+dx)) + 12a(c+dx) + 8a\sinh(2(c+dx)) - 4bdx}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (-4*b*d*x + 12*a*(c + d*x) + 8*a*Sinh[2*(c + d*x)] + (a + b)*Sinh[4*(c + d*
x)])/(32*d)
```

Maple [A] time = 0.039, size = 82, normalized size = 1.3

$$\frac{1}{d}\left(b\left(\frac{\sinh(dx+c)(\cosh(dx+c))^3}{4} - \frac{\cosh(dx+c)\sinh(dx+c)}{8} - \frac{dx}{8} - \frac{c}{8}\right) + a\left(\left(\frac{(\cosh(dx+c))^3}{4} + \frac{3\cosh(dx+c)}{8}\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x)
```

```
[Out] 1/d*(b*(1/4*sinh(d*x+c)*cosh(d*x+c)^3-1/8*cosh(d*x+c)*sinh(d*x+c)-1/8*d*x-1
/8*c)+a*((1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A] time = 1.14363, size = 140, normalized size = 2.22

$$\frac{1}{64}a\left(24x + \frac{e^{(4dx+4c)}}{d} + \frac{8e^{(2dx+2c)}}{d} - \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d}\right) - \frac{1}{64}b\left(\frac{8(dx+c)}{d} - \frac{e^{(4dx+4c)}}{d} + \frac{e^{(-4dx-4c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/64*a*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2*c)
/d - e^(-4*d*x - 4*c)/d) - 1/64*b*(8*(d*x + c)/d - e^(4*d*x + 4*c)/d + e^(-
4*d*x - 4*c)/d)
```

Fricas [A] time = 1.82498, size = 169, normalized size = 2.68

$$\frac{(a+b)\cosh(dx+c)\sinh(dx+c)^3 + (3a-b)dx + \left((a+b)\cosh(dx+c)^3 + 4a\cosh(dx+c)\right)\sinh(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/8*((a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a - b)*d*x + ((a + b)*cosh(
d*x + c)^3 + 4*a*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27921, size = 144, normalized size = 2.29

$$\frac{8(3a-b)dx - \left(18ae^{(4dx+4c)} - 6be^{(4dx+4c)} + 8ae^{(2dx+2c)} + a + b\right)e^{(-4dx-4c)} + \left(ae^{(4dx+12c)} + be^{(4dx+12c)} + 8ae^{(2dx+10c)}\right.}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/64*(8*(3*a - b)*d*x - (18*a*e^(4*d*x + 4*c) - 6*b*e^(4*d*x + 4*c) + 8*a*e
^(2*d*x + 2*c) + a + b)*e^(-4*d*x - 4*c) + (a*e^(4*d*x + 12*c) + b*e^(4*d*x
 + 12*c) + 8*a*e^(2*d*x + 10*c))*e^(-8*c))/d
```

3.82 $\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)dx$

Optimal. Leaf size=30

$$\frac{(a+b)\sinh^3(c+dx)}{3d}+\frac{a\sinh(c+dx)}{d}$$

```
[Out] (a*Sinh[c + d*x])/d + ((a + b)*Sinh[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0355443, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3676}

$$\frac{(a+b)\sinh^3(c+dx)}{3d}+\frac{a\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]

[Out] (a*Sinh[c + d*x])/d + ((a + b)*Sinh[c + d*x]^3)/(3*d)
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)dx &= \frac{\text{Subst}\left(\int \left(a+(a+b)x^2\right)dx,x,\sinh(c+dx)\right)}{d}\\ &= \frac{a\sinh(c+dx)}{d}+\frac{(a+b)\sinh^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.0155267, size = 44, normalized size = 1.47

$$\frac{a\sinh^3(c+dx)}{3d}+\frac{a\sinh(c+dx)}{d}+\frac{b\sinh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]

[Out] (a*Sinh[c + d*x])/d + (a*Sinh[c + d*x]^3)/(3*d) + (b*Sinh[c + d*x]^3)/(3*d)
```

Maple [A] time = 0.036, size = 53, normalized size = 1.8

$$\frac{1}{d}\left(b\left(\frac{(\cosh(dx+c))^2\sinh(dx+c)}{3}-\frac{\sinh(dx+c)}{3}\right)+a\left(\frac{2}{3}+\frac{(\cosh(dx+c))^2}{3}\right)\sinh(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x)
```

```
[Out] 1/d*(b*(1/3*cosh(d*x+c)^2*sinh(d*x+c)-1/3*sinh(d*x+c))+a*(2/3+1/3*cosh(d*x+
c)^2)*sinh(d*x+c))
```

Maxima [B] time = 1.0968, size = 112, normalized size = 3.73

$$\frac{b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3}{24\,d}+\frac{1}{24}\,a\left(\frac{e^{(3\,dx+3\,c)}}{d}+\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}-\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/24*b*(e^(d*x + c) - e^(-d*x - c))^3/d + 1/24*a*(e^(3*d*x + 3*c)/d + 9*e^(
d*x + c)/d - 9*e^(-d*x - c)/d - e^(-3*d*x - 3*c)/d)
```

Fricas [A] time = 1.81256, size = 119, normalized size = 3.97

$$\frac{(a+b)\sinh\left(dx+c\right)^3+3\left((a+b)\cosh\left(dx+c\right)^2+3\,a-b\right)\sinh\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/12*((a + b)*sinh(d*x + c)^3 + 3*((a + b)*cosh(d*x + c)^2 + 3*a - b)*sinh(
d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tanh^2\left(c+dx\right)\right)\cosh^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*cosh(c + d*x)**3, x)
```

Giac [B] time = 1.24649, size = 127, normalized size = 4.23

$$-\frac{\left(9\,ae^{(2\,dx+2\,c)}-3\,be^{(2\,dx+2\,c)}+a+b\right)e^{(-3\,dx-3\,c)}-\left(ae^{(3\,dx+12\,c)}+be^{(3\,dx+12\,c)}+9\,ae^{(dx+10\,c)}-3\,be^{(dx+10\,c)}\right)e^{(-9\,c)}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/24*((9*a*e^(2*d*x + 2*c) - 3*b*e^(2*d*x + 2*c) + a + b)*e^(-3*d*x - 3*c)
 - (a*e^(3*d*x + 12*c) + b*e^(3*d*x + 12*c) + 9*a*e^(d*x + 10*c) - 3*b*e^(d
*x + 10*c))*e^(-9*c))/d
```

3.83 $\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=33

$$\frac{(a+b)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{1}{2}x(a-b)$$

```
[Out] ((a - b)*x)/2 + ((a + b)*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)
```

Rubi [A] time = 0.0390378, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3675, 385, 206}

$$\frac{(a+b)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{1}{2}x(a-b)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((a - b)*x)/2 + ((a + b)*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)dx=\frac{\text{Subst}\left(\int \frac{a+bx^2}{\left(1-x^2\right)^2}\,dx,x,\tanh(c+dx)\right)}{d}$$
$$=\frac{(a+b)\cosh(c+dx)\sinh(c+dx)}{2d}+\frac{(a-b)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\tanh(c+dx)\right)}{2d}$$
$$=\frac{1}{2}(a-b)x+\frac{(a+b)\cosh(c+dx)\sinh(c+dx)}{2d}$$

Mathematica [A] time = 0.056917, size = 32, normalized size = 0.97

$$\frac{2(a-b)(c+dx)+(a+b)\sinh(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (2*(a - b)*(c + d*x) + (a + b)*Sinh[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.033, size = 54, normalized size = 1.6

$$\frac{1}{d}\left(b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x)
```

```
[Out] 1/d*(b*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+a*(1/2*cosh(d*x+c)*sinh(
d*x+c)+1/2*d*x+1/2*c))
```

Maxima [B] time = 1.11015, size = 93, normalized size = 2.82

$$\frac{1}{8}a\left(4x+\frac{e^{(2dx+2c)}}{d}-\frac{e^{(-2dx-2c)}}{d}\right)-\frac{1}{8}b\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/8*a*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) - 1/8*b*(4*x - e^(2*d*
x + 2*c)/d + e^(-2*d*x - 2*c)/d)
```

Fricas [A] time = 1.86134, size = 80, normalized size = 2.42

$$\frac{(a-b)dx+(a+b)\cosh(dx+c)\sinh(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*((a - b)*d*x + (a + b)*cosh(d*x + c)*sinh(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tanh^2(c+dx)\right)\cosh^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*cosh(c + d*x)**2, x)
```

Giac [B] time = 1.21928, size = 109, normalized size = 3.3

$$\frac{4\,(a-b)dx-\left(2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)e^{(-2\,dx-2\,c)}+\left(ae^{(2\,dx+4\,c)}+be^{(2\,dx+4\,c)}\right)e^{(-2\,c)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/8*(4*(a - b)*d*x - (2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)*e^
(-2*d*x - 2*c) + (a*e^(2*d*x + 4*c) + b*e^(2*d*x + 4*c))*e^(-2*c))/d
```

3.84 $\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)dx$

Optimal. Leaf size=27

$$\frac{(a+b)\sinh(c+dx)}{d}-\frac{b\tan^{-1}(\sinh(c+dx))}{d}$$

```
[Out] -((b*ArcTan[Sinh[c + d*x]])/d) + ((a + b)*Sinh[c + d*x])/d
```

Rubi [A] time = 0.0319431, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3676, 388, 203}

$$\frac{(a+b)\sinh(c+dx)}{d}-\frac{b\tan^{-1}(\sinh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -((b*ArcTan[Sinh[c + d*x]])/d) + ((a + b)*Sinh[c + d*x])/d
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)dx&=\frac{\text{Subst}\left(\int \frac{a+(a+b)x^2}{1+x^2}\,dx,x,\sinh(c+dx)\right)}{d}\\&=\frac{(a+b)\sinh(c+dx)}{d}-\frac{b\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sinh(c+dx)\right)}{d}\\&=-\frac{b\tan^{-1}(\sinh(c+dx))}{d}+\frac{(a+b)\sinh(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0289552, size = 47, normalized size = 1.74

$$\frac{a\sinh(c)\cosh(dx)}{d}+\frac{a\cosh(c)\sinh(dx)}{d}+\frac{b\sinh(c+dx)}{d}-\frac{b\tan^{-1}(\sinh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -((b*ArcTan[Sinh[c + d*x]])/d) + (a*Cosh[d*x]*Sinh[c])/d + (a*Cosh[c]*Sinh[
d*x])/d + (b*Sinh[c + d*x])/d
```

Maple [A] time = 0.034, size = 37, normalized size = 1.4

$$\frac{a\sinh(dx+c)}{d}+\frac{b\sinh(dx+c)}{d}-2\,\frac{b\arctan\left(e^{dx+c}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*(a+b*tanh(d*x+c)^2),x)
```

```
[Out] a*sinh(d*x+c)/d+b*sinh(d*x+c)/d-2/d*b*arctan(exp(d*x+c))
```

Maxima [B] time = 1.60957, size = 74, normalized size = 2.74

$$\frac{1}{2}\,b\left(\frac{4\,\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(dx+c)}}{d}-\frac{e^{(-dx-c)}}{d}\right)+\frac{a\sinh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*b*(4*arctan(e^(-d*x - c))/d + e^(d*x + c)/d - e^(-d*x - c)/d) + a*sinh(
d*x + c)/d
```

Fricas [B] time = 1.86466, size = 296, normalized size = 10.96

$$\frac{(a+b)\cosh(dx+c)^2+2\,(a+b)\cosh(dx+c)\sinh(dx+c)+(a+b)\sinh(dx+c)^2-4\,(b\cosh(dx+c)+b\sinh(dx}{2\,(d\cosh(dx+c)+d\sinh(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a +
 b)*sinh(d*x + c)^2 - 4*(b*cosh(d*x + c) + b*sinh(d*x + c))*arctan(cosh(d*x
 + c) + sinh(d*x + c)) - a - b)/(d*cosh(d*x + c) + d*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tanh^2\left(c + dx\right)\right)\cosh\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*cosh(c + d*x), x)
```

Giac [B] time = 1.265, size = 76, normalized size = 2.81

$$-\frac{4\,b\arctan\left(e^{(dx+c)}\right) + (a+b)e^{(-dx-c)} - \left(ae^{(dx+4\,c)} + be^{(dx+4\,c)}\right)e^{(-3\,c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/2*(4*b*arctan(e^(d*x + c)) + (a + b)*e^(-d*x - c) - (a*e^(d*x + 4*c) + b
*e^(d*x + 4*c))*e^(-3*c))/d
```

3.85 $\int \mathbf{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right) dx$

Optimal. Leaf size=40

$$\frac{(2a+b)\tan^{-1}(\sinh(c+dx))}{2d}-\frac{b\tanh(c+dx)\mathbf{sech}(c+dx)}{2d}$$

```
[Out] ((2*a + b)*ArcTan[Sinh[c + d*x]])/(2*d) - (b*Sech[c + d*x]*Tanh[c + d*x])/(
2*d)
```

Rubi [A] time = 0.0310208, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3676, 385, 203}

$$\frac{(2a+b)\tan^{-1}(\sinh(c+dx))}{2d}-\frac{b\tanh(c+dx)\mathbf{sech}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((2*a + b)*ArcTan[Sinh[c + d*x]])/(2*d) - (b*Sech[c + d*x]*Tanh[c + d*x])/(
2*d)
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right) dx &= \frac{\operatorname{Subst}\left(\int \frac{a+(a+b)x^2}{\left(1+x^2\right)^2} dx, x, \sinh(c+dx)\right)}{d}\\ &= -\frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}+\frac{(2a+b)\operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sinh(c+dx\right.}{2d}\\ &= \frac{(2a+b)\tan^{-1}(\sinh(c+dx))}{2d}-\frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.0228812, size = 48, normalized size = 1.2

$$\frac{a\tan^{-1}(\sinh(c+dx))}{d}+\frac{b\tan^{-1}(\sinh(c+dx))}{2d}-\frac{b\tanh(c+dx)\mathrm{sech}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a*ArcTan[Sinh[c + d*x]])/d + (b*ArcTan[Sinh[c + d*x]])/(2*d) - (b*Sech[c +
 d*x]*Tanh[c + d*x])/(2*d)
```

Maple [A] time = 0.026, size = 65, normalized size = 1.6

$$2\frac{a\arctan\left(e^{dx+c}\right)}{d}-\frac{b\sinh(dx+c)}{d\left(\cosh(dx+c)\right)^{2}}+\frac{b\mathrm{sech}(dx+c)\tanh(dx+c)}{2d}+\frac{b\arctan\left(e^{dx+c}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)*(a+b*tanh(d*x+c)^2),x)
```

```
[Out] 2/d*a*arctan(exp(d*x+c))-1/d*b*sinh(d*x+c)/cosh(d*x+c)^2+1/2*b*sech(d*x+c)*
tanh(d*x+c)/d+1/d*b*arctan(exp(d*x+c))
```

Maxima [B] time = 1.64965, size = 108, normalized size = 2.7

$$-b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(-dx-c)}-e^{(-3dx-3c)}}{d\left(2e^{(-2dx-2c)}+e^{(-4dx-4c)}+1\right)}\right)+\frac{a\arctan(\sinh(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -b*(arctan(e^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*
d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a*arctan(sinh(d*x + c))/d
```

Fricas [B] time = 1.9605, size = 883, normalized size = 22.08

$$-\frac{b\cosh(dx+c)^{3}+3b\cosh(dx+c)\sinh(dx+c)^{2}+b\sinh(dx+c)^{3}-\left((2a+b)\cosh(dx+c)^{4}+4(2a+b)\cosh(dx+\right.}{d\,\mathrm{co}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3
 - ((2*a + b)*cosh(d*x + c)^4 + 4*(2*a + b)*cosh(d*x + c)*sinh(d*x + c)^3 +
 (2*a + b)*sinh(d*x + c)^4 + 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*(2*a + b)*c
osh(d*x + c)^2 + 2*a + b)*sinh(d*x + c)^2 + 4*((2*a + b)*cosh(d*x + c)^3 +
(2*a + b)*cosh(d*x + c))*sinh(d*x + c) + 2*a + b)*arctan(cosh(d*x + c) + si
```

```
nh(d*x + c)) - b*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c))/(
d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 +
 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 4*(d*c
osh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tanh^2\left(c + dx\right)\right)\operatorname{sech}\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*sech(c + d*x), x)
```

Giac [A] time = 1.21055, size = 85, normalized size = 2.12

$$\frac{\left(2\,ae^{c} + be^{c}\right)\arctan\left(e^{(dx+c)}\right)e^{(-c)} - \frac{be^{(3\,dx+3\,c)} - be^{(dx+c)}}{\left(e^{(2\,dx+2\,c)}+1\right)^{2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] ((2*a*e^c + b*e^c)*arctan(e^(d*x + c))*e^(-c) - (b*e^(3*d*x + 3*c) - b*e^(d
*x + c))/(e^(2*d*x + 2*c) + 1)^2)/d
```

3.86 $\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=28

$$\frac{a\tanh(c+dx)}{d}+\frac{b\tanh^3(c+dx)}{3d}$$

```
[Out] (a*Tanh[c + d*x])/d + (b*Tanh[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0293381, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3675}

$$\frac{a\tanh(c+dx)}{d}+\frac{b\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a*Tanh[c + d*x])/d + (b*Tanh[c + d*x]^3)/(3*d)
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx &= \frac{\operatorname{Subst}\left(\int \left(a+bx^2\right)\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{a\tanh(c+dx)}{d}+\frac{b\tanh^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.0112985, size = 28, normalized size = 1.

$$\frac{a\tanh(c+dx)}{d}+\frac{b\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a*Tanh[c + d*x])/d + (b*Tanh[c + d*x]^3)/(3*d)
```

Maple [A] time = 0.036, size = 53, normalized size = 1.9

$$\frac{1}{d}\left(a\tanh\left(dx+c\right)+b\left(-\frac{\sinh\left(dx+c\right)}{2\left(\cosh\left(dx+c\right)\right)^{3}}+\frac{\tanh\left(dx+c\right)}{2}\left(\frac{2}{3}+\frac{\left(\mathrm{sech}\left(dx+c\right)\right)^{2}}{3}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2),x)
```

```
[Out] 1/d*(a*tanh(d*x+c)+b*(-1/2*sinh(d*x+c)/cosh(d*x+c)^3+1/2*(2/3+1/3*sech(d*x+
c)^2)*tanh(d*x+c)))
```

Maxima [A] time = 1.13298, size = 46, normalized size = 1.64

$$\frac{b\tanh\left(dx+c\right)^{3}}{3\,d}+\frac{2\,a}{d\left(e^{\left(-2\,dx-2\,c\right)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/3*b*tanh(d*x + c)^3/d + 2*a/(d*(e^(-2*d*x - 2*c) + 1))
```

Fricas [B] time = 1.81801, size = 424, normalized size = 15.14

$$-\frac{4\left(\left(3\,a+2\,b\right)\cosh\left(dx+c\right)^{2}+2\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)+(3\right.}{3\left(d\cosh\left(dx+c\right)^{4}+4\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}+d\sinh\left(dx+c\right)^{4}+4\,d\cosh\left(dx+c\right)^{2}+2\left(3\,d\cosh\left(dx+c\right)^{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -4/3*((3*a + 2*b)*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + (3*a
+ 2*b)*sinh(d*x + c)^2 + 3*a)/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d
*x + c)^3 + d*sinh(d*x + c)^4 + 4*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^
2 + 2*d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x
 + c) + 3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tanh^{2}\left(c+dx\right)\right)\mathrm{sech}^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*sech(c + d*x)**2, x)
```

Giac [B] time = 1.1891, size = 80, normalized size = 2.86

$$-\frac{2\left(3\,ae^{(4\,dx+4\,c)}+3\,be^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}+3\,a+b\right)}{3\,d\left(e^{(2\,dx+2\,c)}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -2/3*(3*a*e^(4*d*x + 4*c) + 3*b*e^(4*d*x + 4*c) + 6*a*e^(2*d*x + 2*c) + 3*a
 + b)/(d*(e^(2*d*x + 2*c) + 1)^3)
```

3.87 $\int \mathbf{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)dx$

Optimal. Leaf size=66

$$\frac{(4a+b)\tan^{-1}(\sinh(c+dx))}{8d}+\frac{(4a+b)\tanh(c+dx)\mathrm{sech}(c+dx)}{8d}-\frac{b\tanh(c+dx)\mathrm{sech}^3(c+dx)}{4d}$$

```
[Out] ((4*a + b)*ArcTan[Sinh[c + d*x]])/(8*d) + ((4*a + b)*Sech[c + d*x]*Tanh[c +
 d*x])/(8*d) - (b*Sech[c + d*x]^3*Tanh[c + d*x])/(4*d)
```

Rubi [A] time = 0.0455378, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3676, 385, 199, 203}

$$\frac{(4a+b)\tan^{-1}(\sinh(c+dx))}{8d}+\frac{(4a+b)\tanh(c+dx)\mathrm{sech}(c+dx)}{8d}-\frac{b\tanh(c+dx)\mathrm{sech}^3(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((4*a + b)*ArcTan[Sinh[c + d*x]])/(8*d) + ((4*a + b)*Sech[c + d*x]*Tanh[c +
 d*x])/(8*d) - (b*Sech[c + d*x]^3*Tanh[c + d*x])/(4*d)
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)dx = \frac{\operatorname{Subst}\left(\int \frac{a+(a+b)x^2}{(1+x^2)^3}dx, x, \sinh(c+dx)\right)}{d}$$
$$= -\frac{b\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4d} + \frac{(4a+b)\operatorname{Subst}\left(\int \frac{1}{(1+x^2)^2}dx, x, \sinh(c+d$$
$$= \frac{(4a+b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8d} - \frac{b\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4d} + \frac{(4a+$$
$$= \frac{(4a+b)\tan^{-1}(\sinh(c+dx))}{8d} + \frac{(4a+b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8d} - \frac{b\operatorname{sech}$$

Mathematica [A] time = 0.0293792, size = 93, normalized size = 1.41

$$\frac{a\tan^{-1}(\sinh(c+dx))}{2d} + \frac{a\tanh(c+dx)\operatorname{sech}(c+dx)}{2d} + \frac{b\tan^{-1}(\sinh(c+dx))}{8d} - \frac{b\tanh(c+dx)\operatorname{sech}^3(c+dx)}{4d} + \frac{b\tanh($$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a*ArcTan[Sinh[c + d*x]])/(2*d) + (b*ArcTan[Sinh[c + d*x]])/(8*d) + (a*Sech
[c + d*x]*Tanh[c + d*x])/(2*d) + (b*Sech[c + d*x]*Tanh[c + d*x])/(8*d) - (b
*Sech[c + d*x]^3*Tanh[c + d*x])/(4*d)
```

Maple [A] time = 0.044, size = 103, normalized size = 1.6

$$\frac{a\operatorname{sech}(dx+c)\tanh(dx+c)}{2d} + \frac{a\arctan\left(e^{dx+c}\right)}{d} - \frac{b\sinh(dx+c)}{3d(\cosh(dx+c))^4} + \frac{b(\operatorname{sech}(dx+c))^3\tanh(dx+c)}{12d} + \frac{b\operatorname{sech}(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2),x)
```

```
[Out] 1/2/d*a*sech(d*x+c)*tanh(d*x+c)+1/d*a*arctan(exp(d*x+c))-1/3/d*b*sinh(d*x+c
)/cosh(d*x+c)^4+1/12*b*sech(d*x+c)^3*tanh(d*x+c)/d+1/8*b*sech(d*x+c)*tanh(d
*x+c)/d+1/4/d*b*arctan(exp(d*x+c))
```

Maxima [B] time = 1.61634, size = 244, normalized size = 3.7

$$-\frac{1}{4}b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)} - 7e^{(-3dx-3c)} + 7e^{(-5dx-5c)} - e^{(-7dx-7c)}}{d\left(4e^{(-2dx-2c)} + 6e^{(-4dx-4c)} + 4e^{(-6dx-6c)} + e^{(-8dx-8c)} + 1\right)}\right) - a\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx}}{d(2e^{(-2dx-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/4*b*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - 7*e^(-3*d*x - 3*c) + 7*e^(
-5*d*x - 5*c) - e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*
c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - a*(arctan(e^(-d*x - c))
```

```
/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x -
 4*c) + 1)))
```

Fricas [B] time = 2.01207, size = 2826, normalized size = 42.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*((4*a + b)*cosh(d*x + c)^7 + 7*(4*a + b)*cosh(d*x + c)*sinh(d*x + c)^6
+ (4*a + b)*sinh(d*x + c)^7 + (4*a - 7*b)*cosh(d*x + c)^5 + (21*(4*a + b)*c
osh(d*x + c)^2 + 4*a - 7*b)*sinh(d*x + c)^5 + 5*(7*(4*a + b)*cosh(d*x + c)^
3 + (4*a - 7*b)*cosh(d*x + c))*sinh(d*x + c)^4 - (4*a - 7*b)*cosh(d*x + c)^
3 + (35*(4*a + b)*cosh(d*x + c)^4 + 10*(4*a - 7*b)*cosh(d*x + c)^2 - 4*a +
7*b)*sinh(d*x + c)^3 + (21*(4*a + b)*cosh(d*x + c)^5 + 10*(4*a - 7*b)*cosh(
d*x + c)^3 - 3*(4*a - 7*b)*cosh(d*x + c))*sinh(d*x + c)^2 + ((4*a + b)*cosh
(d*x + c)^8 + 8*(4*a + b)*cosh(d*x + c)*sinh(d*x + c)^7 + (4*a + b)*sinh(d*
x + c)^8 + 4*(4*a + b)*cosh(d*x + c)^6 + 4*(7*(4*a + b)*cosh(d*x + c)^2 + 4
*a + b)*sinh(d*x + c)^6 + 8*(7*(4*a + b)*cosh(d*x + c)^3 + 3*(4*a + b)*cosh
(d*x + c))*sinh(d*x + c)^5 + 6*(4*a + b)*cosh(d*x + c)^4 + 2*(35*(4*a + b)*
cosh(d*x + c)^4 + 30*(4*a + b)*cosh(d*x + c)^2 + 12*a + 3*b)*sinh(d*x + c)^
4 + 8*(7*(4*a + b)*cosh(d*x + c)^5 + 10*(4*a + b)*cosh(d*x + c)^3 + 3*(4*a
+ b)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(4*a + b)*cosh(d*x + c)^2 + 4*(7*(4
*a + b)*cosh(d*x + c)^6 + 15*(4*a + b)*cosh(d*x + c)^4 + 9*(4*a + b)*cosh(d
*x + c)^2 + 4*a + b)*sinh(d*x + c)^2 + 8*((4*a + b)*cosh(d*x + c)^7 + 3*(4*
a + b)*cosh(d*x + c)^5 + 3*(4*a + b)*cosh(d*x + c)^3 + (4*a + b)*cosh(d*x +
 c))*sinh(d*x + c) + 4*a + b)*arctan(cosh(d*x + c) + sinh(d*x + c)) - (4*a
+ b)*cosh(d*x + c) + (7*(4*a + b)*cosh(d*x + c)^6 + 5*(4*a - 7*b)*cosh(d*x
+ c)^4 - 3*(4*a - 7*b)*cosh(d*x + c)^2 - 4*a - b)*sinh(d*x + c))/(d*cosh(d*
x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 + 4*d*cosh
(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x
 + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*
d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*co
sh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 +
 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 15*d*cosh(d*x + c)^4 + 9*d*
cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 + 3*d*cosh(d*x
+ c)^5 + 3*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tanh^2(c + dx)\right)\operatorname{sech}^3(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*sech(c + d*x)**3, x)
```

Giac [B] time = 1.22584, size = 178, normalized size = 2.7

$$\frac{(4\,ae^{c}+be^{c})\arctan\left(e^{(dx+c)}\right)e^{(-c)}+\frac{4\,ae^{(7\,dx+7\,c)}+be^{(7\,dx+7\,c)}+4\,ae^{(5\,dx+5\,c)}-7\,be^{(5\,dx+5\,c)}-4\,ae^{(3\,dx+3\,c)}+7\,be^{(3\,dx+3\,c)}-4\,ae^{(dx+c)}-be^{(dx+c)}}{\left(e^{(2\,dx+2\,c)}+1\right)^{4}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/4*((4*a*e^c + b*e^c)*arctan(e^(d*x + c))*e^(-c) + (4*a*e^(7*d*x + 7*c) +
b*e^(7*d*x + 7*c) + 4*a*e^(5*d*x + 5*c) - 7*b*e^(5*d*x + 5*c) - 4*a*e^(3*d*
x + 3*c) + 7*b*e^(3*d*x + 3*c) - 4*a*e^(d*x + c) - b*e^(d*x + c))/(e^(2*d*x
 + 2*c) + 1)^4)/d
```

3.88 $\int \text{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=48

$$-\frac{(a-b)\tanh^3(c+dx)}{3d}+\frac{a\tanh(c+dx)}{d}-\frac{b\tanh^5(c+dx)}{5d}$$

```
[Out] (a*Tanh[c + d*x])/d - ((a - b)*Tanh[c + d*x]^3)/(3*d) - (b*Tanh[c + d*x]^5)
/(5*d)
```

Rubi [A] time = 0.0398919, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3675, 373}

$$-\frac{(a-b)\tanh^3(c+dx)}{3d}+\frac{a\tanh(c+dx)}{d}-\frac{b\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a*Tanh[c + d*x])/d - ((a - b)*Tanh[c + d*x]^3)/(3*d) - (b*Tanh[c + d*x]^5)
/(5*d)
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\int \text{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx &= \frac{\text{Subst}\left(\int\left(1-x^2\right)\left(a+bx^2\right)\,dx,x,\tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int\left(a-(a-b)x^2-bx^4\right)\,dx,x,\tanh(c+dx)\right)}{d}\\
&= \frac{a\tanh(c+dx)}{d}-\frac{(a-b)\tanh^3(c+dx)}{3d}-\frac{b\tanh^5(c+dx)}{5d}
\end{aligned}$$

Mathematica [A] time = 0.0475146, size = 86, normalized size = 1.79

$$-\frac{a\tanh^3(c+dx)}{3d}+\frac{a\tanh(c+dx)}{d}+\frac{2b\tanh(c+dx)}{15d}-\frac{b\tanh(c+dx)\text{sech}^4(c+dx)}{5d}+\frac{b\tanh(c+dx)\text{sech}^2(c+dx)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a*Tanh[c + d*x])/d + (2*b*Tanh[c + d*x])/(15*d) + (b*Sech[c + d*x]^2*Tanh[
c + d*x])/(15*d) - (b*Sech[c + d*x]^4*Tanh[c + d*x])/(5*d) - (a*Tanh[c + d*
x]^3)/(3*d)
```

Maple [A] time = 0.044, size = 75, normalized size = 1.6

$$\frac{1}{d}\left(a\left(\frac{2}{3}+\frac{(\operatorname{sech}(dx+c))^2}{3}\right)\tanh(dx+c)+b\left(-\frac{\sinh(dx+c)}{4(\cosh(dx+c))^5}+\frac{\tanh(dx+c)}{4}\left(\frac{8}{15}+\frac{(\operatorname{sech}(dx+c))^4}{5}+\frac{4(\operatorname{sech}(d}{15}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2),x)
```

```
[Out] 1/d*(a*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+b*(-1/4*sinh(d*x+c)/cosh(d*x+c)^
5+1/4*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c)))
```

Maxima [B] time = 1.10757, size = 501, normalized size = 10.44

$$\frac{4}{15}b\left(\frac{5e^{(-2dx-2c)}}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1\right)}-\frac{5e^{(}}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 4/15*b*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 1
0*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) - 5*e^(-
4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x -
6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*e^(-6*d*x - 6*c)/
(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-
8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(
-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*
c) + 1))) + 4/3*a*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x
- 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x -
4*c) + e^(-6*d*x - 6*c) + 1)))
```

Fricas [B] time = 1.91973, size = 934, normalized size = 19.46

$$-\frac{}{15\left(d\cosh(dx+c)^7+7d\cosh(dx+c)\sinh(dx+c)^6+d\sinh(dx+c)^7+5d\cosh(dx+c)^5+\left(21d\cosh(dx+c)^2+5\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -8/15*(2*(5*a + 4*b)*cosh(d*x + c)^3 + 6*(5*a + 4*b)*cosh(d*x + c)*sinh(d*x
 + c)^2 + (5*a + 7*b)*sinh(d*x + c)^3 + 30*a*cosh(d*x + c) + (3*(5*a + 7*b)
*cosh(d*x + c)^2 + 5*a - 5*b)*sinh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*cosh(
```

```
d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x + c)^7 + 5*d*cosh(d*x + c)^5 + (21*d*
cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^5 + 5*(7*d*cosh(d*x + c)^3 + 5*d*cosh(
d*x + c))*sinh(d*x + c)^4 + 11*d*cosh(d*x + c)^3 + (35*d*cosh(d*x + c)^4 +
50*d*cosh(d*x + c)^2 + 9*d)*sinh(d*x + c)^3 + (21*d*cosh(d*x + c)^5 + 50*d*
cosh(d*x + c)^3 + 33*d*cosh(d*x + c))*sinh(d*x + c)^2 + 15*d*cosh(d*x + c)
+ (7*d*cosh(d*x + c)^6 + 25*d*cosh(d*x + c)^4 + 27*d*cosh(d*x + c)^2 + 5*d)
*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tanh^2{(c + dx)}\right)\operatorname{sech}^4{(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*sech(c + d*x)**4, x)
```

Giac [B] time = 1.34274, size = 128, normalized size = 2.67

$$-\frac{4\left(15\,ae^{(6\,dx+6\,c)} + 15\,be^{(6\,dx+6\,c)} + 35\,ae^{(4\,dx+4\,c)} - 5\,be^{(4\,dx+4\,c)} + 25\,ae^{(2\,dx+2\,c)} + 5\,be^{(2\,dx+2\,c)} + 5\,a + b\right)}{15\,d\left(e^{(2\,dx+2\,c)} + 1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -4/15*(15*a*e^(6*d*x + 6*c) + 15*b*e^(6*d*x + 6*c) + 35*a*e^(4*d*x + 4*c) -
 5*b*e^(4*d*x + 4*c) + 25*a*e^(2*d*x + 2*c) + 5*b*e^(2*d*x + 2*c) + 5*a + b
)/(d*(e^(2*d*x + 2*c) + 1)^5)
```

3.89 $\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=85

$$\frac{3\left(a^2-b^2\right)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{1}{8}x\left(3a^2-2ab+3b^2\right)+\frac{(a+b)\sinh(c+dx)\cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)}{4d}$$

```
[Out] ((3*a^2 - 2*a*b + 3*b^2)*x)/8 + (3*(a^2 - b^2)*Cosh[c + d*x]*Sinh[c + d*x])
/(8*d) + ((a + b)*Cosh[c + d*x]^3*Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2))/(4
*d)
```

Rubi [A] time = 0.0868324, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3675, 413, 385, 206}

$$\frac{3\left(a^2-b^2\right)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{1}{8}x\left(3a^2-2ab+3b^2\right)+\frac{(a+b)\sinh(c+dx)\cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((3*a^2 - 2*a*b + 3*b^2)*x)/8 + (3*(a^2 - b^2)*Cosh[c + d*x]*Sinh[c + d*x])
/(8*d) + ((a + b)*Cosh[c + d*x]^3*Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2))/(4
*d)
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{\left(1-x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{(a+b)\cosh^3(c+dx)\sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)}{4d} - \frac{\text{Subst}\left(\int \frac{-}{}\right.}{}$$

$$= \frac{3\left(a^2-b^2\right)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{(a+b)\cosh^3(c+dx)\sinh(c+}{4d}$$

$$= \frac{1}{8}\left(3a^2-2ab+3b^2\right)x + \frac{3\left(a^2-b^2\right)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{(a+b}{}$$

Mathematica [A] time = 0.292658, size = 63, normalized size = 0.74

$$\frac{4\left(3a^2-2ab+3b^2\right)(c+dx)+8\left(a^2-b^2\right)\sinh(2(c+dx))+(a+b)^2\sinh(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (4*(3*a^2 - 2*a*b + 3*b^2)*(c + d*x) + 8*(a^2 - b^2)*Sinh[2*(c + d*x)] + (a
 + b)^2*Sinh[4*(c + d*x)])/(32*d)
```

Maple [A] time = 0.04, size = 124, normalized size = 1.5

$$\frac{1}{d}\left(b^2\left(\left(\frac{(\sinh(dx+c))^3}{4} - \frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c) + \frac{3dx}{8} + \frac{3c}{8}\right) + 2ab\left(1/4\sinh(dx+c)(\cosh(dx+c))^3 - 1/\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(b^2*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+2*
a*b*(1/4*sinh(d*x+c)*cosh(d*x+c)^3-1/8*cosh(d*x+c)*sinh(d*x+c)-1/8*d*x-1/8*
c)+a^2*((1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [B] time = 1.03492, size = 231, normalized size = 2.72

$$\frac{1}{64}a^2\left(24x + \frac{e^{(4dx+4c)}}{d} + \frac{8e^{(2dx+2c)}}{d} - \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d}\right) + \frac{1}{64}b^2\left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/64*a^2*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + 1/64*b^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*
x + 2*c)/d + 8*e^(-2*d*x - 2*c)/d - e^(-4*d*x - 4*c)/d) - 1/32*a*b*(8*(d*x
+ c)/d - e^(4*d*x + 4*c)/d + e^(-4*d*x - 4*c)/d)
```

Fricas [A] time = 1.95999, size = 234, normalized size = 2.75

$$\frac{\left(a^2+2\,ab+b^2\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+\left(3\,a^2-2\,ab+3\,b^2\right)dx+\left(\left(a^2+2\,ab+b^2\right)\cosh\left(dx+c\right)^3+4\left(a^2-b^2\right)\cosh\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2 - 2*a*b + 3
*b^2)*d*x + ((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 4*(a^2 - b^2)*cosh(d*x +
 c))*sinh(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.93304, size = 255, normalized size = 3.

$$\frac{8\left(3\,a^2-2\,ab+3\,b^2\right)dx-\left(18\,a^2e^{(4\,dx+4\,c)}-12\,abe^{(4\,dx+4\,c)}+18\,b^2e^{(4\,dx+4\,c)}+8\,a^2e^{(2\,dx+2\,c)}-8\,b^2e^{(2\,dx+2\,c)}+a^2+2\,ab+b^2\right)\cdots}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/64*(8*(3*a^2 - 2*a*b + 3*b^2)*d*x - (18*a^2*e^(4*d*x + 4*c) - 12*a*b*e^(4
*d*x + 4*c) + 18*b^2*e^(4*d*x + 4*c) + 8*a^2*e^(2*d*x + 2*c) - 8*b^2*e^(2*d
*x + 2*c) + a^2 + 2*a*b + b^2)*e^(-4*d*x - 4*c) + (a^2*e^(4*d*x + 12*c) + 2
*a*b*e^(4*d*x + 12*c) + b^2*e^(4*d*x + 12*c) + 8*a^2*e^(2*d*x + 10*c) - 8*b
^2*e^(2*d*x + 10*c))*e^(-8*c))/d
```

3.90 $\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=54

$$\frac{(a^2-b^2)\sinh(c+dx)}{d}+\frac{(a+b)^2\sinh^3(c+dx)}{3d}+\frac{b^2\tan^{-1}(\sinh(c+dx))}{d}$$

```
[Out] (b^2*ArcTan[Sinh[c + d*x]])/d + ((a^2 - b^2)*Sinh[c + d*x])/d + ((a + b)^2*
Sinh[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0603328, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3676, 390, 203}

$$\frac{(a^2-b^2)\sinh(c+dx)}{d}+\frac{(a+b)^2\sinh^3(c+dx)}{3d}+\frac{b^2\tan^{-1}(\sinh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (b^2*ArcTan[Sinh[c + d*x]])/d + ((a^2 - b^2)*Sinh[c + d*x])/d + ((a + b)^2*
Sinh[c + d*x]^3)/(3*d)
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+(a+b)x^2\right)^2}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2-b^2+(a+b)^2x^2+\frac{b^2}{1+x^2}\right)dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{\left(a^2-b^2\right)\sinh(c+dx)}{d} + \frac{(a+b)^2\sinh^3(c+dx)}{3d} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, s\right.}{d}$$
$$= \frac{b^2\tan^{-1}(\sinh(c+dx))}{d} + \frac{\left(a^2-b^2\right)\sinh(c+dx)}{d} + \frac{(a+b)^2\sinh^3(c+dx)}{3d}$$

Mathematica [A] time = 0.442775, size = 71, normalized size = 1.31

$$\frac{\sinh(c+dx)\left((a+b)((a+b)\cosh(2(c+dx))+5a-7b)+\frac{6b^2\tanh^{-1}\left(\sqrt{-\sinh^2(c+dx)}\right)}{\sqrt{-\sinh^2(c+dx)}}\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (Sinh[c + d*x]*((a + b)*(5*a - 7*b + (a + b)*Cosh[2*(c + d*x)]) + (6*b^2*Ar
cTanh[Sqrt[-Sinh[c + d*x]^2]])/Sqrt[-Sinh[c + d*x]^2]))/(6*d)
```

Maple [B] time = 0.043, size = 117, normalized size = 2.2

$$\frac{2a^2\sinh(dx+c)}{3d} + \frac{a^2\sinh(dx+c)(\cosh(dx+c))^2}{3d} + \frac{2ab(\cosh(dx+c))^2\sinh(dx+c)}{3d} - \frac{2ab\sinh(dx+c)}{3d} + \frac{b^2(\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] 2/3/d*a^2*sinh(d*x+c)+1/3/d*a^2*sinh(d*x+c)*cosh(d*x+c)^2+2/3/d*a*b*cosh(d*
x+c)^2*sinh(d*x+c)-2/3*a*b*sinh(d*x+c)/d+1/3/d*b^2*sinh(d*x+c)^3-1/d*b^2*si
nh(d*x+c)+2/d*b^2*arctan(exp(d*x+c))
```

Maxima [B] time = 1.67404, size = 217, normalized size = 4.02

$$\frac{ab\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3}{12d} - \frac{1}{24}b^2\left(\frac{\left(15e^{(-2dx-2c)}-1\right)e^{(3dx+3c)}}{d} - \frac{15e^{(-dx-c)}-e^{(-3dx-3c)}}{d} + \frac{48\arctan\left(e^{(-dx-c)}\right)}{d}\right) + \frac{1}{24}a^2\left(\frac{e^{(3d}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*a*b*(e^(d*x + c) - e^(-d*x - c))^3/d - 1/24*b^2*((15*e^(-2*d*x - 2*c)
- 1)*e^(3*d*x + 3*c)/d - (15*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + 48*arctan
```

```
(e^(-d*x - c))/d) + 1/24*a^2*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e^(-d
*x - c)/d - e^(-3*d*x - 3*c)/d)
```

Fricas [B] time = 1.92637, size = 1314, normalized size = 24.33

$$\frac{\left(a^2+2\,ab+b^2\right)\cosh\left(dx+c\right)^6+6\left(a^2+2\,ab+b^2\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^5+\left(a^2+2\,ab+b^2\right)\sinh\left(dx+c\right)^6+3\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x
+ c)*sinh(d*x + c)^5 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^6 + 3*(3*a^2 - 2*a
*b - 5*b^2)*cosh(d*x + c)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 3*
a^2 - 2*a*b - 5*b^2)*sinh(d*x + c)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(d*x +
c)^3 + 3*(3*a^2 - 2*a*b - 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 3*(3*a^2
- 2*a*b - 5*b^2)*cosh(d*x + c)^2 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4
 + 6*(3*a^2 - 2*a*b - 5*b^2)*cosh(d*x + c)^2 - 3*a^2 + 2*a*b + 5*b^2)*sinh(
d*x + c)^2 - a^2 - 2*a*b - b^2 + 48*(b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x +
 c)^2*sinh(d*x + c) + 3*b^2*cosh(d*x + c)*sinh(d*x + c)^2 + b^2*sinh(d*x +
c)^3)*arctan(cosh(d*x + c) + sinh(d*x + c)) + 6*((a^2 + 2*a*b + b^2)*cosh(d
*x + c)^5 + 2*(3*a^2 - 2*a*b - 5*b^2)*cosh(d*x + c)^3 - (3*a^2 - 2*a*b - 5*
b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)^2
*sinh(d*x + c) + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + d*sinh(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.79093, size = 221, normalized size = 4.09

$$\frac{48\,b^2\arctan\left(e^{(dx+c)}\right)-\left(9\,a^2e^{(2\,dx+2\,c)}-6\,abe^{(2\,dx+2\,c)}-15\,b^2e^{(2\,dx+2\,c)}+a^2+2\,ab+b^2\right)e^{(-3\,dx-3\,c)}+\left(a^2e^{(3\,dx+18\,c)}+2\,ab\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(48*b^2*arctan(e^(d*x + c)) - (9*a^2*e^(2*d*x + 2*c) - 6*a*b*e^(2*d*x
+ 2*c) - 15*b^2*e^(2*d*x + 2*c) + a^2 + 2*a*b + b^2)*e^(-3*d*x - 3*c) + (a^
2*e^(3*d*x + 18*c) + 2*a*b*e^(3*d*x + 18*c) + b^2*e^(3*d*x + 18*c) + 9*a^2*
e^(d*x + 16*c) - 6*a*b*e^(d*x + 16*c) - 15*b^2*e^(d*x + 16*c))*e^(-15*c))/d
```

3.91 $\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$

Optimal. Leaf size=51

$$\frac{(a+b)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{1}{2}x(a-3b)(a+b)+\frac{b^2\tanh(c+dx)}{d}$$

```
[Out] ((a - 3*b)*(a + b)*x)/2 + ((a + b)^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + (
b^2*Tanh[c + d*x])/d
```

Rubi [A] time = 0.0756621, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3675, 390, 385, 206}

$$\frac{(a+b)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{1}{2}x(a-3b)(a+b)+\frac{b^2\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((a - 3*b)*(a + b)*x)/2 + ((a + b)^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + (
b^2*Tanh[c + d*x])/d
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{(1-x^2)^2} dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(b^2+\frac{a^2-b^2+2b(a+b)x^2}{(1-x^2)^2}\right) dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{b^2\tanh(c+dx)}{d} + \frac{\text{Subst}\left(\int \frac{a^2-b^2+2b(a+b)x^2}{(1-x^2)^2} dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{(a+b)^2\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{b^2\tanh(c+dx)}{d} + \frac{((a-3b)(a+b}{}$$
$$= \frac{1}{2}(a-3b)(a+b)x + \frac{(a+b)^2\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{b^2\tanh(c+d}{d}$$

Mathematica [A] time = 0.370937, size = 54, normalized size = 1.06

$$\frac{(a-3b)(a+b)(c+dx)}{2d} + \frac{(a+b)^2\sinh(2(c+dx))}{4d} + \frac{b^2\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((a - 3*b)*(a + b)*(c + d*x))/(2*d) + ((a + b)^2*Sinh[2*(c + d*x)])/(4*d) +
 (b^2*Tanh[c + d*x])/d
```

Maple [B] time = 0.04, size = 96, normalized size = 1.9

$$\frac{1}{d}\left(a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2\,ab\,(1/2\,\cosh(dx+c)\sinh(dx+c)-1/2\,dx-c/2)+b^2\left(\frac{(\sinh(dx+}{2\,\cosh(dx}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*(1/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c)+2*a*b*(1/2*cosh(d*x+c)
*sinh(d*x+c)-1/2*d*x-1/2*c)+b^2*(1/2*sinh(d*x+c)^3/cosh(d*x+c)-3/2*d*x-3/2*
c+3/2*tanh(d*x+c)))
```

Maxima [B] time = 1.20452, size = 189, normalized size = 3.71

$$\frac{1}{8}a^2\left(4x+\frac{e^{(2dx+2c)}}{d}-\frac{e^{(-2dx-2c)}}{d}\right)-\frac{1}{4}ab\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)-\frac{1}{8}b^2\left(\frac{12(dx+c)}{d}+\frac{e^{(-2dx-2c)}}{d}-\frac{17e^{(-2d}}{d\left(e^{(-2dx-2c)}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/8*a^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) - 1/4*a*b*(4*x - e^(
2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/8*b^2*(12*(d*x + c)/d + e^(-2*d*x
```

```
- 2*c)/d - (17*e^(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c
))))
```

Fricas [B] time = 2.00785, size = 261, normalized size = 5.12

$$\frac{\left(a^2+2\,ab+b^2\right)\sinh\left(dx+c\right)^3+4\left(\left(a^2-2\,ab-3\,b^2\right)dx-2\,b^2\right)\cosh\left(dx+c\right)+\left(3\left(a^2+2\,ab+b^2\right)\cosh\left(dx+c\right)^2+a^2+\right.}{8\,d\cosh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((a^2 + 2*a*b + b^2)*sinh(d*x + c)^3 + 4*((a^2 - 2*a*b - 3*b^2)*d*x - 2
*b^2)*cosh(d*x + c) + (3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b
+ 9*b^2)*sinh(d*x + c))/(d*cosh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tanh^2\left(c+dx\right)\right)^2\cosh^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*cosh(c + d*x)**2, x)
```

Giac [B] time = 1.68859, size = 230, normalized size = 4.51

$$\frac{4\left(a^2-2\,ab-3\,b^2\right)dx+\left(a^2e^{(2\,dx+8\,c)}+2\,abe^{(2\,dx+8\,c)}+b^2e^{(2\,dx+8\,c)}\right)e^{(-6\,c)}-\frac{\left(a^2e^{(4\,dx+4\,c)}-2\,abe^{(4\,dx+4\,c)}-3\,b^2e^{(4\,dx+4\,c)}+2\,a^2e^{(2\,dx+2\,c)}+14\,b^2\right.}{e^{(2\,dx)}+e^{(4\,dx+2\,c)}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*(4*(a^2 - 2*a*b - 3*b^2)*d*x + (a^2*e^(2*d*x + 8*c) + 2*a*b*e^(2*d*x +
8*c) + b^2*e^(2*d*x + 8*c))*e^(-6*c) - (a^2*e^(4*d*x + 4*c) - 2*a*b*e^(4*d*
x + 4*c) - 3*b^2*e^(4*d*x + 4*c) + 2*a^2*e^(2*d*x + 2*c) + 14*b^2*e^(2*d*x
+ 2*c) + a^2 + 2*a*b + b^2)*e^(-2*c)/(e^(2*d*x) + e^(4*d*x + 2*c)))/d
```

3.92 $\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$

Optimal. Leaf size=60

$$\frac{(a+b)^2\sinh(c+dx)}{d}-\frac{b(4a+3b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{b^2\tanh(c+dx)\text{sech}(c+dx)}{2d}$$

```
[Out] -(b*(4*a + 3*b)*ArcTan[Sinh[c + d*x]])/(2*d) + ((a + b)^2*Sinh[c + d*x])/d
+ (b^2*Sech[c + d*x]*Tanh[c + d*x])/(2*d)
```

Rubi [A] time = 0.0824405, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3676, 390, 385, 203}

$$\frac{(a+b)^2\sinh(c+dx)}{d}-\frac{b(4a+3b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{b^2\tanh(c+dx)\text{sech}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -(b*(4*a + 3*b)*ArcTan[Sinh[c + d*x]])/(2*d) + ((a + b)^2*Sinh[c + d*x])/d
+ (b^2*Sech[c + d*x]*Tanh[c + d*x])/(2*d)
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+(a+b)x^2\right)^2}{\left(1+x^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left((a+b)^2 - \frac{b(2a+b)+2b(a+b)x^2}{\left(1+x^2\right)^2}\right)dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{(a+b)^2\sinh(c+dx)}{d} - \frac{\text{Subst}\left(\int \frac{b(2a+b)+2b(a+b)x^2}{\left(1+x^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{(a+b)^2\sinh(c+dx)}{d} + \frac{b^2\textbf{sech}(c+dx)\tanh(c+dx)}{2d} - \frac{(b(4a+3b))\,\text{Subst}}{}$$
$$= -\frac{b(4a+3b)\tan^{-1}(\sinh(c+dx))}{2d} + \frac{(a+b)^2\sinh(c+dx)}{d} + \frac{b^2\textbf{sech}(c+dx)}{2d}$$

Mathematica [A] time = 0.178763, size = 54, normalized size = 0.9

$$\frac{2(a+b)^2\sinh(c+dx) + b\left(b\tanh(c+dx)\textbf{sech}(c+dx) - (4a+3b)\tan^{-1}(\sinh(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (2*(a + b)^2*Sinh[c + d*x] + b*(-((4*a + 3*b)*ArcTan[Sinh[c + d*x]]) + b*Se
ch[c + d*x]*Tanh[c + d*x]))/(2*d)
```

Maple [B] time = 0.043, size = 122, normalized size = 2.

$$\frac{a^2\sinh(dx+c)}{d} + 2\frac{ab\sinh(dx+c)}{d} - 4\frac{ab\arctan\left(e^{dx+c}\right)}{d} + \frac{b^2(\sinh(dx+c))^3}{d(\cosh(dx+c))^2} + 3\frac{b^2\sinh(dx+c)}{d(\cosh(dx+c))^2} - \frac{3b^2\text{sech}(dx+}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*a^2*sinh(d*x+c)+2*a*b*sinh(d*x+c)/d-4/d*a*b*arctan(exp(d*x+c))+1/d*b^2*
sinh(d*x+c)^3/cosh(d*x+c)^2+3/d*b^2*sinh(d*x+c)/cosh(d*x+c)^2-3/2/d*b^2*sec
h(d*x+c)*tanh(d*x+c)-3/d*b^2*arctan(exp(d*x+c))
```

Maxima [B] time = 1.74926, size = 205, normalized size = 3.42

$$\frac{1}{2}b^2\left(\frac{6\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)}}{d} + \frac{4e^{(-2dx-2c)} - e^{(-4dx-4c)} + 1}{d\left(e^{(-dx-c)} + 2e^{(-3dx-3c)} + e^{(-5dx-5c)}\right)}\right) + ab\left(\frac{4\arctan\left(e^{(-dx-c)}\right)}{d} + \frac{e^{(dx+c)}}{d} - \frac{e^{(-dx-c)}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*(6*arctan(e^(-d*x - c))/d - e^(-d*x - c)/d + (4*e^(-2*d*x - 2*c) -
e^(-4*d*x - 4*c) + 1)/(d*(e^(-d*x - c) + 2*e^(-3*d*x - 3*c) + e^(-5*d*x - 5
*c)))) + a*b*(4*arctan(e^(-d*x - c))/d + e^(d*x + c)/d - e^(-d*x - c)/d) +
a^2*sinh(d*x + c)/d
```

Fricas [B] time = 1.99283, size = 1967, normalized size = 32.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x +
 c)*sinh(d*x + c)^5 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^6 + (a^2 + 2*a*b +
3*b^2)*cosh(d*x + c)^4 + (15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*
a*b + 3*b^2)*sinh(d*x + c)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (
a^2 + 2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - (a^2 + 2*a*b + 3*b^2)
*cosh(d*x + c)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 6*(a^2 + 2*a*b
 + 3*b^2)*cosh(d*x + c)^2 - a^2 - 2*a*b - 3*b^2)*sinh(d*x + c)^2 - a^2 - 2*
a*b - b^2 - 2*((4*a*b + 3*b^2)*cosh(d*x + c)^5 + 5*(4*a*b + 3*b^2)*cosh(d*x
 + c)*sinh(d*x + c)^4 + (4*a*b + 3*b^2)*sinh(d*x + c)^5 + 2*(4*a*b + 3*b^2)
*cosh(d*x + c)^3 + 2*(5*(4*a*b + 3*b^2)*cosh(d*x + c)^2 + 4*a*b + 3*b^2)*si
nh(d*x + c)^3 + 2*(5*(4*a*b + 3*b^2)*cosh(d*x + c)^3 + 3*(4*a*b + 3*b^2)*co
sh(d*x + c))*sinh(d*x + c)^2 + (4*a*b + 3*b^2)*cosh(d*x + c) + (5*(4*a*b +
3*b^2)*cosh(d*x + c)^4 + 6*(4*a*b + 3*b^2)*cosh(d*x + c)^2 + 4*a*b + 3*b^2)
*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) + 2*(3*(a^2 + 2*a*b +
 b^2)*cosh(d*x + c)^5 + 2*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^3 - (a^2 + 2*
a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*
x + c)*sinh(d*x + c)^4 + d*sinh(d*x + c)^5 + 2*d*cosh(d*x + c)^3 + 2*(5*d*c
osh(d*x + c)^2 + d)*sinh(d*x + c)^3 + 2*(5*d*cosh(d*x + c)^3 + 3*d*cosh(d*x
 + c))*sinh(d*x + c)^2 + d*cosh(d*x + c) + (5*d*cosh(d*x + c)^4 + 6*d*cosh(
d*x + c)^2 + d)*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tanh^{2}\left(c + dx\right)\right)^{2}\cosh\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*cosh(c + d*x), x)
```

Giac [B] time = 1.66563, size = 185, normalized size = 3.08

$$-\frac{2\left(4\,abe^{c} + 3\,b^{2}e^{c}\right)\arctan\left(e^{(dx+c)}\right)e^{(-c)} + \left(a^{2} + 2\,ab + b^{2}\right)e^{(-dx-c)} - \left(a^{2}e^{(dx+8\,c)} + 2\,abe^{(dx+8\,c)} + b^{2}e^{(dx+8\,c)}\right)e^{(-7\,c)} - \frac{2(b}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(4*a*b*e^c + 3*b^2*e^c)*arctan(e^(d*x + c))*e^(-c) + (a^2 + 2*a*b +
 b^2)*e^(-d*x - c) - (a^2*e^(d*x + 8*c) + 2*a*b*e^(d*x + 8*c) + b^2*e^(d*x
+ 8*c))*e^(-7*c) - 2*(b^2*e^(3*d*x + 3*c) - b^2*e^(d*x + c))/(e^(2*d*x + 2*
c) + 1)^2)/d
```

3.93 $\int \mathbf{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=91

$$\frac{\left(8a^2+8ab+3b^2\right)\tan^{-1}(\sinh(c+dx))}{8d}-\frac{3b(2a+b)\tanh(c+dx)\text{sech}(c+dx)}{8d}-\frac{b\tanh(c+dx)\text{sech}^3(c+dx)\left((a+b\right.}{4d}$$

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTan[Sinh[c + d*x]])/(8*d) - (3*b*(2*a + b)*Sech
[c + d*x]*Tanh[c + d*x])/(8*d) - (b*Sech[c + d*x]^3*(a + (a + b)*Sinh[c + d
*x]^2)*Tanh[c + d*x])/(4*d)
```

Rubi [A] time = 0.085125, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3676, 413, 385, 203}

$$\frac{\left(8a^2+8ab+3b^2\right)\tan^{-1}(\sinh(c+dx))}{8d}-\frac{3b(2a+b)\tanh(c+dx)\text{sech}(c+dx)}{8d}-\frac{b\tanh(c+dx)\text{sech}^3(c+dx)\left((a+b\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTan[Sinh[c + d*x]])/(8*d) - (3*b*(2*a + b)*Sech
[c + d*x]*Tanh[c + d*x])/(8*d) - (b*Sech[c + d*x]^3*(a + (a + b)*Sinh[c + d
*x]^2)*Tanh[c + d*x])/(4*d)
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+(a+b)x^2\right)^2}{\left(1+x^2\right)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{b\operatorname{sech}^3(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)\tanh(c+dx)}{4d} + \frac{\operatorname{Subst}\left(\int \frac{a(4a+}{}\right.}{}$$

$$= -\frac{3b(2a+b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8d} - \frac{b\operatorname{sech}^3(c+dx)\left(a+(a+b)\sinh^2(\right.}{4d}$$

$$= \frac{\left(8a^2+8ab+3b^2\right)\tan^{-1}(\sinh(c+dx))}{8d} - \frac{3b(2a+b)\operatorname{sech}(c+dx)\tanh(c+}{8d}$$

Mathematica [C] time = 7.22038, size = 427, normalized size = 4.69

$$-\frac{\operatorname{csch}^3(c+dx)\left(128\sinh^6(c+dx)\left(a^2\left(5\sinh^4(c+dx)+12\sinh^2(c+dx)+7\right)+2ab\left(5\sinh^2(c+dx)+6\right)\sinh^2(c+dx\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -(Csch[c + d*x]^3*(128*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 9/2},
 -Sinh[c + d*x]^2]*Sinh[c + d*x]^6*(a + a*Sinh[c + d*x]^2 + b*Sinh[c + d*x]
^2)^2 + 128*HypergeometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, -Sinh[c + d*x]^2
]*Sinh[c + d*x]^6*(5*b^2*Sinh[c + d*x]^4 + 2*a*b*Sinh[c + d*x]^2*(6 + 5*Sin
h[c + d*x]^2) + a^2*(7 + 12*Sinh[c + d*x]^2 + 5*Sinh[c + d*x]^4)) + 35*(b^2
*Sinh[c + d*x]^4*(1947 + 485*Sinh[c + d*x]^2) + 2*a*b*Sinh[c + d*x]^2*(2625
 + 2554*Sinh[c + d*x]^2 + 485*Sinh[c + d*x]^4) + a^2*(3375 + 5907*Sinh[c +
d*x]^2 + 3161*Sinh[c + d*x]^4 + 485*Sinh[c + d*x]^6)) - (105*ArcTanh[Sqrt[-
Sinh[c + d*x]^2]]*(b^2*Sinh[c + d*x]^4*(649 + 378*Sinh[c + d*x]^2 + 9*Sinh[
c + d*x]^4) + 2*a*b*Sinh[c + d*x]^2*(875 + 1143*Sinh[c + d*x]^2 + 389*Sinh[
c + d*x]^4 + 9*Sinh[c + d*x]^6) + a^2*(1125 + 2344*Sinh[c + d*x]^2 + 1674*S
inh[c + d*x]^4 + 400*Sinh[c + d*x]^6 + 9*Sinh[c + d*x]^8)))/Sqrt[-Sinh[c +
d*x]^2]))/(6720*d)
```

Maple [B] time = 0.036, size = 173, normalized size = 1.9

$$2\,\frac{a^2\arctan\left(e^{dx+c}\right)}{d} - 2\,\frac{ab\sinh(dx+c)}{d\left(\cosh(dx+c)\right)^2} + \frac{ab\operatorname{sech}(dx+c)\tanh(dx+c)}{d} + 2\,\frac{ab\arctan\left(e^{dx+c}\right)}{d} - \frac{b^2\left(\sinh(dx+c)\right)^3}{d\left(\cosh(dx+c)\right)^4} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] 2/d*a^2*arctan(exp(d*x+c))-2/d*a*b*sinh(d*x+c)/cosh(d*x+c)^2+1/d*a*b*sech(d
*x+c)*tanh(d*x+c)+2/d*a*b*arctan(exp(d*x+c))-1/d*b^2*sinh(d*x+c)^3/cosh(d*x
+c)^4-1/d*b^2*sinh(d*x+c)/cosh(d*x+c)^4+1/4/d*b^2*tanh(d*x+c)*sech(d*x+c)^3
```

+3/8/d*b^2*sech(d*x+c)*tanh(d*x+c)+3/4/d*b^2*arctan(exp(d*x+c))

Maxima [B] time = 1.61505, size = 269, normalized size = 2.96

$$-\frac{1}{4}\,b^2\left(\frac{3\,\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{5\,e^{(-dx-c)}-3\,e^{(-3\,dx-3\,c)}+3\,e^{(-5\,dx-5\,c)}-5\,e^{(-7\,dx-7\,c)}}{d\left(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1\right)}\right)-2\,ab\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}+\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*b^2*(3*arctan(e^(-d*x - c))/d + (5*e^(-d*x - c) - 3*e^(-3*d*x - 3*c) +
 3*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*
d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - 2*a*b*(arctan(e
^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) +
 e^(-4*d*x - 4*c) + 1))) + a^2*arctan(sinh(d*x + c))/d
```

Fricas [B] time = 2.19051, size = 3433, normalized size = 37.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/4*((8*a*b + 5*b^2)*cosh(d*x + c)^7 + 7*(8*a*b + 5*b^2)*cosh(d*x + c)*sin
h(d*x + c)^6 + (8*a*b + 5*b^2)*sinh(d*x + c)^7 + (8*a*b - 3*b^2)*cosh(d*x +
 c)^5 + (21*(8*a*b + 5*b^2)*cosh(d*x + c)^2 + 8*a*b - 3*b^2)*sinh(d*x + c)^
5 + 5*(7*(8*a*b + 5*b^2)*cosh(d*x + c)^3 + (8*a*b - 3*b^2)*cosh(d*x + c))*s
inh(d*x + c)^4 - (8*a*b - 3*b^2)*cosh(d*x + c)^3 + (35*(8*a*b + 5*b^2)*cosh
(d*x + c)^4 + 10*(8*a*b - 3*b^2)*cosh(d*x + c)^2 - 8*a*b + 3*b^2)*sinh(d*x
+ c)^3 + (21*(8*a*b + 5*b^2)*cosh(d*x + c)^5 + 10*(8*a*b - 3*b^2)*cosh(d*x
+ c)^3 - 3*(8*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - ((8*a^2 + 8*a*b
 + 3*b^2)*cosh(d*x + c)^8 + 8*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)*sinh(d*
x + c)^7 + (8*a^2 + 8*a*b + 3*b^2)*sinh(d*x + c)^8 + 4*(8*a^2 + 8*a*b + 3*b
^2)*cosh(d*x + c)^6 + 4*(7*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 8*a^2
+ 8*a*b + 3*b^2)*sinh(d*x + c)^6 + 8*(7*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x +
c)^3 + 3*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(8*a^2
+ 8*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(35*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x +
 c)^4 + 30*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 24*a^2 + 24*a*b + 9*b^
2)*sinh(d*x + c)^4 + 8*(7*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^5 + 10*(8*a
^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^3 + 3*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x +
c))*sinh(d*x + c)^3 + 4*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 4*(7*(8*a
^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^6 + 15*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x +
 c)^4 + 9*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 8*a^2 + 8*a*b + 3*b^2)*
sinh(d*x + c)^2 + 8*a^2 + 8*a*b + 3*b^2 + 8*((8*a^2 + 8*a*b + 3*b^2)*cosh(d
*x + c)^7 + 3*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^5 + 3*(8*a^2 + 8*a*b +
3*b^2)*cosh(d*x + c)^3 + (8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x +
c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - (8*a*b + 5*b^2)*cosh(d*x + c) +
 (7*(8*a*b + 5*b^2)*cosh(d*x + c)^6 + 5*(8*a*b - 3*b^2)*cosh(d*x + c)^4 - 3
*(8*a*b - 3*b^2)*cosh(d*x + c)^2 - 8*a*b - 5*b^2)*sinh(d*x + c))/(d*cosh(d*
x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 + 4*d*cosh
(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x
 + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*
```

```
d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*co
sh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 +
 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 15*d*cosh(d*x + c)^4 + 9*d*
cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 + 3*d*cosh(d*x
+ c)^5 + 3*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tanh^2\left(c + dx\right)\right)^2 \operatorname{sech}\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*sech(c + d*x), x)
```

Giac [A] time = 1.39557, size = 212, normalized size = 2.33

$$\frac{\left(8\,a^2e^c + 8\,abe^c + 3\,b^2e^c\right)\arctan\left(e^{(dx+c)}\right)e^{(-c)} - \frac{8\,abe^{(7\,dx+7\,c)} + 5\,b^2e^{(7\,dx+7\,c)} + 8\,abe^{(5\,dx+5\,c)} - 3\,b^2e^{(5\,dx+5\,c)} - 8\,abe^{(3\,dx+3\,c)} + 3\,b^2e^{(3\,dx+3\,c)} - 8\,abe^{(dx+}}{\left(e^{(2\,dx+2\,c)}+1\right)^4}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/4*((8*a^2*e^c + 8*a*b*e^c + 3*b^2*e^c)*arctan(e^(d*x + c))*e^(-c) - (8*a*
b*e^(7*d*x + 7*c) + 5*b^2*e^(7*d*x + 7*c) + 8*a*b*e^(5*d*x + 5*c) - 3*b^2*e
^(5*d*x + 5*c) - 8*a*b*e^(3*d*x + 3*c) + 3*b^2*e^(3*d*x + 3*c) - 8*a*b*e^(d
*x + c) - 5*b^2*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^4)/d
```

3.94 $\int \mathbf{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=49

$$\frac{a^2\tanh(c+dx)}{d}+\frac{2ab\tanh^3(c+dx)}{3d}+\frac{b^2\tanh^5(c+dx)}{5d}$$

```
[Out] (a^2*Tanh[c + d*x])/d + (2*a*b*Tanh[c + d*x]^3)/(3*d) + (b^2*Tanh[c + d*x]^
5)/(5*d)
```

Rubi [A] time = 0.0530052, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3675, 194}

$$\frac{a^2\tanh(c+dx)}{d}+\frac{2ab\tanh^3(c+dx)}{3d}+\frac{b^2\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Tanh[c + d*x])/d + (2*a*b*Tanh[c + d*x]^3)/(3*d) + (b^2*Tanh[c + d*x]^
5)/(5*d)
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx&=\frac{\operatorname{Subst}\left(\int\left(a+bx^2\right)^2\,dx,x,\tanh(c+dx)\right)}{d}\\&=\frac{\operatorname{Subst}\left(\int\left(a^2+2abx^2+b^2x^4\right)\,dx,x,\tanh(c+dx)\right)}{d}\\&=\frac{a^2\tanh(c+dx)}{d}+\frac{2ab\tanh^3(c+dx)}{3d}+\frac{b^2\tanh^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.179594, size = 49, normalized size = 1.

$$\frac{a^2\tanh(c+dx)}{d}+\frac{2ab\tanh^3(c+dx)}{3d}+\frac{b^2\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Tanh[c + d*x])/d + (2*a*b*Tanh[c + d*x]^3)/(3*d) + (b^2*Tanh[c + d*x]^
5)/(5*d)
```

Maple [B] time = 0.052, size = 126, normalized size = 2.6

$$\frac{1}{d}\left(a^2\tanh(dx+c)+2\,ab\left(-1/2\,\frac{\sinh(dx+c)}{(\cosh(dx+c))^3}+1/2\,\left(2/3+1/3\,(\operatorname{sech}(dx+c))^2\right)\tanh(dx+c)\right)+b^2\left(-\frac{(\sinh(dx+c}{2\,(\cosh(dx+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*tanh(d*x+c)+2*a*b*(-1/2*sinh(d*x+c)/cosh(d*x+c)^3+1/2*(2/3+1/3*sec
h(d*x+c)^2)*tanh(d*x+c))+b^2*(-1/2*sinh(d*x+c)^3/cosh(d*x+c)^5-3/8*sinh(d*x
+c)/cosh(d*x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+
c)))
```

Maxima [A] time = 1.16467, size = 72, normalized size = 1.47

$$\frac{b^2\tanh(dx+c)^5}{5\,d}+\frac{2\,ab\tanh(dx+c)^3}{3\,d}+\frac{2\,a^2}{d\left(e^{(-2\,dx-2\,c)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*tanh(d*x + c)^5/d + 2/3*a*b*tanh(d*x + c)^3/d + 2*a^2/(d*(e^(-2*d*x
 - 2*c) + 1))
```

Fricas [B] time = 1.86226, size = 1023, normalized size = 20.88

$$-\frac{4\left(\left(15\,a^2+20\,ab+9\,b^2\right)\cosh(dx+c)^4+8\left(5\,ab+3\,b^2\right)\cosh(dx+c)\sinh(dx+c)^3+\left(15\,a^2+2\right.\right.}{15\left(d\cosh(dx+c)^6+6\,d\cosh(dx+c)\sinh(dx+c)^5+d\sinh(dx+c)^6+6\,d\cosh(dx+c)^4+3\left(5\,d\cosh(dx+c)^2+2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -4/15*((15*a^2 + 20*a*b + 9*b^2)*cosh(d*x + c)^4 + 8*(5*a*b + 3*b^2)*cosh(d
*x + c)*sinh(d*x + c)^3 + (15*a^2 + 20*a*b + 9*b^2)*sinh(d*x + c)^4 + 20*(3
*a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*(15*a^2 + 20*a*b + 9*b^2)*cosh(d*x + c
)^2 + 30*a^2 + 20*a*b)*sinh(d*x + c)^2 + 45*a^2 + 20*a*b + 15*b^2 + 8*((5*a
*b + 3*b^2)*cosh(d*x + c)^3 + 5*a*b*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d
*x + c)^6 + 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c)^6 + 6*d*cos
h(d*x + c)^4 + 3*(5*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^4 + 4*(5*d*cosh(
d*x + c)^3 + 4*d*cosh(d*x + c))*sinh(d*x + c)^3 + 15*d*cosh(d*x + c)^2 + 3*
(5*d*cosh(d*x + c)^4 + 12*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^2 + 2*(3*d
*cosh(d*x + c)^5 + 8*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c) +
```

```
10*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\tanh^2\left(c+dx\right)\right)^2\operatorname{sech}^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*sech(c + d*x)**2, x)
```

Giac [B] time = 1.41103, size = 228, normalized size = 4.65

$$-\frac{2\left(15\,a^2e^{(8\,dx+8\,c)}+30\,abe^{(8\,dx+8\,c)}+15\,b^2e^{(8\,dx+8\,c)}+60\,a^2e^{(6\,dx+6\,c)}+60\,abe^{(6\,dx+6\,c)}+90\,a^2e^{(4\,dx+4\,c)}+40\,abe^{(4\,dx+4\,c)}\right.}{15\,d\left(e^{(2\,dx+2\,c)}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -2/15*(15*a^2*e^(8*d*x + 8*c) + 30*a*b*e^(8*d*x + 8*c) + 15*b^2*e^(8*d*x +
8*c) + 60*a^2*e^(6*d*x + 6*c) + 60*a*b*e^(6*d*x + 6*c) + 90*a^2*e^(4*d*x +
4*c) + 40*a*b*e^(4*d*x + 4*c) + 30*b^2*e^(4*d*x + 4*c) + 60*a^2*e^(2*d*x +
2*c) + 20*a*b*e^(2*d*x + 2*c) + 15*a^2 + 10*a*b + 3*b^2)/(d*(e^(2*d*x + 2*c
) + 1)^5)
```

3.95 $\int \mathbf{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=125

$$\frac{(8a^2+4ab+b^2)\tan^{-1}(\sinh(c+dx))}{16d}+\frac{(8a^2+4ab+b^2)\tanh(c+dx)\mathrm{sech}(c+dx)}{16d}-\frac{b(8a+3b)\tanh(c+dx)\mathrm{sech}^3(c+}{24d}$$

```
[Out] ((8*a^2 + 4*a*b + b^2)*ArcTan[Sinh[c + d*x]])/(16*d) + ((8*a^2 + 4*a*b + b^
2)*Sech[c + d*x]*Tanh[c + d*x])/(16*d) - (b*(8*a + 3*b)*Sech[c + d*x]^3*Tan
h[c + d*x])/(24*d) - (b*Sech[c + d*x]^5*(a + (a + b)*Sinh[c + d*x]^2)*Tanh[
c + d*x])/(6*d)
```

Rubi [A] time = 0.151915, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3676, 413, 385, 199, 203}

$$\frac{(8a^2+4ab+b^2)\tan^{-1}(\sinh(c+dx))}{16d}+\frac{(8a^2+4ab+b^2)\tanh(c+dx)\mathrm{sech}(c+dx)}{16d}-\frac{b(8a+3b)\tanh(c+dx)\mathrm{sech}^3(c+}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((8*a^2 + 4*a*b + b^2)*ArcTan[Sinh[c + d*x]])/(16*d) + ((8*a^2 + 4*a*b + b^
2)*Sech[c + d*x]*Tanh[c + d*x])/(16*d) - (b*(8*a + 3*b)*Sech[c + d*x]^3*Tan
h[c + d*x])/(24*d) - (b*Sech[c + d*x]^5*(a + (a + b)*Sinh[c + d*x]^2)*Tanh[
c + d*x])/(6*d)
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
```

```
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+(a+b)x^2\right)^2}{\left(1+x^2\right)^4} dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{b\operatorname{sech}^5(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)\tanh(c+dx)}{6d} + \frac{\operatorname{Subst}\left(\int\right.}{}$$

$$= -\frac{b(8a+3b)\operatorname{sech}^3(c+dx)\tanh(c+dx)}{24d} - \frac{b\operatorname{sech}^5(c+dx)\left(a+(a+b)\operatorname{s}\right.}{6d}$$

$$= \frac{\left(8a^2+4ab+b^2\right)\operatorname{sech}(c+dx)\tanh(c+dx)}{16d} - \frac{b(8a+3b)\operatorname{sech}^3(c+dx)}{24d}$$

$$= \frac{\left(8a^2+4ab+b^2\right)\tan^{-1}(\sinh(c+dx))}{16d} + \frac{\left(8a^2+4ab+b^2\right)\operatorname{sech}(c+dx)}{16d}$$

Mathematica [C] time = 8.64102, size = 792, normalized size = 6.34

$$a^2\sinh(c+dx)\left(-\frac{380(a+b)^2\sinh^6(c+dx)\text{HypergeometricPFQ}\left(\left\{\frac{3}{2},2,2,2\right\},\left\{1,1,\frac{9}{2}\right\},-\sinh^2(c+dx)\right)}{a^2} - \frac{128(a+b)^2\sinh^6(c+dx)\text{HypergeometricPFQ}\left(\right.}{a^2}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Sinh[c + d*x]*((-23555*(a + b))/a - (32970*(a + b)^2)/a^2 - 14980*Csch
[c + d*x]^2 - (91875*(a + b)*Csch[c + d*x]^2)/a - 65625*Csch[c + d*x]^4 - (
8855*(a + b)^2*Sinh[c + d*x]^2)/a^2 - 620*HypergeometricPFQ[{3/2, 2, 2, 2},
 {1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^2 - 160*HypergeometricPFQ[{3/
2, 2, 2, 2, 2}, {1, 1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^2 - 16*Hype
rgeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 9/2}, -Sinh[c + d*x]^2]*Si
nh[c + d*x]^2 - (968*(a + b)*HypergeometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2},
 -Sinh[c + d*x]^2]*Sinh[c + d*x]^4)/a - (288*(a + b)*HypergeometricPFQ[{3/2
, 2, 2, 2, 2}, {1, 1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^4)/a - (32*(
a + b)*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 9/2}, -Sinh[c +
 d*x]^2]*Sinh[c + d*x]^4)/a - (380*(a + b)^2*HypergeometricPFQ[{3/2, 2, 2,
2}, {1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6)/a^2 - (128*(a + b)^2*Hy
pergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c
 + d*x]^6)/a^2 - (16*(a + b)^2*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1,
1, 1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6)/a^2 + (65625*ArcTanh[Sqrt
[-Sinh[c + d*x]^2]])/(-Sinh[c + d*x]^2)^(5/2) + (1680*ArcTanh[Sqrt[-Sinh[c
+ d*x]^2]]*Sinh[c + d*x]^4)/(-Sinh[c + d*x]^2)^(5/2) - (36855*ArcTanh[Sqrt[
```

```
-Sinh[c + d*x]^2]])/(-Sinh[c + d*x]^2)^(3/2) - (91875*(a + b)*ArcTanh[Sqrt[
-Sinh[c + d*x]^2]])/(a*(-Sinh[c + d*x]^2)^(3/2)) + (54180*(a + b)*ArcTanh[S
qrt[-Sinh[c + d*x]^2]])/(a*Sqrt[-Sinh[c + d*x]^2]) + (32970*(a + b)^2*ArcTa
nh[Sqrt[-Sinh[c + d*x]^2]])/(a^2*Sqrt[-Sinh[c + d*x]^2]) + (525*(a + b)^2*A
rcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^4)/(a^2*Sqrt[-Sinh[c + d*x]^2]
) - (1365*(a + b)*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sqrt[-Sinh[c + d*x]^2])/a
 - (19845*(a + b)^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sqrt[-Sinh[c + d*x]^2])
/a^2))/(2520*d)
```

Maple [B] time = 0.056, size = 236, normalized size = 1.9

$$\frac{a^{2}\operatorname{sech}\left(dx+c\right)\tanh\left(dx+c\right)}{2d}+\frac{a^{2}\arctan\left(e^{dx+c}\right)}{d}-\frac{2ab\sinh\left(dx+c\right)}{3d\left(\cosh\left(dx+c\right)\right)^{4}}+\frac{ab\tanh\left(dx+c\right)\left(\operatorname{sech}\left(dx+c\right)\right)^{3}}{6d}+\frac{ab\operatorname{sech}(\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] 1/2/d*a^2*sech(d*x+c)*tanh(d*x+c)+1/d*a^2*arctan(exp(d*x+c))-2/3/d*a*b*sinh
(d*x+c)/cosh(d*x+c)^4+1/6/d*a*b*tanh(d*x+c)*sech(d*x+c)^3+1/4/d*a*b*sech(d*
x+c)*tanh(d*x+c)+1/2/d*a*b*arctan(exp(d*x+c))-1/3/d*b^2*sinh(d*x+c)^3/cosh(
d*x+c)^6-1/5/d*b^2*sinh(d*x+c)/cosh(d*x+c)^6+1/30/d*b^2*tanh(d*x+c)*sech(d*
x+c)^5+1/24/d*b^2*tanh(d*x+c)*sech(d*x+c)^3+1/16/d*b^2*sech(d*x+c)*tanh(d*x
+c)+1/8/d*b^2*arctan(exp(d*x+c))
```

Maxima [B] time = 1.71977, size = 466, normalized size = 3.73

$$-\frac{1}{24}b^{2}\left(\frac{3\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{3e^{(-dx-c)}-47e^{(-3dx-3c)}+78e^{(-5dx-5c)}-78e^{(-7dx-7c)}+47e^{(-9dx-9c)}-3e^{(-11dx-11c)}}{d\left(6e^{(-2dx-2c)}+15e^{(-4dx-4c)}+20e^{(-6dx-6c)}+15e^{(-8dx-8c)}+6e^{(-10dx-10c)}+e^{(-12dx-12c)}+\ldots\right.}\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/24*b^2*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) - 47*e^(-3*d*x - 3*c)
 + 78*e^(-5*d*x - 5*c) - 78*e^(-7*d*x - 7*c) + 47*e^(-9*d*x - 9*c) - 3*e^(-
11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x
 - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c) +
 1))) - 1/2*a*b*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - 7*e^(-3*d*x - 3*c
) + 7*e^(-5*d*x - 5*c) - e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4
*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - a^2*(arctan(e^
(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) +
e^(-4*d*x - 4*c) + 1)))
```

Fricas [B] time = 2.29356, size = 7337, normalized size = 58.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^11 + 33*(8*a^2 + 4*a*b + b^2)*c
osh(d*x + c)*sinh(d*x + c)^10 + 3*(8*a^2 + 4*a*b + b^2)*sinh(d*x + c)^11 +
(72*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c)^9 + (165*(8*a^2 + 4*a*b + b^2)*cos
h(d*x + c)^2 + 72*a^2 - 60*a*b - 47*b^2)*sinh(d*x + c)^9 + 9*(55*(8*a^2 + 4
*a*b + b^2)*cosh(d*x + c)^3 + (72*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c))*sin
h(d*x + c)^8 + 6*(8*a^2 - 12*a*b + 13*b^2)*cosh(d*x + c)^7 + 6*(165*(8*a^2
+ 4*a*b + b^2)*cosh(d*x + c)^4 + 6*(72*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c)
^2 + 8*a^2 - 12*a*b + 13*b^2)*sinh(d*x + c)^7 + 42*(33*(8*a^2 + 4*a*b + b^2
)*cosh(d*x + c)^5 + 2*(72*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c)^3 + (8*a^2 -
 12*a*b + 13*b^2)*cosh(d*x + c))*sinh(d*x + c)^6 - 6*(8*a^2 - 12*a*b + 13*b
^2)*cosh(d*x + c)^5 + 6*(231*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^6 + 21*(72
*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c)^4 + 21*(8*a^2 - 12*a*b + 13*b^2)*cosh
(d*x + c)^2 - 8*a^2 + 12*a*b - 13*b^2)*sinh(d*x + c)^5 + 6*(165*(8*a^2 + 4*
a*b + b^2)*cosh(d*x + c)^7 + 21*(72*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c)^5
+ 35*(8*a^2 - 12*a*b + 13*b^2)*cosh(d*x + c)^3 - 5*(8*a^2 - 12*a*b + 13*b^2
)*cosh(d*x + c))*sinh(d*x + c)^4 - (72*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c)
^3 + (495*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^8 + 84*(72*a^2 - 60*a*b - 47*
b^2)*cosh(d*x + c)^6 + 210*(8*a^2 - 12*a*b + 13*b^2)*cosh(d*x + c)^4 - 60*(
8*a^2 - 12*a*b + 13*b^2)*cosh(d*x + c)^2 - 72*a^2 + 60*a*b + 47*b^2)*sinh(d
*x + c)^3 + 3*(55*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^9 + 12*(72*a^2 - 60*a
*b - 47*b^2)*cosh(d*x + c)^7 + 42*(8*a^2 - 12*a*b + 13*b^2)*cosh(d*x + c)^5
 - 20*(8*a^2 - 12*a*b + 13*b^2)*cosh(d*x + c)^3 - (72*a^2 - 60*a*b - 47*b^2
)*cosh(d*x + c))*sinh(d*x + c)^2 + 3*((8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^1
2 + 12*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^11 + (8*a^2 + 4*a*
b + b^2)*sinh(d*x + c)^12 + 6*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^10 + 6*(1
1*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^2 + 8*a^2 + 4*a*b + b^2)*sinh(d*x + c
)^10 + 20*(11*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^3 + 3*(8*a^2 + 4*a*b + b^
2)*cosh(d*x + c))*sinh(d*x + c)^9 + 15*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^
8 + 15*(33*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^4 + 18*(8*a^2 + 4*a*b + b^2)
*cosh(d*x + c)^2 + 8*a^2 + 4*a*b + b^2)*sinh(d*x + c)^8 + 24*(33*(8*a^2 + 4
*a*b + b^2)*cosh(d*x + c)^5 + 30*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^3 + 5*
(8*a^2 + 4*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 20*(8*a^2 + 4*a*b +
b^2)*cosh(d*x + c)^6 + 4*(231*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^6 + 315*(
8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^4 + 105*(8*a^2 + 4*a*b + b^2)*cosh(d*x +
 c)^2 + 40*a^2 + 20*a*b + 5*b^2)*sinh(d*x + c)^6 + 24*(33*(8*a^2 + 4*a*b +
b^2)*cosh(d*x + c)^7 + 63*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^5 + 35*(8*a^2
 + 4*a*b + b^2)*cosh(d*x + c)^3 + 5*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c))*si
nh(d*x + c)^5 + 15*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^4 + 15*(33*(8*a^2 +
4*a*b + b^2)*cosh(d*x + c)^8 + 84*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^6 + 7
0*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^4 + 20*(8*a^2 + 4*a*b + b^2)*cosh(d*x
 + c)^2 + 8*a^2 + 4*a*b + b^2)*sinh(d*x + c)^4 + 20*(11*(8*a^2 + 4*a*b + b^
2)*cosh(d*x + c)^9 + 36*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^7 + 42*(8*a^2 +
 4*a*b + b^2)*cosh(d*x + c)^5 + 20*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^3 +
3*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(8*a^2 + 4*a*b +
 b^2)*cosh(d*x + c)^2 + 6*(11*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^10 + 45*(
8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^8 + 70*(8*a^2 + 4*a*b + b^2)*cosh(d*x +
c)^6 + 50*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^4 + 15*(8*a^2 + 4*a*b + b^2)*
cosh(d*x + c)^2 + 8*a^2 + 4*a*b + b^2)*sinh(d*x + c)^2 + 8*a^2 + 4*a*b + b^
2 + 12*((8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^11 + 5*(8*a^2 + 4*a*b + b^2)*co
sh(d*x + c)^9 + 10*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^7 + 10*(8*a^2 + 4*a*
b + b^2)*cosh(d*x + c)^5 + 5*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^3 + (8*a^2
 + 4*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d
*x + c)) - 3*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c) + 3*(11*(8*a^2 + 4*a*b + b
^2)*cosh(d*x + c)^10 + 3*(72*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c)^8 + 14*(8
*a^2 - 12*a*b + 13*b^2)*cosh(d*x + c)^6 - 10*(8*a^2 - 12*a*b + 13*b^2)*cosh
(d*x + c)^4 - (72*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c)^2 - 8*a^2 - 4*a*b -
b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^12 + 12*d*cosh(d*x + c)*sinh(d*x + c)^
11 + d*sinh(d*x + c)^12 + 6*d*cosh(d*x + c)^10 + 6*(11*d*cosh(d*x + c)^2 +
d)*sinh(d*x + c)^10 + 20*(11*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*
```

```
x + c)^9 + 15*d*cosh(d*x + c)^8 + 15*(33*d*cosh(d*x + c)^4 + 18*d*cosh(d*x
+ c)^2 + d)*sinh(d*x + c)^8 + 24*(33*d*cosh(d*x + c)^5 + 30*d*cosh(d*x + c)
^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^7 + 20*d*cosh(d*x + c)^6 + 4*(231*d*c
osh(d*x + c)^6 + 315*d*cosh(d*x + c)^4 + 105*d*cosh(d*x + c)^2 + 5*d)*sinh(
d*x + c)^6 + 24*(33*d*cosh(d*x + c)^7 + 63*d*cosh(d*x + c)^5 + 35*d*cosh(d*
x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^5 + 15*d*cosh(d*x + c)^4 + 15*(
33*d*cosh(d*x + c)^8 + 84*d*cosh(d*x + c)^6 + 70*d*cosh(d*x + c)^4 + 20*d*c
osh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 20*(11*d*cosh(d*x + c)^9 + 36*d*cosh(
d*x + c)^7 + 42*d*cosh(d*x + c)^5 + 20*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c
))*sinh(d*x + c)^3 + 6*d*cosh(d*x + c)^2 + 6*(11*d*cosh(d*x + c)^10 + 45*d*
cosh(d*x + c)^8 + 70*d*cosh(d*x + c)^6 + 50*d*cosh(d*x + c)^4 + 15*d*cosh(d
*x + c)^2 + d)*sinh(d*x + c)^2 + 12*(d*cosh(d*x + c)^11 + 5*d*cosh(d*x + c)
^9 + 10*d*cosh(d*x + c)^7 + 10*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^3 + d*
cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tanh^2(c + dx)\right)^2 \operatorname{sech}^3(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*sech(c + d*x)**3, x)
```

Giac [B] time = 1.48948, size = 393, normalized size = 3.14

$$3\left(8a^2e^c + 4abe^c + b^2e^c\right)\arctan\left(e^{(dx+c)}\right)e^{(-c)} + \frac{24a^2e^{(11dx+11c)} + 12abe^{(11dx+11c)} + 3b^2e^{(11dx+11c)} + 72a^2e^{(9dx+9c)} - 60abe^{(9dx+9c)} - 47b^2e^{(9dx+9}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(3*(8*a^2*e^c + 4*a*b*e^c + b^2*e^c)*arctan(e^(d*x + c))*e^(-c) + (24*
a^2*e^(11*d*x + 11*c) + 12*a*b*e^(11*d*x + 11*c) + 3*b^2*e^(11*d*x + 11*c)
+ 72*a^2*e^(9*d*x + 9*c) - 60*a*b*e^(9*d*x + 9*c) - 47*b^2*e^(9*d*x + 9*c)
+ 48*a^2*e^(7*d*x + 7*c) - 72*a*b*e^(7*d*x + 7*c) + 78*b^2*e^(7*d*x + 7*c)
- 48*a^2*e^(5*d*x + 5*c) + 72*a*b*e^(5*d*x + 5*c) - 78*b^2*e^(5*d*x + 5*c)
- 72*a^2*e^(3*d*x + 3*c) + 60*a*b*e^(3*d*x + 3*c) + 47*b^2*e^(3*d*x + 3*c)
- 24*a^2*e^(d*x + c) - 12*a*b*e^(d*x + c) - 3*b^2*e^(d*x + c))/(e^(2*d*x +
2*c) + 1)^6)/d
```

3.96 $\int \text{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=76

$$\frac{a^2\tanh(c+dx)}{d}-\frac{b(2a-b)\tanh^5(c+dx)}{5d}-\frac{a(a-2b)\tanh^3(c+dx)}{3d}-\frac{b^2\tanh^7(c+dx)}{7d}$$

```
[Out] (a^2*Tanh[c + d*x])/d - (a*(a - 2*b)*Tanh[c + d*x]^3)/(3*d) - ((2*a - b)*b*
Tanh[c + d*x]^5)/(5*d) - (b^2*Tanh[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.070226, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3675, 373}

$$\frac{a^2\tanh(c+dx)}{d}-\frac{b(2a-b)\tanh^5(c+dx)}{5d}-\frac{a(a-2b)\tanh^3(c+dx)}{3d}-\frac{b^2\tanh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Tanh[c + d*x])/d - (a*(a - 2*b)*Tanh[c + d*x]^3)/(3*d) - ((2*a - b)*b*
Tanh[c + d*x]^5)/(5*d) - (b^2*Tanh[c + d*x]^7)/(7*d)
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \text{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx &= \frac{\text{Subst}\left(\int\left(1-x^2\right)\left(a+bx^2\right)^2\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int\left(a^2-a(a-2b)x^2-(2a-b)bx^4-b^2x^6\right)dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{a^2\tanh(c+dx)}{d}-\frac{a(a-2b)\tanh^3(c+dx)}{3d}-\frac{(2a-b)b\tanh^5(c+dx)}{5d}-\end{aligned}$$

Mathematica [A] time = 0.514829, size = 83, normalized size = 1.09

$$\frac{\tanh(c+dx)\left(\left(35a^2+14ab+3b^2\right)\text{sech}^2(c+dx)+70a^2-6b(7a+4b)\text{sech}^4(c+dx)+28ab+15b^2\text{sech}^6(c+dx)+6\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((70*a^2 + 28*a*b + 6*b^2 + (35*a^2 + 14*a*b + 3*b^2)*Sech[c + d*x]^2 - 6*b
*(7*a + 4*b)*Sech[c + d*x]^4 + 15*b^2*Sech[c + d*x]^6)*Tanh[c + d*x])/(105*
d)
```

Maple [B] time = 0.057, size = 158, normalized size = 2.1

$$\frac{1}{d}\left(a^2\left(\frac{2}{3}+\frac{(\mathrm{sech}\,(dx+c))^2}{3}\right)\tanh(dx+c)+2\,ab\left(-1/4\,\frac{\sinh(dx+c)}{(\cosh(dx+c))^5}+1/4\left(\frac{8}{15}+1/5\,(\mathrm{sech}\,(dx+c))^4+\frac{4\,(\mathrm{sech}\,(dx}{15}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+2*a*b*(-1/4*sinh(d*x+c)/cosh(d
*x+c)^5+1/4*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c))+b^2*(-
1/4*sinh(d*x+c)^3/cosh(d*x+c)^7-1/8*sinh(d*x+c)/cosh(d*x+c)^7+1/8*(16/35+1/
7*sech(d*x+c)^6+6/35*sech(d*x+c)^4+8/35*sech(d*x+c)^2)*tanh(d*x+c)))
```

Maxima [B] time = 1.21135, size = 1253, normalized size = 16.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 4/35*b^2*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) +
 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-
12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 14*e^(-4*d*x - 4*c)/(d*(7*e^(-2
*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*
c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)
) + 70*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e
^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*
x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 35*e^(-8*d*x - 8*c)/(d*(7*e^(-2*d*x
- 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) +
21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 3
5*e^(-10*d*x - 10*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-
6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x -
 12*c) + e^(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x
 - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c)
 + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 8/15*a*b*(5*e^(-2*d*x
 - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c)
+ 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) - 5*e^(-4*d*x - 4*c)/(d*(5*
e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x
- 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*e^(-6*d*x - 6*c)/(d*(5*e^(-2*d*x - 2
*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-
10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*
e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 4/3*a^2
*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*
x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x
```

```
- 6*c) + 1)))
```

Fricas [B] time = 1.90376, size = 1829, normalized size = 24.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -8/105*(2*(35*a^2 + 56*a*b + 27*b^2)*cosh(d*x + c)^5 + 10*(35*a^2 + 56*a*b
+ 27*b^2)*cosh(d*x + c)*sinh(d*x + c)^4 + (35*a^2 + 98*a*b + 51*b^2)*sinh(d
*x + c)^5 + 14*(25*a^2 + 16*a*b - 3*b^2)*cosh(d*x + c)^3 + (10*(35*a^2 + 98
*a*b + 51*b^2)*cosh(d*x + c)^2 + 105*a^2 + 126*a*b - 63*b^2)*sinh(d*x + c)^
3 + 2*(10*(35*a^2 + 56*a*b + 27*b^2)*cosh(d*x + c)^3 + 21*(25*a^2 + 16*a*b
- 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 28*(25*a^2 + 4*a*b + 3*b^2)*cosh(
d*x + c) + (5*(35*a^2 + 98*a*b + 51*b^2)*cosh(d*x + c)^4 + 63*(5*a^2 + 6*a*
b - 3*b^2)*cosh(d*x + c)^2 + 70*a^2 + 28*a*b + 126*b^2)*sinh(d*x + c))/(d*c
osh(d*x + c)^9 + 9*d*cosh(d*x + c)*sinh(d*x + c)^8 + d*sinh(d*x + c)^9 + 7*
d*cosh(d*x + c)^7 + (36*d*cosh(d*x + c)^2 + 7*d)*sinh(d*x + c)^7 + 7*(12*d*
cosh(d*x + c)^3 + 7*d*cosh(d*x + c))*sinh(d*x + c)^6 + 22*d*cosh(d*x + c)^5
 + (126*d*cosh(d*x + c)^4 + 147*d*cosh(d*x + c)^2 + 20*d)*sinh(d*x + c)^5 +
 (126*d*cosh(d*x + c)^5 + 245*d*cosh(d*x + c)^3 + 110*d*cosh(d*x + c))*sinh
(d*x + c)^4 + 42*d*cosh(d*x + c)^3 + (84*d*cosh(d*x + c)^6 + 245*d*cosh(d*x
 + c)^4 + 200*d*cosh(d*x + c)^2 + 28*d)*sinh(d*x + c)^3 + (36*d*cosh(d*x +
c)^7 + 147*d*cosh(d*x + c)^5 + 220*d*cosh(d*x + c)^3 + 126*d*cosh(d*x + c))
*sinh(d*x + c)^2 + 56*d*cosh(d*x + c) + (9*d*cosh(d*x + c)^8 + 49*d*cosh(d*
x + c)^6 + 100*d*cosh(d*x + c)^4 + 84*d*cosh(d*x + c)^2 + 14*d)*sinh(d*x +
c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tanh^2\left(c + dx\right)\right)^2 \operatorname{sech}^4\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*sech(c + d*x)**4, x)
```

Giac [B] time = 1.45603, size = 321, normalized size = 4.22

$$-\frac{4\left(105\,a^2e^{(10\,dx+10\,c)} + 210\,abe^{(10\,dx+10\,c)} + 105\,b^2e^{(10\,dx+10\,c)} + 455\,a^2e^{(8\,dx+8\,c)} + 350\,abe^{(8\,dx+8\,c)} - 105\,b^2e^{(8\,dx+8\,c)} + 77}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -4/105*(105*a^2*e^(10*d*x + 10*c) + 210*a*b*e^(10*d*x + 10*c) + 105*b^2*e^(
10*d*x + 10*c) + 455*a^2*e^(8*d*x + 8*c) + 350*a*b*e^(8*d*x + 8*c) - 105*b^
```

```
2*e^(8*d*x + 8*c) + 770*a^2*e^(6*d*x + 6*c) + 140*a*b*e^(6*d*x + 6*c) + 210
*b^2*e^(6*d*x + 6*c) + 630*a^2*e^(4*d*x + 4*c) + 84*a*b*e^(4*d*x + 4*c) - 4
2*b^2*e^(4*d*x + 4*c) + 245*a^2*e^(2*d*x + 2*c) + 98*a*b*e^(2*d*x + 2*c) +
21*b^2*e^(2*d*x + 2*c) + 35*a^2 + 14*a*b + 3*b^2)/(d*(e^(2*d*x + 2*c) + 1)^
7)
```

3.97 $\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=91

$$\frac{3}{8}x(a+b)\left(a^2-2ab+5b^2\right)+\frac{(a+b)^3\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3(a-3b)(a+b)^2\sinh(c+dx)\cosh(c+dx)}{8d}-\frac{b^3 \text{t}}{}$$

```
[Out] (3*(a + b)*(a^2 - 2*a*b + 5*b^2)*x)/8 + (3*(a - 3*b)*(a + b)^2*Cosh[c + d*x
]*Sinh[c + d*x])/(8*d) + ((a + b)^3*Cosh[c + d*x]^3*Sinh[c + d*x])/(4*d) -
(b^3*Tanh[c + d*x])/d
```

Rubi [A] time = 0.130354, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3675, 390, 1157, 385, 206}

$$\frac{3}{8}x(a+b)\left(a^2-2ab+5b^2\right)+\frac{(a+b)^3\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3(a-3b)(a+b)^2\sinh(c+dx)\cosh(c+dx)}{8d}-\frac{b^3 \text{t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (3*(a + b)*(a^2 - 2*a*b + 5*b^2)*x)/8 + (3*(a - 3*b)*(a + b)^2*Cosh[c + d*x
]*Sinh[c + d*x])/(8*d) + ((a + b)^3*Cosh[c + d*x]^3*Sinh[c + d*x])/(4*d) -
(b^3*Tanh[c + d*x])/d
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
```

```
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^3}{\left(1-x^2\right)^3}dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-b^3+\frac{a^3+b^3+3b\left(a^2-b^2\right)x^2+3b^2(a+b)x^4}{\left(1-x^2\right)^3}\right)dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{b^3\tanh(c+dx)}{d}+\frac{\text{Subst}\left(\int \frac{a^3+b^3+3b\left(a^2-b^2\right)x^2+3b^2(a+b)x^4}{\left(1-x^2\right)^3}dx, x, \tanh(c+dx\right.}{d}$$

$$= \frac{(a+b)^3\cosh^3(c+dx)\sinh(c+dx)}{4d}-\frac{b^3\tanh(c+dx)}{d}-\frac{\text{Subst}\left(\int \frac{-3(a-b)^2}{}\right.}{}$$

$$= \frac{3(a-3b)(a+b)^2\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{(a+b)^3\cosh^3(c+dx)\sinh(}{4d}$$

$$= \frac{3}{8}(a+b)\left(a^2-2ab+5b^2\right)x+\frac{3(a-3b)(a+b)^2\cosh(c+dx)\sinh(c+dx)}{8d}$$

Mathematica [A] time = 0.668448, size = 81, normalized size = 0.89

$$\frac{12\left(-a^2b+a^3+3ab^2+5b^3\right)(c+dx)+8(a-2b)(a+b)^2\sinh(2(c+dx))+(a+b)^3\sinh(4(c+dx))-32b^3\tanh(c+dx)}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (12*(a^3 - a^2*b + 3*a*b^2 + 5*b^3)*(c + d*x) + 8*(a - 2*b)*(a + b)^2*Sinh[
2*(c + d*x)] + (a + b)^3*Sinh[4*(c + d*x)] - 32*b^3*Tanh[c + d*x])/(32*d)
```

Maple [B] time = 0.042, size = 184, normalized size = 2.

$$\frac{1}{d}\left(a^3\left(\left(\frac{(\cosh(dx+c))^3}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+3a^2b\left(1/4\sinh(dx+c)(\cosh(dx+c))^3-1/8\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*((1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh(d*x+c)+3/8*d*x+3/8*c)+3*
a^2*b*(1/4*sinh(d*x+c)*cosh(d*x+c)^3-1/8*cosh(d*x+c)*sinh(d*x+c)-1/8*d*x-1/
```

```
8*c)+3*a*b^2*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c
)+b^3*(1/4*sinh(d*x+c)^5/cosh(d*x+c)-5/8*sinh(d*x+c)^3/cosh(d*x+c)+15/8*d*x
+15/8*c-15/8*tanh(d*x+c)))
```

Maxima [B] time = 1.16712, size = 360, normalized size = 3.96

$$\frac{1}{64}a^3\left(24x + \frac{e^{(4dx+4c)}}{d} + \frac{8e^{(2dx+2c)}}{d} - \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d}\right) + \frac{3}{64}ab^2\left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/64*a^3*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + 3/64*a*b^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*
d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)/d - e^(-4*d*x - 4*c)/d) + 1/64*b^3*(120*(
d*x + c)/d + (16*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c))/d - (15*e^(-2*d*x - 2
*c) + 144*e^(-4*d*x - 4*c) - 1)/(d*(e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))))
- 3/64*a^2*b*(8*(d*x + c)/d - e^(4*d*x + 4*c)/d + e^(-4*d*x - 4*c)/d)
```

Fricas [B] time = 2.0398, size = 539, normalized size = 5.92

$$\frac{\left(a^3 + 3a^2b + 3ab^2 + b^3\right)\sinh(dx+c)^5 + \left(9a^3 + 3a^2b - 21ab^2 - 15b^3 + 10\left(a^3 + 3a^2b + 3ab^2 + b^3\right)\cosh(dx+c)^2\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^5 + (9*a^3 + 3*a^2*b -
21*a*b^2 - 15*b^3 + 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sin
h(d*x + c)^3 + 8*(8*b^3 + 3*(a^3 - a^2*b + 3*a*b^2 + 5*b^3)*d*x)*cosh(d*x +
 c) + (5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 8*a^3 - 24*a*b^2
 - 80*b^3 + 9*(3*a^3 + a^2*b - 7*a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x +
 c))/(d*cosh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.67663, size = 385, normalized size = 4.23

$$\frac{24\left(a^3 - a^2b + 3ab^2 + 5b^3\right)dx + \frac{128b^3}{e^{(2dx+2c)}+1} - \left(18a^3e^{(4dx+4c)} - 18a^2be^{(4dx+4c)} + 54ab^2e^{(4dx+4c)} + 90b^3e^{(4dx+4c)} + 8a^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/64*(24*(a^3 - a^2*b + 3*a*b^2 + 5*b^3)*d*x + 128*b^3/(e^(2*d*x + 2*c) + 1
) - (18*a^3*e^(4*d*x + 4*c) - 18*a^2*b*e^(4*d*x + 4*c) + 54*a*b^2*e^(4*d*x
+ 4*c) + 90*b^3*e^(4*d*x + 4*c) + 8*a^3*e^(2*d*x + 2*c) - 24*a*b^2*e^(2*d*x
 + 2*c) - 16*b^3*e^(2*d*x + 2*c) + a^3 + 3*a^2*b + 3*a*b^2 + b^3)*e^(-4*d*x
 - 4*c) + (a^3*e^(4*d*x + 20*c) + 3*a^2*b*e^(4*d*x + 20*c) + 3*a*b^2*e^(4*d
*x + 20*c) + b^3*e^(4*d*x + 20*c) + 8*a^3*e^(2*d*x + 18*c) - 24*a*b^2*e^(2*
d*x + 18*c) - 16*b^3*e^(2*d*x + 18*c))*e^(-16*c))/d
```

3.98 $\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=87

$$\frac{b^2(6a+5b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{(a+b)^3\sinh^3(c+dx)}{3d}+\frac{(a-2b)(a+b)^2\sinh(c+dx)}{d}-\frac{b^3\tanh(c+dx)\text{sech}(c+$$

```
[Out] (b^2*(6*a + 5*b)*ArcTan[Sinh[c + d*x]])/(2*d) + ((a - 2*b)*(a + b)^2*Sinh[c
 + d*x])/d + ((a + b)^3*Sinh[c + d*x]^3)/(3*d) - (b^3*Sech[c + d*x]*Tanh[c
+ d*x])/(2*d)
```

Rubi [A] time = 0.105108, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3676, 390, 385, 203}

$$\frac{b^2(6a+5b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{(a+b)^3\sinh^3(c+dx)}{3d}+\frac{(a-2b)(a+b)^2\sinh(c+dx)}{d}-\frac{b^3\tanh(c+dx)\text{sech}(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (b^2*(6*a + 5*b)*ArcTan[Sinh[c + d*x]])/(2*d) + ((a - 2*b)*(a + b)^2*Sinh[c
 + d*x])/d + ((a + b)^3*Sinh[c + d*x]^3)/(3*d) - (b^3*Sech[c + d*x]*Tanh[c
+ d*x])/(2*d)
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+(a+b)x^2\right)^3}{\left(1+x^2\right)^2}dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left((a-2b)(a+b)^2+(a+b)^3x^2+\frac{b^2(3a+2b)+3b^2(a+b)x^2}{\left(1+x^2\right)^2}\right)dx, x, \sinh(c\right.}{d}$$

$$= \frac{(a-2b)(a+b)^2\sinh(c+dx)}{d}+\frac{(a+b)^3\sinh^3(c+dx)}{3d}+\frac{\text{Subst}\left(\int \frac{b^2(3a+2b}{(1}\right.}{}$$

$$= \frac{(a-2b)(a+b)^2\sinh(c+dx)}{d}+\frac{(a+b)^3\sinh^3(c+dx)}{3d}-\frac{b^3\text{sech}(c+dx)\,\text{ta}}{2d}$$

$$= \frac{b^2(6a+5b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{(a-2b)(a+b)^2\sinh(c+dx)}{d}+\frac{(a+b)}{}$$

Mathematica [C] time = 6.90058, size = 494, normalized size = 5.68

$$\frac{\text{csch}^5(c+dx)\left(-256\sinh^8(c+dx)\left(a\sinh^2(c+dx)+a+b\sinh^2(c+dx)\right)^3 \text{HypergeometricPFQ}\left(\left\{\frac{3}{2},2,2,2,2\right\},\left\{1,1,\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cosh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (Csch[c + d*x]^5*(-256*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2}
, -Sinh[c + d*x]^2]*Sinh[c + d*x]^8*(a + a*Sinh[c + d*x]^2 + b*Sinh[c + d*x
]^2)^3 - (315*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*(b^3*Sinh[c + d*x]^6*(2161 +
1875*Sinh[c + d*x]^2 + 243*Sinh[c + d*x]^4 + Sinh[c + d*x]^6) + a^3*Cosh[c
+ d*x]^6*(2401 + 1875*Sinh[c + d*x]^2 + 243*Sinh[c + d*x]^4 + Sinh[c + d*x]
^6) + 3*a^2*b*(Sinh[c + d*x] + Sinh[c + d*x]^3)^2*(2401 + 1875*Sinh[c + d*x
]^2 + 243*Sinh[c + d*x]^4 + Sinh[c + d*x]^6) + 3*a*b^2*Sinh[c + d*x]^4*(240
1 + 4180*Sinh[c + d*x]^2 + 2118*Sinh[c + d*x]^4 + 244*Sinh[c + d*x]^6 + Sin
h[c + d*x]^8)))/Sqrt[-Sinh[c + d*x]^2] + 21*(b^3*Sinh[c + d*x]^6*(32415 + 1
7320*Sinh[c + d*x]^2 + 753*Sinh[c + d*x]^4) + 3*a*b^2*Sinh[c + d*x]^4*(3601
5 + 50695*Sinh[c + d*x]^2 + 18073*Sinh[c + d*x]^4 + 753*Sinh[c + d*x]^6) +
3*a^2*b*Sinh[c + d*x]^2*(36015 + 88150*Sinh[c + d*x]^2 + 69728*Sinh[c + d*x
]^4 + 18826*Sinh[c + d*x]^6 + 753*Sinh[c + d*x]^8) + a^3*(36015 + 124165*Si
nh[c + d*x]^2 + 157878*Sinh[c + d*x]^4 + 89514*Sinh[c + d*x]^6 + 19579*Sinh
[c + d*x]^8 + 753*Sinh[c + d*x]^10))))/(30240*d)
```

Maple [B] time = 0.044, size = 227, normalized size = 2.6

$$\frac{2a^3\sinh(dx+c)}{3d}+\frac{a^3\sinh(dx+c)(\cosh(dx+c))^2}{3d}+\frac{a^2b(\cosh(dx+c))^2\sinh(dx+c)}{d}-\frac{a^2b\sinh(dx+c)}{d}+\frac{ab^2(\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 2/3/d*a^3*sinh(d*x+c)+1/3/d*a^3*sinh(d*x+c)*cosh(d*x+c)^2+1/d*a^2*b*cosh(d*
x+c)^2*sinh(d*x+c)-a^2*b*sinh(d*x+c)/d+1/d*a*b^2*sinh(d*x+c)^3+6/d*a*b^2*ar
```

```
ctan(exp(d*x+c))-3/d*a*b^2*sinh(d*x+c)+1/3/d*b^3*sinh(d*x+c)^5/cosh(d*x+c)^
2-5/3/d*b^3*sinh(d*x+c)^3/cosh(d*x+c)^2-5/d*b^3*sinh(d*x+c)/cosh(d*x+c)^2+5
/2*b^3*sech(d*x+c)*tanh(d*x+c)/d+5/d*b^3*arctan(exp(d*x+c))
```

Maxima [B] time = 1.66538, size = 383, normalized size = 4.4

$$\frac{a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3}{8\,d}-\frac{1}{8}\,ab^2\left(\frac{\left(15\,e^{(-2\,dx-2\,c)}-1\right)e^{(3\,dx+3\,c)}}{d}-\frac{15\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}+\frac{48\,\arctan\left(e^{(-dx-c)}\right)}{d}\right)+\frac{1}{24}\,b^3\Big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a^2*b*(e^(d*x + c) - e^(-d*x - c))^3/d - 1/8*a*b^2*((15*e^(-2*d*x - 2*c
) - 1)*e^(3*d*x + 3*c)/d - (15*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + 48*arct
an(e^(-d*x - c))/d) + 1/24*b^3*((27*e^(-d*x - c) - e^(-3*d*x - 3*c))/d - 12
0*arctan(e^(-d*x - c))/d - (25*e^(-2*d*x - 2*c) + 77*e^(-4*d*x - 4*c) + 3*e
^(-6*d*x - 6*c) - 1)/(d*(e^(-3*d*x - 3*c) + 2*e^(-5*d*x - 5*c) + e^(-7*d*x
- 7*c)))) + 1/24*a^3*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e^(-d*x - c)/
d - e^(-3*d*x - 3*c)/d)
```

Fricas [B] time = 2.20745, size = 4523, normalized size = 51.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/24*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^10 + 10*(a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^9 + (a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*sinh(d*x + c)^10 + (11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*cosh(d*x +
c)^8 + (11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3 + 45*(a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*cosh(d*x + c)^3 + (11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*cosh(d*x + c)
)*sinh(d*x + c)^7 + 2*(5*a^3 - 3*a^2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c)^6
 + 2*(105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 5*a^3 - 3*a^2*b
 - 21*a*b^2 - 25*b^3 + 14*(11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*cosh(d*x +
 c)^2)*sinh(d*x + c)^6 + 4*(63*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c
)^5 + 14*(11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*cosh(d*x + c)^3 + 3*(5*a^3
- 3*a^2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(5*a^3 -
3*a^2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c)^4 + 2*(105*(a^3 + 3*a^2*b + 3*a*
b^2 + b^3)*cosh(d*x + c)^6 + 35*(11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*cosh
(d*x + c)^4 - 5*a^3 + 3*a^2*b + 21*a*b^2 + 25*b^3 + 15*(5*a^3 - 3*a^2*b - 2
1*a*b^2 - 25*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(15*(a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 7*(11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*
cosh(d*x + c)^5 + 5*(5*a^3 - 3*a^2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c)^3 -
 (5*a^3 - 3*a^2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - a^3
 - 3*a^2*b - 3*a*b^2 - b^3 - (11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*cosh(d*
x + c)^2 + (45*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 28*(11*a^3
 - 3*a^2*b - 39*a*b^2 - 25*b^3)*cosh(d*x + c)^6 + 30*(5*a^3 - 3*a^2*b - 21*
a*b^2 - 25*b^3)*cosh(d*x + c)^4 - 11*a^3 + 3*a^2*b + 39*a*b^2 + 25*b^3 - 12
*(5*a^3 - 3*a^2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2
4*((6*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 7*(6*a*b^2 + 5*b^3)*cosh(d*x + c)*si
```

```
nh(d*x + c)^6 + (6*a*b^2 + 5*b^3)*sinh(d*x + c)^7 + 2*(6*a*b^2 + 5*b^3)*cos
h(d*x + c)^5 + (12*a*b^2 + 10*b^3 + 21*(6*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*s
inh(d*x + c)^5 + 5*(7*(6*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 2*(6*a*b^2 + 5*b^
3)*cosh(d*x + c))*sinh(d*x + c)^4 + (6*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + (35
*(6*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 6*a*b^2 + 5*b^3 + 20*(6*a*b^2 + 5*b^3)
*cosh(d*x + c)^2)*sinh(d*x + c)^3 + (21*(6*a*b^2 + 5*b^3)*cosh(d*x + c)^5 +
 20*(6*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 3*(6*a*b^2 + 5*b^3)*cosh(d*x + c))*
sinh(d*x + c)^2 + (7*(6*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 10*(6*a*b^2 + 5*b^
3)*cosh(d*x + c)^4 + 3*(6*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))*ar
ctan(cosh(d*x + c) + sinh(d*x + c)) + 2*(5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*
cosh(d*x + c)^9 + 4*(11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*cosh(d*x + c)^7
+ 6*(5*a^3 - 3*a^2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c)^5 - 4*(5*a^3 - 3*a^
2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c)^3 - (11*a^3 - 3*a^2*b - 39*a*b^2 - 2
5*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)
*sinh(d*x + c)^6 + d*sinh(d*x + c)^7 + 2*d*cosh(d*x + c)^5 + (21*d*cosh(d*x
 + c)^2 + 2*d)*sinh(d*x + c)^5 + 5*(7*d*cosh(d*x + c)^3 + 2*d*cosh(d*x + c)
)*sinh(d*x + c)^4 + d*cosh(d*x + c)^3 + (35*d*cosh(d*x + c)^4 + 20*d*cosh(d
*x + c)^2 + d)*sinh(d*x + c)^3 + (21*d*cosh(d*x + c)^5 + 20*d*cosh(d*x + c)
^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + (7*d*cosh(d*x + c)^6 + 10*d*cosh(
d*x + c)^4 + 3*d*cosh(d*x + c)^2)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.4196, size = 377, normalized size = 4.33

$$\frac{24\left(6\,ab^2e^c+5\,b^3e^c\right)\arctan\left(e^{(dx+c)}\right)e^{(-c)}-\left(9\,a^3e^{(2\,dx+2\,c)}-9\,a^2be^{(2\,dx+2\,c)}-45\,ab^2e^{(2\,dx+2\,c)}-27\,b^3e^{(2\,dx+2\,c)}+a^3+3\,a^2b+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/24*(24*(6*a*b^2*e^c + 5*b^3*e^c)*arctan(e^(d*x + c))*e^(-c) - (9*a^3*e^(2
*d*x + 2*c) - 9*a^2*b*e^(2*d*x + 2*c) - 45*a*b^2*e^(2*d*x + 2*c) - 27*b^3*e
^(2*d*x + 2*c) + a^3 + 3*a^2*b + 3*a*b^2 + b^3)*e^(-3*d*x - 3*c) + (a^3*e^(
3*d*x + 30*c) + 3*a^2*b*e^(3*d*x + 30*c) + 3*a*b^2*e^(3*d*x + 30*c) + b^3*e
^(3*d*x + 30*c) + 9*a^3*e^(d*x + 28*c) - 9*a^2*b*e^(d*x + 28*c) - 45*a*b^2*
e^(d*x + 28*c) - 27*b^3*e^(d*x + 28*c))*e^(-27*c) - 24*(b^3*e^(3*d*x + 3*c)
 - b^3*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^2)/d
```

3.99 $\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=78

$$\frac{b^2(3a+2b)\tanh(c+dx)}{d}+\frac{(a+b)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{1}{2}x(a-5b)(a+b)^2+\frac{b^3\tanh^3(c+dx)}{3d}$$

```
[Out] ((a - 5*b)*(a + b)^2*x)/2 + ((a + b)^3*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) +
 (b^2*(3*a + 2*b)*Tanh[c + d*x])/d + (b^3*Tanh[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.089681, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3675, 390, 385, 206}

$$\frac{b^2(3a+2b)\tanh(c+dx)}{d}+\frac{(a+b)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{1}{2}x(a-5b)(a+b)^2+\frac{b^3\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((a - 5*b)*(a + b)^2*x)/2 + ((a + b)^3*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) +
 (b^2*(3*a + 2*b)*Tanh[c + d*x])/d + (b^3*Tanh[c + d*x]^3)/(3*d)
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^3}{(1-x^2)^2}dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(b^2(3a+2b)+b^3x^2+\frac{(a-2b)(a+b)^2+3b(a+b)^2x^2}{(1-x^2)^2}\right)dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b^2(3a+2b)\tanh(c+dx)}{d}+\frac{b^3\tanh^3(c+dx)}{3d}+\frac{\text{Subst}\left(\int \frac{(a-2b)(a+b)^2+3b(a+}{(1-x^2)^2}\right.}{d}$$

$$= \frac{(a+b)^3\cosh(c+dx)\sinh(c+dx)}{2d}+\frac{b^2(3a+2b)\tanh(c+dx)}{d}+\frac{b^3\tanh^3}{3}$$

$$= \frac{1}{2}(a-5b)(a+b)^2x+\frac{(a+b)^3\cosh(c+dx)\sinh(c+dx)}{2d}+\frac{b^2(3a+2b)\tan}{d}$$

Mathematica [A] time = 0.850207, size = 69, normalized size = 0.88

$$\frac{4b^2\tanh(c+dx)\left(9a-b\,\text{sech}^2(c+dx)+7b\right)+6(a-5b)(a+b)^2(c+dx)+3(a+b)^3\sinh(2(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (6*(a - 5*b)*(a + b)^2*(c + d*x) + 3*(a + b)^3*Sinh[2*(c + d*x)] + 4*b^2*(9
*a + 7*b - b*Sech[c + d*x]^2)*Tanh[c + d*x])/(12*d)
```

Maple [B] time = 0.047, size = 148, normalized size = 1.9

$$\frac{1}{d}\left(a^3\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^2b\left(1/2\cosh(dx+c)\sinh(dx+c)-1/2\,dx-c/2\right)+3ab^2\left(1/2\,\frac{(\sinh(}{\cosh(}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*(1/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c)+3*a^2*b*(1/2*cosh(d*x+
c)*sinh(d*x+c)-1/2*d*x-1/2*c)+3*a*b^2*(1/2*sinh(d*x+c)^3/cosh(d*x+c)-3/2*d*
x-3/2*c+3/2*tanh(d*x+c))+b^3*(1/2*sinh(d*x+c)^5/cosh(d*x+c)^3-5/2*d*x-5/2*c
+5/2*tanh(d*x+c)+5/6*tanh(d*x+c)^3))
```

Maxima [B] time = 1.15486, size = 346, normalized size = 4.44

$$\frac{1}{8}a^3\left(4x+\frac{e^{(2dx+2c)}}{d}-\frac{e^{(-2dx-2c)}}{d}\right)-\frac{3}{8}a^2b\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)-\frac{1}{24}b^3\left(\frac{60(dx+c)}{d}+\frac{3e^{(-2dx-2c)}}{d}-\frac{121e^{(-2dx}}{d\left(e^{(-2dx-2c}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a^3*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) - 3/8*a^2*b*(4*x - e
^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/24*b^3*(60*(d*x + c)/d + 3*e^(-2
*d*x - 2*c)/d - (121*e^(-2*d*x - 2*c) + 201*e^(-4*d*x - 4*c) + 147*e^(-6*d*
x - 6*c) + 3)/(d*(e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 3*e^(-6*d*x - 6*c
) + e^(-8*d*x - 8*c)))) - 3/8*a*b^2*(12*(d*x + c)/d + e^(-2*d*x - 2*c)/d -
(17*e^(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c))))
```

Fricas [B] time = 2.0419, size = 903, normalized size = 11.58

$$3\left(a^3+3a^2b+3ab^2+b^3\right)\sinh\left(dx+c\right)^5-4\left(18ab^2+14b^3-3\left(a^3-3a^2b-9ab^2-5b^3\right)dx\right)\cosh\left(dx+c\right)^3-12\left(18\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^5 - 4*(18*a*b^2 + 14*
b^3 - 3*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*d*x)*cosh(d*x + c)^3 - 12*(18*a*b
^2 + 14*b^3 - 3*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*d*x)*cosh(d*x + c)*sinh(d
*x + c)^2 + (9*a^3 + 27*a^2*b + 99*a*b^2 + 65*b^3 + 30*(a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 - 12*(18*a*b^2 + 14*b^3 - 3*(a
^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*d*x)*cosh(d*x + c) + 3*(5*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*cosh(d*x + c)^4 + 2*a^3 + 6*a^2*b + 30*a*b^2 + 10*b^3 + (9*a
^3 + 27*a^2*b + 99*a*b^2 + 65*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(
d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.06105, size = 360, normalized size = 4.62

$$12\left(a^3-3a^2b-9ab^2-5b^3\right)dx-3\left(2a^3e^{(2dx+2c)}-6a^2be^{(2dx+2c)}-18ab^2e^{(2dx+2c)}-10b^3e^{(2dx+2c)}+a^3+3a^2b+3ab\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/24*(12*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*d*x - 3*(2*a^3*e^(2*d*x + 2*c) -
 6*a^2*b*e^(2*d*x + 2*c) - 18*a*b^2*e^(2*d*x + 2*c) - 10*b^3*e^(2*d*x + 2*c
) + a^3 + 3*a^2*b + 3*a*b^2 + b^3)*e^(-2*d*x - 2*c) + 3*(a^3*e^(2*d*x + 12*
c) + 3*a^2*b*e^(2*d*x + 12*c) + 3*a*b^2*e^(2*d*x + 12*c) + b^3*e^(2*d*x + 1
2*c))*e^(-10*c) - 16*(9*a*b^2*e^(4*d*x + 4*c) + 9*b^3*e^(4*d*x + 4*c) + 18*
a*b^2*e^(2*d*x + 2*c) + 12*b^3*e^(2*d*x + 2*c) + 9*a*b^2 + 7*b^3)/(e^(2*d*x
 + 2*c) + 1)^3)/d
```

3.100 $\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=99

$$\frac{3b^2(4a+3b)\tanh(c+dx)\text{sech}(c+dx)}{8d}+\frac{(a+b)^3\sinh(c+dx)}{d}-\frac{3b\left(4(a+b)^2+(2a+b)^2\right)\tan^{-1}(\sinh(c+dx))}{8d}-\frac{b^3\ \text{ta}}{}$$

```
[Out] (-3*b*(4*(a + b)^2 + (2*a + b)^2)*ArcTan[Sinh[c + d*x]])/(8*d) + ((a + b)^3
*Sinh[c + d*x])/d + (3*b^2*(4*a + 3*b)*Sech[c + d*x]*Tanh[c + d*x])/(8*d) -
 (b^3*Sech[c + d*x]^3*Tanh[c + d*x])/(4*d)
```

Rubi [A] time = 0.116297, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3676, 390, 1157, 385, 203}

$$\frac{3b^2(4a+3b)\tanh(c+dx)\text{sech}(c+dx)}{8d}+\frac{(a+b)^3\sinh(c+dx)}{d}-\frac{3b\left(4(a+b)^2+(2a+b)^2\right)\tan^{-1}(\sinh(c+dx))}{8d}-\frac{b^3\ \text{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (-3*b*(4*(a + b)^2 + (2*a + b)^2)*ArcTan[Sinh[c + d*x]])/(8*d) + ((a + b)^3
*Sinh[c + d*x])/d + (3*b^2*(4*a + 3*b)*Sech[c + d*x]*Tanh[c + d*x])/(8*d) -
 (b^3*Sech[c + d*x]^3*Tanh[c + d*x])/(4*d)
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
```

```
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+(a+b)x^2\right)^3}{\left(1+x^2\right)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left((a+b)^3 - \frac{b\left(3a^2+3ab+b^2\right)+3b(a+b)(2a+b)x^2+3b(a+b)^2x^4}{\left(1+x^2\right)^3}\right) dx, x, \sinh(c$$

$$= \frac{(a+b)^3\sinh(c+dx)}{d} - \frac{\text{Subst}\left(\int \frac{b\left(3a^2+3ab+b^2\right)+3b(a+b)(2a+b)x^2+3b(a+b)^2x^4}{\left(1+x^2\right)^3} dx\right.}{d}$$

$$= \frac{(a+b)^3\sinh(c+dx)}{d} - \frac{b^3\text{sech}^3(c+dx)\tanh(c+dx)}{4d} + \frac{\text{Subst}\left(\int \frac{-3b(2a}{}\right.}{}$$

$$= \frac{(a+b)^3\sinh(c+dx)}{d} + \frac{3b^2(4a+3b)\text{sech}(c+dx)\tanh(c+dx)}{8d} - \frac{b^3\text{sec}}{}$$

$$= -\frac{3b\left(4(a+b)^2+(2a+b)^2\right)\tan^{-1}(\sinh(c+dx))}{8d} + \frac{(a+b)^3\sinh(c+dx)}{d}$$

Mathematica [A] time = 0.363403, size = 89, normalized size = 0.9

$$\frac{-3b\left(8a^2+12ab+5b^2\right)\tan^{-1}(\sinh(c+dx)) + 3b^2(4a+3b)\tanh(c+dx)\text{sech}(c+dx) + 8(a+b)^3\sinh(c+dx) - 2b^3}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (-3*b*(8*a^2 + 12*a*b + 5*b^2)*ArcTan[Sinh[c + d*x]] + 8*(a + b)^3*Sinh[c +
 d*x] + 3*b^2*(4*a + 3*b)*Sech[c + d*x]*Tanh[c + d*x] - 2*b^3*Sech[c + d*x]
^3*Tanh[c + d*x])/(8*d)
```

Maple [B] time = 0.048, size = 257, normalized size = 2.6

$$\frac{a^3\sinh(dx+c)}{d} + 3\frac{a^2b\sinh(dx+c)}{d} - 6\frac{a^2b\arctan\left(e^{dx+c}\right)}{d} + 3\frac{ab^2(\sinh(dx+c))^3}{d(\cosh(dx+c))^2} + 9\frac{ab^2\sinh(dx+c)}{d(\cosh(dx+c))^2} - \frac{9\,ab^2\text{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*a^3*sinh(d*x+c)+3*a^2*b*sinh(d*x+c)/d-6/d*a^2*b*arctan(exp(d*x+c))+3/d*
a*b^2*sinh(d*x+c)^3/cosh(d*x+c)^2+9/d*a*b^2*sinh(d*x+c)/cosh(d*x+c)^2-9/2/d
```

```
*a*b^2*sech(d*x+c)*tanh(d*x+c)-9/d*a*b^2*arctan(exp(d*x+c))+1/d*b^3*sinh(d*
x+c)^5/cosh(d*x+c)^4+5/d*b^3*sinh(d*x+c)^3/cosh(d*x+c)^4+5/d*b^3*sinh(d*x+c
)/cosh(d*x+c)^4-5/4*b^3*sech(d*x+c)^3*tanh(d*x+c)/d-15/8*b^3*sech(d*x+c)*ta
nh(d*x+c)/d-15/4/d*b^3*arctan(exp(d*x+c))
```

Maxima [B] time = 1.7006, size = 398, normalized size = 4.02

$$\frac{1}{4}\,b^3\left(\frac{15\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{2\,e^{(-dx-c)}}{d}+\frac{17\,e^{(-2\,dx-2\,c)}+13\,e^{(-4\,dx-4\,c)}+7\,e^{(-6\,dx-6\,c)}-7\,e^{(-8\,dx-8\,c)}+2}{d\left(e^{(-dx-c)}+4\,e^{(-3\,dx-3\,c)}+6\,e^{(-5\,dx-5\,c)}+4\,e^{(-7\,dx-7\,c)}+e^{(-9\,dx-9\,c)}\right)}\right)+\frac{3}{2}\,ab^2\left(\frac{6\ \mathrm{arc}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*(15*arctan(e^(-d*x - c))/d - 2*e^(-d*x - c)/d + (17*e^(-2*d*x - 2*c
) + 13*e^(-4*d*x - 4*c) + 7*e^(-6*d*x - 6*c) - 7*e^(-8*d*x - 8*c) + 2)/(d*(
e^(-d*x - c) + 4*e^(-3*d*x - 3*c) + 6*e^(-5*d*x - 5*c) + 4*e^(-7*d*x - 7*c)
 + e^(-9*d*x - 9*c)))) + 3/2*a*b^2*(6*arctan(e^(-d*x - c))/d - e^(-d*x - c)
/d + (4*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) + 1)/(d*(e^(-d*x - c) + 2*e^(-3
*d*x - 3*c) + e^(-5*d*x - 5*c)))) + 3/2*a^2*b*(4*arctan(e^(-d*x - c))/d + e
^(d*x + c)/d - e^(-d*x - c)/d) + a^3*sinh(d*x + c)/d
```

Fricas [B] time = 2.18889, size = 5933, normalized size = 59.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^10 + 20*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^9 + 2*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*sinh(d*x + c)^10 + 3*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(d*x
 + c)^8 + 3*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3 + 30*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 24*(10*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*cosh(d*x + c)^3 + (2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(d*x +
c))*sinh(d*x + c)^7 + (4*a^3 + 12*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^6
 + (420*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 4*a^3 + 12*a^2*b
+ 24*a*b^2 + 5*b^3 + 84*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^6 + 6*(84*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5
+ 28*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + (4*a^3 + 12*a^2
*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - (4*a^3 + 12*a^2*b +
 24*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + (420*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c
osh(d*x + c)^6 + 210*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(d*x + c)^4 -
 4*a^3 - 12*a^2*b - 24*a*b^2 - 5*b^3 + 15*(4*a^3 + 12*a^2*b + 24*a*b^2 + 5*
b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(60*(a^3 + 3*a^2*b + 3*a*b^2 + b^
3)*cosh(d*x + c)^7 + 42*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(d*x + c)^
5 + 5*(4*a^3 + 12*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^3 - (4*a^3 + 12*a
^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*a^3 - 6*a^2*b -
 6*a*b^2 - 2*b^3 - 3*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(d*x + c)^2 +
 3*(30*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 28*(2*a^3 + 6*a^2*
b + 10*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 5*(4*a^3 + 12*a^2*b + 24*a*b^2 + 5*
b^3)*cosh(d*x + c)^4 - 2*a^3 - 6*a^2*b - 10*a*b^2 - 5*b^3 - 2*(4*a^3 + 12*a
^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 3*((8*a^2*b + 1
```

```
2*a*b^2 + 5*b^3)*cosh(d*x + c)^9 + 9*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x
+ c)*sinh(d*x + c)^8 + (8*a^2*b + 12*a*b^2 + 5*b^3)*sinh(d*x + c)^9 + 4*(8*
a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 4*(8*a^2*b + 12*a*b^2 + 5*b^3 +
 9*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 28*(3*(8
*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + (8*a^2*b + 12*a*b^2 + 5*b^3)*c
osh(d*x + c))*sinh(d*x + c)^6 + 6*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c
)^5 + 6*(21*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 8*a^2*b + 12*a*b
^2 + 5*b^3 + 14*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)
^5 + 2*(63*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 70*(8*a^2*b + 12*
a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 15*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x +
 c))*sinh(d*x + c)^4 + 4*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 4*(
21*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 35*(8*a^2*b + 12*a*b^2 +
5*b^3)*cosh(d*x + c)^4 + 8*a^2*b + 12*a*b^2 + 5*b^3 + 15*(8*a^2*b + 12*a*b^
2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 12*(3*(8*a^2*b + 12*a*b^2 + 5
*b^3)*cosh(d*x + c)^7 + 7*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 5*
(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + (8*a^2*b + 12*a*b^2 + 5*b^3)
*cosh(d*x + c))*sinh(d*x + c)^2 + (8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c
) + (9*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^8 + 28*(8*a^2*b + 12*a*b^
2 + 5*b^3)*cosh(d*x + c)^6 + 30*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^
4 + 8*a^2*b + 12*a*b^2 + 5*b^3 + 12*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x +
 c)^2)*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) + 2*(10*(a^3 +
3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^9 + 12*(2*a^3 + 6*a^2*b + 10*a*b^2 +
 5*b^3)*cosh(d*x + c)^7 + 3*(4*a^3 + 12*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x
+ c)^5 - 2*(4*a^3 + 12*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^3 - 3*(2*a^3
 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x +
c)^9 + 9*d*cosh(d*x + c)*sinh(d*x + c)^8 + d*sinh(d*x + c)^9 + 4*d*cosh(d*x
 + c)^7 + 4*(9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^7 + 28*(3*d*cosh(d*x +
c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^6 + 6*d*cosh(d*x + c)^5 + 6*(21*d*cos
h(d*x + c)^4 + 14*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^5 + 2*(63*d*cosh(d*x
 + c)^5 + 70*d*cosh(d*x + c)^3 + 15*d*cosh(d*x + c))*sinh(d*x + c)^4 + 4*d*
cosh(d*x + c)^3 + 4*(21*d*cosh(d*x + c)^6 + 35*d*cosh(d*x + c)^4 + 15*d*cos
h(d*x + c)^2 + d)*sinh(d*x + c)^3 + 12*(3*d*cosh(d*x + c)^7 + 7*d*cosh(d*x
+ c)^5 + 5*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^2 + d*cosh(d*
x + c) + (9*d*cosh(d*x + c)^8 + 28*d*cosh(d*x + c)^6 + 30*d*cosh(d*x + c)^4
 + 12*d*cosh(d*x + c)^2 + d)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.83198, size = 347, normalized size = 3.51

$$-\frac{3\left(8a^2be^c+12ab^2e^c+5b^3e^c\right)\arctan\left(e^{(dx+c)}\right)e^{(-c)}+2\left(a^3+3a^2b+3ab^2+b^3\right)e^{(-dx-c)}-2\left(a^3e^{(dx+12c)}+3a^2be^{(dx+1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/4*(3*(8*a^2*b*e^c + 12*a*b^2*e^c + 5*b^3*e^c)*arctan(e^(d*x + c))*e^(-c)
 + 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*e^(-d*x - c) - 2*(a^3*e^(d*x + 12*c) +
 3*a^2*b*e^(d*x + 12*c) + 3*a*b^2*e^(d*x + 12*c) + b^3*e^(d*x + 12*c))*e^(-
11*c) - (12*a*b^2*e^(7*d*x + 7*c) + 9*b^3*e^(7*d*x + 7*c) + 12*a*b^2*e^(5*d
*x + 5*c) + b^3*e^(5*d*x + 5*c) - 12*a*b^2*e^(3*d*x + 3*c) - b^3*e^(3*d*x +
 3*c) - 12*a*b^2*e^(d*x + c) - 9*b^3*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^4)/
d
```

3.101 $\int \mathbf{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=149

$$\frac{(2a+b)\left(8a^2+8ab+5b^2\right)\tan^{-1}(\sinh(c+dx))}{16d}-\frac{b\left(44a^2+44ab+15b^2\right)\tanh(c+dx)\mathbf{sech}(c+dx)}{48d}-\frac{b\tanh(c+dx)}{}$$

```
[Out] ((2*a + b)*(8*a^2 + 8*a*b + 5*b^2)*ArcTan[Sinh[c + d*x]])/(16*d) - (b*(44*a
^2 + 44*a*b + 15*b^2)*Sech[c + d*x]*Tanh[c + d*x])/(48*d) - (5*b*(2*a + b)*
Sech[c + d*x]^3*(a + (a + b)*Sinh[c + d*x]^2)*Tanh[c + d*x])/(24*d) - (b*Se
ch[c + d*x]^5*(a + (a + b)*Sinh[c + d*x]^2)^2*Tanh[c + d*x])/(6*d)
```

Rubi [A] time = 0.154391, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3676, 413, 526, 385, 203}

$$\frac{(2a+b)\left(8a^2+8ab+5b^2\right)\tan^{-1}(\sinh(c+dx))}{16d}-\frac{b\left(44a^2+44ab+15b^2\right)\tanh(c+dx)\mathbf{sech}(c+dx)}{48d}-\frac{b\tanh(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```



```
[Out] ((2*a + b)*(8*a^2 + 8*a*b + 5*b^2)*ArcTan[Sinh[c + d*x]])/(16*d) - (b*(44*a
^2 + 44*a*b + 15*b^2)*Sech[c + d*x]*Tanh[c + d*x])/(48*d) - (5*b*(2*a + b)*
Sech[c + d*x]^3*(a + (a + b)*Sinh[c + d*x]^2)*Tanh[c + d*x])/(24*d) - (b*Se
ch[c + d*x]^5*(a + (a + b)*Sinh[c + d*x]^2)^2*Tanh[c + d*x])/(6*d)
```


Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+(a+b)x^2\right)^3}{\left(1+x^2\right)^4}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{b\operatorname{sech}^5(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)^2\tanh(c+dx)}{6d} + \frac{\operatorname{Subst}\left(\int \frac{(a+(\ldots}{\ldots}\right.}{\ldots}$$

$$= -\frac{5b(2a+b)\operatorname{sech}^3(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)\tanh(c+dx)}{24d} - \frac{b\operatorname{sech}\ldots}{\ldots}$$

$$= -\frac{b\left(44a^2+44ab+15b^2\right)\operatorname{sech}(c+dx)\tanh(c+dx)}{48d} - \frac{5b(2a+b)\operatorname{sech}^3(c+\ldots}{\ldots}$$

$$= \frac{(2a+b)\left(8a^2+8ab+5b^2\right)\tan^{-1}(\sinh(c+dx))}{16d} - \frac{b\left(44a^2+44ab+15b^2\right)\operatorname{s}\ldots}{48\ldots}$$

Mathematica [C] time = 17.2886, size = 1341, normalized size = 9.

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (a^3*Sinh[c + d*x]*((9514449*(a + b))/a + (135323370*(a + b)^2)/a^2 + (5800
9455*(a + b)^3)/a^3 + 4093425*Csch[c + d*x]^2 + (168951510*(a + b)*Csch[c +
 d*x]^2)/a + (215549775*(a + b)^2*Csch[c + d*x]^2)/a^2 + 70189350*Csch[c +
d*x]^4 + (274542345*(a + b)*Csch[c + d*x]^4)/a + 117228825*Csch[c + d*x]^6
+ (7808535*(a + b)^2*Sinh[c + d*x]^2)/a^2 + (36772890*(a + b)^3*Sinh[c + d*
x]^2)/a^3 - 75520*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2}, -Si
nh[c + d*x]^2]*Sinh[c + d*x]^2 - 13824*HypergeometricPFQ[{3/2, 2, 2, 2, 2,
2}, {1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^2 - 1024*Hypergeome
tricPFQ[{3/2, 2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*S
inh[c + d*x]^2 + (2160711*(a + b)^3*Sinh[c + d*x]^4)/a^3 - (189696*(a + b)*
HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sin
h[c + d*x]^4)/a - (38400*(a + b)*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1
, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^4)/a - (3072*(a + b)*Hype
rgeometricPFQ[{3/2, 2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 1, 11/2}, -Sinh[c + d*x
]^2]*Sinh[c + d*x]^4)/a - (158976*(a + b)^2*HypergeometricPFQ[{3/2, 2, 2, 2
, 2}, {1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6)/a^2 - (35328*(a +
 b)^2*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2}, -Sinh[c +
 d*x]^2]*Sinh[c + d*x]^6)/a^2 - (3072*(a + b)^2*HypergeometricPFQ[{3/2, 2,
```

```
2, 2, 2, 2, 2}, {1, 1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6)/a
^2 - (44800*(a + b)^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2},
 -Sinh[c + d*x]^2]*Sinh[c + d*x]^8)/a^3 - (10752*(a + b)^3*HypergeometricPF
Q[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]
^8)/a^3 - (1024*(a + b)^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2, 2}, {1, 1,
 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^8)/a^3 + (142065*ArcTanh[S
qrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^8)/(-Sinh[c + d*x]^2)^(9/2) + (1172288
25*ArcTanh[Sqrt[-Sinh[c + d*x]^2]])/(-Sinh[c + d*x]^2)^(7/2) + (17069535*Ar
cTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^4)/(-Sinh[c + d*x]^2)^(7/2) + (
33756345*(a + b)^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^8)/(a^2*(-
Sinh[c + d*x]^2)^(7/2)) + (56109375*(a + b)^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2
]]*Sinh[c + d*x]^8)/(a^3*(-Sinh[c + d*x]^2)^(7/2)) - (109265625*ArcTanh[Sqr
t[-Sinh[c + d*x]^2]])/(-Sinh[c + d*x]^2)^(5/2) - (274542345*(a + b)*ArcTanh
[Sqrt[-Sinh[c + d*x]^2]])/(a*(-Sinh[c + d*x]^2)^(5/2)) + (260465625*(a + b)
*ArcTanh[Sqrt[-Sinh[c + d*x]^2]])/(a*(-Sinh[c + d*x]^2)^(3/2)) + (215549775
*(a + b)^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]])/(a^2*(-Sinh[c + d*x]^2)^(3/2))
+ (174825*(a + b)^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^6)/(a^2*(
-Sinh[c + d*x]^2)^(3/2)) + (9261945*(a + b)^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2
]]*Sinh[c + d*x]^6)/(a^3*(-Sinh[c + d*x]^2)^(3/2)) + (48825*(a + b)^3*ArcTa
nh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^8)/(a^3*(-Sinh[c + d*x]^2)^(3/2))
- (41427855*(a + b)*ArcTanh[Sqrt[-Sinh[c + d*x]^2]])/(a*Sqrt[-Sinh[c + d*x]
^2]) - (207173295*(a + b)^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]])/(a^2*Sqrt[-Sin
h[c + d*x]^2]) - (58009455*(a + b)^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]])/(a^3*
Sqrt[-Sinh[c + d*x]^2]) + (210735*(a + b)*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*S
qrt[-Sinh[c + d*x]^2])/a))/(725760*d)
```

Maple [B] time = 0.041, size = 334, normalized size = 2.2

$$2\,\frac{a^3\arctan\left(e^{dx+c}\right)}{d}-3\,\frac{a^2b\sinh(dx+c)}{d\,(\cosh(dx+c))^2}+\frac{3\,a^2b\,\mathrm{sech}(dx+c)\tanh(dx+c)}{2\,d}+3\,\frac{a^2b\arctan\left(e^{dx+c}\right)}{d}-3\,\frac{ab^2\,(\sinh(d}{d\,(\cosh(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 2/d*a^3*arctan(exp(d*x+c))-3/d*a^2*b*sinh(d*x+c)/cosh(d*x+c)^2+3/2*a^2*b*se
ch(d*x+c)*tanh(d*x+c)/d+3/d*a^2*b*arctan(exp(d*x+c))-3/d*a*b^2*sinh(d*x+c)^
3/cosh(d*x+c)^4-3/d*a*b^2*sinh(d*x+c)/cosh(d*x+c)^4+3/4/d*a*b^2*tanh(d*x+c)
*sech(d*x+c)^3+9/8/d*a*b^2*sech(d*x+c)*tanh(d*x+c)+9/4/d*a*b^2*arctan(exp(d
*x+c))-1/d*b^3*sinh(d*x+c)^5/cosh(d*x+c)^6-5/3/d*b^3*sinh(d*x+c)^3/cosh(d*x
+c)^6-1/d*b^3*sinh(d*x+c)/cosh(d*x+c)^6+1/6/d*b^3*tanh(d*x+c)*sech(d*x+c)^5
+5/24*b^3*sech(d*x+c)^3*tanh(d*x+c)/d+5/16*b^3*sech(d*x+c)*tanh(d*x+c)/d+5/
8/d*b^3*arctan(exp(d*x+c))
```

Maxima [B] time = 1.68702, size = 489, normalized size = 3.28

$$-\frac{1}{24}\,b^3\left(\frac{15\,\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{33\,e^{(-dx-c)}-5\,e^{(-3\,dx-3\,c)}+90\,e^{(-5\,dx-5\,c)}-90\,e^{(-7\,dx-7\,c)}+5\,e^{(-9\,dx-9\,c)}-33\,e^{(-11\,dx-1}}{d\left(6\,e^{(-2\,dx-2\,c)}+15\,e^{(-4\,dx-4\,c)}+20\,e^{(-6\,dx-6\,c)}+15\,e^{(-8\,dx-8\,c)}+6\,e^{(-10\,dx-10\,c)}+e^{(-12\,dx-1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/24*b^3*(15*arctan(e^(-d*x - c))/d + (33*e^(-d*x - c) - 5*e^(-3*d*x - 3*c
) + 90*e^(-5*d*x - 5*c) - 90*e^(-7*d*x - 7*c) + 5*e^(-9*d*x - 9*c) - 33*e^(
-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*
x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c)
+ 1))) - 3/4*a*b^2*(3*arctan(e^(-d*x - c))/d + (5*e^(-d*x - c) - 3*e^(-3*d*
x - 3*c) + 3*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c)
+ 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - 3*a^2
*b*(arctan(e^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*
d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a^3*arctan(sinh(d*x + c))/d
```

Fricas [B] time = 2.32246, size = 8768, normalized size = 58.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/24*(3*(24*a^2*b + 30*a*b^2 + 11*b^3)*cosh(d*x + c)^11 + 33*(24*a^2*b + 3
0*a*b^2 + 11*b^3)*cosh(d*x + c)*sinh(d*x + c)^10 + 3*(24*a^2*b + 30*a*b^2 +
 11*b^3)*sinh(d*x + c)^11 + (216*a^2*b + 126*a*b^2 - 5*b^3)*cosh(d*x + c)^9
 + (216*a^2*b + 126*a*b^2 - 5*b^3 + 165*(24*a^2*b + 30*a*b^2 + 11*b^3)*cosh
(d*x + c)^2)*sinh(d*x + c)^9 + 9*(55*(24*a^2*b + 30*a*b^2 + 11*b^3)*cosh(d*
x + c)^3 + (216*a^2*b + 126*a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^8 +
 18*(8*a^2*b + 2*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 18*(55*(24*a^2*b + 30*a*b
^2 + 11*b^3)*cosh(d*x + c)^4 + 8*a^2*b + 2*a*b^2 + 5*b^3 + 2*(216*a^2*b + 1
26*a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 42*(33*(24*a^2*b + 30*
a*b^2 + 11*b^3)*cosh(d*x + c)^5 + 2*(216*a^2*b + 126*a*b^2 - 5*b^3)*cosh(d*
x + c)^3 + 3*(8*a^2*b + 2*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 - 1
8*(8*a^2*b + 2*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 18*(77*(24*a^2*b + 30*a*b^2
 + 11*b^3)*cosh(d*x + c)^6 + 7*(216*a^2*b + 126*a*b^2 - 5*b^3)*cosh(d*x + c
)^4 - 8*a^2*b - 2*a*b^2 - 5*b^3 + 21*(8*a^2*b + 2*a*b^2 + 5*b^3)*cosh(d*x +
 c)^2)*sinh(d*x + c)^5 + 18*(55*(24*a^2*b + 30*a*b^2 + 11*b^3)*cosh(d*x + c
)^7 + 7*(216*a^2*b + 126*a*b^2 - 5*b^3)*cosh(d*x + c)^5 + 35*(8*a^2*b + 2*a
*b^2 + 5*b^3)*cosh(d*x + c)^3 - 5*(8*a^2*b + 2*a*b^2 + 5*b^3)*cosh(d*x + c)
)*sinh(d*x + c)^4 - (216*a^2*b + 126*a*b^2 - 5*b^3)*cosh(d*x + c)^3 + (495*
(24*a^2*b + 30*a*b^2 + 11*b^3)*cosh(d*x + c)^8 + 84*(216*a^2*b + 126*a*b^2
- 5*b^3)*cosh(d*x + c)^6 + 630*(8*a^2*b + 2*a*b^2 + 5*b^3)*cosh(d*x + c)^4
- 216*a^2*b - 126*a*b^2 + 5*b^3 - 180*(8*a^2*b + 2*a*b^2 + 5*b^3)*cosh(d*x
+ c)^2)*sinh(d*x + c)^3 + 3*(55*(24*a^2*b + 30*a*b^2 + 11*b^3)*cosh(d*x + c
)^9 + 12*(216*a^2*b + 126*a*b^2 - 5*b^3)*cosh(d*x + c)^7 + 126*(8*a^2*b + 2
*a*b^2 + 5*b^3)*cosh(d*x + c)^5 - 60*(8*a^2*b + 2*a*b^2 + 5*b^3)*cosh(d*x +
 c)^3 - (216*a^2*b + 126*a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - 3*
((16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^12 + 12*(16*a^3 + 24*
a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)*sinh(d*x + c)^11 + (16*a^3 + 24*a^2
*b + 18*a*b^2 + 5*b^3)*sinh(d*x + c)^12 + 6*(16*a^3 + 24*a^2*b + 18*a*b^2 +
 5*b^3)*cosh(d*x + c)^10 + 6*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3 + 11*(16
*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 20*
(11*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 3*(16*a^3 + 24
*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 + 15*(16*a^3 + 24
*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^8 + 15*(33*(16*a^3 + 24*a^2*b + 18
*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3 + 18
*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 +
24*(33*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 30*(16*a^3
+ 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 5*(16*a^3 + 24*a^2*b + 18*
a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 20*(16*a^3 + 24*a^2*b + 18*
a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 4*(231*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b
```

```
^3)*cosh(d*x + c)^6 + 315*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x +
 c)^4 + 80*a^3 + 120*a^2*b + 90*a*b^2 + 25*b^3 + 105*(16*a^3 + 24*a^2*b + 1
8*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 24*(33*(16*a^3 + 24*a^2
*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 63*(16*a^3 + 24*a^2*b + 18*a*b^2 +
 5*b^3)*cosh(d*x + c)^5 + 35*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*
x + c)^3 + 5*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x
 + c)^5 + 15*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 15*(3
3*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^8 + 84*(16*a^3 + 24*
a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 70*(16*a^3 + 24*a^2*b + 18*a*b^
2 + 5*b^3)*cosh(d*x + c)^4 + 16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3 + 20*(16*
a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 20*(1
1*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^9 + 36*(16*a^3 + 24*
a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 42*(16*a^3 + 24*a^2*b + 18*a*b^
2 + 5*b^3)*cosh(d*x + c)^5 + 20*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh
(d*x + c)^3 + 3*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(
d*x + c)^3 + 16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3 + 6*(16*a^3 + 24*a^2*b +
18*a*b^2 + 5*b^3)*cosh(d*x + c)^2 + 6*(11*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5
*b^3)*cosh(d*x + c)^10 + 45*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x
 + c)^8 + 70*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 50*(1
6*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 16*a^3 + 24*a^2*b +
18*a*b^2 + 5*b^3 + 15*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^2 + 12*((16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x +
c)^11 + 5*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^9 + 10*(16*a
^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 10*(16*a^3 + 24*a^2*b +
 18*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 5*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^
3)*cosh(d*x + c)^3 + (16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c))*
sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - 3*(24*a^2*b + 30*a*b
^2 + 11*b^3)*cosh(d*x + c) + 3*(11*(24*a^2*b + 30*a*b^2 + 11*b^3)*cosh(d*x
+ c)^10 + 3*(216*a^2*b + 126*a*b^2 - 5*b^3)*cosh(d*x + c)^8 + 42*(8*a^2*b +
 2*a*b^2 + 5*b^3)*cosh(d*x + c)^6 - 30*(8*a^2*b + 2*a*b^2 + 5*b^3)*cosh(d*x
 + c)^4 - 24*a^2*b - 30*a*b^2 - 11*b^3 - (216*a^2*b + 126*a*b^2 - 5*b^3)*co
sh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^12 + 12*d*cosh(d*x + c)*sinh
(d*x + c)^11 + d*sinh(d*x + c)^12 + 6*d*cosh(d*x + c)^10 + 6*(11*d*cosh(d*x
 + c)^2 + d)*sinh(d*x + c)^10 + 20*(11*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c
))*sinh(d*x + c)^9 + 15*d*cosh(d*x + c)^8 + 15*(33*d*cosh(d*x + c)^4 + 18*d
*cosh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 24*(33*d*cosh(d*x + c)^5 + 30*d*cos
h(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^7 + 20*d*cosh(d*x + c)^6 +
4*(231*d*cosh(d*x + c)^6 + 315*d*cosh(d*x + c)^4 + 105*d*cosh(d*x + c)^2 +
5*d)*sinh(d*x + c)^6 + 24*(33*d*cosh(d*x + c)^7 + 63*d*cosh(d*x + c)^5 + 35
*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^5 + 15*d*cosh(d*x + c
)^4 + 15*(33*d*cosh(d*x + c)^8 + 84*d*cosh(d*x + c)^6 + 70*d*cosh(d*x + c)^
4 + 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 20*(11*d*cosh(d*x + c)^9 +
36*d*cosh(d*x + c)^7 + 42*d*cosh(d*x + c)^5 + 20*d*cosh(d*x + c)^3 + 3*d*co
sh(d*x + c))*sinh(d*x + c)^3 + 6*d*cosh(d*x + c)^2 + 6*(11*d*cosh(d*x + c)^
10 + 45*d*cosh(d*x + c)^8 + 70*d*cosh(d*x + c)^6 + 50*d*cosh(d*x + c)^4 + 1
5*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 12*(d*cosh(d*x + c)^11 + 5*d*cos
h(d*x + c)^9 + 10*d*cosh(d*x + c)^7 + 10*d*cosh(d*x + c)^5 + 5*d*cosh(d*x +
 c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tanh^2\left(c + dx\right)\right)^3 \operatorname{sech}\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**3*sech(c + d*x), x)
```

Giac [B] time = 1.58934, size = 433, normalized size = 2.91

$$3\left(16\,a^{3}e^{c}+24\,a^{2}be^{c}+18\,ab^{2}e^{c}+5\,b^{3}e^{c}\right)\arctan\left(e^{(dx+c)}\right)e^{(-c)}-\frac{72\,a^{2}be^{(11\,dx+11\,c)}+90\,ab^{2}e^{(11\,dx+11\,c)}+33\,b^{3}e^{(11\,dx+11\,c)}+216\,a^{2}be^{(9\,dx+9\,c)}+1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/24*(3*(16*a^3*e^c + 24*a^2*b*e^c + 18*a*b^2*e^c + 5*b^3*e^c)*arctan(e^(d*
x + c))*e^(-c) - (72*a^2*b*e^(11*d*x + 11*c) + 90*a*b^2*e^(11*d*x + 11*c) +
 33*b^3*e^(11*d*x + 11*c) + 216*a^2*b*e^(9*d*x + 9*c) + 126*a*b^2*e^(9*d*x
+ 9*c) - 5*b^3*e^(9*d*x + 9*c) + 144*a^2*b*e^(7*d*x + 7*c) + 36*a*b^2*e^(7*
d*x + 7*c) + 90*b^3*e^(7*d*x + 7*c) - 144*a^2*b*e^(5*d*x + 5*c) - 36*a*b^2*
e^(5*d*x + 5*c) - 90*b^3*e^(5*d*x + 5*c) - 216*a^2*b*e^(3*d*x + 3*c) - 126*
a*b^2*e^(3*d*x + 3*c) + 5*b^3*e^(3*d*x + 3*c) - 72*a^2*b*e^(d*x + c) - 90*a
*b^2*e^(d*x + c) - 33*b^3*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^6)/d
```

3.102 $\int \operatorname{sech}^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx$

Optimal. Leaf size=67

$$\frac{a^2 b \tanh^3(c+dx)}{d}+\frac{a^3 \tanh(c+dx)}{d}+\frac{3 a b^2 \tanh^5(c+dx)}{5 d}+\frac{b^3 \tanh^7(c+dx)}{7 d}$$

```
[Out] (a^3*Tanh[c + d*x])/d + (a^2*b*Tanh[c + d*x]^3)/d + (3*a*b^2*Tanh[c + d*x]^
5)/(5*d) + (b^3*Tanh[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.0613605, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3675, 194}

$$\frac{a^2 b \tanh^3(c+dx)}{d}+\frac{a^3 \tanh(c+dx)}{d}+\frac{3 a b^2 \tanh^5(c+dx)}{5 d}+\frac{b^3 \tanh^7(c+dx)}{7 d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (a^3*Tanh[c + d*x])/d + (a^2*b*Tanh[c + d*x]^3)/d + (3*a*b^2*Tanh[c + d*x]^
5)/(5*d) + (b^3*Tanh[c + d*x]^7)/(7*d)
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx &= \frac{\operatorname{Subst}\left(\int \left(a+bx^2\right)^3 dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \left(a^3+3a^2bx^2+3ab^2x^4+b^3x^6\right) dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{a^3 \tanh(c+dx)}{d}+\frac{a^2 b \tanh^3(c+dx)}{d}+\frac{3 a b^2 \tanh^5(c+dx)}{5 d}+\frac{b^3 \tanh^7(c+dx)}{7 d}\end{aligned}$$

Mathematica [A] time = 0.167314, size = 67, normalized size = 1.

$$\frac{a^2 b \tanh^3(c+dx)}{d}+\frac{a^3 \tanh(c+dx)}{d}+\frac{3 a b^2 \tanh^5(c+dx)}{5 d}+\frac{b^3 \tanh^7(c+dx)}{7 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (a^3*Tanh[c + d*x])/d + (a^2*b*Tanh[c + d*x]^3)/d + (3*a*b^2*Tanh[c + d*x]^
5)/(5*d) + (b^3*Tanh[c + d*x]^7)/(7*d)
```

Maple [B] time = 0.065, size = 227, normalized size = 3.4

$$\frac{1}{d}\left(a^3\tanh(dx+c)+3a^2b\left(-1/2\frac{\sinh(dx+c)}{(\cosh(dx+c))^3}+1/2\left(2/3+1/3(\operatorname{sech}(dx+c))^2\right)\tanh(dx+c)\right)+3ab^2\left(-1/2\frac{(\sinh(}{(\cosh(}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*tanh(d*x+c)+3*a^2*b*(-1/2*sinh(d*x+c)/cosh(d*x+c)^3+1/2*(2/3+1/3*s
ech(d*x+c)^2)*tanh(d*x+c))+3*a*b^2*(-1/2*sinh(d*x+c)^3/cosh(d*x+c)^5-3/8*si
nh(d*x+c)/cosh(d*x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tan
h(d*x+c))+b^3*(-1/2*sinh(d*x+c)^5/cosh(d*x+c)^7-5/8*sinh(d*x+c)^3/cosh(d*x+
c)^7-5/16*sinh(d*x+c)/cosh(d*x+c)^7+5/16*(16/35+1/7*sech(d*x+c)^6+6/35*sech
(d*x+c)^4+8/35*sech(d*x+c)^2)*tanh(d*x+c)))
```

Maxima [A] time = 1.05402, size = 96, normalized size = 1.43

$$\frac{b^3\tanh(dx+c)^7}{7d}+\frac{3ab^2\tanh(dx+c)^5}{5d}+\frac{a^2b\tanh(dx+c)^3}{d}+\frac{2a^3}{d\left(e^{(-2dx-2c)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/7*b^3*tanh(d*x + c)^7/d + 3/5*a*b^2*tanh(d*x + c)^5/d + a^2*b*tanh(d*x +
c)^3/d + 2*a^3/(d*(e^(-2*d*x - 2*c) + 1))
```

Fricas [B] time = 1.99027, size = 2034, normalized size = 30.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -4/35*((35*a^3 + 70*a^2*b + 63*a*b^2 + 20*b^3)*cosh(d*x + c)^6 + 6*(35*a^2*
b + 42*a*b^2 + 15*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (35*a^3 + 70*a^2*b +
 63*a*b^2 + 20*b^3)*sinh(d*x + c)^6 + 14*(15*a^3 + 20*a^2*b + 9*a*b^2)*cosh
(d*x + c)^4 + (210*a^3 + 280*a^2*b + 126*a*b^2 + 15*(35*a^3 + 70*a^2*b + 63
*a*b^2 + 20*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(35*a^2*b + 42*a*b
^2 + 15*b^3)*cosh(d*x + c)^3 + 28*(5*a^2*b + 3*a*b^2)*cosh(d*x + c))*sinh(d
*x + c)^3 + 350*a^3 + 280*a^2*b + 210*a*b^2 + 7*(75*a^3 + 70*a^2*b + 39*a*b
^2 + 20*b^3)*cosh(d*x + c)^2 + (15*(35*a^3 + 70*a^2*b + 63*a*b^2 + 20*b^3)*
cosh(d*x + c)^4 + 525*a^3 + 490*a^2*b + 273*a*b^2 + 140*b^3 + 84*(15*a^3 +
20*a^2*b + 9*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(35*a^2*b + 42*
```

```
a*b^2 + 15*b^3)*cosh(d*x + c)^5 + 56*(5*a^2*b + 3*a*b^2)*cosh(d*x + c)^3 +
7*(25*a^2*b + 6*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x +
c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 + 8*d*cosh(d*x
 + c)^6 + 4*(7*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^6 + 4*(14*d*cosh(d*x
+ c)^3 + 9*d*cosh(d*x + c))*sinh(d*x + c)^5 + 28*d*cosh(d*x + c)^4 + 2*(35*
d*cosh(d*x + c)^4 + 60*d*cosh(d*x + c)^2 + 14*d)*sinh(d*x + c)^4 + 8*(7*d*c
osh(d*x + c)^5 + 15*d*cosh(d*x + c)^3 + 7*d*cosh(d*x + c))*sinh(d*x + c)^3
+ 56*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 30*d*cosh(d*x + c)^4 + 42
*d*cosh(d*x + c)^2 + 14*d)*sinh(d*x + c)^2 + 4*(2*d*cosh(d*x + c)^7 + 9*d*c
osh(d*x + c)^5 + 14*d*cosh(d*x + c)^3 + 7*d*cosh(d*x + c))*sinh(d*x + c) +
35*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tanh^2\left(c + dx\right)\right)^3 \operatorname{sech}^2\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**3*sech(c + d*x)**2, x)
```

Giac [B] time = 1.70473, size = 468, normalized size = 6.99

$$-\frac{2\left(35\,a^3e^{(12\,dx+12\,c)} + 105\,a^2be^{(12\,dx+12\,c)} + 105\,ab^2e^{(12\,dx+12\,c)} + 35\,b^3e^{(12\,dx+12\,c)} + 210\,a^3e^{(10\,dx+10\,c)} + 420\,a^2be^{(10\,dx+10\,c)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -2/35*(35*a^3*e^(12*d*x + 12*c) + 105*a^2*b*e^(12*d*x + 12*c) + 105*a*b^2*e
^(12*d*x + 12*c) + 35*b^3*e^(12*d*x + 12*c) + 210*a^3*e^(10*d*x + 10*c) + 4
20*a^2*b*e^(10*d*x + 10*c) + 210*a*b^2*e^(10*d*x + 10*c) + 525*a^3*e^(8*d*x
 + 8*c) + 665*a^2*b*e^(8*d*x + 8*c) + 315*a*b^2*e^(8*d*x + 8*c) + 175*b^3*e
^(8*d*x + 8*c) + 700*a^3*e^(6*d*x + 6*c) + 560*a^2*b*e^(6*d*x + 6*c) + 420*
a*b^2*e^(6*d*x + 6*c) + 525*a^3*e^(4*d*x + 4*c) + 315*a^2*b*e^(4*d*x + 4*c)
 + 231*a*b^2*e^(4*d*x + 4*c) + 105*b^3*e^(4*d*x + 4*c) + 210*a^3*e^(2*d*x +
 2*c) + 140*a^2*b*e^(2*d*x + 2*c) + 42*a*b^2*e^(2*d*x + 2*c) + 35*a^3 + 35*
a^2*b + 21*a*b^2 + 5*b^3)/(d*(e^(2*d*x + 2*c) + 1)^7)
```

3.103 $\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=198

$$\frac{(48a^2b+64a^3+24ab^2+5b^3)\tan^{-1}(\sinh(c+dx))}{128d}-\frac{b(72a^2+52ab+15b^2)\tanh(c+dx)\operatorname{sech}^3(c+dx)}{192d}+\frac{(48a^2b+64a$$

```
[Out] ((64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*ArcTan[Sinh[c + d*x]])/(128*d) + ((
64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*Sech[c + d*x]*Tanh[c + d*x])/(128*d)
- (b*(72*a^2 + 52*a*b + 15*b^2)*Sech[c + d*x]^3*Tanh[c + d*x])/(192*d) - (b
*(12*a + 5*b)*Sech[c + d*x]^5*(a + (a + b)*Sinh[c + d*x]^2)*Tanh[c + d*x])/
(48*d) - (b*Sech[c + d*x]^7*(a + (a + b)*Sinh[c + d*x]^2)^2*Tanh[c + d*x])/
(8*d)
```

Rubi [A] time = 0.239466, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3676, 413, 526, 385, 199, 203}

$$\frac{(48a^2b+64a^3+24ab^2+5b^3)\tan^{-1}(\sinh(c+dx))}{128d}-\frac{b(72a^2+52ab+15b^2)\tanh(c+dx)\operatorname{sech}^3(c+dx)}{192d}+\frac{(48a^2b+64a$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*ArcTan[Sinh[c + d*x]])/(128*d) + ((
64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*Sech[c + d*x]*Tanh[c + d*x])/(128*d)
- (b*(72*a^2 + 52*a*b + 15*b^2)*Sech[c + d*x]^3*Tanh[c + d*x])/(192*d) - (b
*(12*a + 5*b)*Sech[c + d*x]^5*(a + (a + b)*Sinh[c + d*x]^2)*Tanh[c + d*x])/
(48*d) - (b*Sech[c + d*x]^7*(a + (a + b)*Sinh[c + d*x]^2)^2*Tanh[c + d*x])/
(8*d)
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
```

```
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+(a+b)x^2\right)^3}{\left(1+x^2\right)^5}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{b\operatorname{sech}^7(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)^2\tanh(c+dx)}{8d} + \frac{\operatorname{Subst}\left(\int\right.}{}$$

$$= -\frac{b(12a+5b)\operatorname{sech}^5(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)\tanh(c+dx)}{48d} -$$

$$= -\frac{b\left(72a^2+52ab+15b^2\right)\operatorname{sech}^3(c+dx)\tanh(c+dx)}{192d} - \frac{b(12a+5b)\operatorname{sec}}{}$$

$$= \frac{\left(64a^3+48a^2b+24ab^2+5b^3\right)\operatorname{sech}(c+dx)\tanh(c+dx)}{128d} - \frac{b\left(72a^2+5\right.}{}$$

$$= \frac{\left(64a^3+48a^2b+24ab^2+5b^3\right)\tan^{-1}(\sinh(c+dx))}{128d} + \frac{\left(64a^3+48a^2b+\right.}{}$$

Mathematica [A] time = 11.6773, size = 158, normalized size = 0.8

$$\frac{6\left(48a^2b+64a^3+24ab^2+5b^3\right)\tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) - 2b\left(144a^2+168ab+59b^2\right)\tanh(c+dx)\operatorname{sech}^3(c+dx) +}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (6*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*ArcTan[Tanh[(c + d*x)/2]] + 3*(64
*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*Sech[c + d*x]*Tanh[c + d*x] - 2*b*(144*
```

```
a^2 + 168*a*b + 59*b^2)*Sech[c + d*x]^3*Tanh[c + d*x] + 8*b^2*(24*a + 17*b)
*Sech[c + d*x]^5*Tanh[c + d*x] - 48*b^3*Sech[c + d*x]^7*Tanh[c + d*x])/(384
*d)
```

Maple [B] time = 0.071, size = 421, normalized size = 2.1

$$\frac{a^3\mathrm{sech}\,(dx+c)\tanh\,(dx+c)}{2\,d}+\frac{a^3\arctan\left(\mathrm{e}^{dx+c}\right)}{d}-\frac{a^2b\sinh\,(dx+c)}{d\,(\cosh\,(dx+c))^4}+\frac{a^2b\tanh\,(dx+c)\,(\mathrm{sech}\,(dx+c))^3}{4\,d}+\frac{3\,a^2b\mathrm{sech}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 1/2/d*a^3*sech(d*x+c)*tanh(d*x+c)+1/d*a^3*arctan(exp(d*x+c))-1/d*a^2*b*sinh
(d*x+c)/cosh(d*x+c)^4+1/4/d*a^2*b*tanh(d*x+c)*sech(d*x+c)^3+3/8*a^2*b*sech(
d*x+c)*tanh(d*x+c)/d+3/4/d*a^2*b*arctan(exp(d*x+c))-1/d*a*b^2*sinh(d*x+c)^3
/cosh(d*x+c)^6-3/5/d*a*b^2*sinh(d*x+c)/cosh(d*x+c)^6+1/10/d*a*b^2*tanh(d*x+
c)*sech(d*x+c)^5+1/8/d*a*b^2*tanh(d*x+c)*sech(d*x+c)^3+3/16/d*a*b^2*sech(d*
x+c)*tanh(d*x+c)+3/8/d*a*b^2*arctan(exp(d*x+c))-1/3/d*b^3*sinh(d*x+c)^5/cos
h(d*x+c)^8-1/3/d*b^3*sinh(d*x+c)^3/cosh(d*x+c)^8-1/7/d*b^3*sinh(d*x+c)/cosh
(d*x+c)^8+1/56/d*b^3*tanh(d*x+c)*sech(d*x+c)^7+1/48/d*b^3*tanh(d*x+c)*sech(
d*x+c)^5+5/192*b^3*sech(d*x+c)^3*tanh(d*x+c)/d+5/128*b^3*sech(d*x+c)*tanh(d
*x+c)/d+5/64/d*b^3*arctan(exp(d*x+c))
```

Maxima [B] time = 1.66783, size = 747, normalized size = 3.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/192*b^3*(15*arctan(e^(-d*x - c))/d - (15*e^(-d*x - c) - 397*e^(-3*d*x -
3*c) + 895*e^(-5*d*x - 5*c) - 1765*e^(-7*d*x - 7*c) + 1765*e^(-9*d*x - 9*c)
 - 895*e^(-11*d*x - 11*c) + 397*e^(-13*d*x - 13*c) - 15*e^(-15*d*x - 15*c))
/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^
(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d
*x - 14*c) + e^(-16*d*x - 16*c) + 1))) - 1/8*a*b^2*(3*arctan(e^(-d*x - c))/
d - (3*e^(-d*x - c) - 47*e^(-3*d*x - 3*c) + 78*e^(-5*d*x - 5*c) - 78*e^(-7*
d*x - 7*c) + 47*e^(-9*d*x - 9*c) - 3*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x -
2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*
e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c) + 1))) - 3/4*a^2*b*(arctan(e^(-d*x
- c))/d - (e^(-d*x - c) - 7*e^(-3*d*x - 3*c) + 7*e^(-5*d*x - 5*c) - e^(-7*d
*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c)
 + e^(-8*d*x - 8*c) + 1))) - a^3*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) -
e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1)))
```

Fricas [B] time = 2.56761, size = 16058, normalized size = 81.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/192*(3*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^15 + 45*(64*a
^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)*sinh(d*x + c)^14 + 3*(64*a^
3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*sinh(d*x + c)^15 + (960*a^3 - 432*a^2*b -
984*a*b^2 - 397*b^3)*cosh(d*x + c)^13 + (960*a^3 - 432*a^2*b - 984*a*b^2 -
397*b^3 + 315*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(
d*x + c)^13 + 13*(105*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^
3 + (960*a^3 - 432*a^2*b - 984*a*b^2 - 397*b^3)*cosh(d*x + c))*sinh(d*x + c
)^12 + (1728*a^3 - 2160*a^2*b - 312*a*b^2 + 895*b^3)*cosh(d*x + c)^11 + (40
95*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 1728*a^3 - 2160
*a^2*b - 312*a*b^2 + 895*b^3 + 78*(960*a^3 - 432*a^2*b - 984*a*b^2 - 397*b^
3)*cosh(d*x + c)^2)*sinh(d*x + c)^11 + 11*(819*(64*a^3 + 48*a^2*b + 24*a*b^
2 + 5*b^3)*cosh(d*x + c)^5 + 26*(960*a^3 - 432*a^2*b - 984*a*b^2 - 397*b^3)
*cosh(d*x + c)^3 + (1728*a^3 - 2160*a^2*b - 312*a*b^2 + 895*b^3)*cosh(d*x +
 c))*sinh(d*x + c)^10 + (960*a^3 - 1584*a^2*b + 744*a*b^2 - 1765*b^3)*cosh(
d*x + c)^9 + (15015*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^6
+ 715*(960*a^3 - 432*a^2*b - 984*a*b^2 - 397*b^3)*cosh(d*x + c)^4 + 960*a^3
 - 1584*a^2*b + 744*a*b^2 - 1765*b^3 + 55*(1728*a^3 - 2160*a^2*b - 312*a*b^
2 + 895*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^9 + 3*(6435*(64*a^3 + 48*a^2*b
+ 24*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 429*(960*a^3 - 432*a^2*b - 984*a*b^2
- 397*b^3)*cosh(d*x + c)^5 + 55*(1728*a^3 - 2160*a^2*b - 312*a*b^2 + 895*b^
3)*cosh(d*x + c)^3 + 3*(960*a^3 - 1584*a^2*b + 744*a*b^2 - 1765*b^3)*cosh(d
*x + c))*sinh(d*x + c)^8 - (960*a^3 - 1584*a^2*b + 744*a*b^2 - 1765*b^3)*co
sh(d*x + c)^7 + (19305*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)
^8 + 1716*(960*a^3 - 432*a^2*b - 984*a*b^2 - 397*b^3)*cosh(d*x + c)^6 + 330
*(1728*a^3 - 2160*a^2*b - 312*a*b^2 + 895*b^3)*cosh(d*x + c)^4 - 960*a^3 +
1584*a^2*b - 744*a*b^2 + 1765*b^3 + 36*(960*a^3 - 1584*a^2*b + 744*a*b^2 -
1765*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + (15015*(64*a^3 + 48*a^2*b + 24
*a*b^2 + 5*b^3)*cosh(d*x + c)^9 + 1716*(960*a^3 - 432*a^2*b - 984*a*b^2 - 3
97*b^3)*cosh(d*x + c)^7 + 462*(1728*a^3 - 2160*a^2*b - 312*a*b^2 + 895*b^3)
*cosh(d*x + c)^5 + 84*(960*a^3 - 1584*a^2*b + 744*a*b^2 - 1765*b^3)*cosh(d*
x + c)^3 - 7*(960*a^3 - 1584*a^2*b + 744*a*b^2 - 1765*b^3)*cosh(d*x + c))*s
inh(d*x + c)^6 - (1728*a^3 - 2160*a^2*b - 312*a*b^2 + 895*b^3)*cosh(d*x + c
)^5 + (9009*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^10 + 1287*
(960*a^3 - 432*a^2*b - 984*a*b^2 - 397*b^3)*cosh(d*x + c)^8 + 462*(1728*a^3
 - 2160*a^2*b - 312*a*b^2 + 895*b^3)*cosh(d*x + c)^6 + 126*(960*a^3 - 1584*
a^2*b + 744*a*b^2 - 1765*b^3)*cosh(d*x + c)^4 - 1728*a^3 + 2160*a^2*b + 312
*a*b^2 - 895*b^3 - 21*(960*a^3 - 1584*a^2*b + 744*a*b^2 - 1765*b^3)*cosh(d*
x + c)^2)*sinh(d*x + c)^5 + (4095*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*co
sh(d*x + c)^11 + 715*(960*a^3 - 432*a^2*b - 984*a*b^2 - 397*b^3)*cosh(d*x +
 c)^9 + 330*(1728*a^3 - 2160*a^2*b - 312*a*b^2 + 895*b^3)*cosh(d*x + c)^7 +
 126*(960*a^3 - 1584*a^2*b + 744*a*b^2 - 1765*b^3)*cosh(d*x + c)^5 - 35*(96
0*a^3 - 1584*a^2*b + 744*a*b^2 - 1765*b^3)*cosh(d*x + c)^3 - 5*(1728*a^3 -
2160*a^2*b - 312*a*b^2 + 895*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 - (960*a^3
 - 432*a^2*b - 984*a*b^2 - 397*b^3)*cosh(d*x + c)^3 + (1365*(64*a^3 + 48*a^
2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^12 + 286*(960*a^3 - 432*a^2*b - 984*a
*b^2 - 397*b^3)*cosh(d*x + c)^10 + 165*(1728*a^3 - 2160*a^2*b - 312*a*b^2 +
 895*b^3)*cosh(d*x + c)^8 + 84*(960*a^3 - 1584*a^2*b + 744*a*b^2 - 1765*b^3
)*cosh(d*x + c)^6 - 35*(960*a^3 - 1584*a^2*b + 744*a*b^2 - 1765*b^3)*cosh(d
*x + c)^4 - 960*a^3 + 432*a^2*b + 984*a*b^2 + 397*b^3 - 10*(1728*a^3 - 2160
*a^2*b - 312*a*b^2 + 895*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + (315*(64*a
^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^13 + 78*(960*a^3 - 432*a^2*
b - 984*a*b^2 - 397*b^3)*cosh(d*x + c)^11 + 55*(1728*a^3 - 2160*a^2*b - 312
*a*b^2 + 895*b^3)*cosh(d*x + c)^9 + 36*(960*a^3 - 1584*a^2*b + 744*a*b^2 -
1765*b^3)*cosh(d*x + c)^7 - 21*(960*a^3 - 1584*a^2*b + 744*a*b^2 - 1765*b^3
)*cosh(d*x + c)^5 - 10*(1728*a^3 - 2160*a^2*b - 312*a*b^2 + 895*b^3)*cosh(d
*x + c)^3 - 3*(960*a^3 - 432*a^2*b - 984*a*b^2 - 397*b^3)*cosh(d*x + c))*si
nh(d*x + c)^2 + 3*((64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^16
```

+ 16*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)*sinh(d*x + c)^15
+ (64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*sinh(d*x + c)^16 + 8*(64*a^3 + 48*
a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^14 + 8*(64*a^3 + 48*a^2*b + 24*a*b^
2 + 5*b^3 + 15*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh
(d*x + c)^14 + 112*(5*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^
3 + (64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^13
+ 28*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^12 + 28*(65*(64*a
^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 64*a^3 + 48*a^2*b + 24*
a*b^2 + 5*b^3 + 26*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*
sinh(d*x + c)^12 + 112*(39*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x
+ c)^5 + 26*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 3*(64*
a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^11 + 56*(64
*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^10 + 56*(143*(64*a^3 + 48
*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 143*(64*a^3 + 48*a^2*b + 24*a*
b^2 + 5*b^3)*cosh(d*x + c)^4 + 64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3 + 33*(6
4*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 16
(715(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 1001*(64*a^3
 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 385*(64*a^3 + 48*a^2*b +
24*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 35*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^
3)*cosh(d*x + c))*sinh(d*x + c)^9 + 70*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^
3)*cosh(d*x + c)^8 + 2*(6435*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*
x + c)^8 + 12012*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 6
930*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 2240*a^3 + 168
0*a^2*b + 840*a*b^2 + 175*b^3 + 1260*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)
*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 16*(715*(64*a^3 + 48*a^2*b + 24*a*b^2 +
 5*b^3)*cosh(d*x + c)^9 + 1716*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(
d*x + c)^7 + 1386*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^5 +
420*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 35*(64*a^3 + 4
8*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 56*(64*a^3 + 4
8*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 56*(143*(64*a^3 + 48*a^2*b +
24*a*b^2 + 5*b^3)*cosh(d*x + c)^10 + 429*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*
b^3)*cosh(d*x + c)^8 + 462*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x
+ c)^6 + 210*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 64*a^
3 + 48*a^2*b + 24*a*b^2 + 5*b^3 + 35*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)
*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 112*(39*(64*a^3 + 48*a^2*b + 24*a*b^2 +
 5*b^3)*cosh(d*x + c)^11 + 143*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(
d*x + c)^9 + 198*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 1
26*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 35*(64*a^3 + 48
*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 3*(64*a^3 + 48*a^2*b + 24*a*b^
2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 28*(64*a^3 + 48*a^2*b + 24*a*b^
2 + 5*b^3)*cosh(d*x + c)^4 + 28*(65*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*
cosh(d*x + c)^12 + 286*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)
^10 + 495*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^8 + 420*(64*
a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 175*(64*a^3 + 48*a^2*b
 + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3
 + 30*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)
^4 + 112*(5*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^13 + 26*(6
4*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^11 + 55*(64*a^3 + 48*a^2
*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^9 + 60*(64*a^3 + 48*a^2*b + 24*a*b^2 +
 5*b^3)*cosh(d*x + c)^7 + 35*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*
x + c)^5 + 10*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + (64*
a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 64*a^3
+ 48*a^2*b + 24*a*b^2 + 5*b^3 + 8*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*co
sh(d*x + c)^2 + 8*(15*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^
14 + 91*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^12 + 231*(64*a
^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^10 + 315*(64*a^3 + 48*a^2*b
 + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^8 + 245*(64*a^3 + 48*a^2*b + 24*a*b^2 +
5*b^3)*cosh(d*x + c)^6 + 105*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*
x + c)^4 + 64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3 + 21*(64*a^3 + 48*a^2*b + 2

```
4*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 16*((64*a^3 + 48*a^2*b
+ 24*a*b^2 + 5*b^3)*cosh(d*x + c)^15 + 7*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*
b^3)*cosh(d*x + c)^13 + 21*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x
+ c)^11 + 35*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^9 + 35*(6
4*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 21*(64*a^3 + 48*a^2*
b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 7*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5
*b^3)*cosh(d*x + c)^3 + (64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c
))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - 3*(64*a^3 + 48*a^
2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c) + (45*(64*a^3 + 48*a^2*b + 24*a*b^2 +
 5*b^3)*cosh(d*x + c)^14 + 13*(960*a^3 - 432*a^2*b - 984*a*b^2 - 397*b^3)*c
osh(d*x + c)^12 + 11*(1728*a^3 - 2160*a^2*b - 312*a*b^2 + 895*b^3)*cosh(d*x
 + c)^10 + 9*(960*a^3 - 1584*a^2*b + 744*a*b^2 - 1765*b^3)*cosh(d*x + c)^8
- 7*(960*a^3 - 1584*a^2*b + 744*a*b^2 - 1765*b^3)*cosh(d*x + c)^6 - 5*(1728
*a^3 - 2160*a^2*b - 312*a*b^2 + 895*b^3)*cosh(d*x + c)^4 - 192*a^3 - 144*a^
2*b - 72*a*b^2 - 15*b^3 - 3*(960*a^3 - 432*a^2*b - 984*a*b^2 - 397*b^3)*cos
h(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^16 + 16*d*cosh(d*x + c)*sinh(
d*x + c)^15 + d*sinh(d*x + c)^16 + 8*d*cosh(d*x + c)^14 + 8*(15*d*cosh(d*x
+ c)^2 + d)*sinh(d*x + c)^14 + 112*(5*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*
sinh(d*x + c)^13 + 28*d*cosh(d*x + c)^12 + 28*(65*d*cosh(d*x + c)^4 + 26*d*
cosh(d*x + c)^2 + d)*sinh(d*x + c)^12 + 112*(39*d*cosh(d*x + c)^5 + 26*d*co
sh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^11 + 56*d*cosh(d*x + c)^10
 + 56*(143*d*cosh(d*x + c)^6 + 143*d*cosh(d*x + c)^4 + 33*d*cosh(d*x + c)^2
 + d)*sinh(d*x + c)^10 + 16*(715*d*cosh(d*x + c)^7 + 1001*d*cosh(d*x + c)^5
 + 385*d*cosh(d*x + c)^3 + 35*d*cosh(d*x + c))*sinh(d*x + c)^9 + 70*d*cosh(
d*x + c)^8 + 2*(6435*d*cosh(d*x + c)^8 + 12012*d*cosh(d*x + c)^6 + 6930*d*c
osh(d*x + c)^4 + 1260*d*cosh(d*x + c)^2 + 35*d)*sinh(d*x + c)^8 + 16*(715*d
*cosh(d*x + c)^9 + 1716*d*cosh(d*x + c)^7 + 1386*d*cosh(d*x + c)^5 + 420*d*
cosh(d*x + c)^3 + 35*d*cosh(d*x + c))*sinh(d*x + c)^7 + 56*d*cosh(d*x + c)^
6 + 56*(143*d*cosh(d*x + c)^10 + 429*d*cosh(d*x + c)^8 + 462*d*cosh(d*x + c
)^6 + 210*d*cosh(d*x + c)^4 + 35*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 1
12*(39*d*cosh(d*x + c)^11 + 143*d*cosh(d*x + c)^9 + 198*d*cosh(d*x + c)^7 +
 126*d*cosh(d*x + c)^5 + 35*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x
 + c)^5 + 28*d*cosh(d*x + c)^4 + 28*(65*d*cosh(d*x + c)^12 + 286*d*cosh(d*x
 + c)^10 + 495*d*cosh(d*x + c)^8 + 420*d*cosh(d*x + c)^6 + 175*d*cosh(d*x +
 c)^4 + 30*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 112*(5*d*cosh(d*x + c)^
13 + 26*d*cosh(d*x + c)^11 + 55*d*cosh(d*x + c)^9 + 60*d*cosh(d*x + c)^7 +
35*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c
)^3 + 8*d*cosh(d*x + c)^2 + 8*(15*d*cosh(d*x + c)^14 + 91*d*cosh(d*x + c)^1
2 + 231*d*cosh(d*x + c)^10 + 315*d*cosh(d*x + c)^8 + 245*d*cosh(d*x + c)^6
+ 105*d*cosh(d*x + c)^4 + 21*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 16*(d
*cosh(d*x + c)^15 + 7*d*cosh(d*x + c)^13 + 21*d*cosh(d*x + c)^11 + 35*d*cos
h(d*x + c)^9 + 35*d*cosh(d*x + c)^7 + 21*d*cosh(d*x + c)^5 + 7*d*cosh(d*x +
 c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tanh^2\left(c + dx\right)\right)^3 \operatorname{sech}^3\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**3*sech(c + d*x)**3, x)
```

Giac [B] time = 1.70428, size = 698, normalized size = 3.53

$$3\left(64\,a^3e^c+48\,a^2be^c+24\,ab^2e^c+5\,b^3e^c\right)\arctan\left(e^{(dx+c)}\right)e^{(-c)}+\frac{192\,a^3e^{(15\,dx+15\,c)}+144\,a^2be^{(15\,dx+15\,c)}+72\,ab^2e^{(15\,dx+15\,c)}+15\,b^3e^{(15\,dx+15\,c)}\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/192*(3*(64*a^3*e^c + 48*a^2*b*e^c + 24*a*b^2*e^c + 5*b^3*e^c)*arctan(e^(d
*x + c))*e^(-c) + (192*a^3*e^(15*d*x + 15*c) + 144*a^2*b*e^(15*d*x + 15*c)
+ 72*a*b^2*e^(15*d*x + 15*c) + 15*b^3*e^(15*d*x + 15*c) + 960*a^3*e^(13*d*x
 + 13*c) - 432*a^2*b*e^(13*d*x + 13*c) - 984*a*b^2*e^(13*d*x + 13*c) - 397*
b^3*e^(13*d*x + 13*c) + 1728*a^3*e^(11*d*x + 11*c) - 2160*a^2*b*e^(11*d*x +
 11*c) - 312*a*b^2*e^(11*d*x + 11*c) + 895*b^3*e^(11*d*x + 11*c) + 960*a^3*
e^(9*d*x + 9*c) - 1584*a^2*b*e^(9*d*x + 9*c) + 744*a*b^2*e^(9*d*x + 9*c) -
1765*b^3*e^(9*d*x + 9*c) - 960*a^3*e^(7*d*x + 7*c) + 1584*a^2*b*e^(7*d*x +
7*c) - 744*a*b^2*e^(7*d*x + 7*c) + 1765*b^3*e^(7*d*x + 7*c) - 1728*a^3*e^(5
*d*x + 5*c) + 2160*a^2*b*e^(5*d*x + 5*c) + 312*a*b^2*e^(5*d*x + 5*c) - 895*
b^3*e^(5*d*x + 5*c) - 960*a^3*e^(3*d*x + 3*c) + 432*a^2*b*e^(3*d*x + 3*c) +
 984*a*b^2*e^(3*d*x + 3*c) + 397*b^3*e^(3*d*x + 3*c) - 192*a^3*e^(d*x + c)
- 144*a^2*b*e^(d*x + c) - 72*a*b^2*e^(d*x + c) - 15*b^3*e^(d*x + c))/(e^(2*
d*x + 2*c) + 1)^8)/d
```

3.104 $\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=102

$$-\frac{a^2(a-3b)\tanh^3(c+dx)}{3d}+\frac{a^3\tanh(c+dx)}{d}-\frac{b^2(3a-b)\tanh^7(c+dx)}{7d}-\frac{3ab(a-b)\tanh^5(c+dx)}{5d}-\frac{b^3\tanh^9(c+}{9d}$$

```
[Out] (a^3*Tanh[c + d*x])/d - (a^2*(a - 3*b)*Tanh[c + d*x]^3)/(3*d) - (3*a*(a - b
)*b*Tanh[c + d*x]^5)/(5*d) - ((3*a - b)*b^2*Tanh[c + d*x]^7)/(7*d) - (b^3*T
anh[c + d*x]^9)/(9*d)
```

Rubi [A] time = 0.0889386, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3675, 373}

$$-\frac{a^2(a-3b)\tanh^3(c+dx)}{3d}+\frac{a^3\tanh(c+dx)}{d}-\frac{b^2(3a-b)\tanh^7(c+dx)}{7d}-\frac{3ab(a-b)\tanh^5(c+dx)}{5d}-\frac{b^3\tanh^9(c+}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (a^3*Tanh[c + d*x])/d - (a^2*(a - 3*b)*Tanh[c + d*x]^3)/(3*d) - (3*a*(a - b
)*b*Tanh[c + d*x]^5)/(5*d) - ((3*a - b)*b^2*Tanh[c + d*x]^7)/(7*d) - (b^3*T
anh[c + d*x]^9)/(9*d)
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx &= \frac{\operatorname{Subst}\left(\int\left(1-x^2\right)\left(a+bx^2\right)^3\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int\left(a^3-a^2(a-3b)x^2-3a(a-b)bx^4-(3a-b)b^2x^6-b^3x^8\right)dx,x\right.}{d}\\ &= \frac{a^3\tanh(c+dx)}{d}-\frac{a^2(a-3b)\tanh^3(c+dx)}{3d}-\frac{3a(a-b)b\tanh^5(c+dx}{5d}\end{aligned}$$

Mathematica [B] time = 0.825721, size = 218, normalized size = 2.14

$$\tanh(c+dx)\operatorname{sech}^8(c+dx)\left(10\left(-63a^2b+903a^3-27ab^2+107b^3\right)\cosh(2(c+dx))+8\left(126a^2b+525a^3-81ab^2-50\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((5775*a^3 - 1071*a^2*b + 621*a*b^2 - 725*b^3 + 10*(903*a^3 - 63*a^2*b - 27
*a*b^2 + 107*b^3)*Cosh[2*(c + d*x)] + 8*(525*a^3 + 126*a^2*b - 81*a*b^2 - 5
0*b^3)*Cosh[4*(c + d*x)] + 1050*a^3*Cosh[6*(c + d*x)] + 630*a^2*b*Cosh[6*(c
 + d*x)] + 270*a*b^2*Cosh[6*(c + d*x)] + 50*b^3*Cosh[6*(c + d*x)] + 105*a^3
*Cosh[8*(c + d*x)] + 63*a^2*b*Cosh[8*(c + d*x)] + 27*a*b^2*Cosh[8*(c + d*x)
] + 5*b^3*Cosh[8*(c + d*x)])*Sech[c + d*x]^8*Tanh[c + d*x])/(20160*d)
```

Maple [B] time = 0.124, size = 269, normalized size = 2.6

$$\frac{1}{d}\left(a^3\left(\frac{2}{3}+\frac{(\operatorname{sech}(dx+c))^2}{3}\right)\tanh(dx+c)+3a^2b\left(-1/4\frac{\sinh(dx+c)}{(\cosh(dx+c))^5}+1/4\left(\frac{8}{15}+1/5(\operatorname{sech}(dx+c))^4+\frac{4(\operatorname{sech}(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+3*a^2*b*(-1/4*sinh(d*x+c)/cosh
(d*x+c)^5+1/4*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c))+3*a*
b^2*(-1/4*sinh(d*x+c)^3/cosh(d*x+c)^7-1/8*sinh(d*x+c)/cosh(d*x+c)^7+1/8*(16
/35+1/7*sech(d*x+c)^6+6/35*sech(d*x+c)^4+8/35*sech(d*x+c)^2)*tanh(d*x+c))+b
^3*(-1/4*sinh(d*x+c)^5/cosh(d*x+c)^9-5/24*sinh(d*x+c)^3/cosh(d*x+c)^9-5/64*
sinh(d*x+c)/cosh(d*x+c)^9+5/64*(128/315+1/9*sech(d*x+c)^8+8/63*sech(d*x+c)^
6+16/105*sech(d*x+c)^4+64/315*sech(d*x+c)^2)*tanh(d*x+c)))
```

Maxima [B] time = 1.17249, size = 2493, normalized size = 24.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 4/63*b^3*(9*e^(-2*d*x - 2*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) +
 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e
^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d
*x - 18*c) + 1)) - 27*e^(-4*d*x - 4*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*
x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10
*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c)
+ e^(-18*d*x - 18*c) + 1)) + 189*e^(-6*d*x - 6*c)/(d*(9*e^(-2*d*x - 2*c) +
36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-
10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d
*x - 16*c) + e^(-18*d*x - 18*c) + 1)) - 189*e^(-8*d*x - 8*c)/(d*(9*e^(-2*d*
x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c)
 + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) +
 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) + 315*e^(-10*d*x - 10*c)/(
d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(
-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*
d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) - 105*e^(-12*
d*x - 12*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6
*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c)
```

```
 + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1))
+ 63*e^(-14*d*x - 14*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e
^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12
*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x -
18*c) + 1)) + 1/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x
 - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 1
2*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) +
1))) + 12/35*a*b^2*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*
x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c
) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 14*e^(-4*d*x - 4*c)/(
d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-
8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x -
14*c) + 1)) + 70*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4
*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7
*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 35*e^(-8*d*x - 8*c)/(d*(7*
e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x
 - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c)
 + 1)) + 35*e^(-10*d*x - 10*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c)
 + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^
(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*x - 2*c) + 21
*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d
*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 4/5*a^2*b*(
5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*
d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) - 5*e^(-4*d*x -
4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5
*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*e^(-6*d*x - 6*c)/(d*(5*e^
(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x -
8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x -
 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))
) + 4/3*a^3*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c)
 + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) +
 e^(-6*d*x - 6*c) + 1)))
```

Fricas [B] time = 2.01396, size = 3218, normalized size = 31.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -8/315*(2*(105*a^3 + 252*a^2*b + 243*a*b^2 + 80*b^3)*cosh(d*x + c)^7 + 14*(
105*a^3 + 252*a^2*b + 243*a*b^2 + 80*b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + (
105*a^3 + 441*a^2*b + 459*a*b^2 + 155*b^3)*sinh(d*x + c)^7 + 6*(245*a^3 + 3
36*a^2*b + 99*a*b^2 - 40*b^3)*cosh(d*x + c)^5 + 3*(175*a^3 + 483*a^2*b + 11
7*a*b^2 - 95*b^3 + 7*(105*a^3 + 441*a^2*b + 459*a*b^2 + 155*b^3)*cosh(d*x +
 c)^2)*sinh(d*x + c)^5 + 10*(7*(105*a^3 + 252*a^2*b + 243*a*b^2 + 80*b^3)*c
osh(d*x + c)^3 + 3*(245*a^3 + 336*a^2*b + 99*a*b^2 - 40*b^3)*cosh(d*x + c))
*sinh(d*x + c)^4 + 18*(245*a^3 + 168*a^2*b + 27*a*b^2 + 40*b^3)*cosh(d*x +
c)^3 + (35*(105*a^3 + 441*a^2*b + 459*a*b^2 + 155*b^3)*cosh(d*x + c)^4 + 94
5*a^3 + 1701*a^2*b + 459*a*b^2 + 855*b^3 + 30*(175*a^3 + 483*a^2*b + 117*a*
b^2 - 95*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 6*(7*(105*a^3 + 252*a^2*b
+ 243*a*b^2 + 80*b^3)*cosh(d*x + c)^5 + 10*(245*a^3 + 336*a^2*b + 99*a*b^2
- 40*b^3)*cosh(d*x + c)^3 + 9*(245*a^3 + 168*a^2*b + 27*a*b^2 + 40*b^3)*cos
h(d*x + c))*sinh(d*x + c)^2 + 210*(35*a^3 + 12*a^2*b + 9*a*b^2)*cosh(d*x +
c) + (7*(105*a^3 + 441*a^2*b + 459*a*b^2 + 155*b^3)*cosh(d*x + c)^6 + 15*(1
75*a^3 + 483*a^2*b + 117*a*b^2 - 95*b^3)*cosh(d*x + c)^4 + 525*a^3 + 693*a^
```

```
2*b + 567*a*b^2 - 945*b^3 + 27*(105*a^3 + 189*a^2*b + 51*a*b^2 + 95*b^3)*co
sh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^11 + 11*d*cosh(d*x + c)*sinh
(d*x + c)^10 + d*sinh(d*x + c)^11 + 9*d*cosh(d*x + c)^9 + (55*d*cosh(d*x +
c)^2 + 9*d)*sinh(d*x + c)^9 + 3*(55*d*cosh(d*x + c)^3 + 27*d*cosh(d*x + c))
*sinh(d*x + c)^8 + 37*d*cosh(d*x + c)^7 + (330*d*cosh(d*x + c)^4 + 324*d*co
sh(d*x + c)^2 + 35*d)*sinh(d*x + c)^7 + 7*(66*d*cosh(d*x + c)^5 + 108*d*cos
h(d*x + c)^3 + 37*d*cosh(d*x + c))*sinh(d*x + c)^6 + 93*d*cosh(d*x + c)^5 +
 3*(154*d*cosh(d*x + c)^6 + 378*d*cosh(d*x + c)^4 + 245*d*cosh(d*x + c)^2 +
 25*d)*sinh(d*x + c)^5 + (330*d*cosh(d*x + c)^7 + 1134*d*cosh(d*x + c)^5 +
1295*d*cosh(d*x + c)^3 + 465*d*cosh(d*x + c))*sinh(d*x + c)^4 + 162*d*cosh(
d*x + c)^3 + (165*d*cosh(d*x + c)^8 + 756*d*cosh(d*x + c)^6 + 1225*d*cosh(d
*x + c)^4 + 750*d*cosh(d*x + c)^2 + 90*d)*sinh(d*x + c)^3 + (55*d*cosh(d*x
+ c)^9 + 324*d*cosh(d*x + c)^7 + 777*d*cosh(d*x + c)^5 + 930*d*cosh(d*x + c
)^3 + 486*d*cosh(d*x + c))*sinh(d*x + c)^2 + 210*d*cosh(d*x + c) + (11*d*co
sh(d*x + c)^10 + 81*d*cosh(d*x + c)^8 + 245*d*cosh(d*x + c)^6 + 375*d*cosh(
d*x + c)^4 + 270*d*cosh(d*x + c)^2 + 42*d)*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tanh^2\left(c + dx\right)\right)^3 \operatorname{sech}^4\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**3*sech(c + d*x)**4, x)
```

Giac [B] time = 1.82701, size = 603, normalized size = 5.91

$$-\frac{4\left(315\,a^3e^{(14\,dx+14\,c)} + 945\,a^2be^{(14\,dx+14\,c)} + 945\,ab^2e^{(14\,dx+14\,c)} + 315\,b^3e^{(14\,dx+14\,c)} + 1995\,a^3e^{(12\,dx+12\,c)} + 3465\,a^2be^{(12\,dx+1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -4/315*(315*a^3*e^(14*d*x + 14*c) + 945*a^2*b*e^(14*d*x + 14*c) + 945*a*b^2
*e^(14*d*x + 14*c) + 315*b^3*e^(14*d*x + 14*c) + 1995*a^3*e^(12*d*x + 12*c)
 + 3465*a^2*b*e^(12*d*x + 12*c) + 945*a*b^2*e^(12*d*x + 12*c) - 525*b^3*e^(
12*d*x + 12*c) + 5355*a^3*e^(10*d*x + 10*c) + 4725*a^2*b*e^(10*d*x + 10*c)
+ 945*a*b^2*e^(10*d*x + 10*c) + 1575*b^3*e^(10*d*x + 10*c) + 7875*a^3*e^(8*
d*x + 8*c) + 3213*a^2*b*e^(8*d*x + 8*c) + 2457*a*b^2*e^(8*d*x + 8*c) - 945*
b^3*e^(8*d*x + 8*c) + 6825*a^3*e^(6*d*x + 6*c) + 1827*a^2*b*e^(6*d*x + 6*c)
 + 1323*a*b^2*e^(6*d*x + 6*c) + 945*b^3*e^(6*d*x + 6*c) + 3465*a^3*e^(4*d*x
 + 4*c) + 1323*a^2*b*e^(4*d*x + 4*c) + 27*a*b^2*e^(4*d*x + 4*c) - 135*b^3*e
^(4*d*x + 4*c) + 945*a^3*e^(2*d*x + 2*c) + 567*a^2*b*e^(2*d*x + 2*c) + 243*
a*b^2*e^(2*d*x + 2*c) + 45*b^3*e^(2*d*x + 2*c) + 105*a^3 + 63*a^2*b + 27*a*
b^2 + 5*b^3)/(d*(e^(2*d*x + 2*c) + 1)^9)
```

3.105 $\int \frac{\cosh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=120

$$\frac{x(3a^2+10ab+15b^2)}{8(a+b)^3}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a+b)^3}+\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d(a+b)}+\frac{(3a+7b)\sinh(c+dx)\cosh(c+d}{8d(a+b)^2}$$

```
[Out] ((3*a^2 + 10*a*b + 15*b^2)*x)/(8*(a + b)^3) + (b^(5/2)*ArcTan[(Sqrt[b]*Tanh
[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)^3*d) + ((3*a + 7*b)*Cosh[c + d*x]*Sin
h[c + d*x])/(8*(a + b)^2*d) + (Cosh[c + d*x]^3*Sinh[c + d*x])/(4*(a + b)*d)
```

Rubi [A] time = 0.168274, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3675, 414, 527, 522, 206, 205}

$$\frac{x(3a^2+10ab+15b^2)}{8(a+b)^3}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a+b)^3}+\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d(a+b)}+\frac{(3a+7b)\sinh(c+dx)\cosh(c+d}{8d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((3*a^2 + 10*a*b + 15*b^2)*x)/(8*(a + b)^3) + (b^(5/2)*ArcTan[(Sqrt[b]*Tanh
[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)^3*d) + ((3*a + 7*b)*Cosh[c + d*x]*Sin
h[c + d*x])/(8*(a + b)^2*d) + (Cosh[c + d*x]^3*Sinh[c + d*x])/(4*(a + b)*d)
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^3(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d} + \frac{\text{Subst}\left(\int \frac{3a+4b+3bx^2}{(1-x^2)^2(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{4(a+b)d}$$

$$= \frac{(3a+7b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d} + \frac{\text{Subst}\left(\int \frac{3a^2+7ab+8b^2+ \cdots}{(1-x^2)(a+ \cdots}\right.}{8(\cdots}$$

$$= \frac{(3a+7b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d} + \frac{b^3\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, \cdots\right.}{(a+b)^3 \cdots}$$

$$= \frac{\left(3a^2+10ab+15b^2\right)x}{8(a+b)^3} + \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^3d} + \frac{(3a+7b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d} + \cdots$$

Mathematica [A] time = 0.275048, size = 115, normalized size = 0.96

$$\frac{\left(3a^2+10ab+15b^2\right)(c+dx)}{8d(a+b)^3} + \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a+b)^3} + \frac{(a+2b)\sinh(2(c+dx))}{4d(a+b)^2} + \frac{\sinh(4(c+dx))}{32d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((3*a^2 + 10*a*b + 15*b^2)*(c + d*x))/(8*(a + b)^3*d) + (b^(5/2)*ArcTan[(Sq
rt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)^3*d) + ((a + 2*b)*Sinh[2*(c
 + d*x)])/(4*(a + b)^2*d) + Sinh[4*(c + d*x)]/(32*(a + b)*d)
```

Maple [B] time = 0.088, size = 857, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x)
```

```
[Out] -1/2/d/(2*b+2*a)/(tanh(1/2*d*x+1/2*c)+1)^4+2/d/(4*a+4*b)/(tanh(1/2*d*x+1/2*
c)+1)^3+5/8/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)+1)*a+9/8/d/(a+b)^2/(tanh(1/2*d*x
+1/2*c)+1)*b-7/8/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)+1)^2*a-11/8/d/(a+b)^2/(tanh
(1/2*d*x+1/2*c)+1)^2*b+3/8/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)+1)*a^2+5/4/d/(a
+b)^3*ln(tanh(1/2*d*x+1/2*c)+1)*a*b+15/8/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)+1
)*b^2-1/d*b^3/(a+b)^3*a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)
*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/d*b^3
/(a+b)^3/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/
((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-1/d*b^4/(a+b)^3/(b*(a+b))^(1/2)/((2*(b
*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(
1/2)-a-2*b)*a)^(1/2))-1/d*b^3/(a+b)^3*a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)
+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)
^(1/2))-1/d*b^3/(a+b)^3/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1
/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/d*b^4/(a+b)^3/(b*(a+b)
)^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((
2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/2/d/(2*b+2*a)/(tanh(1/2*d*x+1/2*c)-1)^
4+2/d/(4*a+4*b)/(tanh(1/2*d*x+1/2*c)-1)^3+7/8/d/(a+b)^2/(tanh(1/2*d*x+1/2*c
)-1)^2*a+11/8/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)^2*b+5/8/d/(a+b)^2/(tanh(1/2
*d*x+1/2*c)-1)*a+9/8/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)*b-3/8/d/(a+b)^3*ln(t
anh(1/2*d*x+1/2*c)-1)*a^2-5/4/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)-1)*a*b-15/8/
d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)-1)*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.57445, size = 5515, normalized size = 45.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/64*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x
 + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^8 + 8*(3*a^2 + 10
*a*b + 15*b^2)*d*x*cosh(d*x + c)^4 + 8*(a^2 + 3*a*b + 2*b^2)*cosh(d*x + c)^
6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 2*a^2 + 6*a*b + 4*b^2)*sinh(
d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 6*(a^2 + 3*a*b + 2*
b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x +
c)^4 + 4*(3*a^2 + 10*a*b + 15*b^2)*d*x + 60*(a^2 + 3*a*b + 2*b^2)*cosh(d*x
+ c)^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 4*(3*a
^2 + 10*a*b + 15*b^2)*d*x*cosh(d*x + c) + 20*(a^2 + 3*a*b + 2*b^2)*cosh(d*x
 + c)^3)*sinh(d*x + c)^3 - 8*(a^2 + 3*a*b + 2*b^2)*cosh(d*x + c)^2 + 4*(7*(
```

```
a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 12*(3*a^2 + 10*a*b + 15*b^2)*d*x*cosh(
d*x + c)^2 + 30*(a^2 + 3*a*b + 2*b^2)*cosh(d*x + c)^4 - 2*a^2 - 6*a*b - 4*b
^2)*sinh(d*x + c)^2 + 32*(b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)^3*sinh(
d*x + c) + 6*b^2*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^2*cosh(d*x + c)*sinh
(d*x + c)^3 + b^2*sinh(d*x + c)^4)*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh
(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 +
2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 +
2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b
^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*si
nh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*
sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((a +
b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh
(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a
- b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*
sinh(d*x + c) + a + b)) - a^2 - 2*a*b - b^2 + 8*((a^2 + 2*a*b + b^2)*cosh(d
*x + c)^7 + 4*(3*a^2 + 10*a*b + 15*b^2)*d*x*cosh(d*x + c)^3 + 6*(a^2 + 3*a*
b + 2*b^2)*cosh(d*x + c)^5 - 2*(a^2 + 3*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*
x + c))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^4 + 4*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d
*sinh(d*x + c)^4), 1/64*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a
*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)
^8 + 8*(3*a^2 + 10*a*b + 15*b^2)*d*x*cosh(d*x + c)^4 + 8*(a^2 + 3*a*b + 2*b
^2)*cosh(d*x + c)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 2*a^2 + 6*
a*b + 4*b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 6
*(a^2 + 3*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^2 + 2*a*b
+ b^2)*cosh(d*x + c)^4 + 4*(3*a^2 + 10*a*b + 15*b^2)*d*x + 60*(a^2 + 3*a*b
+ 2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d
*x + c)^5 + 4*(3*a^2 + 10*a*b + 15*b^2)*d*x*cosh(d*x + c) + 20*(a^2 + 3*a*b
 + 2*b^2)*cosh(d*x + c)^3)*sinh(d*x + c)^3 - 8*(a^2 + 3*a*b + 2*b^2)*cosh(d
*x + c)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 12*(3*a^2 + 10*a*b +
 15*b^2)*d*x*cosh(d*x + c)^2 + 30*(a^2 + 3*a*b + 2*b^2)*cosh(d*x + c)^4 - 2
*a^2 - 6*a*b - 4*b^2)*sinh(d*x + c)^2 + 64*(b^2*cosh(d*x + c)^4 + 4*b^2*cos
h(d*x + c)^3*sinh(d*x + c) + 6*b^2*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^2*
cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4)*sqrt(b/a)*arctan(1/2*(
(a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*s
inh(d*x + c)^2 + a - b)*sqrt(b/a)/b) - a^2 - 2*a*b - b^2 + 8*((a^2 + 2*a*b
+ b^2)*cosh(d*x + c)^7 + 4*(3*a^2 + 10*a*b + 15*b^2)*d*x*cosh(d*x + c)^3 +
6*(a^2 + 3*a*b + 2*b^2)*cosh(d*x + c)^5 - 2*(a^2 + 3*a*b + 2*b^2)*cosh(d*x
+ c))*sinh(d*x + c))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^4 + 4
*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*d*sinh(d*x + c)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.79531, size = 440, normalized size = 3.67

$$\frac{\frac{64\, b^{3} \arctan\left(\frac{a e^{(2\,dx+2\,c)}+b e^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)\sqrt{ab}}+\frac{8\left(3\,a^{2}+10\,ab+15\,b^{2}\right)dx}{a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}}-\frac{\left(18\,a^{2}e^{(4\,dx+4\,c)}+60\,abe^{(4\,dx+4\,c)}+90\,b^{2}e^{(4\,dx+4\,c)}+8\,a^{2}e^{(2\,dx+2\,c)}+24\,abe^{(2\,dx+2\,c)}+16\,b\right.}{a^{3}e^{(4\,c)}+3\,a^{2}be^{(4\,c)}+3\,ab^{2}e^{(4\,c)}+b^{3}e^{(4\,c)}}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/64*(64*b^3*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqr
t(a*b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b)) + 8*(3*a^2 + 10*a*b + 1
5*b^2)*d*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (18*a^2*e^(4*d*x + 4*c) + 60*a
*b*e^(4*d*x + 4*c) + 90*b^2*e^(4*d*x + 4*c) + 8*a^2*e^(2*d*x + 2*c) + 24*a*
b*e^(2*d*x + 2*c) + 16*b^2*e^(2*d*x + 2*c) + a^2 + 2*a*b + b^2)*e^(-4*d*x)/
(a^3*e^(4*c) + 3*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c) + b^3*e^(4*c)) + (a*e^(4*d
*x + 20*c) + b*e^(4*d*x + 20*c) + 8*a*e^(2*d*x + 18*c) + 16*b*e^(2*d*x + 18
*c))/(a^2*e^(16*c) + 2*a*b*e^(16*c) + b^2*e^(16*c)))/d
```

3.106 $\int \frac{\cosh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=80

$$\frac{b^2\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a+b)^{5/2}}+\frac{\sinh^3(c+dx)}{3d(a+b)}+\frac{(a+2b)\sinh(c+dx)}{d(a+b)^2}$$

```
[Out] (b^2*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(5/2)*d)
 + ((a + 2*b)*Sinh[c + d*x])/((a + b)^2*d) + Sinh[c + d*x]^3/(3*(a + b)*d)
```

Rubi [A] time = 0.109062, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3676, 390, 205}

$$\frac{b^2\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a+b)^{5/2}}+\frac{\sinh^3(c+dx)}{3d(a+b)}+\frac{(a+2b)\sinh(c+dx)}{d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (b^2*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(5/2)*d)
 + ((a + 2*b)*Sinh[c + d*x])/((a + b)^2*d) + Sinh[c + d*x]^3/(3*(a + b)*d)
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{(1+x^2)^2}{a+(a+b)x^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(\frac{a+2b}{(a+b)^2}+\frac{x^2}{a+b}+\frac{b^2}{(a+b)^2(a+(a+b)x^2)}\right)dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{(a+2b)\sinh(c+dx)}{(a+b)^2 d}+\frac{\sinh^3(c+dx)}{3(a+b)d}+\frac{b^2\operatorname{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \sinh(c+dx)\right)}{(a+b)^2 d}$$

$$= \frac{b^2\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{5/2}d}+\frac{(a+2b)\sinh(c+dx)}{(a+b)^2 d}+\frac{\sinh^3(c+dx)}{3(a+b)d}$$

Mathematica [A] time = 0.396, size = 79, normalized size = 0.99

$$\frac{-\frac{12b^2\tan^{-1}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}(a+b)^{5/2}}+\frac{3(3a+7b)\sinh(c+dx)}{(a+b)^2}+\frac{\sinh(3(c+dx))}{a+b}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((-12*b^2*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]])/(Sqrt[a]*(a + b)^(5/
2)) + (3*(3*a + 7*b)*Sinh[c + d*x])/(a + b)^2 + Sinh[3*(c + d*x)]/(a + b))/
(12*d)
```

Maple [B] time = 0.087, size = 468, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x)
```

```
[Out] -2/3/d/(tanh(1/2*d*x+1/2*c)+1)^3/(2*b+2*a)+1/d/(2*b+2*a)/(tanh(1/2*d*x+1/2*
c)+1)^2-1/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)+1)*a-2/d/(a+b)^2/(tanh(1/2*d*x+1/2
*c)+1)*b-1/d*b^2/(a+b)^2/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh
(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/d*b^3/(a+b)^2/(b*(a+
b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)
/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/d*b^2/(a+b)^2/((2*(b*(a+b))^(1/2)+a
+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(
1/2))+1/d*b^3/(a+b)^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*a
rctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-2/3/d/(tan
h(1/2*d*x+1/2*c)-1)^3/(2*b+2*a)-1/d/(2*b+2*a)/(tanh(1/2*d*x+1/2*c)-1)^2-1/d
/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)*a-2/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(\left(ae^{(6c)}+be^{(6c)}\right)e^{(6dx)}+3\left(3ae^{(4c)}+7be^{(4c)}\right)e^{(4dx)}-3\left(3ae^{(2c)}+7be^{(2c)}\right)e^{(2dx)}-a-b\right)e^{(-3dx)}}{24\left(a^2de^{(3c)}+2abde^{(3c)}+b^2de^{(3c)}\right)}+\frac{1}{8}\int \frac{}{a^3+3a^2b+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/24*((a*e^(6*c) + b*e^(6*c))*e^(6*d*x) + 3*(3*a*e^(4*c) + 7*b*e^(4*c))*e^(
4*d*x) - 3*(3*a*e^(2*c) + 7*b*e^(2*c))*e^(2*d*x) - a - b)*e^(-3*d*x)/(a^2*d
*e^(3*c) + 2*a*b*d*e^(3*c) + b^2*d*e^(3*c)) + 1/8*integrate(16*(b^2*e^(3*d*
x + 3*c) + b^2*e^(d*x + c))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3 + (a^3*e^(4*c) +
 3*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c) + b^3*e^(4*c))*e^(4*d*x) + 2*(a^3*e^(2*c
) + a^2*b*e^(2*c) - a*b^2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 2.37351, size = 4604, normalized size = 57.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/24*((a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^6 + 6*(a^3 + 2*a^2*b + a*b^2)*
cosh(d*x + c)*sinh(d*x + c)^5 + (a^3 + 2*a^2*b + a*b^2)*sinh(d*x + c)^6 + 3
*(3*a^3 + 10*a^2*b + 7*a*b^2)*cosh(d*x + c)^4 + 3*(3*a^3 + 10*a^2*b + 7*a*b
^2 + 5*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(a^3
 + 2*a^2*b + a*b^2)*cosh(d*x + c)^3 + 3*(3*a^3 + 10*a^2*b + 7*a*b^2)*cosh(d
*x + c))*sinh(d*x + c)^3 - a^3 - 2*a^2*b - a*b^2 - 3*(3*a^3 + 10*a^2*b + 7*
a*b^2)*cosh(d*x + c)^2 + 3*(5*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^4 - 3*a
^3 - 10*a^2*b - 7*a*b^2 + 6*(3*a^3 + 10*a^2*b + 7*a*b^2)*cosh(d*x + c)^2)*s
inh(d*x + c)^2 - 12*(b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c)^2*sinh(d*x +
 c) + 3*b^2*cosh(d*x + c)*sinh(d*x + c)^2 + b^2*sinh(d*x + c)^3)*sqrt(-a^2
- a*b)*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)
^3 + (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*c
osh(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 - (3
*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)
*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)
- cosh(d*x + c))*sqrt(-a^2 - a*b) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a
+ b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*co
sh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*(
(a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) +
6*((a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^5 + 2*(3*a^3 + 10*a^2*b + 7*a*b^2)
*cosh(d*x + c)^3 - (3*a^3 + 10*a^2*b + 7*a*b^2)*cosh(d*x + c))*sinh(d*x + c
))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^3 + 3*(a^4 + 3*a^3*
b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^4 + 3*a^3*b +
 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^2 + (a^4 + 3*a^3*b + 3*a^
2*b^2 + a*b^3)*d*sinh(d*x + c)^3), 1/24*((a^3 + 2*a^2*b + a*b^2)*cosh(d*x +
 c)^6 + 6*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (a^3 + 2*
a^2*b + a*b^2)*sinh(d*x + c)^6 + 3*(3*a^3 + 10*a^2*b + 7*a*b^2)*cosh(d*x +
c)^4 + 3*(3*a^3 + 10*a^2*b + 7*a*b^2 + 5*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x +
 c)^2)*sinh(d*x + c)^4 + 4*(5*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^3 + 3*(
3*a^3 + 10*a^2*b + 7*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - a^3 - 2*a^2*b
- a*b^2 - 3*(3*a^3 + 10*a^2*b + 7*a*b^2)*cosh(d*x + c)^2 + 3*(5*(a^3 + 2*a^
2*b + a*b^2)*cosh(d*x + c)^4 - 3*a^3 - 10*a^2*b - 7*a*b^2 + 6*(3*a^3 + 10*a
^2*b + 7*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 24*(b^2*cosh(d*x + c)^3
+ 3*b^2*cosh(d*x + c)^2*sinh(d*x + c) + 3*b^2*cosh(d*x + c)*sinh(d*x + c)^2
 + b^2*sinh(d*x + c)^3)*sqrt(a^2 + a*b)*arctan(1/2*((a + b)*cosh(d*x + c)^3
 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (3*a
 - b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + 3*a - b)*sinh(d*x + c))/
sqrt(a^2 + a*b)) + 24*(b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c)^2*sinh(d*x
```

```
 + c) + 3*b^2*cosh(d*x + c)*sinh(d*x + c)^2 + b^2*sinh(d*x + c)^3)*sqrt(a^2
 + a*b)*arctan(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh(d*x + c))/a) + 6*(
(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^5 + 2*(3*a^3 + 10*a^2*b + 7*a*b^2)*co
sh(d*x + c)^3 - (3*a^3 + 10*a^2*b + 7*a*b^2)*cosh(d*x + c))*sinh(d*x + c))/
((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^3 + 3*(a^4 + 3*a^3*b +
 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^4 + 3*a^3*b + 3*
a^2*b^2 + a*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^2 + (a^4 + 3*a^3*b + 3*a^2*b
^2 + a*b^3)*d*sinh(d*x + c)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(cosh(c + d*x)**3/(a + b*tanh(c + d*x)**2), x)
```

Giac [C] time = 2.11494, size = 6807, normalized size = 85.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/24*(12*(3*(2*a*b^3*e^(2*c) + (a*b^2*e^(2*c) - b^3*e^(2*c))*sqrt(-a*b))*co
s(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arcco
s(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a +
b)))) - (2*a*b^3*e^(2*c) + (a*b^2*e^(2*c) - b^3*e^(2*c))*sqrt(-a*b))*cosh(1
/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a
/(a + b) + b/(a + b))))^3 - 9*(2*a*b^3*e^(2*c) + (a*b^2*e^(2*c) - b^3*e^(2*
c))*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1
/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a
/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))
 + 3*(2*a*b^3*e^(2*c) + (a*b^2*e^(2*c) - b^3*e^(2*c))*sqrt(-a*b))*cosh(1/2*
imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a
 + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))
+ 9*(2*a*b^3*e^(2*c) + (a*b^2*e^(2*c) - b^3*e^(2*c))*sqrt(-a*b))*cos(1/2*re
al_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a
+ b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh
(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(2*a*b^3*e^(2*c) + (a
*b^2*e^(2*c) - b^3*e^(2*c))*sqrt(-a*b))*cosh(1/2*imag_part(arccos(-a/(a + b
) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(
1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(2*a*b^3*e^(2*c) + (a*
b^2*e^(2*c) - b^3*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b)
+ b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/
2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (2*a*b^3*e^(2*c) + (a*b^2*
e^(2*c) - b^3*e^(2*c))*sqrt(-a*b))*sin(1/2*real_part(arccos(-a/(a + b) + b/
(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (2*a*b
^3*e^(2*c) + (a*b^2*e^(2*c) - b^3*e^(2*c))*sqrt(-a*b))*cosh(1/2*imag_part(a
rccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a
```

```
+ b)))) - (2*a*b^3*e^(2*c) + (a*b^2*e^(2*c) - b^3*e^(2*c))*sqrt(-a*b))*sin(
1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a
/(a + b) + b/(a + b)))))*arctan((((a^3 + 3*a^2*b + 3*a*b^2 + b^3)/(a^3*e^(4
*c) + 3*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c) + b^3*e^(4*c)))^(1/4)*cos(1/2*arcco
s(-(a - b)/(a + b))) + e^(d*x))/(((a^3 + 3*a^2*b + 3*a*b^2 + b^3)/(a^3*e^(4
*c) + 3*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c) + b^3*e^(4*c)))^(1/4)*sin(1/2*arcco
s(-(a - b)/(a + b)))))/(a^4*b*e^(2*c) + 3*a^3*b^2*e^(2*c) + 3*a^2*b^3*e^(2*
c) + a*b^4*e^(2*c)) + 12*(3*(2*a*b^3*e^(2*c) + (a*b^2*e^(2*c) - b^3*e^(2*c)
)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2
*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(
a + b) + b/(a + b)))) - (2*a*b^3*e^(2*c) + (a*b^2*e^(2*c) - b^3*e^(2*c))*sq
rt(-a*b))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*rea
l_part(arccos(-a/(a + b) + b/(a + b))))^3 - 9*(2*a*b^3*e^(2*c) + (a*b^2*e^(
2*c) - b^3*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a
+ b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*rea
l_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b
) + b/(a + b)))) + 3*(2*a*b^3*e^(2*c) + (a*b^2*e^(2*c) - b^3*e^(2*c))*sqrt(
-a*b))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_p
art(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b)
 + b/(a + b)))) + 9*(2*a*b^3*e^(2*c) + (a*b^2*e^(2*c) - b^3*e^(2*c))*sqrt(-
a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_pa
rt(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b
/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(2*a*
b^3*e^(2*c) + (a*b^2*e^(2*c) - b^3*e^(2*c))*sqrt(-a*b))*cosh(1/2*imag_part(
arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a
 + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(2*a*b
^3*e^(2*c) + (a*b^2*e^(2*c) - b^3*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(ar
ccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a
 + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (2*a*b^3*e
^(2*c) + (a*b^2*e^(2*c) - b^3*e^(2*c))*sqrt(-a*b))*sin(1/2*real_part(arccos
(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a +
b))))^3 + (2*a*b^3*e^(2*c) + (a*b^2*e^(2*c) - b^3*e^(2*c))*sqrt(-a*b))*cosh
(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a
/(a + b) + b/(a + b)))) - (2*a*b^3*e^(2*c) + (a*b^2*e^(2*c) - b^3*e^(2*c))*
sqrt(-a*b))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*ima
g_part(arccos(-a/(a + b) + b/(a + b)))))*arctan(-(((a^3 + 3*a^2*b + 3*a*b^2
 + b^3)/(a^3*e^(4*c) + 3*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c) + b^3*e^(4*c)))^(1
/4)*cos(1/2*arccos(-(a - b)/(a + b))) - e^(d*x))/(((a^3 + 3*a^2*b + 3*a*b^2
 + b^3)/(a^3*e^(4*c) + 3*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c) + b^3*e^(4*c)))^(1
/4)*sin(1/2*arccos(-(a - b)/(a + b)))))/(a^4*b*e^(2*c) + 3*a^3*b^2*e^(2*c)
+ 3*a^2*b^3*e^(2*c) + a*b^4*e^(2*c)) - (9*a*e^(2*d*x + 2*c) + 21*b*e^(2*d*x
 + 2*c) + a + b)*e^(-3*d*x)/(a^2*e^(3*c) + 2*a*b*e^(3*c) + b^2*e^(3*c)) + 6
*((2*a*b^3*e^(2*c) + (a*b^2*e^(2*c) - b^3*e^(2*c))*sqrt(-a*b))*cos(1/2*real
_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a +
b) + b/(a + b))))^3 - 3*(2*a*b^3*e^(2*c) + (a*b^2*e^(2*c) - b^3*e^(2*c))*sq
rt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_
part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b)
 + b/(a + b))))^2 - 3*(2*a*b^3*e^(2*c) + (a*b^2*e^(2*c) - b^3*e^(2*c))*sqrt
(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_
part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b
) + b/(a + b)))) + 9*(2*a*b^3*e^(2*c) + (a*b^2*e^(2*c) - b^3*e^(2*c))*sqrt(
-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_par
t(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) +
b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(2*a
*b^3*e^(2*c) + (a*b^2*e^(2*c) - b^3*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(
arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b
/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 9*(2*a*
b^3*e^(2*c) + (a*b^2*e^(2*c) - b^3*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(a
rccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a
```

```
 + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag
_part(arccos(-a/(a + b) + b/(a + b))))^2 - (2*a*b^3*e^(2*c) + (a*b^2*e^(2*c
) - b^3*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b
))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + 3*(2*a*b^3*e
^(2*c) + (a*b^2*e^(2*c) - b^3*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(arccos
(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))
))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (2*a*b^3*e^(2*
c) + (a*b^2*e^(2*c) - b^3*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/
(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))
 - (2*a*b^3*e^(2*c) + (a*b^2*e^(2*c) - b^3*e^(2*c))*sqrt(-a*b))*cos(1/2*real
_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b)
 + b/(a + b)))))*log(2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)/(a^3*e^(4*c) + 3*a^
2*b*e^(4*c) + 3*a*b^2*e^(4*c) + b^3*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)
/(a + b)))*e^(d*x) + sqrt((a^3 + 3*a^2*b + 3*a*b^2 + b^3)/(a^3*e^(4*c) + 3*
a^2*b*e^(4*c) + 3*a*b^2*e^(4*c) + b^3*e^(4*c))) + e^(2*d*x))/(a^4*b*e^(2*c)
 + 3*a^3*b^2*e^(2*c) + 3*a^2*b^3*e^(2*c) + a*b^4*e^(2*c)) - 6*((2*a*b^3*e^(
2*c) + (a*b^2*e^(2*c) - b^3*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(arccos(-
a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)
)))^3 - 3*(2*a*b^3*e^(2*c) + (a*b^2*e^(2*c) - b^3*e^(2*c))*sqrt(-a*b))*cos(
1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a
/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))
)^2 - 3*(2*a*b^3*e^(2*c) + (a*b^2*e^(2*c) - b^3*e^(2*c))*sqrt(-a*b))*cos(1/
2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a
/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))
)) + 9*(2*a*b^3*e^(2*c) + (a*b^2*e^(2*c) - b^3*e^(2*c))*sqrt(-a*b))*cos(1/2
*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a
 + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2
*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(2*a*b^3*e^(2*c) +
 (a*b^2*e^(2*c) - b^3*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a +
 b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*si
nh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 9*(2*a*b^3*e^(2*c) +
(a*b^2*e^(2*c) - b^3*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a +
b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1
/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-
a/(a + b) + b/(a + b))))^2 - (2*a*b^3*e^(2*c) + (a*b^2*e^(2*c) - b^3*e^(2*c
))*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/
2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + 3*(2*a*b^3*e^(2*c) + (a*b^
2*e^(2*c) - b^3*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) +
b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*
imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (2*a*b^3*e^(2*c) + (a*b^2*e^
(2*c) - b^3*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a
 + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) - (2*a*b^3*e^(
2*c) + (a*b^2*e^(2*c) - b^3*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(arccos(-
a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))
))*log(-2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)/(a^3*e^(4*c) + 3*a^2*b*e^(4*c) +
 3*a*b^2*e^(4*c) + b^3*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b)))*e^
(d*x) + sqrt((a^3 + 3*a^2*b + 3*a*b^2 + b^3)/(a^3*e^(4*c) + 3*a^2*b*e^(4*c)
 + 3*a*b^2*e^(4*c) + b^3*e^(4*c))) + e^(2*d*x))/(a^4*b*e^(2*c) + 3*a^3*b^2*
e^(2*c) + 3*a^2*b^3*e^(2*c) + a*b^4*e^(2*c)) + (a^2*e^(3*d*x + 24*c) + 2*a*
b*e^(3*d*x + 24*c) + b^2*e^(3*d*x + 24*c) + 9*a^2*e^(d*x + 22*c) + 30*a*b*e
^(d*x + 22*c) + 21*b^2*e^(d*x + 22*c))/(a^3*e^(21*c) + 3*a^2*b*e^(21*c) + 3
*a*b^2*e^(21*c) + b^3*e^(21*c)))/d
```

3.107 $\int \frac{\cosh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=77

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a+b)^2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d(a+b)}+\frac{x(a+3b)}{2(a+b)^2}$$

```
[Out] ((a + 3*b)*x)/(2*(a + b)^2) + (b^(3/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[
a]])/(Sqrt[a]*(a + b)^2*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*(a + b)*d)
```

Rubi [A] time = 0.102965, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3675, 414, 522, 206, 205}

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a+b)^2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d(a+b)}+\frac{x(a+3b)}{2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((a + 3*b)*x)/(2*(a + b)^2) + (b^(3/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[
a]])/(Sqrt[a]*(a + b)^2*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*(a + b)*d)
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d} + \frac{\text{Subst}\left(\int \frac{a+2b+bx^2}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{2(a+b)d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)^2 d} + \frac{(a+3b)\,\text{Subst}\left(\int\right.}{2}$$

$$= \frac{(a+3b)x}{2(a+b)^2} + \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^2 d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d}$$

Mathematica [A] time = 0.147657, size = 77, normalized size = 1.

$$\frac{4b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right) + 2\sqrt{a}(a+3b)(c+dx) + \sqrt{a}(a+b)\sinh(2(c+dx))}{4\sqrt{a}d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (2*Sqrt[a]*(a + 3*b)*(c + d*x) + 4*b^(3/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/S
qrt[a]] + Sqrt[a]*(a + b)*Sinh[2*(c + d*x)])/(4*Sqrt[a]*(a + b)^2*d)
```

Maple [B] time = 0.083, size = 608, normalized size = 7.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x)
```

```
[Out] -1/d/(2*b+2*a)/(tanh(1/2*d*x+1/2*c)+1)^2+2/d/(4*a+4*b)/(tanh(1/2*d*x+1/2*c)
+1)+1/2/d/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)+1)*a+3/2/d/(a+b)^2*ln(tanh(1/2*d*x
+1/2*c)+1)*b-1/d*a*b^2/(a+b)^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a
)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+
1/d*b^2/(a+b)^2/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+
1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-1/d*b^3/(a+b)^2/(b*(a+b))^(1/2)
/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(
a+b))^(1/2)-a-2*b)*a)^(1/2))-1/d*a*b^2/(a+b)^2/(b*(a+b))^(1/2)/((2*(b*(a+b)
)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+
2*b)*a)^(1/2))-1/d*b^2/(a+b)^2/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a
```

```
*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/d*b^3/(a+b)^2/(
b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/
2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/d/(2*b+2*a)/(tanh(1/2*d*x+1/2*c
)-1)^2+2/d/(4*a+4*b)/(tanh(1/2*d*x+1/2*c)-1)-1/2/d/(a+b)^2*ln(tanh(1/2*d*x+
1/2*c)-1)*a-3/2/d/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)-1)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.28545, size = 2520, normalized size = 32.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a + 3*b)*d*x*cosh(d*x + c)^2 + (a + b)*cosh(d*x + c)^4 + 4*(a + b)
*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(2*(a + 3*b)*d
*x + 3*(a + b)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^2 + 2*
b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2)*sqrt(-b/a)*log(((a^2 + 2
*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x
+ c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^
2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 +
 a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*c
osh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b
)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sq
rt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^
3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh
(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)
*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 4*(2*(a + 3*b)*d*x*cosh(d*x + c)
+ (a + b)*cosh(d*x + c)^3)*sinh(d*x + c) - a - b)/((a^2 + 2*a*b + b^2)*d*co
sh(d*x + c)^2 + 2*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)*sinh(d*x + c) + (a^2
+ 2*a*b + b^2)*d*sinh(d*x + c)^2), 1/8*(4*(a + 3*b)*d*x*cosh(d*x + c)^2 + (
a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*
sinh(d*x + c)^4 + 2*(2*(a + 3*b)*d*x + 3*(a + b)*cosh(d*x + c)^2)*sinh(d*x
+ c)^2 + 8*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*
x + c)^2)*sqrt(b/a)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*
x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(b/a)/b) + 4*(2
*(a + 3*b)*d*x*cosh(d*x + c) + (a + b)*cosh(d*x + c)^3)*sinh(d*x + c) - a -
 b)/((a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2 + 2*(a^2 + 2*a*b + b^2)*d*cosh(d
*x + c)*sinh(d*x + c) + (a^2 + 2*a*b + b^2)*d*sinh(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(cosh(c + d*x)**2/(a + b*tanh(c + d*x)**2), x)
```

Giac [B] time = 1.85299, size = 232, normalized size = 3.01

$$\frac{\frac{4\,(a+3\,b)dx}{a^2+2\,ab+b^2}+\frac{8\,b^2\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\left(a^2+2\,ab+b^2\right)\sqrt{ab}}-\frac{\left(2\,ae^{(2\,dx+2\,c)}+6\,be^{(2\,dx+2\,c)}+a+b\right)e^{(-2\,dx)}}{a^2e^{(2\,c)}+2\,abe^{(2\,c)}+b^2e^{(2\,c)}}+\frac{e^{(2\,dx+8\,c)}}{ae^{(6\,c)}+be^{(6\,c)}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/8*(4*(a + 3*b)*d*x/(a^2 + 2*a*b + b^2) + 8*b^2*arctan(1/2*(a*e^(2*d*x + 2
*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b))
 - (2*a*e^(2*d*x + 2*c) + 6*b*e^(2*d*x + 2*c) + a + b)*e^(-2*d*x)/(a^2*e^(2
*c) + 2*a*b*e^(2*c) + b^2*e^(2*c)) + e^(2*d*x + 8*c)/(a*e^(6*c) + b*e^(6*c)
))/d
```

3.108 $\int \frac{\cosh(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=53

$$\frac{\sinh(c+dx)}{d(a+b)}+\frac{b\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a+b)^{3/2}}$$

```
[Out] (b*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(3/2)*d) +
 Sinh[c + d*x]/((a + b)*d)
```

Rubi [A] time = 0.0704951, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3676, 388, 205}

$$\frac{\sinh(c+dx)}{d(a+b)}+\frac{b\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (b*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(3/2)*d) +
 Sinh[c + d*x]/((a + b)*d)
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{a+(a+b)x^2}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{\sinh(c+dx)}{(a+b)d} + \frac{b\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \sinh(c+dx)\right)}{(a+b)d}$$
$$= \frac{b\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{3/2}d} + \frac{\sinh(c+dx)}{(a+b)d}$$

Mathematica [A] time = 0.0923188, size = 53, normalized size = 1.

$$\frac{\sinh(c+dx)}{d(a+b)} + \frac{b\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (b*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(3/2)*d) +
 Sinh[c + d*x]/((a + b)*d)
```

Maple [B] time = 0.071, size = 315, normalized size = 5.9

$$-\frac{b}{d(a+b)}\text{Artanh}\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\frac{1}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}\right)\frac{1}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}} + \frac{b^2}{d(a+b)}\text{Artanh}\left(a\,t\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+b*tanh(d*x+c)^2),x)
```

```
[Out] -1/d*b/(a+b)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2
*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/d*b^2/(a+b)/(b*(a+b))^(1/2)/((2*
(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))
^(1/2)-a-2*b)*a)^(1/2))+1/d*b/(a+b)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arc
tan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/d*b^2/(a+b
)/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x
+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-2/d/(2*b+2*a)/(tanh(1/2*d*x+1/
2*c)-1)-2/d/(2*b+2*a)/(tanh(1/2*d*x+1/2*c)+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx)}}{2\left(ade^{c}+bde^{c}\right)} + \frac{1}{2}\int \frac{4\left(be^{(3\,dx+3\,c)}+be^{(dx+c)}\right)}{a^2+2\,ab+b^2+\left(a^2e^{(4\,c)}+2\,abe^{(4\,c)}+b^2e^{(4\,c)}\right)e^{(4\,dx)}+2\left(a^2e^{(2\,c)}-b^2e^{(2\,c)}\right)e^{(2\,dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x)/(a*d*e^c + b*d*e^c) + 1/2*integrate(4*(b
*e^(3*d*x + 3*c) + b*e^(d*x + c))/(a^2 + 2*a*b + b^2 + (a^2*e^(4*c) + 2*a*b
*e^(4*c) + b^2*e^(4*c))*e^(4*d*x) + 2*(a^2*e^(2*c) - b^2*e^(2*c))*e^(2*d*x)
), x)
```

Fricas [B] time = 2.46219, size = 2040, normalized size = 38.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x +
c) + (a^2 + a*b)*sinh(d*x + c)^2 - sqrt(-a^2 - a*b)*(b*cosh(d*x + c) + b*si
nh(d*x + c))*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*
x + c)^3 + (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a
+ b)*cosh(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^
3 - (3*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*
x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x
 + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a + b)/((a + b)*cosh(d*x + c)^4 +
 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a -
 b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2
 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a +
b)) - a^2 - a*b)/((a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c) + (a^3 + 2*a^2*b
+ a*b^2)*d*sinh(d*x + c)), 1/2*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)
*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + 2*sqrt(a^2 + a
*b)*(b*cosh(d*x + c) + b*sinh(d*x + c))*arctan(1/2*((a + b)*cosh(d*x + c)^3
 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (3*a
 - b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + 3*a - b)*sinh(d*x + c))/
sqrt(a^2 + a*b)) + 2*sqrt(a^2 + a*b)*(b*cosh(d*x + c) + b*sinh(d*x + c))*ar
ctan(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh(d*x + c))/a) - a^2 - a*b)/((
a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c) + (a^3 + 2*a^2*b + a*b^2)*d*sinh(d*x
 + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(cosh(c + d*x)/(a + b*tanh(c + d*x)**2), x)
```

Giac [C] time = 1.58398, size = 6017, normalized size = 113.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/4*(2*(3*(a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*cos(1/2*real_part
(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) +
b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) - (a^2*b*
e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a +
b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3 -
9*(a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*cos(1/2*real_part(arccos(
-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b
))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_par
t(arccos(-a/(a + b) + b/(a + b)))) + 3*(a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b
^3*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*r
eal_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a
 + b) + b/(a + b)))) + 9*(a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*co
s(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arcco
s(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)
)))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(a^2*b*e^(4*c
) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b
/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*i
mag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(a^2*b*e^(4*c) + 2*a*b^2*e^
(4*c) + b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*s
in(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos
(-a/(a + b) + b/(a + b))))^3 + (a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*
c))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part
(arccos(-a/(a + b) + b/(a + b))))^3 + (a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^
3*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real
_part(arccos(-a/(a + b) + b/(a + b)))) - (a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) +
 b^3*e^(4*c))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*i
mag_part(arccos(-a/(a + b) + b/(a + b)))))*arctan((((a^2 + 2*a*b + b^2)/(a^
2*e^(4*c) + 2*a*b*e^(4*c) + b^2*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a
+ b))) + e^(d*x))/(((a^2 + 2*a*b + b^2)/(a^2*e^(4*c) + 2*a*b*e^(4*c) + b^2*
e^(4*c)))^(1/4)*sin(1/2*arccos(-(a - b)/(a + b)))))/(2*(a*e^(2*c) + b*e^(2*
c))^2*a*b + (a^2*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b)*abs(-a*e^(2*c) - b*e^(2*
c))) + 2*(3*(a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*cos(1/2*real_pa
rt(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b)
+ b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) - (a^2*
b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a
+ b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3
- 9*(a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*cos(1/2*real_part(arcco
s(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a +
 b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_p
art(arccos(-a/(a + b) + b/(a + b)))) + 3*(a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) +
 b^3*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2
*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/
(a + b) + b/(a + b)))) + 9*(a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*
cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arc
cos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a +
b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(a^2*b*e^(4
*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) +
 b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2
*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(a^2*b*e^(4*c) + 2*a*b^2*
e^(4*c) + b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2
*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arcc
os(-a/(a + b) + b/(a + b))))^3 + (a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(
4*c))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_pa
rt(arccos(-a/(a + b) + b/(a + b))))^3 + (a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) +
b^3*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*re
al_part(arccos(-a/(a + b) + b/(a + b)))) - (a^2*b*e^(4*c) + 2*a*b^2*e^(4*c)
 + b^3*e^(4*c))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2
```

```
*imag_part(arccos(-a/(a + b) + b/(a + b)))))*arctan(-(((a^2 + 2*a*b + b^2)/
(a^2*e^(4*c) + 2*a*b*e^(4*c) + b^2*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/
(a + b))) - e^(d*x))/(((a^2 + 2*a*b + b^2)/(a^2*e^(4*c) + 2*a*b*e^(4*c) + b
^2*e^(4*c)))^(1/4)*sin(1/2*arccos(-(a - b)/(a + b)))))/(2*(a*e^(2*c) + b*e^
(2*c))^2*a*b + (a^2*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b)*abs(-a*e^(2*c) - b*e^
(2*c))) + ((a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*cos(1/2*real_par
t(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) +
 b/(a + b))))^3 - 3*(a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*cos(1/2
*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a
 + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2
 - 3*(a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*cos(1/2*real_part(arcc
os(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a
+ b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(a^2*b*e^
(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b)
+ b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/
2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a
/(a + b) + b/(a + b)))) + 3*(a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))
*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(ar
ccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a
+ b))))^2 - 9*(a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*cos(1/2*real_
part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b)
+ b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/
2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - (a^2*b*e^(4*c) + 2*a*b^2*e
^(4*c) + b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*
sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + 3*(a^2*b*e^(4*c) +
2*a*b^2*e^(4*c) + b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a +
 b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_p
art(arccos(-a/(a + b) + b/(a + b))))^3 + (a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) +
 b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*i
mag_part(arccos(-a/(a + b) + b/(a + b)))) - (a^2*b*e^(4*c) + 2*a*b^2*e^(4*c
) + b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/
2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*log(2*((a^2 + 2*a*b + b^2)/(a
^2*e^(4*c) + 2*a*b*e^(4*c) + b^2*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a
 + b)))*e^(d*x) + sqrt((a^2 + 2*a*b + b^2)/(a^2*e^(4*c) + 2*a*b*e^(4*c) + b
^2*e^(4*c))) + e^(2*d*x))/(2*(a*e^(2*c) + b*e^(2*c))^2*a*b + (a^2*e^(2*c) -
 b^2*e^(2*c))*sqrt(-a*b)*abs(-a*e^(2*c) - b*e^(2*c))) - ((a^2*b*e^(4*c) + 2
*a*b^2*e^(4*c) + b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a +
b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 - 3*(a^2*b*e^
(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b)
+ b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/
2*real_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(a^2*b*e^(4*c) + 2*a*b^2
*e^(4*c) + b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^
3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(
arccos(-a/(a + b) + b/(a + b)))) + 9*(a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3
*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_
part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b)
 + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(
a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/
(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))
)*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 9*(a^2*b*e^(4*c)
+ 2*a*b^2*e^(4*c) + b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a
 + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_p
art(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b)
 + b/(a + b))))^2 - (a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*cos(1/2
*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/
(a + b) + b/(a + b))))^3 + 3*(a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c)
)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arcc
os(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a
+ b))))^3 + (a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*cos(1/2*real_pa
```

```
rt(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) +
b/(a + b)))) - (a^2*b*e^(4*c) + 2*a*b^2*e^(4*c) + b^3*e^(4*c))*cos(1/2*real
_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b)
 + b/(a + b)))))*log(-2*((a^2 + 2*a*b + b^2)/(a^2*e^(4*c) + 2*a*b*e^(4*c) +
 b^2*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b)))*e^(d*x) + sqrt((a^2
+ 2*a*b + b^2)/(a^2*e^(4*c) + 2*a*b*e^(4*c) + b^2*e^(4*c))) + e^(2*d*x))/(2
*(a*e^(2*c) + b*e^(2*c))^2*a*b + (a^2*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b)*abs
(-a*e^(2*c) - b*e^(2*c))) + 2*e^(d*x + 6*c)/(a*e^(5*c) + b*e^(5*c)) - 2*e^(
-d*x)/(a*e^c + b*e^c))/d
```

3.109 $\int \frac{\operatorname{sech}(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=36

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d\sqrt{a+b}}$$

```
[Out] ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a + b]*d)
```

Rubi [A] time = 0.0443204, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3676, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a + b]*d)
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}(c+dx)}{a+b\tanh^2(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \sinh(c+dx)\right)}{d}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a+b}d}\end{aligned}$$

Mathematica [A] time = 0.0370236, size = 36, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a + b]*d)
```

Maple [B] time = 0.059, size = 235, normalized size = 6.5

$$-\frac{1}{d}\mathrm{Artanh}\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\frac{1}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}\right)\frac{1}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}+\frac{b}{d}\mathrm{Artanh}\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)/(a+b*tanh(d*x+c)^2),x)
```

```
[Out] -1/d/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*
(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/d*b/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-
a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)
^(1/2))+1/d/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c
)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/d*b/(b*(a+b))^(1/2)/((2*(b*(a+b))^
(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*
b)*a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\mathrm{sech}\,(dx+c)}{b\tanh\,(dx+c)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate(sech(d*x + c)/(b*tanh(d*x + c)^2 + a), x)
```

Fricas [B] time = 2.39758, size = 1385, normalized size = 38.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a^2 - a*b)*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x +
c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2
+ 2*(3*(a + b)*cosh(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh
(d*x + c)^3 - (3*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 +
 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1
)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a + b)/((a + b)*cosh(d*
x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^
4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(
d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x +
 c) + a + b))/((a^2 + a*b)*d), (sqrt(a^2 + a*b)*arctan(1/2*((a + b)*cosh(d*
```

```
x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^
3 + (3*a - b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + 3*a - b)*sinh(d*
x + c))/sqrt(a^2 + a*b)) + sqrt(a^2 + a*b)*arctan(1/2*sqrt(a^2 + a*b)*(cosh
(d*x + c) + sinh(d*x + c))/a))/((a^2 + a*b)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)/(a + b*tanh(c + d*x)**2), x)
```

Giac [C] time = 1.56991, size = 6607, normalized size = 183.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/4*(2*(3*(2*a^2*b*e^(2*c) + 2*a*b^2*e^(2*c) - (a^2*e^(2*c) - b^2*e^(2*c))*
sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*i
mag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a
+ b) + b/(a + b)))) - (2*a^2*b*e^(2*c) + 2*a*b^2*e^(2*c) - (a^2*e^(2*c) - b
^2*e^(2*c))*sqrt(-a*b))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))
^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3 - 9*(2*a^2*b*e^(2*c
) + 2*a*b^2*e^(2*c) - (a^2*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cos(1/2*real_
part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b
) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(
1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(2*a^2*b*e^(2*c) + 2*a*b
^2*e^(2*c) - (a^2*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cosh(1/2*imag_part(arc
cos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a
+ b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(2*a^2*b*
e^(2*c) + 2*a*b^2*e^(2*c) - (a^2*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cos(1/2
*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/
(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*s
inh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(2*a^2*b*e^(2*c) +
 2*a*b^2*e^(2*c) - (a^2*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cosh(1/2*imag_pa
rt(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b
/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(2*
a^2*b*e^(2*c) + 2*a*b^2*e^(2*c) - (a^2*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*c
os(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arcco
s(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b
))))^3 + (2*a^2*b*e^(2*c) + 2*a*b^2*e^(2*c) - (a^2*e^(2*c) - b^2*e^(2*c))*s
qrt(-a*b))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*im
ag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (2*a^2*b*e^(2*c) + 2*a*b^2*e^(
2*c) - (a^2*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cosh(1/2*imag_part(arccos(-a
/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))
- (2*a^2*b*e^(2*c) + 2*a*b^2*e^(2*c) - (a^2*e^(2*c) - b^2*e^(2*c))*sqrt(-a*
b))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(a
rccos(-a/(a + b) + b/(a + b)))))*arctan((((a + b)/(a*e^(4*c) + b*e^(4*c)))^
```

```
(1/4)*cos(1/2*arccos(-(a - b)/(a + b))) + e^(d*x))/(((a + b)/(a*e^(4*c) + b
*e^(4*c)))^(1/4)*sin(1/2*arccos(-(a - b)/(a + b)))))/(a^3*b*e^(2*c) + 2*a^2
*b^2*e^(2*c) + a*b^3*e^(2*c)) + 2*(3*(2*a^2*b*e^(2*c) + 2*a*b^2*e^(2*c) - (
a^2*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b)
+ b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(
1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) - (2*a^2*b*e^(2*c) + 2*a*b^2
*e^(2*c) - (a^2*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cosh(1/2*imag_part(arcco
s(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a +
b))))^3 - 9*(2*a^2*b*e^(2*c) + 2*a*b^2*e^(2*c) - (a^2*e^(2*c) - b^2*e^(2*c)
)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2
*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(
a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) +
 3*(2*a^2*b*e^(2*c) + 2*a*b^2*e^(2*c) - (a^2*e^(2*c) - b^2*e^(2*c))*sqrt(-a
*b))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_par
t(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) +
 b/(a + b)))) + 9*(2*a^2*b*e^(2*c) + 2*a*b^2*e^(2*c) - (a^2*e^(2*c) - b^2*e
^(2*c))*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*co
sh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(
-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))
))^2 - 3*(2*a^2*b*e^(2*c) + 2*a*b^2*e^(2*c) - (a^2*e^(2*c) - b^2*e^(2*c))*s
qrt(-a*b))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real
_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a +
b) + b/(a + b))))^2 - 3*(2*a^2*b*e^(2*c) + 2*a*b^2*e^(2*c) - (a^2*e^(2*c) -
 b^2*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))
)^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(a
rccos(-a/(a + b) + b/(a + b))))^3 + (2*a^2*b*e^(2*c) + 2*a*b^2*e^(2*c) - (a
^2*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*sin(1/2*real_part(arccos(-a/(a + b) +
 b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (2*
a^2*b*e^(2*c) + 2*a*b^2*e^(2*c) - (a^2*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*c
osh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos
(-a/(a + b) + b/(a + b)))) - (2*a^2*b*e^(2*c) + 2*a*b^2*e^(2*c) - (a^2*e^(2
*c) - b^2*e^(2*c))*sqrt(-a*b))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a +
 b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*arctan(-(((a +
b)/(a*e^(4*c) + b*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b))) - e^(d*
x))/(((a + b)/(a*e^(4*c) + b*e^(4*c)))^(1/4)*sin(1/2*arccos(-(a - b)/(a + b
)))))/(a^3*b*e^(2*c) + 2*a^2*b^2*e^(2*c) + a*b^3*e^(2*c)) + ((2*a^2*b*e^(2*
c) + 2*a*b^2*e^(2*c) - (a^2*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cos(1/2*real
_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a +
b) + b/(a + b))))^3 - 3*(2*a^2*b*e^(2*c) + 2*a*b^2*e^(2*c) - (a^2*e^(2*c) -
 b^2*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))
)*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(a
rccos(-a/(a + b) + b/(a + b))))^2 - 3*(2*a^2*b*e^(2*c) + 2*a*b^2*e^(2*c) -
(a^2*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b)
 + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin
h(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(2*a^2*b*e^(2*c) + 2*a
*b^2*e^(2*c) - (a^2*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(ar
ccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a
+ b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*ima
g_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(2*a^2*b*e^(2*c) + 2*a*b^2*e^(2
*c) - (a^2*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(
a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))
*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 9*(2*a^2*b*e^(2*c)
 + 2*a*b^2*e^(2*c) - (a^2*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cos(1/2*real_p
art(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) +
 b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2
*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - (2*a^2*b*e^(2*c) + 2*a*b^2*
e^(2*c) - (a^2*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(arccos(
-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b
))))^3 + 3*(2*a^2*b*e^(2*c) + 2*a*b^2*e^(2*c) - (a^2*e^(2*c) - b^2*e^(2*c))
```

```
*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*rea
l_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a +
 b) + b/(a + b))))^3 + (2*a^2*b*e^(2*c) + 2*a*b^2*e^(2*c) - (a^2*e^(2*c) -
b^2*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))
*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) - (2*a^2*b*e^(2*c) + 2
*a*b^2*e^(2*c) - (a^2*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(
arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(
a + b)))))*log(2*((a + b)/(a*e^(4*c) + b*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a
 - b)/(a + b)))*e^(d*x) + sqrt((a + b)/(a*e^(4*c) + b*e^(4*c))) + e^(2*d*x)
)/(a^3*b*e^(2*c) + 2*a^2*b^2*e^(2*c) + a*b^3*e^(2*c)) - ((2*a^2*b*e^(2*c) +
 2*a*b^2*e^(2*c) - (a^2*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cos(1/2*real_par
t(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) +
 b/(a + b))))^3 - 3*(2*a^2*b*e^(2*c) + 2*a*b^2*e^(2*c) - (a^2*e^(2*c) - b^2
*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*co
sh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arcco
s(-a/(a + b) + b/(a + b))))^2 - 3*(2*a^2*b*e^(2*c) + 2*a*b^2*e^(2*c) - (a^2
*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b
/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/
2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(2*a^2*b*e^(2*c) + 2*a*b^2
*e^(2*c) - (a^2*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(arccos
(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)
)))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_pa
rt(arccos(-a/(a + b) + b/(a + b)))) + 3*(2*a^2*b*e^(2*c) + 2*a*b^2*e^(2*c)
- (a^2*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a +
b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin
h(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 9*(2*a^2*b*e^(2*c) + 2
*a*b^2*e^(2*c) - (a^2*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(
arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(
a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*ima
g_part(arccos(-a/(a + b) + b/(a + b))))^2 - (2*a^2*b*e^(2*c) + 2*a*b^2*e^(2
*c) - (a^2*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(
a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))
^3 + 3*(2*a^2*b*e^(2*c) + 2*a*b^2*e^(2*c) - (a^2*e^(2*c) - b^2*e^(2*c))*sqr
t(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_pa
rt(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b)
+ b/(a + b))))^3 + (2*a^2*b*e^(2*c) + 2*a*b^2*e^(2*c) - (a^2*e^(2*c) - b^2*
e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cos
h(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) - (2*a^2*b*e^(2*c) + 2*a*b
^2*e^(2*c) - (a^2*e^(2*c) - b^2*e^(2*c))*sqrt(-a*b))*cos(1/2*real_part(arcc
os(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a +
b)))))*log(-2*((a + b)/(a*e^(4*c) + b*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a -
b)/(a + b)))*e^(d*x) + sqrt((a + b)/(a*e^(4*c) + b*e^(4*c))) + e^(2*d*x))/(
a^3*b*e^(2*c) + 2*a^2*b^2*e^(2*c) + a*b^3*e^(2*c)))/d
```

3.110 $\int \frac{\text{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=32

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

```
[Out] ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Rubi [A] time = 0.056264, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3675, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}\end{aligned}$$

Mathematica [A] time = 0.0533549, size = 32, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Maple [B] time = 0.07, size = 363, normalized size = 11.3

$$-\frac{a}{d}\mathrm{Artanh}\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\frac{1}{\sqrt{\left(2\sqrt{b\left(a+b\right)}-a-2b\right)a}}\right)\frac{1}{\sqrt{b\left(a+b\right)}}\frac{1}{\sqrt{\left(2\sqrt{b\left(a+b\right)}-a-2b\right)a}}+\frac{1}{d}\mathrm{Artanh}\left(a\tanh\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x)
```

```
[Out] -a/d/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2
*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/d/((2*(b*(a+b))^(1/2)-a-
2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(
1/2))-1/d*b/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*t
anh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-a/d/(b*(a+b))^(1/2)
/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a
+b))^(1/2)+a+2*b)*a)^(1/2))-1/d/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(
a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/d*b/(b*(a+b))^
(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*
(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.39622, size = 1199, normalized size = 37.47

$$\left[-\frac{\sqrt{-ab}\log\left(\frac{\left(a^2+2\,ab+b^2\right)\cosh(dx+c)^4+4\left(a^2+2\,ab+b^2\right)\cosh(dx+c)\sinh(dx+c)^3+\left(a^2+2\,ab+b^2\right)\sinh(dx+c)^4+2\left(a^2-b^2\right)\cosh(dx+c)^2+2\left(3\left(a^2+2\,ab+b^2\right)\right.}{(a+b)\cosh(dx+c)^4+4\,(a+b)\cosh(dx+c)\sinh(dx+c)^3+(a+b)\sinh(dx+c)}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b
+ b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4
+ 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2
```

```
+ a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*c
osh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh
(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)
^2 + a - b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*
sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(
3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x +
c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b))/(a*b*d), sqrt(a*b)*ar
ctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) +
 (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(a*b)/(a*b))/(a*b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)**2/(a + b*tanh(c + d*x)**2), x)
```

Giac [A] time = 1.40056, size = 59, normalized size = 1.84

$$\frac{\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\sqrt{ab}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/(sqrt
(a*b)*d)
```

3.111 $\int \frac{\operatorname{sech}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=55

$$\frac{\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}bd}-\frac{\tan^{-1}(\sinh(c+dx))}{bd}$$

```
[Out] -(ArcTan[Sinh[c + d*x]]/(b*d)) + (Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Sinh[c +
d*x])/Sqrt[a]])/(Sqrt[a]*b*d)
```

Rubi [A] time = 0.0729596, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3676, 391, 203, 205}

$$\frac{\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}bd}-\frac{\tan^{-1}(\sinh(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -(ArcTan[Sinh[c + d*x]]/(b*d)) + (Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Sinh[c +
d*x])/Sqrt[a]])/(Sqrt[a]*b*d)
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)(a+(a+b)x^2)}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= -\frac{\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{bd} + \frac{(a+b)\operatorname{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \sinh(c+dx)\right)}{bd}$$
$$= -\frac{\tan^{-1}(\sinh(c+dx))}{bd} + \frac{\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}bd}$$

Mathematica [A] time = 0.194912, size = 55, normalized size = 1.

$$-\frac{\frac{\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}} + 2\tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -(((Sqrt[a + b]*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]])/Sqrt[a] + 2*Ar
cTan[Tanh[(c + d*x)/2]])/(b*d))
```

Maple [B] time = 0.073, size = 494, normalized size = 9.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2),x)
```

```
[Out] -1/d*a/b/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/
((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+a/d/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2
)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*
a)^(1/2))+1/d*a/b/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x
+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+a/d/(b*(a+b))^(1/2)/((2*(b*(a+
b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+
a+2*b)*a)^(1/2))-1/d/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2
*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/d*b/(b*(a+b))^(1/2)/((2*
(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))
^(1/2)-a-2*b)*a)^(1/2))+1/d/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*ta
nh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/d*b/(b*(a+b))^(1/2
)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(
a+b))^(1/2)+a+2*b)*a)^(1/2))-2/d/b*arctan(tanh(1/2*d*x+1/2*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\arctan\left(e^{(dx+c)}\right)}{bd} + 8\int \frac{\left(ae^{(3c)} + be^{(3c)}\right)e^{(3dx)} + \left(ae^{c} + be^{c}\right)e^{(dx)}}{4\left(ab + b^2 + \left(abe^{(4c)} + b^2e^{(4c)}\right)e^{(4dx)} + 2\left(abe^{(2c)} - b^2e^{(2c)}\right)e^{(2dx)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -2*arctan(e^(d*x + c))/(b*d) + 8*integrate(1/4*((a*e^(3*c) + b*e^(3*c))*e^(
3*d*x) + (a*e^c + b*e^c)*e^(d*x))/(a*b + b^2 + (a*b*e^(4*c) + b^2*e^(4*c))*
e^(4*d*x) + 2*(a*b*e^(2*c) - b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 2.58427, size = 1494, normalized size = 27.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-(a + b)/a)*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x +
c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2
+ 2*(3*(a + b)*cosh(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh
(d*x + c)^3 - (3*a + b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3
 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 - a*cosh(d*x + c)
+ (3*a*cosh(d*x + c)^2 - a)*sinh(d*x + c))*sqrt(-(a + b)/a) + a + b)/((a +
b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh
(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a
- b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*
sinh(d*x + c) + a + b)) - 4*arctan(cosh(d*x + c) + sinh(d*x + c)))/(b*d), (
sqrt((a + b)/a)*arctan(1/2*sqrt((a + b)/a)*(cosh(d*x + c) + sinh(d*x + c)))
 + sqrt((a + b)/a)*arctan(1/2*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x
 + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (3*a - b)*cosh(d*x + c) +
 (3*(a + b)*cosh(d*x + c)^2 + 3*a - b)*sinh(d*x + c))*sqrt((a + b)/a)/(a +
b)) - 2*arctan(cosh(d*x + c) + sinh(d*x + c)))/(b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)**3/(a + b*tanh(c + d*x)**2), x)
```

Giac [C] time = 1.79286, size = 4938, normalized size = 89.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/4*(2*(3*(a^2*b + 2*a*b^2 + b^3)*cos(1/2*real_part(arccos(-a/(a + b) + b/(
a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*r
```

```
eal_part(arccos(-a/(a + b) + b/(a + b)))) - (a^2*b + 2*a*b^2 + b^3)*cosh(1/
2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/
(a + b) + b/(a + b))))^3 - 9*(a^2*b + 2*a*b^2 + b^3)*cos(1/2*real_part(arcc
os(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a
+ b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_
part(arccos(-a/(a + b) + b/(a + b)))) + 3*(a^2*b + 2*a*b^2 + b^3)*cosh(1/2*
imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a
 + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))
+ 9*(a^2*b + 2*a*b^2 + b^3)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b)
)))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part
(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/
(a + b))))^2 - 3*(a^2*b + 2*a*b^2 + b^3)*cosh(1/2*imag_part(arccos(-a/(a +
b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh
(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(a^2*b + 2*a*b^2 + b^
3)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(a
rccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a
 + b))))^3 + (a^2*b + 2*a*b^2 + b^3)*sin(1/2*real_part(arccos(-a/(a + b) +
b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (a^2
*b + 2*a*b^2 + b^3)*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin
(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) - (a^2*b + 2*a*b^2 + b^3)*s
in(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos
(-a/(a + b) + b/(a + b)))))*arctan((((a*b + b^2)/(a*b*e^(4*c) + b^2*e^(4*c)
))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b))) + e^(d*x))/(((a*b + b^2)/(a*b*e^
(4*c) + b^2*e^(4*c)))^(1/4)*sin(1/2*arccos(-(a - b)/(a + b)))))/(2*a*b^3 +
(a*b - b^2)*sqrt(-a*b)*abs(b)) + 2*(3*(a^2*b + 2*a*b^2 + b^3)*cos(1/2*real_
part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b
) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) - (a^
2*b + 2*a*b^2 + b^3)*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*
sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3 - 9*(a^2*b + 2*a*b^2 +
 b^3)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_pa
rt(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) +
 b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(a^2*
b + 2*a*b^2 + b^3)*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*si
n(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arcco
s(-a/(a + b) + b/(a + b)))) + 9*(a^2*b + 2*a*b^2 + b^3)*cos(1/2*real_part(a
rccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/
(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag
_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(a^2*b + 2*a*b^2 + b^3)*cosh(1
/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(
a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))
^2 - 3*(a^2*b + 2*a*b^2 + b^3)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a +
 b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_p
art(arccos(-a/(a + b) + b/(a + b))))^3 + (a^2*b + 2*a*b^2 + b^3)*sin(1/2*re
al_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a
+ b) + b/(a + b))))^3 + (a^2*b + 2*a*b^2 + b^3)*cosh(1/2*imag_part(arccos(-
a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))
 - (a^2*b + 2*a*b^2 + b^3)*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))
))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*arctan(-(((a*b + b^
2)/(a*b*e^(4*c) + b^2*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b))) - e
^(d*x))/(((a*b + b^2)/(a*b*e^(4*c) + b^2*e^(4*c)))^(1/4)*sin(1/2*arccos(-(a
 - b)/(a + b)))))/(2*a*b^3 + (a*b - b^2)*sqrt(-a*b)*abs(b)) + ((a^2*b + 2*a
*b^2 + b^3)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*i
mag_part(arccos(-a/(a + b) + b/(a + b))))^3 - 3*(a^2*b + 2*a*b^2 + b^3)*cos
(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-
a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))
))^2 - 3*(a^2*b + 2*a*b^2 + b^3)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a
 + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*i
mag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(a^2*b + 2*a*b^2 + b^3)*cos(1
/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/
```

```
(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))
^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(a^2*b + 2*a*b^2
 + b^3)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_
part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b)
+ b/(a + b))))^2 - 9*(a^2*b + 2*a*b^2 + b^3)*cos(1/2*real_part(arccos(-a/(a
 + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*si
n(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arcco
s(-a/(a + b) + b/(a + b))))^2 - (a^2*b + 2*a*b^2 + b^3)*cos(1/2*real_part(a
rccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/
(a + b))))^3 + 3*(a^2*b + 2*a*b^2 + b^3)*cos(1/2*real_part(arccos(-a/(a + b
) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(
1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (a^2*b + 2*a*b^2 + b^3)*
cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arcco
s(-a/(a + b) + b/(a + b)))) - (a^2*b + 2*a*b^2 + b^3)*cos(1/2*real_part(arc
cos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a +
 b)))))*log(2*((a*b + b^2)/(a*b*e^(4*c) + b^2*e^(4*c)))^(1/4)*cos(1/2*arcco
s(-(a - b)/(a + b)))*e^(d*x) + sqrt((a*b + b^2)/(a*b*e^(4*c) + b^2*e^(4*c))
) + e^(2*d*x))/(2*a*b^3 + (a*b - b^2)*sqrt(-a*b)*abs(b)) - ((a^2*b + 2*a*b^
2 + b^3)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag
_part(arccos(-a/(a + b) + b/(a + b))))^3 - 3*(a^2*b + 2*a*b^2 + b^3)*cos(1/
2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(
a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^
2 - 3*(a^2*b + 2*a*b^2 + b^3)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a +
b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag
_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(a^2*b + 2*a*b^2 + b^3)*cos(1/2*
real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a
+ b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*
sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(a^2*b + 2*a*b^2 +
b^3)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_par
t(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b
/(a + b))))^2 - 9*(a^2*b + 2*a*b^2 + b^3)*cos(1/2*real_part(arccos(-a/(a +
b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1
/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-
a/(a + b) + b/(a + b))))^2 - (a^2*b + 2*a*b^2 + b^3)*cos(1/2*real_part(arcc
os(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a
+ b))))^3 + 3*(a^2*b + 2*a*b^2 + b^3)*cos(1/2*real_part(arccos(-a/(a + b) +
 b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2
*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (a^2*b + 2*a*b^2 + b^3)*cos
(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-
a/(a + b) + b/(a + b)))) - (a^2*b + 2*a*b^2 + b^3)*cos(1/2*real_part(arccos
(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)
))))*log(-2*((a*b + b^2)/(a*b*e^(4*c) + b^2*e^(4*c)))^(1/4)*cos(1/2*arccos(
-(a - b)/(a + b)))*e^(d*x) + sqrt((a*b + b^2)/(a*b*e^(4*c) + b^2*e^(4*c)))
+ e^(2*d*x))/(2*a*b^3 + (a*b - b^2)*sqrt(-a*b)*abs(b)) - 8*arctan(e^(d*x +
c))/b)/d
```

3.112 $\int \frac{\operatorname{sech}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=50

$$\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}d}-\frac{\tanh(c+dx)}{bd}$$

```
[Out] ((a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(3/2)*d) - Tan
h[c + d*x]/(b*d)
```

Rubi [A] time = 0.0695992, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3675, 388, 205}

$$\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}d}-\frac{\tanh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(3/2)*d) - Tan
h[c + d*x]/(b*d)
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\frac{\operatorname{Subst}\left(\int \frac{1-x^2}{a+bx^2}\,dx,x,\tanh(c+dx)\right)}{d}$$
$$=-\frac{\tanh(c+dx)}{bd}+\frac{(a+b)\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\tanh(c+dx)\right)}{bd}$$
$$=\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}d}-\frac{\tanh(c+dx)}{bd}$$

Mathematica [A] time = 0.133055, size = 50, normalized size = 1.

$$\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}d}-\frac{\tanh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(3/2)*d) - Tan
h[c + d*x]/(b*d)
```

Maple [B] time = 0.068, size = 648, normalized size = 13.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2),x)
```

```
[Out] -1/d*a^2/b/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*ta
nh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/d*a/b/((2*(b*(a+b)
)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a
-2*b)*a)^(1/2))-2*a/d/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*a
rctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-1/d*a^2/b
/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+
1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/d*a/b/((2*(b*(a+b))^(1/2)+a+2
*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/
2))-2*a/d/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh
(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/d/((2*(b*(a+b))^(1/2
)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*
a)^(1/2))-1/d*b/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh
(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-1/d/((2*(b*(a+b
))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a
+2*b)*a)^(1/2))-1/d*b/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*a
rctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-2/d/b*tanh
(1/2*d*x+1/2*c)/(tanh(1/2*d*x+1/2*c)^2+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.42194, size = 1750, normalized size = 35.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (
a + b)*sinh(d*x + c)^2 + a + b)*sqrt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*
x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a
*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a
*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2
+ 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(
d*x + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x +
c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)^4
+ 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a
- b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^
2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a +
 b)) - 4*a*b)/(a*b^2*d*cosh(d*x + c)^2 + 2*a*b^2*d*cosh(d*x + c)*sinh(d*x +
 c) + a*b^2*d*sinh(d*x + c)^2 + a*b^2*d), (((a + b)*cosh(d*x + c)^2 + 2*(a
+ b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a + b)*sqrt(a*
b)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x +
 c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(a*b)/(a*b)) + 2*a*b)/(a*b^2*d*c
osh(d*x + c)^2 + 2*a*b^2*d*cosh(d*x + c)*sinh(d*x + c) + a*b^2*d*sinh(d*x +
 c)^2 + a*b^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)**4/(a + b*tanh(c + d*x)**2), x)
```

Giac [A] time = 1.37988, size = 113, normalized size = 2.26

$$\frac{\frac{\left(ae^{(2c)}+be^{(2c)}\right)\arctan\left(\frac{ae^{(2dx+2c)}+be^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)e^{(-2c)}}{\sqrt{ab}b}+\frac{2}{b\left(e^{(2dx+2c)}+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] ((a*e^(2*c) + b*e^(2*c))*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c)
+ a - b)/sqrt(a*b))*e^(-2*c)/(sqrt(a*b)*b) + 2/(b*(e^(2*d*x + 2*c) + 1)))/d
```

3.113 $\int \frac{\operatorname{sech}^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=86

$$\frac{(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^2d}-\frac{(2a+3b)\tan^{-1}(\sinh(c+dx))}{2b^2d}-\frac{\tanh(c+dx)\operatorname{sech}(c+dx)}{2bd}$$

```
[Out] -((2*a + 3*b)*ArcTan[Sinh[c + d*x]])/(2*b^2*d) + ((a + b)^(3/2)*ArcTan[(Sqr
t[a + b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^2*d) - (Sech[c + d*x]*Tanh[c +
 d*x])/(2*b*d)
```

Rubi [A] time = 0.120484, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3676, 414, 522, 203, 205}

$$\frac{(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^2d}-\frac{(2a+3b)\tan^{-1}(\sinh(c+dx))}{2b^2d}-\frac{\tanh(c+dx)\operatorname{sech}(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^5/(a + b*Tanh[c + d*x]^2),x]
```


```
[Out] -((2*a + 3*b)*ArcTan[Sinh[c + d*x]])/(2*b^2*d) + ((a + b)^(3/2)*ArcTan[(Sqr
t[a + b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^2*d) - (Sech[c + d*x]*Tanh[c +
 d*x])/(2*b*d)
```


Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)^2(a+(a+b)x^2)}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2bd} + \frac{\operatorname{Subst}\left(\int \frac{a+2b+(-a-b)x^2}{(1+x^2)(a+(a+b)x^2)}\,dx, x, \sinh(c+dx)\right)}{2bd}$$

$$= -\frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2bd} + \frac{(a+b)^2\operatorname{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \sinh(c+dx)\right)}{b^2d} - \frac{(2a+3b)\,\mathrm{S}}{}$$

$$= -\frac{(2a+3b)\tan^{-1}(\sinh(c+dx))}{2b^2d} + \frac{(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^2d} - \frac{\operatorname{sech}(c+dx)\tanh(c+}{2bd}$$

Mathematica [A] time = 0.557462, size = 79, normalized size = 0.92

$$-\frac{2(2a+3b)\tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) + \frac{2(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}} + b\tanh(c+dx)\operatorname{sech}(c+dx)}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^5/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -((2*(a + b)^(3/2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]])/Sqrt[a] + 2
*(2*a + 3*b)*ArcTan[Tanh[(c + d*x)/2]] + b*Sech[c + d*x]*Tanh[c + d*x])/(2*
b^2*d)
```

Maple [B] time = 0.076, size = 836, normalized size = 9.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2),x)
```

```
[Out] -1/d*a^2/b^2/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2
*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-2/d*a/b/((2*(b*(a+b))^(1/2)-a-2*b)
*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)
)-1/d/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2
*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/d*a^2/b/(b*(a+b))^(1/2)/((2*(b*(a+b))^(
1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*
b)*a)^(1/2))+2*a/d/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arct
anh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/d*b/(b*(a+
b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)
```

```
/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/d*a^2/b^2/((2*(b*(a+b))^(1/2)+a+2*b
)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)
)+2/d*a/b/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/
((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/d/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2
)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/d*a^2
/b/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*
x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+2*a/d/(b*(a+b))^(1/2)/((2*(b*
(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/
2)+a+2*b)*a)^(1/2))+1/d*b/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/
2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/d/b/
(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d*x+1/2*c)^3-1/d/b/(tanh(1/2*d*x+1/2*c
)^2+1)^2*tanh(1/2*d*x+1/2*c)-3/d/b*arctan(tanh(1/2*d*x+1/2*c))-2/d/b^2*arct
an(tanh(1/2*d*x+1/2*c))*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{e^{(3\,dx+3\,c)}-e^{(dx+c)}}{bde^{(4\,dx+4\,c)}+2\,bde^{(2\,dx+2\,c)}+bd}-\frac{(2\,ae^{c}+3\,be^{c})\arctan\left(e^{(dx+c)}\right)e^{(-c)}}{b^{2}d}+32\int\frac{\left(a^{2}e^{(3\,c)}+2\,abe^{(3\,c)}+b^{2}e^{(3\,c)}\right)e^{(3\,dx)}}{16\left(ab^{2}+b^{3}+\left(ab^{2}e^{(4\,c)}+b^{3}e^{(4\,c)}\right)e^{(4\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -(e^(3*d*x + 3*c) - e^(d*x + c))/(b*d*e^(4*d*x + 4*c) + 2*b*d*e^(2*d*x + 2*
c) + b*d) - (2*a*e^c + 3*b*e^c)*arctan(e^(d*x + c))*e^(-c)/(b^2*d) + 32*int
egrate(1/16*((a^2*e^(3*c) + 2*a*b*e^(3*c) + b^2*e^(3*c))*e^(3*d*x) + (a^2*e
^c + 2*a*b*e^c + b^2*e^c)*e^(d*x))/(a*b^2 + b^3 + (a*b^2*e^(4*c) + b^3*e^(4
*c))*e^(4*d*x) + 2*(a*b^2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 2.82544, size = 4316, normalized size = 50.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*b*cosh(d*x + c)^3 + 6*b*cosh(d*x + c)*sinh(d*x + c)^2 + 2*b*sinh(d
*x + c)^3 - ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c
)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*co
sh(d*x + c)^2 + a + b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a +
b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*sqrt(-(a + b)/a)*log(((a + b)*cosh
(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x +
c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 - 3*a - b
)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 - (3*a + b)*cosh(d*x + c))*s
inh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a
*sinh(d*x + c)^3 - a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 - a)*sinh(d*x + c
))*sqrt(-(a + b)/a) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x
+ c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2
+ 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d
*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 2*((2*a + 3*b)
*cosh(d*x + c)^4 + 4*(2*a + 3*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a + 3*b
)*sinh(d*x + c)^4 + 2*(2*a + 3*b)*cosh(d*x + c)^2 + 2*(3*(2*a + 3*b)*cosh(d
*x + c)^2 + 2*a + 3*b)*sinh(d*x + c)^2 + 4*((2*a + 3*b)*cosh(d*x + c)^3 + (
```

```
2*a + 3*b)*cosh(d*x + c))*sinh(d*x + c) + 2*a + 3*b)*arctan(cosh(d*x + c) +
 sinh(d*x + c)) - 2*b*cosh(d*x + c) + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*x
+ c))/(b^2*d*cosh(d*x + c)^4 + 4*b^2*d*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*
d*sinh(d*x + c)^4 + 2*b^2*d*cosh(d*x + c)^2 + b^2*d + 2*(3*b^2*d*cosh(d*x +
 c)^2 + b^2*d)*sinh(d*x + c)^2 + 4*(b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x
+ c))*sinh(d*x + c)), -(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)
^2 + b*sinh(d*x + c)^3 - ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)
*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a + b)*cosh(d*x + c)^2 + 2*
(3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x +
 c)^3 + (a + b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*sqrt((a + b)/a)*arcta
n(1/2*sqrt((a + b)/a)*(cosh(d*x + c) + sinh(d*x + c))) - ((a + b)*cosh(d*x
+ c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4
+ 2*(a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*
x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + b)*cosh(d*x + c))*sinh(d*x + c
) + a + b)*sqrt((a + b)/a)*arctan(1/2*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*
cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (3*a - b)*cosh(d*
x + c) + (3*(a + b)*cosh(d*x + c)^2 + 3*a - b)*sinh(d*x + c))*sqrt((a + b)/
a)/(a + b)) + ((2*a + 3*b)*cosh(d*x + c)^4 + 4*(2*a + 3*b)*cosh(d*x + c)*si
nh(d*x + c)^3 + (2*a + 3*b)*sinh(d*x + c)^4 + 2*(2*a + 3*b)*cosh(d*x + c)^2
 + 2*(3*(2*a + 3*b)*cosh(d*x + c)^2 + 2*a + 3*b)*sinh(d*x + c)^2 + 4*((2*a
+ 3*b)*cosh(d*x + c)^3 + (2*a + 3*b)*cosh(d*x + c))*sinh(d*x + c) + 2*a + 3
*b)*arctan(cosh(d*x + c) + sinh(d*x + c)) - b*cosh(d*x + c) + (3*b*cosh(d*x
 + c)^2 - b)*sinh(d*x + c))/(b^2*d*cosh(d*x + c)^4 + 4*b^2*d*cosh(d*x + c)*
sinh(d*x + c)^3 + b^2*d*sinh(d*x + c)^4 + 2*b^2*d*cosh(d*x + c)^2 + b^2*d +
 2*(3*b^2*d*cosh(d*x + c)^2 + b^2*d)*sinh(d*x + c)^2 + 4*(b^2*d*cosh(d*x +
c)^3 + b^2*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**5/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)**5/(a + b*tanh(c + d*x)**2), x)
```

Giac [C] time = 1.84435, size = 5747, normalized size = 66.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/4*(4*(2*a*e^c + 3*b*e^c)*arctan(e^(d*x + c))*e^(-c)/b^2 + 4*(e^(3*d*x +
3*c) - e^(d*x + c))/(b*(e^(2*d*x + 2*c) + 1)^2) - 2*(3*(2*a^2*b + 2*a*b^2 -
 (a^2 - b^2)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))
^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(
arccos(-a/(a + b) + b/(a + b)))) - (2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a
*b))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_par
t(arccos(-a/(a + b) + b/(a + b))))^3 - 9*(2*a^2*b + 2*a*b^2 - (a^2 - b^2)*s
qrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*im
```

```
ag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a +
 b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*
(2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*cosh(1/2*imag_part(arccos(-a/(
a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^
3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(2*a^2*b + 2*a*b^
2 - (a^2 - b^2)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b)
)))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part
(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/
(a + b))))^2 - 3*(2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*cosh(1/2*imag
_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b)
+ b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*
(2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a
 + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*s
inh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (2*a^2*b + 2*a*b^2 -
 (a^2 - b^2)*sqrt(-a*b))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))
^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (2*a^2*b + 2*a*b
^2 - (a^2 - b^2)*sqrt(-a*b))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a +
b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) - (2*a^2*b + 2*a*b
^2 - (a^2 - b^2)*sqrt(-a*b))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b
))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*arctan((((a*b^2 +
 b^3)/(a*b^2*e^(4*c) + b^3*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b))
) + e^(d*x))/(((a*b^2 + b^3)/(a*b^2*e^(4*c) + b^3*e^(4*c)))^(1/4)*sin(1/2*a
rccos(-(a - b)/(a + b)))))/(a*b^3) - 2*(3*(2*a^2*b + 2*a*b^2 - (a^2 - b^2)*
sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*i
mag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a
+ b) + b/(a + b)))) - (2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*cosh(1/2
*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(
a + b) + b/(a + b))))^3 - 9*(2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*co
s(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arcco
s(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a +
b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(2*a^2*b + 2*
a*b^2 - (a^2 - b^2)*sqrt(-a*b))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a
 + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*im
ag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(2*a^2*b + 2*a*b^2 - (a^2 - b^
2)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/
2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a
 + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2
- 3*(2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*cosh(1/2*imag_part(arccos(
-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))
)^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(2*a^2*b + 2*
a*b^2 - (a^2 - b^2)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a
+ b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_
part(arccos(-a/(a + b) + b/(a + b))))^3 + (2*a^2*b + 2*a*b^2 - (a^2 - b^2)*
sqrt(-a*b))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*i
mag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (2*a^2*b + 2*a*b^2 - (a^2 - b
^2)*sqrt(-a*b))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2
*real_part(arccos(-a/(a + b) + b/(a + b)))) - (2*a^2*b + 2*a*b^2 - (a^2 - b
^2)*sqrt(-a*b))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2
*imag_part(arccos(-a/(a + b) + b/(a + b)))))*arctan(-(((a*b^2 + b^3)/(a*b^2
*e^(4*c) + b^3*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b))) - e^(d*x))
/(((a*b^2 + b^3)/(a*b^2*e^(4*c) + b^3*e^(4*c)))^(1/4)*sin(1/2*arccos(-(a -
b)/(a + b)))))/(a*b^3) - ((2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*cos(
1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(
-a/(a + b) + b/(a + b))))^3 - 3*(2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b)
)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arc
cos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a
+ b))))^2 - 3*(2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*cos(1/2*real_par
t(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) +
 b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(2*
```

```
a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a +
b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin
(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos
(-a/(a + b) + b/(a + b)))) + 3*(2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))
*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(ar
ccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a
+ b))))^2 - 9*(2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*cos(1/2*real_par
t(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b
/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*i
mag_part(arccos(-a/(a + b) + b/(a + b))))^2 - (2*a^2*b + 2*a*b^2 - (a^2 - b
^2)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1
/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + 3*(2*a^2*b + 2*a*b^2 - (a
^2 - b^2)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*si
n(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arcco
s(-a/(a + b) + b/(a + b))))^3 + (2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b)
)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arc
cos(-a/(a + b) + b/(a + b)))) - (2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b)
)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arc
cos(-a/(a + b) + b/(a + b)))))*log(2*((a*b^2 + b^3)/(a*b^2*e^(4*c) + b^3*e^
(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b)))*e^(d*x) + sqrt((a*b^2 + b^3
)/(a*b^2*e^(4*c) + b^3*e^(4*c))) + e^(2*d*x))/(a*b^3) + ((2*a^2*b + 2*a*b^2
 - (a^2 - b^2)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))
))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 - 3*(2*a^2*b + 2
*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a
 + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real
_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(2*a^2*b + 2*a*b^2 - (a^2 - b^
2)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/
2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a
/(a + b) + b/(a + b)))) + 9*(2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*co
s(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(
-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)
)))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(2*a^2*b + 2*
a*b^2 - (a^2 - b^2)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a
+ b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag
_part(arccos(-a/(a + b) + b/(a + b))))^2 - 9*(2*a^2*b + 2*a*b^2 - (a^2 - b^
2)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*
imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a +
 b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2
 - (2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/
(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))
)^3 + 3*(2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*cos(1/2*real_part(arcc
os(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b
))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (2*a^2*b + 2
*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a
 + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) - (2*a^2*b + 2
*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a
 + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*log(-2*((a*b^
2 + b^3)/(a*b^2*e^(4*c) + b^3*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a +
b)))*e^(d*x) + sqrt((a*b^2 + b^3)/(a*b^2*e^(4*c) + b^3*e^(4*c))) + e^(2*d*x
))/(a*b^3))/d
```

3.114 $\int \frac{\text{sech}^6(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=75

$$-\frac{(a+2b)\tanh(c+dx)}{b^2d}+\frac{(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}d}+\frac{\tanh^3(c+dx)}{3bd}$$

```
[Out] ((a + b)^2*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(5/2)*d) - (
(a + 2*b)*Tanh[c + d*x])/(b^2*d) + Tanh[c + d*x]^3/(3*b*d)
```

Rubi [A] time = 0.0915841, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3675, 390, 205}

$$-\frac{(a+2b)\tanh(c+dx)}{b^2d}+\frac{(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}d}+\frac{\tanh^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^6/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((a + b)^2*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(5/2)*d) - (
(a + 2*b)*Tanh[c + d*x])/(b^2*d) + Tanh[c + d*x]^3/(3*b*d)
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^2}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(-\frac{a+2b}{b^2}+\frac{x^2}{b}+\frac{a^2+2ab+b^2}{b^2(a+bx^2)}\right)dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{(a+2b)\tanh(c+dx)}{b^2d}+\frac{\tanh^3(c+dx)}{3bd}+\frac{(a+b)^2\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{b^2d}$$
$$= \frac{(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}d}-\frac{(a+2b)\tanh(c+dx)}{b^2d}+\frac{\tanh^3(c+dx)}{3bd}$$

Mathematica [A] time = 0.34114, size = 71, normalized size = 0.95

$$\frac{(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}d}-\frac{\tanh(c+dx)\left(3a+b\operatorname{sech}^2(c+dx)+5b\right)}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^6/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((a + b)^2*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(5/2)*d) - (
(3*a + 5*b + b*Sech[c + d*x]^2)*Tanh[c + d*x])/(3*b^2*d)
```

Maple [B] time = 0.078, size = 1077, normalized size = 14.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2),x)
```

```
[Out] -1/d*a^3/b^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*
tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-3/d*a^2/b/(b*(a+b)
)^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/(
(2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-3*a/d/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/
2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)
*a)^(1/2))+1/d*a^2/b^2/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1
/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+2/d*a/b/((2*(b*(a+b))^(1
/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b
)*a)^(1/2))+1/d/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+
1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-1/d*b/(b*(a+b))^(1/2)/((2*(b*(a
+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2
)-a-2*b)*a)^(1/2))-1/d*a^3/b^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a
)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-3
/d*a^2/b/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(
1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-3*a/d/(b*(a+b))^(1/2)/(
(2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b
))^(1/2)+a+2*b)*a)^(1/2))-1/d*a^2/b^2/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*a
rctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-2/d*a/b/((
2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b)
```

```
)^(1/2)+a+2*b)*a)^(1/2))-1/d/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*t
anh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/d*b/(b*(a+b))^(1/
2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*
(a+b))^(1/2)+a+2*b)*a)^(1/2))-2/d/b^2/(tanh(1/2*d*x+1/2*c)^2+1)^3*tanh(1/2*
d*x+1/2*c)^5*a-4/d/b/(tanh(1/2*d*x+1/2*c)^2+1)^3*tanh(1/2*d*x+1/2*c)^5-4/d/
b^2/(tanh(1/2*d*x+1/2*c)^2+1)^3*tanh(1/2*d*x+1/2*c)^3*a-16/3/d/b/(tanh(1/2*
d*x+1/2*c)^2+1)^3*tanh(1/2*d*x+1/2*c)^3-2/d/b^2/(tanh(1/2*d*x+1/2*c)^2+1)^3
*tanh(1/2*d*x+1/2*c)*a-4/d/b/(tanh(1/2*d*x+1/2*c)^2+1)^3*tanh(1/2*d*x+1/2*c
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.64971, size = 5189, normalized size = 69.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(12*(a^2*b + a*b^2)*cosh(d*x + c)^4 + 48*(a^2*b + a*b^2)*cosh(d*x + c)
*sinh(d*x + c)^3 + 12*(a^2*b + a*b^2)*sinh(d*x + c)^4 + 12*a^2*b + 20*a*b^2
 + 24*(a^2*b + 2*a*b^2)*cosh(d*x + c)^2 + 24*(a^2*b + 2*a*b^2 + 3*(a^2*b +
a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 3*((a^2 + 2*a*b + b^2)*cosh(d*x +
 c)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (a^2 + 2*a*b
+ b^2)*sinh(d*x + c)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 3*(5*(a^2
+ 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 4*(5*
(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*
sinh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 3*(5*(a^2 + 2*a*b
 + b^2)*cosh(d*x + c)^4 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a
*b + b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 6*((a^2 + 2*a*b + b^2)*cosh
(d*x + c)^5 + 2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*c
osh(d*x + c))*sinh(d*x + c))*sqrt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x +
 c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b
+ b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b
+ b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4
*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x
 + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c)
+ (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)^4 + 4
*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b
)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 +
 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)
) + 48*((a^2*b + a*b^2)*cosh(d*x + c)^3 + (a^2*b + 2*a*b^2)*cosh(d*x + c))*
sinh(d*x + c))/(a*b^3*d*cosh(d*x + c)^6 + 6*a*b^3*d*cosh(d*x + c)*sinh(d*x
+ c)^5 + a*b^3*d*sinh(d*x + c)^6 + 3*a*b^3*d*cosh(d*x + c)^4 + 3*a*b^3*d*co
sh(d*x + c)^2 + a*b^3*d + 3*(5*a*b^3*d*cosh(d*x + c)^2 + a*b^3*d)*sinh(d*x
+ c)^4 + 4*(5*a*b^3*d*cosh(d*x + c)^3 + 3*a*b^3*d*cosh(d*x + c))*sinh(d*x +
```

```
c)^3 + 3*(5*a*b^3*d*cosh(d*x + c)^4 + 6*a*b^3*d*cosh(d*x + c)^2 + a*b^3*d)
*sinh(d*x + c)^2 + 6*(a*b^3*d*cosh(d*x + c)^5 + 2*a*b^3*d*cosh(d*x + c)^3 +
 a*b^3*d*cosh(d*x + c))*sinh(d*x + c)), 1/3*(6*(a^2*b + a*b^2)*cosh(d*x + c
)^4 + 24*(a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 6*(a^2*b + a*b^2)*
sinh(d*x + c)^4 + 6*a^2*b + 10*a*b^2 + 12*(a^2*b + 2*a*b^2)*cosh(d*x + c)^2
 + 12*(a^2*b + 2*a*b^2 + 3*(a^2*b + a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2
 + 3*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x
+ c)*sinh(d*x + c)^5 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^6 + 3*(a^2 + 2*a*b
 + b^2)*cosh(d*x + c)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 +
2*a*b + b^2)*sinh(d*x + c)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3
*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)
*cosh(d*x + c)^2 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 6*(a^2 + 2*a*
b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b
 + b^2 + 6*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 2*(a^2 + 2*a*b + b^2)*cos
h(d*x + c)^3 + (a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b)*
arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c)
 + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(a*b)/(a*b)) + 24*((a^2*b + a*b^2)*
cosh(d*x + c)^3 + (a^2*b + 2*a*b^2)*cosh(d*x + c))*sinh(d*x + c))/(a*b^3*d*
cosh(d*x + c)^6 + 6*a*b^3*d*cosh(d*x + c)*sinh(d*x + c)^5 + a*b^3*d*sinh(d*
x + c)^6 + 3*a*b^3*d*cosh(d*x + c)^4 + 3*a*b^3*d*cosh(d*x + c)^2 + a*b^3*d
+ 3*(5*a*b^3*d*cosh(d*x + c)^2 + a*b^3*d)*sinh(d*x + c)^4 + 4*(5*a*b^3*d*co
sh(d*x + c)^3 + 3*a*b^3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*a*b^3*d*cos
h(d*x + c)^4 + 6*a*b^3*d*cosh(d*x + c)^2 + a*b^3*d)*sinh(d*x + c)^2 + 6*(a*
b^3*d*cosh(d*x + c)^5 + 2*a*b^3*d*cosh(d*x + c)^3 + a*b^3*d*cosh(d*x + c))*
sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**6/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)**6/(a + b*tanh(c + d*x)**2), x)
```

Giac [B] time = 1.44931, size = 207, normalized size = 2.76

$$\frac{\frac{3\left(a^2e^{(2c)}+2\,abe^{(2c)}+b^2e^{(2c)}\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\sqrt{ab}}\right)e^{(-2\,c)}}{\sqrt{ab}b^2}+\frac{2\left(3\,ae^{(4\,dx+4\,c)}+3\,be^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}+12\,be^{(2\,dx+2\,c)}+3\,a+5\,b\right)}{b^2\left(e^{(2\,dx+2\,c)}+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*(a^2*e^(2*c) + 2*a*b*e^(2*c) + b^2*e^(2*c))*arctan(1/2*(a*e^(2*d*x +
 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))*e^(-2*c)/(sqrt(a*b)*b^2) + 2*
(3*a*e^(4*d*x + 4*c) + 3*b*e^(4*d*x + 4*c) + 6*a*e^(2*d*x + 2*c) + 12*b*e^(
2*d*x + 2*c) + 3*a + 5*b)/(b^2*(e^(2*d*x + 2*c) + 1)^3))/d
```

3.115 $\int \frac{\cosh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=128

$$\frac{b^2(6a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a+b)^{7/2}}+\frac{b^3\sinh(c+dx)}{2ad(a+b)^3\left((a+b)\sinh^2(c+dx)+a\right)}+\frac{\sinh^3(c+dx)}{3d(a+b)^2}+\frac{(a+3b)\sinh(c+dx)}{d(a+b)^3}$$

```
[Out] (b^2*(6*a + b)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a +
 b)^(7/2)*d) + ((a + 3*b)*Sinh[c + d*x])/((a + b)^3*d) + Sinh[c + d*x]^3/(3
*(a + b)^2*d) + (b^3*Sinh[c + d*x])/(2*a*(a + b)^3*d*(a + (a + b)*Sinh[c +
d*x]^2))
```

Rubi [A] time = 0.181511, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3676, 390, 385, 205}

$$\frac{b^2(6a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a+b)^{7/2}}+\frac{b^3\sinh(c+dx)}{2ad(a+b)^3\left((a+b)\sinh^2(c+dx)+a\right)}+\frac{\sinh^3(c+dx)}{3d(a+b)^2}+\frac{(a+3b)\sinh(c+dx)}{d(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (b^2*(6*a + b)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a +
 b)^(7/2)*d) + ((a + 3*b)*Sinh[c + d*x])/((a + b)^3*d) + Sinh[c + d*x]^3/(3
*(a + b)^2*d) + (b^3*Sinh[c + d*x])/(2*a*(a + b)^3*d*(a + (a + b)*Sinh[c +
d*x]^2))
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{\left(a+(a+b)x^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a+3b}{(a+b)^3}+\frac{x^2}{(a+b)^2}+\frac{b^2(3a+b)+3b^2(a+b)x^2}{(a+b)^3\left(a+(a+b)x^2\right)^2}\right)dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{(a+3b)\sinh(c+dx)}{(a+b)^3d}+\frac{\sinh^3(c+dx)}{3(a+b)^2d}+\frac{\text{Subst}\left(\int \frac{b^2(3a+b)+3b^2(a+b)x^2}{\left(a+(a+b)x^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{(a+b)^3d}$$

$$= \frac{(a+3b)\sinh(c+dx)}{(a+b)^3d}+\frac{\sinh^3(c+dx)}{3(a+b)^2d}+\frac{b^3\sinh(c+dx)}{2a(a+b)^3d\left(a+(a+b)\sinh^2(c+dx)\right)}+\frac{\left(b^2(6\right.}{}$$

$$= \frac{b^2(6a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{7/2}d}+\frac{(a+3b)\sinh(c+dx)}{(a+b)^3d}+\frac{\sinh^3(c+dx)}{3(a+b)^2d}+\frac{}{2a(a+b)^3d}$$

Mathematica [A] time = 1.05302, size = 111, normalized size = 0.87

$$\frac{-\frac{6b^2(6a+b)\tan^{-1}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)}{a^{3/2}(a+b)^{7/2}}+\frac{3\sinh(c+dx)\left(\frac{4b^3}{a((a+b)\cosh(2(c+dx))+a-b)}+3a+11b\right)}{(a+b)^3}+\frac{\sinh(3(c+dx))}{(a+b)^2}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((-6*b^2*(6*a + b)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]])/(a^(3/2)*(a
 + b)^(7/2)) + (3*(3*a + 11*b + (4*b^3)/(a*(a - b + (a + b)*Cosh[2*(c + d*x
)]))))*Sinh[c + d*x])/(a + b)^3 + Sinh[3*(c + d*x)]/(a + b)^2)/(12*d)
```

Maple [B] time = 0.105, size = 875, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] -1/3/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)+1)^3+1/2/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)
+1)^2-1/d/(a+b)^3/(tanh(1/2*d*x+1/2*c)+1)*a-3/d/(a+b)^3/(tanh(1/2*d*x+1/2*c
)+1)*b-1/d*b^3/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4
*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)^3+1/d*b^3/(a+b)^3/(tanh(1
/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*
tanh(1/2*d*x+1/2*c)-3/d*b^2/(a+b)^3/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arc
tanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+3/d*b^3/(a+
b)^3/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2
```

```
*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+3/d*b^2/(a+b)^3/((2*(b*(a+
b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+
a+2*b)*a)^(1/2))+3/d*b^3/(a+b)^3/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)
*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))
-1/2/d*b^3/(a+b)^3/a/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2
*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/2/d*b^4/(a+b)^3/a/(b*(a+
b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)
/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/2/d*b^3/(a+b)^3/a/((2*(b*(a+b))^(1/
2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*
a)^(1/2))+1/2/d*b^4/(a+b)^3/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)
^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/
3/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)^3-1/2/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)
^2-1/d/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)*a-3/d/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1
)*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/24*(a^3 + 2*a^2*b + a*b^2 - (a^3*e^(10*c) + 2*a^2*b*e^(10*c) + a*b^2*e^(
10*c))*e^(10*d*x) - (11*a^3*e^(8*c) + 42*a^2*b*e^(8*c) + 31*a*b^2*e^(8*c))*
e^(8*d*x) - 2*(5*a^3*e^(6*c) + 4*a^2*b*e^(6*c) - 49*a*b^2*e^(6*c) + 12*b^3*
e^(6*c))*e^(6*d*x) + 2*(5*a^3*e^(4*c) + 4*a^2*b*e^(4*c) - 49*a*b^2*e^(4*c)
+ 12*b^3*e^(4*c))*e^(4*d*x) + (11*a^3*e^(2*c) + 42*a^2*b*e^(2*c) + 31*a*b^2
*e^(2*c))*e^(2*d*x))/((a^5*d*e^(7*c) + 4*a^4*b*d*e^(7*c) + 6*a^3*b^2*d*e^(7
*c) + 4*a^2*b^3*d*e^(7*c) + a*b^4*d*e^(7*c))*e^(7*d*x) + 2*(a^5*d*e^(5*c) +
 2*a^4*b*d*e^(5*c) - 2*a^2*b^3*d*e^(5*c) - a*b^4*d*e^(5*c))*e^(5*d*x) + (a^
5*d*e^(3*c) + 4*a^4*b*d*e^(3*c) + 6*a^3*b^2*d*e^(3*c) + 4*a^2*b^3*d*e^(3*c)
 + a*b^4*d*e^(3*c))*e^(3*d*x)) + 1/8*integrate(8*((6*a*b^2*e^(3*c) + b^3*e^
(3*c))*e^(3*d*x) + (6*a*b^2*e^c + b^3*e^c)*e^(d*x))/(a^5 + 4*a^4*b + 6*a^3*
b^2 + 4*a^2*b^3 + a*b^4 + (a^5*e^(4*c) + 4*a^4*b*e^(4*c) + 6*a^3*b^2*e^(4*c
) + 4*a^2*b^3*e^(4*c) + a*b^4*e^(4*c))*e^(4*d*x) + 2*(a^5*e^(2*c) + 2*a^4*b
*e^(2*c) - 2*a^2*b^3*e^(2*c) - a*b^4*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 3.21489, size = 16069, normalized size = 125.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/24*((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^10 + 10*(a^5 + 3
*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)*sinh(d*x + c)^9 + (a^5 + 3*a^4*
b + 3*a^3*b^2 + a^2*b^3)*sinh(d*x + c)^10 + (11*a^5 + 53*a^4*b + 73*a^3*b^2
 + 31*a^2*b^3)*cosh(d*x + c)^8 + (11*a^5 + 53*a^4*b + 73*a^3*b^2 + 31*a^2*b
^3 + 45*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c
)^8 + 8*(15*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^3 + (11*a^5
 + 53*a^4*b + 73*a^3*b^2 + 31*a^2*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(
5*a^5 + 9*a^4*b - 45*a^3*b^2 - 37*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^6 + 2*(
5*a^5 + 9*a^4*b - 45*a^3*b^2 - 37*a^2*b^3 + 12*a*b^4 + 105*(a^5 + 3*a^4*b +
```

```
3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^4 + 14*(11*a^5 + 53*a^4*b + 73*a^3*b^2
+ 31*a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(a^5 + 3*a^4*b + 3*a
^3*b^2 + a^2*b^3)*cosh(d*x + c)^5 + 14*(11*a^5 + 53*a^4*b + 73*a^3*b^2 + 31
*a^2*b^3)*cosh(d*x + c)^3 + 3*(5*a^5 + 9*a^4*b - 45*a^3*b^2 - 37*a^2*b^3 +
12*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 - a^5 - 3*a^4*b - 3*a^3*b^2 - a^2*
b^3 - 2*(5*a^5 + 9*a^4*b - 45*a^3*b^2 - 37*a^2*b^3 + 12*a*b^4)*cosh(d*x + c
)^4 + 2*(105*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^6 - 5*a^5
- 9*a^4*b + 45*a^3*b^2 + 37*a^2*b^3 - 12*a*b^4 + 35*(11*a^5 + 53*a^4*b + 73
*a^3*b^2 + 31*a^2*b^3)*cosh(d*x + c)^4 + 15*(5*a^5 + 9*a^4*b - 45*a^3*b^2 -
 37*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(15*(a^5 + 3*a
^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^7 + 7*(11*a^5 + 53*a^4*b + 73*a^3
*b^2 + 31*a^2*b^3)*cosh(d*x + c)^5 + 5*(5*a^5 + 9*a^4*b - 45*a^3*b^2 - 37*a
^2*b^3 + 12*a*b^4)*cosh(d*x + c)^3 - (5*a^5 + 9*a^4*b - 45*a^3*b^2 - 37*a^2
*b^3 + 12*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 - (11*a^5 + 53*a^4*b + 73*a
^3*b^2 + 31*a^2*b^3)*cosh(d*x + c)^2 + (45*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2
*b^3)*cosh(d*x + c)^8 + 28*(11*a^5 + 53*a^4*b + 73*a^3*b^2 + 31*a^2*b^3)*co
sh(d*x + c)^6 - 11*a^5 - 53*a^4*b - 73*a^3*b^2 - 31*a^2*b^3 + 30*(5*a^5 + 9
*a^4*b - 45*a^3*b^2 - 37*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^4 - 12*(5*a^5 +
9*a^4*b - 45*a^3*b^2 - 37*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c
)^2 - 6*((6*a^2*b^2 + 7*a*b^3 + b^4)*cosh(d*x + c)^7 + 7*(6*a^2*b^2 + 7*a*b
^3 + b^4)*cosh(d*x + c)*sinh(d*x + c)^6 + (6*a^2*b^2 + 7*a*b^3 + b^4)*sinh(
d*x + c)^7 + 2*(6*a^2*b^2 - 5*a*b^3 - b^4)*cosh(d*x + c)^5 + (12*a^2*b^2 -
10*a*b^3 - 2*b^4 + 21*(6*a^2*b^2 + 7*a*b^3 + b^4)*cosh(d*x + c)^2)*sinh(d*x
 + c)^5 + 5*(7*(6*a^2*b^2 + 7*a*b^3 + b^4)*cosh(d*x + c)^3 + 2*(6*a^2*b^2 -
 5*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c)^4 + (6*a^2*b^2 + 7*a*b^3 + b^4
)*cosh(d*x + c)^3 + (35*(6*a^2*b^2 + 7*a*b^3 + b^4)*cosh(d*x + c)^4 + 6*a^2
*b^2 + 7*a*b^3 + b^4 + 20*(6*a^2*b^2 - 5*a*b^3 - b^4)*cosh(d*x + c)^2)*sinh
(d*x + c)^3 + (21*(6*a^2*b^2 + 7*a*b^3 + b^4)*cosh(d*x + c)^5 + 20*(6*a^2*b
^2 - 5*a*b^3 - b^4)*cosh(d*x + c)^3 + 3*(6*a^2*b^2 + 7*a*b^3 + b^4)*cosh(d*
x + c))*sinh(d*x + c)^2 + (7*(6*a^2*b^2 + 7*a*b^3 + b^4)*cosh(d*x + c)^6 +
10*(6*a^2*b^2 - 5*a*b^3 - b^4)*cosh(d*x + c)^4 + 3*(6*a^2*b^2 + 7*a*b^3 + b
^4)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(-a^2 - a*b)*log(((a + b)*cosh(d*x
+ c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4
- 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 - 3*a - b)*sin
h(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 - (3*a + b)*cosh(d*x + c))*sinh(d
*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x +
 c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 -
a*b) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x +
 c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*
cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a
- b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 2*(5*(a^5 + 3*a^4*b + 3*a^3*b
^2 + a^2*b^3)*cosh(d*x + c)^9 + 4*(11*a^5 + 53*a^4*b + 73*a^3*b^2 + 31*a^2*
b^3)*cosh(d*x + c)^7 + 6*(5*a^5 + 9*a^4*b - 45*a^3*b^2 - 37*a^2*b^3 + 12*a*
b^4)*cosh(d*x + c)^5 - 4*(5*a^5 + 9*a^4*b - 45*a^3*b^2 - 37*a^2*b^3 + 12*a*
b^4)*cosh(d*x + c)^3 - (11*a^5 + 53*a^4*b + 73*a^3*b^2 + 31*a^2*b^3)*cosh(d
*x + c))*sinh(d*x + c))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b
^4 + a^2*b^5)*d*cosh(d*x + c)^7 + 7*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^
3 + 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)*sinh(d*x + c)^6 + (a^7 + 5*a^6*b +
 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*sinh(d*x + c)^7 + 2*(a^7
+ 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^5
+ (21*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*cos
h(d*x + c)^2 + 2*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b
^5)*d)*sinh(d*x + c)^5 + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b
^4 + a^2*b^5)*d*cosh(d*x + c)^3 + 5*(7*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4
*b^3 + 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^3 + 2*(a^7 + 3*a^6*b + 2*a^5*b^
2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^4 + (35
*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x
 + c)^4 + 20*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*
d*cosh(d*x + c)^2 + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 +
```

a^2*b^5)*d)*sinh(d*x + c)^3 + (21*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^5 + 20*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^3 + 3*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^2 + (7*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^6 + 10*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^4 + 3*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^2)*sinh(d*x + c)), 1/24*((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^10 + 10*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)*sinh(d*x + c)^9 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sinh(d*x + c)^10 + (11*a^5 + 53*a^4*b + 73*a^3*b^2 + 31*a^2*b^3)*cosh(d*x + c)^8 + (11*a^5 + 53*a^4*b + 73*a^3*b^2 + 31*a^2*b^3 + 45*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^3 + (11*a^5 + 53*a^4*b + 73*a^3*b^2 + 31*a^2*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(5*a^5 + 9*a^4*b - 45*a^3*b^2 - 37*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^6 + 2*(5*a^5 + 9*a^4*b - 45*a^3*b^2 - 37*a^2*b^3 + 12*a*b^4 + 105*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^4 + 14*(11*a^5 + 53*a^4*b + 73*a^3*b^2 + 31*a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^5 + 14*(11*a^5 + 53*a^4*b + 73*a^3*b^2 + 31*a^2*b^3)*cosh(d*x + c)^3 + 3*(5*a^5 + 9*a^4*b - 45*a^3*b^2 - 37*a^2*b^3 + 12*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 - a^5 - 3*a^4*b - 3*a^3*b^2 - a^2*b^3 - 2*(5*a^5 + 9*a^4*b - 45*a^3*b^2 - 37*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^4 + 2*(105*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^6 - 5*a^5 - 9*a^4*b + 45*a^3*b^2 + 37*a^2*b^3 - 12*a*b^4 + 35*(11*a^5 + 53*a^4*b + 73*a^3*b^2 + 31*a^2*b^3)*cosh(d*x + c)^4 + 15*(5*a^5 + 9*a^4*b - 45*a^3*b^2 - 37*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(15*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^7 + 7*(11*a^5 + 53*a^4*b + 73*a^3*b^2 + 31*a^2*b^3)*cosh(d*x + c)^5 + 5*(5*a^5 + 9*a^4*b - 45*a^3*b^2 - 37*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^3 - (5*a^5 + 9*a^4*b - 45*a^3*b^2 - 37*a^2*b^3 + 12*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 - (11*a^5 + 53*a^4*b + 73*a^3*b^2 + 31*a^2*b^3)*cosh(d*x + c)^2 + (45*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^8 + 28*(11*a^5 + 53*a^4*b + 73*a^3*b^2 + 31*a^2*b^3)*cosh(d*x + c)^6 - 11*a^5 - 53*a^4*b - 73*a^3*b^2 - 31*a^2*b^3 + 30*(5*a^5 + 9*a^4*b - 45*a^3*b^2 - 37*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^4 - 12*(5*a^5 + 9*a^4*b - 45*a^3*b^2 - 37*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 12*((6*a^2*b^2 + 7*a*b^3 + b^4)*cosh(d*x + c)^7 + 7*(6*a^2*b^2 + 7*a*b^3 + b^4)*cosh(d*x + c)*sinh(d*x + c)^6 + (6*a^2*b^2 + 7*a*b^3 + b^4)*sinh(d*x + c)^7 + 2*(6*a^2*b^2 - 5*a*b^3 - b^4)*cosh(d*x + c)^5 + (12*a^2*b^2 - 10*a*b^3 - 2*b^4 + 21*(6*a^2*b^2 + 7*a*b^3 + b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 5*(7*(6*a^2*b^2 + 7*a*b^3 + b^4)*cosh(d*x + c)^3 + 2*(6*a^2*b^2 - 5*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c)^4 + (6*a^2*b^2 + 7*a*b^3 + b^4)*cosh(d*x + c)^3 + (35*(6*a^2*b^2 + 7*a*b^3 + b^4)*cosh(d*x + c)^4 + 6*a^2*b^2 + 7*a*b^3 + b^4 + 20*(6*a^2*b^2 - 5*a*b^3 - b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + (21*(6*a^2*b^2 + 7*a*b^3 + b^4)*cosh(d*x + c)^5 + 20*(6*a^2*b^2 - 5*a*b^3 - b^4)*cosh(d*x + c)^3 + 3*(6*a^2*b^2 + 7*a*b^3 + b^4)*cosh(d*x + c))*sinh(d*x + c)^2 + (7*(6*a^2*b^2 + 7*a*b^3 + b^4)*cosh(d*x + c)^6 + 10*(6*a^2*b^2 - 5*a*b^3 - b^4)*cosh(d*x + c)^4 + 3*(6*a^2*b^2 + 7*a*b^3 + b^4)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(a^2 + a*b)*arctan(1/2*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (3*a - b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + 3*a - b)*sinh(d*x + c))/sqrt(a^2 + a*b)) + 12*((6*a^2*b^2 + 7*a*b^3 + b^4)*cosh(d*x + c)^7 + 7*(6*a^2*b^2 + 7*a*b^3 + b^4)*cosh(d*x + c)*sinh(d*x + c)^6 + (6*a^2*b^2 + 7*a*b^3 + b^4)*sinh(d*x + c)^7 + 2*(6*a^2*b^2 - 5*a*b^3 - b^4)*cosh(d*x + c)^5 + (12*a^2*b^2 - 10*a*b^3 - 2*b^4 + 21*(6*a^2*b^2 + 7*a*b^3 + b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 5*(7*(6*a^2*b^2 + 7*a*b^3 + b^4)*cosh(d*x + c)^3 + 2*(6*a^2*b^2 - 5*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c)^4 + (6*a^2*b^2 + 7*a*b^3 + b^4)*cosh(d*x + c)^3 + (35*(6*a^2*b^2 + 7*a*b^3 + b^4)*cosh(d*x + c)^4 + 6*a^2*b^2 + 7*a*b^3 + b^4 + 20*(6*a^2*b^2 - 5*a*b^3 - b^4)*cosh(d*x + c)^2)*s

```
inh(d*x + c)^3 + (21*(6*a^2*b^2 + 7*a*b^3 + b^4)*cosh(d*x + c)^5 + 20*(6*a^
2*b^2 - 5*a*b^3 - b^4)*cosh(d*x + c)^3 + 3*(6*a^2*b^2 + 7*a*b^3 + b^4)*cosh
(d*x + c))*sinh(d*x + c)^2 + (7*(6*a^2*b^2 + 7*a*b^3 + b^4)*cosh(d*x + c)^6
 + 10*(6*a^2*b^2 - 5*a*b^3 - b^4)*cosh(d*x + c)^4 + 3*(6*a^2*b^2 + 7*a*b^3
+ b^4)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(a^2 + a*b)*arctan(1/2*sqrt(a^2
+ a*b)*(cosh(d*x + c) + sinh(d*x + c))/a) + 2*(5*(a^5 + 3*a^4*b + 3*a^3*b^2
 + a^2*b^3)*cosh(d*x + c)^9 + 4*(11*a^5 + 53*a^4*b + 73*a^3*b^2 + 31*a^2*b^
3)*cosh(d*x + c)^7 + 6*(5*a^5 + 9*a^4*b - 45*a^3*b^2 - 37*a^2*b^3 + 12*a*b^
4)*cosh(d*x + c)^5 - 4*(5*a^5 + 9*a^4*b - 45*a^3*b^2 - 37*a^2*b^3 + 12*a*b^
4)*cosh(d*x + c)^3 - (11*a^5 + 53*a^4*b + 73*a^3*b^2 + 31*a^2*b^3)*cosh(d*x
 + c))*sinh(d*x + c))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4
 + a^2*b^5)*d*cosh(d*x + c)^7 + 7*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3
+ 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)*sinh(d*x + c)^6 + (a^7 + 5*a^6*b + 1
0*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*sinh(d*x + c)^7 + 2*(a^7 +
3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^5 +
(21*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*cosh(
d*x + c)^2 + 2*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5
)*d)*sinh(d*x + c)^5 + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4
 + a^2*b^5)*d*cosh(d*x + c)^3 + 5*(7*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b
^3 + 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^3 + 2*(a^7 + 3*a^6*b + 2*a^5*b^2
- 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^4 + (35*(
a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x +
 c)^4 + 20*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*d*
cosh(d*x + c)^2 + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^
2*b^5)*d)*sinh(d*x + c)^3 + (21*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 +
5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^5 + 20*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*
a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^3 + 3*(a^7 + 5*a^6*b + 10*a^
5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^2
+ (7*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*cosh
(d*x + c)^6 + 10*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b
^5)*d*cosh(d*x + c)^4 + 3*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*
b^4 + a^2*b^5)*d*cosh(d*x + c)^2)*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [C] time = 2.51348, size = 9686, normalized size = 75.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(6*(3*(6*a^6*b^2*e^(4*c) + 25*a^5*b^3*e^(4*c) + 40*a^4*b^4*e^(4*c) + 3
0*a^3*b^5*e^(4*c) + 10*a^2*b^6*e^(4*c) + a*b^7*e^(4*c))*cos(1/2*real_part(a
rccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/
```

```
(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) - (6*a^6*b^
2*e^(4*c) + 25*a^5*b^3*e^(4*c) + 40*a^4*b^4*e^(4*c) + 30*a^3*b^5*e^(4*c) +
10*a^2*b^6*e^(4*c) + a*b^7*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) +
b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3 - 9*(6*
a^6*b^2*e^(4*c) + 25*a^5*b^3*e^(4*c) + 40*a^4*b^4*e^(4*c) + 30*a^3*b^5*e^(4
*c) + 10*a^2*b^6*e^(4*c) + a*b^7*e^(4*c))*cos(1/2*real_part(arccos(-a/(a +
b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*s
in(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos
(-a/(a + b) + b/(a + b)))) + 3*(6*a^6*b^2*e^(4*c) + 25*a^5*b^3*e^(4*c) + 40
*a^4*b^4*e^(4*c) + 30*a^3*b^5*e^(4*c) + 10*a^2*b^6*e^(4*c) + a*b^7*e^(4*c))
*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(ar
ccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(
a + b)))) + 9*(6*a^6*b^2*e^(4*c) + 25*a^5*b^3*e^(4*c) + 40*a^4*b^4*e^(4*c)
+ 30*a^3*b^5*e^(4*c) + 10*a^2*b^6*e^(4*c) + a*b^7*e^(4*c))*cos(1/2*real_par
t(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) +
 b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*i
mag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(6*a^6*b^2*e^(4*c) + 25*a^5
*b^3*e^(4*c) + 40*a^4*b^4*e^(4*c) + 30*a^3*b^5*e^(4*c) + 10*a^2*b^6*e^(4*c)
 + a*b^7*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1
/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-
a/(a + b) + b/(a + b))))^2 - 3*(6*a^6*b^2*e^(4*c) + 25*a^5*b^3*e^(4*c) + 40
*a^4*b^4*e^(4*c) + 30*a^3*b^5*e^(4*c) + 10*a^2*b^6*e^(4*c) + a*b^7*e^(4*c))
*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arc
cos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a +
 b))))^3 + (6*a^6*b^2*e^(4*c) + 25*a^5*b^3*e^(4*c) + 40*a^4*b^4*e^(4*c) + 3
0*a^3*b^5*e^(4*c) + 10*a^2*b^6*e^(4*c) + a*b^7*e^(4*c))*sin(1/2*real_part(a
rccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/
(a + b))))^3 + (6*a^6*b^2*e^(4*c) + 25*a^5*b^3*e^(4*c) + 40*a^4*b^4*e^(4*c)
 + 30*a^3*b^5*e^(4*c) + 10*a^2*b^6*e^(4*c) + a*b^7*e^(4*c))*cosh(1/2*imag_p
art(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) +
b/(a + b)))) - (6*a^6*b^2*e^(4*c) + 25*a^5*b^3*e^(4*c) + 40*a^4*b^4*e^(4*c)
 + 30*a^3*b^5*e^(4*c) + 10*a^2*b^6*e^(4*c) + a*b^7*e^(4*c))*sin(1/2*real_pa
rt(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) +
b/(a + b)))))*arctan((((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)/(a^5
*e^(4*c) + 4*a^4*b*e^(4*c) + 6*a^3*b^2*e^(4*c) + 4*a^2*b^3*e^(4*c) + a*b^4*
e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b))) + e^(d*x))/(((a^5 + 4*a^4
*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)/(a^5*e^(4*c) + 4*a^4*b*e^(4*c) + 6*a^3*
b^2*e^(4*c) + 4*a^2*b^3*e^(4*c) + a*b^4*e^(4*c)))^(1/4)*sin(1/2*arccos(-(a
- b)/(a + b)))))/(2*(a^4*e^(2*c) + 3*a^3*b*e^(2*c) + 3*a^2*b^2*e^(2*c) + a*
b^3*e^(2*c))^2*a*b + (a^5*e^(2*c) + 2*a^4*b*e^(2*c) - 2*a^2*b^3*e^(2*c) - a
*b^4*e^(2*c))*sqrt(-a*b)*abs(-a^4*e^(2*c) - 3*a^3*b*e^(2*c) - 3*a^2*b^2*e^(
2*c) - a*b^3*e^(2*c))) + 6*(3*(6*a^6*b^2*e^(4*c) + 25*a^5*b^3*e^(4*c) + 40*
a^4*b^4*e^(4*c) + 30*a^3*b^5*e^(4*c) + 10*a^2*b^6*e^(4*c) + a*b^7*e^(4*c))*
cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arc
cos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a
+ b)))) - (6*a^6*b^2*e^(4*c) + 25*a^5*b^3*e^(4*c) + 40*a^4*b^4*e^(4*c) + 30
*a^3*b^5*e^(4*c) + 10*a^2*b^6*e^(4*c) + a*b^7*e^(4*c))*cosh(1/2*imag_part(a
rccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(
a + b))))^3 - 9*(6*a^6*b^2*e^(4*c) + 25*a^5*b^3*e^(4*c) + 40*a^4*b^4*e^(4*c
) + 30*a^3*b^5*e^(4*c) + 10*a^2*b^6*e^(4*c) + a*b^7*e^(4*c))*cos(1/2*real_p
art(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b)
 + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1
/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(6*a^6*b^2*e^(4*c) + 25*a
^5*b^3*e^(4*c) + 40*a^4*b^4*e^(4*c) + 30*a^3*b^5*e^(4*c) + 10*a^2*b^6*e^(4*
c) + a*b^7*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*s
in(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arcc
os(-a/(a + b) + b/(a + b)))) + 9*(6*a^6*b^2*e^(4*c) + 25*a^5*b^3*e^(4*c) +
40*a^4*b^4*e^(4*c) + 30*a^3*b^5*e^(4*c) + 10*a^2*b^6*e^(4*c) + a*b^7*e^(4*c
))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(
```

arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(6*a^6*b^2*e^(4*c) + 25*a^5*b^3*e^(4*c) + 40*a^4*b^4*e^(4*c) + 30*a^3*b^5*e^(4*c) + 10*a^2*b^6*e^(4*c) + a*b^7*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(6*a^6*b^2*e^(4*c) + 25*a^5*b^3*e^(4*c) + 40*a^4*b^4*e^(4*c) + 30*a^3*b^5*e^(4*c) + 10*a^2*b^6*e^(4*c) + a*b^7*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (6*a^6*b^2*e^(4*c) + 25*a^5*b^3*e^(4*c) + 40*a^4*b^4*e^(4*c) + 30*a^3*b^5*e^(4*c) + 10*a^2*b^6*e^(4*c) + a*b^7*e^(4*c))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (6*a^6*b^2*e^(4*c) + 25*a^5*b^3*e^(4*c) + 40*a^4*b^4*e^(4*c) + 30*a^3*b^5*e^(4*c) + 10*a^2*b^6*e^(4*c) + a*b^7*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) - (6*a^6*b^2*e^(4*c) + 25*a^5*b^3*e^(4*c) + 40*a^4*b^4*e^(4*c) + 30*a^3*b^5*e^(4*c) + 10*a^2*b^6*e^(4*c) + a*b^7*e^(4*c))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*arctan(-(((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)/(a^5*e^(4*c) + 4*a^4*b*e^(4*c) + 6*a^3*b^2*e^(4*c) + 4*a^2*b^3*e^(4*c) + a*b^4*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b))) - e^(d*x))/(((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)/(a^5*e^(4*c) + 4*a^4*b*e^(4*c) + 6*a^3*b^2*e^(4*c) + 4*a^2*b^3*e^(4*c) + a*b^4*e^(4*c)))^(1/4)*sin(1/2*arccos(-(a - b)/(a + b)))))/(2*(a^4*e^(2*c) + 3*a^3*b*e^(2*c) + 3*a^2*b^2*e^(2*c) + a*b^3*e^(2*c))^2*a*b + (a^5*e^(2*c) + 2*a^4*b*e^(2*c) - 2*a^2*b^3*e^(2*c) - a*b^4*e^(2*c))*sqrt(-a*b)*abs(-a^4*e^(2*c) - 3*a^3*b*e^(2*c) - 3*a^2*b^2*e^(2*c) - a*b^3*e^(2*c))) - (9*a*e^(2*d*x + 2*c) + 33*b*e^(2*d*x + 2*c) + a + b)*e^(-3*d*x)/(a^3*e^(3*c) + 3*a^2*b*e^(3*c) + 3*a*b^2*e^(3*c) + b^3*e^(3*c)) + 3*((6*a^6*b^2*e^(4*c) + 25*a^5*b^3*e^(4*c) + 40*a^4*b^4*e^(4*c) + 30*a^3*b^5*e^(4*c) + 10*a^2*b^6*e^(4*c) + a*b^7*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 - 3*(6*a^6*b^2*e^(4*c) + 25*a^5*b^3*e^(4*c) + 40*a^4*b^4*e^(4*c) + 30*a^3*b^5*e^(4*c) + 10*a^2*b^6*e^(4*c) + a*b^7*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(6*a^6*b^2*e^(4*c) + 25*a^5*b^3*e^(4*c) + 40*a^4*b^4*e^(4*c) + 30*a^3*b^5*e^(4*c) + 10*a^2*b^6*e^(4*c) + a*b^7*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(6*a^6*b^2*e^(4*c) + 25*a^5*b^3*e^(4*c) + 40*a^4*b^4*e^(4*c) + 30*a^3*b^5*e^(4*c) + 10*a^2*b^6*e^(4*c) + a*b^7*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(6*a^6*b^2*e^(4*c) + 25*a^5*b^3*e^(4*c) + 40*a^4*b^4*e^(4*c) + 30*a^3*b^5*e^(4*c) + 10*a^2*b^6*e^(4*c) + a*b^7*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 9*(6*a^6*b^2*e^(4*c) + 25*a^5*b^3*e^(4*c) + 40*a^4*b^4*e^(4*c) + 30*a^3*b^5*e^(4*c) + 10*a^2*b^6*e^(4*c) + a*b^7*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - (6*a^6*b^2*e^(4*c) + 25*a^5*b^3*e^(4*c) + 40*a^4*b^4*e^(4*c) + 30*a^3*b^5*e^(4*c) + 10*a^2*b^6*e^(4*c) + a*b^7*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + 3*(6*a^6*b^2*e^(4*c) + 25*a^5*b^3*e^(4*c) + 40*a^4*b^4*e^(4*c) + 30*a^3*b^5*e^(4*c) + 10*a^2*b^6*e^(4*c) + a*b^7*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (6*a^6*b^2*e^(4*c) + 25*a^5*b^3*e^(4*c) +

```
40*a^4*b^4*e^(4*c) + 30*a^3*b^5*e^(4*c) + 10*a^2*b^6*e^(4*c) + a*b^7*e^(4*
c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(a
rccos(-a/(a + b) + b/(a + b)))) - (6*a^6*b^2*e^(4*c) + 25*a^5*b^3*e^(4*c) +
 40*a^4*b^4*e^(4*c) + 30*a^3*b^5*e^(4*c) + 10*a^2*b^6*e^(4*c) + a*b^7*e^(4*
c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(a
rccos(-a/(a + b) + b/(a + b)))))*log(2*((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*
b^3 + a*b^4)/(a^5*e^(4*c) + 4*a^4*b*e^(4*c) + 6*a^3*b^2*e^(4*c) + 4*a^2*b^3
*e^(4*c) + a*b^4*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b)))*e^(d*x)
+ sqrt((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)/(a^5*e^(4*c) + 4*a^4
*b*e^(4*c) + 6*a^3*b^2*e^(4*c) + 4*a^2*b^3*e^(4*c) + a*b^4*e^(4*c))) + e^(2
*d*x))/(2*(a^4*e^(2*c) + 3*a^3*b*e^(2*c) + 3*a^2*b^2*e^(2*c) + a*b^3*e^(2*c
))^2*a*b + (a^5*e^(2*c) + 2*a^4*b*e^(2*c) - 2*a^2*b^3*e^(2*c) - a*b^4*e^(2*
c))*sqrt(-a*b)*abs(-a^4*e^(2*c) - 3*a^3*b*e^(2*c) - 3*a^2*b^2*e^(2*c) - a*b
^3*e^(2*c))) - 3*((6*a^6*b^2*e^(4*c) + 25*a^5*b^3*e^(4*c) + 40*a^4*b^4*e^(4
*c) + 30*a^3*b^5*e^(4*c) + 10*a^2*b^6*e^(4*c) + a*b^7*e^(4*c))*cos(1/2*real
_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a +
b) + b/(a + b))))^3 - 3*(6*a^6*b^2*e^(4*c) + 25*a^5*b^3*e^(4*c) + 40*a^4*b^
4*e^(4*c) + 30*a^3*b^5*e^(4*c) + 10*a^2*b^6*e^(4*c) + a*b^7*e^(4*c))*cos(1/
2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(
a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^
2 - 3*(6*a^6*b^2*e^(4*c) + 25*a^5*b^3*e^(4*c) + 40*a^4*b^4*e^(4*c) + 30*a^3
*b^5*e^(4*c) + 10*a^2*b^6*e^(4*c) + a*b^7*e^(4*c))*cos(1/2*real_part(arccos
(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a +
b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(6*a^6*b^2*
e^(4*c) + 25*a^5*b^3*e^(4*c) + 40*a^4*b^4*e^(4*c) + 30*a^3*b^5*e^(4*c) + 10
*a^2*b^6*e^(4*c) + a*b^7*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(
a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*rea
l_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a +
 b) + b/(a + b)))) + 3*(6*a^6*b^2*e^(4*c) + 25*a^5*b^3*e^(4*c) + 40*a^4*b^4
*e^(4*c) + 30*a^3*b^5*e^(4*c) + 10*a^2*b^6*e^(4*c) + a*b^7*e^(4*c))*cos(1/2
*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/
(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^
2 - 9*(6*a^6*b^2*e^(4*c) + 25*a^5*b^3*e^(4*c) + 40*a^4*b^4*e^(4*c) + 30*a^3
*b^5*e^(4*c) + 10*a^2*b^6*e^(4*c) + a*b^7*e^(4*c))*cos(1/2*real_part(arccos
(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)
)))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part
(arccos(-a/(a + b) + b/(a + b))))^2 - (6*a^6*b^2*e^(4*c) + 25*a^5*b^3*e^(4*
c) + 40*a^4*b^4*e^(4*c) + 30*a^3*b^5*e^(4*c) + 10*a^2*b^6*e^(4*c) + a*b^7*e
^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_
part(arccos(-a/(a + b) + b/(a + b))))^3 + 3*(6*a^6*b^2*e^(4*c) + 25*a^5*b^3
*e^(4*c) + 40*a^4*b^4*e^(4*c) + 30*a^3*b^5*e^(4*c) + 10*a^2*b^6*e^(4*c) + a
*b^7*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*re
al_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a
+ b) + b/(a + b))))^3 + (6*a^6*b^2*e^(4*c) + 25*a^5*b^3*e^(4*c) + 40*a^4*b^
4*e^(4*c) + 30*a^3*b^5*e^(4*c) + 10*a^2*b^6*e^(4*c) + a*b^7*e^(4*c))*cos(1/
2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(
a + b) + b/(a + b)))) - (6*a^6*b^2*e^(4*c) + 25*a^5*b^3*e^(4*c) + 40*a^4*b^
4*e^(4*c) + 30*a^3*b^5*e^(4*c) + 10*a^2*b^6*e^(4*c) + a*b^7*e^(4*c))*cos(1/
2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(
a + b) + b/(a + b)))))*log(-2*((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b
^4)/(a^5*e^(4*c) + 4*a^4*b*e^(4*c) + 6*a^3*b^2*e^(4*c) + 4*a^2*b^3*e^(4*c)
+ a*b^4*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b)))*e^(d*x) + sqrt((a
^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)/(a^5*e^(4*c) + 4*a^4*b*e^(4*c
) + 6*a^3*b^2*e^(4*c) + 4*a^2*b^3*e^(4*c) + a*b^4*e^(4*c))) + e^(2*d*x))/(2
*(a^4*e^(2*c) + 3*a^3*b*e^(2*c) + 3*a^2*b^2*e^(2*c) + a*b^3*e^(2*c))^2*a*b
+ (a^5*e^(2*c) + 2*a^4*b*e^(2*c) - 2*a^2*b^3*e^(2*c) - a*b^4*e^(2*c))*sqrt(
-a*b)*abs(-a^4*e^(2*c) - 3*a^3*b*e^(2*c) - 3*a^2*b^2*e^(2*c) - a*b^3*e^(2*c
))) + (a^4*e^(3*d*x + 36*c) + 4*a^3*b*e^(3*d*x + 36*c) + 6*a^2*b^2*e^(3*d*x
 + 36*c) + 4*a*b^3*e^(3*d*x + 36*c) + b^4*e^(3*d*x + 36*c) + 9*a^4*e^(d*x +
```

```
 34*c) + 60*a^3*b*e^(d*x + 34*c) + 126*a^2*b^2*e^(d*x + 34*c) + 108*a*b^3*e
^(d*x + 34*c) + 33*b^4*e^(d*x + 34*c))/(a^6*e^(33*c) + 6*a^5*b*e^(33*c) + 1
5*a^4*b^2*e^(33*c) + 20*a^3*b^3*e^(33*c) + 15*a^2*b^4*e^(33*c) + 6*a*b^5*e^
(33*c) + b^6*e^(33*c)) + 24*(b^3*e^(3*d*x + 3*c) - b^3*e^(d*x + c))/((a^4 +
 3*a^3*b + 3*a^2*b^2 + a*b^3)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*
e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)))/d
```

3.116 $\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=140

$$\frac{b^{3/2}(5a+b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a+b)^3}-\frac{b(a-b)\tanh(c+dx)}{2ad(a+b)^2\left(a+b\tanh^2(c+dx)\right)}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d(a+b)\left(a+b\tanh^2(c+dx)\right)}+\frac{x(a+5b)}{2(a+b)^3}$$

```
[Out] ((a + 5*b)*x)/(2*(a + b)^3) + (b^(3/2)*(5*a + b)*ArcTan[(Sqrt[b]*Tanh[c + d
*x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^3*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*(
a + b)*d*(a + b*Tanh[c + d*x]^2)) - ((a - b)*b*Tanh[c + d*x])/(2*a*(a + b)^
2*d*(a + b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.193778, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3675, 414, 527, 522, 206, 205}

$$\frac{b^{3/2}(5a+b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a+b)^3}-\frac{b(a-b)\tanh(c+dx)}{2ad(a+b)^2\left(a+b\tanh^2(c+dx)\right)}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d(a+b)\left(a+b\tanh^2(c+dx)\right)}+\frac{x(a+5b)}{2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((a + 5*b)*x)/(2*(a + b)^3) + (b^(3/2)*(5*a + b)*ArcTan[(Sqrt[b]*Tanh[c + d
*x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^3*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*(
a + b)*d*(a + b*Tanh[c + d*x]^2)) - ((a - b)*b*Tanh[c + d*x])/(2*a*(a + b)^
2*d*(a + b*Tanh[c + d*x]^2))
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
```

```
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\left(a+bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{a+2b+3bx^2}{\left(1-x^2\right)\left(a+bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{2(a+b)d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{(a-b)b\tanh(c+dx)}{2a(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-2\left(a^2+4a\right.}{(1-}\right.}{}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{(a-b)b\tanh(c+dx)}{2a(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{\left(b^2(5a+b)\right)\text{Sub}}{}$$

$$= \frac{(a+5b)x}{2(a+b)^3} + \frac{b^{3/2}(5a+b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^3d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{}{2a(a}$$

Mathematica [A] time = 0.747527, size = 110, normalized size = 0.79

$$\frac{\frac{2b^{3/2}(5a+b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}} + \frac{2b^2(a+b)\sinh(2(c+dx))}{a((a+b)\cosh(2(c+dx))+a-b)} + 2(a+5b)(c+dx) + (a+b)\sinh(2(c+dx))}{4d(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (2*(a + 5*b)*(c + d*x) + (2*b^(3/2)*(5*a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x]
)/Sqrt[a]])/a^(3/2) + (a + b)*Sinh[2*(c + d*x)] + (2*b^2*(a + b)*Sinh[2*(c
+ d*x)])/(a*(a - b + (a + b)*Cosh[2*(c + d*x)])))/(4*(a + b)^3*d)
```

Maple [B] time = 0.107, size = 1146, normalized size = 8.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] -1/2/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)
+1)+1/2/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)+1)*a+5/2/d/(a+b)^3*ln(tanh(1/2*d*x
+1/2*c)+1)*b+1/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)
^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*tanh(1/2*d*x+1/2*c)^3+1/d*b^3/(a+b)^3/(ta
nh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a
)/a*tanh(1/2*d*x+1/2*c)^3+1/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1
/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*tanh(1/2*d*x+1/2*c)+1/d*b^3/
(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1
/2*c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)-5/2/d*b^2/(a+b)^3*a/(b*(a+b))^(1/2)/((2*
(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))
^(1/2)-a-2*b)*a)^(1/2))+5/2/d*b^2/(a+b)^3/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/
2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-3/d*b
^3/(a+b)^3/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*ta
nh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-5/2/d*b^2/(a+b)^3*a/
(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1
/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-5/2/d*b^2/(a+b)^3/((2*(b*(a+b))^
(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*
b)*a)^(1/2))-3/d*b^3/(a+b)^3/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^
(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/2
/d*b^3/(a+b)^3/a/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x
+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-1/2/d*b^4/(a+b)^3/a/(b*(a+b))^
(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2
*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-1/2/d*b^3/(a+b)^3/a/((2*(b*(a+b))^(1/2)+a
+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(
1/2))-1/2/d*b^4/(a+b)^3/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/
2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/2/d/
(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)-1/2
/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)-1)*a-5/2/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)
-1)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.86065, size = 10118, normalized size = 72.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*((a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*c
osh(d*x + c)*sinh(d*x + c)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(d*x + c)^8 + 2*
(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^6 + 2*(a^3 -
a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(d
*x + c)^2)*sinh(d*x + c)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^3
+ 3*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x
 + c)^5 - 8*(a*b^2 - b^3 - (a^3 + 4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^4 +
 2*(35*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^4 - 4*a*b^2 + 4*b^3 + 4*(a^3 +
 4*a^2*b - 5*a*b^2)*d*x + 15*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x
)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x
+ c)^5 + 5*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^3
- 4*(a*b^2 - b^3 - (a^3 + 4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x +
 c)^3 - a^3 - 2*a^2*b - a*b^2 - 2*(a^3 + 3*a*b^2 + 4*b^3 - 2*(a^3 + 6*a^2*b
 + 5*a*b^2)*d*x)*cosh(d*x + c)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x +
 c)^6 + 15*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^4
- a^3 - 3*a*b^2 - 4*b^3 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x - 24*(a*b^2 - b^3
 - (a^3 + 4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*((5*
a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^6 + 6*(5*a^2*b + 6*a*b^2 + b^3)*cosh(d
*x + c)*sinh(d*x + c)^5 + (5*a^2*b + 6*a*b^2 + b^3)*sinh(d*x + c)^6 + 2*(5*
a^2*b - 4*a*b^2 - b^3)*cosh(d*x + c)^4 + (10*a^2*b - 8*a*b^2 - 2*b^3 + 15*(
5*a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(5*a^2*b +
 6*a*b^2 + b^3)*cosh(d*x + c)^3 + 2*(5*a^2*b - 4*a*b^2 - b^3)*cosh(d*x + c)
)*sinh(d*x + c)^3 + (5*a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^2 + (15*(5*a^2*
b + 6*a*b^2 + b^3)*cosh(d*x + c)^4 + 5*a^2*b + 6*a*b^2 + b^3 + 12*(5*a^2*b
- 4*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(5*a^2*b + 6*a*b^2
 + b^3)*cosh(d*x + c)^5 + 4*(5*a^2*b - 4*a*b^2 - b^3)*cosh(d*x + c)^3 + (5*
a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/a)*log(((a^2 +
 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*
x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c
)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2
 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)
*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a
*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*
sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c
)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*co
sh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a -
b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*co
sh(d*x + c)^7 + 3*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x
+ c)^5 - 8*(a*b^2 - b^3 - (a^3 + 4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^3 -
(a^3 + 3*a*b^2 + 4*b^3 - 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c))*si
nh(d*x + c))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d*cosh(d*x +
c)^6 + 6*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)*si
nh(d*x + c)^5 + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d*sinh(d*x
+ c)^6 + 2*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^4 + (15*(a^5
 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + 2*(a^5 + 2*
a^4*b - 2*a^2*b^3 - a*b^4)*d)*sinh(d*x + c)^4 + (a^5 + 4*a^4*b + 6*a^3*b^2
+ 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + 4*(5*(a^5 + 4*a^4*b + 6*a^3*b^2 +
4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^3 + 2*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4
)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (15*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2
*b^3 + a*b^4)*d*cosh(d*x + c)^4 + 12*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*d*
cosh(d*x + c)^2 + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d)*sinh(d
*x + c)^2 + 2*(3*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d*cosh(d*x
 + c)^5 + 4*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^3 + (a^5 +
4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c))*sinh(d*x + c)), 1
/8*((a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cos
h(d*x + c)*sinh(d*x + c)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(d*x + c)^8 + 2*(a
^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^6 + 2*(a^3 - a*
```

```
b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x
 + c)^2)*sinh(d*x + c)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^3 +
3*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x +
 c)^5 - 8*(a*b^2 - b^3 - (a^3 + 4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^4 + 2
*(35*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^4 - 4*a*b^2 + 4*b^3 + 4*(a^3 + 4
*a^2*b - 5*a*b^2)*d*x + 15*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*
cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x +
c)^5 + 5*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^3 -
4*(a*b^2 - b^3 - (a^3 + 4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c
)^3 - a^3 - 2*a^2*b - a*b^2 - 2*(a^3 + 3*a*b^2 + 4*b^3 - 2*(a^3 + 6*a^2*b +
 5*a*b^2)*d*x)*cosh(d*x + c)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c
)^6 + 15*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^4 -
a^3 - 3*a*b^2 - 4*b^3 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x - 24*(a*b^2 - b^3 -
 (a^3 + 4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((5*a^
2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^6 + 6*(5*a^2*b + 6*a*b^2 + b^3)*cosh(d*x
 + c)*sinh(d*x + c)^5 + (5*a^2*b + 6*a*b^2 + b^3)*sinh(d*x + c)^6 + 2*(5*a^
2*b - 4*a*b^2 - b^3)*cosh(d*x + c)^4 + (10*a^2*b - 8*a*b^2 - 2*b^3 + 15*(5*
a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(5*a^2*b + 6
*a*b^2 + b^3)*cosh(d*x + c)^3 + 2*(5*a^2*b - 4*a*b^2 - b^3)*cosh(d*x + c))*
sinh(d*x + c)^3 + (5*a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^2 + (15*(5*a^2*b
+ 6*a*b^2 + b^3)*cosh(d*x + c)^4 + 5*a^2*b + 6*a*b^2 + b^3 + 12*(5*a^2*b -
4*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(5*a^2*b + 6*a*b^2 +
 b^3)*cosh(d*x + c)^5 + 4*(5*a^2*b - 4*a*b^2 - b^3)*cosh(d*x + c)^3 + (5*a^
2*b + 6*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*((a
 + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sin
h(d*x + c)^2 + a - b)*sqrt(b/a)/b) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x
+ c)^7 + 3*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^5
- 8*(a*b^2 - b^3 - (a^3 + 4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^3 - (a^3 +
3*a*b^2 + 4*b^3 - 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x
+ c))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^6 +
6*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)*sinh(d*x
+ c)^5 + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d*sinh(d*x + c)^6
+ 2*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^4 + (15*(a^5 + 4*a^
4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + 2*(a^5 + 2*a^4*b -
 2*a^2*b^3 - a*b^4)*d)*sinh(d*x + c)^4 + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2
*b^3 + a*b^4)*d*cosh(d*x + c)^2 + 4*(5*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b
^3 + a*b^4)*d*cosh(d*x + c)^3 + 2*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*d*cos
h(d*x + c))*sinh(d*x + c)^3 + (15*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 +
a*b^4)*d*cosh(d*x + c)^4 + 12*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*d*cosh(d*
x + c)^2 + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d)*sinh(d*x + c)
^2 + 2*(3*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^5
 + 4*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^3 + (a^5 + 4*a^4*b
 + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.3075, size = 606, normalized size = 4.33

$$\frac{\frac{12\,(a+5\,b)dx}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{12\left(5\,ab^2e^{(2\,c)}+b^3e^{(2\,c)}\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)e^{(-2\,c)}}{\left(a^4+3\,a^3b+3\,a^2b^2+ab^3\right)\sqrt{ab}}+\frac{3\,e^{(2\,dx+12\,c)}}{a^2e^{(10\,c)}+2\,abe^{(10\,c)}+b^2e^{(10\,c)}}-\frac{2\,a^3e^{(6\,dx+6\,c)}+12\,a^2be^{(6\,dx+6\,c)}+10\,a}{}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(12*(a + 5*b)*d*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 12*(5*a*b^2*e^(2*c
) + b^3*e^(2*c))*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)
/sqrt(a*b))*e^(-2*c)/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)) + 3*e^
(2*d*x + 12*c)/(a^2*e^(10*c) + 2*a*b*e^(10*c) + b^2*e^(10*c)) - (2*a^3*e^(6
*d*x + 6*c) + 12*a^2*b*e^(6*d*x + 6*c) + 10*a*b^2*e^(6*d*x + 6*c) + 7*a^3*e
^(4*d*x + 4*c) + 22*a^2*b*e^(4*d*x + 4*c) + 7*a*b^2*e^(4*d*x + 4*c) - 24*b^
3*e^(4*d*x + 4*c) + 8*a^3*e^(2*d*x + 2*c) + 12*a^2*b*e^(2*d*x + 2*c) + 28*a
*b^2*e^(2*d*x + 2*c) + 24*b^3*e^(2*d*x + 2*c) + 3*a^3 + 6*a^2*b + 3*a*b^2)/
((a^4*e^(2*c) + 3*a^3*b*e^(2*c) + 3*a^2*b^2*e^(2*c) + a*b^3*e^(2*c))*(a*e^(
2*d*x) + b*e^(2*d*x) + a*e^(6*d*x + 4*c) + b*e^(6*d*x + 4*c) + 2*a*e^(4*d*x
 + 2*c) - 2*b*e^(4*d*x + 2*c))))/d
```

3.117 $\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=101

$$\frac{b(4a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a+b)^{5/2}}+\frac{b^2\sinh(c+dx)}{2ad(a+b)^2\left((a+b)\sinh^2(c+dx)+a\right)}+\frac{\sinh(c+dx)}{d(a+b)^2}$$

```
[Out] (b*(4*a + b)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a + b
)^(5/2)*d) + Sinh[c + d*x]/((a + b)^2*d) + (b^2*Sinh[c + d*x])/(2*a*(a + b)
^2*d*(a + (a + b)*Sinh[c + d*x]^2))
```

Rubi [A] time = 0.139196, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3676, 390, 385, 205}

$$\frac{b(4a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a+b)^{5/2}}+\frac{b^2\sinh(c+dx)}{2ad(a+b)^2\left((a+b)\sinh^2(c+dx)+a\right)}+\frac{\sinh(c+dx)}{d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (b*(4*a + b)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a + b
)^(5/2)*d) + Sinh[c + d*x]/((a + b)^2*d) + (b^2*Sinh[c + d*x])/(2*a*(a + b)
^2*d*(a + (a + b)*Sinh[c + d*x]^2))
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{\left(a+(a+b)x^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)^2}+\frac{b(2a+b)+2b(a+b)x^2}{(a+b)^2\left(a+(a+b)x^2\right)^2}\right)dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\sinh(c+dx)}{(a+b)^2d}+\frac{\text{Subst}\left(\int \frac{b(2a+b)+2b(a+b)x^2}{\left(a+(a+b)x^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{(a+b)^2d}$$

$$= \frac{\sinh(c+dx)}{(a+b)^2d}+\frac{b^2\sinh(c+dx)}{2a(a+b)^2d\left(a+(a+b)\sinh^2(c+dx)\right)}+\frac{(b(4a+b))\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\right.}{2a(a+b)^2d}$$

$$= \frac{b(4a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{5/2}d}+\frac{\sinh(c+dx)}{(a+b)^2d}+\frac{b^2\sinh(c+dx)}{2a(a+b)^2d\left(a+(a+b)\sinh^2(c+dx)\right)}$$

Mathematica [A] time = 0.697244, size = 84, normalized size = 0.83

$$\frac{\frac{b(4a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}(a+b)^{5/2}}+\frac{\sinh(c+dx)\left(\frac{b^2}{a\left((a+b)\sinh^2(c+dx)+a\right)}+2\right)}{(a+b)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((b*(4*a + b)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(a^(3/2)*(a + b)
^(5/2)) + (Sinh[c + d*x]*(2 + b^2/(a*(a + (a + b)*Sinh[c + d*x]^2))))/(a +
b)^2)/(2*d)
```

Maple [B] time = 0.096, size = 729, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] -1/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)+1)-1/d*b^2/(a+b)^2/(tanh(1/2*d*x+1/2*c)^4
*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*x+1/
2*c)^3+1/d*b^2/(a+b)^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4
*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)-2/d*b/(a+b)^2/((2*(b*(a+b
))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-
a-2*b)*a)^(1/2))+2/d*b^2/(a+b)^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)
*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)
)+2/d*b/(a+b)^2/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1
/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+2/d*b^2/(a+b)^2/(b*(a+b))^(1/2)/
((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+
b))^(1/2)+a+2*b)*a)^(1/2))-1/2/d*b^2/(a+b)^2/a/((2*(b*(a+b))^(1/2)-a-2*b)*a
```

```
)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+
1/2/d*b^3/(a+b)^2/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arc
tanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/2/d*b^2/(
a+b)^2/a/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/(
(2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/2/d*b^3/(a+b)^2/a/(b*(a+b))^(1/2)/((2
*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))
^(1/2)+a+2*b)*a)^(1/2))-1/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2 + ab - \left(a^2e^{(6\,c)} + abe^{(6\,c)}\right)e^{(6\,dx)} - \left(a^2e^{(4\,c)} - 3\,abe^{(4\,c)} + 2\,b^2e^{(4\,c)}\right)e^{(4\,dx)} + \left(a^2e^{(2\,c)} - 3\,abe^{(2\,c}\right.}{2\left(\left(a^4de^{(5\,c)} + 3\,a^3bde^{(5\,c)} + 3\,a^2b^2de^{(5\,c)} + ab^3de^{(5\,c)}\right)e^{(5\,dx)} + 2\left(a^4de^{(3\,c)} + a^3bde^{(3\,c)} - a^2b^2de^{(3\,c)} - ab^3de^{(3\,c)}\right)e^{(3\,dx)} + \left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a^2 + a*b - (a^2*e^(6*c) + a*b*e^(6*c))*e^(6*d*x) - (a^2*e^(4*c) - 3*
a*b*e^(4*c) + 2*b^2*e^(4*c))*e^(4*d*x) + (a^2*e^(2*c) - 3*a*b*e^(2*c) + 2*b
^2*e^(2*c))*e^(2*d*x))/((a^4*d*e^(5*c) + 3*a^3*b*d*e^(5*c) + 3*a^2*b^2*d*e^
(5*c) + a*b^3*d*e^(5*c))*e^(5*d*x) + 2*(a^4*d*e^(3*c) + a^3*b*d*e^(3*c) - a
^2*b^2*d*e^(3*c) - a*b^3*d*e^(3*c))*e^(3*d*x) + (a^4*d*e^c + 3*a^3*b*d*e^c
+ 3*a^2*b^2*d*e^c + a*b^3*d*e^c)*e^(d*x)) + 1/2*integrate(2*((4*a*b*e^(3*c)
 + b^2*e^(3*c))*e^(3*d*x) + (4*a*b*e^c + b^2*e^c)*e^(d*x))/(a^4 + 3*a^3*b +
 3*a^2*b^2 + a*b^3 + (a^4*e^(4*c) + 3*a^3*b*e^(4*c) + 3*a^2*b^2*e^(4*c) + a
*b^3*e^(4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) + a^3*b*e^(2*c) - a^2*b^2*e^(2*c)
- a*b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 2.5325, size = 8197, normalized size = 81.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^6 + 12*(a^4 + 2*a^3*b + a^2
*b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(a^4 + 2*a^3*b + a^2*b^2)*sinh(d*x
+ c)^6 + 2*(a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^4 + 2*(a^4 - 2
*a^3*b - a^2*b^2 + 2*a*b^3 + 15*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^2)*
sinh(d*x + c)^4 - 2*a^4 - 4*a^3*b - 2*a^2*b^2 + 8*(5*(a^4 + 2*a^3*b + a^2*b
^2)*cosh(d*x + c)^3 + (a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*cosh(d*x + c))*si
nh(d*x + c)^3 - 2*(a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^2 + 2*(
15*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^4 - a^4 + 2*a^3*b + a^2*b^2 - 2*
a*b^3 + 6*(a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c
)^2 - ((4*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c)^5 + 5*(4*a^2*b + 5*a*b^2 + b
^3)*cosh(d*x + c)*sinh(d*x + c)^4 + (4*a^2*b + 5*a*b^2 + b^3)*sinh(d*x + c)
^5 + 2*(4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^3 + 2*(4*a^2*b - 3*a*b^2 - b
^3 + 5*(4*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(5*(4
*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c)^3 + 3*(4*a^2*b - 3*a*b^2 - b^3)*cosh(
d*x + c))*sinh(d*x + c)^2 + (4*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c) + (5*(4
*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c)^4 + 4*a^2*b + 5*a*b^2 + b^3 + 6*(4*a^
2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(-a^2 - a*b)*log((
(a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)
```

```
*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)
^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 - (3*a + b)*cosh
(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x +
c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x +
 c))*sqrt(-a^2 - a*b) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*
x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^
2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh
(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 4*(3*(a^4 +
2*a^3*b + a^2*b^2)*cosh(d*x + c)^5 + 2*(a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*
cosh(d*x + c)^3 - (a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*cosh(d*x + c))*sinh(d
*x + c))/((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)
^5 + 5*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)*si
nh(d*x + c)^4 + (a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*sinh(d*
x + c)^5 + 2*(a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^3 + 2*(5
*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^2 + (a^6
 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*d)*sinh(d*x + c)^3 + (a^6 + 4*a^5*b + 6*a
^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c) + 2*(5*(a^6 + 4*a^5*b + 6*a^4
*b^2 + 4*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^3 + 3*(a^6 + 2*a^5*b - 2*a^3*b^
3 - a^2*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^2 + (5*(a^6 + 4*a^5*b + 6*a^4*b
^2 + 4*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^4 + 6*(a^6 + 2*a^5*b - 2*a^3*b^3
- a^2*b^4)*d*cosh(d*x + c)^2 + (a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2
*b^4)*d)*sinh(d*x + c)), 1/2*((a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^6 + 6
*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (a^4 + 2*a^3*b +
 a^2*b^2)*sinh(d*x + c)^6 + (a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*cosh(d*x +
c)^4 + (a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3 + 15*(a^4 + 2*a^3*b + a^2*b^2)*co
sh(d*x + c)^2)*sinh(d*x + c)^4 - a^4 - 2*a^3*b - a^2*b^2 + 4*(5*(a^4 + 2*a^
3*b + a^2*b^2)*cosh(d*x + c)^3 + (a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*cosh(d
*x + c))*sinh(d*x + c)^3 - (a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*cosh(d*x + c
)^2 + (15*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^4 - a^4 + 2*a^3*b + a^2*b
^2 - 2*a*b^3 + 6*(a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^2)*sinh(
d*x + c)^2 + ((4*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c)^5 + 5*(4*a^2*b + 5*a*
b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^4 + (4*a^2*b + 5*a*b^2 + b^3)*sinh(d
*x + c)^5 + 2*(4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^3 + 2*(4*a^2*b - 3*a*
b^2 - b^3 + 5*(4*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 +
2*(5*(4*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c)^3 + 3*(4*a^2*b - 3*a*b^2 - b^3
)*cosh(d*x + c))*sinh(d*x + c)^2 + (4*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c)
+ (5*(4*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c)^4 + 4*a^2*b + 5*a*b^2 + b^3 +
6*(4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(a^2 + a*b)
*arctan(1/2*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c
)^2 + (a + b)*sinh(d*x + c)^3 + (3*a - b)*cosh(d*x + c) + (3*(a + b)*cosh(d
*x + c)^2 + 3*a - b)*sinh(d*x + c))/sqrt(a^2 + a*b)) + ((4*a^2*b + 5*a*b^2
+ b^3)*cosh(d*x + c)^5 + 5*(4*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x
 + c)^4 + (4*a^2*b + 5*a*b^2 + b^3)*sinh(d*x + c)^5 + 2*(4*a^2*b - 3*a*b^2
- b^3)*cosh(d*x + c)^3 + 2*(4*a^2*b - 3*a*b^2 - b^3 + 5*(4*a^2*b + 5*a*b^2
+ b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(5*(4*a^2*b + 5*a*b^2 + b^3)*co
sh(d*x + c)^3 + 3*(4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c)^2
+ (4*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c) + (5*(4*a^2*b + 5*a*b^2 + b^3)*co
sh(d*x + c)^4 + 4*a^2*b + 5*a*b^2 + b^3 + 6*(4*a^2*b - 3*a*b^2 - b^3)*cosh(
d*x + c)^2)*sinh(d*x + c))*sqrt(a^2 + a*b)*arctan(1/2*sqrt(a^2 + a*b)*(cosh
(d*x + c) + sinh(d*x + c))/a) + 2*(3*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c
)^5 + 2*(a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^3 - (a^4 - 2*a^3*
b - a^2*b^2 + 2*a*b^3)*cosh(d*x + c))*sinh(d*x + c))/((a^6 + 4*a^5*b + 6*a^
4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^5 + 5*(a^6 + 4*a^5*b + 6*a^4*b
^2 + 4*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^4 + (a^6 + 4*a^5*b
+ 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*sinh(d*x + c)^5 + 2*(a^6 + 2*a^5*b - 2
*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^3 + 2*(5*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4
*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^2 + (a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^
4)*d)*sinh(d*x + c)^3 + (a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d
*cosh(d*x + c) + 2*(5*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*c
```

```
osh(d*x + c)^3 + 3*(a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c))*s
inh(d*x + c)^2 + (5*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*cos
h(d*x + c)^4 + 6*(a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^2 +
(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d)*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)**2)**2,x)

[Out] Timed out
```

Giac [C] time = 1.88005, size = 8317, normalized size = 82.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

[Out] 1/8*(2*(3*(4*a^5*b*e^(4*c) + 13*a^4*b^2*e^(4*c) + 15*a^3*b^3*e^(4*c) + 7*a^
2*b^4*e^(4*c) + a*b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a +
 b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real
_part(arccos(-a/(a + b) + b/(a + b)))) - (4*a^5*b*e^(4*c) + 13*a^4*b^2*e^(4
*c) + 15*a^3*b^3*e^(4*c) + 7*a^2*b^4*e^(4*c) + a*b^5*e^(4*c))*cosh(1/2*imag
_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b
) + b/(a + b))))^3 - 9*(4*a^5*b*e^(4*c) + 13*a^4*b^2*e^(4*c) + 15*a^3*b^3*e
^(4*c) + 7*a^2*b^4*e^(4*c) + a*b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a
+ b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2
*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arcc
os(-a/(a + b) + b/(a + b)))) + 3*(4*a^5*b*e^(4*c) + 13*a^4*b^2*e^(4*c) + 15
*a^3*b^3*e^(4*c) + 7*a^2*b^4*e^(4*c) + a*b^5*e^(4*c))*cosh(1/2*imag_part(ar
ccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a
 + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(4*a^5*b
*e^(4*c) + 13*a^4*b^2*e^(4*c) + 15*a^3*b^3*e^(4*c) + 7*a^2*b^4*e^(4*c) + a*
b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*
imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a +
 b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 -
3*(4*a^5*b*e^(4*c) + 13*a^4*b^2*e^(4*c) + 15*a^3*b^3*e^(4*c) + 7*a^2*b^4*e^
(4*c) + a*b^5*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*
sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arc
cos(-a/(a + b) + b/(a + b))))^2 - 3*(4*a^5*b*e^(4*c) + 13*a^4*b^2*e^(4*c) +
 15*a^3*b^3*e^(4*c) + 7*a^2*b^4*e^(4*c) + a*b^5*e^(4*c))*cos(1/2*real_part(
arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/
(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (4*a^5*b
*e^(4*c) + 13*a^4*b^2*e^(4*c) + 15*a^3*b^3*e^(4*c) + 7*a^2*b^4*e^(4*c) + a*
b^5*e^(4*c))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*
imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (4*a^5*b*e^(4*c) + 13*a^4*b^
2*e^(4*c) + 15*a^3*b^3*e^(4*c) + 7*a^2*b^4*e^(4*c) + a*b^5*e^(4*c))*cosh(1/
2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a
 + b) + b/(a + b)))) - (4*a^5*b*e^(4*c) + 13*a^4*b^2*e^(4*c) + 15*a^3*b^3*e
```

```
^(4*c) + 7*a^2*b^4*e^(4*c) + a*b^5*e^(4*c))*sin(1/2*real_part(arccos(-a/(a
+ b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*ar
ctan((((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)/(a^4*e^(4*c) + 3*a^3*b*e^(4*c) +
 3*a^2*b^2*e^(4*c) + a*b^3*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b))
) + e^(d*x))/(((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)/(a^4*e^(4*c) + 3*a^3*b*e
^(4*c) + 3*a^2*b^2*e^(4*c) + a*b^3*e^(4*c)))^(1/4)*sin(1/2*arccos(-(a - b)/
(a + b)))))/(2*(a^3*e^(2*c) + 2*a^2*b*e^(2*c) + a*b^2*e^(2*c))^2*a*b + (a^4
*e^(2*c) + a^3*b*e^(2*c) - a^2*b^2*e^(2*c) - a*b^3*e^(2*c))*sqrt(-a*b)*abs(
a^3*e^(2*c) + 2*a^2*b*e^(2*c) + a*b^2*e^(2*c))) + 2*(3*(4*a^5*b*e^(4*c) + 1
3*a^4*b^2*e^(4*c) + 15*a^3*b^3*e^(4*c) + 7*a^2*b^4*e^(4*c) + a*b^5*e^(4*c))
*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(ar
ccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a
 + b)))) - (4*a^5*b*e^(4*c) + 13*a^4*b^2*e^(4*c) + 15*a^3*b^3*e^(4*c) + 7*a
^2*b^4*e^(4*c) + a*b^5*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a
 + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3 - 9*(4*a^5*
b*e^(4*c) + 13*a^4*b^2*e^(4*c) + 15*a^3*b^3*e^(4*c) + 7*a^2*b^4*e^(4*c) + a
*b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2
*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(
a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) +
 3*(4*a^5*b*e^(4*c) + 13*a^4*b^2*e^(4*c) + 15*a^3*b^3*e^(4*c) + 7*a^2*b^4*e
^(4*c) + a*b^5*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))
^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(
arccos(-a/(a + b) + b/(a + b)))) + 9*(4*a^5*b*e^(4*c) + 13*a^4*b^2*e^(4*c)
+ 15*a^3*b^3*e^(4*c) + 7*a^2*b^4*e^(4*c) + a*b^5*e^(4*c))*cos(1/2*real_part
(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) +
b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*im
ag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(4*a^5*b*e^(4*c) + 13*a^4*b^
2*e^(4*c) + 15*a^3*b^3*e^(4*c) + 7*a^2*b^4*e^(4*c) + a*b^5*e^(4*c))*cosh(1/
2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a
 + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^
2 - 3*(4*a^5*b*e^(4*c) + 13*a^4*b^2*e^(4*c) + 15*a^3*b^3*e^(4*c) + 7*a^2*b^
4*e^(4*c) + a*b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))
))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(
arccos(-a/(a + b) + b/(a + b))))^3 + (4*a^5*b*e^(4*c) + 13*a^4*b^2*e^(4*c)
+ 15*a^3*b^3*e^(4*c) + 7*a^2*b^4*e^(4*c) + a*b^5*e^(4*c))*sin(1/2*real_part
(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) +
b/(a + b))))^3 + (4*a^5*b*e^(4*c) + 13*a^4*b^2*e^(4*c) + 15*a^3*b^3*e^(4*c)
 + 7*a^2*b^4*e^(4*c) + a*b^5*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b)
+ b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) - (4*a^5*
b*e^(4*c) + 13*a^4*b^2*e^(4*c) + 15*a^3*b^3*e^(4*c) + 7*a^2*b^4*e^(4*c) + a
*b^5*e^(4*c))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*i
mag_part(arccos(-a/(a + b) + b/(a + b)))))*arctan(-(((a^4 + 3*a^3*b + 3*a^2
*b^2 + a*b^3)/(a^4*e^(4*c) + 3*a^3*b*e^(4*c) + 3*a^2*b^2*e^(4*c) + a*b^3*e^
(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b))) - e^(d*x))/(((a^4 + 3*a^3*b
 + 3*a^2*b^2 + a*b^3)/(a^4*e^(4*c) + 3*a^3*b*e^(4*c) + 3*a^2*b^2*e^(4*c) +
a*b^3*e^(4*c)))^(1/4)*sin(1/2*arccos(-(a - b)/(a + b)))))/(2*(a^3*e^(2*c) +
 2*a^2*b*e^(2*c) + a*b^2*e^(2*c))^2*a*b + (a^4*e^(2*c) + a^3*b*e^(2*c) - a^
2*b^2*e^(2*c) - a*b^3*e^(2*c))*sqrt(-a*b)*abs(a^3*e^(2*c) + 2*a^2*b*e^(2*c)
 + a*b^2*e^(2*c))) + ((4*a^5*b*e^(4*c) + 13*a^4*b^2*e^(4*c) + 15*a^3*b^3*e^
(4*c) + 7*a^2*b^4*e^(4*c) + a*b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a +
 b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3
- 3*(4*a^5*b*e^(4*c) + 13*a^4*b^2*e^(4*c) + 15*a^3*b^3*e^(4*c) + 7*a^2*b^4*
e^(4*c) + a*b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))
*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(ar
ccos(-a/(a + b) + b/(a + b))))^2 - 3*(4*a^5*b*e^(4*c) + 13*a^4*b^2*e^(4*c)
+ 15*a^3*b^3*e^(4*c) + 7*a^2*b^4*e^(4*c) + a*b^5*e^(4*c))*cos(1/2*real_part
(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) +
b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(4*a
^5*b*e^(4*c) + 13*a^4*b^2*e^(4*c) + 15*a^3*b^3*e^(4*c) + 7*a^2*b^4*e^(4*c)
```

```
+ a*b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/
2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/
(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))
) + 3*(4*a^5*b*e^(4*c) + 13*a^4*b^2*e^(4*c) + 15*a^3*b^3*e^(4*c) + 7*a^2*b^
4*e^(4*c) + a*b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))
))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part
(arccos(-a/(a + b) + b/(a + b))))^2 - 9*(4*a^5*b*e^(4*c) + 13*a^4*b^2*e^(4*
c) + 15*a^3*b^3*e^(4*c) + 7*a^2*b^4*e^(4*c) + a*b^5*e^(4*c))*cos(1/2*real_p
art(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) +
 b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2
*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - (4*a^5*b*e^(4*c) + 13*a^4*b
^2*e^(4*c) + 15*a^3*b^3*e^(4*c) + 7*a^2*b^4*e^(4*c) + a*b^5*e^(4*c))*cos(1/
2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a
/(a + b) + b/(a + b))))^3 + 3*(4*a^5*b*e^(4*c) + 13*a^4*b^2*e^(4*c) + 15*a^
3*b^3*e^(4*c) + 7*a^2*b^4*e^(4*c) + a*b^5*e^(4*c))*cos(1/2*real_part(arccos
(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))
))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (4*a^5*b*e^(4*
c) + 13*a^4*b^2*e^(4*c) + 15*a^3*b^3*e^(4*c) + 7*a^2*b^4*e^(4*c) + a*b^5*e^
(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_par
t(arccos(-a/(a + b) + b/(a + b)))) - (4*a^5*b*e^(4*c) + 13*a^4*b^2*e^(4*c)
+ 15*a^3*b^3*e^(4*c) + 7*a^2*b^4*e^(4*c) + a*b^5*e^(4*c))*cos(1/2*real_part
(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/
(a + b)))))*log(2*((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)/(a^4*e^(4*c) + 3*a^3
*b*e^(4*c) + 3*a^2*b^2*e^(4*c) + a*b^3*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a -
 b)/(a + b)))*e^(d*x) + sqrt((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)/(a^4*e^(4*
c) + 3*a^3*b*e^(4*c) + 3*a^2*b^2*e^(4*c) + a*b^3*e^(4*c))) + e^(2*d*x))/(2*
(a^3*e^(2*c) + 2*a^2*b*e^(2*c) + a*b^2*e^(2*c))^2*a*b + (a^4*e^(2*c) + a^3*
b*e^(2*c) - a^2*b^2*e^(2*c) - a*b^3*e^(2*c))*sqrt(-a*b)*abs(a^3*e^(2*c) + 2
*a^2*b*e^(2*c) + a*b^2*e^(2*c))) - ((4*a^5*b*e^(4*c) + 13*a^4*b^2*e^(4*c) +
 15*a^3*b^3*e^(4*c) + 7*a^2*b^4*e^(4*c) + a*b^5*e^(4*c))*cos(1/2*real_part(
arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b
/(a + b))))^3 - 3*(4*a^5*b*e^(4*c) + 13*a^4*b^2*e^(4*c) + 15*a^3*b^3*e^(4*c
) + 7*a^2*b^4*e^(4*c) + a*b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b)
+ b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/
2*real_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(4*a^5*b*e^(4*c) + 13*a^
4*b^2*e^(4*c) + 15*a^3*b^3*e^(4*c) + 7*a^2*b^4*e^(4*c) + a*b^5*e^(4*c))*cos
(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos
(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a +
b)))) + 9*(4*a^5*b*e^(4*c) + 13*a^4*b^2*e^(4*c) + 15*a^3*b^3*e^(4*c) + 7*a^
2*b^4*e^(4*c) + a*b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a +
 b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_p
art(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b)
 + b/(a + b)))) + 3*(4*a^5*b*e^(4*c) + 13*a^4*b^2*e^(4*c) + 15*a^3*b^3*e^(4
*c) + 7*a^2*b^4*e^(4*c) + a*b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b
) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sinh
(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 9*(4*a^5*b*e^(4*c) + 13
*a^4*b^2*e^(4*c) + 15*a^3*b^3*e^(4*c) + 7*a^2*b^4*e^(4*c) + a*b^5*e^(4*c))*
cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arcco
s(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)
)))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - (4*a^5*b*e^(4
*c) + 13*a^4*b^2*e^(4*c) + 15*a^3*b^3*e^(4*c) + 7*a^2*b^4*e^(4*c) + a*b^5*e
^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_
part(arccos(-a/(a + b) + b/(a + b))))^3 + 3*(4*a^5*b*e^(4*c) + 13*a^4*b^2*e
^(4*c) + 15*a^3*b^3*e^(4*c) + 7*a^2*b^4*e^(4*c) + a*b^5*e^(4*c))*cos(1/2*re
al_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b
) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 +
(4*a^5*b*e^(4*c) + 13*a^4*b^2*e^(4*c) + 15*a^3*b^3*e^(4*c) + 7*a^2*b^4*e^(4
*c) + a*b^5*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cos
h(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) - (4*a^5*b*e^(4*c) + 13*a^
```

```
4*b^2*e^(4*c) + 15*a^3*b^3*e^(4*c) + 7*a^2*b^4*e^(4*c) + a*b^5*e^(4*c))*cos
(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-
a/(a + b) + b/(a + b)))))*log(-2*((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)/(a^4*
e^(4*c) + 3*a^3*b*e^(4*c) + 3*a^2*b^2*e^(4*c) + a*b^3*e^(4*c)))^(1/4)*cos(1
/2*arccos(-(a - b)/(a + b)))*e^(d*x) + sqrt((a^4 + 3*a^3*b + 3*a^2*b^2 + a*
b^3)/(a^4*e^(4*c) + 3*a^3*b*e^(4*c) + 3*a^2*b^2*e^(4*c) + a*b^3*e^(4*c))) +
 e^(2*d*x))/(2*(a^3*e^(2*c) + 2*a^2*b*e^(2*c) + a*b^2*e^(2*c))^2*a*b + (a^4
*e^(2*c) + a^3*b*e^(2*c) - a^2*b^2*e^(2*c) - a*b^3*e^(2*c))*sqrt(-a*b)*abs(
a^3*e^(2*c) + 2*a^2*b*e^(2*c) + a*b^2*e^(2*c))) + 4*e^(d*x + 10*c)/(a^2*e^(
9*c) + 2*a*b*e^(9*c) + b^2*e^(9*c)) - 4*(a^2*e^(4*d*x + 4*c) + a*b*e^(4*d*x
 + 4*c) - 2*b^2*e^(4*d*x + 4*c) + 2*a^2*e^(2*d*x + 2*c) - 2*a*b*e^(2*d*x +
2*c) + 2*b^2*e^(2*d*x + 2*c) + a^2 + a*b)/((a^3*e^c + 2*a^2*b*e^c + a*b^2*e
^c)*(a*e^(5*d*x + 4*c) + b*e^(5*d*x + 4*c) + 2*a*e^(3*d*x + 2*c) - 2*b*e^(3
*d*x + 2*c) + a*e^(d*x) + b*e^(d*x))))/d
```

3.118 $\int \frac{\text{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=83

$$\frac{(2a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a+b)^{3/2}}+\frac{b\sinh(c+dx)}{2ad(a+b)\left((a+b)\sinh^2(c+dx)+a\right)}$$

```
[Out] ((2*a + b)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^
(3/2)*d) + (b*Sinh[c + d*x])/(2*a*(a + b)*d*(a + (a + b)*Sinh[c + d*x]^2))
```

Rubi [A] time = 0.0708357, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3676, 385, 205}

$$\frac{(2a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a+b)^{3/2}}+\frac{b\sinh(c+dx)}{2ad(a+b)\left((a+b)\sinh^2(c+dx)+a\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((2*a + b)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^
(3/2)*d) + (b*Sinh[c + d*x])/(2*a*(a + b)*d*(a + (a + b)*Sinh[c + d*x]^2))
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{\left(a+(a+b)x^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{b\sinh(c+dx)}{2a(a+b)d\left(a+(a+b)\sinh^2(c+dx)\right)} + \frac{(2a+b)\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \sinh(c+dx)\right)}{2a(a+b)d}$$

$$= \frac{(2a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{3/2}d} + \frac{b\sinh(c+dx)}{2a(a+b)d\left(a+(a+b)\sinh^2(c+dx)\right)}$$

Mathematica [A] time = 0.274639, size = 78, normalized size = 0.94

$$\frac{\frac{(2a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a+b}} + \frac{b\sinh(c+dx)}{a\left((a+b)\sinh^2(c+dx)+a\right)}}{2d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (((2*a + b)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(a^(3/2)*Sqrt[a +
b]) + (b*Sinh[c + d*x])/(a*(a + (a + b)*Sinh[c + d*x]^2)))/(2*(a + b)*d)
```

Maple [B] time = 0.082, size = 666, normalized size = 8.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] -1/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*
c)^2*b+a)*b/a/(a+b)*tanh(1/2*d*x+1/2*c)^3+1/d/(tanh(1/2*d*x+1/2*c)^4*a+2*ta
nh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*b/a/(a+b)*tanh(1/2*d*x+1
/2*c)-1/d/(a+b)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+
1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/d/(a+b)/(b*(a+b))^(1/2)/((2*(
b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^
(1/2)-a-2*b)*a)^(1/2))*b+1/d/(a+b)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arct
an(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/d/(a+b)/(b*
(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*
c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))*b-1/2/d/(a+b)*b/a/((2*(b*(a+b))^(1/
2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)
*a)^(1/2))+1/2/d/(a+b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2
)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))*b^2+1/
2/d/(a+b)*b/a/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2
*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/2/d/(a+b)/a/(b*(a+b))^(1/2)/((2*
(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^
(1/2)+a+2*b)*a)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{be^{(3\,dx+3\,c)} - be^{(dx+c)}}{a^3d + 2\,a^2bd + ab^2d + \left(a^3de^{(4\,c)} + 2\,a^2bde^{(4\,c)} + ab^2de^{(4\,c)}\right)e^{(4\,dx)} + 2\left(a^3de^{(2\,c)} - ab^2de^{(2\,c)}\right)e^{(2\,dx)}} + 2\int \frac{}{2\left(a^3 + 2\,a^2b + \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] (b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^3*d + 2*a^2*b*d + a*b^2*d + (a^3*d*e
^(4*c) + 2*a^2*b*d*e^(4*c) + a*b^2*d*e^(4*c))*e^(4*d*x) + 2*(a^3*d*e^(2*c)
- a*b^2*d*e^(2*c))*e^(2*d*x)) + 2*integrate(1/2*((2*a*e^(3*c) + b*e^(3*c))*
e^(3*d*x) + (2*a*e^c + b*e^c)*e^(d*x))/(a^3 + 2*a^2*b + a*b^2 + (a^3*e^(4*c
) + 2*a^2*b*e^(4*c) + a*b^2*e^(4*c))*e^(4*d*x) + 2*(a^3*e^(2*c) - a*b^2*e^(
2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 2.30218, size = 4917, normalized size = 59.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^2*b + a*b^2)*cosh(d*x + c)^3 + 12*(a^2*b + a*b^2)*cosh(d*x + c)*
sinh(d*x + c)^2 + 4*(a^2*b + a*b^2)*sinh(d*x + c)^3 - ((2*a^2 + 3*a*b + b^2
)*cosh(d*x + c)^4 + 4*(2*a^2 + 3*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 +
 (2*a^2 + 3*a*b + b^2)*sinh(d*x + c)^4 + 2*(2*a^2 - a*b - b^2)*cosh(d*x + c
)^2 + 2*(3*(2*a^2 + 3*a*b + b^2)*cosh(d*x + c)^2 + 2*a^2 - a*b - b^2)*sinh(
d*x + c)^2 + 2*a^2 + 3*a*b + b^2 + 4*((2*a^2 + 3*a*b + b^2)*cosh(d*x + c)^3
 + (2*a^2 - a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 - a*b)*log((
(a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)
*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)
^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 - (3*a + b)*cosh
(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x +
c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x +
 c))*sqrt(-a^2 - a*b) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*
x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^
2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh
(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) - 4*(a^2*b + a
*b^2)*cosh(d*x + c) - 4*(a^2*b + a*b^2 - 3*(a^2*b + a*b^2)*cosh(d*x + c)^2)
*sinh(d*x + c))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^4 +
4*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (
a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*sinh(d*x + c)^4 + 2*(a^5 + a^4*b - a
^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2
*b^3)*d*cosh(d*x + c)^2 + (a^5 + a^4*b - a^3*b^2 - a^2*b^3)*d)*sinh(d*x + c
)^2 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d + 4*((a^5 + 3*a^4*b + 3*a^3*b
^2 + a^2*b^3)*d*cosh(d*x + c)^3 + (a^5 + a^4*b - a^3*b^2 - a^2*b^3)*d*cosh(
d*x + c))*sinh(d*x + c)), 1/2*(2*(a^2*b + a*b^2)*cosh(d*x + c)^3 + 6*(a^2*b
 + a*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(a^2*b + a*b^2)*sinh(d*x + c)^3
 + ((2*a^2 + 3*a*b + b^2)*cosh(d*x + c)^4 + 4*(2*a^2 + 3*a*b + b^2)*cosh(d*
x + c)*sinh(d*x + c)^3 + (2*a^2 + 3*a*b + b^2)*sinh(d*x + c)^4 + 2*(2*a^2 -
 a*b - b^2)*cosh(d*x + c)^2 + 2*(3*(2*a^2 + 3*a*b + b^2)*cosh(d*x + c)^2 +
2*a^2 - a*b - b^2)*sinh(d*x + c)^2 + 2*a^2 + 3*a*b + b^2 + 4*((2*a^2 + 3*a*
b + b^2)*cosh(d*x + c)^3 + (2*a^2 - a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)
)*sqrt(a^2 + a*b)*arctan(1/2*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x
```

```
+ c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (3*a - b)*cosh(d*x + c) +
(3*(a + b)*cosh(d*x + c)^2 + 3*a - b)*sinh(d*x + c))/sqrt(a^2 + a*b)) + ((2
*a^2 + 3*a*b + b^2)*cosh(d*x + c)^4 + 4*(2*a^2 + 3*a*b + b^2)*cosh(d*x + c)
*sinh(d*x + c)^3 + (2*a^2 + 3*a*b + b^2)*sinh(d*x + c)^4 + 2*(2*a^2 - a*b -
 b^2)*cosh(d*x + c)^2 + 2*(3*(2*a^2 + 3*a*b + b^2)*cosh(d*x + c)^2 + 2*a^2
- a*b - b^2)*sinh(d*x + c)^2 + 2*a^2 + 3*a*b + b^2 + 4*((2*a^2 + 3*a*b + b^
2)*cosh(d*x + c)^3 + (2*a^2 - a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt
(a^2 + a*b)*arctan(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh(d*x + c))/a) -
 2*(a^2*b + a*b^2)*cosh(d*x + c) - 2*(a^2*b + a*b^2 - 3*(a^2*b + a*b^2)*cos
h(d*x + c)^2)*sinh(d*x + c))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(
d*x + c)^4 + 4*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)*sinh(d
*x + c)^3 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*sinh(d*x + c)^4 + 2*(a^
5 + a^4*b - a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^5 + 3*a^4*b + 3*
a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + (a^5 + a^4*b - a^3*b^2 - a^2*b^3)*d)
*sinh(d*x + c)^2 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d + 4*((a^5 + 3*a^
4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^3 + (a^5 + a^4*b - a^3*b^2 - a^2
*b^3)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(sech(c + d*x)/(a + b*tanh(c + d*x)**2)**2, x)
```

Giac [C] time = 1.68103, size = 6892, normalized size = 83.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*(2*(3*(2*a^4*e^(4*c) + 5*a^3*b*e^(4*c) + 4*a^2*b^2*e^(4*c) + a*b^3*e^(4
*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_par
t(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) +
b/(a + b)))) - (2*a^4*e^(4*c) + 5*a^3*b*e^(4*c) + 4*a^2*b^2*e^(4*c) + a*b^3
*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*rea
l_part(arccos(-a/(a + b) + b/(a + b))))^3 - 9*(2*a^4*e^(4*c) + 5*a^3*b*e^(4
*c) + 4*a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b
) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*si
n(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(
-a/(a + b) + b/(a + b)))) + 3*(2*a^4*e^(4*c) + 5*a^3*b*e^(4*c) + 4*a^2*b^2*
e^(4*c) + a*b^3*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))
)^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part
(arccos(-a/(a + b) + b/(a + b)))) + 9*(2*a^4*e^(4*c) + 5*a^3*b*e^(4*c) + 4*
a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a
 + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real
_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b)
 + b/(a + b))))^2 - 3*(2*a^4*e^(4*c) + 5*a^3*b*e^(4*c) + 4*a^2*b^2*e^(4*c)
```

+ a*b^3*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(2*a^4*e^(4*c) + 5*a^3*b*e^(4*c) + 4*a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (2*a^4*e^(4*c) + 5*a^3*b*e^(4*c) + 4*a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (2*a^4*e^(4*c) + 5*a^3*b*e^(4*c) + 4*a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) - (2*a^4*e^(4*c) + 5*a^3*b*e^(4*c) + 4*a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*arctan((((a^3 + 2*a^2*b + a*b^2)/(a^3*e^(4*c) + 2*a^2*b*e^(4*c) + a*b^2*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b))) + e^(d*x))/(((a^3 + 2*a^2*b + a*b^2)/(a^3*e^(4*c) + 2*a^2*b*e^(4*c) + a*b^2*e^(4*c)))^(1/4)*sin(1/2*arccos(-(a - b)/(a + b)))))/(2*(a^2*e^(2*c) + a*b*e^(2*c))^2*a*b - (a^3*e^(2*c) - a*b^2*e^(2*c))*sqrt(-a*b)*abs(-a^2*e^(2*c) - a*b*e^(2*c))) + 2*(3*(2*a^4*e^(4*c) + 5*a^3*b*e^(4*c) + 4*a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) - (2*a^4*e^(4*c) + 5*a^3*b*e^(4*c) + 4*a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3 - 9*(2*a^4*e^(4*c) + 5*a^3*b*e^(4*c) + 4*a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(2*a^4*e^(4*c) + 5*a^3*b*e^(4*c) + 4*a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(2*a^4*e^(4*c) + 5*a^3*b*e^(4*c) + 4*a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(2*a^4*e^(4*c) + 5*a^3*b*e^(4*c) + 4*a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(2*a^4*e^(4*c) + 5*a^3*b*e^(4*c) + 4*a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (2*a^4*e^(4*c) + 5*a^3*b*e^(4*c) + 4*a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (2*a^4*e^(4*c) + 5*a^3*b*e^(4*c) + 4*a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) - (2*a^4*e^(4*c) + 5*a^3*b*e^(4*c) + 4*a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*arctan(-(((a^3 + 2*a^2*b + a*b^2)/(a^3*e^(4*c) + 2*a^2*b*e^(4*c) + a*b^2*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b))) - e^(d*x))/(((a^3 + 2*a^2*b + a*b^2)/(a^3*e^(4*c) + 2*a^2*b*e^(4*c) + a*b^2*e^(4*c)))^(1/4)*sin(1/2*arccos(-(a - b)/(a + b)))))/(2*(a^2*e^(2*c) + a*b*e^(2*c))^2*a*b - (a^3*e^(2*c) - a*b^2*e^(2*c))*sqrt(-a*b)*abs(-a^2*e^(2*c) - a*b*e^(2*c))) + ((2*a^4*e^(4*c) + 5*a^3*b*e^(4*c) + 4*a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 - 3*(2*a^4*e^(4*c) + 5*a^3*b*e^(4*c) + 4*a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(2*a^4*e^(4*c) + 5*a^3*b*e^(4*c) + 4*a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*im ag_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(2*a^4*e^(4*c) + 5*a^3*b*e^(4*c) + 4*a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(2*a^4*e^(4*c) + 5*a^3*b*e^(4*c) + 4*a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 9*(2*a^4*e^(4*c) + 5*a^3*b*e^(4*c) + 4*a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - (2*a^4*e^(4*c) + 5*a^3*b*e^(4*c) + 4*a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + 3*(2*a^4*e^(4*c) + 5*a^3*b*e^(4*c) + 4*a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (2*a^4*e^(4*c) + 5*a^3*b*e^(4*c) + 4*a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) - (2*a^4*e^(4*c) + 5*a^3*b*e^(4*c) + 4*a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*log(2*((a^3 + 2*a^2*b + a*b^2)/(a^3*e^(4*c) + 2*a^2*b*e^(4*c) + a*b^2*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b)))*e^(d*x) + sqrt((a^3 + 2*a^2*b + a*b^2)/(a^3*e^(4*c) + 2*a^2*b*e^(4*c) + a*b^2*e^(4*c))) + e^(2*d*x))/(2*(a^2*e^(2*c) + a*b*e^(2*c))^2*a*b - (a^3*e^(2*c) - a*b^2*e^(2*c))*sqrt(-a*b)*abs(-a^2*e^(2*c) - a*b*e^(2*c))) - ((2*a^4*e^(4*c) + 5*a^3*b*e^(4*c) + 4*a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 - 3*(2*a^4*e^(4*c) + 5*a^3*b*e^(4*c) + 4*a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(2*a^4*e^(4*c) + 5*a^3*b*e^(4*c) + 4*a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(2*a^4*e^(4*c) + 5*a^3*b*e^(4*c) + 4*a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(2*a^4*e^(4*c) + 5*a^3*b*e^(4*c) + 4*a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 9*(2*a^4*e^(4*c) + 5*a^3*b*e^(4*c) + 4*a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - (2*a^4*e^(4*c) + 5*a^3*b*e^(4*c) + 4*a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + 3*(2*a^4*e^(4*c) + 5*a^3*b*e^(4*c) + 4*a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (2*a^4*e^(4*c) + 5*a^3*b*e^(4*c) + 4*a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) - (2*a^4*e^(4*c) + 5*a^3*b*e^(4*c) + 4*a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*log(-2*((a^3 + 2*a^2*b + a*b^2)/(a^3*e^(4*c) + 2*a^2*b*e^(4*c) + a*b^2*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b)))*e^(d*x) + sqrt((a^3 + 2*a^2*b + a*b^2)/(a^3*e^(4*c) + 2*a^2*b*e^(4*c) + a*b^2*e^(4*c))) + e^(2*d*x))/(2*

```
(a^2*e^(2*c) + a*b*e^(2*c))^2*a*b - (a^3*e^(2*c) - a*b^2*e^(2*c))*sqrt(-a*b
)*abs(-a^2*e^(2*c) - a*b*e^(2*c))) + 8*(b*e^(3*d*x + 3*c) - b*e^(d*x + c))/
((a^2 + a*b)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) -
 2*b*e^(2*d*x + 2*c) + a + b)))/d
```

3.119 $\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=66

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}d}+\frac{\tanh(c+dx)}{2ad\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]*d) + Tanh[c + d*
x]/(2*a*d*(a + b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.0660707, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3675, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}d}+\frac{\tanh(c+dx)}{2ad\left(a+b\tanh^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]*d) + Tanh[c + d*
x]/(2*a*d*(a + b*Tanh[c + d*x]^2))
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\mathrm{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\mathrm{Subst}\left(\int \frac{1}{(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\tanh(c+dx)}{2ad\left(a+b\tanh^2(c+dx)\right)} + \frac{\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{2ad}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}d} + \frac{\tanh(c+dx)}{2ad\left(a+b\tanh^2(c+dx)\right)}$$

Mathematica [A] time = 0.239028, size = 63, normalized size = 0.95

$$\frac{\frac{\sqrt{a}\tanh(c+dx)}{a+b\tanh^2(c+dx)} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}}}{2a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]]/Sqrt[b] + (Sqrt[a]*Tanh[c + d*x])/
(a + b*Tanh[c + d*x]^2))/(2*a^(3/2)*d)
```

Maple [B] time = 0.096, size = 498, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] 1/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c
)^2*b+a)/a*tanh(1/2*d*x+1/2*c)^3+1/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*
x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)-1/2/d/(b*(a
+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c
)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/2/d/a/((2*(b*(a+b))^(1/2)-a-2*b)*a
)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-
1/2/d/(b*(a+b))^(1/2)/a/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(
1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))*b-1/2/d/(b*(a+b))^(1/2)
/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a
+b))^(1/2)+a+2*b)*a)^(1/2))-1/2/d/a/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arc
tan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/2/d/(b*(a+
b))^(1/2)/a/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c
)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.30832, size = 3663, normalized size = 55.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a^2*b + 4*a*b^2 + 4*(a^2*b - a*b^2)*cosh(d*x + c)^2 + 8*(a^2*b - a
*b^2)*cosh(d*x + c)*sinh(d*x + c) + 4*(a^2*b - a*b^2)*sinh(d*x + c)^2 + ((a
^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sin
h(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x
 + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x +
c)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 -
b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh
(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 +
2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 +
2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b
^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*si
nh(d*x + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x
 + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)
^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*
(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x +
c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) +
a + b)))/((a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^4 + 4*(a^4*b + 2*a^
3*b^2 + a^2*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b + 2*a^3*b^2 + a^2
*b^3)*d*sinh(d*x + c)^4 + 2*(a^4*b - a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^4
*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + (a^4*b - a^2*b^3)*d)*sinh(d*x
 + c)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*d + 4*((a^4*b + 2*a^3*b^2 + a^2*b^3
)*d*cosh(d*x + c)^3 + (a^4*b - a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), -1
/2*(2*a^2*b + 2*a*b^2 + 2*(a^2*b - a*b^2)*cosh(d*x + c)^2 + 4*(a^2*b - a*b^
2)*cosh(d*x + c)*sinh(d*x + c) + 2*(a^2*b - a*b^2)*sinh(d*x + c)^2 - ((a^2
+ 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d
*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x +
c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^
2 + a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2
)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b)*arctan(1/2*((a + b)*cosh(d*x + c)
^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a -
b)*sqrt(a*b)/(a*b)))/((a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^4 + 4*(
a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b + 2*a
^3*b^2 + a^2*b^3)*d*sinh(d*x + c)^4 + 2*(a^4*b - a^2*b^3)*d*cosh(d*x + c)^2
 + 2*(3*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + (a^4*b - a^2*b^3)
*d)*sinh(d*x + c)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*d + 4*((a^4*b + 2*a^3*b
^2 + a^2*b^3)*d*cosh(d*x + c)^3 + (a^4*b - a^2*b^3)*d*cosh(d*x + c))*sinh(d
*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(sech(c + d*x)**2/(a + b*tanh(c + d*x)**2)**2, x)
```

Giac [B] time = 1.61842, size = 186, normalized size = 2.82

$$\frac{\frac{\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\sqrt{ab}a}-\frac{2\,\left(ae^{(2\,dx+2\,c)}-be^{(2\,dx+2\,c)}+a+b\right)}{\left(a^2+ab\right)\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/
(sqrt(a*b)*a) - 2*(a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) + a + b)/((a^2 + a
*b)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2
*d*x + 2*c) + a + b)))/d
```

3.120 $\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=72

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d\sqrt{a+b}}+\frac{\sinh(c+dx)}{2ad\left((a+b)\sinh^2(c+dx)+a\right)}$$

```
[Out] ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[a + b]*d) + Sin
h[c + d*x]/(2*a*d*(a + (a + b)*Sinh[c + d*x]^2))
```

Rubi [A] time = 0.0765269, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3676, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d\sqrt{a+b}}+\frac{\sinh(c+dx)}{2ad\left((a+b)\sinh^2(c+dx)+a\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[a + b]*d) + Sin
h[c + d*x]/(2*a*d*(a + (a + b)*Sinh[c + d*x]^2))
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{\left(a+(a+b)x^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\sinh(c+dx)}{2ad\left(a+(a+b)\sinh^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \sinh(c+dx)\right)}{2ad}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a+b}d} + \frac{\sinh(c+dx)}{2ad\left(a+(a+b)\sinh^2(c+dx)\right)}$$

Mathematica [A] time = 0.138047, size = 69, normalized size = 0.96

$$\frac{\frac{\sqrt{a}\sinh(c+dx)}{(a+b)\sinh^2(c+dx)+a} + \frac{\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a+b}}}{2a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]]/Sqrt[a + b] + (Sqrt[a]*Sinh[c
 + d*x])/(a + (a + b)*Sinh[c + d*x]^2))/(2*a^(3/2)*d)
```

Maple [B] time = 0.101, size = 375, normalized size = 5.2

$$-\frac{1}{da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4 a + 2\left(\tanh(1/2\,dx+c/2)\right)^2 a + 4\left(\tanh(1/2\,dx+c/2)\right)^2 b + a\right)^{-1} + \frac{1}{da}\tan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] -1/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*
c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)^3+1/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d
*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)-1/2/d/a/((
2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b
))^(1/2)-a-2*b)*a)^(1/2))+1/2/d/(b*(a+b))^(1/2)/a/((2*(b*(a+b))^(1/2)-a-2*b
)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2
))*b+1/2/d/a/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*
c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/2/d/(b*(a+b))^(1/2)/a/((2*(b*(a+b
))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a
+2*b)*a)^(1/2))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(ae^{(3c)} + be^{(3c)}\right)e^{(3dx)} - \left(ae^{c} + be^{c}\right)e^{(dx)}}{a^3d + 2a^2bd + ab^2d + \left(a^3de^{(4c)} + 2a^2bde^{(4c)} + ab^2de^{(4c)}\right)e^{(4dx)} + 2\left(a^3de^{(2c)} - ab^2de^{(2c)}\right)e^{(2dx)}} + 8\int \frac{}{8\left(a^2 + ab + \left(a^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] ((a*e^(3*c) + b*e^(3*c))*e^(3*d*x) - (a*e^c + b*e^c)*e^(d*x))/(a^3*d + 2*a^
2*b*d + a*b^2*d + (a^3*d*e^(4*c) + 2*a^2*b*d*e^(4*c) + a*b^2*d*e^(4*c))*e^(
4*d*x) + 2*(a^3*d*e^(2*c) - a*b^2*d*e^(2*c))*e^(2*d*x)) + 8*integrate(1/8*(
e^(3*d*x + 3*c) + e^(d*x + c))/(a^2 + a*b + (a^2*e^(4*c) + a*b*e^(4*c))*e^(
4*d*x) + 2*(a^2*e^(2*c) - a*b*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 2.27416, size = 4026, normalized size = 55.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^2 + a*b)*cosh(d*x + c)^3 + 12*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x
 + c)^2 + 4*(a^2 + a*b)*sinh(d*x + c)^3 - ((a + b)*cosh(d*x + c)^4 + 4*(a +
 b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cos
h(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((
a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*sqrt
(-a^2 - a*b)*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*
x + c)^3 + (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a
+ b)*cosh(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^
3 - (3*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*
x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x
 + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a + b)/((a + b)*cosh(d*x + c)^4 +
 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a -
 b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2
 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a +
b)) - 4*(a^2 + a*b)*cosh(d*x + c) + 4*(3*(a^2 + a*b)*cosh(d*x + c)^2 - a^2
- a*b)*sinh(d*x + c))/((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^4 + 4*(a^4
 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^3*b + a^
2*b^2)*d*sinh(d*x + c)^4 + 2*(a^4 - a^2*b^2)*d*cosh(d*x + c)^2 + 2*(3*(a^4
+ 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 + (a^4 - a^2*b^2)*d)*sinh(d*x + c)^2
 + (a^4 + 2*a^3*b + a^2*b^2)*d + 4*((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x +
c)^3 + (a^4 - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*(a^2 + a*b)*
cosh(d*x + c)^3 + 6*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(a^2 + a*
b)*sinh(d*x + c)^3 + ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sin
h(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(
a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^
3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*sqrt(a^2 + a*b)*arctan(1/
2*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a +
 b)*sinh(d*x + c)^3 + (3*a - b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2
+ 3*a - b)*sinh(d*x + c))/sqrt(a^2 + a*b)) + ((a + b)*cosh(d*x + c)^4 + 4*(
a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*
cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4
*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*s
qrt(a^2 + a*b)*arctan(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh(d*x + c))/a
) - 2*(a^2 + a*b)*cosh(d*x + c) + 2*(3*(a^2 + a*b)*cosh(d*x + c)^2 - a^2 -
a*b)*sinh(d*x + c))/((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^4 + 4*(a^4 +
 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^3*b + a^2*
b^2)*d*sinh(d*x + c)^4 + 2*(a^4 - a^2*b^2)*d*cosh(d*x + c)^2 + 2*(3*(a^4 +
2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 + (a^4 - a^2*b^2)*d)*sinh(d*x + c)^2 +
 (a^4 + 2*a^3*b + a^2*b^2)*d + 4*((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)
^3 + (a^4 - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(sech(c + d*x)**3/(a + b*tanh(c + d*x)**2)**2, x)
```

Giac [C] time = 1.9383, size = 4585, normalized size = 63.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*(2*(3*(a^2 + a*b)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*
cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arc
cos(-a/(a + b) + b/(a + b)))) - (a^2 + a*b)*cosh(1/2*imag_part(arccos(-a/(a
 + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3
 - 9*(a^2 + a*b)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(
1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-
a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))
) + 3*(a^2 + a*b)*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin
(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos
(-a/(a + b) + b/(a + b)))) + 9*(a^2 + a*b)*cos(1/2*real_part(arccos(-a/(a +
 b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*si
n(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(
-a/(a + b) + b/(a + b))))^2 - 3*(a^2 + a*b)*cosh(1/2*imag_part(arccos(-a/(a
 + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*s
inh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(a^2 + a*b)*cos(1/
2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/
(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^
3 + (a^2 + a*b)*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1
/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (a^2 + a*b)*cosh(1/2*imag
_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b)
+ b/(a + b)))) - (a^2 + a*b)*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b
))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*arctan((((a^2 + a
*b)/(a^2*e^(4*c) + a*b*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b))) +
e^(d*x))/(((a^2 + a*b)/(a^2*e^(4*c) + a*b*e^(4*c)))^(1/4)*sin(1/2*arccos(-(
a - b)/(a + b)))))/(2*a^3*b + (a^2 - a*b)*sqrt(-a*b)*abs(a)) + 2*(3*(a^2 +
a*b)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_par
t(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) +
b/(a + b)))) - (a^2 + a*b)*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)
)))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3 - 9*(a^2 + a*b)*
cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arc
cos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a
+ b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(a^2 + a*b)
*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(ar
ccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(
a + b)))) + 9*(a^2 + a*b)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))
```

```
)^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(a
rccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a
 + b))))^2 - 3*(a^2 + a*b)*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)
)))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part
(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(a^2 + a*b)*cos(1/2*real_part(arcco
s(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a +
b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (a^2 + a*b)*s
in(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arcc
os(-a/(a + b) + b/(a + b))))^3 + (a^2 + a*b)*cosh(1/2*imag_part(arccos(-a/(
a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) -
(a^2 + a*b)*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*ima
g_part(arccos(-a/(a + b) + b/(a + b)))))*arctan(-(((a^2 + a*b)/(a^2*e^(4*c)
 + a*b*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b))) - e^(d*x))/(((a^2
+ a*b)/(a^2*e^(4*c) + a*b*e^(4*c)))^(1/4)*sin(1/2*arccos(-(a - b)/(a + b)))
))/(2*a^3*b + (a^2 - a*b)*sqrt(-a*b)*abs(a)) + ((a^2 + a*b)*cos(1/2*real_pa
rt(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b)
+ b/(a + b))))^3 - 3*(a^2 + a*b)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a
 + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real
_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(a^2 + a*b)*cos(1/2*real_part(
arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b
/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(a^2
+ a*b)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_par
t(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) +
b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(a^2
 + a*b)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_
part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b)
+ b/(a + b))))^2 - 9*(a^2 + a*b)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a
 + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_p
art(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b)
 + b/(a + b))))^2 - (a^2 + a*b)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a
+ b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + 3*(a^2 +
a*b)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(a
rccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/
(a + b))))^3 + (a^2 + a*b)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))
))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) - (a^2 + a*b)*cos(1/
2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(
a + b) + b/(a + b)))))*log(2*((a^2 + a*b)/(a^2*e^(4*c) + a*b*e^(4*c)))^(1/4
)*cos(1/2*arccos(-(a - b)/(a + b)))*e^(d*x) + sqrt((a^2 + a*b)/(a^2*e^(4*c)
 + a*b*e^(4*c))) + e^(2*d*x))/(2*a^3*b + (a^2 - a*b)*sqrt(-a*b)*abs(a)) - (
(a^2 + a*b)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*i
mag_part(arccos(-a/(a + b) + b/(a + b))))^3 - 3*(a^2 + a*b)*cos(1/2*real_pa
rt(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) +
b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(a^
2 + a*b)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag
_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a +
b) + b/(a + b)))) + 9*(a^2 + a*b)*cos(1/2*real_part(arccos(-a/(a + b) + b/(
a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*rea
l_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a +
 b) + b/(a + b)))) + 3*(a^2 + a*b)*cos(1/2*real_part(arccos(-a/(a + b) + b/
(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*i
mag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 9*(a^2 + a*b)*cos(1/2*real_pa
rt(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) +
b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*
imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - (a^2 + a*b)*cos(1/2*real_par
t(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) +
 b/(a + b))))^3 + 3*(a^2 + a*b)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a
+ b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_
part(arccos(-a/(a + b) + b/(a + b))))^3 + (a^2 + a*b)*cos(1/2*real_part(arc
cos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a +
```

```
 b)))) - (a^2 + a*b)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sin
h(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*log(-2*((a^2 + a*b)/(a^2*
e^(4*c) + a*b*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b)))*e^(d*x) + s
qrt((a^2 + a*b)/(a^2*e^(4*c) + a*b*e^(4*c))) + e^(2*d*x))/(2*a^3*b + (a^2 -
 a*b)*sqrt(-a*b)*abs(a)) + 8*(e^(3*d*x + 3*c) - e^(d*x + c))/((a*e^(4*d*x +
 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a +
 b)*a))/d
```

3.121 $\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2} dx$

Optimal. Leaf size=77

$$\frac{(a+b)\tanh(c+dx)}{2abd\left(a+b\tanh^2(c+dx)\right)}-\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}d}$$

```
[Out] -((a - b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^(3/2)*d) +
((a + b)*Tanh[c + d*x])/(2*a*b*d*(a + b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.0788741, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3675, 385, 205}

$$\frac{(a+b)\tanh(c+dx)}{2abd\left(a+b\tanh^2(c+dx)\right)}-\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -((a - b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^(3/2)*d) +
((a + b)*Tanh[c + d*x])/(2*a*b*d*(a + b*Tanh[c + d*x]^2))
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{\left(a+bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{(a+b)\tanh(c+dx)}{2abd\left(a+b\tanh^2(c+dx)\right)} - \frac{(a-b)\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{2abd}$$
$$= -\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}d} + \frac{(a+b)\tanh(c+dx)}{2abd\left(a+b\tanh^2(c+dx)\right)}$$

Mathematica [A] time = 0.279209, size = 83, normalized size = 1.08

$$\frac{(b-a)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right) + \frac{\sqrt{a}\sqrt{b}(a+b)\sinh(2(c+dx))}{(a+b)\cosh(2(c+dx))+a-b}}{2a^{3/2}b^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((-a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] + (Sqrt[a]*Sqrt[b]*(a + b
)*Sinh[2*(c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)]))/(2*a^(3/2)*b^(3/2
)*d)
```

Maple [B] time = 0.095, size = 746, normalized size = 9.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] 1/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c
)^2*b+a)/b*tanh(1/2*d*x+1/2*c)^3+1/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*
x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)^3+1/d/(tanh
(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/
b*tanh(1/2*d*x+1/2*c)+1/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*
a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)+1/2/d/b*a/(b*(a+b))^(1
/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(
b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-1/2/d/b/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)
*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/2/d/b
*a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*
x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/2/d/b/((2*(b*(a+b))^(1/2)+a
+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(
1/2))+1/2/d/a/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/
2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-1/2/d/(b*(a+b))^(1/2)/a/((2*(b*(a
+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2
)-a-2*b)*a)^(1/2))*b-1/2/d/a/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*t
anh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/2/d/(b*(a+b))^(1/
2)/a/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(
b*(a+b))^(1/2)+a+2*b)*a)^(1/2))*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.22099, size = 3452, normalized size = 44.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a^2*b + 4*a*b^2 + 4*(a^2*b - a*b^2)*cosh(d*x + c)^2 + 8*(a^2*b - a
*b^2)*cosh(d*x + c)*sinh(d*x + c) + 4*(a^2*b - a*b^2)*sinh(d*x + c)^2 - ((a
^2 - b^2)*cosh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (
a^2 - b^2)*sinh(d*x + c)^4 + 2*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(
a^2 - b^2)*cosh(d*x + c)^2 + a^2 - 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 - b^2
 + 4*((a^2 - b^2)*cosh(d*x + c)^3 + (a^2 - 2*a*b + b^2)*cosh(d*x + c))*sinh
(d*x + c))*sqrt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2
*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x +
c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x +
c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b
^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)
*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x
 + c)^2 + a - b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x
+ c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2
+ 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d
*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)))/((a^3*b^2 + a^2
*b^3)*d*cosh(d*x + c)^4 + 4*(a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)*sinh(d*x +
c)^3 + (a^3*b^2 + a^2*b^3)*d*sinh(d*x + c)^4 + 2*(a^3*b^2 - a^2*b^3)*d*cosh
(d*x + c)^2 + 2*(3*(a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + (a^3*b^2 - a^2*b
^3)*d)*sinh(d*x + c)^2 + (a^3*b^2 + a^2*b^3)*d + 4*((a^3*b^2 + a^2*b^3)*d*c
osh(d*x + c)^3 + (a^3*b^2 - a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*
(2*a^2*b + 2*a*b^2 + 2*(a^2*b - a*b^2)*cosh(d*x + c)^2 + 4*(a^2*b - a*b^2)*
cosh(d*x + c)*sinh(d*x + c) + 2*(a^2*b - a*b^2)*sinh(d*x + c)^2 + ((a^2 - b
^2)*cosh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 -
b^2)*sinh(d*x + c)^4 + 2*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 -
b^2)*cosh(d*x + c)^2 + a^2 - 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 - b^2 + 4*(
(a^2 - b^2)*cosh(d*x + c)^3 + (a^2 - 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x +
 c))*sqrt(a*b)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c
)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(a*b)/(a*b)))/((a^3*
b^2 + a^2*b^3)*d*cosh(d*x + c)^4 + 4*(a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)*si
nh(d*x + c)^3 + (a^3*b^2 + a^2*b^3)*d*sinh(d*x + c)^4 + 2*(a^3*b^2 - a^2*b^
3)*d*cosh(d*x + c)^2 + 2*(3*(a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + (a^3*b^
2 - a^2*b^3)*d)*sinh(d*x + c)^2 + (a^3*b^2 + a^2*b^3)*d + 4*((a^3*b^2 + a^2
*b^3)*d*cosh(d*x + c)^3 + (a^3*b^2 - a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c
))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(sech(c + d*x)**4/(a + b*tanh(c + d*x)**2)**2, x)
```

Giac [B] time = 1.66815, size = 211, normalized size = 2.74

$$-\frac{\frac{\left(ae^{(2c)}-be^{(2c)}\right)\arctan\left(\frac{ae^{(2dx+2c)}+be^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)e^{(-2c)}}{\sqrt{ab}ab}+\frac{2\left(ae^{(2dx+2c)}-be^{(2dx+2c)}+a+b\right)}{\left(ae^{(4dx+4c)}+be^{(4dx+4c)}+2ae^{(2dx+2c)}-2be^{(2dx+2c)}+a+b\right)ab}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((a*e^(2*c) - b*e^(2*c))*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x +
2*c) + a - b)/sqrt(a*b))*e^(-2*c)/(sqrt(a*b)*a*b) + 2*(a*e^(2*d*x + 2*c) -
b*e^(2*d*x + 2*c) + a + b)/((a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^
(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)*a*b))/d
```

3.122 $\int \frac{\text{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=102

$$-\frac{(2a-b)\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^2d}+\frac{(a+b)\sinh(c+dx)}{2abd\left((a+b)\sinh^2(c+dx)+a\right)}+\frac{\tan^{-1}(\sinh(c+dx))}{b^2d}$$

```
[Out] ArcTan[Sinh[c + d*x]]/(b^2*d) - ((2*a - b)*Sqrt[a + b]*ArcTan[(Sqrt[a + b]*
Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^2*d) + ((a + b)*Sinh[c + d*x])/(2*a*b
*d*(a + (a + b)*Sinh[c + d*x]^2))
```

Rubi [A] time = 0.127104, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3676, 414, 522, 203, 205}

$$-\frac{(2a-b)\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^2d}+\frac{(a+b)\sinh(c+dx)}{2abd\left((a+b)\sinh^2(c+dx)+a\right)}+\frac{\tan^{-1}(\sinh(c+dx))}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ArcTan[Sinh[c + d*x]]/(b^2*d) - ((2*a - b)*Sqrt[a + b]*ArcTan[(Sqrt[a + b]*
Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^2*d) + ((a + b)*Sinh[c + d*x])/(2*a*b
*d*(a + (a + b)*Sinh[c + d*x]^2))
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\left(a+(a+b)x^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{(a+b)\sinh(c+dx)}{2abd\left(a+(a+b)\sinh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{a-b+(-a-b)x^2}{(1+x^2)\left(a+(a+b)x^2\right)}\,dx, x, \sinh(c+dx)\right)}{2abd}$$

$$= \frac{(a+b)\sinh(c+dx)}{2abd\left(a+(a+b)\sinh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{b^2d} - \frac{((2a-b)(a+$$

$$= \frac{\tan^{-1}(\sinh(c+dx))}{b^2d} - \frac{(2a-b)\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^2d} + \frac{(a+b)\sinh(c-}{2abd\left(a+(a+b)\sinh\right.}$$

Mathematica [A] time = 0.483468, size = 203, normalized size = 1.99

$$\frac{(a-b)\left(\left(2a^2+ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)+4a^{3/2}\sqrt{a+b}\tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)\right)+(a+b)\cosh(2(c+dx))\left(\left(2a^2\right.\right.}{2a^{3/2}b^2d\sqrt{a+b}((a+b)\cosh(2(}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((a - b)*((2*a^2 + a*b - b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]] +
 4*a^(3/2)*Sqrt[a + b]*ArcTan[Tanh[(c + d*x)/2]]) + (a + b)*((2*a^2 + a*b -
 b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]] + 4*a^(3/2)*Sqrt[a + b]*A
rcTan[Tanh[(c + d*x)/2]])*Cosh[2*(c + d*x)] + 2*Sqrt[a]*b*(a + b)^(3/2)*Sin
h[c + d*x])/(2*a^(3/2)*b^2*Sqrt[a + b]*d*(a - b + (a + b)*Cosh[2*(c + d*x)]
))
```

Maple [B] time = 0.099, size = 1007, normalized size = 9.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] -1/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*
c)^2*b+a)/b*tanh(1/2*d*x+1/2*c)^3-1/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d
*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)^3+1/d/(tan
```

```
h(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)
/b*tanh(1/2*d*x+1/2*c)+1/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2
*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)+1/d/b^2*a/((2*(b*(a+b
))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-
a-2*b)*a)^(1/2))-1/d/b*a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2
)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-1/d/b^
2*a/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b
*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/d/b*a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+
a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^
(1/2))+1/2/d/b/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1
/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-1/2/d/(b*(a+b))^(1/2)/((2*(b*(a+
b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)
-a-2*b)*a)^(1/2))-1/2/d/b/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh
(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/2/d/(b*(a+b))^(1/2)/
((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+
b))^(1/2)+a+2*b)*a)^(1/2))-1/2/d/a/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arct
anh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/2/d/(b*(a+
b))^(1/2)/a/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*
c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))*b+1/2/d/a/((2*(b*(a+b))^(1/2)+a+2*b
)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)
)+1/2/d/(b*(a+b))^(1/2)/a/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh
(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))*b+2/d/b^2*arctan(tanh(
1/2*d*x+1/2*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(ae^{(3c)}+be^{(3c)}\right)e^{(3dx)}-\left(ae^{c}+be^{c}\right)e^{(dx)}}{a^2bd+ab^2d+\left(a^2bde^{(4c)}+ab^2de^{(4c)}\right)e^{(4dx)}+2\left(a^2bde^{(2c)}-ab^2de^{(2c)}\right)e^{(2dx)}}+\frac{2\arctan\left(e^{(dx+c)}\right)}{b^2d}-32\int\frac{\left(2a^2e^{(3c)}\right.}{32\left(a^2b^2+ab^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] ((a*e^(3*c) + b*e^(3*c))*e^(3*d*x) - (a*e^c + b*e^c)*e^(d*x))/(a^2*b*d + a*
b^2*d + (a^2*b*d*e^(4*c) + a*b^2*d*e^(4*c))*e^(4*d*x) + 2*(a^2*b*d*e^(2*c)
- a*b^2*d*e^(2*c))*e^(2*d*x)) + 2*arctan(e^(d*x + c))/(b^2*d) - 32*integrat
e(1/32*((2*a^2*e^(3*c) + a*b*e^(3*c) - b^2*e^(3*c))*e^(3*d*x) + (2*a^2*e^c
+ a*b*e^c - b^2*e^c)*e^(d*x))/(a^2*b^2 + a*b^3 + (a^2*b^2*e^(4*c) + a*b^3*e
^(4*c))*e^(4*d*x) + 2*(a^2*b^2*e^(2*c) - a*b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 2.58701, size = 5292, normalized size = 51.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a*b + b^2)*cosh(d*x + c)^3 + 12*(a*b + b^2)*cosh(d*x + c)*sinh(d*x
 + c)^2 + 4*(a*b + b^2)*sinh(d*x + c)^3 - ((2*a^2 + a*b - b^2)*cosh(d*x + c
)^4 + 4*(2*a^2 + a*b - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2 + a*b -
b^2)*sinh(d*x + c)^4 + 2*(2*a^2 - 3*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(2*a^
2 + a*b - b^2)*cosh(d*x + c)^2 + 2*a^2 - 3*a*b + b^2)*sinh(d*x + c)^2 + 2*a
^2 + a*b - b^2 + 4*((2*a^2 + a*b - b^2)*cosh(d*x + c)^3 + (2*a^2 - 3*a*b +
```

```
b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-(a + b)/a)*log(((a + b)*cosh(d*x +
 c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 -
 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 - 3*a - b)*sinh
(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 - (3*a + b)*cosh(d*x + c))*sinh(d*
x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(
d*x + c)^3 - a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 - a)*sinh(d*x + c))*sqr
t(-(a + b)/a) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*s
inh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3
*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c
)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 8*((a^2 + a*b)*cosh(
d*x + c)^4 + 4*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + a*b)*sinh
(d*x + c)^4 + 2*(a^2 - a*b)*cosh(d*x + c)^2 + 2*(3*(a^2 + a*b)*cosh(d*x + c
)^2 + a^2 - a*b)*sinh(d*x + c)^2 + a^2 + a*b + 4*((a^2 + a*b)*cosh(d*x + c)
^3 + (a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(
d*x + c)) - 4*(a*b + b^2)*cosh(d*x + c) + 4*(3*(a*b + b^2)*cosh(d*x + c)^2
- a*b - b^2)*sinh(d*x + c))/((a^2*b^2 + a*b^3)*d*cosh(d*x + c)^4 + 4*(a^2*b
^2 + a*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2*b^2 + a*b^3)*d*sinh(d*x
+ c)^4 + 2*(a^2*b^2 - a*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^2*b^2 + a*b^3)*d*c
osh(d*x + c)^2 + (a^2*b^2 - a*b^3)*d)*sinh(d*x + c)^2 + (a^2*b^2 + a*b^3)*d
 + 4*((a^2*b^2 + a*b^3)*d*cosh(d*x + c)^3 + (a^2*b^2 - a*b^3)*d*cosh(d*x +
c))*sinh(d*x + c)), 1/2*(2*(a*b + b^2)*cosh(d*x + c)^3 + 6*(a*b + b^2)*cosh
(d*x + c)*sinh(d*x + c)^2 + 2*(a*b + b^2)*sinh(d*x + c)^3 - ((2*a^2 + a*b -
 b^2)*cosh(d*x + c)^4 + 4*(2*a^2 + a*b - b^2)*cosh(d*x + c)*sinh(d*x + c)^3
 + (2*a^2 + a*b - b^2)*sinh(d*x + c)^4 + 2*(2*a^2 - 3*a*b + b^2)*cosh(d*x +
 c)^2 + 2*(3*(2*a^2 + a*b - b^2)*cosh(d*x + c)^2 + 2*a^2 - 3*a*b + b^2)*sin
h(d*x + c)^2 + 2*a^2 + a*b - b^2 + 4*((2*a^2 + a*b - b^2)*cosh(d*x + c)^3 +
 (2*a^2 - 3*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a + b)/a)*arctan
(1/2*sqrt((a + b)/a)*(cosh(d*x + c) + sinh(d*x + c))) - ((2*a^2 + a*b - b^2
)*cosh(d*x + c)^4 + 4*(2*a^2 + a*b - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (
2*a^2 + a*b - b^2)*sinh(d*x + c)^4 + 2*(2*a^2 - 3*a*b + b^2)*cosh(d*x + c)^
2 + 2*(3*(2*a^2 + a*b - b^2)*cosh(d*x + c)^2 + 2*a^2 - 3*a*b + b^2)*sinh(d*
x + c)^2 + 2*a^2 + a*b - b^2 + 4*((2*a^2 + a*b - b^2)*cosh(d*x + c)^3 + (2*
a^2 - 3*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a + b)/a)*arctan(1/2
*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a +
b)*sinh(d*x + c)^3 + (3*a - b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 +
 3*a - b)*sinh(d*x + c))*sqrt((a + b)/a)/(a + b)) + 4*((a^2 + a*b)*cosh(d*x
 + c)^4 + 4*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + a*b)*sinh(d*
x + c)^4 + 2*(a^2 - a*b)*cosh(d*x + c)^2 + 2*(3*(a^2 + a*b)*cosh(d*x + c)^2
 + a^2 - a*b)*sinh(d*x + c)^2 + a^2 + a*b + 4*((a^2 + a*b)*cosh(d*x + c)^3
+ (a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x
 + c)) - 2*(a*b + b^2)*cosh(d*x + c) + 2*(3*(a*b + b^2)*cosh(d*x + c)^2 - a
*b - b^2)*sinh(d*x + c))/((a^2*b^2 + a*b^3)*d*cosh(d*x + c)^4 + 4*(a^2*b^2
+ a*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2*b^2 + a*b^3)*d*sinh(d*x + c
)^4 + 2*(a^2*b^2 - a*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^2*b^2 + a*b^3)*d*cosh
(d*x + c)^2 + (a^2*b^2 - a*b^3)*d)*sinh(d*x + c)^2 + (a^2*b^2 + a*b^3)*d +
4*((a^2*b^2 + a*b^3)*d*cosh(d*x + c)^3 + (a^2*b^2 - a*b^3)*d*cosh(d*x + c))
*sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**5/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(sech(c + d*x)**5/(a + b*tanh(c + d*x)**2)**2, x)
```

Giac [C] time = 2.1604, size = 5392, normalized size = 52.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/8*(2*(3*(2*a^4 + 3*a^3*b - a*b^3)*cos(1/2*real_part(arccos(-a/(a + b) +
b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/
2*real_part(arccos(-a/(a + b) + b/(a + b)))) - (2*a^4 + 3*a^3*b - a*b^3)*co
sh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arcco
s(-a/(a + b) + b/(a + b))))^3 - 9*(2*a^4 + 3*a^3*b - a*b^3)*cos(1/2*real_pa
rt(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b)
+ b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/
2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(2*a^4 + 3*a^3*b - a*b^3)*
cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arc
cos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a
 + b)))) + 9*(2*a^4 + 3*a^3*b - a*b^3)*cos(1/2*real_part(arccos(-a/(a + b)
+ b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/
2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(
a + b) + b/(a + b))))^2 - 3*(2*a^4 + 3*a^3*b - a*b^3)*cosh(1/2*imag_part(ar
ccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a +
 b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(2*a^4 +
 3*a^3*b - a*b^3)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(
1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a
/(a + b) + b/(a + b))))^3 + (2*a^4 + 3*a^3*b - a*b^3)*sin(1/2*real_part(arc
cos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a
 + b))))^3 + (2*a^4 + 3*a^3*b - a*b^3)*cosh(1/2*imag_part(arccos(-a/(a + b)
 + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) - (2*a^4
 + 3*a^3*b - a*b^3)*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh
(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*arctan((((a^2*b^2 + a*b^3)
/(a^2*b^2*e^(4*c) + a*b^3*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b)))
 + e^(d*x))/(((a^2*b^2 + a*b^3)/(a^2*b^2*e^(4*c) + a*b^3*e^(4*c)))^(1/4)*si
n(1/2*arccos(-(a - b)/(a + b)))))/(2*a^3*b^3 + (a^2*b^2 - a*b^3)*sqrt(-a*b)
*abs(a)) + 2*(3*(2*a^4 + 3*a^3*b - a*b^3)*cos(1/2*real_part(arccos(-a/(a +
b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*s
in(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) - (2*a^4 + 3*a^3*b - a*b^
3)*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(
arccos(-a/(a + b) + b/(a + b))))^3 - 9*(2*a^4 + 3*a^3*b - a*b^3)*cos(1/2*re
al_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a
+ b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*si
nh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(2*a^4 + 3*a^3*b - a*
b^3)*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_par
t(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) +
 b/(a + b)))) + 9*(2*a^4 + 3*a^3*b - a*b^3)*cos(1/2*real_part(arccos(-a/(a
+ b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*s
in(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos
(-a/(a + b) + b/(a + b))))^2 - 3*(2*a^4 + 3*a^3*b - a*b^3)*cosh(1/2*imag_pa
rt(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b
/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(2*
a^4 + 3*a^3*b - a*b^3)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2
*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arcc
os(-a/(a + b) + b/(a + b))))^3 + (2*a^4 + 3*a^3*b - a*b^3)*sin(1/2*real_par
t(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) +
```

```
b/(a + b))))^3 + (2*a^4 + 3*a^3*b - a*b^3)*cosh(1/2*imag_part(arccos(-a/(a
 + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) - (
2*a^4 + 3*a^3*b - a*b^3)*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))
*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*arctan(-(((a^2*b^2 +
a*b^3)/(a^2*b^2*e^(4*c) + a*b^3*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a
+ b))) - e^(d*x))/(((a^2*b^2 + a*b^3)/(a^2*b^2*e^(4*c) + a*b^3*e^(4*c)))^(1
/4)*sin(1/2*arccos(-(a - b)/(a + b)))))/(2*a^3*b^3 + (a^2*b^2 - a*b^3)*sqrt
(-a*b)*abs(a)) + ((2*a^4 + 3*a^3*b - a*b^3)*cos(1/2*real_part(arccos(-a/(a
+ b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3
 - 3*(2*a^4 + 3*a^3*b - a*b^3)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a +
 b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_p
art(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(2*a^4 + 3*a^3*b - a*b^3)*cos(1/
2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a
/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))
)) + 9*(2*a^4 + 3*a^3*b - a*b^3)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a
 + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real
_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a +
b) + b/(a + b)))) + 3*(2*a^4 + 3*a^3*b - a*b^3)*cos(1/2*real_part(arccos(-a
/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))
))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 9*(2*a^4 + 3*a^3
*b - a*b^3)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*ima
g_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b)
 + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - (
2*a^4 + 3*a^3*b - a*b^3)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))
^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + 3*(2*a^4 + 3*a^3
*b - a*b^3)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real
_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a +
b) + b/(a + b))))^3 + (2*a^4 + 3*a^3*b - a*b^3)*cos(1/2*real_part(arccos(-a
/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))
 - (2*a^4 + 3*a^3*b - a*b^3)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b
))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*log(2*((a^2*b^2 +
 a*b^3)/(a^2*b^2*e^(4*c) + a*b^3*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a
 + b)))*e^(d*x) + sqrt((a^2*b^2 + a*b^3)/(a^2*b^2*e^(4*c) + a*b^3*e^(4*c)))
 + e^(2*d*x))/(2*a^3*b^3 + (a^2*b^2 - a*b^3)*sqrt(-a*b)*abs(a)) - ((2*a^4 +
 3*a^3*b - a*b^3)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh
(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 - 3*(2*a^4 + 3*a^3*b - a*
b^3)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(
arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/
(a + b))))^2 - 3*(2*a^4 + 3*a^3*b - a*b^3)*cos(1/2*real_part(arccos(-a/(a +
 b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*
sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(2*a^4 + 3*a^3*b -
a*b^3)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_par
t(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) +
b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(2*a
^4 + 3*a^3*b - a*b^3)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*
cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arcc
os(-a/(a + b) + b/(a + b))))^2 - 9*(2*a^4 + 3*a^3*b - a*b^3)*cos(1/2*real_p
art(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) +
 b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2
*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - (2*a^4 + 3*a^3*b - a*b^3)*c
os(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arcc
os(-a/(a + b) + b/(a + b))))^3 + 3*(2*a^4 + 3*a^3*b - a*b^3)*cos(1/2*real_p
art(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) +
b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (2*a
^4 + 3*a^3*b - a*b^3)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*co
sh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) - (2*a^4 + 3*a^3*b - a*b^
3)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(ar
ccos(-a/(a + b) + b/(a + b)))))*log(-2*((a^2*b^2 + a*b^3)/(a^2*b^2*e^(4*c)
+ a*b^3*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b)))*e^(d*x) + sqrt((a
```

```
^2*b^2 + a*b^3)/(a^2*b^2*e^(4*c) + a*b^3*e^(4*c))) + e^(2*d*x))/(2*a^3*b^3
+ (a^2*b^2 - a*b^3)*sqrt(-a*b)*abs(a)) - 16*arctan(e^(d*x + c))/b^2 - 8*(a*
e^(3*d*x + 3*c) + b*e^(3*d*x + 3*c) - a*e^(d*x + c) - b*e^(d*x + c))/((a*e^
(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*
c) + a + b)*a*b))/d
```

3.123 $\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=97

$$-\frac{(3a-b)(a+b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}d}+\frac{(a+b)^2\tanh(c+dx)}{2ab^2d\left(a+b\tanh^2(c+dx)\right)}+\frac{\tanh(c+dx)}{b^2d}$$

```
[Out] -((3*a - b)*(a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^(
5/2)*d) + Tanh[c + d*x]/(b^2*d) + ((a + b)^2*Tanh[c + d*x])/(2*a*b^2*d*(a +
 b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.1295, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3675, 390, 385, 205}

$$-\frac{(3a-b)(a+b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}d}+\frac{(a+b)^2\tanh(c+dx)}{2ab^2d\left(a+b\tanh^2(c+dx)\right)}+\frac{\tanh(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^6/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -((3*a - b)*(a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^(
5/2)*d) + Tanh[c + d*x]/(b^2*d) + ((a + b)^2*Tanh[c + d*x])/(2*a*b^2*d*(a +
 b*Tanh[c + d*x]^2))
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^2}{(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{b^2}-\frac{a^2-b^2+2b(a+b)x^2}{b^2(a+bx^2)^2}\right)dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\tanh(c+dx)}{b^2d}-\frac{\operatorname{Subst}\left(\int \frac{a^2-b^2+2b(a+b)x^2}{(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{b^2d}$$

$$= \frac{\tanh(c+dx)}{b^2d}+\frac{(a+b)^2\tanh(c+dx)}{2ab^2d\left(a+b\tanh^2(c+dx)\right)}-\frac{((3a-b)(a+b))\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{2ab^2d}$$

$$= -\frac{(3a-b)(a+b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}d}+\frac{\tanh(c+dx)}{b^2d}+\frac{(a+b)^2\tanh(c+dx)}{2ab^2d\left(a+b\tanh^2(c+dx)\right)}$$

Mathematica [A] time = 0.58541, size = 102, normalized size = 1.05

$$\frac{-\frac{(3a-b)(a+b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}}+\frac{\sqrt{b}(a+b)^2\sinh(2(c+dx))}{a((a+b)\cosh(2(c+dx))+a-b)}+2\sqrt{b}\tanh(c+dx)}{2b^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^6/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (-(((3*a - b)*(a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/a^(3/2)) + (
Sqrt[b]*(a + b)^2*Sinh[2*(c + d*x)])/(a*(a - b + (a + b)*Cosh[2*(c + d*x)])
) + 2*Sqrt[b]*Tanh[c + d*x])/(2*b^(5/2)*d)
```

Maple [B] time = 0.092, size = 1283, normalized size = 13.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] 1/d/b^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1
/2*c)^2*b+a)*a*tanh(1/2*d*x+1/2*c)^3+2/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/
2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/b*tanh(1/2*d*x+1/2*c)^3+1/d/(
tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b
+a)/a*tanh(1/2*d*x+1/2*c)^3+1/d/b^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x
+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*a*tanh(1/2*d*x+1/2*c)+2/d/(tanh(1/
2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/b*t
anh(1/2*d*x+1/2*c)+1/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4
```

```
*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)+3/2/d/b^2*a^2/(b*(a+b))^(
1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*
(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-3/2/d/b^2*a/((2*(b*(a+b))^(1/2)-a-2*b)*a)^
(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+5/
2/d/b*a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(
1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+3/2/d/b^2*a^2/(b*(a+b))
^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2
*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+3/2/d/b^2*a/((2*(b*(a+b))^(1/2)+a+2*b)*a)
^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+5/
2/d/b*a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1
/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/d/b/((2*(b*(a+b))^(1/2
)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*
a)^(1/2))+1/2/d/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh
(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/d/b/((2*(b*(a
+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)
+a+2*b)*a)^(1/2))+1/2/d/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)
*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/2/d/a/
((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a
+b))^(1/2)-a-2*b)*a)^(1/2))-1/2/d/(b*(a+b))^(1/2)/a/((2*(b*(a+b))^(1/2)-a-2
*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1
/2))*b-1/2/d/a/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/
2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/2/d/(b*(a+b))^(1/2)/a/((2*(b*(a
+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)
+a+2*b)*a)^(1/2))*b+2/d/b^2*tanh(1/2*d*x+1/2*c)/(tanh(1/2*d*x+1/2*c)^2+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.59689, size = 6589, normalized size = 67.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(3*a^3*b + 2*a^2*b^2 - a*b^3)*cosh(d*x + c)^4 + 16*(3*a^3*b + 2*a^
2*b^2 - a*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + 4*(3*a^3*b + 2*a^2*b^2 - a*b
^3)*sinh(d*x + c)^4 + 12*a^3*b + 16*a^2*b^2 + 4*a*b^3 + 8*(3*a^3*b - a^2*b^
2)*cosh(d*x + c)^2 + 8*(3*a^3*b - a^2*b^2 + 3*(3*a^3*b + 2*a^2*b^2 - a*b^3)
*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((3*a^3 + 5*a^2*b + a*b^2 - b^3)*cosh(d
*x + c)^6 + 6*(3*a^3 + 5*a^2*b + a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x + c)^5
 + (3*a^3 + 5*a^2*b + a*b^2 - b^3)*sinh(d*x + c)^6 + (9*a^3 + 3*a^2*b - 5*a
*b^2 + b^3)*cosh(d*x + c)^4 + (9*a^3 + 3*a^2*b - 5*a*b^2 + b^3 + 15*(3*a^3
+ 5*a^2*b + a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(3*a^3 + 5
*a^2*b + a*b^2 - b^3)*cosh(d*x + c)^3 + (9*a^3 + 3*a^2*b - 5*a*b^2 + b^3)*c
osh(d*x + c))*sinh(d*x + c)^3 + 3*a^3 + 5*a^2*b + a*b^2 - b^3 + (9*a^3 + 3*
a^2*b - 5*a*b^2 + b^3)*cosh(d*x + c)^2 + (15*(3*a^3 + 5*a^2*b + a*b^2 - b^3
```

```
)*cosh(d*x + c)^4 + 9*a^3 + 3*a^2*b - 5*a*b^2 + b^3 + 6*(9*a^3 + 3*a^2*b -
5*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(3*a^3 + 5*a^2*b + a
*b^2 - b^3)*cosh(d*x + c)^5 + 2*(9*a^3 + 3*a^2*b - 5*a*b^2 + b^3)*cosh(d*x
+ c)^3 + (9*a^3 + 3*a^2*b - 5*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c))*sq
rt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2
 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 -
b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x
+ c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c
)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a -
 b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x
 + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b
)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (
a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 16*((3*a^3*b + 2*a^2*b^2 -
a*b^3)*cosh(d*x + c)^3 + (3*a^3*b - a^2*b^2)*cosh(d*x + c))*sinh(d*x + c))/
((a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^6 + 6*(a^3*b^3 + a^2*b^4)*d*cosh(d*x +
 c)*sinh(d*x + c)^5 + (a^3*b^3 + a^2*b^4)*d*sinh(d*x + c)^6 + (3*a^3*b^3 -
a^2*b^4)*d*cosh(d*x + c)^4 + (15*(a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^2 + (3
*a^3*b^3 - a^2*b^4)*d)*sinh(d*x + c)^4 + (3*a^3*b^3 - a^2*b^4)*d*cosh(d*x +
 c)^2 + 4*(5*(a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^3 + (3*a^3*b^3 - a^2*b^4)*
d*cosh(d*x + c))*sinh(d*x + c)^3 + (15*(a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^
4 + 6*(3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^2 + (3*a^3*b^3 - a^2*b^4)*d)*si
nh(d*x + c)^2 + (a^3*b^3 + a^2*b^4)*d + 2*(3*(a^3*b^3 + a^2*b^4)*d*cosh(d*x
 + c)^5 + 2*(3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^3 + (3*a^3*b^3 - a^2*b^4)
*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*(3*a^3*b + 2*a^2*b^2 - a*b^3)*cos
h(d*x + c)^4 + 8*(3*a^3*b + 2*a^2*b^2 - a*b^3)*cosh(d*x + c)*sinh(d*x + c)^
3 + 2*(3*a^3*b + 2*a^2*b^2 - a*b^3)*sinh(d*x + c)^4 + 6*a^3*b + 8*a^2*b^2 +
 2*a*b^3 + 4*(3*a^3*b - a^2*b^2)*cosh(d*x + c)^2 + 4*(3*a^3*b - a^2*b^2 + 3
*(3*a^3*b + 2*a^2*b^2 - a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((3*a^3 +
 5*a^2*b + a*b^2 - b^3)*cosh(d*x + c)^6 + 6*(3*a^3 + 5*a^2*b + a*b^2 - b^3)
*cosh(d*x + c)*sinh(d*x + c)^5 + (3*a^3 + 5*a^2*b + a*b^2 - b^3)*sinh(d*x +
 c)^6 + (9*a^3 + 3*a^2*b - 5*a*b^2 + b^3)*cosh(d*x + c)^4 + (9*a^3 + 3*a^2*
b - 5*a*b^2 + b^3 + 15*(3*a^3 + 5*a^2*b + a*b^2 - b^3)*cosh(d*x + c)^2)*sin
h(d*x + c)^4 + 4*(5*(3*a^3 + 5*a^2*b + a*b^2 - b^3)*cosh(d*x + c)^3 + (9*a^
3 + 3*a^2*b - 5*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*a^3 + 5*a^2
*b + a*b^2 - b^3 + (9*a^3 + 3*a^2*b - 5*a*b^2 + b^3)*cosh(d*x + c)^2 + (15*
(3*a^3 + 5*a^2*b + a*b^2 - b^3)*cosh(d*x + c)^4 + 9*a^3 + 3*a^2*b - 5*a*b^2
 + b^3 + 6*(9*a^3 + 3*a^2*b - 5*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)
^2 + 2*(3*(3*a^3 + 5*a^2*b + a*b^2 - b^3)*cosh(d*x + c)^5 + 2*(9*a^3 + 3*a^
2*b - 5*a*b^2 + b^3)*cosh(d*x + c)^3 + (9*a^3 + 3*a^2*b - 5*a*b^2 + b^3)*co
sh(d*x + c))*sinh(d*x + c))*sqrt(a*b)*arctan(1/2*((a + b)*cosh(d*x + c)^2 +
 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*s
qrt(a*b)/(a*b)) + 8*((3*a^3*b + 2*a^2*b^2 - a*b^3)*cosh(d*x + c)^3 + (3*a^3
*b - a^2*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^3*b^3 + a^2*b^4)*d*cosh(d*x
 + c)^6 + 6*(a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^3*b^3
+ a^2*b^4)*d*sinh(d*x + c)^6 + (3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^4 + (1
5*(a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^2 + (3*a^3*b^3 - a^2*b^4)*d)*sinh(d*x
 + c)^4 + (3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^2 + 4*(5*(a^3*b^3 + a^2*b^4
)*d*cosh(d*x + c)^3 + (3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^
3 + (15*(a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^4 + 6*(3*a^3*b^3 - a^2*b^4)*d*c
osh(d*x + c)^2 + (3*a^3*b^3 - a^2*b^4)*d)*sinh(d*x + c)^2 + (a^3*b^3 + a^2*
b^4)*d + 2*(3*(a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^5 + 2*(3*a^3*b^3 - a^2*b^
4)*d*cosh(d*x + c)^3 + (3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c))*sinh(d*x + c)
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**6/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.66475, size = 335, normalized size = 3.45

$$-\frac{\frac{\left(3\,a^2e^{(2\,c)}+2\,abe^{(2\,c)}-b^2e^{(2\,c)}\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)e^{(-2\,c)}}{\sqrt{ab}ab^2}+\frac{2\left(3\,a^2e^{(4\,dx+4\,c)}+2\,abe^{(4\,dx+4\,c)}-b^2e^{(4\,dx+4\,c)}+6\,a^2e^{(2\,dx+2\,c)}-2\,abe^{(2\,dx+2\,c)}+3\,a^2+\right.}{\left(ae^{(6\,dx+6\,c)}+be^{(6\,dx+6\,c)}+3\,ae^{(4\,dx+4\,c)}-be^{(4\,dx+4\,c)}+3\,ae^{(2\,dx+2\,c)}-be^{(2\,dx+2\,c)}+a\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((3*a^2*e^(2*c) + 2*a*b*e^(2*c) - b^2*e^(2*c))*arctan(1/2*(a*e^(2*d*x
+ 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))*e^(-2*c)/(sqrt(a*b)*a*b^2) +
 2*(3*a^2*e^(4*d*x + 4*c) + 2*a*b*e^(4*d*x + 4*c) - b^2*e^(4*d*x + 4*c) + 6
*a^2*e^(2*d*x + 2*c) - 2*a*b*e^(2*d*x + 2*c) + 3*a^2 + 4*a*b + b^2)/((a*e^(
6*d*x + 6*c) + b*e^(6*d*x + 6*c) + 3*a*e^(4*d*x + 4*c) - b*e^(4*d*x + 4*c)
+ 3*a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) + a + b)*a*b^2))/d
```

3.124 $\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=155

$$-\frac{(4a-b)(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^3d}+\frac{(2a+b)(a+b)\sinh(c+dx)}{2ab^2d\left((a+b)\sinh^2(c+dx)+a\right)}+\frac{(4a+5b)\tan^{-1}(\sinh(c+dx))}{2b^3d}-\frac{\tanh\cdots}{2bd\left((a\cdots\right.}$$

```
[Out] ((4*a + 5*b)*ArcTan[Sinh[c + d*x]])/(2*b^3*d) - ((4*a - b)*(a + b)^(3/2)*Ar
cTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^3*d) + ((a + b)*(2*
a + b)*Sinh[c + d*x])/(2*a*b^2*d*(a + (a + b)*Sinh[c + d*x]^2)) - (Sech[c +
 d*x]*Tanh[c + d*x])/(2*b*d*(a + (a + b)*Sinh[c + d*x]^2))
```

Rubi [A] time = 0.24918, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3676, 414, 527, 522, 203, 205}

$$-\frac{(4a-b)(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^3d}+\frac{(2a+b)(a+b)\sinh(c+dx)}{2ab^2d\left((a+b)\sinh^2(c+dx)+a\right)}+\frac{(4a+5b)\tan^{-1}(\sinh(c+dx))}{2b^3d}-\frac{\tanh\cdots}{2bd\left((a\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^7/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((4*a + 5*b)*ArcTan[Sinh[c + d*x]])/(2*b^3*d) - ((4*a - b)*(a + b)^(3/2)*Ar
cTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^3*d) + ((a + b)*(2*
a + b)*Sinh[c + d*x])/(2*a*b^2*d*(a + (a + b)*Sinh[c + d*x]^2)) - (Sech[c +
 d*x]*Tanh[c + d*x])/(2*b*d*(a + (a + b)*Sinh[c + d*x]^2))
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^2\left(a+(a+b)x^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2bd\left(a+(a+b)\sinh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{a+2b-3(a+b)x^2}{\left(1+x^2\right)\left(a+(a+b)x^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{2bd}$$

$$= \frac{(a+b)(2a+b)\sinh(c+dx)}{2ab^2d\left(a+(a+b)\sinh^2(c+dx)\right)} - \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2bd\left(a+(a+b)\sinh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{2\left(2a^2\right.}{}\right.}{}$$

$$= \frac{(a+b)(2a+b)\sinh(c+dx)}{2ab^2d\left(a+(a+b)\sinh^2(c+dx)\right)} - \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2bd\left(a+(a+b)\sinh^2(c+dx)\right)} - \frac{\left((4a-b)(a+b\right.}{}$$

$$= \frac{(4a+5b)\tan^{-1}(\sinh(c+dx))}{2b^3d} - \frac{(4a-b)(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^3d} + \frac{(a+b)}{2ab^2d\left(a\right.}$$

Mathematica [A] time = 1.30308, size = 265, normalized size = 1.71

$$\frac{(a-b)\left(2a^{3/2}(4a+5b)\sqrt{a+b}\tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+a^{3/2}b\sqrt{a+b}\tanh(c+dx)\operatorname{sech}(c+dx)+(4a-b)(a+b)^2\operatorname{ta}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^7/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (2*Sqrt[a]*b*(a + b)^(5/2)*Sinh[c + d*x] + (a - b)*((4*a - b)*(a + b)^2*Arc
Tan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]] + 2*a^(3/2)*Sqrt[a + b]*(4*a + 5*b
)*ArcTan[Tanh[(c + d*x)/2]] + a^(3/2)*b*Sqrt[a + b]*Sech[c + d*x]*Tanh[c +
d*x]) + (a + b)*Cosh[2*(c + d*x)]*((4*a - b)*(a + b)^2*ArcTan[(Sqrt[a]*Csch
[c + d*x])/Sqrt[a + b]] + 2*a^(3/2)*Sqrt[a + b]*(4*a + 5*b)*ArcTan[Tanh[(c
+ d*x)/2]] + a^(3/2)*b*Sqrt[a + b]*Sech[c + d*x]*Tanh[c + d*x]))/(2*a^(3/2)
```

```
*b^3*Sqrt[a + b]*d*(a - b + (a + b)*Cosh[2*(c + d*x)]))
```

Maple [B] time = 0.102, size = 1477, normalized size = 9.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] -2/d/b^2*a^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*
tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-2/d/b^2*a^2/(b*(a+
b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/
((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/2/d/(b*(a+b))^(1/2)/a/((2*(b*(a+b))^
(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2
*b)*a)^(1/2))*b-2/d/b^3*a^2/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*ta
nh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+5/d/b^2*arctan(tanh(
1/2*d*x+1/2*c))-1/d/b^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+
4*tanh(1/2*d*x+1/2*c)^2*b+a)*a*tanh(1/2*d*x+1/2*c)^3-2/d/(tanh(1/2*d*x+1/2*
c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/b*tanh(1/2*d*
x+1/2*c)^3+2/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/
2*d*x+1/2*c)^2*b+a)/b*tanh(1/2*d*x+1/2*c)+1/d/b/((2*(b*(a+b))^(1/2)-a-2*b)*
a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))
-1/d/b/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2
*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/2/d/(b*(a+b))^(1/2)/a/((2*(b*(a+b))^(1/
2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*
a)^(1/2))*b-1/d/b^2/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d*x+1/2*c)^3+1/d/b
^2/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d*x+1/2*c)+4/d/b^3*arctan(tanh(1/2*
d*x+1/2*c))*a-7/2/d/b^2*a/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh
(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/d/(tanh(1/2*d*x+1/2*
c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*
x+1/2*c)^3+1/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/
2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)-1/d/(b*(a+b))^(1/2)/((2*(b*(a+b))
^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-
2*b)*a)^(1/2))-1/2/d/a/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1
/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-1/d/(b*(a+b))^(1/2)/((2*
(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^
(1/2)+a+2*b)*a)^(1/2))+1/2/d/a/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a
*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-7/2/d/b*a/(b*(a+b
))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/
((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-7/2/d/b*a/(b*(a+b))^(1/2)/((2*(b*(a+b)
)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+
2*b)*a)^(1/2))+7/2/d/b^2*a/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*ta
nh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/d/b^2/(tanh(1/2*d*
x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*a*tanh(
1/2*d*x+1/2*c)+2/d/b^3*a^2/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*ta
nh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(2\,a^{2}e^{(7\,c)}+3\,abe^{(7\,c)}+b^{2}e^{(7\,c)}\right)e^{(7\,dx)}+\left(2\,a^{2}e^{(5\,c)}-abe^{(5\,c)}+b^{2}e^{(5\,c)}\right)e^{(5\,dx)}-\left(2\,a^{2}e^{(3\,c)}-abe^{(3\,c)}+b^{2}e^{(3\,c)}\right)e^{(3\,dx)}-\left(2\,a^{2}e^{c}+\right.}{4\,a^{2}b^{2}de^{(6\,dx+6\,c)}+4\,a^{2}b^{2}de^{(2\,dx+2\,c)}+a^{2}b^{2}d+ab^{3}d+\left(a^{2}b^{2}de^{(8\,c)}+ab^{3}de^{(8\,c)}\right)e^{(8\,dx)}+2\left(3\,a^{2}b^{2}de^{(4\,c)}-ab^{3}de^{(4\,c)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] ((2*a^2*e^(7*c) + 3*a*b*e^(7*c) + b^2*e^(7*c))*e^(7*d*x) + (2*a^2*e^(5*c) -
 a*b*e^(5*c) + b^2*e^(5*c))*e^(5*d*x) - (2*a^2*e^(3*c) - a*b*e^(3*c) + b^2*
e^(3*c))*e^(3*d*x) - (2*a^2*e^c + 3*a*b*e^c + b^2*e^c)*e^(d*x))/(4*a^2*b^2*
d*e^(6*d*x + 6*c) + 4*a^2*b^2*d*e^(2*d*x + 2*c) + a^2*b^2*d + a*b^3*d + (a^
2*b^2*d*e^(8*c) + a*b^3*d*e^(8*c))*e^(8*d*x) + 2*(3*a^2*b^2*d*e^(4*c) - a*b
^3*d*e^(4*c))*e^(4*d*x)) + (4*a*e^c + 5*b*e^c)*arctan(e^(d*x + c))*e^(-c)/(
b^3*d) - 128*integrate(1/128*((4*a^3*e^(3*c) + 7*a^2*b*e^(3*c) + 2*a*b^2*e^
(3*c) - b^3*e^(3*c))*e^(3*d*x) + (4*a^3*e^c + 7*a^2*b*e^c + 2*a*b^2*e^c - b
^3*e^c)*e^(d*x))/(a^2*b^3 + a*b^4 + (a^2*b^3*e^(4*c) + a*b^4*e^(4*c))*e^(4*
d*x) + 2*(a^2*b^3*e^(2*c) - a*b^4*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 3.93312, size = 15050, normalized size = 97.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(2*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 28*(2*a^2*b + 3*a*b^2 +
 b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + 4*(2*a^2*b + 3*a*b^2 + b^3)*sinh(d*x
+ c)^7 + 4*(2*a^2*b - a*b^2 + b^3)*cosh(d*x + c)^5 + 4*(2*a^2*b - a*b^2 + b
^3 + 21*(2*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(7*
(2*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (2*a^2*b - a*b^2 + b^3)*cosh(d*
x + c))*sinh(d*x + c)^4 - 4*(2*a^2*b - a*b^2 + b^3)*cosh(d*x + c)^3 + 4*(35
*(2*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 - 2*a^2*b + a*b^2 - b^3 + 10*(2*
a^2*b - a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 4*(21*(2*a^2*b + 3*
a*b^2 + b^3)*cosh(d*x + c)^5 + 10*(2*a^2*b - a*b^2 + b^3)*cosh(d*x + c)^3 -
 3*(2*a^2*b - a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - ((4*a^3 + 7*a^2
*b + 2*a*b^2 - b^3)*cosh(d*x + c)^8 + 8*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*c
osh(d*x + c)*sinh(d*x + c)^7 + (4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*sinh(d*x +
 c)^8 + 4*(4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)^6 + 4*(4*a^3 + 3*a^2*b -
a*b^2 + 7*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^
6 + 8*(7*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c)^3 + 3*(4*a^3 + 3*a
^2*b - a*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(12*a^3 + 5*a^2*b - 6*a*b^
2 + b^3)*cosh(d*x + c)^4 + 2*(35*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d*x
 + c)^4 + 12*a^3 + 5*a^2*b - 6*a*b^2 + b^3 + 30*(4*a^3 + 3*a^2*b - a*b^2)*c
osh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*co
sh(d*x + c)^5 + 10*(4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)^3 + (12*a^3 + 5*
a^2*b - 6*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*a^3 + 7*a^2*b + 2
*a*b^2 - b^3 + 4*(4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)^2 + 4*(7*(4*a^3 +
7*a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c)^6 + 15*(4*a^3 + 3*a^2*b - a*b^2)*cos
h(d*x + c)^4 + 4*a^3 + 3*a^2*b - a*b^2 + 3*(12*a^3 + 5*a^2*b - 6*a*b^2 + b^
3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*
cosh(d*x + c)^7 + 3*(4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)^5 + (12*a^3 + 5
*a^2*b - 6*a*b^2 + b^3)*cosh(d*x + c)^3 + (4*a^3 + 3*a^2*b - a*b^2)*cosh(d*
x + c))*sinh(d*x + c))*sqrt(-(a + b)/a)*log(((a + b)*cosh(d*x + c)^4 + 4*(a
 + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)
*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2
+ 4*((a + b)*cosh(d*x + c)^3 - (3*a + b)*cosh(d*x + c))*sinh(d*x + c) + 4*(
a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 -
 a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 - a)*sinh(d*x + c))*sqrt(-(a + b)/a
) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)
^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cos
h(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b
```

```
)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 4*((4*a^3 + 9*a^2*b + 5*a*b^2)*c
osh(d*x + c)^8 + 8*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^
7 + (4*a^3 + 9*a^2*b + 5*a*b^2)*sinh(d*x + c)^8 + 4*(4*a^3 + 5*a^2*b)*cosh(
d*x + c)^6 + 4*(4*a^3 + 5*a^2*b + 7*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x +
c)^2)*sinh(d*x + c)^6 + 8*(7*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^3 +
3*(4*a^3 + 5*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(12*a^3 + 11*a^2*b -
 5*a*b^2)*cosh(d*x + c)^4 + 2*(35*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)
^4 + 12*a^3 + 11*a^2*b - 5*a*b^2 + 30*(4*a^3 + 5*a^2*b)*cosh(d*x + c)^2)*si
nh(d*x + c)^4 + 8*(7*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^5 + 10*(4*a^
3 + 5*a^2*b)*cosh(d*x + c)^3 + (12*a^3 + 11*a^2*b - 5*a*b^2)*cosh(d*x + c))
*sinh(d*x + c)^3 + 4*a^3 + 9*a^2*b + 5*a*b^2 + 4*(4*a^3 + 5*a^2*b)*cosh(d*x
 + c)^2 + 4*(7*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^6 + 15*(4*a^3 + 5*
a^2*b)*cosh(d*x + c)^4 + 4*a^3 + 5*a^2*b + 3*(12*a^3 + 11*a^2*b - 5*a*b^2)*
cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x
+ c)^7 + 3*(4*a^3 + 5*a^2*b)*cosh(d*x + c)^5 + (12*a^3 + 11*a^2*b - 5*a*b^2
)*cosh(d*x + c)^3 + (4*a^3 + 5*a^2*b)*cosh(d*x + c))*sinh(d*x + c))*arctan(
cosh(d*x + c) + sinh(d*x + c)) - 4*(2*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)
+ 4*(7*(2*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 5*(2*a^2*b - a*b^2 + b^3
)*cosh(d*x + c)^4 - 2*a^2*b - 3*a*b^2 - b^3 - 3*(2*a^2*b - a*b^2 + b^3)*cos
h(d*x + c)^2)*sinh(d*x + c))/(4*a^2*b^3*d*cosh(d*x + c)^6 + (a^2*b^3 + a*b^
4)*d*cosh(d*x + c)^8 + 8*(a^2*b^3 + a*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^7
+ (a^2*b^3 + a*b^4)*d*sinh(d*x + c)^8 + 4*a^2*b^3*d*cosh(d*x + c)^2 + 4*(a^
2*b^3*d + 7*(a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 2*(3*a^2
*b^3 - a*b^4)*d*cosh(d*x + c)^4 + 8*(3*a^2*b^3*d*cosh(d*x + c) + 7*(a^2*b^3
 + a*b^4)*d*cosh(d*x + c)^3)*sinh(d*x + c)^5 + 2*(30*a^2*b^3*d*cosh(d*x + c
)^2 + 35*(a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4 + (3*a^2*b^3 - a*b^4)*d)*sinh(
d*x + c)^4 + 8*(10*a^2*b^3*d*cosh(d*x + c)^3 + 7*(a^2*b^3 + a*b^4)*d*cosh(d
*x + c)^5 + (3*a^2*b^3 - a*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(15*a^
2*b^3*d*cosh(d*x + c)^4 + 7*(a^2*b^3 + a*b^4)*d*cosh(d*x + c)^6 + a^2*b^3*d
 + 3*(3*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^2)*sinh(d*x + c)^2 + (a^2*b^3 + a*
b^4)*d + 8*(3*a^2*b^3*d*cosh(d*x + c)^5 + (a^2*b^3 + a*b^4)*d*cosh(d*x + c)
^7 + a^2*b^3*d*cosh(d*x + c) + (3*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^3)*sinh(
d*x + c)), 1/2*(2*(2*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 14*(2*a^2*b +
 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + 2*(2*a^2*b + 3*a*b^2 + b^3)
*sinh(d*x + c)^7 + 2*(2*a^2*b - a*b^2 + b^3)*cosh(d*x + c)^5 + 2*(2*a^2*b -
 a*b^2 + b^3 + 21*(2*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^
5 + 10*(7*(2*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (2*a^2*b - a*b^2 + b^
3)*cosh(d*x + c))*sinh(d*x + c)^4 - 2*(2*a^2*b - a*b^2 + b^3)*cosh(d*x + c)
^3 + 2*(35*(2*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 - 2*a^2*b + a*b^2 - b^
3 + 10*(2*a^2*b - a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(21*(2*
a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 10*(2*a^2*b - a*b^2 + b^3)*cosh(d*
x + c)^3 - 3*(2*a^2*b - a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - ((4*a
^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c)^8 + 8*(4*a^3 + 7*a^2*b + 2*a*b^
2 - b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*
sinh(d*x + c)^8 + 4*(4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)^6 + 4*(4*a^3 +
3*a^2*b - a*b^2 + 7*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh
(d*x + c)^6 + 8*(7*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c)^3 + 3*(4
*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(12*a^3 + 5*a^2*
b - 6*a*b^2 + b^3)*cosh(d*x + c)^4 + 2*(35*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3
)*cosh(d*x + c)^4 + 12*a^3 + 5*a^2*b - 6*a*b^2 + b^3 + 30*(4*a^3 + 3*a^2*b
- a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(4*a^3 + 7*a^2*b + 2*a*b^2
 - b^3)*cosh(d*x + c)^5 + 10*(4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)^3 + (1
2*a^3 + 5*a^2*b - 6*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*a^3 + 7
*a^2*b + 2*a*b^2 - b^3 + 4*(4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)^2 + 4*(7
*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c)^6 + 15*(4*a^3 + 3*a^2*b -
a*b^2)*cosh(d*x + c)^4 + 4*a^3 + 3*a^2*b - a*b^2 + 3*(12*a^3 + 5*a^2*b - 6*
a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((4*a^3 + 7*a^2*b + 2*a*b
^2 - b^3)*cosh(d*x + c)^7 + 3*(4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)^5 + (
12*a^3 + 5*a^2*b - 6*a*b^2 + b^3)*cosh(d*x + c)^3 + (4*a^3 + 3*a^2*b - a*b^
```

```
2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a + b)/a)*arctan(1/2*sqrt((a + b)/a)
*(cosh(d*x + c) + sinh(d*x + c))) - ((4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh
(d*x + c)^8 + 8*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x +
c)^7 + (4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*sinh(d*x + c)^8 + 4*(4*a^3 + 3*a^2
*b - a*b^2)*cosh(d*x + c)^6 + 4*(4*a^3 + 3*a^2*b - a*b^2 + 7*(4*a^3 + 7*a^2
*b + 2*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(4*a^3 + 7*a^2*
b + 2*a*b^2 - b^3)*cosh(d*x + c)^3 + 3*(4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x +
 c))*sinh(d*x + c)^5 + 2*(12*a^3 + 5*a^2*b - 6*a*b^2 + b^3)*cosh(d*x + c)^4
 + 2*(35*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c)^4 + 12*a^3 + 5*a^2
*b - 6*a*b^2 + b^3 + 30*(4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)^2)*sinh(d*x
 + c)^4 + 8*(7*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c)^5 + 10*(4*a^
3 + 3*a^2*b - a*b^2)*cosh(d*x + c)^3 + (12*a^3 + 5*a^2*b - 6*a*b^2 + b^3)*c
osh(d*x + c))*sinh(d*x + c)^3 + 4*a^3 + 7*a^2*b + 2*a*b^2 - b^3 + 4*(4*a^3
+ 3*a^2*b - a*b^2)*cosh(d*x + c)^2 + 4*(7*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)
*cosh(d*x + c)^6 + 15*(4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)^4 + 4*a^3 + 3
*a^2*b - a*b^2 + 3*(12*a^3 + 5*a^2*b - 6*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh
(d*x + c)^2 + 8*((4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c)^7 + 3*(4*a
^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)^5 + (12*a^3 + 5*a^2*b - 6*a*b^2 + b^3)*
cosh(d*x + c)^3 + (4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c))*sinh(d*x + c))*s
qrt((a + b)/a)*arctan(1/2*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c
)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (3*a - b)*cosh(d*x + c) + (3*
(a + b)*cosh(d*x + c)^2 + 3*a - b)*sinh(d*x + c))*sqrt((a + b)/a)/(a + b))
+ 2*((4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^8 + 8*(4*a^3 + 9*a^2*b + 5*a
*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (4*a^3 + 9*a^2*b + 5*a*b^2)*sinh(d*x
+ c)^8 + 4*(4*a^3 + 5*a^2*b)*cosh(d*x + c)^6 + 4*(4*a^3 + 5*a^2*b + 7*(4*a^
3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(4*a^3 + 9*a
^2*b + 5*a*b^2)*cosh(d*x + c)^3 + 3*(4*a^3 + 5*a^2*b)*cosh(d*x + c))*sinh(d
*x + c)^5 + 2*(12*a^3 + 11*a^2*b - 5*a*b^2)*cosh(d*x + c)^4 + 2*(35*(4*a^3
+ 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^4 + 12*a^3 + 11*a^2*b - 5*a*b^2 + 30*(4*
a^3 + 5*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(4*a^3 + 9*a^2*b + 5
*a*b^2)*cosh(d*x + c)^5 + 10*(4*a^3 + 5*a^2*b)*cosh(d*x + c)^3 + (12*a^3 +
11*a^2*b - 5*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*a^3 + 9*a^2*b + 5*a*
b^2 + 4*(4*a^3 + 5*a^2*b)*cosh(d*x + c)^2 + 4*(7*(4*a^3 + 9*a^2*b + 5*a*b^2
)*cosh(d*x + c)^6 + 15*(4*a^3 + 5*a^2*b)*cosh(d*x + c)^4 + 4*a^3 + 5*a^2*b
+ 3*(12*a^3 + 11*a^2*b - 5*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((4*
a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^7 + 3*(4*a^3 + 5*a^2*b)*cosh(d*x + c
)^5 + (12*a^3 + 11*a^2*b - 5*a*b^2)*cosh(d*x + c)^3 + (4*a^3 + 5*a^2*b)*cos
h(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - 2*(2*a^2
*b + 3*a*b^2 + b^3)*cosh(d*x + c) + 2*(7*(2*a^2*b + 3*a*b^2 + b^3)*cosh(d*x
 + c)^6 + 5*(2*a^2*b - a*b^2 + b^3)*cosh(d*x + c)^4 - 2*a^2*b - 3*a*b^2 - b
^3 - 3*(2*a^2*b - a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(4*a^2*b^3*d
*cosh(d*x + c)^6 + (a^2*b^3 + a*b^4)*d*cosh(d*x + c)^8 + 8*(a^2*b^3 + a*b^4
)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2*b^3 + a*b^4)*d*sinh(d*x + c)^8 + 4
*a^2*b^3*d*cosh(d*x + c)^2 + 4*(a^2*b^3*d + 7*(a^2*b^3 + a*b^4)*d*cosh(d*x
+ c)^2)*sinh(d*x + c)^6 + 2*(3*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^4 + 8*(3*a^
2*b^3*d*cosh(d*x + c) + 7*(a^2*b^3 + a*b^4)*d*cosh(d*x + c)^3)*sinh(d*x + c
)^5 + 2*(30*a^2*b^3*d*cosh(d*x + c)^2 + 35*(a^2*b^3 + a*b^4)*d*cosh(d*x + c
)^4 + (3*a^2*b^3 - a*b^4)*d)*sinh(d*x + c)^4 + 8*(10*a^2*b^3*d*cosh(d*x + c
)^3 + 7*(a^2*b^3 + a*b^4)*d*cosh(d*x + c)^5 + (3*a^2*b^3 - a*b^4)*d*cosh(d*
x + c))*sinh(d*x + c)^3 + 4*(15*a^2*b^3*d*cosh(d*x + c)^4 + 7*(a^2*b^3 + a*
b^4)*d*cosh(d*x + c)^6 + a^2*b^3*d + 3*(3*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^
2)*sinh(d*x + c)^2 + (a^2*b^3 + a*b^4)*d + 8*(3*a^2*b^3*d*cosh(d*x + c)^5 +
 (a^2*b^3 + a*b^4)*d*cosh(d*x + c)^7 + a^2*b^3*d*cosh(d*x + c) + (3*a^2*b^3
 - a*b^4)*d*cosh(d*x + c)^3)*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**7/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [C] time = 2.15842, size = 6431, normalized size = 41.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/8*(2*(3*(4*a^4*b^2 + 11*a^3*b^3 + 9*a^2*b^4 + a*b^5 - b^6)*cos(1/2*real_
part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b
) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) - (4*
a^4*b^2 + 11*a^3*b^3 + 9*a^2*b^4 + a*b^5 - b^6)*cosh(1/2*imag_part(arccos(-
a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))
))^3 - 9*(4*a^4*b^2 + 11*a^3*b^3 + 9*a^2*b^4 + a*b^5 - b^6)*cos(1/2*real_pa
rt(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b)
+ b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/
2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(4*a^4*b^2 + 11*a^3*b^3 +
9*a^2*b^4 + a*b^5 - b^6)*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))
)^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part
(arccos(-a/(a + b) + b/(a + b)))) + 9*(4*a^4*b^2 + 11*a^3*b^3 + 9*a^2*b^4 +
 a*b^5 - b^6)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2
*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a
+ b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 -
 3*(4*a^4*b^2 + 11*a^3*b^3 + 9*a^2*b^4 + a*b^5 - b^6)*cosh(1/2*imag_part(ar
ccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a +
 b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(4*a^4*b
^2 + 11*a^3*b^3 + 9*a^2*b^4 + a*b^5 - b^6)*cos(1/2*real_part(arccos(-a/(a +
 b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sin
h(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (4*a^4*b^2 + 11*a^3*b^
3 + 9*a^2*b^4 + a*b^5 - b^6)*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b
))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (4*a^4*b^2 +
 11*a^3*b^3 + 9*a^2*b^4 + a*b^5 - b^6)*cosh(1/2*imag_part(arccos(-a/(a + b)
 + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) - (4*a^4
*b^2 + 11*a^3*b^3 + 9*a^2*b^4 + a*b^5 - b^6)*sin(1/2*real_part(arccos(-a/(a
 + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*a
rctan((((a^2*b^3 + a*b^4)/(a^2*b^3*e^(4*c) + a*b^4*e^(4*c)))^(1/4)*cos(1/2*
arccos(-(a - b)/(a + b))) + e^(d*x))/(((a^2*b^3 + a*b^4)/(a^2*b^3*e^(4*c) +
 a*b^4*e^(4*c)))^(1/4)*sin(1/2*arccos(-(a - b)/(a + b)))))/(2*a^2*b^6 + (a*
b^2 - b^3)*sqrt(-a*b)*b^2*abs(a)*abs(b)) + 2*(3*(4*a^4*b^2 + 11*a^3*b^3 + 9
*a^2*b^4 + a*b^5 - b^6)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^
2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(a
rccos(-a/(a + b) + b/(a + b)))) - (4*a^4*b^2 + 11*a^3*b^3 + 9*a^2*b^4 + a*b
^5 - b^6)*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*rea
l_part(arccos(-a/(a + b) + b/(a + b))))^3 - 9*(4*a^4*b^2 + 11*a^3*b^3 + 9*a
^2*b^4 + a*b^5 - b^6)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*
cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arc
cos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a +
 b)))) + 3*(4*a^4*b^2 + 11*a^3*b^3 + 9*a^2*b^4 + a*b^5 - b^6)*cosh(1/2*imag
_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b
) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*
(4*a^4*b^2 + 11*a^3*b^3 + 9*a^2*b^4 + a*b^5 - b^6)*cos(1/2*real_part(arccos
```

```
(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a +
b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part
(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(4*a^4*b^2 + 11*a^3*b^3 + 9*a^2*b^4
 + a*b^5 - b^6)*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2
*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/
(a + b) + b/(a + b))))^2 - 3*(4*a^4*b^2 + 11*a^3*b^3 + 9*a^2*b^4 + a*b^5 -
b^6)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part
(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/
(a + b))))^3 + (4*a^4*b^2 + 11*a^3*b^3 + 9*a^2*b^4 + a*b^5 - b^6)*sin(1/2*r
eal_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a
 + b) + b/(a + b))))^3 + (4*a^4*b^2 + 11*a^3*b^3 + 9*a^2*b^4 + a*b^5 - b^6)
*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arcc
os(-a/(a + b) + b/(a + b)))) - (4*a^4*b^2 + 11*a^3*b^3 + 9*a^2*b^4 + a*b^5
- b^6)*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_par
t(arccos(-a/(a + b) + b/(a + b)))))*arctan(-(((a^2*b^3 + a*b^4)/(a^2*b^3*e^
(4*c) + a*b^4*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b))) - e^(d*x))/
(((a^2*b^3 + a*b^4)/(a^2*b^3*e^(4*c) + a*b^4*e^(4*c)))^(1/4)*sin(1/2*arccos
(-(a - b)/(a + b)))))/(2*a^2*b^6 + (a*b^2 - b^3)*sqrt(-a*b)*b^2*abs(a)*abs(
b)) + ((4*a^4*b^2 + 11*a^3*b^3 + 9*a^2*b^4 + a*b^5 - b^6)*cos(1/2*real_part
(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) +
b/(a + b))))^3 - 3*(4*a^4*b^2 + 11*a^3*b^3 + 9*a^2*b^4 + a*b^5 - b^6)*cos(1
/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/
(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))
^2 - 3*(4*a^4*b^2 + 11*a^3*b^3 + 9*a^2*b^4 + a*b^5 - b^6)*cos(1/2*real_part
(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) +
b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(4*a
^4*b^2 + 11*a^3*b^3 + 9*a^2*b^4 + a*b^5 - b^6)*cos(1/2*real_part(arccos(-a/
(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^
2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(a
rccos(-a/(a + b) + b/(a + b)))) + 3*(4*a^4*b^2 + 11*a^3*b^3 + 9*a^2*b^4 + a
*b^5 - b^6)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*i
mag_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a +
 b) + b/(a + b))))^2 - 9*(4*a^4*b^2 + 11*a^3*b^3 + 9*a^2*b^4 + a*b^5 - b^6)
*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arcc
os(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b
))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - (4*a^4*b^2 +
 11*a^3*b^3 + 9*a^2*b^4 + a*b^5 - b^6)*cos(1/2*real_part(arccos(-a/(a + b)
+ b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + 3*
(4*a^4*b^2 + 11*a^3*b^3 + 9*a^2*b^4 + a*b^5 - b^6)*cos(1/2*real_part(arccos
(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))
))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (4*a^4*b^2 + 1
1*a^3*b^3 + 9*a^2*b^4 + a*b^5 - b^6)*cos(1/2*real_part(arccos(-a/(a + b) +
b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) - (4*a^4*b
^2 + 11*a^3*b^3 + 9*a^2*b^4 + a*b^5 - b^6)*cos(1/2*real_part(arccos(-a/(a +
 b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*log
(2*((a^2*b^3 + a*b^4)/(a^2*b^3*e^(4*c) + a*b^4*e^(4*c)))^(1/4)*cos(1/2*arcc
os(-(a - b)/(a + b)))*e^(d*x) + sqrt((a^2*b^3 + a*b^4)/(a^2*b^3*e^(4*c) + a
*b^4*e^(4*c))) + e^(2*d*x))/(2*a^2*b^6 + (a*b^2 - b^3)*sqrt(-a*b)*b^2*abs(a
)*abs(b)) - ((4*a^4*b^2 + 11*a^3*b^3 + 9*a^2*b^4 + a*b^5 - b^6)*cos(1/2*rea
l_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a +
 b) + b/(a + b))))^3 - 3*(4*a^4*b^2 + 11*a^3*b^3 + 9*a^2*b^4 + a*b^5 - b^6)
*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arcc
os(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a +
 b))))^2 - 3*(4*a^4*b^2 + 11*a^3*b^3 + 9*a^2*b^4 + a*b^5 - b^6)*cos(1/2*rea
l_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a +
 b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) +
9*(4*a^4*b^2 + 11*a^3*b^3 + 9*a^2*b^4 + a*b^5 - b^6)*cos(1/2*real_part(arcc
os(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a +
b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_
```

```
part(arccos(-a/(a + b) + b/(a + b)))) + 3*(4*a^4*b^2 + 11*a^3*b^3 + 9*a^2*b
^4 + a*b^5 - b^6)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh
(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-
a/(a + b) + b/(a + b))))^2 - 9*(4*a^4*b^2 + 11*a^3*b^3 + 9*a^2*b^4 + a*b^5
- b^6)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_par
t(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/
(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - (4*a^4
*b^2 + 11*a^3*b^3 + 9*a^2*b^4 + a*b^5 - b^6)*cos(1/2*real_part(arccos(-a/(a
 + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^
3 + 3*(4*a^4*b^2 + 11*a^3*b^3 + 9*a^2*b^4 + a*b^5 - b^6)*cos(1/2*real_part(
arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a
 + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (4*a^4*b
^2 + 11*a^3*b^3 + 9*a^2*b^4 + a*b^5 - b^6)*cos(1/2*real_part(arccos(-a/(a +
 b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) - (4
*a^4*b^2 + 11*a^3*b^3 + 9*a^2*b^4 + a*b^5 - b^6)*cos(1/2*real_part(arccos(-
a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))
))*log(-2*((a^2*b^3 + a*b^4)/(a^2*b^3*e^(4*c) + a*b^4*e^(4*c)))^(1/4)*cos(1
/2*arccos(-(a - b)/(a + b)))*e^(d*x) + sqrt((a^2*b^3 + a*b^4)/(a^2*b^3*e^(4
*c) + a*b^4*e^(4*c))) + e^(2*d*x))/(2*a^2*b^6 + (a*b^2 - b^3)*sqrt(-a*b)*b^
2*abs(a)*abs(b)) - 8*(4*a*e^c + 5*b*e^c)*arctan(e^(d*x + c))*e^(-c)/b^3 - 8
*(a^2*e^(3*d*x + 3*c) + 2*a*b*e^(3*d*x + 3*c) + b^2*e^(3*d*x + 3*c) - a^2*e
^(d*x + c) - 2*a*b*e^(d*x + c) - b^2*e^(d*x + c))/((a*e^(4*d*x + 4*c) + b*e
^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)*a*b^2)
- 8*(e^(3*d*x + 3*c) - e^(d*x + c))/(b^2*(e^(2*d*x + 2*c) + 1)^2))/d
```

3.125 $\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=198

$$\frac{b^{3/2}\left(35a^2+14ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a+b)^4}-\frac{b(a-3b)(4a+b)\tanh(c+dx)}{8a^2d(a+b)^3\left(a+b\tanh^2(c+dx)\right)}-\frac{b(2a-b)\tanh(c+dx)}{4ad(a+b)^2\left(a+b\tanh^2(c+dx\right.}$$

```
[Out] ((a + 7*b)*x)/(2*(a + b)^4) + (b^(3/2)*(35*a^2 + 14*a*b + 3*b^2)*ArcTan[(Sq
rt[b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*(a + b)^4*d) + (Cosh[c + d*x]*Sin
h[c + d*x])/(2*(a + b)*d*(a + b*Tanh[c + d*x]^2)^2) - ((2*a - b)*b*Tanh[c +
 d*x])/(4*a*(a + b)^2*d*(a + b*Tanh[c + d*x]^2)^2) - ((a - 3*b)*b*(4*a + b)
*Tanh[c + d*x])/(8*a^2*(a + b)^3*d*(a + b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.309667, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3675, 414, 527, 522, 206, 205}

$$\frac{b^{3/2}\left(35a^2+14ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a+b)^4}-\frac{b(a-3b)(4a+b)\tanh(c+dx)}{8a^2d(a+b)^3\left(a+b\tanh^2(c+dx)\right)}-\frac{b(2a-b)\tanh(c+dx)}{4ad(a+b)^2\left(a+b\tanh^2(c+dx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((a + 7*b)*x)/(2*(a + b)^4) + (b^(3/2)*(35*a^2 + 14*a*b + 3*b^2)*ArcTan[(Sq
rt[b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*(a + b)^4*d) + (Cosh[c + d*x]*Sin
h[c + d*x])/(2*(a + b)*d*(a + b*Tanh[c + d*x]^2)^2) - ((2*a - b)*b*Tanh[c +
 d*x])/(4*a*(a + b)^2*d*(a + b*Tanh[c + d*x]^2)^2) - ((a - 3*b)*b*(4*a + b)
*Tanh[c + d*x])/(8*a^2*(a + b)^3*d*(a + b*Tanh[c + d*x]^2))
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\left(a+bx^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{a+2b+5bx^2}{\left(1-x^2\right)\left(a+bx^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{2(a+b)d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(2a-b)b\tanh(c+dx)}{4a(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-2(2a^{\cdots}}{\cdots}\right.}{\cdots}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(2a-b)b\tanh(c+dx)}{4a(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(a-3b)b(4a\cdots}{8a^2(a+b)^3d\left(a\cdots\right.}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(2a-b)b\tanh(c+dx)}{4a(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(a-3b)b(4a\cdots}{8a^2(a+b)^3d\left(a\cdots\right.}$$

$$= \frac{(a+7b)x}{2(a+b)^4} + \frac{b^{3/2}\left(35a^2+14ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^4d} + \frac{\cosh(c+dx)\sinh(c+d\cdots}{2(a+b)d\left(a+b\tanh^2(c+\cdots\right.}$$

Mathematica [A] time = 1.27913, size = 164, normalized size = 0.83

$$\frac{\frac{b^{3/2}\left(35a^2+14ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}} + \frac{b^2(a+b)(13a+3b)\sinh(2(c+dx))}{a^2((a+b)\cosh(2(c+dx))+a-b)} + \frac{4b^3(a+b)\sinh(2(c+dx))}{a((a+b)\cosh(2(c+dx))+a-b)^2} + 4(a+7b)(c+dx) + 2(a+b)\sin\cdots}{8d(a+b)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (4*(a + 7*b)*(c + d*x) + (b^(3/2)*(35*a^2 + 14*a*b + 3*b^2)*ArcTan[(Sqrt[b]
*Tanh[c + d*x])/Sqrt[a]])/a^(5/2) + 2*(a + b)*Sinh[2*(c + d*x)] + (4*b^3*(a
 + b)*Sinh[2*(c + d*x)])/(a*(a - b + (a + b)*Cosh[2*(c + d*x)])^2) + (b^2*(
a + b)*(13*a + 3*b)*Sinh[2*(c + d*x)])/(a^2*(a - b + (a + b)*Cosh[2*(c + d*
x)])))/(8*(a + b)^4*d)
```

Maple [B] time = 0.121, size = 2132, normalized size = 10.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] -35/8/d*b^2/(a+b)^4*a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*a
rctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-35/8/d*b^
2/(a+b)^4*a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*ta
nh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-17/8/d*b^4/(a+b)^4/(
b*(a+b))^(1/2)/a/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+
1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-17/8/d*b^4/(a+b)^4/(b*(a+b))^(1
/2)/a/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2
*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-3/8/d*b^5/(a+b)^4/a^2/(b*(a+b))^(1/2)/((2
*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))
^(1/2)+a+2*b)*a)^(1/2))-3/8/d*b^5/(a+b)^4/a^2/(b*(a+b))^(1/2)/((2*(b*(a+b))
^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-
2*b)*a)^(1/2))+71/4/d*b^4/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1
/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^5-35/8/d*b^2
/(a+b)^4/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/(
(2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+9/2/d*b^3/(a+b)^4/(tanh(1/2*d*x+1/2*c)^
4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1
/2*c)^7+49/2/d*b^3/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2
*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5+49/2/d*b^3/(a+b)^4/
(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*
b+a)^2*tanh(1/2*d*x+1/2*c)^3+9/2/d*b^3/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+2*t
anh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)-7
/4/d*b^3/(a+b)^4/a/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*
x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+39/4/d*b^2/(a+b)^4/(tanh(1/2*
d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a*t
anh(1/2*d*x+1/2*c)^3+71/4/d*b^4/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2
*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^3+13/4
/d*b^2/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/
2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)*a+1/2/d/(a+b)^4*ln(tanh(1/2*d*x+1
/2*c)+1)*a+7/2/d/(a+b)^4*ln(tanh(1/2*d*x+1/2*c)+1)*b-1/2/d/(a+b)^4*ln(tanh(
1/2*d*x+1/2*c)-1)*a-7/2/d/(a+b)^4*ln(tanh(1/2*d*x+1/2*c)-1)*b-1/2/d/(a+b)^3
/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/d/(a+b)^3/(tanh(1/2*d*x+1/2*c)+1)+1/2/d/(a+b
)^3/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)-49/8/d*
b^3/(a+b)^4/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*t
anh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-49/8/d*b^3/(a+b)^4/
(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1
/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+7/4/d*b^3/(a+b)^4/a/((2*(b*(a+b)
)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a
-2*b)*a)^(1/2))+35/8/d*b^2/(a+b)^4/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arct
anh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+13/4/d*b^2/(
a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/
2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^7*a+39/4/d*b^2/(a+b)^4/(tanh(1/2*d*x+1/2*
```

```
c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a*tanh(1/2*
d*x+1/2*c)^5+5/4/d*b^4/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*
c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^7+3/d*b^5/(a+b)
^4/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)
^2*b+a)^2/a^2*tanh(1/2*d*x+1/2*c)^5+3/d*b^5/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*
a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2*tanh(1/2*d*x
+1/2*c)^3+3/8/d*b^4/(a+b)^4/a^2/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh
(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-3/8/d*b^4/(a+b)
^4/a^2/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2
*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+5/4/d*b^4/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*
a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1
/2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.52959, size = 31513, normalized size = 159.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^12 + 24*(a^5 +
 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)*sinh(d*x + c)^11 + 2*(a^5 + 3
*a^4*b + 3*a^3*b^2 + a^2*b^3)*sinh(d*x + c)^12 + 8*(a^5 + a^4*b - a^3*b^2 -
 a^2*b^3 + (a^5 + 9*a^4*b + 15*a^3*b^2 + 7*a^2*b^3)*d*x)*cosh(d*x + c)^10 +
 4*(2*a^5 + 2*a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + 2*(a^5 + 9*a^4*b + 15*a^3*b^2
 + 7*a^2*b^3)*d*x + 33*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^10 + 40*(11*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x
 + c)^3 + 2*(a^5 + a^4*b - a^3*b^2 - a^2*b^3 + (a^5 + 9*a^4*b + 15*a^3*b^2
+ 7*a^2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^9 + 2*(5*a^5 - a^4*b - 27*a^
3*b^2 + 7*a^2*b^3 + 34*a*b^4 + 6*b^5 + 16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^2*
b^3)*d*x)*cosh(d*x + c)^8 + 2*(5*a^5 - a^4*b - 27*a^3*b^2 + 7*a^2*b^3 + 34*
a*b^4 + 6*b^5 + 495*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^4 +
 16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^2*b^3)*d*x + 180*(a^5 + a^4*b - a^3*b^2
- a^2*b^3 + (a^5 + 9*a^4*b + 15*a^3*b^2 + 7*a^2*b^3)*d*x)*cosh(d*x + c)^2)*
sinh(d*x + c)^8 + 16*(99*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c
)^5 + 60*(a^5 + a^4*b - a^3*b^2 - a^2*b^3 + (a^5 + 9*a^4*b + 15*a^3*b^2 + 7
*a^2*b^3)*d*x)*cosh(d*x + c)^3 + (5*a^5 - a^4*b - 27*a^3*b^2 + 7*a^2*b^3 +
34*a*b^4 + 6*b^5 + 16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^2*b^3)*d*x)*cosh(d*x +
 c))*sinh(d*x + c)^7 - 4*(39*a^3*b^2 - 17*a^2*b^3 + 33*a*b^4 + 9*b^5 - 4*(3
*a^5 + 19*a^4*b - 11*a^3*b^2 + 21*a^2*b^3)*d*x)*cosh(d*x + c)^6 + 4*(462*(a
^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^6 - 39*a^3*b^2 + 17*a^2*b
^3 - 33*a*b^4 - 9*b^5 + 420*(a^5 + a^4*b - a^3*b^2 - a^2*b^3 + (a^5 + 9*a^4
*b + 15*a^3*b^2 + 7*a^2*b^3)*d*x)*cosh(d*x + c)^4 + 4*(3*a^5 + 19*a^4*b - 1
1*a^3*b^2 + 21*a^2*b^3)*d*x + 14*(5*a^5 - a^4*b - 27*a^3*b^2 + 7*a^2*b^3 +
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34*a*b^4 + 6*b^5 + 16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^2*b^3)*d*x)*cosh(d*x +
 c)^2)*sinh(d*x + c)^6 + 8*(198*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(
d*x + c)^7 + 252*(a^5 + a^4*b - a^3*b^2 - a^2*b^3 + (a^5 + 9*a^4*b + 15*a^3
*b^2 + 7*a^2*b^3)*d*x)*cosh(d*x + c)^5 + 14*(5*a^5 - a^4*b - 27*a^3*b^2 + 7
*a^2*b^3 + 34*a*b^4 + 6*b^5 + 16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^2*b^3)*d*x)
*cosh(d*x + c)^3 - 3*(39*a^3*b^2 - 17*a^2*b^3 + 33*a*b^4 + 9*b^5 - 4*(3*a^5
 + 19*a^4*b - 11*a^3*b^2 + 21*a^2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5
- 2*a^5 - 6*a^4*b - 6*a^3*b^2 - 2*a^2*b^3 - 2*(5*a^5 - a^4*b + 77*a^3*b^2 +
 31*a^2*b^3 - 70*a*b^4 - 18*b^5 - 16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^2*b^3)*
d*x)*cosh(d*x + c)^4 + 2*(495*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*
x + c)^8 + 840*(a^5 + a^4*b - a^3*b^2 - a^2*b^3 + (a^5 + 9*a^4*b + 15*a^3*b
^2 + 7*a^2*b^3)*d*x)*cosh(d*x + c)^6 - 5*a^5 + a^4*b - 77*a^3*b^2 - 31*a^2*
b^3 + 70*a*b^4 + 18*b^5 + 70*(5*a^5 - a^4*b - 27*a^3*b^2 + 7*a^2*b^3 + 34*a
*b^4 + 6*b^5 + 16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^2*b^3)*d*x)*cosh(d*x + c)^
4 + 16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^2*b^3)*d*x - 30*(39*a^3*b^2 - 17*a^2*
b^3 + 33*a*b^4 + 9*b^5 - 4*(3*a^5 + 19*a^4*b - 11*a^3*b^2 + 21*a^2*b^3)*d*x
)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(55*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2
*b^3)*cosh(d*x + c)^9 + 120*(a^5 + a^4*b - a^3*b^2 - a^2*b^3 + (a^5 + 9*a^4
*b + 15*a^3*b^2 + 7*a^2*b^3)*d*x)*cosh(d*x + c)^7 + 14*(5*a^5 - a^4*b - 27*
a^3*b^2 + 7*a^2*b^3 + 34*a*b^4 + 6*b^5 + 16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^
2*b^3)*d*x)*cosh(d*x + c)^5 - 10*(39*a^3*b^2 - 17*a^2*b^3 + 33*a*b^4 + 9*b^
5 - 4*(3*a^5 + 19*a^4*b - 11*a^3*b^2 + 21*a^2*b^3)*d*x)*cosh(d*x + c)^3 - (
5*a^5 - a^4*b + 77*a^3*b^2 + 31*a^2*b^3 - 70*a*b^4 - 18*b^5 - 16*(a^5 + 7*a
^4*b - a^3*b^2 - 7*a^2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(2*a^5
+ 2*a^4*b + 11*a^3*b^2 + 27*a^2*b^3 + 19*a*b^4 + 3*b^5 - 2*(a^5 + 9*a^4*b +
 15*a^3*b^2 + 7*a^2*b^3)*d*x)*cosh(d*x + c)^2 + 4*(33*(a^5 + 3*a^4*b + 3*a^
3*b^2 + a^2*b^3)*cosh(d*x + c)^10 + 90*(a^5 + a^4*b - a^3*b^2 - a^2*b^3 + (
a^5 + 9*a^4*b + 15*a^3*b^2 + 7*a^2*b^3)*d*x)*cosh(d*x + c)^8 + 14*(5*a^5 -
a^4*b - 27*a^3*b^2 + 7*a^2*b^3 + 34*a*b^4 + 6*b^5 + 16*(a^5 + 7*a^4*b - a^3
*b^2 - 7*a^2*b^3)*d*x)*cosh(d*x + c)^6 - 2*a^5 - 2*a^4*b - 11*a^3*b^2 - 27*
a^2*b^3 - 19*a*b^4 - 3*b^5 - 15*(39*a^3*b^2 - 17*a^2*b^3 + 33*a*b^4 + 9*b^5
 - 4*(3*a^5 + 19*a^4*b - 11*a^3*b^2 + 21*a^2*b^3)*d*x)*cosh(d*x + c)^4 + 2*
(a^5 + 9*a^4*b + 15*a^3*b^2 + 7*a^2*b^3)*d*x - 3*(5*a^5 - a^4*b + 77*a^3*b^
2 + 31*a^2*b^3 - 70*a*b^4 - 18*b^5 - 16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^2*b^
3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((35*a^4*b + 84*a^3*b^2 + 66*a^2
*b^3 + 20*a*b^4 + 3*b^5)*cosh(d*x + c)^10 + 10*(35*a^4*b + 84*a^3*b^2 + 66*
a^2*b^3 + 20*a*b^4 + 3*b^5)*cosh(d*x + c)*sinh(d*x + c)^9 + (35*a^4*b + 84*
a^3*b^2 + 66*a^2*b^3 + 20*a*b^4 + 3*b^5)*sinh(d*x + c)^10 + 4*(35*a^4*b + 1
4*a^3*b^2 - 32*a^2*b^3 - 14*a*b^4 - 3*b^5)*cosh(d*x + c)^8 + (140*a^4*b + 5
6*a^3*b^2 - 128*a^2*b^3 - 56*a*b^4 - 12*b^5 + 45*(35*a^4*b + 84*a^3*b^2 + 6
6*a^2*b^3 + 20*a*b^4 + 3*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(35*
a^4*b + 84*a^3*b^2 + 66*a^2*b^3 + 20*a*b^4 + 3*b^5)*cosh(d*x + c)^3 + 4*(35
*a^4*b + 14*a^3*b^2 - 32*a^2*b^3 - 14*a*b^4 - 3*b^5)*cosh(d*x + c))*sinh(d*
x + c)^7 + 2*(105*a^4*b - 28*a^3*b^2 + 86*a^2*b^3 + 36*a*b^4 + 9*b^5)*cosh(
d*x + c)^6 + 2*(105*a^4*b - 28*a^3*b^2 + 86*a^2*b^3 + 36*a*b^4 + 9*b^5 + 10
5*(35*a^4*b + 84*a^3*b^2 + 66*a^2*b^3 + 20*a*b^4 + 3*b^5)*cosh(d*x + c)^4 +
 56*(35*a^4*b + 14*a^3*b^2 - 32*a^2*b^3 - 14*a*b^4 - 3*b^5)*cosh(d*x + c)^2
)*sinh(d*x + c)^6 + 4*(63*(35*a^4*b + 84*a^3*b^2 + 66*a^2*b^3 + 20*a*b^4 +
3*b^5)*cosh(d*x + c)^5 + 56*(35*a^4*b + 14*a^3*b^2 - 32*a^2*b^3 - 14*a*b^4
- 3*b^5)*cosh(d*x + c)^3 + 3*(105*a^4*b - 28*a^3*b^2 + 86*a^2*b^3 + 36*a*b^
4 + 9*b^5)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*(35*a^4*b + 14*a^3*b^2 - 32*a
^2*b^3 - 14*a*b^4 - 3*b^5)*cosh(d*x + c)^4 + 2*(105*(35*a^4*b + 84*a^3*b^2
+ 66*a^2*b^3 + 20*a*b^4 + 3*b^5)*cosh(d*x + c)^6 + 70*a^4*b + 28*a^3*b^2 -
64*a^2*b^3 - 28*a*b^4 - 6*b^5 + 140*(35*a^4*b + 14*a^3*b^2 - 32*a^2*b^3 - 1
4*a*b^4 - 3*b^5)*cosh(d*x + c)^4 + 15*(105*a^4*b - 28*a^3*b^2 + 86*a^2*b^3
+ 36*a*b^4 + 9*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(15*(35*a^4*b + 84
*a^3*b^2 + 66*a^2*b^3 + 20*a*b^4 + 3*b^5)*cosh(d*x + c)^7 + 28*(35*a^4*b +
14*a^3*b^2 - 32*a^2*b^3 - 14*a*b^4 - 3*b^5)*cosh(d*x + c)^5 + 5*(105*a^4*b
- 28*a^3*b^2 + 86*a^2*b^3 + 36*a*b^4 + 9*b^5)*cosh(d*x + c)^3 + 2*(35*a^4*b
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 + 14*a^3*b^2 - 32*a^2*b^3 - 14*a*b^4 - 3*b^5)*cosh(d*x + c))*sinh(d*x + c)
^3 + (35*a^4*b + 84*a^3*b^2 + 66*a^2*b^3 + 20*a*b^4 + 3*b^5)*cosh(d*x + c)^
2 + (45*(35*a^4*b + 84*a^3*b^2 + 66*a^2*b^3 + 20*a*b^4 + 3*b^5)*cosh(d*x +
c)^8 + 112*(35*a^4*b + 14*a^3*b^2 - 32*a^2*b^3 - 14*a*b^4 - 3*b^5)*cosh(d*x
 + c)^6 + 35*a^4*b + 84*a^3*b^2 + 66*a^2*b^3 + 20*a*b^4 + 3*b^5 + 30*(105*a
^4*b - 28*a^3*b^2 + 86*a^2*b^3 + 36*a*b^4 + 9*b^5)*cosh(d*x + c)^4 + 24*(35
*a^4*b + 14*a^3*b^2 - 32*a^2*b^3 - 14*a*b^4 - 3*b^5)*cosh(d*x + c)^2)*sinh(
d*x + c)^2 + 2*(5*(35*a^4*b + 84*a^3*b^2 + 66*a^2*b^3 + 20*a*b^4 + 3*b^5)*c
osh(d*x + c)^9 + 16*(35*a^4*b + 14*a^3*b^2 - 32*a^2*b^3 - 14*a*b^4 - 3*b^5)
*cosh(d*x + c)^7 + 6*(105*a^4*b - 28*a^3*b^2 + 86*a^2*b^3 + 36*a*b^4 + 9*b^
5)*cosh(d*x + c)^5 + 8*(35*a^4*b + 14*a^3*b^2 - 32*a^2*b^3 - 14*a*b^4 - 3*b
^5)*cosh(d*x + c)^3 + (35*a^4*b + 84*a^3*b^2 + 66*a^2*b^3 + 20*a*b^4 + 3*b^
5)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(d
*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*
a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*
a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2
 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh
(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*si
nh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((a + b)
*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d
*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a -
b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*si
nh(d*x + c) + a + b)) + 8*(3*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x
 + c)^11 + 10*(a^5 + a^4*b - a^3*b^2 - a^2*b^3 + (a^5 + 9*a^4*b + 15*a^3*b^
2 + 7*a^2*b^3)*d*x)*cosh(d*x + c)^9 + 2*(5*a^5 - a^4*b - 27*a^3*b^2 + 7*a^2
*b^3 + 34*a*b^4 + 6*b^5 + 16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^2*b^3)*d*x)*cos
h(d*x + c)^7 - 3*(39*a^3*b^2 - 17*a^2*b^3 + 33*a*b^4 + 9*b^5 - 4*(3*a^5 + 1
9*a^4*b - 11*a^3*b^2 + 21*a^2*b^3)*d*x)*cosh(d*x + c)^5 - (5*a^5 - a^4*b +
77*a^3*b^2 + 31*a^2*b^3 - 70*a*b^4 - 18*b^5 - 16*(a^5 + 7*a^4*b - a^3*b^2 -
 7*a^2*b^3)*d*x)*cosh(d*x + c)^3 - (2*a^5 + 2*a^4*b + 11*a^3*b^2 + 27*a^2*b
^3 + 19*a*b^4 + 3*b^5 - 2*(a^5 + 9*a^4*b + 15*a^3*b^2 + 7*a^2*b^3)*d*x)*cos
h(d*x + c))*sinh(d*x + c))/((a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a
^4*b^4 + 6*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^10 + 10*(a^8 + 6*a^7*b + 15*a
^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)*sin
h(d*x + c)^9 + (a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^
3*b^5 + a^2*b^6)*d*sinh(d*x + c)^10 + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*
b^4 - 4*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)^8 + (45*(a^8 + 6*a^7*b + 15*a^6*
b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^2 + 4*
(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*d)*sinh(d*x +
 c)^8 + 2*(3*a^8 + 10*a^7*b + 13*a^6*b^2 + 12*a^5*b^3 + 13*a^4*b^4 + 10*a^3
*b^5 + 3*a^2*b^6)*d*cosh(d*x + c)^6 + 8*(15*(a^8 + 6*a^7*b + 15*a^6*b^2 + 2
0*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^3 + 4*(a^8 +
4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c))*sin
h(d*x + c)^7 + 2*(105*(a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4
 + 6*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^4 + 56*(a^8 + 4*a^7*b + 5*a^6*b^2 -
 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)^2 + (3*a^8 + 10*a^7*b + 1
3*a^6*b^2 + 12*a^5*b^3 + 13*a^4*b^4 + 10*a^3*b^5 + 3*a^2*b^6)*d)*sinh(d*x +
 c)^6 + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*d*c
osh(d*x + c)^4 + 4*(63*(a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^
4 + 6*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^5 + 56*(a^8 + 4*a^7*b + 5*a^6*b^2
- 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)^3 + 3*(3*a^8 + 10*a^7*b
+ 13*a^6*b^2 + 12*a^5*b^3 + 13*a^4*b^4 + 10*a^3*b^5 + 3*a^2*b^6)*d*cosh(d*x
 + c))*sinh(d*x + c)^5 + 2*(105*(a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 +
15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^6 + 140*(a^8 + 4*a^7*b +
5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)^4 + 15*(3*a^8
+ 10*a^7*b + 13*a^6*b^2 + 12*a^5*b^3 + 13*a^4*b^4 + 10*a^3*b^5 + 3*a^2*b^6)
*d*cosh(d*x + c)^2 + 2*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 -
 a^2*b^6)*d)*sinh(d*x + c)^4 + (a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 1
5*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^2 + 8*(15*(a^8 + 6*a^7*b +
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15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c
)^7 + 28*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*d*co
sh(d*x + c)^5 + 5*(3*a^8 + 10*a^7*b + 13*a^6*b^2 + 12*a^5*b^3 + 13*a^4*b^4
+ 10*a^3*b^5 + 3*a^2*b^6)*d*cosh(d*x + c)^3 + 2*(a^8 + 4*a^7*b + 5*a^6*b^2
- 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (45*(
a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)
*d*cosh(d*x + c)^8 + 112*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5
- a^2*b^6)*d*cosh(d*x + c)^6 + 30*(3*a^8 + 10*a^7*b + 13*a^6*b^2 + 12*a^5*
b^3 + 13*a^4*b^4 + 10*a^3*b^5 + 3*a^2*b^6)*d*cosh(d*x + c)^4 + 24*(a^8 + 4*
a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)^2 + (a
^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*
d)*sinh(d*x + c)^2 + 2*(5*(a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4
*b^4 + 6*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^9 + 16*(a^8 + 4*a^7*b + 5*a^6*b
^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)^7 + 6*(3*a^8 + 10*a^7
*b + 13*a^6*b^2 + 12*a^5*b^3 + 13*a^4*b^4 + 10*a^3*b^5 + 3*a^2*b^6)*d*cosh(
d*x + c)^5 + 8*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6
)*d*cosh(d*x + c)^3 + (a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4
+ 6*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*((a^5 + 3*a^4*
b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^12 + 12*(a^5 + 3*a^4*b + 3*a^3*b^2 +
a^2*b^3)*cosh(d*x + c)*sinh(d*x + c)^11 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2
*b^3)*sinh(d*x + c)^12 + 4*(a^5 + a^4*b - a^3*b^2 - a^2*b^3 + (a^5 + 9*a^4*
b + 15*a^3*b^2 + 7*a^2*b^3)*d*x)*cosh(d*x + c)^10 + 2*(2*a^5 + 2*a^4*b - 2*
a^3*b^2 - 2*a^2*b^3 + 2*(a^5 + 9*a^4*b + 15*a^3*b^2 + 7*a^2*b^3)*d*x + 33*(
a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 20
(11(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^3 + 2*(a^5 + a^4*b
- a^3*b^2 - a^2*b^3 + (a^5 + 9*a^4*b + 15*a^3*b^2 + 7*a^2*b^3)*d*x)*cosh(d
*x + c))*sinh(d*x + c)^9 + (5*a^5 - a^4*b - 27*a^3*b^2 + 7*a^2*b^3 + 34*a*b
^4 + 6*b^5 + 16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^2*b^3)*d*x)*cosh(d*x + c)^8
+ (5*a^5 - a^4*b - 27*a^3*b^2 + 7*a^2*b^3 + 34*a*b^4 + 6*b^5 + 495*(a^5 + 3
*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^4 + 16*(a^5 + 7*a^4*b - a^3*b^2
- 7*a^2*b^3)*d*x + 180*(a^5 + a^4*b - a^3*b^2 - a^2*b^3 + (a^5 + 9*a^4*b +
15*a^3*b^2 + 7*a^2*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(99*(a^5
+ 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^5 + 60*(a^5 + a^4*b - a^3*b
^2 - a^2*b^3 + (a^5 + 9*a^4*b + 15*a^3*b^2 + 7*a^2*b^3)*d*x)*cosh(d*x + c)^
3 + (5*a^5 - a^4*b - 27*a^3*b^2 + 7*a^2*b^3 + 34*a*b^4 + 6*b^5 + 16*(a^5 +
7*a^4*b - a^3*b^2 - 7*a^2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(39*
a^3*b^2 - 17*a^2*b^3 + 33*a*b^4 + 9*b^5 - 4*(3*a^5 + 19*a^4*b - 11*a^3*b^2
+ 21*a^2*b^3)*d*x)*cosh(d*x + c)^6 + 2*(462*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^
2*b^3)*cosh(d*x + c)^6 - 39*a^3*b^2 + 17*a^2*b^3 - 33*a*b^4 - 9*b^5 + 420*(
a^5 + a^4*b - a^3*b^2 - a^2*b^3 + (a^5 + 9*a^4*b + 15*a^3*b^2 + 7*a^2*b^3)*
d*x)*cosh(d*x + c)^4 + 4*(3*a^5 + 19*a^4*b - 11*a^3*b^2 + 21*a^2*b^3)*d*x +
14*(5*a^5 - a^4*b - 27*a^3*b^2 + 7*a^2*b^3 + 34*a*b^4 + 6*b^5 + 16*(a^5 +
7*a^4*b - a^3*b^2 - 7*a^2*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(1
98*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^7 + 252*(a^5 + a^4*b
- a^3*b^2 - a^2*b^3 + (a^5 + 9*a^4*b + 15*a^3*b^2 + 7*a^2*b^3)*d*x)*cosh(d
x + c)^5 + 14(5*a^5 - a^4*b - 27*a^3*b^2 + 7*a^2*b^3 + 34*a*b^4 + 6*b^5 +
16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^2*b^3)*d*x)*cosh(d*x + c)^3 - 3*(39*a^3*
b^2 - 17*a^2*b^3 + 33*a*b^4 + 9*b^5 - 4*(3*a^5 + 19*a^4*b - 11*a^3*b^2 + 21
*a^2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - a^5 - 3*a^4*b - 3*a^3*b^2 -
a^2*b^3 - (5*a^5 - a^4*b + 77*a^3*b^2 + 31*a^2*b^3 - 70*a*b^4 - 18*b^5 - 1
6*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^2*b^3)*d*x)*cosh(d*x + c)^4 + (495*(a^5 +
3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^8 + 840*(a^5 + a^4*b - a^3*b^2
- a^2*b^3 + (a^5 + 9*a^4*b + 15*a^3*b^2 + 7*a^2*b^3)*d*x)*cosh(d*x + c)^6
- 5*a^5 + a^4*b - 77*a^3*b^2 - 31*a^2*b^3 + 70*a*b^4 + 18*b^5 + 70*(5*a^5 -
a^4*b - 27*a^3*b^2 + 7*a^2*b^3 + 34*a*b^4 + 6*b^5 + 16*(a^5 + 7*a^4*b - a^
3*b^2 - 7*a^2*b^3)*d*x)*cosh(d*x + c)^4 + 16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a
^2*b^3)*d*x - 30*(39*a^3*b^2 - 17*a^2*b^3 + 33*a*b^4 + 9*b^5 - 4*(3*a^5 + 1
9*a^4*b - 11*a^3*b^2 + 21*a^2*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 +
4*(55*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^9 + 120*(a^5 + a^

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4*b - a^3*b^2 - a^2*b^3 + (a^5 + 9*a^4*b + 15*a^3*b^2 + 7*a^2*b^3)*d*x)*cos
h(d*x + c)^7 + 14*(5*a^5 - a^4*b - 27*a^3*b^2 + 7*a^2*b^3 + 34*a*b^4 + 6*b^
5 + 16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^2*b^3)*d*x)*cosh(d*x + c)^5 - 10*(39*
a^3*b^2 - 17*a^2*b^3 + 33*a*b^4 + 9*b^5 - 4*(3*a^5 + 19*a^4*b - 11*a^3*b^2
+ 21*a^2*b^3)*d*x)*cosh(d*x + c)^3 - (5*a^5 - a^4*b + 77*a^3*b^2 + 31*a^2*b
^3 - 70*a*b^4 - 18*b^5 - 16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^2*b^3)*d*x)*cosh
(d*x + c))*sinh(d*x + c)^3 - 2*(2*a^5 + 2*a^4*b + 11*a^3*b^2 + 27*a^2*b^3 +
 19*a*b^4 + 3*b^5 - 2*(a^5 + 9*a^4*b + 15*a^3*b^2 + 7*a^2*b^3)*d*x)*cosh(d*
x + c)^2 + 2*(33*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^10 + 9
0*(a^5 + a^4*b - a^3*b^2 - a^2*b^3 + (a^5 + 9*a^4*b + 15*a^3*b^2 + 7*a^2*b^
3)*d*x)*cosh(d*x + c)^8 + 14*(5*a^5 - a^4*b - 27*a^3*b^2 + 7*a^2*b^3 + 34*a
*b^4 + 6*b^5 + 16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^2*b^3)*d*x)*cosh(d*x + c)^
6 - 2*a^5 - 2*a^4*b - 11*a^3*b^2 - 27*a^2*b^3 - 19*a*b^4 - 3*b^5 - 15*(39*a
^3*b^2 - 17*a^2*b^3 + 33*a*b^4 + 9*b^5 - 4*(3*a^5 + 19*a^4*b - 11*a^3*b^2 +
 21*a^2*b^3)*d*x)*cosh(d*x + c)^4 + 2*(a^5 + 9*a^4*b + 15*a^3*b^2 + 7*a^2*b
^3)*d*x - 3*(5*a^5 - a^4*b + 77*a^3*b^2 + 31*a^2*b^3 - 70*a*b^4 - 18*b^5 -
16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^2*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c
)^2 + ((35*a^4*b + 84*a^3*b^2 + 66*a^2*b^3 + 20*a*b^4 + 3*b^5)*cosh(d*x + c
)^10 + 10*(35*a^4*b + 84*a^3*b^2 + 66*a^2*b^3 + 20*a*b^4 + 3*b^5)*cosh(d*x
+ c)*sinh(d*x + c)^9 + (35*a^4*b + 84*a^3*b^2 + 66*a^2*b^3 + 20*a*b^4 + 3*b
^5)*sinh(d*x + c)^10 + 4*(35*a^4*b + 14*a^3*b^2 - 32*a^2*b^3 - 14*a*b^4 - 3
*b^5)*cosh(d*x + c)^8 + (140*a^4*b + 56*a^3*b^2 - 128*a^2*b^3 - 56*a*b^4 -
12*b^5 + 45*(35*a^4*b + 84*a^3*b^2 + 66*a^2*b^3 + 20*a*b^4 + 3*b^5)*cosh(d*
x + c)^2)*sinh(d*x + c)^8 + 8*(15*(35*a^4*b + 84*a^3*b^2 + 66*a^2*b^3 + 20*
a*b^4 + 3*b^5)*cosh(d*x + c)^3 + 4*(35*a^4*b + 14*a^3*b^2 - 32*a^2*b^3 - 14
*a*b^4 - 3*b^5)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*a^4*b - 28*a^3*b^2
+ 86*a^2*b^3 + 36*a*b^4 + 9*b^5)*cosh(d*x + c)^6 + 2*(105*a^4*b - 28*a^3*b^
2 + 86*a^2*b^3 + 36*a*b^4 + 9*b^5 + 105*(35*a^4*b + 84*a^3*b^2 + 66*a^2*b^3
 + 20*a*b^4 + 3*b^5)*cosh(d*x + c)^4 + 56*(35*a^4*b + 14*a^3*b^2 - 32*a^2*b
^3 - 14*a*b^4 - 3*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(35*a^4*b +
 84*a^3*b^2 + 66*a^2*b^3 + 20*a*b^4 + 3*b^5)*cosh(d*x + c)^5 + 56*(35*a^4*b
 + 14*a^3*b^2 - 32*a^2*b^3 - 14*a*b^4 - 3*b^5)*cosh(d*x + c)^3 + 3*(105*a^4
*b - 28*a^3*b^2 + 86*a^2*b^3 + 36*a*b^4 + 9*b^5)*cosh(d*x + c))*sinh(d*x +
c)^5 + 4*(35*a^4*b + 14*a^3*b^2 - 32*a^2*b^3 - 14*a*b^4 - 3*b^5)*cosh(d*x +
 c)^4 + 2*(105*(35*a^4*b + 84*a^3*b^2 + 66*a^2*b^3 + 20*a*b^4 + 3*b^5)*cosh
(d*x + c)^6 + 70*a^4*b + 28*a^3*b^2 - 64*a^2*b^3 - 28*a*b^4 - 6*b^5 + 140*(
35*a^4*b + 14*a^3*b^2 - 32*a^2*b^3 - 14*a*b^4 - 3*b^5)*cosh(d*x + c)^4 + 15
*(105*a^4*b - 28*a^3*b^2 + 86*a^2*b^3 + 36*a*b^4 + 9*b^5)*cosh(d*x + c)^2)*
sinh(d*x + c)^4 + 8*(15*(35*a^4*b + 84*a^3*b^2 + 66*a^2*b^3 + 20*a*b^4 + 3*
b^5)*cosh(d*x + c)^7 + 28*(35*a^4*b + 14*a^3*b^2 - 32*a^2*b^3 - 14*a*b^4 -
3*b^5)*cosh(d*x + c)^5 + 5*(105*a^4*b - 28*a^3*b^2 + 86*a^2*b^3 + 36*a*b^4
+ 9*b^5)*cosh(d*x + c)^3 + 2*(35*a^4*b + 14*a^3*b^2 - 32*a^2*b^3 - 14*a*b^4
 - 3*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 + (35*a^4*b + 84*a^3*b^2 + 66*a^2*
b^3 + 20*a*b^4 + 3*b^5)*cosh(d*x + c)^2 + (45*(35*a^4*b + 84*a^3*b^2 + 66*a
^2*b^3 + 20*a*b^4 + 3*b^5)*cosh(d*x + c)^8 + 112*(35*a^4*b + 14*a^3*b^2 - 3
2*a^2*b^3 - 14*a*b^4 - 3*b^5)*cosh(d*x + c)^6 + 35*a^4*b + 84*a^3*b^2 + 66*
a^2*b^3 + 20*a*b^4 + 3*b^5 + 30*(105*a^4*b - 28*a^3*b^2 + 86*a^2*b^3 + 36*a
*b^4 + 9*b^5)*cosh(d*x + c)^4 + 24*(35*a^4*b + 14*a^3*b^2 - 32*a^2*b^3 - 14
*a*b^4 - 3*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(5*(35*a^4*b + 84*a^3*
b^2 + 66*a^2*b^3 + 20*a*b^4 + 3*b^5)*cosh(d*x + c)^9 + 16*(35*a^4*b + 14*a^
3*b^2 - 32*a^2*b^3 - 14*a*b^4 - 3*b^5)*cosh(d*x + c)^7 + 6*(105*a^4*b - 28*
a^3*b^2 + 86*a^2*b^3 + 36*a*b^4 + 9*b^5)*cosh(d*x + c)^5 + 8*(35*a^4*b + 14
*a^3*b^2 - 32*a^2*b^3 - 14*a*b^4 - 3*b^5)*cosh(d*x + c)^3 + (35*a^4*b + 84*
a^3*b^2 + 66*a^2*b^3 + 20*a*b^4 + 3*b^5)*cosh(d*x + c))*sinh(d*x + c))*sqrt
(b/a)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*
x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(b/a)/b) + 4*(3*(a^5 + 3*a^4*
b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^11 + 10*(a^5 + a^4*b - a^3*b^2 - a^2
*b^3 + (a^5 + 9*a^4*b + 15*a^3*b^2 + 7*a^2*b^3)*d*x)*cosh(d*x + c)^9 + 2*(5
*a^5 - a^4*b - 27*a^3*b^2 + 7*a^2*b^3 + 34*a*b^4 + 6*b^5 + 16*(a^5 + 7*a^4*
```

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b - a^3*b^2 - 7*a^2*b^3)*d*x)*cosh(d*x + c)^7 - 3*(39*a^3*b^2 - 17*a^2*b^3
+ 33*a*b^4 + 9*b^5 - 4*(3*a^5 + 19*a^4*b - 11*a^3*b^2 + 21*a^2*b^3)*d*x)*co
sh(d*x + c)^5 - (5*a^5 - a^4*b + 77*a^3*b^2 + 31*a^2*b^3 - 70*a*b^4 - 18*b^
5 - 16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^2*b^3)*d*x)*cosh(d*x + c)^3 - (2*a^5
+ 2*a^4*b + 11*a^3*b^2 + 27*a^2*b^3 + 19*a*b^4 + 3*b^5 - 2*(a^5 + 9*a^4*b +
 15*a^3*b^2 + 7*a^2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^8 + 6*a^7*b
 + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*d*cosh(d*x +
 c)^10 + 10*(a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b
^5 + a^2*b^6)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^8 + 6*a^7*b + 15*a^6*b^2
 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*d*sinh(d*x + c)^10 + 4*(a
^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)
^8 + (45*(a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5
+ a^2*b^6)*d*cosh(d*x + c)^2 + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4
*a^3*b^5 - a^2*b^6)*d)*sinh(d*x + c)^8 + 2*(3*a^8 + 10*a^7*b + 13*a^6*b^2 +
 12*a^5*b^3 + 13*a^4*b^4 + 10*a^3*b^5 + 3*a^2*b^6)*d*cosh(d*x + c)^6 + 8*(1
5*(a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b
^6)*d*cosh(d*x + c)^3 + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^
5 - a^2*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^8 + 6*a^7*b + 15*
a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^4
+ 56*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*d*cosh(d
*x + c)^2 + (3*a^8 + 10*a^7*b + 13*a^6*b^2 + 12*a^5*b^3 + 13*a^4*b^4 + 10*a
^3*b^5 + 3*a^2*b^6)*d)*sinh(d*x + c)^6 + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a
^4*b^4 - 4*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)^4 + 4*(63*(a^8 + 6*a^7*b + 15
*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^5
 + 56*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*d*cosh(
d*x + c)^3 + 3*(3*a^8 + 10*a^7*b + 13*a^6*b^2 + 12*a^5*b^3 + 13*a^4*b^4 + 1
0*a^3*b^5 + 3*a^2*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^8 + 6*a
^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*d*cosh(d
*x + c)^6 + 140*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^
6)*d*cosh(d*x + c)^4 + 15*(3*a^8 + 10*a^7*b + 13*a^6*b^2 + 12*a^5*b^3 + 13*
a^4*b^4 + 10*a^3*b^5 + 3*a^2*b^6)*d*cosh(d*x + c)^2 + 2*(a^8 + 4*a^7*b + 5*
a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*d)*sinh(d*x + c)^4 + (a^8 + 6*a^
7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*d*cosh(d*
x + c)^2 + 8*(15*(a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*
a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^7 + 28*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^
4*b^4 - 4*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)^5 + 5*(3*a^8 + 10*a^7*b + 13*a
^6*b^2 + 12*a^5*b^3 + 13*a^4*b^4 + 10*a^3*b^5 + 3*a^2*b^6)*d*cosh(d*x + c)^
3 + 2*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*d*cosh(
d*x + c))*sinh(d*x + c)^3 + (45*(a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 +
15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^8 + 112*(a^8 + 4*a^7*b +
5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)^6 + 30*(3*a^8
+ 10*a^7*b + 13*a^6*b^2 + 12*a^5*b^3 + 13*a^4*b^4 + 10*a^3*b^5 + 3*a^2*b^6)
*d*cosh(d*x + c)^4 + 24*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5
- a^2*b^6)*d*cosh(d*x + c)^2 + (a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 1
5*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*d)*sinh(d*x + c)^2 + 2*(5*(a^8 + 6*a^7*b +
 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c
)^9 + 16*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*d*co
sh(d*x + c)^7 + 6*(3*a^8 + 10*a^7*b + 13*a^6*b^2 + 12*a^5*b^3 + 13*a^4*b^4
+ 10*a^3*b^5 + 3*a^2*b^6)*d*cosh(d*x + c)^5 + 8*(a^8 + 4*a^7*b + 5*a^6*b^2
- 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)^3 + (a^8 + 6*a^7*b + 15*
a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c))*s
inh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.20652, size = 803, normalized size = 4.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(4*(a + 7*b)*d*x/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + (35*a^2*
b^2*e^(2*c) + 14*a*b^3*e^(2*c) + 3*b^4*e^(2*c))*arctan(1/2*(a*e^(2*d*x + 2*
c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))*e^(-2*c)/((a^6 + 4*a^5*b + 6*a^4
*b^2 + 4*a^3*b^3 + a^2*b^4)*sqrt(a*b)) - (2*a*e^(2*d*x + 2*c) + 14*b*e^(2*d
*x + 2*c) + a + b)*e^(-2*d*x)/(a^4*e^(2*c) + 4*a^3*b*e^(2*c) + 6*a^2*b^2*e^
(2*c) + 4*a*b^3*e^(2*c) + b^4*e^(2*c)) + e^(2*d*x + 16*c)/(a^3*e^(14*c) + 3
*a^2*b*e^(14*c) + 3*a*b^2*e^(14*c) + b^3*e^(14*c)) - 2*(13*a^3*b^2*e^(6*d*x
 + 6*c) - a^2*b^3*e^(6*d*x + 6*c) - 17*a*b^4*e^(6*d*x + 6*c) - 3*b^5*e^(6*d
*x + 6*c) + 39*a^3*b^2*e^(4*d*x + 4*c) - 17*a^2*b^3*e^(4*d*x + 4*c) + 33*a*
b^4*e^(4*d*x + 4*c) + 9*b^5*e^(4*d*x + 4*c) + 39*a^3*b^2*e^(2*d*x + 2*c) +
13*a^2*b^3*e^(2*d*x + 2*c) - 35*a*b^4*e^(2*d*x + 2*c) - 9*b^5*e^(2*d*x + 2*
c) + 13*a^3*b^2 + 29*a^2*b^3 + 19*a*b^4 + 3*b^5)/((a^6 + 4*a^5*b + 6*a^4*b^
2 + 4*a^3*b^3 + a^2*b^4)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*
d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2))/d
```

3.126 $\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=154

$$\frac{3b^2(4a+b)\sinh(c+dx)}{8a^2d(a+b)^3\left((a+b)\sinh^2(c+dx)+a\right)}+\frac{3b\left(8a^2+4ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a+b)^{7/2}}+\frac{b^3\sinh(c+dx)}{4ad(a+b)^3\left((a+b)\sinh^2(c-\right.}$$

```
[Out] (3*b*(8*a^2 + 4*a*b + b^2)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(8*
a^(5/2)*(a + b)^(7/2)*d) + Sinh[c + d*x]/((a + b)^3*d) + (b^3*Sinh[c + d*x]
)/(4*a*(a + b)^3*d*(a + (a + b)*Sinh[c + d*x]^2)^2) + (3*b^2*(4*a + b)*Sinh
[c + d*x])/(8*a^2*(a + b)^3*d*(a + (a + b)*Sinh[c + d*x]^2))
```

Rubi [A] time = 0.222993, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3676, 390, 1157, 385, 205}

$$\frac{3b^2(4a+b)\sinh(c+dx)}{8a^2d(a+b)^3\left((a+b)\sinh^2(c+dx)+a\right)}+\frac{3b\left(8a^2+4ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a+b)^{7/2}}+\frac{b^3\sinh(c+dx)}{4ad(a+b)^3\left((a+b)\sinh^2(c-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```


```
[Out] (3*b*(8*a^2 + 4*a*b + b^2)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(8*
a^(5/2)*(a + b)^(7/2)*d) + Sinh[c + d*x]/((a + b)^3*d) + (b^3*Sinh[c + d*x]
)/(4*a*(a + b)^3*d*(a + (a + b)*Sinh[c + d*x]^2)^2) + (3*b^2*(4*a + b)*Sinh
[c + d*x])/(8*a^2*(a + b)^3*d*(a + (a + b)*Sinh[c + d*x]^2))
```


Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{\left(a+(a+b)x^2\right)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)^3}+\frac{b\left(3a^2+3ab+b^2\right)+3b(a+b)(2a+b)x^2+3b(a+b)^2x^4}{(a+b)^3\left(a+(a+b)x^2\right)^3}\right)dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\sinh(c+dx)}{(a+b)^3d}+\frac{\text{Subst}\left(\int \frac{b\left(3a^2+3ab+b^2\right)+3b(a+b)(2a+b)x^2+3b(a+b)^2x^4}{\left(a+(a+b)x^2\right)^3}\,dx, x, \sinh(c+dx)\right)}{(a+b)^3d}$$

$$= \frac{\sinh(c+dx)}{(a+b)^3d}+\frac{b^3\sinh(c+dx)}{4a(a+b)^3d\left(a+(a+b)\sinh^2(c+dx)\right)^2}-\frac{\text{Subst}\left(\int \frac{-3b(2a+b)^2-12ab(a+b)x^2}{\left(a+(a+b)x^2\right)^2}\right.}{4a(a+b)^3}$$

$$= \frac{\sinh(c+dx)}{(a+b)^3d}+\frac{b^3\sinh(c+dx)}{4a(a+b)^3d\left(a+(a+b)\sinh^2(c+dx)\right)^2}+\frac{3b^2(4a+b)\sinh(c+}{8a^2(a+b)^3d\left(a+(a+b)\sinh\right.}$$

$$= \frac{3b\left(8a^2+4ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{7/2}d}+\frac{\sinh(c+dx)}{(a+b)^3d}+\frac{b^3\sinh(c+dx)}{4a(a+b)^3d\left(a+(a+b)\sin\right.}$$

Mathematica [A] time = 1.85671, size = 136, normalized size = 0.88

$$\frac{\frac{\sinh(c+dx)\left(\frac{3b^3}{a^2\left((a+b)\sinh^2(c+dx)+a\right)}+\frac{2b^2\left(6(a+b)\sinh^2(c+dx)+6a+b\right)}{a\left((a+b)\sinh^2(c+dx)+a\right)^2}+8\right)}{(a+b)^3}+\frac{3b\left(8a^2+4ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}(a+b)^{7/2}}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((3*b*(8*a^2 + 4*a*b + b^2)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(a
^(5/2)*(a + b)^(7/2)) + (Sinh[c + d*x]*(8 + (3*b^3)/(a^2*(a + (a + b)*Sinh[
c + d*x]^2)) + (2*b^2*(6*a + b + 6*(a + b)*Sinh[c + d*x]^2))/(a*(a + (a + b
)*Sinh[c + d*x]^2)^2)))/(a + b)^3)/(8*d)
```

Maple [B] time = 0.111, size = 1570, normalized size = 10.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] -1/d/(a+b)^3/(tanh(1/2*d*x+1/2*c)+1)-3/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4
*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/
2*c)^7-5/4/d*b^3/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a
+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^7-3/d*b^2/(a+b)^3/(ta
nh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a
)^2*tanh(1/2*d*x+1/2*c)^5-45/4/d*b^3/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tan
h(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^5
-3/d*b^4/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(
1/2*d*x+1/2*c)^2*b+a)^2/a^2*tanh(1/2*d*x+1/2*c)^5+3/d*b^2/(a+b)^3/(tanh(1/2
*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*ta
nh(1/2*d*x+1/2*c)^3+45/4/d*b^3/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*
d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^3+3/d*b
^4/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*
x+1/2*c)^2*b+a)^2/a^2*tanh(1/2*d*x+1/2*c)^3+3/d*b^2/(a+b)^3/(tanh(1/2*d*x+1
/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2
*d*x+1/2*c)+5/4/d*b^3/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c
)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)-3/d*b/(a+b)^3/((
2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b
))^(1/2)-a-2*b)*a)^(1/2))-3/2/d*b^2/(a+b)^3/a/((2*(b*(a+b))^(1/2)-a-2*b)*a)
^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-3
/8/d*b^3/(a+b)^3/a^2/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2
*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+3/d*b^2/(a+b)^3/(b*(a+b))^
(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2
*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+3/2/d*b^3/(a+b)^3/a/(b*(a+b))^(1/2)/((2*(
b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^
(1/2)-a-2*b)*a)^(1/2))+3/8/d*b^4/(a+b)^3/a^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^
(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2
*b)*a)^(1/2))+3/d*b/(a+b)^3/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*ta
nh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+3/2/d*b^2/(a+b)^3/a/
((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+
b))^(1/2)+a+2*b)*a)^(1/2))+3/8/d*b^3/(a+b)^3/a^2/((2*(b*(a+b))^(1/2)+a+2*b)
*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))
+3/d*b^2/(a+b)^3/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan
(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+3/2/d*b^3/(a+b)
^3/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*
d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+3/8/d*b^4/(a+b)^3/a^2/(b*(a
+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)
/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/d/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*a^4 + 4*a^3*b + 2*a^2*b^2 - 2*(a^4*e^(10*c) + 2*a^3*b*e^(10*c) + a^
2*b^2*e^(10*c))*e^(10*d*x) - (6*a^4*e^(8*c) - 4*a^3*b*e^(8*c) + 2*a^2*b^2*e
```

```
^(8*c) + 15*a*b^3*e^(8*c) + 3*b^4*e^(8*c))*e^(8*d*x) - (4*a^4*e^(6*c) - 8*a
^3*b*e^(6*c) + 32*a^2*b^2*e^(6*c) - 25*a*b^3*e^(6*c) - 9*b^4*e^(6*c))*e^(6*
d*x) + (4*a^4*e^(4*c) - 8*a^3*b*e^(4*c) + 32*a^2*b^2*e^(4*c) - 25*a*b^3*e^(
4*c) - 9*b^4*e^(4*c))*e^(4*d*x) + (6*a^4*e^(2*c) - 4*a^3*b*e^(2*c) + 2*a^2*
b^2*e^(2*c) + 15*a*b^3*e^(2*c) + 3*b^4*e^(2*c))*e^(2*d*x))/((a^7*d*e^(9*c)
+ 5*a^6*b*d*e^(9*c) + 10*a^5*b^2*d*e^(9*c) + 10*a^4*b^3*d*e^(9*c) + 5*a^3*b
^4*d*e^(9*c) + a^2*b^5*d*e^(9*c))*e^(9*d*x) + 4*(a^7*d*e^(7*c) + 3*a^6*b*d*
e^(7*c) + 2*a^5*b^2*d*e^(7*c) - 2*a^4*b^3*d*e^(7*c) - 3*a^3*b^4*d*e^(7*c) -
 a^2*b^5*d*e^(7*c))*e^(7*d*x) + 2*(3*a^7*d*e^(5*c) + 7*a^6*b*d*e^(5*c) + 6*
a^5*b^2*d*e^(5*c) + 6*a^4*b^3*d*e^(5*c) + 7*a^3*b^4*d*e^(5*c) + 3*a^2*b^5*d
*e^(5*c))*e^(5*d*x) + 4*(a^7*d*e^(3*c) + 3*a^6*b*d*e^(3*c) + 2*a^5*b^2*d*e^
(3*c) - 2*a^4*b^3*d*e^(3*c) - 3*a^3*b^4*d*e^(3*c) - a^2*b^5*d*e^(3*c))*e^(3
*d*x) + (a^7*d*e^c + 5*a^6*b*d*e^c + 10*a^5*b^2*d*e^c + 10*a^4*b^3*d*e^c +
5*a^3*b^4*d*e^c + a^2*b^5*d*e^c)*e^(d*x)) + 1/2*integrate(3/2*((8*a^2*b*e^(
3*c) + 4*a*b^2*e^(3*c) + b^3*e^(3*c))*e^(3*d*x) + (8*a^2*b*e^c + 4*a*b^2*e^
c + b^3*e^c)*e^(d*x))/(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4 + (a
^6*e^(4*c) + 4*a^5*b*e^(4*c) + 6*a^4*b^2*e^(4*c) + 4*a^3*b^3*e^(4*c) + a^2*
b^4*e^(4*c))*e^(4*d*x) + 2*(a^6*e^(2*c) + 2*a^5*b*e^(2*c) - 2*a^3*b^3*e^(2*
c) - a^2*b^4*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 3.78261, size = 25858, normalized size = 167.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(8*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^10 + 80*(a^6 +
 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)*sinh(d*x + c)^9 + 8*(a^6 + 3*
a^5*b + 3*a^4*b^2 + a^3*b^3)*sinh(d*x + c)^10 + 4*(6*a^6 + 2*a^5*b - 2*a^4*
b^2 + 17*a^3*b^3 + 18*a^2*b^4 + 3*a*b^5)*cosh(d*x + c)^8 + 4*(6*a^6 + 2*a^5
*b - 2*a^4*b^2 + 17*a^3*b^3 + 18*a^2*b^4 + 3*a*b^5 + 90*(a^6 + 3*a^5*b + 3*
a^4*b^2 + a^3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 32*(30*(a^6 + 3*a^5*b
 + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^3 + (6*a^6 + 2*a^5*b - 2*a^4*b^2 + 17
*a^3*b^3 + 18*a^2*b^4 + 3*a*b^5)*cosh(d*x + c))*sinh(d*x + c)^7 + 4*(4*a^6
- 4*a^5*b + 24*a^4*b^2 + 7*a^3*b^3 - 34*a^2*b^4 - 9*a*b^5)*cosh(d*x + c)^6
+ 4*(4*a^6 - 4*a^5*b + 24*a^4*b^2 + 7*a^3*b^3 - 34*a^2*b^4 - 9*a*b^5 + 420*
(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^4 + 28*(6*a^6 + 2*a^5*b
 - 2*a^4*b^2 + 17*a^3*b^3 + 18*a^2*b^4 + 3*a*b^5)*cosh(d*x + c)^2)*sinh(d*x
 + c)^6 - 8*a^6 - 24*a^5*b - 24*a^4*b^2 - 8*a^3*b^3 + 8*(252*(a^6 + 3*a^5*b
 + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^5 + 28*(6*a^6 + 2*a^5*b - 2*a^4*b^2 +
 17*a^3*b^3 + 18*a^2*b^4 + 3*a*b^5)*cosh(d*x + c)^3 + 3*(4*a^6 - 4*a^5*b +
24*a^4*b^2 + 7*a^3*b^3 - 34*a^2*b^4 - 9*a*b^5)*cosh(d*x + c))*sinh(d*x + c)
^5 - 4*(4*a^6 - 4*a^5*b + 24*a^4*b^2 + 7*a^3*b^3 - 34*a^2*b^4 - 9*a*b^5)*co
sh(d*x + c)^4 + 4*(420*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^
6 - 4*a^6 + 4*a^5*b - 24*a^4*b^2 - 7*a^3*b^3 + 34*a^2*b^4 + 9*a*b^5 + 70*(6
*a^6 + 2*a^5*b - 2*a^4*b^2 + 17*a^3*b^3 + 18*a^2*b^4 + 3*a*b^5)*cosh(d*x +
c)^4 + 15*(4*a^6 - 4*a^5*b + 24*a^4*b^2 + 7*a^3*b^3 - 34*a^2*b^4 - 9*a*b^5)
*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(60*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3
*b^3)*cosh(d*x + c)^7 + 14*(6*a^6 + 2*a^5*b - 2*a^4*b^2 + 17*a^3*b^3 + 18*a
^2*b^4 + 3*a*b^5)*cosh(d*x + c)^5 + 5*(4*a^6 - 4*a^5*b + 24*a^4*b^2 + 7*a^3
*b^3 - 34*a^2*b^4 - 9*a*b^5)*cosh(d*x + c)^3 - (4*a^6 - 4*a^5*b + 24*a^4*b^
2 + 7*a^3*b^3 - 34*a^2*b^4 - 9*a*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(6
*a^6 + 2*a^5*b - 2*a^4*b^2 + 17*a^3*b^3 + 18*a^2*b^4 + 3*a*b^5)*cosh(d*x +
c)^2 + 4*(90*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^8 + 28*(6*
a^6 + 2*a^5*b - 2*a^4*b^2 + 17*a^3*b^3 + 18*a^2*b^4 + 3*a*b^5)*cosh(d*x + c
```

)^6 - 6*a^6 - 2*a^5*b + 2*a^4*b^2 - 17*a^3*b^3 - 18*a^2*b^4 - 3*a*b^5 + 15*
(4*a^6 - 4*a^5*b + 24*a^4*b^2 + 7*a^3*b^3 - 34*a^2*b^4 - 9*a*b^5)*cosh(d*x
+ c)^4 - 6*(4*a^6 - 4*a^5*b + 24*a^4*b^2 + 7*a^3*b^3 - 34*a^2*b^4 - 9*a*b^5
)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 3*((8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3
+ 6*a*b^4 + b^5)*cosh(d*x + c)^9 + 9*(8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6
*a*b^4 + b^5)*cosh(d*x + c)*sinh(d*x + c)^8 + (8*a^4*b + 20*a^3*b^2 + 17*a^
2*b^3 + 6*a*b^4 + b^5)*sinh(d*x + c)^9 + 4*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3
 - 4*a*b^4 - b^5)*cosh(d*x + c)^7 + 4*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*
a*b^4 - b^5 + 9*(8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*b^4 + b^5)*cosh(d*
x + c)^2)*sinh(d*x + c)^7 + 28*(3*(8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*
b^4 + b^5)*cosh(d*x + c)^3 + (8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4 - b
^5)*cosh(d*x + c))*sinh(d*x + c)^6 + 2*(24*a^4*b - 4*a^3*b^2 + 19*a^2*b^3 +
 10*a*b^4 + 3*b^5)*cosh(d*x + c)^5 + 2*(24*a^4*b - 4*a^3*b^2 + 19*a^2*b^3 +
 10*a*b^4 + 3*b^5 + 63*(8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*b^4 + b^5)*
cosh(d*x + c)^4 + 42*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4 - b^5)*cosh
(d*x + c)^2)*sinh(d*x + c)^5 + 2*(63*(8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6
*a*b^4 + b^5)*cosh(d*x + c)^5 + 70*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b
^4 - b^5)*cosh(d*x + c)^3 + 5*(24*a^4*b - 4*a^3*b^2 + 19*a^2*b^3 + 10*a*b^4
 + 3*b^5)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b
^3 - 4*a*b^4 - b^5)*cosh(d*x + c)^3 + 4*(21*(8*a^4*b + 20*a^3*b^2 + 17*a^2*
b^3 + 6*a*b^4 + b^5)*cosh(d*x + c)^6 + 8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*
a*b^4 - b^5 + 35*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4 - b^5)*cosh(d*x
 + c)^4 + 5*(24*a^4*b - 4*a^3*b^2 + 19*a^2*b^3 + 10*a*b^4 + 3*b^5)*cosh(d*x
 + c)^2)*sinh(d*x + c)^3 + 4*(9*(8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*b^
4 + b^5)*cosh(d*x + c)^7 + 21*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4 -
b^5)*cosh(d*x + c)^5 + 5*(24*a^4*b - 4*a^3*b^2 + 19*a^2*b^3 + 10*a*b^4 + 3*
b^5)*cosh(d*x + c)^3 + 3*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4 - b^5)*
cosh(d*x + c))*sinh(d*x + c)^2 + (8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*b
^4 + b^5)*cosh(d*x + c) + (9*(8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*b^4 +
 b^5)*cosh(d*x + c)^8 + 28*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4 - b^5
)*cosh(d*x + c)^6 + 8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*b^4 + b^5 + 10*
(24*a^4*b - 4*a^3*b^2 + 19*a^2*b^3 + 10*a*b^4 + 3*b^5)*cosh(d*x + c)^4 + 12
*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4 - b^5)*cosh(d*x + c)^2)*sinh(d*
x + c))*sqrt(-a^2 - a*b)*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x
+ c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^
2 + 2*(3*(a + b)*cosh(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*co
sh(d*x + c)^3 - (3*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3
 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 -
 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a + b)/((a + b)*cosh(
d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c
)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sin
h(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x
 + c) + a + b)) + 8*(10*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)
^9 + 4*(6*a^6 + 2*a^5*b - 2*a^4*b^2 + 17*a^3*b^3 + 18*a^2*b^4 + 3*a*b^5)*co
sh(d*x + c)^7 + 3*(4*a^6 - 4*a^5*b + 24*a^4*b^2 + 7*a^3*b^3 - 34*a^2*b^4 -
9*a*b^5)*cosh(d*x + c)^5 - 2*(4*a^6 - 4*a^5*b + 24*a^4*b^2 + 7*a^3*b^3 - 34
*a^2*b^4 - 9*a*b^5)*cosh(d*x + c)^3 - (6*a^6 + 2*a^5*b - 2*a^4*b^2 + 17*a^3
*b^3 + 18*a^2*b^4 + 3*a*b^5)*cosh(d*x + c))*sinh(d*x + c))/((a^9 + 6*a^8*b
+ 15*a^7*b^2 + 20*a^6*b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a^3*b^6)*d*cosh(d*x +
c)^9 + 9*(a^9 + 6*a^8*b + 15*a^7*b^2 + 20*a^6*b^3 + 15*a^5*b^4 + 6*a^4*b^5
+ a^3*b^6)*d*cosh(d*x + c)*sinh(d*x + c)^8 + (a^9 + 6*a^8*b + 15*a^7*b^2 +
20*a^6*b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a^3*b^6)*d*sinh(d*x + c)^9 + 4*(a^9 +
 4*a^8*b + 5*a^7*b^2 - 5*a^5*b^4 - 4*a^4*b^5 - a^3*b^6)*d*cosh(d*x + c)^7 +
 4*(9*(a^9 + 6*a^8*b + 15*a^7*b^2 + 20*a^6*b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a
^3*b^6)*d*cosh(d*x + c)^2 + (a^9 + 4*a^8*b + 5*a^7*b^2 - 5*a^5*b^4 - 4*a^4*
b^5 - a^3*b^6)*d)*sinh(d*x + c)^7 + 2*(3*a^9 + 10*a^8*b + 13*a^7*b^2 + 12*a
^6*b^3 + 13*a^5*b^4 + 10*a^4*b^5 + 3*a^3*b^6)*d*cosh(d*x + c)^5 + 28*(3*(a^
9 + 6*a^8*b + 15*a^7*b^2 + 20*a^6*b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a^3*b^6)*d
*cosh(d*x + c)^3 + (a^9 + 4*a^8*b + 5*a^7*b^2 - 5*a^5*b^4 - 4*a^4*b^5 - a^3

```
*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^6 + 2*(63*(a^9 + 6*a^8*b + 15*a^7*b^2
+ 20*a^6*b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^4 + 42*(a^
9 + 4*a^8*b + 5*a^7*b^2 - 5*a^5*b^4 - 4*a^4*b^5 - a^3*b^6)*d*cosh(d*x + c)^
2 + (3*a^9 + 10*a^8*b + 13*a^7*b^2 + 12*a^6*b^3 + 13*a^5*b^4 + 10*a^4*b^5 +
 3*a^3*b^6)*d)*sinh(d*x + c)^5 + 4*(a^9 + 4*a^8*b + 5*a^7*b^2 - 5*a^5*b^4 -
 4*a^4*b^5 - a^3*b^6)*d*cosh(d*x + c)^3 + 2*(63*(a^9 + 6*a^8*b + 15*a^7*b^2
 + 20*a^6*b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^5 + 70*(a
^9 + 4*a^8*b + 5*a^7*b^2 - 5*a^5*b^4 - 4*a^4*b^5 - a^3*b^6)*d*cosh(d*x + c)
^3 + 5*(3*a^9 + 10*a^8*b + 13*a^7*b^2 + 12*a^6*b^3 + 13*a^5*b^4 + 10*a^4*b^
5 + 3*a^3*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(21*(a^9 + 6*a^8*b + 15
*a^7*b^2 + 20*a^6*b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^6
 + 35*(a^9 + 4*a^8*b + 5*a^7*b^2 - 5*a^5*b^4 - 4*a^4*b^5 - a^3*b^6)*d*cosh(
d*x + c)^4 + 5*(3*a^9 + 10*a^8*b + 13*a^7*b^2 + 12*a^6*b^3 + 13*a^5*b^4 + 1
0*a^4*b^5 + 3*a^3*b^6)*d*cosh(d*x + c)^2 + (a^9 + 4*a^8*b + 5*a^7*b^2 - 5*a
^5*b^4 - 4*a^4*b^5 - a^3*b^6)*d)*sinh(d*x + c)^3 + (a^9 + 6*a^8*b + 15*a^7*
b^2 + 20*a^6*b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c) + 4*(9
*(a^9 + 6*a^8*b + 15*a^7*b^2 + 20*a^6*b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a^3*b^
6)*d*cosh(d*x + c)^7 + 21*(a^9 + 4*a^8*b + 5*a^7*b^2 - 5*a^5*b^4 - 4*a^4*b^
5 - a^3*b^6)*d*cosh(d*x + c)^5 + 5*(3*a^9 + 10*a^8*b + 13*a^7*b^2 + 12*a^6*
b^3 + 13*a^5*b^4 + 10*a^4*b^5 + 3*a^3*b^6)*d*cosh(d*x + c)^3 + 3*(a^9 + 4*a
^8*b + 5*a^7*b^2 - 5*a^5*b^4 - 4*a^4*b^5 - a^3*b^6)*d*cosh(d*x + c))*sinh(d
*x + c)^2 + (9*(a^9 + 6*a^8*b + 15*a^7*b^2 + 20*a^6*b^3 + 15*a^5*b^4 + 6*a^
4*b^5 + a^3*b^6)*d*cosh(d*x + c)^8 + 28*(a^9 + 4*a^8*b + 5*a^7*b^2 - 5*a^5*
b^4 - 4*a^4*b^5 - a^3*b^6)*d*cosh(d*x + c)^6 + 10*(3*a^9 + 10*a^8*b + 13*a^
7*b^2 + 12*a^6*b^3 + 13*a^5*b^4 + 10*a^4*b^5 + 3*a^3*b^6)*d*cosh(d*x + c)^4
 + 12*(a^9 + 4*a^8*b + 5*a^7*b^2 - 5*a^5*b^4 - 4*a^4*b^5 - a^3*b^6)*d*cosh(
d*x + c)^2 + (a^9 + 6*a^8*b + 15*a^7*b^2 + 20*a^6*b^3 + 15*a^5*b^4 + 6*a^4*
b^5 + a^3*b^6)*d)*sinh(d*x + c)), 1/8*(4*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b
^3)*cosh(d*x + c)^10 + 40*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x +
c)*sinh(d*x + c)^9 + 4*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sinh(d*x + c)^
10 + 2*(6*a^6 + 2*a^5*b - 2*a^4*b^2 + 17*a^3*b^3 + 18*a^2*b^4 + 3*a*b^5)*co
sh(d*x + c)^8 + 2*(6*a^6 + 2*a^5*b - 2*a^4*b^2 + 17*a^3*b^3 + 18*a^2*b^4 +
3*a*b^5 + 90*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^2)*sinh(d*
x + c)^8 + 16*(30*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^3 + (
6*a^6 + 2*a^5*b - 2*a^4*b^2 + 17*a^3*b^3 + 18*a^2*b^4 + 3*a*b^5)*cosh(d*x +
 c))*sinh(d*x + c)^7 + 2*(4*a^6 - 4*a^5*b + 24*a^4*b^2 + 7*a^3*b^3 - 34*a^2
*b^4 - 9*a*b^5)*cosh(d*x + c)^6 + 2*(4*a^6 - 4*a^5*b + 24*a^4*b^2 + 7*a^3*b
^3 - 34*a^2*b^4 - 9*a*b^5 + 420*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(
d*x + c)^4 + 28*(6*a^6 + 2*a^5*b - 2*a^4*b^2 + 17*a^3*b^3 + 18*a^2*b^4 + 3*
a*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^6 - 4*a^6 - 12*a^5*b - 12*a^4*b^2 - 4
*a^3*b^3 + 4*(252*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^5 + 2
8*(6*a^6 + 2*a^5*b - 2*a^4*b^2 + 17*a^3*b^3 + 18*a^2*b^4 + 3*a*b^5)*cosh(d*
x + c)^3 + 3*(4*a^6 - 4*a^5*b + 24*a^4*b^2 + 7*a^3*b^3 - 34*a^2*b^4 - 9*a*b
^5)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(4*a^6 - 4*a^5*b + 24*a^4*b^2 + 7*a^
3*b^3 - 34*a^2*b^4 - 9*a*b^5)*cosh(d*x + c)^4 + 2*(420*(a^6 + 3*a^5*b + 3*a
^4*b^2 + a^3*b^3)*cosh(d*x + c)^6 - 4*a^6 + 4*a^5*b - 24*a^4*b^2 - 7*a^3*b^
3 + 34*a^2*b^4 + 9*a*b^5 + 70*(6*a^6 + 2*a^5*b - 2*a^4*b^2 + 17*a^3*b^3 + 1
8*a^2*b^4 + 3*a*b^5)*cosh(d*x + c)^4 + 15*(4*a^6 - 4*a^5*b + 24*a^4*b^2 + 7
*a^3*b^3 - 34*a^2*b^4 - 9*a*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(60*(
a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^7 + 14*(6*a^6 + 2*a^5*b
- 2*a^4*b^2 + 17*a^3*b^3 + 18*a^2*b^4 + 3*a*b^5)*cosh(d*x + c)^5 + 5*(4*a^6
 - 4*a^5*b + 24*a^4*b^2 + 7*a^3*b^3 - 34*a^2*b^4 - 9*a*b^5)*cosh(d*x + c)^3
 - (4*a^6 - 4*a^5*b + 24*a^4*b^2 + 7*a^3*b^3 - 34*a^2*b^4 - 9*a*b^5)*cosh(d
*x + c))*sinh(d*x + c)^3 - 2*(6*a^6 + 2*a^5*b - 2*a^4*b^2 + 17*a^3*b^3 + 18
*a^2*b^4 + 3*a*b^5)*cosh(d*x + c)^2 + 2*(90*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^
3*b^3)*cosh(d*x + c)^8 + 28*(6*a^6 + 2*a^5*b - 2*a^4*b^2 + 17*a^3*b^3 + 18*
a^2*b^4 + 3*a*b^5)*cosh(d*x + c)^6 - 6*a^6 - 2*a^5*b + 2*a^4*b^2 - 17*a^3*b
^3 - 18*a^2*b^4 - 3*a*b^5 + 15*(4*a^6 - 4*a^5*b + 24*a^4*b^2 + 7*a^3*b^3 -
34*a^2*b^4 - 9*a*b^5)*cosh(d*x + c)^4 - 6*(4*a^6 - 4*a^5*b + 24*a^4*b^2 + 7
```

*a^3*b^3 - 34*a^2*b^4 - 9*a*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*((8*a
^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*b^4 + b^5)*cosh(d*x + c)^9 + 9*(8*a^4*
b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*b^4 + b^5)*cosh(d*x + c)*sinh(d*x + c)^8
+ (8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*b^4 + b^5)*sinh(d*x + c)^9 + 4*(
8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4 - b^5)*cosh(d*x + c)^7 + 4*(8*a^4
*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4 - b^5 + 9*(8*a^4*b + 20*a^3*b^2 + 17*a
^2*b^3 + 6*a*b^4 + b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 28*(3*(8*a^4*b +
 20*a^3*b^2 + 17*a^2*b^3 + 6*a*b^4 + b^5)*cosh(d*x + c)^3 + (8*a^4*b + 4*a^
3*b^2 - 7*a^2*b^3 - 4*a*b^4 - b^5)*cosh(d*x + c))*sinh(d*x + c)^6 + 2*(24*a
^4*b - 4*a^3*b^2 + 19*a^2*b^3 + 10*a*b^4 + 3*b^5)*cosh(d*x + c)^5 + 2*(24*a
^4*b - 4*a^3*b^2 + 19*a^2*b^3 + 10*a*b^4 + 3*b^5 + 63*(8*a^4*b + 20*a^3*b^2
 + 17*a^2*b^3 + 6*a*b^4 + b^5)*cosh(d*x + c)^4 + 42*(8*a^4*b + 4*a^3*b^2 -
7*a^2*b^3 - 4*a*b^4 - b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 2*(63*(8*a^4*
b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*b^4 + b^5)*cosh(d*x + c)^5 + 70*(8*a^4*b
+ 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4 - b^5)*cosh(d*x + c)^3 + 5*(24*a^4*b - 4*
a^3*b^2 + 19*a^2*b^3 + 10*a*b^4 + 3*b^5)*cosh(d*x + c))*sinh(d*x + c)^4 + 4
*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4 - b^5)*cosh(d*x + c)^3 + 4*(21*
(8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*b^4 + b^5)*cosh(d*x + c)^6 + 8*a^4
*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4 - b^5 + 35*(8*a^4*b + 4*a^3*b^2 - 7*a^
2*b^3 - 4*a*b^4 - b^5)*cosh(d*x + c)^4 + 5*(24*a^4*b - 4*a^3*b^2 + 19*a^2*b
^3 + 10*a*b^4 + 3*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 4*(9*(8*a^4*b + 2
0*a^3*b^2 + 17*a^2*b^3 + 6*a*b^4 + b^5)*cosh(d*x + c)^7 + 21*(8*a^4*b + 4*a
^3*b^2 - 7*a^2*b^3 - 4*a*b^4 - b^5)*cosh(d*x + c)^5 + 5*(24*a^4*b - 4*a^3*b
^2 + 19*a^2*b^3 + 10*a*b^4 + 3*b^5)*cosh(d*x + c)^3 + 3*(8*a^4*b + 4*a^3*b^
2 - 7*a^2*b^3 - 4*a*b^4 - b^5)*cosh(d*x + c))*sinh(d*x + c)^2 + (8*a^4*b +
20*a^3*b^2 + 17*a^2*b^3 + 6*a*b^4 + b^5)*cosh(d*x + c) + (9*(8*a^4*b + 20*a
^3*b^2 + 17*a^2*b^3 + 6*a*b^4 + b^5)*cosh(d*x + c)^8 + 28*(8*a^4*b + 4*a^3*
b^2 - 7*a^2*b^3 - 4*a*b^4 - b^5)*cosh(d*x + c)^6 + 8*a^4*b + 20*a^3*b^2 + 1
7*a^2*b^3 + 6*a*b^4 + b^5 + 10*(24*a^4*b - 4*a^3*b^2 + 19*a^2*b^3 + 10*a*b^
4 + 3*b^5)*cosh(d*x + c)^4 + 12*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4
- b^5)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(a^2 + a*b)*arctan(1/2*((a + b)*
cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*
x + c)^3 + (3*a - b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + 3*a - b)*
sinh(d*x + c))/sqrt(a^2 + a*b)) + 3*((8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6
*a*b^4 + b^5)*cosh(d*x + c)^9 + 9*(8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*
b^4 + b^5)*cosh(d*x + c)*sinh(d*x + c)^8 + (8*a^4*b + 20*a^3*b^2 + 17*a^2*b
^3 + 6*a*b^4 + b^5)*sinh(d*x + c)^9 + 4*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 -
4*a*b^4 - b^5)*cosh(d*x + c)^7 + 4*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b
^4 - b^5 + 9*(8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*b^4 + b^5)*cosh(d*x +
 c)^2)*sinh(d*x + c)^7 + 28*(3*(8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*b^4
 + b^5)*cosh(d*x + c)^3 + (8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4 - b^5)
*cosh(d*x + c))*sinh(d*x + c)^6 + 2*(24*a^4*b - 4*a^3*b^2 + 19*a^2*b^3 + 10
*a*b^4 + 3*b^5)*cosh(d*x + c)^5 + 2*(24*a^4*b - 4*a^3*b^2 + 19*a^2*b^3 + 10
*a*b^4 + 3*b^5 + 63*(8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*b^4 + b^5)*cos
h(d*x + c)^4 + 42*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4 - b^5)*cosh(d*
x + c)^2)*sinh(d*x + c)^5 + 2*(63*(8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*
b^4 + b^5)*cosh(d*x + c)^5 + 70*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4
- b^5)*cosh(d*x + c)^3 + 5*(24*a^4*b - 4*a^3*b^2 + 19*a^2*b^3 + 10*a*b^4 +
3*b^5)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3
- 4*a*b^4 - b^5)*cosh(d*x + c)^3 + 4*(21*(8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3
 + 6*a*b^4 + b^5)*cosh(d*x + c)^6 + 8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b
^4 - b^5 + 35*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4 - b^5)*cosh(d*x +
c)^4 + 5*(24*a^4*b - 4*a^3*b^2 + 19*a^2*b^3 + 10*a*b^4 + 3*b^5)*cosh(d*x +
c)^2)*sinh(d*x + c)^3 + 4*(9*(8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*b^4 +
 b^5)*cosh(d*x + c)^7 + 21*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4 - b^5
)*cosh(d*x + c)^5 + 5*(24*a^4*b - 4*a^3*b^2 + 19*a^2*b^3 + 10*a*b^4 + 3*b^5
)*cosh(d*x + c)^3 + 3*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4 - b^5)*cos
h(d*x + c))*sinh(d*x + c)^2 + (8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*b^4
+ b^5)*cosh(d*x + c) + (9*(8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*b^4 + b^

```
5)*cosh(d*x + c)^8 + 28*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4 - b^5)*c
osh(d*x + c)^6 + 8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*b^4 + b^5 + 10*(24
*a^4*b - 4*a^3*b^2 + 19*a^2*b^3 + 10*a*b^4 + 3*b^5)*cosh(d*x + c)^4 + 12*(8
*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4 - b^5)*cosh(d*x + c)^2)*sinh(d*x +
 c))*sqrt(a^2 + a*b)*arctan(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh(d*x +
 c))/a) + 4*(10*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^9 + 4*(
6*a^6 + 2*a^5*b - 2*a^4*b^2 + 17*a^3*b^3 + 18*a^2*b^4 + 3*a*b^5)*cosh(d*x +
 c)^7 + 3*(4*a^6 - 4*a^5*b + 24*a^4*b^2 + 7*a^3*b^3 - 34*a^2*b^4 - 9*a*b^5)
*cosh(d*x + c)^5 - 2*(4*a^6 - 4*a^5*b + 24*a^4*b^2 + 7*a^3*b^3 - 34*a^2*b^4
 - 9*a*b^5)*cosh(d*x + c)^3 - (6*a^6 + 2*a^5*b - 2*a^4*b^2 + 17*a^3*b^3 + 1
8*a^2*b^4 + 3*a*b^5)*cosh(d*x + c))*sinh(d*x + c))/((a^9 + 6*a^8*b + 15*a^7
*b^2 + 20*a^6*b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^9 + 9
*(a^9 + 6*a^8*b + 15*a^7*b^2 + 20*a^6*b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a^3*b^
6)*d*cosh(d*x + c)*sinh(d*x + c)^8 + (a^9 + 6*a^8*b + 15*a^7*b^2 + 20*a^6*b
^3 + 15*a^5*b^4 + 6*a^4*b^5 + a^3*b^6)*d*sinh(d*x + c)^9 + 4*(a^9 + 4*a^8*b
 + 5*a^7*b^2 - 5*a^5*b^4 - 4*a^4*b^5 - a^3*b^6)*d*cosh(d*x + c)^7 + 4*(9*(a
^9 + 6*a^8*b + 15*a^7*b^2 + 20*a^6*b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a^3*b^6)*
d*cosh(d*x + c)^2 + (a^9 + 4*a^8*b + 5*a^7*b^2 - 5*a^5*b^4 - 4*a^4*b^5 - a^
3*b^6)*d)*sinh(d*x + c)^7 + 2*(3*a^9 + 10*a^8*b + 13*a^7*b^2 + 12*a^6*b^3 +
 13*a^5*b^4 + 10*a^4*b^5 + 3*a^3*b^6)*d*cosh(d*x + c)^5 + 28*(3*(a^9 + 6*a^
8*b + 15*a^7*b^2 + 20*a^6*b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a^3*b^6)*d*cosh(d*
x + c)^3 + (a^9 + 4*a^8*b + 5*a^7*b^2 - 5*a^5*b^4 - 4*a^4*b^5 - a^3*b^6)*d*
cosh(d*x + c))*sinh(d*x + c)^6 + 2*(63*(a^9 + 6*a^8*b + 15*a^7*b^2 + 20*a^6
*b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^4 + 42*(a^9 + 4*a^
8*b + 5*a^7*b^2 - 5*a^5*b^4 - 4*a^4*b^5 - a^3*b^6)*d*cosh(d*x + c)^2 + (3*a
^9 + 10*a^8*b + 13*a^7*b^2 + 12*a^6*b^3 + 13*a^5*b^4 + 10*a^4*b^5 + 3*a^3*b
^6)*d)*sinh(d*x + c)^5 + 4*(a^9 + 4*a^8*b + 5*a^7*b^2 - 5*a^5*b^4 - 4*a^4*b
^5 - a^3*b^6)*d*cosh(d*x + c)^3 + 2*(63*(a^9 + 6*a^8*b + 15*a^7*b^2 + 20*a^
6*b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^5 + 70*(a^9 + 4*a
^8*b + 5*a^7*b^2 - 5*a^5*b^4 - 4*a^4*b^5 - a^3*b^6)*d*cosh(d*x + c)^3 + 5*(
3*a^9 + 10*a^8*b + 13*a^7*b^2 + 12*a^6*b^3 + 13*a^5*b^4 + 10*a^4*b^5 + 3*a^
3*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(21*(a^9 + 6*a^8*b + 15*a^7*b^2
 + 20*a^6*b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^6 + 35*(a
^9 + 4*a^8*b + 5*a^7*b^2 - 5*a^5*b^4 - 4*a^4*b^5 - a^3*b^6)*d*cosh(d*x + c)
^4 + 5*(3*a^9 + 10*a^8*b + 13*a^7*b^2 + 12*a^6*b^3 + 13*a^5*b^4 + 10*a^4*b^
5 + 3*a^3*b^6)*d*cosh(d*x + c)^2 + (a^9 + 4*a^8*b + 5*a^7*b^2 - 5*a^5*b^4 -
 4*a^4*b^5 - a^3*b^6)*d)*sinh(d*x + c)^3 + (a^9 + 6*a^8*b + 15*a^7*b^2 + 20
*a^6*b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c) + 4*(9*(a^9 +
6*a^8*b + 15*a^7*b^2 + 20*a^6*b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a^3*b^6)*d*cos
h(d*x + c)^7 + 21*(a^9 + 4*a^8*b + 5*a^7*b^2 - 5*a^5*b^4 - 4*a^4*b^5 - a^3*
b^6)*d*cosh(d*x + c)^5 + 5*(3*a^9 + 10*a^8*b + 13*a^7*b^2 + 12*a^6*b^3 + 13
*a^5*b^4 + 10*a^4*b^5 + 3*a^3*b^6)*d*cosh(d*x + c)^3 + 3*(a^9 + 4*a^8*b + 5
*a^7*b^2 - 5*a^5*b^4 - 4*a^4*b^5 - a^3*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^
2 + (9*(a^9 + 6*a^8*b + 15*a^7*b^2 + 20*a^6*b^3 + 15*a^5*b^4 + 6*a^4*b^5 +
a^3*b^6)*d*cosh(d*x + c)^8 + 28*(a^9 + 4*a^8*b + 5*a^7*b^2 - 5*a^5*b^4 - 4*
a^4*b^5 - a^3*b^6)*d*cosh(d*x + c)^6 + 10*(3*a^9 + 10*a^8*b + 13*a^7*b^2 +
12*a^6*b^3 + 13*a^5*b^4 + 10*a^4*b^5 + 3*a^3*b^6)*d*cosh(d*x + c)^4 + 12*(a
^9 + 4*a^8*b + 5*a^7*b^2 - 5*a^5*b^4 - 4*a^4*b^5 - a^3*b^6)*d*cosh(d*x + c)
^2 + (a^9 + 6*a^8*b + 15*a^7*b^2 + 20*a^6*b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a^
3*b^6)*d)*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 2.14369, size = 10338, normalized size = 67.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/32*(6*(3*(8*a^8*b*e^(4*c) + 36*a^7*b^2*e^(4*c) + 65*a^6*b^3*e^(4*c) + 60*
a^5*b^4*e^(4*c) + 30*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c) + a^2*b^7*e^(4*c))
*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(ar
ccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a
 + b)))) - (8*a^8*b*e^(4*c) + 36*a^7*b^2*e^(4*c) + 65*a^6*b^3*e^(4*c) + 60*
a^5*b^4*e^(4*c) + 30*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c) + a^2*b^7*e^(4*c))
*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(ar
ccos(-a/(a + b) + b/(a + b))))^3 - 9*(8*a^8*b*e^(4*c) + 36*a^7*b^2*e^(4*c)
+ 65*a^6*b^3*e^(4*c) + 60*a^5*b^4*e^(4*c) + 30*a^4*b^5*e^(4*c) + 8*a^3*b^6*
e^(4*c) + a^2*b^7*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))
))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_par
t(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b
/(a + b)))) + 3*(8*a^8*b*e^(4*c) + 36*a^7*b^2*e^(4*c) + 65*a^6*b^3*e^(4*c)
+ 60*a^5*b^4*e^(4*c) + 30*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c) + a^2*b^7*e^(
4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_pa
rt(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b)
+ b/(a + b)))) + 9*(8*a^8*b*e^(4*c) + 36*a^7*b^2*e^(4*c) + 65*a^6*b^3*e^(4*
c) + 60*a^5*b^4*e^(4*c) + 30*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c) + a^2*b^7*
e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag
_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b)
+ b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(8
*a^8*b*e^(4*c) + 36*a^7*b^2*e^(4*c) + 65*a^6*b^3*e^(4*c) + 60*a^5*b^4*e^(4*
c) + 30*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c) + a^2*b^7*e^(4*c))*cosh(1/2*ima
g_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b)
 + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3
*(8*a^8*b*e^(4*c) + 36*a^7*b^2*e^(4*c) + 65*a^6*b^3*e^(4*c) + 60*a^5*b^4*e^
(4*c) + 30*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c) + a^2*b^7*e^(4*c))*cos(1/2*r
eal_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a
+ b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 +
 (8*a^8*b*e^(4*c) + 36*a^7*b^2*e^(4*c) + 65*a^6*b^3*e^(4*c) + 60*a^5*b^4*e^
(4*c) + 30*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c) + a^2*b^7*e^(4*c))*sin(1/2*r
eal_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a
 + b) + b/(a + b))))^3 + (8*a^8*b*e^(4*c) + 36*a^7*b^2*e^(4*c) + 65*a^6*b^3
*e^(4*c) + 60*a^5*b^4*e^(4*c) + 30*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c) + a^
2*b^7*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*
real_part(arccos(-a/(a + b) + b/(a + b)))) - (8*a^8*b*e^(4*c) + 36*a^7*b^2*
e^(4*c) + 65*a^6*b^3*e^(4*c) + 60*a^5*b^4*e^(4*c) + 30*a^4*b^5*e^(4*c) + 8*
a^3*b^6*e^(4*c) + a^2*b^7*e^(4*c))*sin(1/2*real_part(arccos(-a/(a + b) + b/
(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*arctan((((a
^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)/(a^6*e^(4*c) + 4*a^5*b*e^(4
*c) + 6*a^4*b^2*e^(4*c) + 4*a^3*b^3*e^(4*c) + a^2*b^4*e^(4*c)))^(1/4)*cos(1
/2*arccos(-(a - b)/(a + b))) + e^(d*x))/(((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^
3*b^3 + a^2*b^4)/(a^6*e^(4*c) + 4*a^5*b*e^(4*c) + 6*a^4*b^2*e^(4*c) + 4*a^3
*b^3*e^(4*c) + a^2*b^4*e^(4*c)))^(1/4)*sin(1/2*arccos(-(a - b)/(a + b)))))/
(2*(a^5*e^(2*c) + 3*a^4*b*e^(2*c) + 3*a^3*b^2*e^(2*c) + a^2*b^3*e^(2*c))^2*
a*b + (a^6*e^(2*c) + 2*a^5*b*e^(2*c) - 2*a^3*b^3*e^(2*c) - a^2*b^4*e^(2*c))
```

```
*sqrt(-a*b)*abs(-a^5*e^(2*c) - 3*a^4*b*e^(2*c) - 3*a^3*b^2*e^(2*c) - a^2*b^
3*e^(2*c))) + 6*(3*(8*a^8*b*e^(4*c) + 36*a^7*b^2*e^(4*c) + 65*a^6*b^3*e^(4*
c) + 60*a^5*b^4*e^(4*c) + 30*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c) + a^2*b^7*
e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag
_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b
) + b/(a + b)))) - (8*a^8*b*e^(4*c) + 36*a^7*b^2*e^(4*c) + 65*a^6*b^3*e^(4*
c) + 60*a^5*b^4*e^(4*c) + 30*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c) + a^2*b^7*
e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real
_part(arccos(-a/(a + b) + b/(a + b))))^3 - 9*(8*a^8*b*e^(4*c) + 36*a^7*b^2*
e^(4*c) + 65*a^6*b^3*e^(4*c) + 60*a^5*b^4*e^(4*c) + 30*a^4*b^5*e^(4*c) + 8*
a^3*b^6*e^(4*c) + a^2*b^7*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/
(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*
real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a
+ b) + b/(a + b)))) + 3*(8*a^8*b*e^(4*c) + 36*a^7*b^2*e^(4*c) + 65*a^6*b^3*
e^(4*c) + 60*a^5*b^4*e^(4*c) + 30*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c) + a^2
*b^7*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2
*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/
(a + b) + b/(a + b)))) + 9*(8*a^8*b*e^(4*c) + 36*a^7*b^2*e^(4*c) + 65*a^6*b
^3*e^(4*c) + 60*a^5*b^4*e^(4*c) + 30*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c) +
a^2*b^7*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(
1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/
(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^
2 - 3*(8*a^8*b*e^(4*c) + 36*a^7*b^2*e^(4*c) + 65*a^6*b^3*e^(4*c) + 60*a^5*b
^4*e^(4*c) + 30*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c) + a^2*b^7*e^(4*c))*cosh
(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a
/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))
))^2 - 3*(8*a^8*b*e^(4*c) + 36*a^7*b^2*e^(4*c) + 65*a^6*b^3*e^(4*c) + 60*a^
5*b^4*e^(4*c) + 30*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c) + a^2*b^7*e^(4*c))*c
os(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arcco
s(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b
))))^3 + (8*a^8*b*e^(4*c) + 36*a^7*b^2*e^(4*c) + 65*a^6*b^3*e^(4*c) + 60*a^
5*b^4*e^(4*c) + 30*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c) + a^2*b^7*e^(4*c))*s
in(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arcc
os(-a/(a + b) + b/(a + b))))^3 + (8*a^8*b*e^(4*c) + 36*a^7*b^2*e^(4*c) + 65
*a^6*b^3*e^(4*c) + 60*a^5*b^4*e^(4*c) + 30*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4
*c) + a^2*b^7*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*
sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) - (8*a^8*b*e^(4*c) + 36*
a^7*b^2*e^(4*c) + 65*a^6*b^3*e^(4*c) + 60*a^5*b^4*e^(4*c) + 30*a^4*b^5*e^(4
*c) + 8*a^3*b^6*e^(4*c) + a^2*b^7*e^(4*c))*sin(1/2*real_part(arccos(-a/(a +
 b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*arc
tan(-(((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)/(a^6*e^(4*c) + 4*a
^5*b*e^(4*c) + 6*a^4*b^2*e^(4*c) + 4*a^3*b^3*e^(4*c) + a^2*b^4*e^(4*c)))^(1
/4)*cos(1/2*arccos(-(a - b)/(a + b))) - e^(d*x))/(((a^6 + 4*a^5*b + 6*a^4*b
^2 + 4*a^3*b^3 + a^2*b^4)/(a^6*e^(4*c) + 4*a^5*b*e^(4*c) + 6*a^4*b^2*e^(4*c
) + 4*a^3*b^3*e^(4*c) + a^2*b^4*e^(4*c)))^(1/4)*sin(1/2*arccos(-(a - b)/(a
+ b)))))/(2*(a^5*e^(2*c) + 3*a^4*b*e^(2*c) + 3*a^3*b^2*e^(2*c) + a^2*b^3*e^
(2*c))^2*a*b + (a^6*e^(2*c) + 2*a^5*b*e^(2*c) - 2*a^3*b^3*e^(2*c) - a^2*b^4
*e^(2*c))*sqrt(-a*b)*abs(-a^5*e^(2*c) - 3*a^4*b*e^(2*c) - 3*a^3*b^2*e^(2*c)
 - a^2*b^3*e^(2*c))) + 3*((8*a^8*b*e^(4*c) + 36*a^7*b^2*e^(4*c) + 65*a^6*b^
3*e^(4*c) + 60*a^5*b^4*e^(4*c) + 30*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c) + a
^2*b^7*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1
/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 - 3*(8*a^8*b*e^(4*c) + 36*a
^7*b^2*e^(4*c) + 65*a^6*b^3*e^(4*c) + 60*a^5*b^4*e^(4*c) + 30*a^4*b^5*e^(4*
c) + 8*a^3*b^6*e^(4*c) + a^2*b^7*e^(4*c))*cos(1/2*real_part(arccos(-a/(a +
b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin
(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(8*a^8*b*e^(4*c) + 36
*a^7*b^2*e^(4*c) + 65*a^6*b^3*e^(4*c) + 60*a^5*b^4*e^(4*c) + 30*a^4*b^5*e^(
4*c) + 8*a^3*b^6*e^(4*c) + a^2*b^7*e^(4*c))*cos(1/2*real_part(arccos(-a/(a
+ b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2
```

```
*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(8*a^8*b*e^(4*c) +
 36*a^7*b^2*e^(4*c) + 65*a^6*b^3*e^(4*c) + 60*a^5*b^4*e^(4*c) + 30*a^4*b^5*
e^(4*c) + 8*a^3*b^6*e^(4*c) + a^2*b^7*e^(4*c))*cos(1/2*real_part(arccos(-a/
(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^
2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(a
rccos(-a/(a + b) + b/(a + b)))) + 3*(8*a^8*b*e^(4*c) + 36*a^7*b^2*e^(4*c) +
 65*a^6*b^3*e^(4*c) + 60*a^5*b^4*e^(4*c) + 30*a^4*b^5*e^(4*c) + 8*a^3*b^6*e
^(4*c) + a^2*b^7*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))
)^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(
arccos(-a/(a + b) + b/(a + b))))^2 - 9*(8*a^8*b*e^(4*c) + 36*a^7*b^2*e^(4*c
) + 65*a^6*b^3*e^(4*c) + 60*a^5*b^4*e^(4*c) + 30*a^4*b^5*e^(4*c) + 8*a^3*b^
6*e^(4*c) + a^2*b^7*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b
))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(
arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b
/(a + b))))^2 - (8*a^8*b*e^(4*c) + 36*a^7*b^2*e^(4*c) + 65*a^6*b^3*e^(4*c)
+ 60*a^5*b^4*e^(4*c) + 30*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c) + a^2*b^7*e^(
4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_pa
rt(arccos(-a/(a + b) + b/(a + b))))^3 + 3*(8*a^8*b*e^(4*c) + 36*a^7*b^2*e^(
4*c) + 65*a^6*b^3*e^(4*c) + 60*a^5*b^4*e^(4*c) + 30*a^4*b^5*e^(4*c) + 8*a^3
*b^6*e^(4*c) + a^2*b^7*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a
+ b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_
part(arccos(-a/(a + b) + b/(a + b))))^3 + (8*a^8*b*e^(4*c) + 36*a^7*b^2*e^(
4*c) + 65*a^6*b^3*e^(4*c) + 60*a^5*b^4*e^(4*c) + 30*a^4*b^5*e^(4*c) + 8*a^3
*b^6*e^(4*c) + a^2*b^7*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a
+ b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) - (8*a^8*b*e^(4
*c) + 36*a^7*b^2*e^(4*c) + 65*a^6*b^3*e^(4*c) + 60*a^5*b^4*e^(4*c) + 30*a^4
*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c) + a^2*b^7*e^(4*c))*cos(1/2*real_part(arcco
s(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b
)))))*log(2*((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)/(a^6*e^(4*c)
 + 4*a^5*b*e^(4*c) + 6*a^4*b^2*e^(4*c) + 4*a^3*b^3*e^(4*c) + a^2*b^4*e^(4*c
)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b)))*e^(d*x) + sqrt((a^6 + 4*a^5*b +
 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)/(a^6*e^(4*c) + 4*a^5*b*e^(4*c) + 6*a^4*b^
2*e^(4*c) + 4*a^3*b^3*e^(4*c) + a^2*b^4*e^(4*c))) + e^(2*d*x))/(2*(a^5*e^(2
*c) + 3*a^4*b*e^(2*c) + 3*a^3*b^2*e^(2*c) + a^2*b^3*e^(2*c))^2*a*b + (a^6*e
^(2*c) + 2*a^5*b*e^(2*c) - 2*a^3*b^3*e^(2*c) - a^2*b^4*e^(2*c))*sqrt(-a*b)*
abs(-a^5*e^(2*c) - 3*a^4*b*e^(2*c) - 3*a^3*b^2*e^(2*c) - a^2*b^3*e^(2*c)))
- 3*((8*a^8*b*e^(4*c) + 36*a^7*b^2*e^(4*c) + 65*a^6*b^3*e^(4*c) + 60*a^5*b^
4*e^(4*c) + 30*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c) + a^2*b^7*e^(4*c))*cos(1
/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-
a/(a + b) + b/(a + b))))^3 - 3*(8*a^8*b*e^(4*c) + 36*a^7*b^2*e^(4*c) + 65*a
^6*b^3*e^(4*c) + 60*a^5*b^4*e^(4*c) + 30*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c
) + a^2*b^7*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cos
h(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos
(-a/(a + b) + b/(a + b))))^2 - 3*(8*a^8*b*e^(4*c) + 36*a^7*b^2*e^(4*c) + 65
*a^6*b^3*e^(4*c) + 60*a^5*b^4*e^(4*c) + 30*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4
*c) + a^2*b^7*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3
*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(a
rccos(-a/(a + b) + b/(a + b)))) + 9*(8*a^8*b*e^(4*c) + 36*a^7*b^2*e^(4*c) +
 65*a^6*b^3*e^(4*c) + 60*a^5*b^4*e^(4*c) + 30*a^4*b^5*e^(4*c) + 8*a^3*b^6*e
^(4*c) + a^2*b^7*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))
)*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(a
rccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/
(a + b)))) + 3*(8*a^8*b*e^(4*c) + 36*a^7*b^2*e^(4*c) + 65*a^6*b^3*e^(4*c) +
 60*a^5*b^4*e^(4*c) + 30*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c) + a^2*b^7*e^(4
*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_par
t(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b
/(a + b))))^2 - 9*(8*a^8*b*e^(4*c) + 36*a^7*b^2*e^(4*c) + 65*a^6*b^3*e^(4*c
) + 60*a^5*b^4*e^(4*c) + 30*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c) + a^2*b^7*e
^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_pa
```

```
rt(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b
/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - (8*a^
8*b*e^(4*c) + 36*a^7*b^2*e^(4*c) + 65*a^6*b^3*e^(4*c) + 60*a^5*b^4*e^(4*c)
+ 30*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c) + a^2*b^7*e^(4*c))*cos(1/2*real_pa
rt(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b)
+ b/(a + b))))^3 + 3*(8*a^8*b*e^(4*c) + 36*a^7*b^2*e^(4*c) + 65*a^6*b^3*e^(
4*c) + 60*a^5*b^4*e^(4*c) + 30*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c) + a^2*b^
7*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_
part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b
) + b/(a + b))))^3 + (8*a^8*b*e^(4*c) + 36*a^7*b^2*e^(4*c) + 65*a^6*b^3*e^(
4*c) + 60*a^5*b^4*e^(4*c) + 30*a^4*b^5*e^(4*c) + 8*a^3*b^6*e^(4*c) + a^2*b^
7*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag
_part(arccos(-a/(a + b) + b/(a + b)))) - (8*a^8*b*e^(4*c) + 36*a^7*b^2*e^(4
*c) + 65*a^6*b^3*e^(4*c) + 60*a^5*b^4*e^(4*c) + 30*a^4*b^5*e^(4*c) + 8*a^3*
b^6*e^(4*c) + a^2*b^7*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a +
 b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*log(-2*((a^6 +
4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)/(a^6*e^(4*c) + 4*a^5*b*e^(4*c) +
 6*a^4*b^2*e^(4*c) + 4*a^3*b^3*e^(4*c) + a^2*b^4*e^(4*c)))^(1/4)*cos(1/2*ar
ccos(-(a - b)/(a + b)))*e^(d*x) + sqrt((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b
^3 + a^2*b^4)/(a^6*e^(4*c) + 4*a^5*b*e^(4*c) + 6*a^4*b^2*e^(4*c) + 4*a^3*b^
3*e^(4*c) + a^2*b^4*e^(4*c))) + e^(2*d*x))/(2*(a^5*e^(2*c) + 3*a^4*b*e^(2*c
) + 3*a^3*b^2*e^(2*c) + a^2*b^3*e^(2*c))^2*a*b + (a^6*e^(2*c) + 2*a^5*b*e^(
2*c) - 2*a^3*b^3*e^(2*c) - a^2*b^4*e^(2*c))*sqrt(-a*b)*abs(-a^5*e^(2*c) - 3
*a^4*b*e^(2*c) - 3*a^3*b^2*e^(2*c) - a^2*b^3*e^(2*c))) + 16*e^(d*x + 14*c)/
(a^3*e^(13*c) + 3*a^2*b*e^(13*c) + 3*a*b^2*e^(13*c) + b^3*e^(13*c)) - 16*e^
(-d*x)/(a^3*e^c + 3*a^2*b*e^c + 3*a*b^2*e^c + b^3*e^c) + 8*(12*a^2*b^2*e^(7
*d*x + 7*c) + 15*a*b^3*e^(7*d*x + 7*c) + 3*b^4*e^(7*d*x + 7*c) + 12*a^2*b^2
*e^(5*d*x + 5*c) - 25*a*b^3*e^(5*d*x + 5*c) - 9*b^4*e^(5*d*x + 5*c) - 12*a^
2*b^2*e^(3*d*x + 3*c) + 25*a*b^3*e^(3*d*x + 3*c) + 9*b^4*e^(3*d*x + 3*c) -
12*a^2*b^2*e^(d*x + c) - 15*a*b^3*e^(d*x + c) - 3*b^4*e^(d*x + c))/((a^5 +
3*a^4*b + 3*a^3*b^2 + a^2*b^3)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a
*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2))/d
```

3.127 $\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=144

$$\frac{\left(8a^2+8ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a+b)^{5/2}}+\frac{3b(2a+b)\sinh(c+dx)}{8a^2d(a+b)^2\left((a+b)\sinh^2(c+dx)+a\right)}+\frac{b\sinh(c+dx)\cosh^2(c+}{4ad(a+b)\left((a+b)\sinh^2(c+d\right.}$$

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^
(5/2)*(a + b)^(5/2)*d) + (b*Cosh[c + d*x]^2*Sinh[c + d*x])/(4*a*(a + b)*d*(
a + (a + b)*Sinh[c + d*x]^2)^2) + (3*b*(2*a + b)*Sinh[c + d*x])/(8*a^2*(a +
 b)^2*d*(a + (a + b)*Sinh[c + d*x]^2))
```

Rubi [A] time = 0.141426, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3676, 413, 385, 205}

$$\frac{\left(8a^2+8ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a+b)^{5/2}}+\frac{3b(2a+b)\sinh(c+dx)}{8a^2d(a+b)^2\left((a+b)\sinh^2(c+dx)+a\right)}+\frac{b\sinh(c+dx)\cosh^2(c+}{4ad(a+b)\left((a+b)\sinh^2(c+d\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^
(5/2)*(a + b)^(5/2)*d) + (b*Cosh[c + d*x]^2*Sinh[c + d*x])/(4*a*(a + b)*d*(
a + (a + b)*Sinh[c + d*x]^2)^2) + (3*b*(2*a + b)*Sinh[c + d*x])/(8*a^2*(a +
 b)^2*d*(a + (a + b)*Sinh[c + d*x]^2))
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{\left(a+(a+b)x^2\right)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{b\cosh^2(c+dx)\sinh(c+dx)}{4a(a+b)d\left(a+(a+b)\sinh^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a+3b+(4a+b)x^2}{\left(a+(a+b)x^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{4a(a+b)d}$$

$$= \frac{b\cosh^2(c+dx)\sinh(c+dx)}{4a(a+b)d\left(a+(a+b)\sinh^2(c+dx)\right)^2} + \frac{3b(2a+b)\sinh(c+dx)}{8a^2(a+b)^2d\left(a+(a+b)\sinh^2(c+dx)\right)} + \frac{(8a}{}$$

$$= \frac{\left(8a^2+8ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{5/2}d} + \frac{b\cosh^2(c+dx)\sinh(c+dx)}{4a(a+b)d\left(a+(a+b)\sinh^2(c+dx)\right)^2} + \frac{}{8a}$$

Mathematica [A] time = 0.943102, size = 134, normalized size = 0.93

$$\frac{\frac{2\sqrt{a}b\sinh(c+dx)\left(\left(8a^2+11ab+3b^2\right)\cosh(2(c+dx))+8a^2-ab-3b^2\right)}{(a+b)^2((a+b)\cosh(2(c+dx))+a-b)^2} - \frac{\left(8a^2+8ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}}{8a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (-(((8*a^2 + 8*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]])/(a
 + b)^(5/2)) + (2*Sqrt[a]*b*(8*a^2 - a*b - 3*b^2 + (8*a^2 + 11*a*b + 3*b^2)
*Cosh[2*(c + d*x)])*Sinh[c + d*x])/((a + b)^2*(a - b + (a + b)*Cosh[2*(c +
d*x)])^2))/(8*a^(5/2)*d)
```

Maple [B] time = 0.086, size = 1676, normalized size = 11.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] -2/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*
c)^2*b+a)^2*b/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7-5/4/d/(tanh(1/2*d*x+1/2
*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*b^2/a/(a^2
+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7-2/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d
*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*b/(a^2+2*a*b+b^2)*tanh(1/2*d*x
+1/2*c)^5-29/4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(
1/2*d*x+1/2*c)^2*b+a)^2/a*b^2/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5-3/d/(ta
nh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a
```

```
)^2/a^2*b^3/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5+2/d/(tanh(1/2*d*x+1/2*c)^
4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*b/(a^2+2*a*b+b
^2)*tanh(1/2*d*x+1/2*c)^3+29/4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/
2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*b^2/(a^2+2*a*b+b^2)*tanh(1/2*d*x+
1/2*c)^3+3/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*
d*x+1/2*c)^2*b+a)^2/a^2*b^3/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3+2/d/(tanh
(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^
2*b/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)+5/4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*ta
nh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*b^2/a/(a^2+2*a*b+b^2)*
tanh(1/2*d*x+1/2*c)-1/d/(a^2+2*a*b+b^2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)
*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-1/d/a/(
a^2+2*a*b+b^2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1
/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))*b-3/8/d/a^2/(a^2+2*a*b+b^2)/((2*
(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))
^(1/2)-a-2*b)*a)^(1/2))*b^2+1/d/(a^2+2*a*b+b^2)/(b*(a+b))^(1/2)/((2*(b*(a+b
))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-
a-2*b)*a)^(1/2))*b+1/d/a/(a^2+2*a*b+b^2)/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2
)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*
a)^(1/2))*b^2+3/8/d/a^2/(a^2+2*a*b+b^2)/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)
-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a
)^(1/2))*b^3+1/d/(a^2+2*a*b+b^2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan
(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/d/a/(a^2+2*a*
b+b^2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2
*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))*b+3/8/d/a^2/(a^2+2*a*b+b^2)/((2*(b*(a+b))
^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2
*b)*a)^(1/2))*b^2+1/d/(a^2+2*a*b+b^2)/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a
+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(
1/2))*b+1/d/a/(a^2+2*a*b+b^2)/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)
^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))*b^
2+3/8/d/a^2/(a^2+2*a*b+b^2)/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(
1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((8*a^2*b*e^(7*c) + 11*a*b^2*e^(7*c) + 3*b^3*e^(7*c))*e^(7*d*x) + (8*a^
2*b*e^(5*c) - 13*a*b^2*e^(5*c) - 9*b^3*e^(5*c))*e^(5*d*x) - (8*a^2*b*e^(3*c
) - 13*a*b^2*e^(3*c) - 9*b^3*e^(3*c))*e^(3*d*x) - (8*a^2*b*e^c + 11*a*b^2*e
^c + 3*b^3*e^c)*e^(d*x))/(a^6*d + 4*a^5*b*d + 6*a^4*b^2*d + 4*a^3*b^3*d + a
^2*b^4*d + (a^6*d*e^(8*c) + 4*a^5*b*d*e^(8*c) + 6*a^4*b^2*d*e^(8*c) + 4*a^3
*b^3*d*e^(8*c) + a^2*b^4*d*e^(8*c))*e^(8*d*x) + 4*(a^6*d*e^(6*c) + 2*a^5*b*
d*e^(6*c) - 2*a^3*b^3*d*e^(6*c) - a^2*b^4*d*e^(6*c))*e^(6*d*x) + 2*(3*a^6*d
*e^(4*c) + 4*a^5*b*d*e^(4*c) + 2*a^4*b^2*d*e^(4*c) + 4*a^3*b^3*d*e^(4*c) +
3*a^2*b^4*d*e^(4*c))*e^(4*d*x) + 4*(a^6*d*e^(2*c) + 2*a^5*b*d*e^(2*c) - 2*a
^3*b^3*d*e^(2*c) - a^2*b^4*d*e^(2*c))*e^(2*d*x)) + 2*integrate(1/8*((8*a^2*
e^(3*c) + 8*a*b*e^(3*c) + 3*b^2*e^(3*c))*e^(3*d*x) + (8*a^2*e^c + 8*a*b*e^c
 + 3*b^2*e^c)*e^(d*x))/(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 + (a^5*e^(4*c)
+ 3*a^4*b*e^(4*c) + 3*a^3*b^2*e^(4*c) + a^2*b^3*e^(4*c))*e^(4*d*x) + 2*(a^5
*e^(2*c) + a^4*b*e^(2*c) - a^3*b^2*e^(2*c) - a^2*b^3*e^(2*c))*e^(2*d*x)), x
)
```

Fricas [B] time = 3.184, size = 18148, normalized size = 126.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(8*a^4*b + 19*a^3*b^2 + 14*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^7 + 28
*(8*a^4*b + 19*a^3*b^2 + 14*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^
6 + 4*(8*a^4*b + 19*a^3*b^2 + 14*a^2*b^3 + 3*a*b^4)*sinh(d*x + c)^7 + 4*(8*
a^4*b - 5*a^3*b^2 - 22*a^2*b^3 - 9*a*b^4)*cosh(d*x + c)^5 + 4*(8*a^4*b - 5*
a^3*b^2 - 22*a^2*b^3 - 9*a*b^4 + 21*(8*a^4*b + 19*a^3*b^2 + 14*a^2*b^3 + 3*
a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(7*(8*a^4*b + 19*a^3*b^2 + 14*
a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^3 + (8*a^4*b - 5*a^3*b^2 - 22*a^2*b^3 - 9*
a*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 - 4*(8*a^4*b - 5*a^3*b^2 - 22*a^2*b^3
 - 9*a*b^4)*cosh(d*x + c)^3 - 4*(8*a^4*b - 5*a^3*b^2 - 22*a^2*b^3 - 9*a*b^4
 - 35*(8*a^4*b + 19*a^3*b^2 + 14*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^4 - 10*(8
*a^4*b - 5*a^3*b^2 - 22*a^2*b^3 - 9*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^3
 + 4*(21*(8*a^4*b + 19*a^3*b^2 + 14*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^5 + 10
*(8*a^4*b - 5*a^3*b^2 - 22*a^2*b^3 - 9*a*b^4)*cosh(d*x + c)^3 - 3*(8*a^4*b
- 5*a^3*b^2 - 22*a^2*b^3 - 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 - ((8*a^
4 + 24*a^3*b + 27*a^2*b^2 + 14*a*b^3 + 3*b^4)*cosh(d*x + c)^8 + 8*(8*a^4 +
24*a^3*b + 27*a^2*b^2 + 14*a*b^3 + 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (
8*a^4 + 24*a^3*b + 27*a^2*b^2 + 14*a*b^3 + 3*b^4)*sinh(d*x + c)^8 + 4*(8*a^
4 + 8*a^3*b - 5*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^6 + 4*(8*a^4 + 8*a
^3*b - 5*a^2*b^2 - 8*a*b^3 - 3*b^4 + 7*(8*a^4 + 24*a^3*b + 27*a^2*b^2 + 14*
a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(8*a^4 + 24*a^3*b +
27*a^2*b^2 + 14*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + 3*(8*a^4 + 8*a^3*b - 5*a^2
*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(24*a^4 + 8*a^3*
b + 17*a^2*b^2 + 18*a*b^3 + 9*b^4)*cosh(d*x + c)^4 + 2*(35*(8*a^4 + 24*a^3*
b + 27*a^2*b^2 + 14*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 24*a^4 + 8*a^3*b + 17*
a^2*b^2 + 18*a*b^3 + 9*b^4 + 30*(8*a^4 + 8*a^3*b - 5*a^2*b^2 - 8*a*b^3 - 3*
b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*a^4 + 24*a^3*b + 27*a^2*b^2 + 14*
a*b^3 + 3*b^4 + 8*(7*(8*a^4 + 24*a^3*b + 27*a^2*b^2 + 14*a*b^3 + 3*b^4)*cos
h(d*x + c)^5 + 10*(8*a^4 + 8*a^3*b - 5*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x
+ c)^3 + (24*a^4 + 8*a^3*b + 17*a^2*b^2 + 18*a*b^3 + 9*b^4)*cosh(d*x + c))*
sinh(d*x + c)^3 + 4*(8*a^4 + 8*a^3*b - 5*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*
x + c)^2 + 4*(7*(8*a^4 + 24*a^3*b + 27*a^2*b^2 + 14*a*b^3 + 3*b^4)*cosh(d*x
 + c)^6 + 15*(8*a^4 + 8*a^3*b - 5*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^
4 + 8*a^4 + 8*a^3*b - 5*a^2*b^2 - 8*a*b^3 - 3*b^4 + 3*(24*a^4 + 8*a^3*b + 1
7*a^2*b^2 + 18*a*b^3 + 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((8*a^4
+ 24*a^3*b + 27*a^2*b^2 + 14*a*b^3 + 3*b^4)*cosh(d*x + c)^7 + 3*(8*a^4 + 8*
a^3*b - 5*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^5 + (24*a^4 + 8*a^3*b +
17*a^2*b^2 + 18*a*b^3 + 9*b^4)*cosh(d*x + c)^3 + (8*a^4 + 8*a^3*b - 5*a^2*b
^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 - a*b)*log(((
a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*
sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^
2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 - (3*a + b)*cosh(
d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c
)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x +
c))*sqrt(-a^2 - a*b) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x
 + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2
 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(
d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) - 4*(8*a^4*b +
19*a^3*b^2 + 14*a^2*b^3 + 3*a*b^4)*cosh(d*x + c) + 4*(7*(8*a^4*b + 19*a^3*b
^2 + 14*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^6 - 8*a^4*b - 19*a^3*b^2 - 14*a^2*
b^3 - 3*a*b^4 + 5*(8*a^4*b - 5*a^3*b^2 - 22*a^2*b^3 - 9*a*b^4)*cosh(d*x + c
)^4 - 3*(8*a^4*b - 5*a^3*b^2 - 22*a^2*b^3 - 9*a*b^4)*cosh(d*x + c)^2)*sinh(
```

d*x + c))/((a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*
d*cosh(d*x + c)^8 + 8*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4
+ a^3*b^5)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^8 + 5*a^7*b + 10*a^6*b^2 +
10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*sinh(d*x + c)^8 + 4*(a^8 + 3*a^7*b + 2*
a^6*b^2 - 2*a^5*b^3 - 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^6 + 4*(7*(a^8 +
5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^2
+ (a^8 + 3*a^7*b + 2*a^6*b^2 - 2*a^5*b^3 - 3*a^4*b^4 - a^3*b^5)*d)*sinh(d*x
+ c)^6 + 2*(3*a^8 + 7*a^7*b + 6*a^6*b^2 + 6*a^5*b^3 + 7*a^4*b^4 + 3*a^3*b^
5)*d*cosh(d*x + c)^4 + 8*(7*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^
4*b^4 + a^3*b^5)*d*cosh(d*x + c)^3 + 3*(a^8 + 3*a^7*b + 2*a^6*b^2 - 2*a^5*b
^3 - 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^8 + 5
*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^4 +
30*(a^8 + 3*a^7*b + 2*a^6*b^2 - 2*a^5*b^3 - 3*a^4*b^4 - a^3*b^5)*d*cosh(d*
x + c)^2 + (3*a^8 + 7*a^7*b + 6*a^6*b^2 + 6*a^5*b^3 + 7*a^4*b^4 + 3*a^3*b^5
)*d)*sinh(d*x + c)^4 + 4*(a^8 + 3*a^7*b + 2*a^6*b^2 - 2*a^5*b^3 - 3*a^4*b^4
- a^3*b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b
^3 + 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^5 + 10*(a^8 + 3*a^7*b + 2*a^6*b^2
- 2*a^5*b^3 - 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^3 + (3*a^8 + 7*a^7*b +
6*a^6*b^2 + 6*a^5*b^3 + 7*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x +
c)^3 + 4*(7*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)
*d*cosh(d*x + c)^6 + 15*(a^8 + 3*a^7*b + 2*a^6*b^2 - 2*a^5*b^3 - 3*a^4*b^4
- a^3*b^5)*d*cosh(d*x + c)^4 + 3*(3*a^8 + 7*a^7*b + 6*a^6*b^2 + 6*a^5*b^3 +
7*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^2 + (a^8 + 3*a^7*b + 2*a^6*b^2 - 2*
a^5*b^3 - 3*a^4*b^4 - a^3*b^5)*d)*sinh(d*x + c)^2 + (a^8 + 5*a^7*b + 10*a^6
*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d + 8*((a^8 + 5*a^7*b + 10*a^6*b^2
+ 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^7 + 3*(a^8 + 3*a^7*b +
2*a^6*b^2 - 2*a^5*b^3 - 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^5 + (3*a^8 +
7*a^7*b + 6*a^6*b^2 + 6*a^5*b^3 + 7*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^3
+ (a^8 + 3*a^7*b + 2*a^6*b^2 - 2*a^5*b^3 - 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x
+ c))*sinh(d*x + c)), 1/8*(2*(8*a^4*b + 19*a^3*b^2 + 14*a^2*b^3 + 3*a*b^4)*
cosh(d*x + c)^7 + 14*(8*a^4*b + 19*a^3*b^2 + 14*a^2*b^3 + 3*a*b^4)*cosh(d*x
+ c)*sinh(d*x + c)^6 + 2*(8*a^4*b + 19*a^3*b^2 + 14*a^2*b^3 + 3*a*b^4)*sin
h(d*x + c)^7 + 2*(8*a^4*b - 5*a^3*b^2 - 22*a^2*b^3 - 9*a*b^4)*cosh(d*x + c)
^5 + 2*(8*a^4*b - 5*a^3*b^2 - 22*a^2*b^3 - 9*a*b^4 + 21*(8*a^4*b + 19*a^3*b
^2 + 14*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 10*(7*(8*a^4*
b + 19*a^3*b^2 + 14*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^3 + (8*a^4*b - 5*a^3*b
^2 - 22*a^2*b^3 - 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 - 2*(8*a^4*b - 5*
a^3*b^2 - 22*a^2*b^3 - 9*a*b^4)*cosh(d*x + c)^3 - 2*(8*a^4*b - 5*a^3*b^2 -
22*a^2*b^3 - 9*a*b^4 - 35*(8*a^4*b + 19*a^3*b^2 + 14*a^2*b^3 + 3*a*b^4)*cos
h(d*x + c)^4 - 10*(8*a^4*b - 5*a^3*b^2 - 22*a^2*b^3 - 9*a*b^4)*cosh(d*x + c
)^2)*sinh(d*x + c)^3 + 2*(21*(8*a^4*b + 19*a^3*b^2 + 14*a^2*b^3 + 3*a*b^4)*
cosh(d*x + c)^5 + 10*(8*a^4*b - 5*a^3*b^2 - 22*a^2*b^3 - 9*a*b^4)*cosh(d*x
+ c)^3 - 3*(8*a^4*b - 5*a^3*b^2 - 22*a^2*b^3 - 9*a*b^4)*cosh(d*x + c))*sinh
(d*x + c)^2 + ((8*a^4 + 24*a^3*b + 27*a^2*b^2 + 14*a*b^3 + 3*b^4)*cosh(d*x
+ c)^8 + 8*(8*a^4 + 24*a^3*b + 27*a^2*b^2 + 14*a*b^3 + 3*b^4)*cosh(d*x + c)
*sinh(d*x + c)^7 + (8*a^4 + 24*a^3*b + 27*a^2*b^2 + 14*a*b^3 + 3*b^4)*sinh(
d*x + c)^8 + 4*(8*a^4 + 8*a^3*b - 5*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c
)^6 + 4*(8*a^4 + 8*a^3*b - 5*a^2*b^2 - 8*a*b^3 - 3*b^4 + 7*(8*a^4 + 24*a^3*
b + 27*a^2*b^2 + 14*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*
(8*a^4 + 24*a^3*b + 27*a^2*b^2 + 14*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + 3*(8*a
^4 + 8*a^3*b - 5*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^5
+ 2*(24*a^4 + 8*a^3*b + 17*a^2*b^2 + 18*a*b^3 + 9*b^4)*cosh(d*x + c)^4 + 2*
(35*(8*a^4 + 24*a^3*b + 27*a^2*b^2 + 14*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 24
*a^4 + 8*a^3*b + 17*a^2*b^2 + 18*a*b^3 + 9*b^4 + 30*(8*a^4 + 8*a^3*b - 5*a^
2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*a^4 + 24*a^3*
b + 27*a^2*b^2 + 14*a*b^3 + 3*b^4 + 8*(7*(8*a^4 + 24*a^3*b + 27*a^2*b^2 + 1
4*a*b^3 + 3*b^4)*cosh(d*x + c)^5 + 10*(8*a^4 + 8*a^3*b - 5*a^2*b^2 - 8*a*b^
3 - 3*b^4)*cosh(d*x + c)^3 + (24*a^4 + 8*a^3*b + 17*a^2*b^2 + 18*a*b^3 + 9*
b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(8*a^4 + 8*a^3*b - 5*a^2*b^2 - 8*a*

b^3 - 3*b^4)*cosh(d*x + c)^2 + 4*(7*(8*a^4 + 24*a^3*b + 27*a^2*b^2 + 14*a*b ^3 + 3*b^4)*cosh(d*x + c)^6 + 15*(8*a^4 + 8*a^3*b - 5*a^2*b^2 - 8*a*b^3 - 3 *b^4)*cosh(d*x + c)^4 + 8*a^4 + 8*a^3*b - 5*a^2*b^2 - 8*a*b^3 - 3*b^4 + 3*(24*a^4 + 8*a^3*b + 17*a^2*b^2 + 18*a*b^3 + 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((8*a^4 + 24*a^3*b + 27*a^2*b^2 + 14*a*b^3 + 3*b^4)*cosh(d*x + c)^7 + 3*(8*a^4 + 8*a^3*b - 5*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^5 + (24*a^4 + 8*a^3*b + 17*a^2*b^2 + 18*a*b^3 + 9*b^4)*cosh(d*x + c)^3 + (8*a^4 + 8*a^3*b - 5*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqr t(a^2 + a*b)*arctan(1/2*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)* sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (3*a - b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + 3*a - b)*sinh(d*x + c))/sqrt(a^2 + a*b)) + ((8*a^4 + 24*a^3*b + 27*a^2*b^2 + 14*a*b^3 + 3*b^4)*cosh(d*x + c)^8 + 8*(8*a^4 + 24 *a^3*b + 27*a^2*b^2 + 14*a*b^3 + 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (8* a^4 + 24*a^3*b + 27*a^2*b^2 + 14*a*b^3 + 3*b^4)*sinh(d*x + c)^8 + 4*(8*a^4 + 8*a^3*b - 5*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^6 + 4*(8*a^4 + 8*a^3 *b - 5*a^2*b^2 - 8*a*b^3 - 3*b^4 + 7*(8*a^4 + 24*a^3*b + 27*a^2*b^2 + 14*a* b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(8*a^4 + 24*a^3*b + 27 *a^2*b^2 + 14*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + 3*(8*a^4 + 8*a^3*b - 5*a^2*b ^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(24*a^4 + 8*a^3*b + 17*a^2*b^2 + 18*a*b^3 + 9*b^4)*cosh(d*x + c)^4 + 2*(35*(8*a^4 + 24*a^3*b + 27*a^2*b^2 + 14*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 24*a^4 + 8*a^3*b + 17*a^ 2*b^2 + 18*a*b^3 + 9*b^4 + 30*(8*a^4 + 8*a^3*b - 5*a^2*b^2 - 8*a*b^3 - 3*b^ 4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*a^4 + 24*a^3*b + 27*a^2*b^2 + 14*a* b^3 + 3*b^4 + 8*(7*(8*a^4 + 24*a^3*b + 27*a^2*b^2 + 14*a*b^3 + 3*b^4)*cosh(d*x + c)^5 + 10*(8*a^4 + 8*a^3*b - 5*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^3 + (24*a^4 + 8*a^3*b + 17*a^2*b^2 + 18*a*b^3 + 9*b^4)*cosh(d*x + c))*si nh(d*x + c)^3 + 4*(8*a^4 + 8*a^3*b - 5*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^2 + 4*(7*(8*a^4 + 24*a^3*b + 27*a^2*b^2 + 14*a*b^3 + 3*b^4)*cosh(d*x + c)^6 + 15*(8*a^4 + 8*a^3*b - 5*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^4 + 8*a^4 + 8*a^3*b - 5*a^2*b^2 - 8*a*b^3 - 3*b^4 + 3*(24*a^4 + 8*a^3*b + 17* a^2*b^2 + 18*a*b^3 + 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((8*a^4 + 24*a^3*b + 27*a^2*b^2 + 14*a*b^3 + 3*b^4)*cosh(d*x + c)^7 + 3*(8*a^4 + 8*a^ 3*b - 5*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^5 + (24*a^4 + 8*a^3*b + 17 *a^2*b^2 + 18*a*b^3 + 9*b^4)*cosh(d*x + c)^3 + (8*a^4 + 8*a^3*b - 5*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + a*b)*arctan(1/ 2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh(d*x + c))/a) - 2*(8*a^4*b + 19*a^3* b^2 + 14*a^2*b^3 + 3*a*b^4)*cosh(d*x + c) + 2*(7*(8*a^4*b + 19*a^3*b^2 + 14 *a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^6 - 8*a^4*b - 19*a^3*b^2 - 14*a^2*b^3 - 3 *a*b^4 + 5*(8*a^4*b - 5*a^3*b^2 - 22*a^2*b^3 - 9*a*b^4)*cosh(d*x + c)^4 - 3 *(8*a^4*b - 5*a^3*b^2 - 22*a^2*b^3 - 9*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^8 + 8*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b ^5)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5* b^3 + 5*a^4*b^4 + a^3*b^5)*d*sinh(d*x + c)^8 + 4*(a^8 + 3*a^7*b + 2*a^6*b^2 - 2*a^5*b^3 - 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^6 + 4*(7*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^2 + (a^8 + 3*a^7*b + 2*a^6*b^2 - 2*a^5*b^3 - 3*a^4*b^4 - a^3*b^5)*d)*sinh(d*x + c)^6 + 2*(3*a^8 + 7*a^7*b + 6*a^6*b^2 + 6*a^5*b^3 + 7*a^4*b^4 + 3*a^3*b^5)*d*co sh(d*x + c)^4 + 8*(7*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^3 + 3*(a^8 + 3*a^7*b + 2*a^6*b^2 - 2*a^5*b^3 - 3* a^4*b^4 - a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^4 + 30*(a^ 8 + 3*a^7*b + 2*a^6*b^2 - 2*a^5*b^3 - 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^ 2 + (3*a^8 + 7*a^7*b + 6*a^6*b^2 + 6*a^5*b^3 + 7*a^4*b^4 + 3*a^3*b^5)*d)*si nh(d*x + c)^4 + 4*(a^8 + 3*a^7*b + 2*a^6*b^2 - 2*a^5*b^3 - 3*a^4*b^4 - a^3* b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5* a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^5 + 10*(a^8 + 3*a^7*b + 2*a^6*b^2 - 2*a^ 5*b^3 - 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^3 + (3*a^8 + 7*a^7*b + 6*a^6*b ^2 + 6*a^5*b^3 + 7*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 +

```
4*(7*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*cosh
(d*x + c)^6 + 15*(a^8 + 3*a^7*b + 2*a^6*b^2 - 2*a^5*b^3 - 3*a^4*b^4 - a^3*b
^5)*d*cosh(d*x + c)^4 + 3*(3*a^8 + 7*a^7*b + 6*a^6*b^2 + 6*a^5*b^3 + 7*a^4*
b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^2 + (a^8 + 3*a^7*b + 2*a^6*b^2 - 2*a^5*b^3
 - 3*a^4*b^4 - a^3*b^5)*d)*sinh(d*x + c)^2 + (a^8 + 5*a^7*b + 10*a^6*b^2 +
10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d + 8*((a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a
^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^7 + 3*(a^8 + 3*a^7*b + 2*a^6*
b^2 - 2*a^5*b^3 - 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^5 + (3*a^8 + 7*a^7*b
 + 6*a^6*b^2 + 6*a^5*b^3 + 7*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^3 + (a^8
+ 3*a^7*b + 2*a^6*b^2 - 2*a^5*b^3 - 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c))*s
inh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.89588, size = 7417, normalized size = 51.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/32*(2*(3*(16*a^3*b + 16*a^2*b^2 + 6*a*b^3 - (8*a^3 - 5*a*b^2 - 3*b^3)*sqr
t(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag
_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b
) + b/(a + b)))) - (16*a^3*b + 16*a^2*b^2 + 6*a*b^3 - (8*a^3 - 5*a*b^2 - 3*
b^3)*sqrt(-a*b))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(
1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3 - 9*(16*a^3*b + 16*a^2*b^2
 + 6*a*b^3 - (8*a^3 - 5*a*b^2 - 3*b^3)*sqrt(-a*b))*cos(1/2*real_part(arccos
(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a +
b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_pa
rt(arccos(-a/(a + b) + b/(a + b)))) + 3*(16*a^3*b + 16*a^2*b^2 + 6*a*b^3 -
(8*a^3 - 5*a*b^2 - 3*b^3)*sqrt(-a*b))*cosh(1/2*imag_part(arccos(-a/(a + b)
+ b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(
1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(16*a^3*b + 16*a^2*b^2 +
 6*a*b^3 - (8*a^3 - 5*a*b^2 - 3*b^3)*sqrt(-a*b))*cos(1/2*real_part(arccos(-
a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)
)))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(a
rccos(-a/(a + b) + b/(a + b))))^2 - 3*(16*a^3*b + 16*a^2*b^2 + 6*a*b^3 - (8
*a^3 - 5*a*b^2 - 3*b^3)*sqrt(-a*b))*cosh(1/2*imag_part(arccos(-a/(a + b) +
b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*
imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(16*a^3*b + 16*a^2*b^2 + 6
*a*b^3 - (8*a^3 - 5*a*b^2 - 3*b^3)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/
(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))
*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (16*a^3*b + 16*a^2
*b^2 + 6*a*b^3 - (8*a^3 - 5*a*b^2 - 3*b^3)*sqrt(-a*b))*sin(1/2*real_part(ar
```

```
ccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(
a + b))))^3 + (16*a^3*b + 16*a^2*b^2 + 6*a*b^3 - (8*a^3 - 5*a*b^2 - 3*b^3)*
sqrt(-a*b))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*rea
l_part(arccos(-a/(a + b) + b/(a + b)))) - (16*a^3*b + 16*a^2*b^2 + 6*a*b^3
- (8*a^3 - 5*a*b^2 - 3*b^3)*sqrt(-a*b))*sin(1/2*real_part(arccos(-a/(a + b)
 + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*arctan
((((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)/(a^5*e^(4*c) + 3*a^4*b*e^(4*c) + 3
*a^3*b^2*e^(4*c) + a^2*b^3*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b))
) + e^(d*x))/(((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)/(a^5*e^(4*c) + 3*a^4*b
*e^(4*c) + 3*a^3*b^2*e^(4*c) + a^2*b^3*e^(4*c)))^(1/4)*sin(1/2*arccos(-(a -
 b)/(a + b)))))/(a^6*b + 3*a^5*b^2 + 3*a^4*b^3 + a^3*b^4) + 2*(3*(16*a^3*b
+ 16*a^2*b^2 + 6*a*b^3 - (8*a^3 - 5*a*b^2 - 3*b^3)*sqrt(-a*b))*cos(1/2*real
_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a +
b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) - (1
6*a^3*b + 16*a^2*b^2 + 6*a*b^3 - (8*a^3 - 5*a*b^2 - 3*b^3)*sqrt(-a*b))*cosh
(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(
-a/(a + b) + b/(a + b))))^3 - 9*(16*a^3*b + 16*a^2*b^2 + 6*a*b^3 - (8*a^3 -
 5*a*b^2 - 3*b^3)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a +
b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_
part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b)
+ b/(a + b)))) + 3*(16*a^3*b + 16*a^2*b^2 + 6*a*b^3 - (8*a^3 - 5*a*b^2 - 3*
b^3)*sqrt(-a*b))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(
1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(
-a/(a + b) + b/(a + b)))) + 9*(16*a^3*b + 16*a^2*b^2 + 6*a*b^3 - (8*a^3 - 5
*a*b^2 - 3*b^3)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b)
)))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part
(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/
(a + b))))^2 - 3*(16*a^3*b + 16*a^2*b^2 + 6*a*b^3 - (8*a^3 - 5*a*b^2 - 3*b^
3)*sqrt(-a*b))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*
real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(
a + b) + b/(a + b))))^2 - 3*(16*a^3*b + 16*a^2*b^2 + 6*a*b^3 - (8*a^3 - 5*a
*b^2 - 3*b^3)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))
)^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(a
rccos(-a/(a + b) + b/(a + b))))^3 + (16*a^3*b + 16*a^2*b^2 + 6*a*b^3 - (8*a
^3 - 5*a*b^2 - 3*b^3)*sqrt(-a*b))*sin(1/2*real_part(arccos(-a/(a + b) + b/(
a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (16*a^3
*b + 16*a^2*b^2 + 6*a*b^3 - (8*a^3 - 5*a*b^2 - 3*b^3)*sqrt(-a*b))*cosh(1/2*
imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a +
 b) + b/(a + b)))) - (16*a^3*b + 16*a^2*b^2 + 6*a*b^3 - (8*a^3 - 5*a*b^2 -
3*b^3)*sqrt(-a*b))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(
1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*arctan(-(((a^5 + 3*a^4*b +
3*a^3*b^2 + a^2*b^3)/(a^5*e^(4*c) + 3*a^4*b*e^(4*c) + 3*a^3*b^2*e^(4*c) + a
^2*b^3*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b))) - e^(d*x))/(((a^5
+ 3*a^4*b + 3*a^3*b^2 + a^2*b^3)/(a^5*e^(4*c) + 3*a^4*b*e^(4*c) + 3*a^3*b^2
*e^(4*c) + a^2*b^3*e^(4*c)))^(1/4)*sin(1/2*arccos(-(a - b)/(a + b)))))/(a^6
*b + 3*a^5*b^2 + 3*a^4*b^3 + a^3*b^4) + ((16*a^3*b + 16*a^2*b^2 + 6*a*b^3 -
 (8*a^3 - 5*a*b^2 - 3*b^3)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b)
+ b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 - 3*
(16*a^3*b + 16*a^2*b^2 + 6*a*b^3 - (8*a^3 - 5*a*b^2 - 3*b^3)*sqrt(-a*b))*co
s(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(
-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)
)))^2 - 3*(16*a^3*b + 16*a^2*b^2 + 6*a*b^3 - (8*a^3 - 5*a*b^2 - 3*b^3)*sqrt
(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_
part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b
) + b/(a + b)))) + 9*(16*a^3*b + 16*a^2*b^2 + 6*a*b^3 - (8*a^3 - 5*a*b^2 -
3*b^3)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(
1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-
a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)
))) + 3*(16*a^3*b + 16*a^2*b^2 + 6*a*b^3 - (8*a^3 - 5*a*b^2 - 3*b^3)*sqrt(-
```

```
a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_pa
rt(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) +
b/(a + b))))^2 - 9*(16*a^3*b + 16*a^2*b^2 + 6*a*b^3 - (8*a^3 - 5*a*b^2 - 3*
b^3)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/
2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a
 + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^
2 - (16*a^3*b + 16*a^2*b^2 + 6*a*b^3 - (8*a^3 - 5*a*b^2 - 3*b^3)*sqrt(-a*b)
)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(a
rccos(-a/(a + b) + b/(a + b))))^3 + 3*(16*a^3*b + 16*a^2*b^2 + 6*a*b^3 - (8
*a^3 - 5*a*b^2 - 3*b^3)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b
/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*i
mag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (16*a^3*b + 16*a^2*b^2 + 6*a*
b^3 - (8*a^3 - 5*a*b^2 - 3*b^3)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a
+ b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) - (
16*a^3*b + 16*a^2*b^2 + 6*a*b^3 - (8*a^3 - 5*a*b^2 - 3*b^3)*sqrt(-a*b))*cos
(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-
a/(a + b) + b/(a + b)))))*log(2*((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)/(a^5
*e^(4*c) + 3*a^4*b*e^(4*c) + 3*a^3*b^2*e^(4*c) + a^2*b^3*e^(4*c)))^(1/4)*co
s(1/2*arccos(-(a - b)/(a + b)))*e^(d*x) + sqrt((a^5 + 3*a^4*b + 3*a^3*b^2 +
 a^2*b^3)/(a^5*e^(4*c) + 3*a^4*b*e^(4*c) + 3*a^3*b^2*e^(4*c) + a^2*b^3*e^(4
*c))) + e^(2*d*x))/(a^6*b + 3*a^5*b^2 + 3*a^4*b^3 + a^3*b^4) - ((16*a^3*b +
 16*a^2*b^2 + 6*a*b^3 - (8*a^3 - 5*a*b^2 - 3*b^3)*sqrt(-a*b))*cos(1/2*real_
part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b
) + b/(a + b))))^3 - 3*(16*a^3*b + 16*a^2*b^2 + 6*a*b^3 - (8*a^3 - 5*a*b^2
- 3*b^3)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cos
h(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos
(-a/(a + b) + b/(a + b))))^2 - 3*(16*a^3*b + 16*a^2*b^2 + 6*a*b^3 - (8*a^3
- 5*a*b^2 - 3*b^3)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a +
 b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*ima
g_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(16*a^3*b + 16*a^2*b^2 + 6*a*b^
3 - (8*a^3 - 5*a*b^2 - 3*b^3)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a +
b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin
(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos
(-a/(a + b) + b/(a + b)))) + 3*(16*a^3*b + 16*a^2*b^2 + 6*a*b^3 - (8*a^3 -
5*a*b^2 - 3*b^3)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b
))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_pa
rt(arccos(-a/(a + b) + b/(a + b))))^2 - 9*(16*a^3*b + 16*a^2*b^2 + 6*a*b^3
- (8*a^3 - 5*a*b^2 - 3*b^3)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b)
 + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2
*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/
(a + b) + b/(a + b))))^2 - (16*a^3*b + 16*a^2*b^2 + 6*a*b^3 - (8*a^3 - 5*a*
b^2 - 3*b^3)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))
^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + 3*(16*a^3*b + 16
*a^2*b^2 + 6*a*b^3 - (8*a^3 - 5*a*b^2 - 3*b^3)*sqrt(-a*b))*cos(1/2*real_par
t(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/
(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (16*a^
3*b + 16*a^2*b^2 + 6*a*b^3 - (8*a^3 - 5*a*b^2 - 3*b^3)*sqrt(-a*b))*cos(1/2*
real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a
+ b) + b/(a + b)))) - (16*a^3*b + 16*a^2*b^2 + 6*a*b^3 - (8*a^3 - 5*a*b^2 -
 3*b^3)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh
(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*log(-2*((a^5 + 3*a^4*b + 3
*a^3*b^2 + a^2*b^3)/(a^5*e^(4*c) + 3*a^4*b*e^(4*c) + 3*a^3*b^2*e^(4*c) + a^
2*b^3*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b)))*e^(d*x) + sqrt((a^5
 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)/(a^5*e^(4*c) + 3*a^4*b*e^(4*c) + 3*a^3*b^
2*e^(4*c) + a^2*b^3*e^(4*c))) + e^(2*d*x))/(a^6*b + 3*a^5*b^2 + 3*a^4*b^3 +
 a^3*b^4) + 8*(8*a^2*b*e^(7*d*x + 7*c) + 11*a*b^2*e^(7*d*x + 7*c) + 3*b^3*e
^(7*d*x + 7*c) + 8*a^2*b*e^(5*d*x + 5*c) - 13*a*b^2*e^(5*d*x + 5*c) - 9*b^3
*e^(5*d*x + 5*c) - 8*a^2*b*e^(3*d*x + 3*c) + 13*a*b^2*e^(3*d*x + 3*c) + 9*b
^3*e^(3*d*x + 3*c) - 8*a^2*b*e^(d*x + c) - 11*a*b^2*e^(d*x + c) - 3*b^3*e^(
```

```
d*x + c))/((a^4 + 2*a^3*b + a^2*b^2)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c)
 + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2))/d
```

3.128 $\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=96

$$\frac{3\tanh(c+dx)}{8a^2d\left(a+b\tanh^2(c+dx)\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}d}+\frac{\tanh(c+dx)}{4ad\left(a+b\tanh^2(c+dx)\right)^2}$$

```
[Out] (3*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*Sqrt[b]*d) + Tanh[c
+ d*x]/(4*a*d*(a + b*Tanh[c + d*x]^2)^2) + (3*Tanh[c + d*x])/(8*a^2*d*(a +
b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.0754865, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3675, 199, 205}

$$\frac{3\tanh(c+dx)}{8a^2d\left(a+b\tanh^2(c+dx)\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}d}+\frac{\tanh(c+dx)}{4ad\left(a+b\tanh^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```


```
[Out] (3*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*Sqrt[b]*d) + Tanh[c
+ d*x]/(4*a*d*(a + b*Tanh[c + d*x]^2)^2) + (3*Tanh[c + d*x])/(8*a^2*d*(a +
b*Tanh[c + d*x]^2))
```


Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\tanh(c+dx)}{4ad\left(a+b\tanh^2(c+dx)\right)^2} + \frac{3\operatorname{Subst}\left(\int \frac{1}{(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{4ad}$$

$$= \frac{\tanh(c+dx)}{4ad\left(a+b\tanh^2(c+dx)\right)^2} + \frac{3\tanh(c+dx)}{8a^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{3\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh\right.}{8a^2d}$$

$$= \frac{3\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}d} + \frac{\tanh(c+dx)}{4ad\left(a+b\tanh^2(c+dx)\right)^2} + \frac{3\tanh(c+dx)}{8a^2d\left(a+b\tanh^2(c+dx)\right)}$$

Mathematica [A] time = 0.748001, size = 77, normalized size = 0.8

$$\frac{\frac{\tanh(c+dx)\left(5a+3b\tanh^2(c+dx)\right)}{a^2\left(a+b\tanh^2(c+dx)\right)^2} + \frac{3\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{b}}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((3*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(5/2)*Sqrt[b]) + (Tanh[c +
d*x]*(5*a + 3*b*Tanh[c + d*x]^2))/(a^2*(a + b*Tanh[c + d*x]^2)^2))/(8*d)
```

Maple [B] time = 0.108, size = 764, normalized size = 8.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 5/4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2
*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^7+15/4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh
(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^5+
3/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x
+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5+15/4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*ta
nh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^
3+3/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d
*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^3+5/4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*t
anh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)
-3/8/d/(b*(a+b))^(1/2)/a/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh
(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+3/8/d/a^2/((2*(b*(a+b)
)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a
-2*b)*a)^(1/2))-3/8/d/a^2*b/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(
1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-3/8
/d/(b*(a+b))^(1/2)/a/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*
d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-3/8/d/a^2/((2*(b*(a+b))^(1/
```

```
2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*
a)^(1/2))-3/8/d/a^2*b/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*a
rctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 3.04773, size = 13154, normalized size = 137.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [-1/16*(4*(5*a^4*b - a^3*b^2 - 9*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^6 + 24*(5
*a^4*b - a^3*b^2 - 9*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^5 + 4*(
5*a^4*b - a^3*b^2 - 9*a^2*b^3 - 3*a*b^4)*sinh(d*x + c)^6 + 20*a^4*b + 52*a^
3*b^2 + 44*a^2*b^3 + 12*a*b^4 + 4*(15*a^4*b - a^3*b^2 + 9*a^2*b^3 + 9*a*b^4
)*cosh(d*x + c)^4 + 4*(15*a^4*b - a^3*b^2 + 9*a^2*b^3 + 9*a*b^4 + 15*(5*a^4
*b - a^3*b^2 - 9*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(
5*(5*a^4*b - a^3*b^2 - 9*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^3 + (15*a^4*b - a
^3*b^2 + 9*a^2*b^3 + 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(15*a^4*b
+ 13*a^3*b^2 - 11*a^2*b^3 - 9*a*b^4)*cosh(d*x + c)^2 + 4*(15*a^4*b + 13*a^3
*b^2 - 11*a^2*b^3 - 9*a*b^4 + 15*(5*a^4*b - a^3*b^2 - 9*a^2*b^3 - 3*a*b^4)*
cosh(d*x + c)^4 + 6*(15*a^4*b - a^3*b^2 + 9*a^2*b^3 + 9*a*b^4)*cosh(d*x + c
)^2)*sinh(d*x + c)^2 + 3*((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(
d*x + c)^8 + 8*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)*si
nh(d*x + c)^7 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sinh(d*x + c)^8
 + 4*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c)^6 + 4*(a^4 + 2*a^3*b - 2
*a*b^3 - b^4 + 7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^
2)*sinh(d*x + c)^6 + 8*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(
d*x + c)^3 + 3*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c)
^5 + 2*(3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 2*
(35*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^4 + 3*a^4 + 4
*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4 + 30*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*c
osh(d*x + c)^2)*sinh(d*x + c)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4
 + 8*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^5 + 10*(a
^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c)^3 + (3*a^4 + 4*a^3*b + 2*a^2*b^
2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^4 + 2*a^3*b - 2*
a*b^3 - b^4)*cosh(d*x + c)^2 + 4*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 +
b^4)*cosh(d*x + c)^6 + 15*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c)^4 +
 a^4 + 2*a^3*b - 2*a*b^3 - b^4 + 3*(3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 +
 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^4 + 4*a^3*b + 6*a^2*b^2 +
4*a*b^3 + b^4)*cosh(d*x + c)^7 + 3*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x
 + c)^5 + (3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^3 +
 (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b)*l
og(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x +
```

c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*co
sh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(
d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (
a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(a
 + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-
a*b))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 +
(a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x
 + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cos
h(d*x + c))*sinh(d*x + c) + a + b)) + 8*(3*(5*a^4*b - a^3*b^2 - 9*a^2*b^3 -
 3*a*b^4)*cosh(d*x + c)^5 + 2*(15*a^4*b - a^3*b^2 + 9*a^2*b^3 + 9*a*b^4)*co
sh(d*x + c)^3 + (15*a^4*b + 13*a^3*b^2 - 11*a^2*b^3 - 9*a*b^4)*cosh(d*x + c
))*sinh(d*x + c))/((a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4 + a^3*b^5)*d*
cosh(d*x + c)^8 + 8*(a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4 + a^3*b^5)*d
*cosh(d*x + c)*sinh(d*x + c)^7 + (a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4
 + a^3*b^5)*d*sinh(d*x + c)^8 + 4*(a^7*b + 2*a^6*b^2 - 2*a^4*b^4 - a^3*b^5)
*d*cosh(d*x + c)^6 + 4*(7*(a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4 + a^3*
b^5)*d*cosh(d*x + c)^2 + (a^7*b + 2*a^6*b^2 - 2*a^4*b^4 - a^3*b^5)*d)*sinh(
d*x + c)^6 + 2*(3*a^7*b + 4*a^6*b^2 + 2*a^5*b^3 + 4*a^4*b^4 + 3*a^3*b^5)*d*
cosh(d*x + c)^4 + 8*(7*(a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4 + a^3*b^5
)*d*cosh(d*x + c)^3 + 3*(a^7*b + 2*a^6*b^2 - 2*a^4*b^4 - a^3*b^5)*d*cosh(d*
x + c))*sinh(d*x + c)^5 + 2*(35*(a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4
 + a^3*b^5)*d*cosh(d*x + c)^4 + 30*(a^7*b + 2*a^6*b^2 - 2*a^4*b^4 - a^3*b^5)
*d*cosh(d*x + c)^2 + (3*a^7*b + 4*a^6*b^2 + 2*a^5*b^3 + 4*a^4*b^4 + 3*a^3*b
^5)*d)*sinh(d*x + c)^4 + 4*(a^7*b + 2*a^6*b^2 - 2*a^4*b^4 - a^3*b^5)*d*cosh
(d*x + c)^2 + 8*(7*(a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4 + a^3*b^5)*d*
cosh(d*x + c)^5 + 10*(a^7*b + 2*a^6*b^2 - 2*a^4*b^4 - a^3*b^5)*d*cosh(d*x +
 c)^3 + (3*a^7*b + 4*a^6*b^2 + 2*a^5*b^3 + 4*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*
x + c))*sinh(d*x + c)^3 + 4*(7*(a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4 +
 a^3*b^5)*d*cosh(d*x + c)^6 + 15*(a^7*b + 2*a^6*b^2 - 2*a^4*b^4 - a^3*b^5)*
d*cosh(d*x + c)^4 + 3*(3*a^7*b + 4*a^6*b^2 + 2*a^5*b^3 + 4*a^4*b^4 + 3*a^3*
b^5)*d*cosh(d*x + c)^2 + (a^7*b + 2*a^6*b^2 - 2*a^4*b^4 - a^3*b^5)*d)*sinh(
d*x + c)^2 + (a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4 + a^3*b^5)*d + 8*((
a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^7 + 3*
(a^7*b + 2*a^6*b^2 - 2*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^5 + (3*a^7*b + 4*
a^6*b^2 + 2*a^5*b^3 + 4*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^3 + (a^7*b + 2
*a^6*b^2 - 2*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)), -1/8*(2*(5
*a^4*b - a^3*b^2 - 9*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^6 + 12*(5*a^4*b - a^3
*b^2 - 9*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(5*a^4*b - a^
3*b^2 - 9*a^2*b^3 - 3*a*b^4)*sinh(d*x + c)^6 + 10*a^4*b + 26*a^3*b^2 + 22*a
^2*b^3 + 6*a*b^4 + 2*(15*a^4*b - a^3*b^2 + 9*a^2*b^3 + 9*a*b^4)*cosh(d*x +
c)^4 + 2*(15*a^4*b - a^3*b^2 + 9*a^2*b^3 + 9*a*b^4 + 15*(5*a^4*b - a^3*b^2
 - 9*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(5*(5*a^4*b - a
^3*b^2 - 9*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^3 + (15*a^4*b - a^3*b^2 + 9*a^2
*b^3 + 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(15*a^4*b + 13*a^3*b^2 -
 11*a^2*b^3 - 9*a*b^4)*cosh(d*x + c)^2 + 2*(15*a^4*b + 13*a^3*b^2 - 11*a^2*
b^3 - 9*a*b^4 + 15*(5*a^4*b - a^3*b^2 - 9*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^
4 + 6*(15*a^4*b - a^3*b^2 + 9*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^2)*sinh(d*x
+ c)^2 - 3*((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^8 + 8
*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)*sinh(d*x + c)^7
+ (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sinh(d*x + c)^8 + 4*(a^4 + 2*
a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c)^6 + 4*(a^4 + 2*a^3*b - 2*a*b^3 - b^4 +
 7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^2)*sinh(d*x +
c)^6 + 8*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^3 + 3
*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^4
+ 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 2*(35*(a^4 + 4*a
^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^4 + 3*a^4 + 4*a^3*b + 2*a^2
*b^2 + 4*a*b^3 + 3*b^4 + 30*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c)^2
)*sinh(d*x + c)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 + 8*(7*(a^4 +
 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^5 + 10*(a^4 + 2*a^3*b -

```
 2*a*b^3 - b^4)*cosh(d*x + c)^3 + (3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 +
3*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*c
osh(d*x + c)^2 + 4*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x
+ c)^6 + 15*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c)^4 + a^4 + 2*a^3*b
 - 2*a*b^3 - b^4 + 3*(3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d
*x + c)^2)*sinh(d*x + c)^2 + 8*((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)
*cosh(d*x + c)^7 + 3*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c)^5 + (3*a
^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + (a^4 + 2*a^3*
b - 2*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b)*arctan(1/2*((a +
 b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(
d*x + c)^2 + a - b)*sqrt(a*b)/(a*b)) + 4*(3*(5*a^4*b - a^3*b^2 - 9*a^2*b^3
- 3*a*b^4)*cosh(d*x + c)^5 + 2*(15*a^4*b - a^3*b^2 + 9*a^2*b^3 + 9*a*b^4)*c
osh(d*x + c)^3 + (15*a^4*b + 13*a^3*b^2 - 11*a^2*b^3 - 9*a*b^4)*cosh(d*x +
c))*sinh(d*x + c))/((a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4 + a^3*b^5)*d
*cosh(d*x + c)^8 + 8*(a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4 + a^3*b^5)*
d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^
4 + a^3*b^5)*d*sinh(d*x + c)^8 + 4*(a^7*b + 2*a^6*b^2 - 2*a^4*b^4 - a^3*b^5
)*d*cosh(d*x + c)^6 + 4*(7*(a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4 + a^3
*b^5)*d*cosh(d*x + c)^2 + (a^7*b + 2*a^6*b^2 - 2*a^4*b^4 - a^3*b^5)*d)*sinh
(d*x + c)^6 + 2*(3*a^7*b + 4*a^6*b^2 + 2*a^5*b^3 + 4*a^4*b^4 + 3*a^3*b^5)*d
*cosh(d*x + c)^4 + 8*(7*(a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4 + a^3*b^
5)*d*cosh(d*x + c)^3 + 3*(a^7*b + 2*a^6*b^2 - 2*a^4*b^4 - a^3*b^5)*d*cosh(d
*x + c))*sinh(d*x + c)^5 + 2*(35*(a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4
 + a^3*b^5)*d*cosh(d*x + c)^4 + 30*(a^7*b + 2*a^6*b^2 - 2*a^4*b^4 - a^3*b^5
)*d*cosh(d*x + c)^2 + (3*a^7*b + 4*a^6*b^2 + 2*a^5*b^3 + 4*a^4*b^4 + 3*a^3*
b^5)*d)*sinh(d*x + c)^4 + 4*(a^7*b + 2*a^6*b^2 - 2*a^4*b^4 - a^3*b^5)*d*cos
h(d*x + c)^2 + 8*(7*(a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4 + a^3*b^5)*d
*cosh(d*x + c)^5 + 10*(a^7*b + 2*a^6*b^2 - 2*a^4*b^4 - a^3*b^5)*d*cosh(d*x
+ c)^3 + (3*a^7*b + 4*a^6*b^2 + 2*a^5*b^3 + 4*a^4*b^4 + 3*a^3*b^5)*d*cosh(d
*x + c))*sinh(d*x + c)^3 + 4*(7*(a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4
+ a^3*b^5)*d*cosh(d*x + c)^6 + 15*(a^7*b + 2*a^6*b^2 - 2*a^4*b^4 - a^3*b^5)
*d*cosh(d*x + c)^4 + 3*(3*a^7*b + 4*a^6*b^2 + 2*a^5*b^3 + 4*a^4*b^4 + 3*a^3
*b^5)*d*cosh(d*x + c)^2 + (a^7*b + 2*a^6*b^2 - 2*a^4*b^4 - a^3*b^5)*d)*sinh
(d*x + c)^2 + (a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4 + a^3*b^5)*d + 8*(
(a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^7 + 3
*(a^7*b + 2*a^6*b^2 - 2*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^5 + (3*a^7*b + 4
*a^6*b^2 + 2*a^5*b^3 + 4*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^3 + (a^7*b +
2*a^6*b^2 - 2*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2/(a+b*tanh(d*x+c)**2)**3,x)

[Out] Timed out
```

Giac [B] time = 1.91676, size = 432, normalized size = 4.5

$$\frac{\frac{3\arctan\left(\frac{ae^{(2dx+2c)}+be^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}a^2} - \frac{2\left(5a^3e^{(6dx+6c)}-a^2be^{(6dx+6c)}-9ab^2e^{(6dx+6c)}-3b^3e^{(6dx+6c)}+15a^3e^{(4dx+4c)}-a^2be^{(4dx+4c)}+9ab^2e^{(4dx+4c)}+9b^3e^{(4}\right.}{\left(a^4+2a^3b+a^2b^2\right)\left(ae^{(4dx+4c)}+be^{(4dx+4c)}+2ae\right.}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b)
)/(sqrt(a*b)*a^2) - 2*(5*a^3*e^(6*d*x + 6*c) - a^2*b*e^(6*d*x + 6*c) - 9*a*
b^2*e^(6*d*x + 6*c) - 3*b^3*e^(6*d*x + 6*c) + 15*a^3*e^(4*d*x + 4*c) - a^2*
b*e^(4*d*x + 4*c) + 9*a*b^2*e^(4*d*x + 4*c) + 9*b^3*e^(4*d*x + 4*c) + 15*a^
3*e^(2*d*x + 2*c) + 13*a^2*b*e^(2*d*x + 2*c) - 11*a*b^2*e^(2*d*x + 2*c) - 9
*b^3*e^(2*d*x + 2*c) + 5*a^3 + 13*a^2*b + 11*a*b^2 + 3*b^3)/((a^4 + 2*a^3*b
 + a^2*b^2)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) -
2*b*e^(2*d*x + 2*c) + a + b)^2))/d
```

3.129 $\int \frac{\mathrm{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=129

$$\frac{(4a+3b)\sinh(c+dx)}{8a^2d(a+b)\left((a+b)\sinh^2(c+dx)+a\right)}+\frac{(4a+3b)\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a+b)^{3/2}}+\frac{b\sinh(c+dx)}{4ad(a+b)\left((a+b)\sinh^2(c+dx)+a\right)^2}$$

```
[Out] ((4*a + 3*b)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*(a + b
)^(3/2)*d) + (b*Sinh[c + d*x])/(4*a*(a + b)*d*(a + (a + b)*Sinh[c + d*x]^2)
^2) + ((4*a + 3*b)*Sinh[c + d*x])/(8*a^2*(a + b)*d*(a + (a + b)*Sinh[c + d*
x]^2))
```

Rubi [A] time = 0.119973, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3676, 385, 199, 205}

$$\frac{(4a+3b)\sinh(c+dx)}{8a^2d(a+b)\left((a+b)\sinh^2(c+dx)+a\right)}+\frac{(4a+3b)\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a+b)^{3/2}}+\frac{b\sinh(c+dx)}{4ad(a+b)\left((a+b)\sinh^2(c+dx)+a\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((4*a + 3*b)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*(a + b
)^(3/2)*d) + (b*Sinh[c + d*x])/(4*a*(a + b)*d*(a + (a + b)*Sinh[c + d*x]^2)
^2) + ((4*a + 3*b)*Sinh[c + d*x])/(8*a^2*(a + b)*d*(a + (a + b)*Sinh[c + d*
x]^2))
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\text{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{\left(a+(a+b)x^2\right)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{b\sinh(c+dx)}{4a(a+b)d\left(a+(a+b)\sinh^2(c+dx)\right)^2} + \frac{\left(\frac{3}{a}+\frac{1}{a+b}\right)\text{Subst}\left(\int \frac{1}{\left(a+(a+b)x^2\right)^2}\,dx, x, \sinh(c+\right.}{4d}$$

$$= \frac{b\sinh(c+dx)}{4a(a+b)d\left(a+(a+b)\sinh^2(c+dx)\right)^2} + \frac{(4a+3b)\sinh(c+dx)}{8a^2(a+b)d\left(a+(a+b)\sinh^2(c+dx)\right)} + \frac{(4a -}{}$$

$$= \frac{(4a+3b)\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{3/2}d} + \frac{b\sinh(c+dx)}{4a(a+b)d\left(a+(a+b)\sinh^2(c+dx)\right)^2} + \frac{(4a}{8a^2(a+b)d}$$

Mathematica [A] time = 0.742373, size = 123, normalized size = 0.95

$$\frac{(4a+3b)\left(\frac{3(a+b)\sinh^3(c+dx)+5a\sinh(c+dx)}{a^2\left((a+b)\sinh^2(c+dx)+a\right)^2}+\frac{3\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a+b}}\right)-\frac{8\sinh(c+dx)}{\left((a+b)\sinh^2(c+dx)+a\right)^2}}{24d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((-8*Sinh[c + d*x])/(a + (a + b)*Sinh[c + d*x]^2)^2 + (4*a + 3*b)*((3*ArcTa
n[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(a^(5/2)*Sqrt[a + b]) + (5*a*Sinh[c
 + d*x] + 3*(a + b)*Sinh[c + d*x]^3)/(a^2*(a + (a + b)*Sinh[c + d*x]^2)^2))
)/(24*(a + b)*d)
```

Maple [B] time = 0.12, size = 1226, normalized size = 9.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] -1/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*
c)^2*b+a)^2/(a+b)*tanh(1/2*d*x+1/2*c)^7-5/4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*ta
nh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a/(a+b)*tanh(1/2*d*x+1
/2*c)^7*b-1/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2
*d*x+1/2*c)^2*b+a)^2/(a+b)*tanh(1/2*d*x+1/2*c)^5-13/4/d/(tanh(1/2*d*x+1/2*c
)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a/(a+b)*tanh
(1/2*d*x+1/2*c)^5*b-3/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+
```

```
4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2/(a+b)*tanh(1/2*d*x+1/2*c)^5*b^2+1/d/(tan
h(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)
^2/(a+b)*tanh(1/2*d*x+1/2*c)^3+13/4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d
*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a/(a+b)*tanh(1/2*d*x+1/2*c)^3*
b+3/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2
*c)^2*b+a)^2/a^2/(a+b)*tanh(1/2*d*x+1/2*c)^3*b^2+1/d/(tanh(1/2*d*x+1/2*c)^4
*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a+b)*tanh(1/2*
d*x+1/2*c)+5/4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(
1/2*d*x+1/2*c)^2*b+a)^2/a/(a+b)*tanh(1/2*d*x+1/2*c)*b-1/2/d/a/(a+b)/((2*(b*
(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1
/2)-a-2*b)*a)^(1/2))+1/2/d/a/(a+b)/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*
b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/
2))*b+1/2/d/a/(a+b)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d
*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/2/d/a/(a+b)/(b*(a+b))^(1/2
)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(
a+b))^(1/2)+a+2*b)*a)^(1/2))*b-3/8/d/a^2/(a+b)*b/((2*(b*(a+b))^(1/2)-a-2*b)
*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)
)+3/8/d/a^2/(a+b)/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arcta
nh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))*b^2+3/8/d/a^2
/(a+b)*b/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/(
(2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+3/8/d/a^2/(a+b)/(b*(a+b))^(1/2)/((2*(b*
(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/
2)+a+2*b)*a)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(4\,a^2e^{(7\,c)} + 7\,abe^{(7\,c)} + 3\,b^2e^{(7\,c)}\right)e^{(7\,dx)} + \left(4\,a^2e^{(5\,c)} - abe^{(5\,c} \cdots}{4\left(a^5d + 3\,a^4bd + 3\,a^3b^2d + a^2b^3d + \left(a^5de^{(8\,c)} + 3\,a^4bde^{(8\,c)} + 3\,a^3b^2de^{(8\,c)} + a^2b^3de^{(8\,c)}\right)e^{(8\,dx)} + 4\left(a^5de^{(6\,c)} + a^4bde^{(6\,c)} - \cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((4*a^2*e^(7*c) + 7*a*b*e^(7*c) + 3*b^2*e^(7*c))*e^(7*d*x) + (4*a^2*e^(
5*c) - a*b*e^(5*c) - 9*b^2*e^(5*c))*e^(5*d*x) - (4*a^2*e^(3*c) - a*b*e^(3*c
) - 9*b^2*e^(3*c))*e^(3*d*x) - (4*a^2*e^c + 7*a*b*e^c + 3*b^2*e^c)*e^(d*x))
/(a^5*d + 3*a^4*b*d + 3*a^3*b^2*d + a^2*b^3*d + (a^5*d*e^(8*c) + 3*a^4*b*d*
e^(8*c) + 3*a^3*b^2*d*e^(8*c) + a^2*b^3*d*e^(8*c))*e^(8*d*x) + 4*(a^5*d*e^(
6*c) + a^4*b*d*e^(6*c) - a^3*b^2*d*e^(6*c) - a^2*b^3*d*e^(6*c))*e^(6*d*x) +
 2*(3*a^5*d*e^(4*c) + a^4*b*d*e^(4*c) + a^3*b^2*d*e^(4*c) + 3*a^2*b^3*d*e^(
4*c))*e^(4*d*x) + 4*(a^5*d*e^(2*c) + a^4*b*d*e^(2*c) - a^3*b^2*d*e^(2*c) -
a^2*b^3*d*e^(2*c))*e^(2*d*x)) + 8*integrate(1/32*((4*a*e^(3*c) + 3*b*e^(3*c
))*e^(3*d*x) + (4*a*e^c + 3*b*e^c)*e^(d*x))/(a^4 + 2*a^3*b + a^2*b^2 + (a^4
*e^(4*c) + 2*a^3*b*e^(4*c) + a^2*b^2*e^(4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) -
a^2*b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 2.87703, size = 15408, normalized size = 119.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(4*a^4 + 11*a^3*b + 10*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^7 + 28*(4*
a^4 + 11*a^3*b + 10*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + 4*(4
*a^4 + 11*a^3*b + 10*a^2*b^2 + 3*a*b^3)*sinh(d*x + c)^7 + 4*(4*a^4 + 3*a^3*
b - 10*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^5 + 4*(4*a^4 + 3*a^3*b - 10*a^2*b^2
 - 9*a*b^3 + 21*(4*a^4 + 11*a^3*b + 10*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^2)*
sinh(d*x + c)^5 + 20*(7*(4*a^4 + 11*a^3*b + 10*a^2*b^2 + 3*a*b^3)*cosh(d*x
+ c)^3 + (4*a^4 + 3*a^3*b - 10*a^2*b^2 - 9*a*b^3)*cosh(d*x + c))*sinh(d*x +
 c)^4 - 4*(4*a^4 + 3*a^3*b - 10*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^3 + 4*(35*
(4*a^4 + 11*a^3*b + 10*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^4 - 4*a^4 - 3*a^3*b
 + 10*a^2*b^2 + 9*a*b^3 + 10*(4*a^4 + 3*a^3*b - 10*a^2*b^2 - 9*a*b^3)*cosh(
d*x + c)^2)*sinh(d*x + c)^3 + 4*(21*(4*a^4 + 11*a^3*b + 10*a^2*b^2 + 3*a*b^
3)*cosh(d*x + c)^5 + 10*(4*a^4 + 3*a^3*b - 10*a^2*b^2 - 9*a*b^3)*cosh(d*x +
 c)^3 - 3*(4*a^4 + 3*a^3*b - 10*a^2*b^2 - 9*a*b^3)*cosh(d*x + c))*sinh(d*x
+ c)^2 - ((4*a^3 + 11*a^2*b + 10*a*b^2 + 3*b^3)*cosh(d*x + c)^8 + 8*(4*a^3
+ 11*a^2*b + 10*a*b^2 + 3*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (4*a^3 + 11*
a^2*b + 10*a*b^2 + 3*b^3)*sinh(d*x + c)^8 + 4*(4*a^3 + 3*a^2*b - 4*a*b^2 -
3*b^3)*cosh(d*x + c)^6 + 4*(4*a^3 + 3*a^2*b - 4*a*b^2 - 3*b^3 + 7*(4*a^3 +
11*a^2*b + 10*a*b^2 + 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(4*a^3
 + 11*a^2*b + 10*a*b^2 + 3*b^3)*cosh(d*x + c)^3 + 3*(4*a^3 + 3*a^2*b - 4*a*
b^2 - 3*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(12*a^3 + a^2*b + 6*a*b^2 +
 9*b^3)*cosh(d*x + c)^4 + 2*(35*(4*a^3 + 11*a^2*b + 10*a*b^2 + 3*b^3)*cosh(
d*x + c)^4 + 12*a^3 + a^2*b + 6*a*b^2 + 9*b^3 + 30*(4*a^3 + 3*a^2*b - 4*a*b
^2 - 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(4*a^3 + 11*a^2*b + 10*
a*b^2 + 3*b^3)*cosh(d*x + c)^5 + 10*(4*a^3 + 3*a^2*b - 4*a*b^2 - 3*b^3)*cos
h(d*x + c)^3 + (12*a^3 + a^2*b + 6*a*b^2 + 9*b^3)*cosh(d*x + c))*sinh(d*x +
 c)^3 + 4*a^3 + 11*a^2*b + 10*a*b^2 + 3*b^3 + 4*(4*a^3 + 3*a^2*b - 4*a*b^2
- 3*b^3)*cosh(d*x + c)^2 + 4*(7*(4*a^3 + 11*a^2*b + 10*a*b^2 + 3*b^3)*cosh(
d*x + c)^6 + 15*(4*a^3 + 3*a^2*b - 4*a*b^2 - 3*b^3)*cosh(d*x + c)^4 + 4*a^3
 + 3*a^2*b - 4*a*b^2 - 3*b^3 + 3*(12*a^3 + a^2*b + 6*a*b^2 + 9*b^3)*cosh(d*
x + c)^2)*sinh(d*x + c)^2 + 8*((4*a^3 + 11*a^2*b + 10*a*b^2 + 3*b^3)*cosh(d
*x + c)^7 + 3*(4*a^3 + 3*a^2*b - 4*a*b^2 - 3*b^3)*cosh(d*x + c)^5 + (12*a^3
 + a^2*b + 6*a*b^2 + 9*b^3)*cosh(d*x + c)^3 + (4*a^3 + 3*a^2*b - 4*a*b^2 -
3*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 - a*b)*log(((a + b)*cosh(d*x
 + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4
 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 - 3*a - b)*si
nh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 - (3*a + b)*cosh(d*x + c))*sinh(
d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x
+ c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 -
 a*b) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x
+ c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)
*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a
 - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) - 4*(4*a^4 + 11*a^3*b + 10*a^2
*b^2 + 3*a*b^3)*cosh(d*x + c) + 4*(7*(4*a^4 + 11*a^3*b + 10*a^2*b^2 + 3*a*b
^3)*cosh(d*x + c)^6 + 5*(4*a^4 + 3*a^3*b - 10*a^2*b^2 - 9*a*b^3)*cosh(d*x +
 c)^4 - 4*a^4 - 11*a^3*b - 10*a^2*b^2 - 3*a*b^3 - 3*(4*a^4 + 3*a^3*b - 10*a
^2*b^2 - 9*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^7 + 4*a^6*b + 6*a^5*b
^2 + 4*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^8 + 8*(a^7 + 4*a^6*b + 6*a^5*b^2
+ 4*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^7 + 4*a^6*b + 6
*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*sinh(d*x + c)^8 + 4*(a^7 + 2*a^6*b - 2*a^
4*b^3 - a^3*b^4)*d*cosh(d*x + c)^6 + 4*(7*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^
4*b^3 + a^3*b^4)*d*cosh(d*x + c)^2 + (a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*
d)*sinh(d*x + c)^6 + 2*(3*a^7 + 4*a^6*b + 2*a^5*b^2 + 4*a^4*b^3 + 3*a^3*b^4
)*d*cosh(d*x + c)^4 + 8*(7*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4
)*d*cosh(d*x + c)^3 + 3*(a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*cosh(d*x +
c))*sinh(d*x + c)^5 + 2*(35*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^
4)*d*cosh(d*x + c)^4 + 30*(a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*cosh(d*x
+ c)^2 + (3*a^7 + 4*a^6*b + 2*a^5*b^2 + 4*a^4*b^3 + 3*a^3*b^4)*d)*sinh(d*x
+ c)^4 + 4*(a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^2 + 8*(7*(
a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^5 + 10*(a^
```

7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^3 + (3*a^7 + 4*a^6*b + 2
*a^5*b^2 + 4*a^4*b^3 + 3*a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(
a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^6 + 15*(a^
7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^4 + 3*(3*a^7 + 4*a^6*b +
 2*a^5*b^2 + 4*a^4*b^3 + 3*a^3*b^4)*d*cosh(d*x + c)^2 + (a^7 + 2*a^6*b - 2*
a^4*b^3 - a^3*b^4)*d)*sinh(d*x + c)^2 + (a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*
b^3 + a^3*b^4)*d + 8*((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*c
osh(d*x + c)^7 + 3*(a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^5
+ (3*a^7 + 4*a^6*b + 2*a^5*b^2 + 4*a^4*b^3 + 3*a^3*b^4)*d*cosh(d*x + c)^3 +
 (a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8
(2(4*a^4 + 11*a^3*b + 10*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^7 + 14*(4*a^4 +
 11*a^3*b + 10*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + 2*(4*a^4
+ 11*a^3*b + 10*a^2*b^2 + 3*a*b^3)*sinh(d*x + c)^7 + 2*(4*a^4 + 3*a^3*b - 1
0*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^5 + 2*(4*a^4 + 3*a^3*b - 10*a^2*b^2 - 9*
a*b^3 + 21*(4*a^4 + 11*a^3*b + 10*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^2)*sinh(
d*x + c)^5 + 10*(7*(4*a^4 + 11*a^3*b + 10*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^
3 + (4*a^4 + 3*a^3*b - 10*a^2*b^2 - 9*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^4
 - 2*(4*a^4 + 3*a^3*b - 10*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^3 + 2*(35*(4*a^
4 + 11*a^3*b + 10*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^4 - 4*a^4 - 3*a^3*b + 10
*a^2*b^2 + 9*a*b^3 + 10*(4*a^4 + 3*a^3*b - 10*a^2*b^2 - 9*a*b^3)*cosh(d*x +
 c)^2)*sinh(d*x + c)^3 + 2*(21*(4*a^4 + 11*a^3*b + 10*a^2*b^2 + 3*a*b^3)*co
sh(d*x + c)^5 + 10*(4*a^4 + 3*a^3*b - 10*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^3
 - 3*(4*a^4 + 3*a^3*b - 10*a^2*b^2 - 9*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^
2 + ((4*a^3 + 11*a^2*b + 10*a*b^2 + 3*b^3)*cosh(d*x + c)^8 + 8*(4*a^3 + 11*
a^2*b + 10*a*b^2 + 3*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (4*a^3 + 11*a^2*b
 + 10*a*b^2 + 3*b^3)*sinh(d*x + c)^8 + 4*(4*a^3 + 3*a^2*b - 4*a*b^2 - 3*b^3
)*cosh(d*x + c)^6 + 4*(4*a^3 + 3*a^2*b - 4*a*b^2 - 3*b^3 + 7*(4*a^3 + 11*a^
2*b + 10*a*b^2 + 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(4*a^3 + 11
*a^2*b + 10*a*b^2 + 3*b^3)*cosh(d*x + c)^3 + 3*(4*a^3 + 3*a^2*b - 4*a*b^2 -
 3*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(12*a^3 + a^2*b + 6*a*b^2 + 9*b^
3)*cosh(d*x + c)^4 + 2*(35*(4*a^3 + 11*a^2*b + 10*a*b^2 + 3*b^3)*cosh(d*x +
 c)^4 + 12*a^3 + a^2*b + 6*a*b^2 + 9*b^3 + 30*(4*a^3 + 3*a^2*b - 4*a*b^2 -
3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(4*a^3 + 11*a^2*b + 10*a*b^2
 + 3*b^3)*cosh(d*x + c)^5 + 10*(4*a^3 + 3*a^2*b - 4*a*b^2 - 3*b^3)*cosh(d*x
 + c)^3 + (12*a^3 + a^2*b + 6*a*b^2 + 9*b^3)*cosh(d*x + c))*sinh(d*x + c)^3
 + 4*a^3 + 11*a^2*b + 10*a*b^2 + 3*b^3 + 4*(4*a^3 + 3*a^2*b - 4*a*b^2 - 3*b
^3)*cosh(d*x + c)^2 + 4*(7*(4*a^3 + 11*a^2*b + 10*a*b^2 + 3*b^3)*cosh(d*x +
 c)^6 + 15*(4*a^3 + 3*a^2*b - 4*a*b^2 - 3*b^3)*cosh(d*x + c)^4 + 4*a^3 + 3*
a^2*b - 4*a*b^2 - 3*b^3 + 3*(12*a^3 + a^2*b + 6*a*b^2 + 9*b^3)*cosh(d*x + c
)^2)*sinh(d*x + c)^2 + 8*((4*a^3 + 11*a^2*b + 10*a*b^2 + 3*b^3)*cosh(d*x +
c)^7 + 3*(4*a^3 + 3*a^2*b - 4*a*b^2 - 3*b^3)*cosh(d*x + c)^5 + (12*a^3 + a^
2*b + 6*a*b^2 + 9*b^3)*cosh(d*x + c)^3 + (4*a^3 + 3*a^2*b - 4*a*b^2 - 3*b^3
)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + a*b)*arctan(1/2*((a + b)*cosh(d*
x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^
3 + (3*a - b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + 3*a - b)*sinh(d*
x + c))/sqrt(a^2 + a*b)) + ((4*a^3 + 11*a^2*b + 10*a*b^2 + 3*b^3)*cosh(d*x
+ c)^8 + 8*(4*a^3 + 11*a^2*b + 10*a*b^2 + 3*b^3)*cosh(d*x + c)*sinh(d*x + c
)^7 + (4*a^3 + 11*a^2*b + 10*a*b^2 + 3*b^3)*sinh(d*x + c)^8 + 4*(4*a^3 + 3*
a^2*b - 4*a*b^2 - 3*b^3)*cosh(d*x + c)^6 + 4*(4*a^3 + 3*a^2*b - 4*a*b^2 - 3
b^3 + 7(4*a^3 + 11*a^2*b + 10*a*b^2 + 3*b^3)*cosh(d*x + c)^2)*sinh(d*x +
c)^6 + 8*(7*(4*a^3 + 11*a^2*b + 10*a*b^2 + 3*b^3)*cosh(d*x + c)^3 + 3*(4*a^
3 + 3*a^2*b - 4*a*b^2 - 3*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(12*a^3 +
 a^2*b + 6*a*b^2 + 9*b^3)*cosh(d*x + c)^4 + 2*(35*(4*a^3 + 11*a^2*b + 10*a*
b^2 + 3*b^3)*cosh(d*x + c)^4 + 12*a^3 + a^2*b + 6*a*b^2 + 9*b^3 + 30*(4*a^3
 + 3*a^2*b - 4*a*b^2 - 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(4*a^
3 + 11*a^2*b + 10*a*b^2 + 3*b^3)*cosh(d*x + c)^5 + 10*(4*a^3 + 3*a^2*b - 4*
a*b^2 - 3*b^3)*cosh(d*x + c)^3 + (12*a^3 + a^2*b + 6*a*b^2 + 9*b^3)*cosh(d*
x + c))*sinh(d*x + c)^3 + 4*a^3 + 11*a^2*b + 10*a*b^2 + 3*b^3 + 4*(4*a^3 +
3*a^2*b - 4*a*b^2 - 3*b^3)*cosh(d*x + c)^2 + 4*(7*(4*a^3 + 11*a^2*b + 10*a*

```
b^2 + 3*b^3)*cosh(d*x + c)^6 + 15*(4*a^3 + 3*a^2*b - 4*a*b^2 - 3*b^3)*cosh(
d*x + c)^4 + 4*a^3 + 3*a^2*b - 4*a*b^2 - 3*b^3 + 3*(12*a^3 + a^2*b + 6*a*b^
2 + 9*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((4*a^3 + 11*a^2*b + 10*a*b
^2 + 3*b^3)*cosh(d*x + c)^7 + 3*(4*a^3 + 3*a^2*b - 4*a*b^2 - 3*b^3)*cosh(d*
x + c)^5 + (12*a^3 + a^2*b + 6*a*b^2 + 9*b^3)*cosh(d*x + c)^3 + (4*a^3 + 3*
a^2*b - 4*a*b^2 - 3*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + a*b)*arct
an(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh(d*x + c))/a) - 2*(4*a^4 + 11*a
^3*b + 10*a^2*b^2 + 3*a*b^3)*cosh(d*x + c) + 2*(7*(4*a^4 + 11*a^3*b + 10*a^
2*b^2 + 3*a*b^3)*cosh(d*x + c)^6 + 5*(4*a^4 + 3*a^3*b - 10*a^2*b^2 - 9*a*b^
3)*cosh(d*x + c)^4 - 4*a^4 - 11*a^3*b - 10*a^2*b^2 - 3*a*b^3 - 3*(4*a^4 + 3
*a^3*b - 10*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^7 + 4*a^
6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^8 + 8*(a^7 + 4*a^6*b
 + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^7
+ 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*sinh(d*x + c)^8 + 4*(a^7 + 2
*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^6 + 4*(7*(a^7 + 4*a^6*b + 6*a
^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^2 + (a^7 + 2*a^6*b - 2*a^4*b^
3 - a^3*b^4)*d)*sinh(d*x + c)^6 + 2*(3*a^7 + 4*a^6*b + 2*a^5*b^2 + 4*a^4*b^
3 + 3*a^3*b^4)*d*cosh(d*x + c)^4 + 8*(7*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*
b^3 + a^3*b^4)*d*cosh(d*x + c)^3 + 3*(a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*
d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4
*b^3 + a^3*b^4)*d*cosh(d*x + c)^4 + 30*(a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4
)*d*cosh(d*x + c)^2 + (3*a^7 + 4*a^6*b + 2*a^5*b^2 + 4*a^4*b^3 + 3*a^3*b^4)
*d)*sinh(d*x + c)^4 + 4*(a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*cosh(d*x +
c)^2 + 8*(7*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*cosh(d*x +
c)^5 + 10*(a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^3 + (3*a^7
+ 4*a^6*b + 2*a^5*b^2 + 4*a^4*b^3 + 3*a^3*b^4)*d*cosh(d*x + c))*sinh(d*x +
c)^3 + 4*(7*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*cosh(d*x +
c)^6 + 15*(a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^4 + 3*(3*a^
7 + 4*a^6*b + 2*a^5*b^2 + 4*a^4*b^3 + 3*a^3*b^4)*d*cosh(d*x + c)^2 + (a^7 +
 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d)*sinh(d*x + c)^2 + (a^7 + 4*a^6*b + 6*a^5
*b^2 + 4*a^4*b^3 + a^3*b^4)*d + 8*((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 +
 a^3*b^4)*d*cosh(d*x + c)^7 + 3*(a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*cos
h(d*x + c)^5 + (3*a^7 + 4*a^6*b + 2*a^5*b^2 + 4*a^4*b^3 + 3*a^3*b^4)*d*cosh
(d*x + c)^3 + (a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c))*sinh(d
*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 2.42807, size = 7308, normalized size = 56.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/32*(2*(3*(4*a^5*e^(4*c) + 11*a^4*b*e^(4*c) + 10*a^3*b^2*e^(4*c) + 3*a^2*b
^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*i
mag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a
+ b) + b/(a + b)))) - (4*a^5*e^(4*c) + 11*a^4*b*e^(4*c) + 10*a^3*b^2*e^(4*c
) + 3*a^2*b^3*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^
3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3 - 9*(4*a^5*e^(4*c) +
 11*a^4*b*e^(4*c) + 10*a^3*b^2*e^(4*c) + 3*a^2*b^3*e^(4*c))*cos(1/2*real_pa
rt(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b)
+ b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/
2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(4*a^5*e^(4*c) + 11*a^4*b*
e^(4*c) + 10*a^3*b^2*e^(4*c) + 3*a^2*b^3*e^(4*c))*cosh(1/2*imag_part(arccos
(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b
))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(4*a^5*e^(4*
c) + 11*a^4*b*e^(4*c) + 10*a^3*b^2*e^(4*c) + 3*a^2*b^3*e^(4*c))*cos(1/2*rea
l_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a +
 b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(
1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(4*a^5*e^(4*c) + 11*a^
4*b*e^(4*c) + 10*a^3*b^2*e^(4*c) + 3*a^2*b^3*e^(4*c))*cosh(1/2*imag_part(ar
ccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a +
 b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(4*a^5*e
^(4*c) + 11*a^4*b*e^(4*c) + 10*a^3*b^2*e^(4*c) + 3*a^2*b^3*e^(4*c))*cos(1/2
*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(
a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3
 + (4*a^5*e^(4*c) + 11*a^4*b*e^(4*c) + 10*a^3*b^2*e^(4*c) + 3*a^2*b^3*e^(4*
c))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part
(arccos(-a/(a + b) + b/(a + b))))^3 + (4*a^5*e^(4*c) + 11*a^4*b*e^(4*c) + 1
0*a^3*b^2*e^(4*c) + 3*a^2*b^3*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(a + b)
 + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) - (4*a^5
*e^(4*c) + 11*a^4*b*e^(4*c) + 10*a^3*b^2*e^(4*c) + 3*a^2*b^3*e^(4*c))*sin(1
/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/
(a + b) + b/(a + b)))))*arctan((((a^4 + 2*a^3*b + a^2*b^2)/(a^4*e^(4*c) + 2
*a^3*b*e^(4*c) + a^2*b^2*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b)))
+ e^(d*x))/(((a^4 + 2*a^3*b + a^2*b^2)/(a^4*e^(4*c) + 2*a^3*b*e^(4*c) + a^2
*b^2*e^(4*c)))^(1/4)*sin(1/2*arccos(-(a - b)/(a + b)))))/(2*(a^3*e^(2*c) +
a^2*b*e^(2*c))^2*a*b + (a^4*e^(2*c) - a^2*b^2*e^(2*c))*sqrt(-a*b)*abs(-a^3*
e^(2*c) - a^2*b*e^(2*c))) + 2*(3*(4*a^5*e^(4*c) + 11*a^4*b*e^(4*c) + 10*a^3
*b^2*e^(4*c) + 3*a^2*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(
a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*r
eal_part(arccos(-a/(a + b) + b/(a + b)))) - (4*a^5*e^(4*c) + 11*a^4*b*e^(4*
c) + 10*a^3*b^2*e^(4*c) + 3*a^2*b^3*e^(4*c))*cosh(1/2*imag_part(arccos(-a/(
a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^
3 - 9*(4*a^5*e^(4*c) + 11*a^4*b*e^(4*c) + 10*a^3*b^2*e^(4*c) + 3*a^2*b^3*e^
(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_p
art(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b)
+ b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(4*a
^5*e^(4*c) + 11*a^4*b*e^(4*c) + 10*a^3*b^2*e^(4*c) + 3*a^2*b^3*e^(4*c))*cos
h(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos
(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a +
b)))) + 9*(4*a^5*e^(4*c) + 11*a^4*b*e^(4*c) + 10*a^3*b^2*e^(4*c) + 3*a^2*b^
3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*im
ag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b
) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*
(4*a^5*e^(4*c) + 11*a^4*b*e^(4*c) + 10*a^3*b^2*e^(4*c) + 3*a^2*b^3*e^(4*c))
*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arcc
os(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a
+ b))))^2 - 3*(4*a^5*e^(4*c) + 11*a^4*b*e^(4*c) + 10*a^3*b^2*e^(4*c) + 3*a^
2*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2
*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a
 + b) + b/(a + b))))^3 + (4*a^5*e^(4*c) + 11*a^4*b*e^(4*c) + 10*a^3*b^2*e^(
```

```
4*c) + 3*a^2*b^3*e^(4*c))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))
)^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (4*a^5*e^(4*c)
+ 11*a^4*b*e^(4*c) + 10*a^3*b^2*e^(4*c) + 3*a^2*b^3*e^(4*c))*cosh(1/2*imag_
part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) +
 b/(a + b)))) - (4*a^5*e^(4*c) + 11*a^4*b*e^(4*c) + 10*a^3*b^2*e^(4*c) + 3*
a^2*b^3*e^(4*c))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/
2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*arctan(-(((a^4 + 2*a^3*b + a^
2*b^2)/(a^4*e^(4*c) + 2*a^3*b*e^(4*c) + a^2*b^2*e^(4*c)))^(1/4)*cos(1/2*arc
cos(-(a - b)/(a + b))) - e^(d*x))/(((a^4 + 2*a^3*b + a^2*b^2)/(a^4*e^(4*c)
+ 2*a^3*b*e^(4*c) + a^2*b^2*e^(4*c)))^(1/4)*sin(1/2*arccos(-(a - b)/(a + b)
))))/(2*(a^3*e^(2*c) + a^2*b*e^(2*c))^2*a*b + (a^4*e^(2*c) - a^2*b^2*e^(2*c
))*sqrt(-a*b)*abs(-a^3*e^(2*c) - a^2*b*e^(2*c))) + ((4*a^5*e^(4*c) + 11*a^4
*b*e^(4*c) + 10*a^3*b^2*e^(4*c) + 3*a^2*b^3*e^(4*c))*cos(1/2*real_part(arcc
os(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a
+ b))))^3 - 3*(4*a^5*e^(4*c) + 11*a^4*b*e^(4*c) + 10*a^3*b^2*e^(4*c) + 3*a^
2*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*
imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a
 + b) + b/(a + b))))^2 - 3*(4*a^5*e^(4*c) + 11*a^4*b*e^(4*c) + 10*a^3*b^2*e
^(4*c) + 3*a^2*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b)
)))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_p
art(arccos(-a/(a + b) + b/(a + b)))) + 9*(4*a^5*e^(4*c) + 11*a^4*b*e^(4*c)
+ 10*a^3*b^2*e^(4*c) + 3*a^2*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a +
b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin
(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos
(-a/(a + b) + b/(a + b)))) + 3*(4*a^5*e^(4*c) + 11*a^4*b*e^(4*c) + 10*a^3*b
^2*e^(4*c) + 3*a^2*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a
+ b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag
_part(arccos(-a/(a + b) + b/(a + b))))^2 - 9*(4*a^5*e^(4*c) + 11*a^4*b*e^(4
*c) + 10*a^3*b^2*e^(4*c) + 3*a^2*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(
a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*s
in(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arcc
os(-a/(a + b) + b/(a + b))))^2 - (4*a^5*e^(4*c) + 11*a^4*b*e^(4*c) + 10*a^3
*b^2*e^(4*c) + 3*a^2*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(
a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + 3*(4*a^
5*e^(4*c) + 11*a^4*b*e^(4*c) + 10*a^3*b^2*e^(4*c) + 3*a^2*b^3*e^(4*c))*cos(
1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/
(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))
)^3 + (4*a^5*e^(4*c) + 11*a^4*b*e^(4*c) + 10*a^3*b^2*e^(4*c) + 3*a^2*b^3*e^
(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_par
t(arccos(-a/(a + b) + b/(a + b)))) - (4*a^5*e^(4*c) + 11*a^4*b*e^(4*c) + 10
*a^3*b^2*e^(4*c) + 3*a^2*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) +
 b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*log(2*((
a^4 + 2*a^3*b + a^2*b^2)/(a^4*e^(4*c) + 2*a^3*b*e^(4*c) + a^2*b^2*e^(4*c)))
^(1/4)*cos(1/2*arccos(-(a - b)/(a + b)))*e^(d*x) + sqrt((a^4 + 2*a^3*b + a^
2*b^2)/(a^4*e^(4*c) + 2*a^3*b*e^(4*c) + a^2*b^2*e^(4*c))) + e^(2*d*x))/(2*(
a^3*e^(2*c) + a^2*b*e^(2*c))^2*a*b + (a^4*e^(2*c) - a^2*b^2*e^(2*c))*sqrt(-
a*b)*abs(-a^3*e^(2*c) - a^2*b*e^(2*c))) - ((4*a^5*e^(4*c) + 11*a^4*b*e^(4*c
) + 10*a^3*b^2*e^(4*c) + 3*a^2*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a
+ b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3
 - 3*(4*a^5*e^(4*c) + 11*a^4*b*e^(4*c) + 10*a^3*b^2*e^(4*c) + 3*a^2*b^3*e^(
4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part
(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b
/(a + b))))^2 - 3*(4*a^5*e^(4*c) + 11*a^4*b*e^(4*c) + 10*a^3*b^2*e^(4*c) +
3*a^2*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cos
h(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arcco
s(-a/(a + b) + b/(a + b)))) + 9*(4*a^5*e^(4*c) + 11*a^4*b*e^(4*c) + 10*a^3*
b^2*e^(4*c) + 3*a^2*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a
 + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real
_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a +
```

```
b) + b/(a + b)))) + 3*(4*a^5*e^(4*c) + 11*a^4*b*e^(4*c) + 10*a^3*b^2*e^(4*c
) + 3*a^2*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3
*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arc
cos(-a/(a + b) + b/(a + b))))^2 - 9*(4*a^5*e^(4*c) + 11*a^4*b*e^(4*c) + 10*
a^3*b^2*e^(4*c) + 3*a^2*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) +
b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*re
al_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a
+ b) + b/(a + b))))^2 - (4*a^5*e^(4*c) + 11*a^4*b*e^(4*c) + 10*a^3*b^2*e^(4
*c) + 3*a^2*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))
^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + 3*(4*a^5*e^(4*c)
 + 11*a^4*b*e^(4*c) + 10*a^3*b^2*e^(4*c) + 3*a^2*b^3*e^(4*c))*cos(1/2*real_
part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) +
 b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + (4*
a^5*e^(4*c) + 11*a^4*b*e^(4*c) + 10*a^3*b^2*e^(4*c) + 3*a^2*b^3*e^(4*c))*co
s(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(
-a/(a + b) + b/(a + b)))) - (4*a^5*e^(4*c) + 11*a^4*b*e^(4*c) + 10*a^3*b^2*
e^(4*c) + 3*a^2*b^3*e^(4*c))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b
))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*log(-2*((a^4 + 2*
a^3*b + a^2*b^2)/(a^4*e^(4*c) + 2*a^3*b*e^(4*c) + a^2*b^2*e^(4*c)))^(1/4)*c
os(1/2*arccos(-(a - b)/(a + b)))*e^(d*x) + sqrt((a^4 + 2*a^3*b + a^2*b^2)/(
a^4*e^(4*c) + 2*a^3*b*e^(4*c) + a^2*b^2*e^(4*c))) + e^(2*d*x))/(2*(a^3*e^(2
*c) + a^2*b*e^(2*c))^2*a*b + (a^4*e^(2*c) - a^2*b^2*e^(2*c))*sqrt(-a*b)*abs
(-a^3*e^(2*c) - a^2*b*e^(2*c))) + 8*(4*a^2*e^(7*d*x + 7*c) + 7*a*b*e^(7*d*x
 + 7*c) + 3*b^2*e^(7*d*x + 7*c) + 4*a^2*e^(5*d*x + 5*c) - a*b*e^(5*d*x + 5*
c) - 9*b^2*e^(5*d*x + 5*c) - 4*a^2*e^(3*d*x + 3*c) + a*b*e^(3*d*x + 3*c) +
9*b^2*e^(3*d*x + 3*c) - 4*a^2*e^(d*x + c) - 7*a*b*e^(d*x + c) - 3*b^2*e^(d*
x + c))/((a^3 + a^2*b)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*
x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2))/d
```

3.130 $\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=115

$$-\frac{(a-3b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}d}-\frac{(a-3b)\tanh(c+dx)}{8a^2bd\left(a+b\tanh^2(c+dx)\right)}+\frac{(a+b)\tanh(c+dx)}{4abd\left(a+b\tanh^2(c+dx)\right)^2}$$

```
[Out] -((a - 3*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*b^(3/2)*d)
+ ((a + b)*Tanh[c + d*x])/(4*a*b*d*(a + b*Tanh[c + d*x]^2)^2) - ((a - 3*b)*
Tanh[c + d*x])/(8*a^2*b*d*(a + b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.0968225, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3675, 385, 199, 205}

$$-\frac{(a-3b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}d}-\frac{(a-3b)\tanh(c+dx)}{8a^2bd\left(a+b\tanh^2(c+dx)\right)}+\frac{(a+b)\tanh(c+dx)}{4abd\left(a+b\tanh^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -((a - 3*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*b^(3/2)*d)
+ ((a + b)*Tanh[c + d*x])/(4*a*b*d*(a + b*Tanh[c + d*x]^2)^2) - ((a - 3*b)*
Tanh[c + d*x])/(8*a^2*b*d*(a + b*Tanh[c + d*x]^2))
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{\left(a+bx^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{(a+b)\tanh(c+dx)}{4abd\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(a-3b)\operatorname{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{4abd}$$

$$= \frac{(a+b)\tanh(c+dx)}{4abd\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(a-3b)\tanh(c+dx)}{8a^2bd\left(a+b\tanh^2(c+dx)\right)} - \frac{(a-3b)\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\right.}{8a^2b}$$

$$= -\frac{(a-3b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}d} + \frac{(a+b)\tanh(c+dx)}{4abd\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(a-3b)\tanh(c+}{8a^2bd\left(a+b\tanh^2(c\right.}$$

Mathematica [A] time = 0.962973, size = 115, normalized size = 1.

$$\frac{\frac{\sqrt{a}\sinh(2(c+dx))\left(\left(a^2+4ab+3b^2\right)\cosh(2(c+dx))+a^2+6ab-3b^2\right)}{b((a+b)\cosh(2(c+dx))+a-b)^2} + \frac{(3b-a)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{b^{3/2}}}{8a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (((-a + 3*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/b^(3/2) + (Sqrt[a]*(a
^2 + 6*a*b - 3*b^2 + (a^2 + 4*a*b + 3*b^2)*Cosh[2*(c + d*x)])*Sinh[2*(c + d
*x)])/(b*(a - b + (a + b)*Cosh[2*(c + d*x)])^2))/(8*a^(5/2)*d)
```

Maple [B] time = 0.111, size = 1270, normalized size = 11.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 1/4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2
*c)^2*b+a)^2/b*tanh(1/2*d*x+1/2*c)^7+5/4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(
1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^7+3
/4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*
c)^2*b+a)^2/b*tanh(1/2*d*x+1/2*c)^5+11/4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(
1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^5+3
/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+
1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5+3/4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh
(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/b*tanh(1/2*d*x+1/2*c)^3+
11/4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/
```

```
2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^3+3/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+2*
tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^
3+1/4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1
/2*c)^2*b+a)^2/b*tanh(1/2*d*x+1/2*c)+5/4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(
1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)+1/8
/d/b/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2
*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-1/8/d/a/b/((2*(b*(a+b))^(1
/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b
)*a)^(1/2))-1/4/d/(b*(a+b))^(1/2)/a/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arc
tanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/8/d/b/(b*
(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*
c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/8/d/a/b/((2*(b*(a+b))^(1/2)+a+2*b
)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)
)-1/4/d/(b*(a+b))^(1/2)/a/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh
(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+3/8/d/a^2/((2*(b*(a+b)
)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a
-2*b)*a)^(1/2))-3/8/d/a^2*b/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(
1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-3/8
/d/a^2/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2
*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-3/8/d/a^2*b/(b*(a+b))^(1/2)/((2*(b*(a+b))
^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2
*b)*a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.97443, size = 12604, normalized size = 109.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(a^4*b - a^3*b^2 - 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^6 + 24*(a^4
*b - a^3*b^2 - 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^5 + 4*(a^4*
b - a^3*b^2 - 5*a^2*b^3 - 3*a*b^4)*sinh(d*x + c)^6 + 4*a^4*b + 20*a^3*b^2 +
 28*a^2*b^3 + 12*a*b^4 + 4*(3*a^4*b + 7*a^3*b^2 - 3*a^2*b^3 + 9*a*b^4)*cosh
(d*x + c)^4 + 4*(3*a^4*b + 7*a^3*b^2 - 3*a^2*b^3 + 9*a*b^4 + 15*(a^4*b - a^
3*b^2 - 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(5*(a^4*
b - a^3*b^2 - 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^3 + (3*a^4*b + 7*a^3*b^2 -
 3*a^2*b^3 + 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(3*a^4*b + 13*a^3*
b^2 + a^2*b^3 - 9*a*b^4)*cosh(d*x + c)^2 + 4*(3*a^4*b + 13*a^3*b^2 + a^2*b^
3 - 9*a*b^4 + 15*(a^4*b - a^3*b^2 - 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^4 +
6*(3*a^4*b + 7*a^3*b^2 - 3*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c
)^2 - ((a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^8 + 8*(a^4 - 6*a^2
*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^4 - 6*a^2*b^2 -
8*a*b^3 - 3*b^4)*sinh(d*x + c)^8 + 4*(a^4 - 2*a^3*b - 4*a^2*b^2 + 2*a*b^3 +
```

3*b^4)*cosh(d*x + c)^6 + 4*(a^4 - 2*a^3*b - 4*a^2*b^2 + 2*a*b^3 + 3*b^4 + 7*(a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^3 + 3*(a^4 - 2*a^3*b - 4*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^4 - 8*a^3*b - 2*a^2*b^2 - 9*b^4)*cosh(d*x + c)^4 + 2*(35*(a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^4 + 3*a^4 - 8*a^3*b - 2*a^2*b^2 - 9*b^4 + 30*(a^4 - 2*a^3*b - 4*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4 + 8*(7*(a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^5 + 10*(a^4 - 2*a^3*b - 4*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + (3*a^4 - 8*a^3*b - 2*a^2*b^2 - 9*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^4 - 2*a^3*b - 4*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^2 + 4*(7*(a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^6 + 15*(a^4 - 2*a^3*b - 4*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + a^4 - 2*a^3*b - 4*a^2*b^2 + 2*a*b^3 + 3*b^4 + 3*(3*a^4 - 8*a^3*b - 2*a^2*b^2 - 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^7 + 3*(a^4 - 2*a^3*b - 4*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^5 + (3*a^4 - 8*a^3*b - 2*a^2*b^2 - 9*b^4)*cosh(d*x + c)^3 + (a^4 - 2*a^3*b - 4*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 8*(3*(a^4*b - a^3*b^2 - 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^5 + 2*(3*a^4*b + 7*a^3*b^2 - 3*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^3 + (3*a^4*b + 13*a^3*b^2 + a^2*b^3 - 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^8 + 8*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*sinh(d*x + c)^8 + 4*(a^6*b^2 + a^5*b^3 - a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^6 + 4*(7*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^2 + (a^6*b^2 + a^5*b^3 - a^4*b^4 - a^3*b^5)*d)*sinh(d*x + c)^6 + 2*(3*a^6*b^2 + a^5*b^3 + a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^3 + 3*(a^6*b^2 + a^5*b^3 - a^4*b^4 - a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^4 + 30*(a^6*b^2 + a^5*b^3 - a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^2 + (3*a^6*b^2 + a^5*b^3 + a^4*b^4 + 3*a^3*b^5)*d)*sinh(d*x + c)^4 + 4*(a^6*b^2 + a^5*b^3 - a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^5 + 10*(a^6*b^2 + a^5*b^3 - a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^3 + (3*a^6*b^2 + a^5*b^3 + a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^6 + 15*(a^6*b^2 + a^5*b^3 - a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^4 + 3*(3*a^6*b^2 + a^5*b^3 + a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^2 + (a^6*b^2 + a^5*b^3 - a^4*b^4 - a^3*b^5)*d)*sinh(d*x + c)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d + 8*((a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^7 + 3*(a^6*b^2 + a^5*b^3 - a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^5 + (3*a^6*b^2 + a^5*b^3 + a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^3 + (a^6*b^2 + a^5*b^3 - a^4*b^4 - a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)), -1/8*(2*(a^4*b - a^3*b^2 - 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^6 + 12*(a^4*b - a^3*b^2 - 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(a^4*b - a^3*b^2 - 5*a^2*b^3 - 3*a*b^4)*sinh(d*x + c)^6 + 2*a^4*b + 10*a^3*b^2 + 14*a^2*b^3 + 6*a*b^4 + 2*(3*a^4*b + 7*a^3*b^2 - 3*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^4 + 2*(3*a^4*b + 7*a^3*b^2 - 3*a^2*b^3 + 9*a*b^4 + 15*(a^4*b - a^3*b^2 - 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(5*(a^4*b - a^3*b^2 - 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^3 + (3*a^4*b + 7*a^

```
3*b^2 - 3*a^2*b^3 + 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(3*a^4*b +
13*a^3*b^2 + a^2*b^3 - 9*a*b^4)*cosh(d*x + c)^2 + 2*(3*a^4*b + 13*a^3*b^2 +
 a^2*b^3 - 9*a*b^4 + 15*(a^4*b - a^3*b^2 - 5*a^2*b^3 - 3*a*b^4)*cosh(d*x +
c)^4 + 6*(3*a^4*b + 7*a^3*b^2 - 3*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^2)*sinh(
d*x + c)^2 + ((a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^8 + 8*(a^4
- 6*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^4 - 6*a^2
*b^2 - 8*a*b^3 - 3*b^4)*sinh(d*x + c)^8 + 4*(a^4 - 2*a^3*b - 4*a^2*b^2 + 2*
a*b^3 + 3*b^4)*cosh(d*x + c)^6 + 4*(a^4 - 2*a^3*b - 4*a^2*b^2 + 2*a*b^3 + 3
*b^4 + 7*(a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)
^6 + 8*(7*(a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^3 + 3*(a^4 - 2*
a^3*b - 4*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*
a^4 - 8*a^3*b - 2*a^2*b^2 - 9*b^4)*cosh(d*x + c)^4 + 2*(35*(a^4 - 6*a^2*b^2
 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^4 + 3*a^4 - 8*a^3*b - 2*a^2*b^2 - 9*b^4 +
 30*(a^4 - 2*a^3*b - 4*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x
 + c)^4 + a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4 + 8*(7*(a^4 - 6*a^2*b^2 - 8*a*b
^3 - 3*b^4)*cosh(d*x + c)^5 + 10*(a^4 - 2*a^3*b - 4*a^2*b^2 + 2*a*b^3 + 3*b
^4)*cosh(d*x + c)^3 + (3*a^4 - 8*a^3*b - 2*a^2*b^2 - 9*b^4)*cosh(d*x + c))*
sinh(d*x + c)^3 + 4*(a^4 - 2*a^3*b - 4*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x
+ c)^2 + 4*(7*(a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^6 + 15*(a^4
 - 2*a^3*b - 4*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + a^4 - 2*a^3*b -
 4*a^2*b^2 + 2*a*b^3 + 3*b^4 + 3*(3*a^4 - 8*a^3*b - 2*a^2*b^2 - 9*b^4)*cosh
(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(
d*x + c)^7 + 3*(a^4 - 2*a^3*b - 4*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^
5 + (3*a^4 - 8*a^3*b - 2*a^2*b^2 - 9*b^4)*cosh(d*x + c)^3 + (a^4 - 2*a^3*b
- 4*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b)*arct
an(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (
a + b)*sinh(d*x + c)^2 + a - b)*sqrt(a*b)/(a*b)) + 4*(3*(a^4*b - a^3*b^2 -
5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^5 + 2*(3*a^4*b + 7*a^3*b^2 - 3*a^2*b^3 +
 9*a*b^4)*cosh(d*x + c)^3 + (3*a^4*b + 13*a^3*b^2 + a^2*b^3 - 9*a*b^4)*cosh
(d*x + c))*sinh(d*x + c))/((a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*co
sh(d*x + c)^8 + 8*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x +
c)*sinh(d*x + c)^7 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*sinh(d*x
 + c)^8 + 4*(a^6*b^2 + a^5*b^3 - a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^6 + 4*(
7*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^2 + (a^6*b^2
+ a^5*b^3 - a^4*b^4 - a^3*b^5)*d)*sinh(d*x + c)^6 + 2*(3*a^6*b^2 + a^5*b^3
+ a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^6*b^2 + 3*a^5*b^3 + 3*a^
4*b^4 + a^3*b^5)*d*cosh(d*x + c)^3 + 3*(a^6*b^2 + a^5*b^3 - a^4*b^4 - a^3*b
^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b
^4 + a^3*b^5)*d*cosh(d*x + c)^4 + 30*(a^6*b^2 + a^5*b^3 - a^4*b^4 - a^3*b^5
)*d*cosh(d*x + c)^2 + (3*a^6*b^2 + a^5*b^3 + a^4*b^4 + 3*a^3*b^5)*d)*sinh(d
*x + c)^4 + 4*(a^6*b^2 + a^5*b^3 - a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^2 + 8
*(7*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^5 + 10*(a^6
*b^2 + a^5*b^3 - a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^3 + (3*a^6*b^2 + a^5*b^
3 + a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^6*b^2 +
 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^6 + 15*(a^6*b^2 + a^5*b^3
 - a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^4 + 3*(3*a^6*b^2 + a^5*b^3 + a^4*b^4
+ 3*a^3*b^5)*d*cosh(d*x + c)^2 + (a^6*b^2 + a^5*b^3 - a^4*b^4 - a^3*b^5)*d)
*sinh(d*x + c)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d + 8*((a^6*
b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^7 + 3*(a^6*b^2 + a^5
*b^3 - a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^5 + (3*a^6*b^2 + a^5*b^3 + a^4*b^
4 + 3*a^3*b^5)*d*cosh(d*x + c)^3 + (a^6*b^2 + a^5*b^3 - a^4*b^4 - a^3*b^5)*
d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.9162, size = 448, normalized size = 3.9

$$-\frac{\frac{\left(ae^{(2c)}-3\,be^{(2c)}\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)e^{(-2\,c)}}{\sqrt{ab}a^2b}+\frac{2\left(a^3e^{(6\,dx+6\,c)}-a^2be^{(6\,dx+6\,c)}-5\,ab^2e^{(6\,dx+6\,c)}-3\,b^3e^{(6\,dx+6\,c)}+3\,a^3e^{(4\,dx+4\,c)}+7\,a^2be^{(4\,dx+4\,c)}-3\cdots\right.}{\left(a^3b+a^2b^2\right)\left(ae^{(4\,dx+4\,c)}+\cdots\right.}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((a*e^(2*c) - 3*b*e^(2*c))*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x
+ 2*c) + a - b)/sqrt(a*b))*e^(-2*c)/(sqrt(a*b)*a^2*b) + 2*(a^3*e^(6*d*x + 6
*c) - a^2*b*e^(6*d*x + 6*c) - 5*a*b^2*e^(6*d*x + 6*c) - 3*b^3*e^(6*d*x + 6*
c) + 3*a^3*e^(4*d*x + 4*c) + 7*a^2*b*e^(4*d*x + 4*c) - 3*a*b^2*e^(4*d*x + 4
*c) + 9*b^3*e^(4*d*x + 4*c) + 3*a^3*e^(2*d*x + 2*c) + 13*a^2*b*e^(2*d*x + 2
*c) + a*b^2*e^(2*d*x + 2*c) - 9*b^3*e^(2*d*x + 2*c) + a^3 + 5*a^2*b + 7*a*b
^2 + 3*b^3)/((a^3*b + a^2*b^2)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a
*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2))/d
```

3.131 $\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=104

$$\frac{3\sinh(c+dx)}{8a^2d\left((a+b)\sinh^2(c+dx)+a\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d\sqrt{a+b}}+\frac{\sinh(c+dx)}{4ad\left((a+b)\sinh^2(c+dx)+a\right)^2}$$

```
[Out] (3*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*Sqrt[a + b]*d) +
 Sinh[c + d*x]/(4*a*d*(a + (a + b)*Sinh[c + d*x]^2)^2) + (3*Sinh[c + d*x])/
(8*a^2*d*(a + (a + b)*Sinh[c + d*x]^2))
```

Rubi [A] time = 0.0905743, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3676, 199, 205}

$$\frac{3\sinh(c+dx)}{8a^2d\left((a+b)\sinh^2(c+dx)+a\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d\sqrt{a+b}}+\frac{\sinh(c+dx)}{4ad\left((a+b)\sinh^2(c+dx)+a\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (3*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*Sqrt[a + b]*d) +
 Sinh[c + d*x]/(4*a*d*(a + (a + b)*Sinh[c + d*x]^2)^2) + (3*Sinh[c + d*x])/
(8*a^2*d*(a + (a + b)*Sinh[c + d*x]^2))
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{\left(a+(a+b)x^2\right)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\sinh(c+dx)}{4ad\left(a+(a+b)\sinh^2(c+dx)\right)^2} + \frac{3\operatorname{Subst}\left(\int \frac{1}{\left(a+(a+b)x^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{4ad}$$

$$= \frac{\sinh(c+dx)}{4ad\left(a+(a+b)\sinh^2(c+dx)\right)^2} + \frac{3\sinh(c+dx)}{8a^2d\left(a+(a+b)\sinh^2(c+dx)\right)} + \frac{3\operatorname{Subst}\left(\int \frac{1}{a+(\cdots}\right.}{}$$

$$= \frac{3\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{a+b}d} + \frac{\sinh(c+dx)}{4ad\left(a+(a+b)\sinh^2(c+dx)\right)^2} + \frac{3\sinh(c+dx)}{8a^2d\left(a+(a+b)\sinh^2\cdots\right.}$$

Mathematica [A] time = 0.29467, size = 88, normalized size = 0.85

$$\frac{\frac{3\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a+b}} + \frac{3(a+b)\sinh^3(c+dx)+5a\sinh(c+dx)}{a\left((a+b)\sinh^2(c+dx)+a\right)^2}}{8ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((3*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(a^(3/2)*Sqrt[a + b]) + (5
*a*Sinh[c + d*x] + 3*(a + b)*Sinh[c + d*x]^3)/(a*(a + (a + b)*Sinh[c + d*x]
^2)^2))/(8*a*d)
```

Maple [B] time = 0.116, size = 634, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] -5/4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/
2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^7+3/4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh
(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^5-
3/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x
+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5-3/4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tan
h(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^3
+3/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*
x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^3+5/4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*ta
nh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)-
3/8/d/a^2/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)
/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+3/8/d/a^2*b/(b*(a+b))^(1/2)/((2*(b*(a
+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2
)-a-2*b)*a)^(1/2))+3/8/d/a^2/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*t
anh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+3/8/d/a^2*b/(b*(a+b
))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/(
```

```
(2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))
```

Maximia [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\left(ae^{(7\,c)}+be^{(7\,c)}\right)e^{(7\,dx)}+\left(11\,ae^{(5\,c)}-9\,be^{(5\,c)}\right)e^{(5\,dx)}-\left(11\,ae^{(3\,c)}-9\,be^{(3\,c)}\right)e^{(3\,dx)}}{4\left(a^4d+2\,a^3bd+a^2b^2d+\left(a^4de^{(8\,c)}+2\,a^3bde^{(8\,c)}+a^2b^2de^{(8\,c)}\right)e^{(8\,dx)}+4\left(a^4de^{(6\,c)}-a^2b^2de^{(6\,c)}\right)e^{(6\,dx)}+2\left(3\,a^4de^{(4\,c)}-2\,a^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(3*(a*e^(7*c) + b*e^(7*c))*e^(7*d*x) + (11*a*e^(5*c) - 9*b*e^(5*c))*e^(
5*d*x) - (11*a*e^(3*c) - 9*b*e^(3*c))*e^(3*d*x) - 3*(a*e^c + b*e^c)*e^(d*x)
)/(a^4*d + 2*a^3*b*d + a^2*b^2*d + (a^4*d*e^(8*c) + 2*a^3*b*d*e^(8*c) + a^2
*b^2*d*e^(8*c))*e^(8*d*x) + 4*(a^4*d*e^(6*c) - a^2*b^2*d*e^(6*c))*e^(6*d*x)
 + 2*(3*a^4*d*e^(4*c) - 2*a^3*b*d*e^(4*c) + 3*a^2*b^2*d*e^(4*c))*e^(4*d*x)
+ 4*(a^4*d*e^(2*c) - a^2*b^2*d*e^(2*c))*e^(2*d*x)) + 32*integrate(3/128*(e^
(3*d*x + 3*c) + e^(d*x + c))/(a^3 + a^2*b + (a^3*e^(4*c) + a^2*b*e^(4*c))*e
^(4*d*x) + 2*(a^3*e^(2*c) - a^2*b*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 2.78232, size = 12057, normalized size = 115.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(12*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^7 + 84*(a^3 + 2*a^2*b + a*b
^2)*cosh(d*x + c)*sinh(d*x + c)^6 + 12*(a^3 + 2*a^2*b + a*b^2)*sinh(d*x + c
)^7 + 4*(11*a^3 + 2*a^2*b - 9*a*b^2)*cosh(d*x + c)^5 + 4*(11*a^3 + 2*a^2*b
- 9*a*b^2 + 63*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 2
0*(21*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^3 + (11*a^3 + 2*a^2*b - 9*a*b^2
)*cosh(d*x + c))*sinh(d*x + c)^4 - 4*(11*a^3 + 2*a^2*b - 9*a*b^2)*cosh(d*x
+ c)^3 + 4*(105*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^4 - 11*a^3 - 2*a^2*b
+ 9*a*b^2 + 10*(11*a^3 + 2*a^2*b - 9*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^
3 + 4*(63*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^5 + 10*(11*a^3 + 2*a^2*b -
9*a*b^2)*cosh(d*x + c)^3 - 3*(11*a^3 + 2*a^2*b - 9*a*b^2)*cosh(d*x + c))*si
nh(d*x + c)^2 - 3*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b
^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^8 + 4
*(a^2 - b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a
^2 - b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a
^2 - b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^2 - 2*a*b + 3*b^2)*cosh(d
*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 30*(a^2 - b^2)*cosh
(d*x + c)^2 + 3*a^2 - 2*a*b + 3*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b +
b^2)*cosh(d*x + c)^5 + 10*(a^2 - b^2)*cosh(d*x + c)^3 + (3*a^2 - 2*a*b + 3*
b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^2 - b^2)*cosh(d*x + c)^2 + 4*(7*
(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 15*(a^2 - b^2)*cosh(d*x + c)^4 + 3*(3
*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 +
2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 3*(a^2 - b^2)*cosh(d
*x + c)^5 + (3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x
+ c))*sinh(d*x + c))*sqrt(-a^2 - a*b)*log(((a + b)*cosh(d*x + c)^4 + 4*(a +
 b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)*c
osh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2 +
```

```
4*((a + b)*cosh(d*x + c)^3 - (3*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(co
sh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh
(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a + b)/(
(a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)
*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2
 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x +
 c))*sinh(d*x + c) + a + b)) - 12*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c) + 4
*(21*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^6 + 5*(11*a^3 + 2*a^2*b - 9*a*b^
2)*cosh(d*x + c)^4 - 3*a^3 - 6*a^2*b - 3*a*b^2 - 3*(11*a^3 + 2*a^2*b - 9*a*
b^2)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)
*d*cosh(d*x + c)^8 + 8*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c
)*sinh(d*x + c)^7 + (a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*sinh(d*x + c)^8
 + 4*(a^6 + a^5*b - a^4*b^2 - a^3*b^3)*d*cosh(d*x + c)^6 + 4*(7*(a^6 + 3*a^
5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^2 + (a^6 + a^5*b - a^4*b^2 - a^3
*b^3)*d)*sinh(d*x + c)^6 + 2*(3*a^6 + a^5*b + a^4*b^2 + 3*a^3*b^3)*d*cosh(d
*x + c)^4 + 8*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^3 +
3*(a^6 + a^5*b - a^4*b^2 - a^3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3
5*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^4 + 30*(a^6 + a^5*b
 - a^4*b^2 - a^3*b^3)*d*cosh(d*x + c)^2 + (3*a^6 + a^5*b + a^4*b^2 + 3*a^3*
b^3)*d)*sinh(d*x + c)^4 + 4*(a^6 + a^5*b - a^4*b^2 - a^3*b^3)*d*cosh(d*x +
c)^2 + 8*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^5 + 10*(a
^6 + a^5*b - a^4*b^2 - a^3*b^3)*d*cosh(d*x + c)^3 + (3*a^6 + a^5*b + a^4*b^
2 + 3*a^3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^6 + 3*a^5*b + 3*a
^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^6 + 15*(a^6 + a^5*b - a^4*b^2 - a^3*b^3)*
d*cosh(d*x + c)^4 + 3*(3*a^6 + a^5*b + a^4*b^2 + 3*a^3*b^3)*d*cosh(d*x + c)
^2 + (a^6 + a^5*b - a^4*b^2 - a^3*b^3)*d)*sinh(d*x + c)^2 + (a^6 + 3*a^5*b
+ 3*a^4*b^2 + a^3*b^3)*d + 8*((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(
d*x + c)^7 + 3*(a^6 + a^5*b - a^4*b^2 - a^3*b^3)*d*cosh(d*x + c)^5 + (3*a^6
 + a^5*b + a^4*b^2 + 3*a^3*b^3)*d*cosh(d*x + c)^3 + (a^6 + a^5*b - a^4*b^2
- a^3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*(6*(a^3 + 2*a^2*b + a*b^2)*
cosh(d*x + c)^7 + 42*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c)^6
+ 6*(a^3 + 2*a^2*b + a*b^2)*sinh(d*x + c)^7 + 2*(11*a^3 + 2*a^2*b - 9*a*b^2
)*cosh(d*x + c)^5 + 2*(11*a^3 + 2*a^2*b - 9*a*b^2 + 63*(a^3 + 2*a^2*b + a*b
^2)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 10*(21*(a^3 + 2*a^2*b + a*b^2)*cosh(
d*x + c)^3 + (11*a^3 + 2*a^2*b - 9*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 -
2*(11*a^3 + 2*a^2*b - 9*a*b^2)*cosh(d*x + c)^3 + 2*(105*(a^3 + 2*a^2*b + a*
b^2)*cosh(d*x + c)^4 - 11*a^3 - 2*a^2*b + 9*a*b^2 + 10*(11*a^3 + 2*a^2*b -
9*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(63*(a^3 + 2*a^2*b + a*b^2)*c
osh(d*x + c)^5 + 10*(11*a^3 + 2*a^2*b - 9*a*b^2)*cosh(d*x + c)^3 - 3*(11*a^
3 + 2*a^2*b - 9*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 3*((a^2 + 2*a*b + b
^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 +
 (a^2 + 2*a*b + b^2)*sinh(d*x + c)^8 + 4*(a^2 - b^2)*cosh(d*x + c)^6 + 4*(7
*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^6 + 8*(7*(a
^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 - b^2)*cosh(d*x + c))*sinh(d*x +
 c)^5 + 2*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^
2)*cosh(d*x + c)^4 + 30*(a^2 - b^2)*cosh(d*x + c)^2 + 3*a^2 - 2*a*b + 3*b^2
)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 10*(a^2 - b^
2)*cosh(d*x + c)^3 + (3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3
 + 4*(a^2 - b^2)*cosh(d*x + c)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6
 + 15*(a^2 - b^2)*cosh(d*x + c)^4 + 3*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)
^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2
)*cosh(d*x + c)^7 + 3*(a^2 - b^2)*cosh(d*x + c)^5 + (3*a^2 - 2*a*b + 3*b^2)
*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + a*b
)*arctan(1/2*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x +
c)^2 + (a + b)*sinh(d*x + c)^3 + (3*a - b)*cosh(d*x + c) + (3*(a + b)*cosh(
d*x + c)^2 + 3*a - b)*sinh(d*x + c))/sqrt(a^2 + a*b)) + 3*((a^2 + 2*a*b + b
^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 +
 (a^2 + 2*a*b + b^2)*sinh(d*x + c)^8 + 4*(a^2 - b^2)*cosh(d*x + c)^6 + 4*(7
*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^6 + 8*(7*(a
```

```
^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 - b^2)*cosh(d*x + c))*sinh(d*x +
 c)^5 + 2*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^
2)*cosh(d*x + c)^4 + 30*(a^2 - b^2)*cosh(d*x + c)^2 + 3*a^2 - 2*a*b + 3*b^2
)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 10*(a^2 - b^
2)*cosh(d*x + c)^3 + (3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3
 + 4*(a^2 - b^2)*cosh(d*x + c)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6
 + 15*(a^2 - b^2)*cosh(d*x + c)^4 + 3*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)
^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2
)*cosh(d*x + c)^7 + 3*(a^2 - b^2)*cosh(d*x + c)^5 + (3*a^2 - 2*a*b + 3*b^2)
*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + a*b
)*arctan(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh(d*x + c))/a) - 6*(a^3 +
2*a^2*b + a*b^2)*cosh(d*x + c) + 2*(21*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c
)^6 + 5*(11*a^3 + 2*a^2*b - 9*a*b^2)*cosh(d*x + c)^4 - 3*a^3 - 6*a^2*b - 3*
a*b^2 - 3*(11*a^3 + 2*a^2*b - 9*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^
6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^8 + 8*(a^6 + 3*a^5*b + 3
*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^6 + 3*a^5*b + 3*a^
4*b^2 + a^3*b^3)*d*sinh(d*x + c)^8 + 4*(a^6 + a^5*b - a^4*b^2 - a^3*b^3)*d*
cosh(d*x + c)^6 + 4*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c
)^2 + (a^6 + a^5*b - a^4*b^2 - a^3*b^3)*d)*sinh(d*x + c)^6 + 2*(3*a^6 + a^5
*b + a^4*b^2 + 3*a^3*b^3)*d*cosh(d*x + c)^4 + 8*(7*(a^6 + 3*a^5*b + 3*a^4*b
^2 + a^3*b^3)*d*cosh(d*x + c)^3 + 3*(a^6 + a^5*b - a^4*b^2 - a^3*b^3)*d*cos
h(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d
*cosh(d*x + c)^4 + 30*(a^6 + a^5*b - a^4*b^2 - a^3*b^3)*d*cosh(d*x + c)^2 +
 (3*a^6 + a^5*b + a^4*b^2 + 3*a^3*b^3)*d)*sinh(d*x + c)^4 + 4*(a^6 + a^5*b
- a^4*b^2 - a^3*b^3)*d*cosh(d*x + c)^2 + 8*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 +
a^3*b^3)*d*cosh(d*x + c)^5 + 10*(a^6 + a^5*b - a^4*b^2 - a^3*b^3)*d*cosh(d*
x + c)^3 + (3*a^6 + a^5*b + a^4*b^2 + 3*a^3*b^3)*d*cosh(d*x + c))*sinh(d*x
+ c)^3 + 4*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^6 + 15*
(a^6 + a^5*b - a^4*b^2 - a^3*b^3)*d*cosh(d*x + c)^4 + 3*(3*a^6 + a^5*b + a^
4*b^2 + 3*a^3*b^3)*d*cosh(d*x + c)^2 + (a^6 + a^5*b - a^4*b^2 - a^3*b^3)*d)
*sinh(d*x + c)^2 + (a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d + 8*((a^6 + 3*a^
5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^7 + 3*(a^6 + a^5*b - a^4*b^2 - a
^3*b^3)*d*cosh(d*x + c)^5 + (3*a^6 + a^5*b + a^4*b^2 + 3*a^3*b^3)*d*cosh(d*
x + c)^3 + (a^6 + a^5*b - a^4*b^2 - a^3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**5/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 2.29725, size = 5954, normalized size = 57.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

[Out] 1/32*(6*(3*(2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*cos(1/2*real_part(a
rccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/
(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) - (2*a^2*b
+ 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*cosh(1/2*imag_part(arccos(-a/(a + b) +
b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3 - 9*(2*
a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a +
b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*s
in(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos
(-a/(a + b) + b/(a + b)))) + 3*(2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))
*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(ar
ccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(
a + b)))) + 9*(2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*cos(1/2*real_par
t(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) +
 b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*i
mag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(2*a^2*b + 2*a*b^2 - (a^2 -
 b^2)*sqrt(-a*b))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1
/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-
a/(a + b) + b/(a + b))))^2 - 3*(2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))
*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arc
cos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a +
 b))))^3 + (2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*sin(1/2*real_part(a
rccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/
(a + b))))^3 + (2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*cosh(1/2*imag_p
art(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) +
b/(a + b)))) - (2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*sin(1/2*real_pa
rt(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) +
b/(a + b)))))*arctan((((a^3 + a^2*b)/(a^3*e^(4*c) + a^2*b*e^(4*c)))^(1/4)*c
os(1/2*arccos(-(a - b)/(a + b))) + e^(d*x))/(((a^3 + a^2*b)/(a^3*e^(4*c) +
a^2*b*e^(4*c)))^(1/4)*sin(1/2*arccos(-(a - b)/(a + b)))))/(a^5*b + 2*a^4*b^
2 + a^3*b^3) + 6*(3*(2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*cos(1/2*re
al_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a
+ b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) -
(2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*cosh(1/2*imag_part(arccos(-a/(
a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^
3 - 9*(2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*cos(1/2*real_part(arccos
(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a +
b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_pa
rt(arccos(-a/(a + b) + b/(a + b)))) + 3*(2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sq
rt(-a*b))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*rea
l_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a +
 b) + b/(a + b)))) + 9*(2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*cos(1/2
*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/
(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*s
inh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(2*a^2*b + 2*a*b^2
 - (a^2 - b^2)*sqrt(-a*b))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)
)))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part
(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sq
rt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real
_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b)
 + b/(a + b))))^3 + (2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*sin(1/2*re
al_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a
+ b) + b/(a + b))))^3 + (2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*cosh(1
/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(-a/(
a + b) + b/(a + b)))) - (2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*sin(1/
2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(
a + b) + b/(a + b)))))*arctan(-(((a^3 + a^2*b)/(a^3*e^(4*c) + a^2*b*e^(4*c)
))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b))) - e^(d*x))/(((a^3 + a^2*b)/(a^3*
e^(4*c) + a^2*b*e^(4*c)))^(1/4)*sin(1/2*arccos(-(a - b)/(a + b)))))/(a^5*b
+ 2*a^4*b^2 + a^3*b^3) + 3*((2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*co

```
s(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arcco
s(-a/(a + b) + b/(a + b))))^3 - 3*(2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*
b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(a
rccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(
a + b))))^2 - 3*(2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*cos(1/2*real_p
art(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b)
 + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(
2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a
+ b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*s
in(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arcc
os(-a/(a + b) + b/(a + b)))) + 3*(2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b
))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(
arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(
a + b))))^2 - 9*(2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*cos(1/2*real_p
art(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) +
 b/(a + b))))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2
*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - (2*a^2*b + 2*a*b^2 - (a^2 -
 b^2)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh
(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3 + 3*(2*a^2*b + 2*a*b^2 -
(a^2 - b^2)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*
sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arc
cos(-a/(a + b) + b/(a + b))))^3 + (2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*
b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(a
rccos(-a/(a + b) + b/(a + b)))) - (2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*
b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(a
rccos(-a/(a + b) + b/(a + b)))))*log(2*((a^3 + a^2*b)/(a^3*e^(4*c) + a^2*b*
e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b)))*e^(d*x) + sqrt((a^3 + a^2
*b)/(a^3*e^(4*c) + a^2*b*e^(4*c))) + e^(2*d*x))/(a^5*b + 2*a^4*b^2 + a^3*b^
3) - 3*((2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*cos(1/2*real_part(arcc
os(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a
+ b))))^3 - 3*(2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*cos(1/2*real_par
t(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b
/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2 - 3*(2*a
^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b
) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*si
nh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(2*a^2*b + 2*a*b^2 -
(a^2 - b^2)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*
cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arc
cos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a
 + b)))) + 3*(2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*cos(1/2*real_part
(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) +
b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 9*(2*a
^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*cos(1/2*real_part(arccos(-a/(a + b
) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/
2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a
/(a + b) + b/(a + b))))^2 - (2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*co
s(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arcco
s(-a/(a + b) + b/(a + b))))^3 + 3*(2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*
b))*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(ar
ccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(
a + b))))^3 + (2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*cos(1/2*real_par
t(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b
/(a + b)))) - (2*a^2*b + 2*a*b^2 - (a^2 - b^2)*sqrt(-a*b))*cos(1/2*real_par
t(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b
/(a + b)))))*log(-2*((a^3 + a^2*b)/(a^3*e^(4*c) + a^2*b*e^(4*c)))^(1/4)*cos
(1/2*arccos(-(a - b)/(a + b)))*e^(d*x) + sqrt((a^3 + a^2*b)/(a^3*e^(4*c) +
a^2*b*e^(4*c))) + e^(2*d*x))/(a^5*b + 2*a^4*b^2 + a^3*b^3) + 8*(3*a*e^(7*d*
x + 7*c) + 3*b*e^(7*d*x + 7*c) + 11*a*e^(5*d*x + 5*c) - 9*b*e^(5*d*x + 5*c)
 - 11*a*e^(3*d*x + 3*c) + 9*b*e^(3*d*x + 3*c) - 3*a*e^(d*x + c) - 3*b*e^(d*
```

```
x + c))/((a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b
*e^(2*d*x + 2*c) + a + b)^2*a^2))/d
```

3.132 $\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=131

$$\frac{3\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\tanh(c+dx)}{8d\left(a+b\tanh^2(c+dx)\right)}+\frac{\left(3a^2-2ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}d}+\frac{(a+b)\tanh(c+dx)\operatorname{sech}^2(c+dx)}{4abd\left(a+b\tanh^2(c+dx)\right)^2}$$

```
[Out] ((3*a^2 - 2*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(5/2
)*b^(5/2)*d) + ((a + b)*Sech[c + d*x]^2*Tanh[c + d*x])/(4*a*b*d*(a + b*Tanh
[c + d*x]^2)^2) + (3*(a^(-2) - b^(-2))*Tanh[c + d*x])/(8*d*(a + b*Tanh[c +
d*x]^2))
```

Rubi [A] time = 0.123816, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3675, 413, 385, 205}

$$\frac{3\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\tanh(c+dx)}{8d\left(a+b\tanh^2(c+dx)\right)}+\frac{\left(3a^2-2ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}d}+\frac{(a+b)\tanh(c+dx)\operatorname{sech}^2(c+dx)}{4abd\left(a+b\tanh^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^6/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((3*a^2 - 2*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(5/2
)*b^(5/2)*d) + ((a + b)*Sech[c + d*x]^2*Tanh[c + d*x])/(4*a*b*d*(a + b*Tanh
[c + d*x]^2)^2) + (3*(a^(-2) - b^(-2))*Tanh[c + d*x])/(8*d*(a + b*Tanh[c +
d*x]^2))
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
```

```
p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a+bx^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{(a+b)\operatorname{sech}^2(c+dx)\tanh(c+dx)}{4abd\left(a+b\tanh^2(c+dx)\right)^2} + \frac{\operatorname{Subst}\left(\int \frac{-a+3b+(3a-b)x^2}{\left(a+bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{4abd}$$

$$= \frac{(a+b)\operatorname{sech}^2(c+dx)\tanh(c+dx)}{4abd\left(a+b\tanh^2(c+dx)\right)^2} - \frac{3\left(a^2-b^2\right)\tanh(c+dx)}{8a^2b^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{\left(3a^2-2ab+3b\right.}{}$$

$$= \frac{\left(3a^2-2ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}d} + \frac{(a+b)\operatorname{sech}^2(c+dx)\tanh(c+dx)}{4abd\left(a+b\tanh^2(c+dx)\right)^2} - \frac{3\left(\right.}{8a^2b^2}$$

Mathematica [A] time = 0.893839, size = 128, normalized size = 0.98

$$\frac{\left(3a^2-2ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right) - \frac{\sqrt{a}\sqrt{b}(a+b)\sinh(2(c+dx))\left(3\left(a^2-b^2\right)\cosh(2(c+dx))+3a^2-10ab+3b^2\right)}{((a+b)\cosh(2(c+dx))+a-b)^2}}{8a^{5/2}b^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^6/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((3*a^2 - 2*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] - (Sqrt[a]
*Sqrt[b]*(a + b)*(3*a^2 - 10*a*b + 3*b^2 + 3*(a^2 - b^2)*Cosh[2*(c + d*x)])
*Sinh[2*(c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)])^2)/(8*a^(5/2)*b^(5/
2)*d)
```

Maple [B] time = 0.114, size = 1776, normalized size = 13.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] -3/8/d/a^2*b/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*
tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-1/8/d/(b*(a+b))^(1
/2)/a/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2
*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-1/8/d/(b*(a+b))^(1/2)/a/((2*(b*(a+b))^(1/
2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*
a)^(1/2))-1/8/d/b/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arcta
```

```
nh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-1/4/d/a/b/((2
*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b)
)^(1/2)-a-2*b)*a)^(1/2))-1/8/d/b/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)
*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))
+1/4/d/a/b/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)
/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-3/8/d/a^2*b/(b*(a+b))^(1/2)/((2*(b*(a
+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)
+a+2*b)*a)^(1/2))-3/8/d*a/b^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)
^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-3
/8/d*a/b^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tan
h(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-9/4/d/(tanh(1/2*d*x+1
/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a/b^2*ta
nh(1/2*d*x+1/2*c)^5-9/4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*
a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a/b^2*tanh(1/2*d*x+1/2*c)^3+3/d/a^2*b/(tan
h(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)
^2*tanh(1/2*d*x+1/2*c)^3+3/8/d/a^2/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arct
anh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-3/8/d/a^2/((
2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b)
)^(1/2)+a+2*b)*a)^(1/2))+3/8/d/b^2/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arct
anh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-3/8/d/b^2/((
2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b)
)^(1/2)+a+2*b)*a)^(1/2))+5/4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*
c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^7+7/4/d/(tanh(1
/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/
a*tanh(1/2*d*x+1/2*c)^5+7/4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c
)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^3+5/4/d/(tanh(1/
2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a
*tanh(1/2*d*x+1/2*c)-7/2/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2
*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/b*tanh(1/2*d*x+1/2*c)^3+1/2/d/(tanh(1/2*d
*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/b*ta
nh(1/2*d*x+1/2*c)+1/2/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+
4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/b*tanh(1/2*d*x+1/2*c)^7-7/2/d/(tanh(1/2*d*x+
1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/b*tanh(
1/2*d*x+1/2*c)^5+3/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2
*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5-3/4/d/(tanh(1/2*d*x
+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a/b^2*
tanh(1/2*d*x+1/2*c)-3/4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*
a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a/b^2*tanh(1/2*d*x+1/2*c)^7
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.86921, size = 11889, normalized size = 90.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [1/16*(4*(3*a^4*b + a^3*b^2 + a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^6 + 24*(3*a^
4*b + a^3*b^2 + a^2*b^3 + 3*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^5 + 4*(3*a^4
*b + a^3*b^2 + a^2*b^3 + 3*a*b^4)*sinh(d*x + c)^6 + 12*a^4*b + 12*a^3*b^2 -
 12*a^2*b^3 - 12*a*b^4 + 12*(3*a^4*b - 5*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4)*cos
h(d*x + c)^4 + 12*(3*a^4*b - 5*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4 + 5*(3*a^4*b +
 a^3*b^2 + a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(5*(3*a
^4*b + a^3*b^2 + a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^3 + 3*(3*a^4*b - 5*a^3*b^
2 + 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(9*a^4*b - 13*a
^3*b^2 - 13*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^2 + 4*(9*a^4*b - 13*a^3*b^2 -
13*a^2*b^3 + 9*a*b^4 + 15*(3*a^4*b + a^3*b^2 + a^2*b^3 + 3*a*b^4)*cosh(d*x
+ c)^4 + 18*(3*a^4*b - 5*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^2)*si
nh(d*x + c)^2 - ((3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x +
 c)^8 + 8*(3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)*sin
h(d*x + c)^7 + (3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*sinh(d*x + c
)^8 + 4*(3*a^4 - 2*a^3*b + 2*a*b^3 - 3*b^4)*cosh(d*x + c)^6 + 4*(3*a^4 - 2*
a^3*b + 2*a*b^3 - 3*b^4 + 7*(3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)
*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a
*b^3 + 3*b^4)*cosh(d*x + c)^3 + 3*(3*a^4 - 2*a^3*b + 2*a*b^3 - 3*b^4)*cosh(
d*x + c))*sinh(d*x + c)^5 + 2*(9*a^4 - 12*a^3*b + 22*a^2*b^2 - 12*a*b^3 + 9
*b^4)*cosh(d*x + c)^4 + 2*(35*(3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^
4)*cosh(d*x + c)^4 + 9*a^4 - 12*a^3*b + 22*a^2*b^2 - 12*a*b^3 + 9*b^4 + 30*
(3*a^4 - 2*a^3*b + 2*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 3*a^
4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4 + 8*(7*(3*a^4 + 4*a^3*b + 2*a^2*b
^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^5 + 10*(3*a^4 - 2*a^3*b + 2*a*b^3 - 3*b
^4)*cosh(d*x + c)^3 + (9*a^4 - 12*a^3*b + 22*a^2*b^2 - 12*a*b^3 + 9*b^4)*co
sh(d*x + c))*sinh(d*x + c)^3 + 4*(3*a^4 - 2*a^3*b + 2*a*b^3 - 3*b^4)*cosh(d
*x + c)^2 + 4*(7*(3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x +
 c)^6 + 15*(3*a^4 - 2*a^3*b + 2*a*b^3 - 3*b^4)*cosh(d*x + c)^4 + 3*a^4 - 2*
a^3*b + 2*a*b^3 - 3*b^4 + 3*(9*a^4 - 12*a^3*b + 22*a^2*b^2 - 12*a*b^3 + 9*b
^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*
a*b^3 + 3*b^4)*cosh(d*x + c)^7 + 3*(3*a^4 - 2*a^3*b + 2*a*b^3 - 3*b^4)*cosh
(d*x + c)^5 + (9*a^4 - 12*a^3*b + 22*a^2*b^2 - 12*a*b^3 + 9*b^4)*cosh(d*x +
 c)^3 + (3*a^4 - 2*a^3*b + 2*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*s
qrt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*
cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^
2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 -
 b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x
 + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x +
c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a
- b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*
x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a +
b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 +
(a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 8*(3*(3*a^4*b + a^3*b^2 +
a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^5 + 6*(3*a^4*b - 5*a^3*b^2 + 5*a^2*b^3 - 3
*a*b^4)*cosh(d*x + c)^3 + (9*a^4*b - 13*a^3*b^2 - 13*a^2*b^3 + 9*a*b^4)*cos
h(d*x + c))*sinh(d*x + c))/((a^5*b^3 + 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)
^8 + 8*(a^5*b^3 + 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a
^5*b^3 + 2*a^4*b^4 + a^3*b^5)*d*sinh(d*x + c)^8 + 4*(a^5*b^3 - a^3*b^5)*d*c
osh(d*x + c)^6 + 4*(7*(a^5*b^3 + 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^2 + (
a^5*b^3 - a^3*b^5)*d)*sinh(d*x + c)^6 + 2*(3*a^5*b^3 - 2*a^4*b^4 + 3*a^3*b^
5)*d*cosh(d*x + c)^4 + 8*(7*(a^5*b^3 + 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)
^3 + 3*(a^5*b^3 - a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^5*b^
3 + 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^4 + 30*(a^5*b^3 - a^3*b^5)*d*cosh(
d*x + c)^2 + (3*a^5*b^3 - 2*a^4*b^4 + 3*a^3*b^5)*d)*sinh(d*x + c)^4 + 4*(a^
5*b^3 - a^3*b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^5*b^3 + 2*a^4*b^4 + a^3*b^5)*d
*cosh(d*x + c)^5 + 10*(a^5*b^3 - a^3*b^5)*d*cosh(d*x + c)^3 + (3*a^5*b^3 -
2*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^5*b^3 + 2
```

```
*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^6 + 15*(a^5*b^3 - a^3*b^5)*d*cosh(d*x +
 c)^4 + 3*(3*a^5*b^3 - 2*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^2 + (a^5*b^3
- a^3*b^5)*d)*sinh(d*x + c)^2 + (a^5*b^3 + 2*a^4*b^4 + a^3*b^5)*d + 8*((a^5
*b^3 + 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^7 + 3*(a^5*b^3 - a^3*b^5)*d*cos
h(d*x + c)^5 + (3*a^5*b^3 - 2*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^3 + (a^5
*b^3 - a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*(2*(3*a^4*b + a^3*b^2
+ a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^6 + 12*(3*a^4*b + a^3*b^2 + a^2*b^3 + 3*
a*b^4)*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(3*a^4*b + a^3*b^2 + a^2*b^3 + 3*a
*b^4)*sinh(d*x + c)^6 + 6*a^4*b + 6*a^3*b^2 - 6*a^2*b^3 - 6*a*b^4 + 6*(3*a^
4*b - 5*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^4 + 6*(3*a^4*b - 5*a^3
*b^2 + 5*a^2*b^3 - 3*a*b^4 + 5*(3*a^4*b + a^3*b^2 + a^2*b^3 + 3*a*b^4)*cosh
(d*x + c)^2)*sinh(d*x + c)^4 + 8*(5*(3*a^4*b + a^3*b^2 + a^2*b^3 + 3*a*b^4)
*cosh(d*x + c)^3 + 3*(3*a^4*b - 5*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4)*cosh(d*x +
 c))*sinh(d*x + c)^3 + 2*(9*a^4*b - 13*a^3*b^2 - 13*a^2*b^3 + 9*a*b^4)*cosh
(d*x + c)^2 + 2*(9*a^4*b - 13*a^3*b^2 - 13*a^2*b^3 + 9*a*b^4 + 15*(3*a^4*b
+ a^3*b^2 + a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^4 + 18*(3*a^4*b - 5*a^3*b^2 +
5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((3*a^4 + 4*a^3*b +
 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^8 + 8*(3*a^4 + 4*a^3*b + 2*a^2*
b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^4 + 4*a^3*b + 2
*a^2*b^2 + 4*a*b^3 + 3*b^4)*sinh(d*x + c)^8 + 4*(3*a^4 - 2*a^3*b + 2*a*b^3
- 3*b^4)*cosh(d*x + c)^6 + 4*(3*a^4 - 2*a^3*b + 2*a*b^3 - 3*b^4 + 7*(3*a^4
+ 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 +
 8*(7*(3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + 3*(
3*a^4 - 2*a^3*b + 2*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(9*a^
4 - 12*a^3*b + 22*a^2*b^2 - 12*a*b^3 + 9*b^4)*cosh(d*x + c)^4 + 2*(35*(3*a^
4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 9*a^4 - 12*a^3
*b + 22*a^2*b^2 - 12*a*b^3 + 9*b^4 + 30*(3*a^4 - 2*a^3*b + 2*a*b^3 - 3*b^4)
*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 +
 3*b^4 + 8*(7*(3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)
^5 + 10*(3*a^4 - 2*a^3*b + 2*a*b^3 - 3*b^4)*cosh(d*x + c)^3 + (9*a^4 - 12*a
^3*b + 22*a^2*b^2 - 12*a*b^3 + 9*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(3
*a^4 - 2*a^3*b + 2*a*b^3 - 3*b^4)*cosh(d*x + c)^2 + 4*(7*(3*a^4 + 4*a^3*b +
 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^6 + 15*(3*a^4 - 2*a^3*b + 2*a*b
^3 - 3*b^4)*cosh(d*x + c)^4 + 3*a^4 - 2*a^3*b + 2*a*b^3 - 3*b^4 + 3*(9*a^4
- 12*a^3*b + 22*a^2*b^2 - 12*a*b^3 + 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^
2 + 8*((3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^7 + 3*
(3*a^4 - 2*a^3*b + 2*a*b^3 - 3*b^4)*cosh(d*x + c)^5 + (9*a^4 - 12*a^3*b + 2
2*a^2*b^2 - 12*a*b^3 + 9*b^4)*cosh(d*x + c)^3 + (3*a^4 - 2*a^3*b + 2*a*b^3
- 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b)*arctan(1/2*((a + b)*cosh(d
*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2
 + a - b)*sqrt(a*b)/(a*b)) + 4*(3*(3*a^4*b + a^3*b^2 + a^2*b^3 + 3*a*b^4)*c
osh(d*x + c)^5 + 6*(3*a^4*b - 5*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c
)^3 + (9*a^4*b - 13*a^3*b^2 - 13*a^2*b^3 + 9*a*b^4)*cosh(d*x + c))*sinh(d*x
 + c))/((a^5*b^3 + 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^8 + 8*(a^5*b^3 + 2*
a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^5*b^3 + 2*a^4*b^4 +
 a^3*b^5)*d*sinh(d*x + c)^8 + 4*(a^5*b^3 - a^3*b^5)*d*cosh(d*x + c)^6 + 4*(
7*(a^5*b^3 + 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^2 + (a^5*b^3 - a^3*b^5)*d
)*sinh(d*x + c)^6 + 2*(3*a^5*b^3 - 2*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^4
 + 8*(7*(a^5*b^3 + 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^3 + 3*(a^5*b^3 - a^
3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^5*b^3 + 2*a^4*b^4 + a^3*
b^5)*d*cosh(d*x + c)^4 + 30*(a^5*b^3 - a^3*b^5)*d*cosh(d*x + c)^2 + (3*a^5*
b^3 - 2*a^4*b^4 + 3*a^3*b^5)*d)*sinh(d*x + c)^4 + 4*(a^5*b^3 - a^3*b^5)*d*c
osh(d*x + c)^2 + 8*(7*(a^5*b^3 + 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^5 + 1
0*(a^5*b^3 - a^3*b^5)*d*cosh(d*x + c)^3 + (3*a^5*b^3 - 2*a^4*b^4 + 3*a^3*b^
5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^5*b^3 + 2*a^4*b^4 + a^3*b^5)*
d*cosh(d*x + c)^6 + 15*(a^5*b^3 - a^3*b^5)*d*cosh(d*x + c)^4 + 3*(3*a^5*b^3
 - 2*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^2 + (a^5*b^3 - a^3*b^5)*d)*sinh(d
*x + c)^2 + (a^5*b^3 + 2*a^4*b^4 + a^3*b^5)*d + 8*((a^5*b^3 + 2*a^4*b^4 + a
^3*b^5)*d*cosh(d*x + c)^7 + 3*(a^5*b^3 - a^3*b^5)*d*cosh(d*x + c)^5 + (3*a^
```

```
5*b^3 - 2*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^3 + (a^5*b^3 - a^3*b^5)*d*co
sh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**6/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.93837, size = 456, normalized size = 3.48

$$\frac{\frac{\left(3\,a^2e^{(2\,c)}-2\,abe^{(2\,c)}+3\,b^2e^{(2\,c)}\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)e^{(-2\,c)}}{\sqrt{ab}a^2b^2}+\frac{2\left(3\,a^3e^{(6\,dx+6\,c)}+a^2be^{(6\,dx+6\,c)}+ab^2e^{(6\,dx+6\,c)}+3\,b^3e^{(6\,dx+6\,c)}+9\,a^3e^{(4\,dx+4\,c)}-15\,a^2\cdots\right.}{\left(ae^{(4\,d\cdots}\right.}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((3*a^2*e^(2*c) - 2*a*b*e^(2*c) + 3*b^2*e^(2*c))*arctan(1/2*(a*e^(2*d*x
 + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))*e^(-2*c)/(sqrt(a*b)*a^2*b^2
) + 2*(3*a^3*e^(6*d*x + 6*c) + a^2*b*e^(6*d*x + 6*c) + a*b^2*e^(6*d*x + 6*c
) + 3*b^3*e^(6*d*x + 6*c) + 9*a^3*e^(4*d*x + 4*c) - 15*a^2*b*e^(4*d*x + 4*c
) + 15*a*b^2*e^(4*d*x + 4*c) - 9*b^3*e^(4*d*x + 4*c) + 9*a^3*e^(2*d*x + 2*c
) - 13*a^2*b*e^(2*d*x + 2*c) - 13*a*b^2*e^(2*d*x + 2*c) + 9*b^3*e^(2*d*x +
2*c) + 3*a^3 + 3*a^2*b - 3*a*b^2 - 3*b^3)/((a*e^(4*d*x + 4*c) + b*e^(4*d*x
+ 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2*a^2*b^2))/d
```

3.133 $\int \frac{\text{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=156

$$-\frac{(4a-3b)(a+b)\sinh(c+dx)}{8a^2b^2d\left((a+b)\sinh^2(c+dx)+a\right)}+\frac{\sqrt{a+b}\left(8a^2-4ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^3d}+\frac{(a+b)\sinh(c+dx)}{4abd\left((a+b)\sinh^2(c+dx)+a\right.}$$

```
[Out] -(ArcTan[Sinh[c + d*x]]/(b^3*d)) + (Sqrt[a + b]*(8*a^2 - 4*a*b + 3*b^2)*Arc
Tan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*b^3*d) + ((a + b)*Sinh
[c + d*x])/(4*a*b*d*(a + (a + b)*Sinh[c + d*x]^2)^2) - ((4*a - 3*b)*(a + b)
*Sinh[c + d*x])/(8*a^2*b^2*d*(a + (a + b)*Sinh[c + d*x]^2))
```

Rubi [A] time = 0.227987, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3676, 414, 527, 522, 203, 205}

$$-\frac{(4a-3b)(a+b)\sinh(c+dx)}{8a^2b^2d\left((a+b)\sinh^2(c+dx)+a\right)}+\frac{\sqrt{a+b}\left(8a^2-4ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^3d}+\frac{(a+b)\sinh(c+dx)}{4abd\left((a+b)\sinh^2(c+dx)+a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^7/(a + b*Tanh[c + d*x]^2)^3,x]
```



```
[Out] -(ArcTan[Sinh[c + d*x]]/(b^3*d)) + (Sqrt[a + b]*(8*a^2 - 4*a*b + 3*b^2)*Arc
Tan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*b^3*d) + ((a + b)*Sinh
[c + d*x])/(4*a*b*d*(a + (a + b)*Sinh[c + d*x]^2)^2) - ((4*a - 3*b)*(a + b)
*Sinh[c + d*x])/(8*a^2*b^2*d*(a + (a + b)*Sinh[c + d*x]^2))
```


Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
```

```
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)\left(a+(a+b)x^2\right)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{(a+b)\sinh(c+dx)}{4abd\left(a+(a+b)\sinh^2(c+dx)\right)^2} - \frac{\operatorname{Subst}\left(\int \frac{a-3b-3(a+b)x^2}{(1+x^2)\left(a+(a+b)x^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{4abd}$$

$$= \frac{(a+b)\sinh(c+dx)}{4abd\left(a+(a+b)\sinh^2(c+dx)\right)^2} - \frac{(4a-3b)(a+b)\sinh(c+dx)}{8a^2b^2d\left(a+(a+b)\sinh^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{4a \cdots}{\cdots}\right.}{\cdots}$$

$$= \frac{(a+b)\sinh(c+dx)}{4abd\left(a+(a+b)\sinh^2(c+dx)\right)^2} - \frac{(4a-3b)(a+b)\sinh(c+dx)}{8a^2b^2d\left(a+(a+b)\sinh^2(c+dx)\right)} - \frac{\operatorname{Subst}\left(\int \frac{\cdots}{1+\cdots}\right.}{\cdots}$$

$$= -\frac{\tan^{-1}(\sinh(c+dx))}{b^3d} + \frac{\sqrt{a+b}\left(8a^2-4ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^3d} + \frac{(a \cdots}{4abd\left(a+\cdots\right.}$$

Mathematica [C] time = 3.17087, size = 317, normalized size = 2.03

$$-\frac{\frac{i\sqrt{a+b}\left(8a^2-4ab+3b^2\right)\log((a+b)\cosh(2(c+dx))+a-b)}{a^{5/2}} - \frac{i\left(4a^2b+8a^3-ab^2+3b^3\right)\log((a+b)\cosh(2(c+dx))+a-b)}{a^{5/2}\sqrt{a+b}} + \frac{8b\left(4a^2+ab-3b^2\right)\sinh(c+dx)}{a^2((a+b)\cosh(2(c+dx))+a-b)} + \frac{2\sqrt{a\cdots}}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^7/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -((2*Sqrt[a + b]*(8*a^2 - 4*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqr
t[a + b]])/a^(5/2) + (2*(8*a^3 + 4*a^2*b - a*b^2 + 3*b^3)*ArcTan[(Sqrt[a]*C
sch[c + d*x])/Sqrt[a + b]])/(a^(5/2)*Sqrt[a + b]) + 64*ArcTan[Tanh[(c + d*x
```

```
)/2]] + (I*Sqrt[a + b]*(8*a^2 - 4*a*b + 3*b^2)*Log[a - b + (a + b)*Cosh[2*(
c + d*x)]])/a^(5/2) - (I*(8*a^3 + 4*a^2*b - a*b^2 + 3*b^3)*Log[a - b + (a +
 b)*Cosh[2*(c + d*x)]])/(a^(5/2)*Sqrt[a + b]) - (32*b^2*(a + b)*Sinh[c + d*
x])/(a*(a - b + (a + b)*Cosh[2*(c + d*x)])^2) + (8*b*(4*a^2 + a*b - 3*b^2)*
Sinh[c + d*x])/(a^2*(a - b + (a + b)*Cosh[2*(c + d*x)])))/(32*b^3*d)
```

Maple [B] time = 0.112, size = 1907, normalized size = 12.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 3/8/d/a^2*b/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*t
anh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-1/8/d/(b*(a+b))^(1/
2)/a/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*
(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-1/8/d/(b*(a+b))^(1/2)/a/((2*(b*(a+b))^(1/2
)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a
)^(1/2))-1/d/b^3*a/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d
*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/d/b^3*a/((2*(b*(a+b))^(1/2
)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a
)^(1/2))-2/d/b^3*arctan(tanh(1/2*d*x+1/2*c))+1/2/d/b/(b*(a+b))^(1/2)/((2*(b
*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(
1/2)-a-2*b)*a)^(1/2))+1/8/d/a/b/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh
(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/2/d/b/(b*(a+b
))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/(
(2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/8/d/a/b/((2*(b*(a+b))^(1/2)+a+2*b)*a)
^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+3/
8/d/a^2*b/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh
(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/d*a/b^2/(b*(a+b))^(1
/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(
b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/d*a/b^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/
2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*
a)^(1/2))+1/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2
*d*x+1/2*c)^2*b+a)^2*a/b^2*tanh(1/2*d*x+1/2*c)^5-1/d/(tanh(1/2*d*x+1/2*c)^4
*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a/b^2*tanh(1/2*
d*x+1/2*c)^3+3/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4
*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^3-3/8/d/a^2/((2*(b*(a+b))
^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-
2*b)*a)^(1/2))+3/8/d/a^2/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(
1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/2/d/b^2/((2*(b*(a+b))
^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-
2*b)*a)^(1/2))+1/2/d/b^2/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(
1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-5/4/d/(tanh(1/2*d*x+1/2
*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2
*d*x+1/2*c)^7+7/4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*ta
nh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^5-7/4/d/(tanh(1/2*d*x+1/2*
c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*
d*x+1/2*c)^3+5/4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tan
h(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)-23/4/d/(tanh(1/2*d*x+1/2*c)
^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/b*tanh(1/2*d*
x+1/2*c)^3+1/4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(
1/2*d*x+1/2*c)^2*b+a)^2/b*tanh(1/2*d*x+1/2*c)-1/4/d/(tanh(1/2*d*x+1/2*c)^4*
a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/b*tanh(1/2*d*x+1
/2*c)^7+23/4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/
2*d*x+1/2*c)^2*b+a)^2/b*tanh(1/2*d*x+1/2*c)^5-3/d/a^2*b/(tanh(1/2*d*x+1/2*c
```

```
)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x
+1/2*c)^5-1/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2
*d*x+1/2*c)^2*b+a)^2*a/b^2*tanh(1/2*d*x+1/2*c)+1/d/(tanh(1/2*d*x+1/2*c)^4*a
+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a/b^2*tanh(1/2*d*
x+1/2*c)^7
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 3.3531, size = 18573, normalized size = 119.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(4*a^3*b + 5*a^2*b^2 - 2*a*b^3 - 3*b^4)*cosh(d*x + c)^7 + 28*(4*a
^3*b + 5*a^2*b^2 - 2*a*b^3 - 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^6 + 4*(4*a^
3*b + 5*a^2*b^2 - 2*a*b^3 - 3*b^4)*sinh(d*x + c)^7 + 4*(4*a^3*b - 19*a^2*b^
2 - 14*a*b^3 + 9*b^4)*cosh(d*x + c)^5 + 4*(4*a^3*b - 19*a^2*b^2 - 14*a*b^3
+ 9*b^4 + 21*(4*a^3*b + 5*a^2*b^2 - 2*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(
d*x + c)^5 + 20*(7*(4*a^3*b + 5*a^2*b^2 - 2*a*b^3 - 3*b^4)*cosh(d*x + c)^3
+ (4*a^3*b - 19*a^2*b^2 - 14*a*b^3 + 9*b^4)*cosh(d*x + c))*sinh(d*x + c)^4
- 4*(4*a^3*b - 19*a^2*b^2 - 14*a*b^3 + 9*b^4)*cosh(d*x + c)^3 + 4*(35*(4*a^
3*b + 5*a^2*b^2 - 2*a*b^3 - 3*b^4)*cosh(d*x + c)^4 - 4*a^3*b + 19*a^2*b^2 +
 14*a*b^3 - 9*b^4 + 10*(4*a^3*b - 19*a^2*b^2 - 14*a*b^3 + 9*b^4)*cosh(d*x +
 c)^2)*sinh(d*x + c)^3 + 4*(21*(4*a^3*b + 5*a^2*b^2 - 2*a*b^3 - 3*b^4)*cosh
(d*x + c)^5 + 10*(4*a^3*b - 19*a^2*b^2 - 14*a*b^3 + 9*b^4)*cosh(d*x + c)^3
- 3*(4*a^3*b - 19*a^2*b^2 - 14*a*b^3 + 9*b^4)*cosh(d*x + c))*sinh(d*x + c)^
2 - ((8*a^4 + 12*a^3*b + 3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^8 + 8*(
8*a^4 + 12*a^3*b + 3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)*sinh(d*x + c)
^7 + (8*a^4 + 12*a^3*b + 3*a^2*b^2 + 2*a*b^3 + 3*b^4)*sinh(d*x + c)^8 + 4*(
8*a^4 - 4*a^3*b - 5*a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)^6 + 4*(8*a^4 -
 4*a^3*b - 5*a^2*b^2 + 4*a*b^3 - 3*b^4 + 7*(8*a^4 + 12*a^3*b + 3*a^2*b^2 +
2*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(8*a^4 + 12*a^3*b
+ 3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + 3*(8*a^4 - 4*a^3*b - 5*a^2
*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(24*a^4 - 28*a^3
*b + 41*a^2*b^2 - 18*a*b^3 + 9*b^4)*cosh(d*x + c)^4 + 2*(35*(8*a^4 + 12*a^3
*b + 3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 24*a^4 - 28*a^3*b + 41*
a^2*b^2 - 18*a*b^3 + 9*b^4 + 30*(8*a^4 - 4*a^3*b - 5*a^2*b^2 + 4*a*b^3 - 3*
b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*a^4 + 12*a^3*b + 3*a^2*b^2 + 2*a*
b^3 + 3*b^4 + 8*(7*(8*a^4 + 12*a^3*b + 3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*
x + c)^5 + 10*(8*a^4 - 4*a^3*b - 5*a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)
^3 + (24*a^4 - 28*a^3*b + 41*a^2*b^2 - 18*a*b^3 + 9*b^4)*cosh(d*x + c))*sin
h(d*x + c)^3 + 4*(8*a^4 - 4*a^3*b - 5*a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x +
 c)^2 + 4*(7*(8*a^4 + 12*a^3*b + 3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)
^6 + 15*(8*a^4 - 4*a^3*b - 5*a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)^4 + 8
```

```
*a^4 - 4*a^3*b - 5*a^2*b^2 + 4*a*b^3 - 3*b^4 + 3*(24*a^4 - 28*a^3*b + 41*a^
2*b^2 - 18*a*b^3 + 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((8*a^4 + 12
*a^3*b + 3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^7 + 3*(8*a^4 - 4*a^3*b
- 5*a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)^5 + (24*a^4 - 28*a^3*b + 41*a^
2*b^2 - 18*a*b^3 + 9*b^4)*cosh(d*x + c)^3 + (8*a^4 - 4*a^3*b - 5*a^2*b^2 +
4*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-(a + b)/a)*log(((a + b
)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(
d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 - 3
*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 - (3*a + b)*cosh(d*x +
 c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)
^2 + a*sinh(d*x + c)^3 - a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 - a)*sinh(d
*x + c))*sqrt(-(a + b)/a) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cos
h(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x +
 c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*
cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 32*((a^4
 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^8 + 8*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*
x + c)*sinh(d*x + c)^7 + (a^4 + 2*a^3*b + a^2*b^2)*sinh(d*x + c)^8 + 4*(a^4
 - a^2*b^2)*cosh(d*x + c)^6 + 4*(a^4 - a^2*b^2 + 7*(a^4 + 2*a^3*b + a^2*b^2
)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*
x + c)^3 + 3*(a^4 - a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^4 - 2*
a^3*b + 3*a^2*b^2)*cosh(d*x + c)^4 + 2*(35*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d
*x + c)^4 + 3*a^4 - 2*a^3*b + 3*a^2*b^2 + 30*(a^4 - a^2*b^2)*cosh(d*x + c)^
2)*sinh(d*x + c)^4 + a^4 + 2*a^3*b + a^2*b^2 + 8*(7*(a^4 + 2*a^3*b + a^2*b^
2)*cosh(d*x + c)^5 + 10*(a^4 - a^2*b^2)*cosh(d*x + c)^3 + (3*a^4 - 2*a^3*b
+ 3*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^4 - a^2*b^2)*cosh(d*x +
c)^2 + 4*(7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^6 + 15*(a^4 - a^2*b^2)*
cosh(d*x + c)^4 + a^4 - a^2*b^2 + 3*(3*a^4 - 2*a^3*b + 3*a^2*b^2)*cosh(d*x
+ c)^2)*sinh(d*x + c)^2 + 8*((a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^7 + 3*
(a^4 - a^2*b^2)*cosh(d*x + c)^5 + (3*a^4 - 2*a^3*b + 3*a^2*b^2)*cosh(d*x +
c)^3 + (a^4 - a^2*b^2)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) +
 sinh(d*x + c)) - 4*(4*a^3*b + 5*a^2*b^2 - 2*a*b^3 - 3*b^4)*cosh(d*x + c) +
 4*(7*(4*a^3*b + 5*a^2*b^2 - 2*a*b^3 - 3*b^4)*cosh(d*x + c)^6 + 5*(4*a^3*b
- 19*a^2*b^2 - 14*a*b^3 + 9*b^4)*cosh(d*x + c)^4 - 4*a^3*b - 5*a^2*b^2 + 2*
a*b^3 + 3*b^4 - 3*(4*a^3*b - 19*a^2*b^2 - 14*a*b^3 + 9*b^4)*cosh(d*x + c)^2
)*sinh(d*x + c))/((a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^8 + 8*(a^
4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^4*b^3 + 2
*a^3*b^4 + a^2*b^5)*d*sinh(d*x + c)^8 + 4*(a^4*b^3 - a^2*b^5)*d*cosh(d*x +
c)^6 + 4*(7*(a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^2 + (a^4*b^3 -
a^2*b^5)*d)*sinh(d*x + c)^6 + 2*(3*a^4*b^3 - 2*a^3*b^4 + 3*a^2*b^5)*d*cosh(
d*x + c)^4 + 8*(7*(a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^3 + 3*(a^
4*b^3 - a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^4*b^3 + 2*a^3*
b^4 + a^2*b^5)*d*cosh(d*x + c)^4 + 30*(a^4*b^3 - a^2*b^5)*d*cosh(d*x + c)^2
 + (3*a^4*b^3 - 2*a^3*b^4 + 3*a^2*b^5)*d)*sinh(d*x + c)^4 + 4*(a^4*b^3 - a^
2*b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*cosh(d*x
+ c)^5 + 10*(a^4*b^3 - a^2*b^5)*d*cosh(d*x + c)^3 + (3*a^4*b^3 - 2*a^3*b^4
+ 3*a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^4*b^3 + 2*a^3*b^4 +
 a^2*b^5)*d*cosh(d*x + c)^6 + 15*(a^4*b^3 - a^2*b^5)*d*cosh(d*x + c)^4 + 3*
(3*a^4*b^3 - 2*a^3*b^4 + 3*a^2*b^5)*d*cosh(d*x + c)^2 + (a^4*b^3 - a^2*b^5)
*d)*sinh(d*x + c)^2 + (a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d + 8*((a^4*b^3 + 2*a
^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^7 + 3*(a^4*b^3 - a^2*b^5)*d*cosh(d*x + c)
^5 + (3*a^4*b^3 - 2*a^3*b^4 + 3*a^2*b^5)*d*cosh(d*x + c)^3 + (a^4*b^3 - a^2
*b^5)*d*cosh(d*x + c))*sinh(d*x + c)), -1/8*(2*(4*a^3*b + 5*a^2*b^2 - 2*a*b
^3 - 3*b^4)*cosh(d*x + c)^7 + 14*(4*a^3*b + 5*a^2*b^2 - 2*a*b^3 - 3*b^4)*co
sh(d*x + c)*sinh(d*x + c)^6 + 2*(4*a^3*b + 5*a^2*b^2 - 2*a*b^3 - 3*b^4)*sin
h(d*x + c)^7 + 2*(4*a^3*b - 19*a^2*b^2 - 14*a*b^3 + 9*b^4)*cosh(d*x + c)^5
+ 2*(4*a^3*b - 19*a^2*b^2 - 14*a*b^3 + 9*b^4 + 21*(4*a^3*b + 5*a^2*b^2 - 2*
a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 10*(7*(4*a^3*b + 5*a^2*b^
2 - 2*a*b^3 - 3*b^4)*cosh(d*x + c)^3 + (4*a^3*b - 19*a^2*b^2 - 14*a*b^3 + 9
*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 - 2*(4*a^3*b - 19*a^2*b^2 - 14*a*b^3 +
```

```
9*b^4)*cosh(d*x + c)^3 + 2*(35*(4*a^3*b + 5*a^2*b^2 - 2*a*b^3 - 3*b^4)*cos
h(d*x + c)^4 - 4*a^3*b + 19*a^2*b^2 + 14*a*b^3 - 9*b^4 + 10*(4*a^3*b - 19*a
^2*b^2 - 14*a*b^3 + 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(21*(4*a^3*
b + 5*a^2*b^2 - 2*a*b^3 - 3*b^4)*cosh(d*x + c)^5 + 10*(4*a^3*b - 19*a^2*b^2
 - 14*a*b^3 + 9*b^4)*cosh(d*x + c)^3 - 3*(4*a^3*b - 19*a^2*b^2 - 14*a*b^3 +
 9*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 - ((8*a^4 + 12*a^3*b + 3*a^2*b^2 + 2
*a*b^3 + 3*b^4)*cosh(d*x + c)^8 + 8*(8*a^4 + 12*a^3*b + 3*a^2*b^2 + 2*a*b^3
 + 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (8*a^4 + 12*a^3*b + 3*a^2*b^2 + 2
*a*b^3 + 3*b^4)*sinh(d*x + c)^8 + 4*(8*a^4 - 4*a^3*b - 5*a^2*b^2 + 4*a*b^3
 - 3*b^4)*cosh(d*x + c)^6 + 4*(8*a^4 - 4*a^3*b - 5*a^2*b^2 + 4*a*b^3 - 3*b^4
 + 7*(8*a^4 + 12*a^3*b + 3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh
(d*x + c)^6 + 8*(7*(8*a^4 + 12*a^3*b + 3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*
x + c)^3 + 3*(8*a^4 - 4*a^3*b - 5*a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c))
*sinh(d*x + c)^5 + 2*(24*a^4 - 28*a^3*b + 41*a^2*b^2 - 18*a*b^3 + 9*b^4)*co
sh(d*x + c)^4 + 2*(35*(8*a^4 + 12*a^3*b + 3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh
(d*x + c)^4 + 24*a^4 - 28*a^3*b + 41*a^2*b^2 - 18*a*b^3 + 9*b^4 + 30*(8*a^4
 - 4*a^3*b - 5*a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4
 + 8*a^4 + 12*a^3*b + 3*a^2*b^2 + 2*a*b^3 + 3*b^4 + 8*(7*(8*a^4 + 12*a^3*b +
 3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^5 + 10*(8*a^4 - 4*a^3*b - 5*a^2
*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)^3 + (24*a^4 - 28*a^3*b + 41*a^2*b^2 -
 18*a*b^3 + 9*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(8*a^4 - 4*a^3*b - 5*
a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)^2 + 4*(7*(8*a^4 + 12*a^3*b + 3*a^2
*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^6 + 15*(8*a^4 - 4*a^3*b - 5*a^2*b^2 +
 4*a*b^3 - 3*b^4)*cosh(d*x + c)^4 + 8*a^4 - 4*a^3*b - 5*a^2*b^2 + 4*a*b^3 -
 3*b^4 + 3*(24*a^4 - 28*a^3*b + 41*a^2*b^2 - 18*a*b^3 + 9*b^4)*cosh(d*x + c
)^2)*sinh(d*x + c)^2 + 8*((8*a^4 + 12*a^3*b + 3*a^2*b^2 + 2*a*b^3 + 3*b^4)*
cosh(d*x + c)^7 + 3*(8*a^4 - 4*a^3*b - 5*a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*
x + c)^5 + (24*a^4 - 28*a^3*b + 41*a^2*b^2 - 18*a*b^3 + 9*b^4)*cosh(d*x + c
)^3 + (8*a^4 - 4*a^3*b - 5*a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d
*x + c))*sqrt((a + b)/a)*arctan(1/2*sqrt((a + b)/a)*(cosh(d*x + c) + sinh(d
*x + c))) - ((8*a^4 + 12*a^3*b + 3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)
^8 + 8*(8*a^4 + 12*a^3*b + 3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)*sinh(
d*x + c)^7 + (8*a^4 + 12*a^3*b + 3*a^2*b^2 + 2*a*b^3 + 3*b^4)*sinh(d*x + c)
^8 + 4*(8*a^4 - 4*a^3*b - 5*a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)^6 + 4*
(8*a^4 - 4*a^3*b - 5*a^2*b^2 + 4*a*b^3 - 3*b^4 + 7*(8*a^4 + 12*a^3*b + 3*a^
2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(8*a^4 + 1
2*a^3*b + 3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + 3*(8*a^4 - 4*a^3*b
 - 5*a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(24*a^4
- 28*a^3*b + 41*a^2*b^2 - 18*a*b^3 + 9*b^4)*cosh(d*x + c)^4 + 2*(35*(8*a^4
+ 12*a^3*b + 3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 24*a^4 - 28*a^3
*b + 41*a^2*b^2 - 18*a*b^3 + 9*b^4 + 30*(8*a^4 - 4*a^3*b - 5*a^2*b^2 + 4*a*
b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*a^4 + 12*a^3*b + 3*a^2*b^
2 + 2*a*b^3 + 3*b^4 + 8*(7*(8*a^4 + 12*a^3*b + 3*a^2*b^2 + 2*a*b^3 + 3*b^4)
*cosh(d*x + c)^5 + 10*(8*a^4 - 4*a^3*b - 5*a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(
d*x + c)^3 + (24*a^4 - 28*a^3*b + 41*a^2*b^2 - 18*a*b^3 + 9*b^4)*cosh(d*x +
 c))*sinh(d*x + c)^3 + 4*(8*a^4 - 4*a^3*b - 5*a^2*b^2 + 4*a*b^3 - 3*b^4)*co
sh(d*x + c)^2 + 4*(7*(8*a^4 + 12*a^3*b + 3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(
d*x + c)^6 + 15*(8*a^4 - 4*a^3*b - 5*a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x +
c)^4 + 8*a^4 - 4*a^3*b - 5*a^2*b^2 + 4*a*b^3 - 3*b^4 + 3*(24*a^4 - 28*a^3*b
 + 41*a^2*b^2 - 18*a*b^3 + 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((8*
a^4 + 12*a^3*b + 3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^7 + 3*(8*a^4 -
4*a^3*b - 5*a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)^5 + (24*a^4 - 28*a^3*b
 + 41*a^2*b^2 - 18*a*b^3 + 9*b^4)*cosh(d*x + c)^3 + (8*a^4 - 4*a^3*b - 5*a^
2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a + b)/a)*arct
an(1/2*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 +
 (a + b)*sinh(d*x + c)^3 + (3*a - b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x +
c)^2 + 3*a - b)*sinh(d*x + c))*sqrt((a + b)/a)/(a + b)) + 16*((a^4 + 2*a^3*
b + a^2*b^2)*cosh(d*x + c)^8 + 8*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)*si
nh(d*x + c)^7 + (a^4 + 2*a^3*b + a^2*b^2)*sinh(d*x + c)^8 + 4*(a^4 - a^2*b^
```

```
2)*cosh(d*x + c)^6 + 4*(a^4 - a^2*b^2 + 7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*
x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^3
+ 3*(a^4 - a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^4 - 2*a^3*b + 3
*a^2*b^2)*cosh(d*x + c)^4 + 2*(35*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^4
 + 3*a^4 - 2*a^3*b + 3*a^2*b^2 + 30*(a^4 - a^2*b^2)*cosh(d*x + c)^2)*sinh(d
*x + c)^4 + a^4 + 2*a^3*b + a^2*b^2 + 8*(7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d
*x + c)^5 + 10*(a^4 - a^2*b^2)*cosh(d*x + c)^3 + (3*a^4 - 2*a^3*b + 3*a^2*b
^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^4 - a^2*b^2)*cosh(d*x + c)^2 + 4*
(7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^6 + 15*(a^4 - a^2*b^2)*cosh(d*x
+ c)^4 + a^4 - a^2*b^2 + 3*(3*a^4 - 2*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^2)*s
inh(d*x + c)^2 + 8*((a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^7 + 3*(a^4 - a^
2*b^2)*cosh(d*x + c)^5 + (3*a^4 - 2*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^3 + (a
^4 - a^2*b^2)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x
 + c)) - 2*(4*a^3*b + 5*a^2*b^2 - 2*a*b^3 - 3*b^4)*cosh(d*x + c) + 2*(7*(4*
a^3*b + 5*a^2*b^2 - 2*a*b^3 - 3*b^4)*cosh(d*x + c)^6 + 5*(4*a^3*b - 19*a^2*
b^2 - 14*a*b^3 + 9*b^4)*cosh(d*x + c)^4 - 4*a^3*b - 5*a^2*b^2 + 2*a*b^3 + 3
*b^4 - 3*(4*a^3*b - 19*a^2*b^2 - 14*a*b^3 + 9*b^4)*cosh(d*x + c)^2)*sinh(d*
x + c))/((a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^8 + 8*(a^4*b^3 + 2
*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^4*b^3 + 2*a^3*b^4
+ a^2*b^5)*d*sinh(d*x + c)^8 + 4*(a^4*b^3 - a^2*b^5)*d*cosh(d*x + c)^6 + 4*
(7*(a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^2 + (a^4*b^3 - a^2*b^5)*
d)*sinh(d*x + c)^6 + 2*(3*a^4*b^3 - 2*a^3*b^4 + 3*a^2*b^5)*d*cosh(d*x + c)^
4 + 8*(7*(a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^3 + 3*(a^4*b^3 - a
^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^4*b^3 + 2*a^3*b^4 + a^2
*b^5)*d*cosh(d*x + c)^4 + 30*(a^4*b^3 - a^2*b^5)*d*cosh(d*x + c)^2 + (3*a^4
*b^3 - 2*a^3*b^4 + 3*a^2*b^5)*d)*sinh(d*x + c)^4 + 4*(a^4*b^3 - a^2*b^5)*d*
cosh(d*x + c)^2 + 8*(7*(a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^5 +
10*(a^4*b^3 - a^2*b^5)*d*cosh(d*x + c)^3 + (3*a^4*b^3 - 2*a^3*b^4 + 3*a^2*b
^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^4*b^3 + 2*a^3*b^4 + a^2*b^5)
*d*cosh(d*x + c)^6 + 15*(a^4*b^3 - a^2*b^5)*d*cosh(d*x + c)^4 + 3*(3*a^4*b^
3 - 2*a^3*b^4 + 3*a^2*b^5)*d*cosh(d*x + c)^2 + (a^4*b^3 - a^2*b^5)*d)*sinh(
d*x + c)^2 + (a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d + 8*((a^4*b^3 + 2*a^3*b^4 +
a^2*b^5)*d*cosh(d*x + c)^7 + 3*(a^4*b^3 - a^2*b^5)*d*cosh(d*x + c)^5 + (3*a
^4*b^3 - 2*a^3*b^4 + 3*a^2*b^5)*d*cosh(d*x + c)^3 + (a^4*b^3 - a^2*b^5)*d*c
osh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**7/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 2.46578, size = 6677, normalized size = 42.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/32*(2*(3*(8*a^4*b^3 + 12*a^3*b^4 + 3*a^2*b^5 + 2*a*b^6 + 3*b^7)*cos(1/2*r
eal_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a
 + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b)))) -
 (8*a^4*b^3 + 12*a^3*b^4 + 3*a^2*b^5 + 2*a*b^6 + 3*b^7)*cosh(1/2*imag_part(
arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/
(a + b))))^3 - 9*(8*a^4*b^3 + 12*a^3*b^4 + 3*a^2*b^5 + 2*a*b^6 + 3*b^7)*cos
(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos
(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b
))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(8*a^4*b^3 + 1
2*a^3*b^4 + 3*a^2*b^5 + 2*a*b^6 + 3*b^7)*cosh(1/2*imag_part(arccos(-a/(a +
b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*si
nh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(8*a^4*b^3 + 12*a^3*b
^4 + 3*a^2*b^5 + 2*a*b^6 + 3*b^7)*cos(1/2*real_part(arccos(-a/(a + b) + b/(
a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*rea
l_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b
) + b/(a + b))))^2 - 3*(8*a^4*b^3 + 12*a^3*b^4 + 3*a^2*b^5 + 2*a*b^6 + 3*b^
7)*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(ar
ccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(
a + b))))^2 - 3*(8*a^4*b^3 + 12*a^3*b^4 + 3*a^2*b^5 + 2*a*b^6 + 3*b^7)*cos(
1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-
a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))
)^3 + (8*a^4*b^3 + 12*a^3*b^4 + 3*a^2*b^5 + 2*a*b^6 + 3*b^7)*sin(1/2*real_p
art(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a + b)
 + b/(a + b))))^3 + (8*a^4*b^3 + 12*a^3*b^4 + 3*a^2*b^5 + 2*a*b^6 + 3*b^7)*
cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arcco
s(-a/(a + b) + b/(a + b)))) - (8*a^4*b^3 + 12*a^3*b^4 + 3*a^2*b^5 + 2*a*b^6
 + 3*b^7)*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_
part(arccos(-a/(a + b) + b/(a + b)))))*arctan((((a^3*b^3 + a^2*b^4)/(a^3*b^
3*e^(4*c) + a^2*b^4*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b))) + e^(
d*x))/(((a^3*b^3 + a^2*b^4)/(a^3*b^3*e^(4*c) + a^2*b^4*e^(4*c)))^(1/4)*sin(
1/2*arccos(-(a - b)/(a + b)))))/(2*a^3*b^7 + (a^3*b^3 - a^2*b^4)*sqrt(-a*b)
*b^2*abs(b)) + 2*(3*(8*a^4*b^3 + 12*a^3*b^4 + 3*a^2*b^5 + 2*a*b^6 + 3*b^7)*
cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_part(arc
cos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a
+ b)))) - (8*a^4*b^3 + 12*a^3*b^4 + 3*a^2*b^5 + 2*a*b^6 + 3*b^7)*cosh(1/2*i
mag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a
+ b) + b/(a + b))))^3 - 9*(8*a^4*b^3 + 12*a^3*b^4 + 3*a^2*b^5 + 2*a*b^6 + 3
*b^7)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*cosh(1/2*imag_pa
rt(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) +
 b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(8*a^
4*b^3 + 12*a^3*b^4 + 3*a^2*b^5 + 2*a*b^6 + 3*b^7)*cosh(1/2*imag_part(arccos
(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b
))))^3*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 9*(8*a^4*b^3 +
 12*a^3*b^4 + 3*a^2*b^5 + 2*a*b^6 + 3*b^7)*cos(1/2*real_part(arccos(-a/(a +
 b) + b/(a + b))))^2*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*si
n(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(
-a/(a + b) + b/(a + b))))^2 - 3*(8*a^4*b^3 + 12*a^3*b^4 + 3*a^2*b^5 + 2*a*b
^6 + 3*b^7)*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*rea
l_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a +
 b) + b/(a + b))))^2 - 3*(8*a^4*b^3 + 12*a^3*b^4 + 3*a^2*b^5 + 2*a*b^6 + 3*
b^7)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part
(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/
(a + b))))^3 + (8*a^4*b^3 + 12*a^3*b^4 + 3*a^2*b^5 + 2*a*b^6 + 3*b^7)*sin(1
/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-
a/(a + b) + b/(a + b))))^3 + (8*a^4*b^3 + 12*a^3*b^4 + 3*a^2*b^5 + 2*a*b^6
+ 3*b^7)*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_p
art(arccos(-a/(a + b) + b/(a + b)))) - (8*a^4*b^3 + 12*a^3*b^4 + 3*a^2*b^5
+ 2*a*b^6 + 3*b^7)*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(
1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*arctan(-(((a^3*b^3 + a^2*b^
```

```
4)/(a^3*b^3*e^(4*c) + a^2*b^4*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a +
b))) - e^(d*x))/(((a^3*b^3 + a^2*b^4)/(a^3*b^3*e^(4*c) + a^2*b^4*e^(4*c)))^
(1/4)*sin(1/2*arccos(-(a - b)/(a + b)))))/(2*a^3*b^7 + (a^3*b^3 - a^2*b^4)*
sqrt(-a*b)*b^2*abs(b)) + ((8*a^4*b^3 + 12*a^3*b^4 + 3*a^2*b^5 + 2*a*b^6 + 3
*b^7)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_pa
rt(arccos(-a/(a + b) + b/(a + b))))^3 - 3*(8*a^4*b^3 + 12*a^3*b^4 + 3*a^2*b
^5 + 2*a*b^6 + 3*b^7)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*co
sh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arcco
s(-a/(a + b) + b/(a + b))))^2 - 3*(8*a^4*b^3 + 12*a^3*b^4 + 3*a^2*b^5 + 2*a
*b^6 + 3*b^7)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2
*imag_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/
(a + b) + b/(a + b)))) + 9*(8*a^4*b^3 + 12*a^3*b^4 + 3*a^2*b^5 + 2*a*b^6 +
3*b^7)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_par
t(arccos(-a/(a + b) + b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) +
b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(8*a
^4*b^3 + 12*a^3*b^4 + 3*a^2*b^5 + 2*a*b^6 + 3*b^7)*cos(1/2*real_part(arccos
(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a +
b))))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))^2 - 9*(8*a^4*b^3
+ 12*a^3*b^4 + 3*a^2*b^5 + 2*a*b^6 + 3*b^7)*cos(1/2*real_part(arccos(-a/(a
+ b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin
(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos
(-a/(a + b) + b/(a + b))))^2 - (8*a^4*b^3 + 12*a^3*b^4 + 3*a^2*b^5 + 2*a*b^
6 + 3*b^7)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*im
ag_part(arccos(-a/(a + b) + b/(a + b))))^3 + 3*(8*a^4*b^3 + 12*a^3*b^4 + 3*
a^2*b^5 + 2*a*b^6 + 3*b^7)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))
))*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(
arccos(-a/(a + b) + b/(a + b))))^3 + (8*a^4*b^3 + 12*a^3*b^4 + 3*a^2*b^5 +
2*a*b^6 + 3*b^7)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/
2*imag_part(arccos(-a/(a + b) + b/(a + b)))) - (8*a^4*b^3 + 12*a^3*b^4 + 3*
a^2*b^5 + 2*a*b^6 + 3*b^7)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))
))*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b)))))*log(2*((a^3*b^3 + a
^2*b^4)/(a^3*b^3*e^(4*c) + a^2*b^4*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/
(a + b)))*e^(d*x) + sqrt((a^3*b^3 + a^2*b^4)/(a^3*b^3*e^(4*c) + a^2*b^4*e^(
4*c))) + e^(2*d*x))/(2*a^3*b^7 + (a^3*b^3 - a^2*b^4)*sqrt(-a*b)*b^2*abs(b))
 - ((8*a^4*b^3 + 12*a^3*b^4 + 3*a^2*b^5 + 2*a*b^6 + 3*b^7)*cos(1/2*real_par
t(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) +
 b/(a + b))))^3 - 3*(8*a^4*b^3 + 12*a^3*b^4 + 3*a^2*b^5 + 2*a*b^6 + 3*b^7)*
cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arcco
s(-a/(a + b) + b/(a + b))))^3*sin(1/2*real_part(arccos(-a/(a + b) + b/(a +
b))))^2 - 3*(8*a^4*b^3 + 12*a^3*b^4 + 3*a^2*b^5 + 2*a*b^6 + 3*b^7)*cos(1/2*
real_part(arccos(-a/(a + b) + b/(a + b))))^3*cosh(1/2*imag_part(arccos(-a/(
a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))
 + 9*(8*a^4*b^3 + 12*a^3*b^4 + 3*a^2*b^5 + 2*a*b^6 + 3*b^7)*cos(1/2*real_pa
rt(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/(a + b) +
b/(a + b))))^2*sin(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))^2*sinh(1/
2*imag_part(arccos(-a/(a + b) + b/(a + b)))) + 3*(8*a^4*b^3 + 12*a^3*b^4 +
3*a^2*b^5 + 2*a*b^6 + 3*b^7)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b
))))^3*cosh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_pa
rt(arccos(-a/(a + b) + b/(a + b))))^2 - 9*(8*a^4*b^3 + 12*a^3*b^4 + 3*a^2*b
^5 + 2*a*b^6 + 3*b^7)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*co
sh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(arccos(
-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/(a + b
))))^2 - (8*a^4*b^3 + 12*a^3*b^4 + 3*a^2*b^5 + 2*a*b^6 + 3*b^7)*cos(1/2*rea
l_part(arccos(-a/(a + b) + b/(a + b))))^3*sinh(1/2*imag_part(arccos(-a/(a +
 b) + b/(a + b))))^3 + 3*(8*a^4*b^3 + 12*a^3*b^4 + 3*a^2*b^5 + 2*a*b^6 + 3*
b^7)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sin(1/2*real_part(a
rccos(-a/(a + b) + b/(a + b))))^2*sinh(1/2*imag_part(arccos(-a/(a + b) + b/
(a + b))))^3 + (8*a^4*b^3 + 12*a^3*b^4 + 3*a^2*b^5 + 2*a*b^6 + 3*b^7)*cos(1
/2*real_part(arccos(-a/(a + b) + b/(a + b))))*cosh(1/2*imag_part(arccos(-a/
```

```
(a + b) + b/(a + b)))) - (8*a^4*b^3 + 12*a^3*b^4 + 3*a^2*b^5 + 2*a*b^6 + 3*
b^7)*cos(1/2*real_part(arccos(-a/(a + b) + b/(a + b))))*sinh(1/2*imag_part(
arccos(-a/(a + b) + b/(a + b)))))*log(-2*((a^3*b^3 + a^2*b^4)/(a^3*b^3*e^(4
*c) + a^2*b^4*e^(4*c)))^(1/4)*cos(1/2*arccos(-(a - b)/(a + b)))*e^(d*x) + s
qrt((a^3*b^3 + a^2*b^4)/(a^3*b^3*e^(4*c) + a^2*b^4*e^(4*c))) + e^(2*d*x))/(
2*a^3*b^7 + (a^3*b^3 - a^2*b^4)*sqrt(-a*b)*b^2*abs(b)) - 64*arctan(e^(d*x +
 c))/b^3 - 8*(4*a^3*e^(7*d*x + 7*c) + 5*a^2*b*e^(7*d*x + 7*c) - 2*a*b^2*e^(
7*d*x + 7*c) - 3*b^3*e^(7*d*x + 7*c) + 4*a^3*e^(5*d*x + 5*c) - 19*a^2*b*e^(
5*d*x + 5*c) - 14*a*b^2*e^(5*d*x + 5*c) + 9*b^3*e^(5*d*x + 5*c) - 4*a^3*e^(
3*d*x + 3*c) + 19*a^2*b*e^(3*d*x + 3*c) + 14*a*b^2*e^(3*d*x + 3*c) - 9*b^3*
e^(3*d*x + 3*c) - 4*a^3*e^(d*x + c) - 5*a^2*b*e^(d*x + c) + 2*a*b^2*e^(d*x
+ c) + 3*b^3*e^(d*x + c))/((a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(
2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2*a^2*b^2))/d
```

3.134 $\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=54

$$-\frac{(a+b)\tanh^3(c+dx)}{3d}-\frac{(a+b)\tanh(c+dx)}{d}+x(a+b)-\frac{b\tanh^5(c+dx)}{5d}$$

```
[Out] (a + b)*x - ((a + b)*Tanh[c + d*x])/d - ((a + b)*Tanh[c + d*x]^3)/(3*d) - (
b*Tanh[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0519301, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3631, 3473, 8}

$$-\frac{(a+b)\tanh^3(c+dx)}{3d}-\frac{(a+b)\tanh(c+dx)}{d}+x(a+b)-\frac{b\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a + b)*x - ((a + b)*Tanh[c + d*x])/d - ((a + b)*Tanh[c + d*x]^3)/(3*d) - (
b*Tanh[c + d*x]^5)/(5*d)
```

Rule 3631

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(
m + 1)), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)dx &= -\frac{b\tanh^5(c+dx)}{5d}+(a+b)\int \tanh^4(c+dx)\,dx\\ &= -\frac{(a+b)\tanh^3(c+dx)}{3d}-\frac{b\tanh^5(c+dx)}{5d}+(a+b)\int \tanh^2(c+dx)\,dx\\ &= -\frac{(a+b)\tanh(c+dx)}{d}-\frac{(a+b)\tanh^3(c+dx)}{3d}-\frac{b\tanh^5(c+dx)}{5d}+(a+b)\\ &= (a+b)x-\frac{(a+b)\tanh(c+dx)}{d}-\frac{(a+b)\tanh^3(c+dx)}{3d}-\frac{b\tanh^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.0340293, size = 97, normalized size = 1.8

$$-\frac{a\tanh^3(c+dx)}{3d}+\frac{a\tanh^{-1}(\tanh(c+dx))}{d}-\frac{a\tanh(c+dx)}{d}-\frac{b\tanh^5(c+dx)}{5d}-\frac{b\tanh^3(c+dx)}{3d}+\frac{b\tanh^{-1}(\tanh(c}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a*ArcTanh[Tanh[c + d*x]])/d + (b*ArcTanh[Tanh[c + d*x]])/d - (a*Tanh[c + d
*x])/d - (b*Tanh[c + d*x])/d - (a*Tanh[c + d*x]^3)/(3*d) - (b*Tanh[c + d*x]
^3)/(3*d) - (b*Tanh[c + d*x]^5)/(5*d)
```

Maple [B] time = 0.006, size = 128, normalized size = 2.4

$$-\frac{b(\tanh(dx+c))^5}{5d}-\frac{a(\tanh(dx+c))^3}{3d}-\frac{b(\tanh(dx+c))^3}{3d}-\frac{a\tanh(dx+c)}{d}-\frac{b\tanh(dx+c)}{d}-\frac{\ln(\tanh(dx+c}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x)
```

```
[Out] -1/5*b*tanh(d*x+c)^5/d-1/3/d*a*tanh(d*x+c)^3-1/3*b*tanh(d*x+c)^3/d-a*tanh(d
*x+c)/d-b*tanh(d*x+c)/d-1/2/d*ln(tanh(d*x+c)-1)*a-1/2/d*ln(tanh(d*x+c)-1)*b
+1/2/d*ln(tanh(d*x+c)+1)*a+1/2/d*ln(tanh(d*x+c)+1)*b
```

Maxima [B] time = 1.03624, size = 269, normalized size = 4.98

$$\frac{1}{15}b\left(15x+\frac{15c}{d}-\frac{2\left(70e^{(-2dx-2c)}+140e^{(-4dx-4c)}+90e^{(-6dx-6c)}+45e^{(-8dx-8c)}+23\right)}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1\right)}\right)+\frac{1}{3}a\left(3x+\frac{3c}{d}-\frac{}{d(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/15*b*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 90*
e^(-6*d*x - 6*c) + 45*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*e^
(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10
*c) + 1))) + 1/3*a*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c
) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))
)
```

Fricas [B] time = 1.95337, size = 938, normalized size = 17.37

$$\frac{(15(a+b)dx+20a+23b)\cosh(dx+c)^5+5(15(a+b)dx+20a+23b)\cosh(dx+c)\sinh(dx+c)^4-(20a+23b)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/15*((15*(a + b)*d*x + 20*a + 23*b)*cosh(d*x + c)^5 + 5*(15*(a + b)*d*x +
20*a + 23*b)*cosh(d*x + c)*sinh(d*x + c)^4 - (20*a + 23*b)*sinh(d*x + c)^5
+ 5*(15*(a + b)*d*x + 20*a + 23*b)*cosh(d*x + c)^3 - 5*(2*(20*a + 23*b)*cos
h(d*x + c)^2 + 8*a + 5*b)*sinh(d*x + c)^3 + 5*(2*(15*(a + b)*d*x + 20*a + 2
```

```
3*b)*cosh(d*x + c)^3 + 3*(15*(a + b)*d*x + 20*a + 23*b)*cosh(d*x + c))*sinh
(d*x + c)^2 + 10*(15*(a + b)*d*x + 20*a + 23*b)*cosh(d*x + c) - 5*((20*a +
23*b)*cosh(d*x + c)^4 + 3*(8*a + 5*b)*cosh(d*x + c)^2 + 4*a + 10*b)*sinh(d*
x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cosh(d
*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + 1
0*d*cosh(d*x + c))
```

Sympy [A] time = 0.932585, size = 82, normalized size = 1.52

$$\begin{cases} ax - \frac{a\tanh^3(c+dx)}{3d} - \frac{a\tanh(c+dx)}{d} + bx - \frac{b\tanh^5(c+dx)}{5d} - \frac{b\tanh^3(c+dx)}{3d} - \frac{b\tanh(c+dx)}{d} & \text{for } d \neq 0 \\ x\left(a + b\tanh^2(c)\right)\tanh^4(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**4*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Piecewise((a*x - a*tanh(c + d*x)**3/(3*d) - a*tanh(c + d*x)/d + b*x - b*tan
h(c + d*x)**5/(5*d) - b*tanh(c + d*x)**3/(3*d) - b*tanh(c + d*x)/d, Ne(d, 0
)), (x*(a + b*tanh(c)**2)*tanh(c)**4, True))
```

Giac [B] time = 1.19379, size = 181, normalized size = 3.35

$$\frac{(dx+c)(a+b)}{d} + \frac{2\left(30\,ae^{(8\,dx+8\,c)} + 45\,be^{(8\,dx+8\,c)} + 90\,ae^{(6\,dx+6\,c)} + 90\,be^{(6\,dx+6\,c)} + 110\,ae^{(4\,dx+4\,c)} + 140\,be^{(4\,dx+4\,c)} + 70\,ae^{(\ldots}}{15\,d\left(e^{(2\,dx+2\,c)} + 1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] (d*x + c)*(a + b)/d + 2/15*(30*a*e^(8*d*x + 8*c) + 45*b*e^(8*d*x + 8*c) + 9
0*a*e^(6*d*x + 6*c) + 90*b*e^(6*d*x + 6*c) + 110*a*e^(4*d*x + 4*c) + 140*b*
e^(4*d*x + 4*c) + 70*a*e^(2*d*x + 2*c) + 70*b*e^(2*d*x + 2*c) + 20*a + 23*b
)/(d*(e^(2*d*x + 2*c) + 1)^5)
```

3.135 $\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=49

$$-\frac{(a+b)\tanh^2(c+dx)}{2d}+\frac{(a+b)\log(\cosh(c+dx))}{d}-\frac{b\tanh^4(c+dx)}{4d}$$

```
[Out] ((a + b)*Log[Cosh[c + d*x]])/d - ((a + b)*Tanh[c + d*x]^2)/(2*d) - (b*Tanh[
c + d*x]^4)/(4*d)
```

Rubi [A] time = 0.0594662, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3631, 3473, 3475}

$$-\frac{(a+b)\tanh^2(c+dx)}{2d}+\frac{(a+b)\log(\cosh(c+dx))}{d}-\frac{b\tanh^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((a + b)*Log[Cosh[c + d*x]])/d - ((a + b)*Tanh[c + d*x]^2)/(2*d) - (b*Tanh[
c + d*x]^4)/(4*d)
```

Rule 3631

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(
m + 1)), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx &= -\frac{b\tanh^4(c+dx)}{4d}+(a+b)\int \tanh^3(c+dx)\,dx\\ &= -\frac{(a+b)\tanh^2(c+dx)}{2d}-\frac{b\tanh^4(c+dx)}{4d}+(a+b)\int \tanh(c+dx)\,dx\\ &= \frac{(a+b)\log(\cosh(c+dx))}{d}-\frac{(a+b)\tanh^2(c+dx)}{2d}-\frac{b\tanh^4(c+dx)}{4d}\end{aligned}$$

Mathematica [A] time = 0.235594, size = 43, normalized size = 0.88

$$-\frac{2(a+b)\tanh^2(c+dx)-4(a+b)\log(\cosh(c+dx))+b\tanh^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -(-4*(a + b)*Log[Cosh[c + d*x]] + 2*(a + b)*Tanh[c + d*x]^2 + b*Tanh[c + d*
x]^4)/(4*d)
```

Maple [B] time = 0.004, size = 104, normalized size = 2.1

$$-\frac{b(\tanh(dx+c))^4}{4d}-\frac{a(\tanh(dx+c))^2}{2d}-\frac{b(\tanh(dx+c))^2}{2d}-\frac{\ln(\tanh(dx+c)-1)a}{2d}-\frac{\ln(\tanh(dx+c)-1)b}{2d}-\ln($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x)
```

```
[Out] -1/4*b*tanh(d*x+c)^4/d-1/2/d*a*tanh(d*x+c)^2-1/2*b*tanh(d*x+c)^2/d-1/2/d*ln
(tanh(d*x+c)-1)*a-1/2/d*ln(tanh(d*x+c)-1)*b-1/2/d*ln(tanh(d*x+c)+1)*a-1/2/d
*ln(tanh(d*x+c)+1)*b
```

Maxima [B] time = 1.66453, size = 227, normalized size = 4.63

$$b\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2dx-2c)}+1\right)}{d}+\frac{4\left(e^{(-2dx-2c)}+e^{(-4dx-4c)}+e^{(-6dx-6c)}\right)}{d\left(4e^{(-2dx-2c)}+6e^{(-4dx-4c)}+4e^{(-6dx-6c)}+e^{(-8dx-8c)}+1\right)}\right)+a\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2dx-2c)}\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] b*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 4*(e^(-2*d*x - 2*c) + e^(-4*d*x
- 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*
e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) + a*(x + c/d + log(e^(-2*d*x - 2
*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) +
 1)))
```

Fricas [B] time = 2.1544, size = 3336, normalized size = 68.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -((a + b)*d*x*cosh(d*x + c)^8 + 8*(a + b)*d*x*cosh(d*x + c)*sinh(d*x + c)^7
 + (a + b)*d*x*sinh(d*x + c)^8 + 2*(2*(a + b)*d*x - a - 2*b)*cosh(d*x + c)^
6 + 2*(14*(a + b)*d*x*cosh(d*x + c)^2 + 2*(a + b)*d*x - a - 2*b)*sinh(d*x +
 c)^6 + 4*(14*(a + b)*d*x*cosh(d*x + c)^3 + 3*(2*(a + b)*d*x - a - 2*b)*cos
h(d*x + c))*sinh(d*x + c)^5 + 2*(3*(a + b)*d*x - 2*a - 2*b)*cosh(d*x + c)^4
 + 2*(35*(a + b)*d*x*cosh(d*x + c)^4 + 3*(a + b)*d*x + 15*(2*(a + b)*d*x -
a - 2*b)*cosh(d*x + c)^2 - 2*a - 2*b)*sinh(d*x + c)^4 + 8*(7*(a + b)*d*x*co
sh(d*x + c)^5 + 5*(2*(a + b)*d*x - a - 2*b)*cosh(d*x + c)^3 + (3*(a + b)*d*
```

```
x - 2*a - 2*b)*cosh(d*x + c))*sinh(d*x + c)^3 + (a + b)*d*x + 2*(2*(a + b)*
d*x - a - 2*b)*cosh(d*x + c)^2 + 2*(14*(a + b)*d*x*cosh(d*x + c)^6 + 15*(2*
(a + b)*d*x - a - 2*b)*cosh(d*x + c)^4 + 2*(a + b)*d*x + 6*(3*(a + b)*d*x -
 2*a - 2*b)*cosh(d*x + c)^2 - a - 2*b)*sinh(d*x + c)^2 - ((a + b)*cosh(d*x
+ c)^8 + 8*(a + b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a + b)*sinh(d*x + c)^8
+ 4*(a + b)*cosh(d*x + c)^6 + 4*(7*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*
x + c)^6 + 8*(7*(a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c))*sinh(d*x
 + c)^5 + 6*(a + b)*cosh(d*x + c)^4 + 2*(35*(a + b)*cosh(d*x + c)^4 + 30*(a
 + b)*cosh(d*x + c)^2 + 3*a + 3*b)*sinh(d*x + c)^4 + 8*(7*(a + b)*cosh(d*x
+ c)^5 + 10*(a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c))*sinh(d*x + c
)^3 + 4*(a + b)*cosh(d*x + c)^2 + 4*(7*(a + b)*cosh(d*x + c)^6 + 15*(a + b)
*cosh(d*x + c)^4 + 9*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c)^2 + 8*(
(a + b)*cosh(d*x + c)^7 + 3*(a + b)*cosh(d*x + c)^5 + 3*(a + b)*cosh(d*x +
c)^3 + (a + b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*log(2*cosh(d*x + c)/(c
osh(d*x + c) - sinh(d*x + c))) + 4*(2*(a + b)*d*x*cosh(d*x + c)^7 + 3*(2*(a
 + b)*d*x - a - 2*b)*cosh(d*x + c)^5 + 2*(3*(a + b)*d*x - 2*a - 2*b)*cosh(d
*x + c)^3 + (2*(a + b)*d*x - a - 2*b)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh
(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 + 4*d*c
osh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 8*(7*d*cosh(
d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(
35*d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d
*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^
3 + 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 15*d*cosh(d*x + c)^4 + 9
*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 + 3*d*cosh(d
*x + c)^5 + 3*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [A] time = 0.639558, size = 88, normalized size = 1.8

$$\begin{cases} ax - \frac{a\log(\tanh(c+dx)+1)}{d} - \frac{a\tanh^2(c+dx)}{2d} + bx - \frac{b\log(\tanh(c+dx)+1)}{d} - \frac{b\tanh^4(c+dx)}{4d} - \frac{b\tanh^2(c+dx)}{2d} & \text{for } d \neq 0 \\ x\left(a + b\tanh^2(c)\right)\tanh^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**3*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Piecewise((a*x - a*log(tanh(c + d*x) + 1)/d - a*tanh(c + d*x)**2/(2*d) + b*
x - b*log(tanh(c + d*x) + 1)/d - b*tanh(c + d*x)**4/(4*d) - b*tanh(c + d*x)
**2/(2*d), Ne(d, 0)), (x*(a + b*tanh(c)**2)*tanh(c)**3, True))
```

Giac [B] time = 1.21444, size = 130, normalized size = 2.65

$$-\frac{(dx+c)(a+b)}{d} + \frac{(a+b)\log\left(e^{(2dx+2c)}+1\right)}{d} + \frac{2\left((a+2b)e^{(6dx+6c)} + 2(a+b)e^{(4dx+4c)} + (a+2b)e^{(2dx+2c)}\right)}{d\left(e^{(2dx+2c)}+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -(d*x + c)*(a + b)/d + (a + b)*log(e^(2*d*x + 2*c) + 1)/d + 2*((a + 2*b)*e^
(6*d*x + 6*c) + 2*(a + b)*e^(4*d*x + 4*c) + (a + 2*b)*e^(2*d*x + 2*c))/(d*(
e^(2*d*x + 2*c) + 1)^4)
```

3.136 $\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=36

$$-\frac{(a+b)\tanh(c+dx)}{d}+x(a+b)-\frac{b\tanh^3(c+dx)}{3d}$$

```
[Out] (a + b)*x - ((a + b)*Tanh[c + d*x])/d - (b*Tanh[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0396052, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3631, 3473, 8}

$$-\frac{(a+b)\tanh(c+dx)}{d}+x(a+b)-\frac{b\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a + b)*x - ((a + b)*Tanh[c + d*x])/d - (b*Tanh[c + d*x]^3)/(3*d)
```

Rule 3631

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(
m + 1)), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx &= -\frac{b\tanh^3(c+dx)}{3d}-(-a-b)\int \tanh^2(c+dx)\,dx\\ &= -\frac{(a+b)\tanh(c+dx)}{d}-\frac{b\tanh^3(c+dx)}{3d}-(-a-b)\int 1\,dx\\ &= (a+b)x-\frac{(a+b)\tanh(c+dx)}{d}-\frac{b\tanh^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.0248043, size = 65, normalized size = 1.81

$$\frac{a\tanh^{-1}(\tanh(c+dx))}{d}-\frac{a\tanh(c+dx)}{d}-\frac{b\tanh^3(c+dx)}{3d}+\frac{b\tanh^{-1}(\tanh(c+dx))}{d}-\frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a*ArcTanh[Tanh[c + d*x]])/d + (b*ArcTanh[Tanh[c + d*x]])/d - (a*Tanh[c + d
*x])/d - (b*Tanh[c + d*x])/d - (b*Tanh[c + d*x]^3)/(3*d)
```

Maple [B] time = 0.005, size = 100, normalized size = 2.8

$$-\frac{b(\tanh(dx+c))^3}{3d}-\frac{a\tanh(dx+c)}{d}-\frac{b\tanh(dx+c)}{d}-\frac{\ln(\tanh(dx+c)-1)a}{2d}-\frac{\ln(\tanh(dx+c)-1)b}{2d}+\frac{\ln(\mathrm{ta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x)
```

```
[Out] -1/3*b*tanh(d*x+c)^3/d-a*tanh(d*x+c)/d-b*tanh(d*x+c)/d-1/2/d*ln(tanh(d*x+c)
-1)*a-1/2/d*ln(tanh(d*x+c)-1)*b+1/2/d*ln(tanh(d*x+c)+1)*a+1/2/d*ln(tanh(d*x
+c)+1)*b
```

Maxima [B] time = 1.13756, size = 142, normalized size = 3.94

$$\frac{1}{3}b\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+2\right)}{d\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1\right)}\right)+a\left(x+\frac{c}{d}-\frac{2}{d\left(e^{(-2dx-2c)}+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/3*b*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(3*
e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + a*(x + c/
d - 2/(d*(e^(-2*d*x - 2*c) + 1)))
```

Fricas [B] time = 1.83019, size = 428, normalized size = 11.89

$$\frac{(3(a+b)dx+3a+4b)\cosh(dx+c)^3+3(3(a+b)dx+3a+4b)\cosh(dx+c)\sinh(dx+c)^2-(3a+4b)\sinh(dx}{3\left(d\cosh(dx+c)^3+3d\cosh(dx+c)\sinh(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/3*((3*(a + b)*d*x + 3*a + 4*b)*cosh(d*x + c)^3 + 3*(3*(a + b)*d*x + 3*a +
 4*b)*cosh(d*x + c)*sinh(d*x + c)^2 - (3*a + 4*b)*sinh(d*x + c)^3 + 3*(3*(a
 + b)*d*x + 3*a + 4*b)*cosh(d*x + c) - 3*((3*a + 4*b)*cosh(d*x + c)^2 + a)*
sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d
*cosh(d*x + c))
```

Sympy [A] time = 0.381292, size = 54, normalized size = 1.5

$$\begin{cases} ax-\frac{a\tanh(c+dx)}{d}+bx-\frac{b\tanh^3(c+dx)}{3d}-\frac{b\tanh(c+dx)}{d} & \text{for } d\neq 0\\ x\left(a+b\tanh^2(c)\right)\tanh^2(c) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**2*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Piecewise((a*x - a*tanh(c + d*x)/d + b*x - b*tanh(c + d*x)**3/(3*d) - b*tan
h(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c)**2)*tanh(c)**2, True))
```

Giac [B] time = 1.16641, size = 116, normalized size = 3.22

$$\frac{(dx+c)(a+b)}{d}+\frac{2\left(3\,ae^{(4\,dx+4\,c)}+6\,be^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}+6\,be^{(2\,dx+2\,c)}+3\,a+4\,b\right)}{3\,d\left(e^{(2\,dx+2\,c)}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] (d*x + c)*(a + b)/d + 2/3*(3*a*e^(4*d*x + 4*c) + 6*b*e^(4*d*x + 4*c) + 6*a*
e^(2*d*x + 2*c) + 6*b*e^(2*d*x + 2*c) + 3*a + 4*b)/(d*(e^(2*d*x + 2*c) + 1)
^3)
```

3.137 $\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)dx$

Optimal. Leaf size=31

$$\frac{(a+b)\log(\cosh(c+dx))}{d}-\frac{b\tanh^2(c+dx)}{2d}$$

```
[Out] ((a + b)*Log[Cosh[c + d*x]])/d - (b*Tanh[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.0319005, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3631, 3475}

$$\frac{(a+b)\log(\cosh(c+dx))}{d}-\frac{b\tanh^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((a + b)*Log[Cosh[c + d*x]])/d - (b*Tanh[c + d*x]^2)/(2*d)
```

Rule 3631

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(
m + 1)), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)dx &= -\frac{b\tanh^2(c+dx)}{2d}-(-a-b)\int \tanh(c+dx)\,dx\\ &= \frac{(a+b)\log(\cosh(c+dx))}{d}-\frac{b\tanh^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.0234953, size = 41, normalized size = 1.32

$$\frac{a\log(\cosh(c+dx))}{d}-\frac{b\tanh^2(c+dx)}{2d}+\frac{b\log(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a*Log[Cosh[c + d*x]])/d + (b*Log[Cosh[c + d*x]])/d - (b*Tanh[c + d*x]^2)/(
2*d)
```

Maple [B] time = 0.004, size = 76, normalized size = 2.5

$$-\frac{b(\tanh(dx+c))^2}{2d}-\frac{\ln(\tanh(dx+c)-1)a}{2d}-\frac{\ln(\tanh(dx+c)-1)b}{2d}-\frac{\ln(\tanh(dx+c)+1)a}{2d}-\frac{\ln(\tanh(dx+c)+}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)*(a+b*tanh(d*x+c)^2),x)
```

```
[Out] -1/2*b*tanh(d*x+c)^2/d-1/2/d*ln(tanh(d*x+c)-1)*a-1/2/d*ln(tanh(d*x+c)-1)*b-
1/2/d*ln(tanh(d*x+c)+1)*a-1/2/d*ln(tanh(d*x+c)+1)*b
```

Maxima [B] time = 1.69827, size = 103, normalized size = 3.32

$$b\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2dx-2c)}+1\right)}{d}+\frac{2e^{(-2dx-2c)}}{d\left(2e^{(-2dx-2c)}+e^{(-4dx-4c)}+1\right)}\right)+\frac{a\log(\cosh(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] b*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d
*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a*log(cosh(d*x + c))/d
```

Fricas [B] time = 1.89958, size = 1114, normalized size = 35.94

$$-\frac{(a+b)dx\cosh(dx+c)^4+4(a+b)dx\cosh(dx+c)\sinh(dx+c)^3+(a+b)dx\sinh(dx+c)^4+(a+b)dx+2((a+b)dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -((a + b)*d*x*cosh(d*x + c)^4 + 4*(a + b)*d*x*cosh(d*x + c)*sinh(d*x + c)^3
 + (a + b)*d*x*sinh(d*x + c)^4 + (a + b)*d*x + 2*((a + b)*d*x - b)*cosh(d*x
 + c)^2 + 2*(3*(a + b)*d*x*cosh(d*x + c)^2 + (a + b)*d*x - b)*sinh(d*x + c)
^2 - ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (
a + b)*sinh(d*x + c)^4 + 2*(a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x
+ c)^2 + a + b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + b)*cosh
(d*x + c))*sinh(d*x + c) + a + b)*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh
(d*x + c))) + 4*((a + b)*d*x*cosh(d*x + c)^3 + ((a + b)*d*x - b)*cosh(d*x +
 c))*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3
+ d*sinh(d*x + c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*sin
h(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [A] time = 0.319899, size = 60, normalized size = 1.94

$$\begin{cases} ax-\frac{a\log(\tanh(c+dx)+1)}{d}+bx-\frac{b\log(\tanh(c+dx)+1)}{d}-\frac{b\tanh^2(c+dx)}{2d} & \text{for } d\neq 0 \\ x\left(a+b\tanh^2(c)\right)\tanh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Piecewise((a*x - a*log(tanh(c + d*x) + 1)/d + b*x - b*log(tanh(c + d*x) + 1
)/d - b*tanh(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a + b*tanh(c)**2)*tanh(c), T
rue))
```

Giac [B] time = 1.16461, size = 82, normalized size = 2.65

$$-\frac{(dx+c)(a+b)}{d}+\frac{(a+b)\log\left(e^{(2\,dx+2\,c)}+1\right)}{d}+\frac{2\,be^{(2\,dx+2\,c)}}{d\left(e^{(2\,dx+2\,c)}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -(d*x + c)*(a + b)/d + (a + b)*log(e^(2*d*x + 2*c) + 1)/d + 2*b*e^(2*d*x +
2*c)/(d*(e^(2*d*x + 2*c) + 1)^2)
```

3.138 $\int \left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=19

$$ax-\frac{b\tanh(c+dx)}{d}+bx$$

```
[Out] a*x + b*x - (b*Tanh[c + d*x])/d
```

Rubi [A] time = 0.0132686, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3473, 8}

$$ax-\frac{b\tanh(c+dx)}{d}+bx$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Tanh[c + d*x]^2,x]
```

```
[Out] a*x + b*x - (b*Tanh[c + d*x])/d
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\tanh^2(c+dx)\right)\,dx &= ax+b\int \tanh^2(c+dx)\,dx\\ &= ax-\frac{b\tanh(c+dx)}{d}+b\int 1\,dx\\ &= ax+bx-\frac{b\tanh(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0067343, size = 28, normalized size = 1.47

$$ax+\frac{b\tanh^{-1}(\tanh(c+dx))}{d}-\frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Tanh[c + d*x]^2,x]
```

```
[Out] a*x + (b*ArcTanh[Tanh[c + d*x]])/d - (b*Tanh[c + d*x])/d
```

Maple [B] time = 0.002, size = 47, normalized size = 2.5

$$ax - \frac{b\tanh(dx+c)}{d} - \frac{\ln(\tanh(dx+c)-1)\,b}{2d} + \frac{\ln(\tanh(dx+c)+1)\,b}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*tanh(d*x+c)^2,x)
```

```
[Out] a*x-b*tanh(d*x+c)/d-1/2/d*ln(tanh(d*x+c)-1)*b+1/2/d*ln(tanh(d*x+c)+1)*b
```

Maxima [A] time = 1.09244, size = 42, normalized size = 2.21

$$b\left(x + \frac{c}{d} - \frac{2}{d\left(e^{(-2\,dx-2\,c)}+1\right)}\right) + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tanh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] b*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a*x
```

Fricas [A] time = 2.0544, size = 96, normalized size = 5.05

$$\frac{((a+b)dx+b)\cosh(dx+c) - b\sinh(dx+c)}{d\cosh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tanh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] (((a + b)*d*x + b)*cosh(d*x + c) - b*sinh(d*x + c))/(d*cosh(d*x + c))
```

Sympy [A] time = 0.190726, size = 20, normalized size = 1.05

$$ax + b\left(\begin{cases} x - \frac{\tanh(c+dx)}{d} & \text{for}\ d \neq 0 \\ x\tanh^2(c) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tanh(d*x+c)**2,x)
```

```
[Out] a*x + b*Piecewise((x - tanh(c + d*x)/d, Ne(d, 0)), (x*tanh(c)**2, True))
```

Giac [A] time = 1.1661, size = 46, normalized size = 2.42

$$ax + b\left(\frac{dx+c}{d} + \frac{2}{d\left(e^{(2\,dx+2\,c)}+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tanh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] a*x + b*((d*x + c)/d + 2/(d*(e^(2*d*x + 2*c) + 1)))
```

3.139 $\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=25

$$\frac{a\log(\sinh(c+dx))}{d}+\frac{b\log(\cosh(c+dx))}{d}$$

```
[Out] (b*Log[Cosh[c + d*x]])/d + (a*Log[Sinh[c + d*x]])/d
```

Rubi [A] time = 0.0412611, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3625, 3475}

$$\frac{a\log(\sinh(c+dx))}{d}+\frac{b\log(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (b*Log[Cosh[c + d*x]])/d + (a*Log[Sinh[c + d*x]])/d
```

Rule 3625

```
Int[((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2)/tan[(e_.) + (f_.)*(x_)], x_Sym
bol] :> Dist[A, Int[1/Tan[e + f*x], x], x] + Dist[C, Int[Tan[e + f*x], x],
x] /; FreeQ[{e, f, A, C}, x] && NeQ[A, C]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)dx &= a\int \coth(c+dx)\,dx+b\int \tanh(c+dx)\,dx\\ &= \frac{b\log(\cosh(c+dx))}{d}+\frac{a\log(\sinh(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.0396557, size = 33, normalized size = 1.32

$$\frac{a(\log(\tanh(c+dx))+\log(\cosh(c+dx)))}{d}+\frac{b\log(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (b*Log[Cosh[c + d*x]])/d + (a*(Log[Cosh[c + d*x]] + Log[Tanh[c + d*x]]))/d
```

Maple [A] time = 0.041, size = 26, normalized size = 1.

$$\frac{b\ln\left(\cosh\left(dx+c\right)\right)}{d}+\frac{a\ln\left(\sinh\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)*(a+b*tanh(d*x+c)^2),x)
```

```
[Out] b*ln(cosh(d*x+c))/d+a*ln(sinh(d*x+c))/d
```

Maxima [A] time = 1.10261, size = 47, normalized size = 1.88

$$\frac{b\log\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{d}+\frac{a\log\left(\sinh\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] b*log(e^(d*x + c) + e^(-d*x - c))/d + a*log(sinh(d*x + c))/d
```

Fricas [B] time = 2.02863, size = 178, normalized size = 7.12

$$-\frac{(a+b)dx-b\log\left(\frac{2\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)-a\log\left(\frac{2\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -((a + b)*d*x - b*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) - a*
log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tanh^2\left(c+dx\right)\right)\coth\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*coth(c + d*x), x)
```

Giac [A] time = 1.17575, size = 66, normalized size = 2.64

$$-\frac{(dx+c)(a+b)}{d}+\frac{b\log\left(e^{(2\,dx+2\,c)}+1\right)}{d}+\frac{a\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -(d*x + c)*(a + b)/d + b*log(e^(2*d*x + 2*c) + 1)/d + a*log(abs(e^(2*d*x +
2*c) - 1))/d
```

3.140 $\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=18

$$x(a+b)-\frac{a\coth(c+dx)}{d}$$

```
[Out] (a + b)*x - (a*Coth[c + d*x])/d
```

Rubi [A] time = 0.0290586, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3629, 8}

$$x(a+b)-\frac{a\coth(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a + b)*x - (a*Coth[c + d*x])/d
```

Rule 3629

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m
 + 1))/(b*f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e
 + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /; F
reeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[a
^2 + b^2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx &= -\frac{a\coth(c+dx)}{d}-\int(-a-b)\,dx\\ &= (a+b)x-\frac{a\coth(c+dx)}{d}\end{aligned}$$

Mathematica [C] time = 0.0259963, size = 32, normalized size = 1.78

$$bx-\frac{a\coth(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\tanh^2(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] b*x - (a*Coth[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[c + d*x]^2])/d
```

Maple [A] time = 0.033, size = 28, normalized size = 1.6

$$\frac{a\left(dx+c-\coth\left(dx+c\right)\right)+\left(dx+c\right)b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2),x)
```

```
[Out] 1/d*(a*(d*x+c-coth(d*x+c))+(d*x+c)*b)
```

Maxima [A] time = 1.04148, size = 42, normalized size = 2.33

$$a\left(x+\frac{c}{d}+\frac{2}{d\left(e^{(-2\,dx-2\,c)}-1\right)}\right)+bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] a*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + b*x
```

Fricas [B] time = 1.9345, size = 97, normalized size = 5.39

$$-\frac{a\cosh\left(dx+c\right)-\left((a+b)dx+a\right)\sinh\left(dx+c\right)}{d\sinh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -(a*cosh(d*x + c) - ((a + b)*d*x + a)*sinh(d*x + c))/(d*sinh(d*x + c))
```

Sympy [B] time = 60.3678, size = 49, normalized size = 2.72

$$a\left(\begin{cases} x\coth^{2}\left(c\right) & \text{for}\ d=0\\ \tilde{\infty}x & \text{for}\ c=\log\left(-e^{-dx}\right)\vee c=\log\left(e^{-dx}\right)\\ x-\frac{1}{d\tanh\left(c+dx\right)} & \text{otherwise}\end{cases}\right)+b\left(\begin{cases} x & \text{for}\ |x|<1\\ G_{2,2}^{1,1}\left(\begin{matrix}1 & 2\\ 1 & 0\end{matrix}\middle|x\right)+G_{2,2}^{0,2}\left(\begin{matrix}2,1 & \\ & 1,0\end{matrix}\middle|x\right) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] a*Piecewise((x*coth(c)**2, Eq(d, 0)), (zoo*x, Eq(c, log(exp(-d*x))) | Eq(c,
 log(-exp(-d*x)))), (x - 1/(d*tanh(c + d*x)), True)) + b*Piecewise((x, Abs(
x) < 1), (meijerg(((1,), (2,)), ((1,), (0,)), x) + meijerg(((2, 1), ()), ((
), (1, 0)), x), True))
```

Giac [A] time = 1.21697, size = 43, normalized size = 2.39

$$\frac{(dx+c)(a+b)}{d}-\frac{2\,a}{d\left(e^{(2\,dx+2\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] (d*x + c)*(a + b)/d - 2*a/(d*(e^(2*d*x + 2*c) - 1))
```

3.141 $\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=31

$$\frac{(a+b)\log(\sinh(c+dx))}{d}-\frac{a\coth^2(c+dx)}{2d}$$

```
[Out] -(a*Coth[c + d*x]^2)/(2*d) + ((a + b)*Log[Sinh[c + d*x]])/d
```

Rubi [A] time = 0.0424706, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3629, 12, 3475}

$$\frac{(a+b)\log(\sinh(c+dx))}{d}-\frac{a\coth^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -(a*Coth[c + d*x]^2)/(2*d) + ((a + b)*Log[Sinh[c + d*x]])/d
```

Rule 3629

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m
 + 1))/(b*f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e
 + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /; F
reeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[a
^2 + b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx &= -\frac{a\coth^2(c+dx)}{2d}+\int (a+b)\coth(c+dx)\,dx\\ &= -\frac{a\coth^2(c+dx)}{2d}+(a+b)\int \coth(c+dx)\,dx\\ &= -\frac{a\coth^2(c+dx)}{2d}+\frac{(a+b)\log(\sinh(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.103553, size = 39, normalized size = 1.26

$$\frac{2(a+b)(\log(\tanh(c+dx))+\log(\cosh(c+dx)))-a\coth^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (-(a*Coth[c + d*x]^2) + 2*(a + b)*(Log[Cosh[c + d*x]] + Log[Tanh[c + d*x]])
)/(2*d)
```

Maple [A] time = 0.046, size = 40, normalized size = 1.3

$$\frac{a\ln(\sinh(dx+c))}{d} - \frac{(\coth(dx+c))^2 a}{2d} + \frac{b\ln(\sinh(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2),x)
```

```
[Out] a*ln(sinh(d*x+c))/d-1/2*a*coth(d*x+c)^2/d+1/d*b*ln(sinh(d*x+c))
```

Maxima [B] time = 1.0785, size = 143, normalized size = 4.61

$$a\left(x + \frac{c}{d} + \frac{\log\left(e^{(-dx-c)}+1\right)}{d} + \frac{\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\,e^{(-2\,dx-2\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)} - e^{(-4\,dx-4\,c)} - 1\right)}\right) + \frac{b\log\left(e^{(dx+c)} - e^{(-dx-c)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] a*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*d*
x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + b*log(e^(d*x +
c) - e^(-d*x - c))/d
```

Fricas [B] time = 2.10567, size = 1114, normalized size = 35.94

$$-\frac{(a+b)dx\cosh(dx+c)^4 + 4\,(a+b)dx\cosh(dx+c)\sinh(dx+c)^3 + (a+b)dx\sinh(dx+c)^4 + (a+b)dx - 2\,((a+b)dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -((a + b)*d*x*cosh(d*x + c)^4 + 4*(a + b)*d*x*cosh(d*x + c)*sinh(d*x + c)^3
 + (a + b)*d*x*sinh(d*x + c)^4 + (a + b)*d*x - 2*((a + b)*d*x - a)*cosh(d*x
 + c)^2 + 2*(3*(a + b)*d*x*cosh(d*x + c)^2 - (a + b)*d*x + a)*sinh(d*x + c)
^2 - ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (
a + b)*sinh(d*x + c)^4 - 2*(a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x
+ c)^2 - a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 - (a + b)*cosh
(d*x + c))*sinh(d*x + c) + a + b)*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh
(d*x + c))) + 4*((a + b)*d*x*cosh(d*x + c)^3 - ((a + b)*d*x - a)*cosh(d*x +
 c))*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3
+ d*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 - d)*sin
```

```
h(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18364, size = 84, normalized size = 2.71

$$-\frac{(dx+c)(a+b)}{d}+\frac{(a+b)\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)}{d}-\frac{2\,ae^{(2\,dx+2\,c)}}{d\left(e^{(2\,dx+2\,c)}-1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -(d*x + c)*(a + b)/d + (a + b)*log(abs(e^(2*d*x + 2*c) - 1))/d - 2*a*e^(2*d
*x + 2*c)/(d*(e^(2*d*x + 2*c) - 1)^2)
```

3.142 $\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=36

$$-\frac{(a+b)\coth(c+dx)}{d}+x(a+b)-\frac{a\coth^3(c+dx)}{3d}$$

```
[Out] (a + b)*x - ((a + b)*Coth[c + d*x])/d - (a*Coth[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.038836, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3629, 12, 3473, 8}

$$-\frac{(a+b)\coth(c+dx)}{d}+x(a+b)-\frac{a\coth^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a + b)*x - ((a + b)*Coth[c + d*x])/d - (a*Coth[c + d*x]^3)/(3*d)
```

Rule 3629

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m
 + 1))/(b*f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e
 + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /; F
reeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[a
^2 + b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx &= -\frac{a\coth^3(c+dx)}{3d}+\int (a+b)\coth^2(c+dx)\,dx\\
&= -\frac{a\coth^3(c+dx)}{3d}+(a+b)\int \coth^2(c+dx)\,dx\\
&= -\frac{(a+b)\coth(c+dx)}{d}-\frac{a\coth^3(c+dx)}{3d}+(a+b)\int 1\,dx\\
&= (a+b)x-\frac{(a+b)\coth(c+dx)}{d}-\frac{a\coth^3(c+dx)}{3d}
\end{aligned}
$$

Mathematica [C] time = 0.0360232, size = 61, normalized size = 1.69

$$
-\frac{a\coth^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\tanh^2(c+dx)\right)}{3d}-\frac{b\coth(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\tanh^2(c+dx)\right)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -(a*Coth[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Tanh[c + d*x]^2])/(3*d
) - (b*Coth[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[c + d*x]^2])/d
```

Maple [A] time = 0.042, size = 46, normalized size = 1.3

$$
\frac{1}{d}\left(a\left(dx+c-\coth(dx+c)-\frac{(\coth(dx+c))^3}{3}\right)+b\,(dx+c-\coth(dx+c))\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2),x)
```

```
[Out] 1/d*(a*(d*x+c-coth(d*x+c)-1/3*coth(d*x+c)^3)+b*(d*x+c-coth(d*x+c)))
```

Maxima [B] time = 1.17206, size = 142, normalized size = 3.94

$$
\frac{1}{3}a\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}-2\right)}{d\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1\right)}\right)+b\left(x+\frac{c}{d}+\frac{2}{d\left(e^{(-2dx-2c)}-1\right)}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/3*a*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - 2)/(d*(3*
e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + b*(x + c/
d + 2/(d*(e^(-2*d*x - 2*c) - 1)))
```

Fricas [B] time = 1.91492, size = 408, normalized size = 11.33

$$
-\frac{(4a+3b)\cosh(dx+c)^3+3(4a+3b)\cosh(dx+c)\sinh(dx+c)^2-(3(a+b)dx+4a+3b)\sinh(dx+c)^3-3b\,\mathrm{co}}{3\left(d\sinh(dx+c)^3+3\left(d\cosh(dx+c)^2\right.\right.}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/3*((4*a + 3*b)*cosh(d*x + c)^3 + 3*(4*a + 3*b)*cosh(d*x + c)*sinh(d*x +
c)^2 - (3*(a + b)*d*x + 4*a + 3*b)*sinh(d*x + c)^3 - 3*b*cosh(d*x + c) + 3*
(3*(a + b)*d*x - (3*(a + b)*d*x + 4*a + 3*b)*cosh(d*x + c)^2 + 4*a + 3*b)*s
inh(d*x + c))/(d*sinh(d*x + c)^3 + 3*(d*cosh(d*x + c)^2 - d)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**4*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19785, size = 116, normalized size = 3.22

$$\frac{(dx+c)(a+b)}{d} - \frac{2\left(6\,ae^{(4\,dx+4\,c)} + 3\,be^{(4\,dx+4\,c)} - 6\,ae^{(2\,dx+2\,c)} - 6\,be^{(2\,dx+2\,c)} + 4\,a + 3\,b\right)}{3\,d\left(e^{(2\,dx+2\,c)} - 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] (d*x + c)*(a + b)/d - 2/3*(6*a*e^(4*d*x + 4*c) + 3*b*e^(4*d*x + 4*c) - 6*a*
e^(2*d*x + 2*c) - 6*b*e^(2*d*x + 2*c) + 4*a + 3*b)/(d*(e^(2*d*x + 2*c) - 1)
^3)
```

3.143 $\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=49

$$-\frac{(a+b)\coth^2(c+dx)}{2d}+\frac{(a+b)\log(\sinh(c+dx))}{d}-\frac{a\coth^4(c+dx)}{4d}$$

```
[Out] -((a + b)*Coth[c + d*x]^2)/(2*d) - (a*Coth[c + d*x]^4)/(4*d) + ((a + b)*Log
[Sinh[c + d*x]])/d
```

Rubi [A] time = 0.0618572, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3629, 12, 3473, 3475}

$$-\frac{(a+b)\coth^2(c+dx)}{2d}+\frac{(a+b)\log(\sinh(c+dx))}{d}-\frac{a\coth^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^5*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -((a + b)*Coth[c + d*x]^2)/(2*d) - (a*Coth[c + d*x]^4)/(4*d) + ((a + b)*Log
[Sinh[c + d*x]])/d
```

Rule 3629

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m
 + 1))/(b*f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e
 + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /; F
reeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[a
^2 + b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)dx = -\frac{a\coth^4(c+dx)}{4d} + \int (a+b)\coth^3(c+dx)\,dx$$
$$= -\frac{a\coth^4(c+dx)}{4d} + (a+b)\int \coth^3(c+dx)\,dx$$
$$= -\frac{(a+b)\coth^2(c+dx)}{2d} - \frac{a\coth^4(c+dx)}{4d} + (a+b)\int \coth(c+dx)\,dx$$
$$= -\frac{(a+b)\coth^2(c+dx)}{2d} - \frac{a\coth^4(c+dx)}{4d} + \frac{(a+b)\log(\sinh(c+dx))}{d}$$

Mathematica [A] time = 0.312658, size = 51, normalized size = 1.04

$$-\frac{2(a+b)\coth^2(c+dx) - 4(a+b)(\log(\tanh(c+dx)) + \log(\cosh(c+dx))) + a\coth^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^5*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -(2*(a + b)*Coth[c + d*x]^2 + a*Coth[c + d*x]^4 - 4*(a + b)*(Log[Cosh[c + d
*x]] + Log[Tanh[c + d*x]]))/(4*d)
```

Maple [A] time = 0.046, size = 68, normalized size = 1.4

$$\frac{a\ln(\sinh(dx+c))}{d} - \frac{(\coth(dx+c))^2 a}{2d} - \frac{a(\coth(dx+c))^4}{4d} + \frac{b\ln(\sinh(dx+c))}{d} - \frac{b(\coth(dx+c))^2}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2),x)
```

```
[Out] a*ln(sinh(d*x+c))/d-1/2*a*coth(d*x+c)^2/d-1/4*a*coth(d*x+c)^4/d+1/d*b*ln(si
nh(d*x+c))-1/2/d*b*coth(d*x+c)^2
```

Maxima [B] time = 1.16511, size = 278, normalized size = 5.67

$$a\left(x + \frac{c}{d} + \frac{\log\left(e^{(-dx-c)}+1\right)}{d} + \frac{\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{4\left(e^{(-2dx-2c)} - e^{(-4dx-4c)} + e^{(-6dx-6c)}\right)}{d\left(4e^{(-2dx-2c)} - 6e^{(-4dx-4c)} + 4e^{(-6dx-6c)} - e^{(-8dx-8c)} - 1\right)}\right) + b\left(x + \frac{c}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] a*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 4*(e^(-2*d
*x - 2*c) - e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) - 6
*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + b*(x + c
/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*d*x - 2*c)
/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

Fricas [B] time = 2.17152, size = 3336, normalized size = 68.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -((a + b)*d*x*cosh(d*x + c)^8 + 8*(a + b)*d*x*cosh(d*x + c)*sinh(d*x + c)^7
 + (a + b)*d*x*sinh(d*x + c)^8 - 2*(2*(a + b)*d*x - 2*a - b)*cosh(d*x + c)^
6 + 2*(14*(a + b)*d*x*cosh(d*x + c)^2 - 2*(a + b)*d*x + 2*a + b)*sinh(d*x +
 c)^6 + 4*(14*(a + b)*d*x*cosh(d*x + c)^3 - 3*(2*(a + b)*d*x - 2*a - b)*cos
h(d*x + c))*sinh(d*x + c)^5 + 2*(3*(a + b)*d*x - 2*a - 2*b)*cosh(d*x + c)^4
 + 2*(35*(a + b)*d*x*cosh(d*x + c)^4 + 3*(a + b)*d*x - 15*(2*(a + b)*d*x -
2*a - b)*cosh(d*x + c)^2 - 2*a - 2*b)*sinh(d*x + c)^4 + 8*(7*(a + b)*d*x*co
sh(d*x + c)^5 - 5*(2*(a + b)*d*x - 2*a - b)*cosh(d*x + c)^3 + (3*(a + b)*d*
x - 2*a - 2*b)*cosh(d*x + c))*sinh(d*x + c)^3 + (a + b)*d*x - 2*(2*(a + b)*
d*x - 2*a - b)*cosh(d*x + c)^2 + 2*(14*(a + b)*d*x*cosh(d*x + c)^6 - 15*(2*
(a + b)*d*x - 2*a - b)*cosh(d*x + c)^4 - 2*(a + b)*d*x + 6*(3*(a + b)*d*x -
 2*a - 2*b)*cosh(d*x + c)^2 + 2*a + b)*sinh(d*x + c)^2 - ((a + b)*cosh(d*x
+ c)^8 + 8*(a + b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a + b)*sinh(d*x + c)^8
- 4*(a + b)*cosh(d*x + c)^6 + 4*(7*(a + b)*cosh(d*x + c)^2 - a - b)*sinh(d*
x + c)^6 + 8*(7*(a + b)*cosh(d*x + c)^3 - 3*(a + b)*cosh(d*x + c))*sinh(d*x
 + c)^5 + 6*(a + b)*cosh(d*x + c)^4 + 2*(35*(a + b)*cosh(d*x + c)^4 - 30*(a
 + b)*cosh(d*x + c)^2 + 3*a + 3*b)*sinh(d*x + c)^4 + 8*(7*(a + b)*cosh(d*x
+ c)^5 - 10*(a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c))*sinh(d*x + c
)^3 - 4*(a + b)*cosh(d*x + c)^2 + 4*(7*(a + b)*cosh(d*x + c)^6 - 15*(a + b)
*cosh(d*x + c)^4 + 9*(a + b)*cosh(d*x + c)^2 - a - b)*sinh(d*x + c)^2 + 8*(
(a + b)*cosh(d*x + c)^7 - 3*(a + b)*cosh(d*x + c)^5 + 3*(a + b)*cosh(d*x +
c)^3 - (a + b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*log(2*sinh(d*x + c)/(c
osh(d*x + c) - sinh(d*x + c))) + 4*(2*(a + b)*d*x*cosh(d*x + c)^7 - 3*(2*(a
 + b)*d*x - 2*a - b)*cosh(d*x + c)^5 + 2*(3*(a + b)*d*x - 2*a - 2*b)*cosh(d
*x + c)^3 - (2*(a + b)*d*x - 2*a - b)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh
(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 - 4*d*c
osh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^6 + 8*(7*d*cosh(
d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(
35*d*cosh(d*x + c)^4 - 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d
*cosh(d*x + c)^5 - 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^
3 - 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 - 15*d*cosh(d*x + c)^4 + 9
*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 - 3*d*cosh(d
*x + c)^5 + 3*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**5*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20055, size = 131, normalized size = 2.67

$$-\frac{(dx+c)(a+b)}{d}+\frac{(a+b)\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)}{d}-\frac{2\left((2\,a+b)e^{(6\,dx+6\,c)}-2\,(a+b)e^{(4\,dx+4\,c)}+(2\,a+b)e^{(2\,dx+2\,c)}\right)}{d\left(e^{(2\,dx+2\,c)}-1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -(d*x + c)*(a + b)/d + (a + b)*log(abs(e^(2*d*x + 2*c) - 1))/d - 2*((2*a +
b)*e^(6*d*x + 6*c) - 2*(a + b)*e^(4*d*x + 4*c) + (2*a + b)*e^(2*d*x + 2*c))
/(d*(e^(2*d*x + 2*c) - 1)^4)
```

3.144 $\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=83

$$-\frac{b(2a+b)\tanh^5(c+dx)}{5d}-\frac{(a+b)^2\tanh^3(c+dx)}{3d}-\frac{(a+b)^2\tanh(c+dx)}{d}+x(a+b)^2-\frac{b^2\tanh^7(c+dx)}{7d}$$

```
[Out] (a + b)^2*x - ((a + b)^2*Tanh[c + d*x])/d - ((a + b)^2*Tanh[c + d*x]^3)/(3*
d) - (b*(2*a + b)*Tanh[c + d*x]^5)/(5*d) - (b^2*Tanh[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.0788679, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 461, 206}

$$-\frac{b(2a+b)\tanh^5(c+dx)}{5d}-\frac{(a+b)^2\tanh^3(c+dx)}{3d}-\frac{(a+b)^2\tanh(c+dx)}{d}+x(a+b)^2-\frac{b^2\tanh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (a + b)^2*x - ((a + b)^2*Tanh[c + d*x])/d - ((a + b)^2*Tanh[c + d*x]^3)/(3*
d) - (b*(2*a + b)*Tanh[c + d*x]^5)/(5*d) - (b^2*Tanh[c + d*x]^7)/(7*d)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{x^4(a+bx^2)^2}{1-x^2}dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-(a+b)^2-(a+b)^2x^2-b(2a+b)x^4-b^2x^6+\frac{a^2+2ab+b^2}{1-x^2}\right)dx, x, \text{t}\right.}{d}$$
$$= -\frac{(a+b)^2\tanh(c+dx)}{d}-\frac{(a+b)^2\tanh^3(c+dx)}{3d}-\frac{b(2a+b)\tanh^5(c+dx}{5d}$$
$$= (a+b)^2x-\frac{(a+b)^2\tanh(c+dx)}{d}-\frac{(a+b)^2\tanh^3(c+dx)}{3d}-\frac{b(2a+b)\tanh}{5d}$$

Mathematica [B] time = 0.0767448, size = 190, normalized size = 2.29

$$-\frac{a^2\tanh^3(c+dx)}{3d}+\frac{a^2\tanh^{-1}(\tanh(c+dx))}{d}-\frac{a^2\tanh(c+dx)}{d}-\frac{2ab\tanh^5(c+dx)}{5d}-\frac{2ab\tanh^3(c+dx)}{3d}+\frac{2ab\tanh}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (a^2*ArcTanh[Tanh[c + d*x]])/d + (2*a*b*ArcTanh[Tanh[c + d*x]])/d + (b^2*Ar
cTanh[Tanh[c + d*x]])/d - (a^2*Tanh[c + d*x])/d - (2*a*b*Tanh[c + d*x])/d -
 (b^2*Tanh[c + d*x])/d - (a^2*Tanh[c + d*x]^3)/(3*d) - (2*a*b*Tanh[c + d*x]
^3)/(3*d) - (b^2*Tanh[c + d*x]^3)/(3*d) - (2*a*b*Tanh[c + d*x]^5)/(5*d) - (
b^2*Tanh[c + d*x]^5)/(5*d) - (b^2*Tanh[c + d*x]^7)/(7*d)
```

Maple [B] time = 0.006, size = 236, normalized size = 2.8

$$-\frac{a^2\ln(\tanh(dx+c)-1)}{2d}-\frac{\ln(\tanh(dx+c)-1)ab}{d}-\frac{\ln(\tanh(dx+c)-1)b^2}{2d}-\frac{b^2(\tanh(dx+c))^3}{3d}-\frac{b^2(\tanh(dx+}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] -1/2*a^2/d*ln(tanh(d*x+c)-1)-1/d*ln(tanh(d*x+c)-1)*a*b-1/2/d*ln(tanh(d*x+c)
-1)*b^2-1/3*b^2*tanh(d*x+c)^3/d-1/5*b^2*tanh(d*x+c)^5/d-1/3/d*tanh(d*x+c)^3
*a^2-1/7*b^2*tanh(d*x+c)^7/d-2*a*b*tanh(d*x+c)/d-2/5/d*tanh(d*x+c)^5*a*b-2/
3*a*b*tanh(d*x+c)^3/d+1/2/d*ln(tanh(d*x+c)+1)*a^2+1/d*ln(tanh(d*x+c)+1)*a*b
+1/2/d*ln(tanh(d*x+c)+1)*b^2-a^2*tanh(d*x+c)/d-b^2*tanh(d*x+c)/d
```

Maxima [B] time = 1.16148, size = 498, normalized size = 6.

$$\frac{1}{105}b^2\left(105x+\frac{105c}{d}-\frac{8\left(203e^{(-2dx-2c)}+609e^{(-4dx-4c)}+770e^{(-6dx-6c)}+770e^{(-8dx-8c)}+315e^{(-10dx-10c)}+105e^{(-12}\right.}{d\left(7e^{(-2dx-2c)}+21e^{(-4dx-4c)}+35e^{(-6dx-6c)}+35e^{(-8dx-8c)}+21e^{(-10dx-10c)}+7e^{(-12dx-12c)}+e^{(-}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/105*b^2*(105*x + 105*c/d - 8*(203*e^(-2*d*x - 2*c) + 609*e^(-4*d*x - 4*c)
 + 770*e^(-6*d*x - 6*c) + 770*e^(-8*d*x - 8*c) + 315*e^(-10*d*x - 10*c) + 1
05*e^(-12*d*x - 12*c) + 44)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) +
35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-1
2*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 2/15*a*b*(15*x + 15*c/d - 2*(70
*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 90*e^(-6*d*x - 6*c) + 45*e^(-8*d
*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
 - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 1/3*a^2*(3*x + 3
*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(3*e^(-2*d*x - 2*
c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))
```

Fricas [B] time = 1.97528, size = 2140, normalized size = 25.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/105*((105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d*x
 + c)^7 + 7*(105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cos
h(d*x + c)*sinh(d*x + c)^6 - 2*(70*a^2 + 161*a*b + 88*b^2)*sinh(d*x + c)^7
+ 7*(105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d*x +
c)^5 - 14*(3*(70*a^2 + 161*a*b + 88*b^2)*cosh(d*x + c)^2 + 40*a^2 + 71*a*b
+ 28*b^2)*sinh(d*x + c)^5 + 35*((105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 32
2*a*b + 176*b^2)*cosh(d*x + c)^3 + (105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 +
 322*a*b + 176*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + 21*(105*(a^2 + 2*a*b +
 b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d*x + c)^3 - 14*(5*(70*a^2 +
161*a*b + 88*b^2)*cosh(d*x + c)^4 + 10*(40*a^2 + 71*a*b + 28*b^2)*cosh(d*x
+ c)^2 + 60*a^2 + 123*a*b + 84*b^2)*sinh(d*x + c)^3 + 7*(3*(105*(a^2 + 2*a*
b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d*x + c)^5 + 10*(105*(a^2
+ 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d*x + c)^3 + 9*(105*
(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d*x + c))*sinh(
d*x + c)^2 + 35*(105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)
*cosh(d*x + c) - 14*((70*a^2 + 161*a*b + 88*b^2)*cosh(d*x + c)^6 + 5*(40*a^
2 + 71*a*b + 28*b^2)*cosh(d*x + c)^4 + 9*(20*a^2 + 41*a*b + 28*b^2)*cosh(d*
x + c)^2 + 30*a^2 + 75*a*b)*sinh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*cosh(d*
x + c)*sinh(d*x + c)^6 + 7*d*cosh(d*x + c)^5 + 35*(d*cosh(d*x + c)^3 + d*co
sh(d*x + c))*sinh(d*x + c)^4 + 21*d*cosh(d*x + c)^3 + 7*(3*d*cosh(d*x + c)^
5 + 10*d*cosh(d*x + c)^3 + 9*d*cosh(d*x + c))*sinh(d*x + c)^2 + 35*d*cosh(d
*x + c))
```

Sympy [A] time = 1.51795, size = 165, normalized size = 1.99

$$\begin{cases} a^2x - \frac{a^2\tanh^3(c+dx)}{3d} - \frac{a^2\tanh(c+dx)}{d} + 2abx - \frac{2ab\tanh^5(c+dx)}{5d} - \frac{2ab\tanh^3(c+dx)}{3d} - \frac{2ab\tanh(c+dx)}{d} + b^2x - \frac{b^2\tanh^7(c+dx)}{7d} - \frac{b^2 \mathrm{ta}}{} \\ x\left(a+b\tanh^2(c)\right)^2\tanh^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**4*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((a**2*x - a**2*tanh(c + d*x)**3/(3*d) - a**2*tanh(c + d*x)/d + 2*
a*b*x - 2*a*b*tanh(c + d*x)**5/(5*d) - 2*a*b*tanh(c + d*x)**3/(3*d) - 2*a*b
*tanh(c + d*x)/d + b**2*x - b**2*tanh(c + d*x)**7/(7*d) - b**2*tanh(c + d*x
```

```
)**5/(5*d) - b**2*tanh(c + d*x)**3/(3*d) - b**2*tanh(c + d*x)/d, Ne(d, 0)),
 (x*(a + b*tanh(c)**2)**2*tanh(c)**4, True))
```

Giac [B] time = 1.31415, size = 405, normalized size = 4.88

$$\frac{\left(a^2+2\,ab+b^2\right)(dx+c)}{d}+\frac{4\left(105\,a^2e^{(12\,dx+12\,c)}+315\,abe^{(12\,dx+12\,c)}+210\,b^2e^{(12\,dx+12\,c)}+525\,a^2e^{(10\,dx+10\,c)}+1260\,abe^{(10\,dx}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] (a^2 + 2*a*b + b^2)*(d*x + c)/d + 4/105*(105*a^2*e^(12*d*x + 12*c) + 315*a*
b*e^(12*d*x + 12*c) + 210*b^2*e^(12*d*x + 12*c) + 525*a^2*e^(10*d*x + 10*c)
 + 1260*a*b*e^(10*d*x + 10*c) + 630*b^2*e^(10*d*x + 10*c) + 1120*a^2*e^(8*d
*x + 8*c) + 2555*a*b*e^(8*d*x + 8*c) + 1540*b^2*e^(8*d*x + 8*c) + 1330*a^2*
e^(6*d*x + 6*c) + 3080*a*b*e^(6*d*x + 6*c) + 1540*b^2*e^(6*d*x + 6*c) + 945
*a^2*e^(4*d*x + 4*c) + 2121*a*b*e^(4*d*x + 4*c) + 1218*b^2*e^(4*d*x + 4*c)
+ 385*a^2*e^(2*d*x + 2*c) + 812*a*b*e^(2*d*x + 2*c) + 406*b^2*e^(2*d*x + 2*
c) + 70*a^2 + 161*a*b + 88*b^2)/(d*(e^(2*d*x + 2*c) + 1)^7)
```

3.145 $\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=76

$$-\frac{b(2a+b)\tanh^4(c+dx)}{4d}-\frac{(a+b)^2\tanh^2(c+dx)}{2d}+\frac{(a+b)^2\log(\cosh(c+dx))}{d}-\frac{b^2\tanh^6(c+dx)}{6d}$$

```
[Out] ((a + b)^2*Log[Cosh[c + d*x]])/d - ((a + b)^2*Tanh[c + d*x]^2)/(2*d) - (b*(
2*a + b)*Tanh[c + d*x]^4)/(4*d) - (b^2*Tanh[c + d*x]^6)/(6*d)
```

Rubi [A] time = 0.10963, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 446, 77}

$$-\frac{b(2a+b)\tanh^4(c+dx)}{4d}-\frac{(a+b)^2\tanh^2(c+dx)}{2d}+\frac{(a+b)^2\log(\cosh(c+dx))}{d}-\frac{b^2\tanh^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```


```
[Out] ((a + b)^2*Log[Cosh[c + d*x]])/d - ((a + b)^2*Tanh[c + d*x]^2)/(2*d) - (b*(
2*a + b)*Tanh[c + d*x]^4)/(4*d) - (b^2*Tanh[c + d*x]^6)/(6*d)
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{x^3\left(a+bx^2\right)^2}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{x(a+bx)^2}{1-x}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(-(a+b)^2 - \frac{(a+b)^2}{-1+x} - b(2a+b)x - b^2x^2\right)dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{(a+b)^2\log(\cosh(c+dx))}{d} - \frac{(a+b)^2\tanh^2(c+dx)}{2d} - \frac{b(2a+b)\tanh^4(c+}{4d}$$

Mathematica [A] time = 0.407834, size = 66, normalized size = 0.87

$$-\frac{3b(2a+b)\tanh^4(c+dx)+6(a+b)^2\tanh^2(c+dx)-12(a+b)^2\log(\cosh(c+dx))+2b^2\tanh^6(c+dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -(-12*(a + b)^2*Log[Cosh[c + d*x]] + 6*(a + b)^2*Tanh[c + d*x]^2 + 3*b*(2*a
 + b)*Tanh[c + d*x]^4 + 2*b^2*Tanh[c + d*x]^6)/(12*d)
```

Maple [B] time = 0.004, size = 196, normalized size = 2.6

$$-\frac{\ln(\tanh(dx+c)+1)a^2}{2d} - \frac{\ln(\tanh(dx+c)+1)ab}{d} - \frac{\ln(\tanh(dx+c)+1)b^2}{2d} - \frac{a^2\ln(\tanh(dx+c)-1)}{2d} - \frac{\ln(\tanh}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] -1/2/d*ln(tanh(d*x+c)+1)*a^2-1/d*ln(tanh(d*x+c)+1)*a*b-1/2/d*ln(tanh(d*x+c)
+1)*b^2-1/2*a^2/d*ln(tanh(d*x+c)-1)-1/d*ln(tanh(d*x+c)-1)*a*b-1/2/d*ln(tanh
(d*x+c)-1)*b^2-1/2/d*tanh(d*x+c)^2*a^2-1/2*b^2*tanh(d*x+c)^2/d-1/4/d*tanh(d
*x+c)^4*b^2-1/6*b^2*tanh(d*x+c)^6/d-a*b*tanh(d*x+c)^2/d-1/2/d*tanh(d*x+c)^4
*a*b
```

Maxima [B] time = 1.78954, size = 450, normalized size = 5.92

$$\frac{1}{3}b^2\left(3x+\frac{3c}{d}+\frac{3\log\left(e^{(-2dx-2c)}+1\right)}{d}+\frac{2\left(9e^{(-2dx-2c)}+18e^{(-4dx-4c)}+34e^{(-6dx-6c)}+18e^{(-8dx-8c)}+9e^{(-10dx}\right.}{d\left(6e^{(-2dx-2c)}+15e^{(-4dx-4c)}+20e^{(-6dx-6c)}+15e^{(-8dx-8c)}+6e^{(-10dx-10c)}+e^{(}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*(3*x + 3*c/d + 3*log(e^(-2*d*x - 2*c) + 1)/d + 2*(9*e^(-2*d*x - 2*c
) + 18*e^(-4*d*x - 4*c) + 34*e^(-6*d*x - 6*c) + 18*e^(-8*d*x - 8*c) + 9*e^(
-10*d*x - 10*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*
```

```
x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c)
+ 1))) + 2*a*b*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 4*(e^(-2*d*x - 2*c)
 + e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*
x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) + a^2*(x + c/d + lo
g(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(
-4*d*x - 4*c) + 1)))
```

Fricas [B] time = 2.34171, size = 8982, normalized size = 118.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^12 + 36*(a^2 + 2*a*b + b^2)*d
*x*cosh(d*x + c)*sinh(d*x + c)^11 + 3*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)
^12 + 6*(3*(a^2 + 2*a*b + b^2)*d*x - a^2 - 4*a*b - 3*b^2)*cosh(d*x + c)^10
+ 6*(33*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^2 + 3*(a^2 + 2*a*b + b^2)*d*x
 - a^2 - 4*a*b - 3*b^2)*sinh(d*x + c)^10 + 60*(11*(a^2 + 2*a*b + b^2)*d*x*c
osh(d*x + c)^3 + (3*(a^2 + 2*a*b + b^2)*d*x - a^2 - 4*a*b - 3*b^2)*cosh(d*x
 + c))*sinh(d*x + c)^9 + 3*(15*(a^2 + 2*a*b + b^2)*d*x - 8*a^2 - 24*a*b - 1
2*b^2)*cosh(d*x + c)^8 + 3*(495*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + 1
5*(a^2 + 2*a*b + b^2)*d*x + 90*(3*(a^2 + 2*a*b + b^2)*d*x - a^2 - 4*a*b - 3
*b^2)*cosh(d*x + c)^2 - 8*a^2 - 24*a*b - 12*b^2)*sinh(d*x + c)^8 + 24*(99*(
a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^5 + 30*(3*(a^2 + 2*a*b + b^2)*d*x - a^
2 - 4*a*b - 3*b^2)*cosh(d*x + c)^3 + (15*(a^2 + 2*a*b + b^2)*d*x - 8*a^2 -
24*a*b - 12*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 4*(15*(a^2 + 2*a*b + b^2)
*d*x - 9*a^2 - 24*a*b - 17*b^2)*cosh(d*x + c)^6 + 4*(693*(a^2 + 2*a*b + b^2
)*d*x*cosh(d*x + c)^6 + 315*(3*(a^2 + 2*a*b + b^2)*d*x - a^2 - 4*a*b - 3*b^
2)*cosh(d*x + c)^4 + 15*(a^2 + 2*a*b + b^2)*d*x + 21*(15*(a^2 + 2*a*b + b^2
)*d*x - 8*a^2 - 24*a*b - 12*b^2)*cosh(d*x + c)^2 - 9*a^2 - 24*a*b - 17*b^2)
*sinh(d*x + c)^6 + 24*(99*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^7 + 63*(3*(
a^2 + 2*a*b + b^2)*d*x - a^2 - 4*a*b - 3*b^2)*cosh(d*x + c)^5 + 7*(15*(a^2
+ 2*a*b + b^2)*d*x - 8*a^2 - 24*a*b - 12*b^2)*cosh(d*x + c)^3 + (15*(a^2 +
2*a*b + b^2)*d*x - 9*a^2 - 24*a*b - 17*b^2)*cosh(d*x + c))*sinh(d*x + c)^5
+ 3*(15*(a^2 + 2*a*b + b^2)*d*x - 8*a^2 - 24*a*b - 12*b^2)*cosh(d*x + c)^4
+ 3*(495*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^8 + 420*(3*(a^2 + 2*a*b + b^
2)*d*x - a^2 - 4*a*b - 3*b^2)*cosh(d*x + c)^6 + 70*(15*(a^2 + 2*a*b + b^2)*
d*x - 8*a^2 - 24*a*b - 12*b^2)*cosh(d*x + c)^4 + 15*(a^2 + 2*a*b + b^2)*d*x
 + 20*(15*(a^2 + 2*a*b + b^2)*d*x - 9*a^2 - 24*a*b - 17*b^2)*cosh(d*x + c)^
2 - 8*a^2 - 24*a*b - 12*b^2)*sinh(d*x + c)^4 + 4*(165*(a^2 + 2*a*b + b^2)*d
*x*cosh(d*x + c)^9 + 180*(3*(a^2 + 2*a*b + b^2)*d*x - a^2 - 4*a*b - 3*b^2)*
cosh(d*x + c)^7 + 42*(15*(a^2 + 2*a*b + b^2)*d*x - 8*a^2 - 24*a*b - 12*b^2)
*cosh(d*x + c)^5 + 20*(15*(a^2 + 2*a*b + b^2)*d*x - 9*a^2 - 24*a*b - 17*b^2
)*cosh(d*x + c)^3 + 3*(15*(a^2 + 2*a*b + b^2)*d*x - 8*a^2 - 24*a*b - 12*b^2
)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*d*x + 6*(3*(a^2 +
2*a*b + b^2)*d*x - a^2 - 4*a*b - 3*b^2)*cosh(d*x + c)^2 + 6*(33*(a^2 + 2*a*
b + b^2)*d*x*cosh(d*x + c)^10 + 45*(3*(a^2 + 2*a*b + b^2)*d*x - a^2 - 4*a*b
 - 3*b^2)*cosh(d*x + c)^8 + 14*(15*(a^2 + 2*a*b + b^2)*d*x - 8*a^2 - 24*a*b
 - 12*b^2)*cosh(d*x + c)^6 + 10*(15*(a^2 + 2*a*b + b^2)*d*x - 9*a^2 - 24*a*
b - 17*b^2)*cosh(d*x + c)^4 + 3*(a^2 + 2*a*b + b^2)*d*x + 3*(15*(a^2 + 2*a*
b + b^2)*d*x - 8*a^2 - 24*a*b - 12*b^2)*cosh(d*x + c)^2 - a^2 - 4*a*b - 3*b
^2)*sinh(d*x + c)^2 - 3*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^12 + 12*(a^2 + 2
*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^11 + (a^2 + 2*a*b + b^2)*sinh(d*x +
 c)^12 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^10 + 6*(11*(a^2 + 2*a*b + b^2)
*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^10 + 20*(11*(a^2 + 2*a*
```

```
b + b^2)*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x +
c)^9 + 15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 15*(33*(a^2 + 2*a*b + b^2)*
cosh(d*x + c)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^
2)*sinh(d*x + c)^8 + 24*(33*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 30*(a^2 +
 2*a*b + b^2)*cosh(d*x + c)^3 + 5*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d
*x + c)^7 + 20*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 4*(231*(a^2 + 2*a*b +
b^2)*cosh(d*x + c)^6 + 315*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 105*(a^2 +
 2*a*b + b^2)*cosh(d*x + c)^2 + 5*a^2 + 10*a*b + 5*b^2)*sinh(d*x + c)^6 + 2
4*(33*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 63*(a^2 + 2*a*b + b^2)*cosh(d*x
 + c)^5 + 35*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 5*(a^2 + 2*a*b + b^2)*co
sh(d*x + c))*sinh(d*x + c)^5 + 15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 15*
(33*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 84*(a^2 + 2*a*b + b^2)*cosh(d*x +
 c)^6 + 70*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 20*(a^2 + 2*a*b + b^2)*cos
h(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 20*(11*(a^2 + 2*a*b + b
^2)*cosh(d*x + c)^9 + 36*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 42*(a^2 + 2*
a*b + b^2)*cosh(d*x + c)^5 + 20*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^
2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(a^2 + 2*a*b + b^2)*cos
h(d*x + c)^2 + 6*(11*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^10 + 45*(a^2 + 2*a*b
 + b^2)*cosh(d*x + c)^8 + 70*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 50*(a^2
+ 2*a*b + b^2)*cosh(d*x + c)^4 + 15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a
^2 + 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 12*((a^2 + 2*a*b +
b^2)*cosh(d*x + c)^11 + 5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^9 + 10*(a^2 + 2
*a*b + b^2)*cosh(d*x + c)^7 + 10*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 5*(a
^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh
(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 12*(3*(a^
2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^11 + 5*(3*(a^2 + 2*a*b + b^2)*d*x - a^2
- 4*a*b - 3*b^2)*cosh(d*x + c)^9 + 2*(15*(a^2 + 2*a*b + b^2)*d*x - 8*a^2 -
24*a*b - 12*b^2)*cosh(d*x + c)^7 + 2*(15*(a^2 + 2*a*b + b^2)*d*x - 9*a^2 -
24*a*b - 17*b^2)*cosh(d*x + c)^5 + (15*(a^2 + 2*a*b + b^2)*d*x - 8*a^2 - 24
*a*b - 12*b^2)*cosh(d*x + c)^3 + (3*(a^2 + 2*a*b + b^2)*d*x - a^2 - 4*a*b -
 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^12 + 12*d*cosh(d*x +
 c)*sinh(d*x + c)^11 + d*sinh(d*x + c)^12 + 6*d*cosh(d*x + c)^10 + 6*(11*d*
cosh(d*x + c)^2 + d)*sinh(d*x + c)^10 + 20*(11*d*cosh(d*x + c)^3 + 3*d*cosh
(d*x + c))*sinh(d*x + c)^9 + 15*d*cosh(d*x + c)^8 + 15*(33*d*cosh(d*x + c)^
4 + 18*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 24*(33*d*cosh(d*x + c)^5 +
30*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^7 + 20*d*cosh(d*x +
 c)^6 + 4*(231*d*cosh(d*x + c)^6 + 315*d*cosh(d*x + c)^4 + 105*d*cosh(d*x +
 c)^2 + 5*d)*sinh(d*x + c)^6 + 24*(33*d*cosh(d*x + c)^7 + 63*d*cosh(d*x + c
)^5 + 35*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^5 + 15*d*cosh
(d*x + c)^4 + 15*(33*d*cosh(d*x + c)^8 + 84*d*cosh(d*x + c)^6 + 70*d*cosh(d
*x + c)^4 + 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 20*(11*d*cosh(d*x +
 c)^9 + 36*d*cosh(d*x + c)^7 + 42*d*cosh(d*x + c)^5 + 20*d*cosh(d*x + c)^3
+ 3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 6*d*cosh(d*x + c)^2 + 6*(11*d*cosh(d
*x + c)^10 + 45*d*cosh(d*x + c)^8 + 70*d*cosh(d*x + c)^6 + 50*d*cosh(d*x +
c)^4 + 15*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 12*(d*cosh(d*x + c)^11 +
 5*d*cosh(d*x + c)^9 + 10*d*cosh(d*x + c)^7 + 10*d*cosh(d*x + c)^5 + 5*d*co
sh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [A] time = 1.21659, size = 170, normalized size = 2.24

$$\begin{cases} a^2x - \frac{a^2\log(\tanh(c+dx)+1)}{d} - \frac{a^2\tanh^2(c+dx)}{2d} + 2abx - \frac{2ab\log(\tanh(c+dx)+1)}{d} - \frac{ab\tanh^4(c+dx)}{2d} - \frac{ab\tanh^2(c+dx)}{d} + b^2x - \frac{b^2\log(\tanh(c}{d} \\ x\left(a+b\tanh^2(c)\right)^2\tanh^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**3*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((a**2*x - a**2*log(tanh(c + d*x) + 1)/d - a**2*tanh(c + d*x)**2/(
2*d) + 2*a*b*x - 2*a*b*log(tanh(c + d*x) + 1)/d - a*b*tanh(c + d*x)**4/(2*d
) - a*b*tanh(c + d*x)**2/d + b**2*x - b**2*log(tanh(c + d*x) + 1)/d - b**2*
tanh(c + d*x)**6/(6*d) - b**2*tanh(c + d*x)**4/(4*d) - b**2*tanh(c + d*x)**
2/(2*d), Ne(d, 0)), (x*(a + b*tanh(c)**2)**2*tanh(c)**3, True))
```

Giac [B] time = 1.30806, size = 262, normalized size = 3.45

$$-\frac{\left(a^2+2\,ab+b^2\right)(dx+c)}{d}+\frac{\left(a^2+2\,ab+b^2\right)\log\left(e^{(2\,dx+2\,c)}+1\right)}{d}+\frac{2\left(3\left(a^2+4\,ab+3\,b^2\right)e^{(10\,dx+10\,c)}+6\left(2\,a^2+6\,ab+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -(a^2 + 2*a*b + b^2)*(d*x + c)/d + (a^2 + 2*a*b + b^2)*log(e^(2*d*x + 2*c)
+ 1)/d + 2/3*(3*(a^2 + 4*a*b + 3*b^2)*e^(10*d*x + 10*c) + 6*(2*a^2 + 6*a*b
+ 3*b^2)*e^(8*d*x + 8*c) + 2*(9*a^2 + 24*a*b + 17*b^2)*e^(6*d*x + 6*c) + 6*
(2*a^2 + 6*a*b + 3*b^2)*e^(4*d*x + 4*c) + 3*(a^2 + 4*a*b + 3*b^2)*e^(2*d*x
+ 2*c))/(d*(e^(2*d*x + 2*c) + 1)^6)
```

3.146 $\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=63

$$-\frac{b(2a+b)\tanh^3(c+dx)}{3d}-\frac{(a+b)^2\tanh(c+dx)}{d}+x(a+b)^2-\frac{b^2\tanh^5(c+dx)}{5d}$$

```
[Out] (a + b)^2*x - ((a + b)^2*Tanh[c + d*x])/d - (b*(2*a + b)*Tanh[c + d*x]^3)/(
3*d) - (b^2*Tanh[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0755494, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 461, 206}

$$-\frac{b(2a+b)\tanh^3(c+dx)}{3d}-\frac{(a+b)^2\tanh(c+dx)}{d}+x(a+b)^2-\frac{b^2\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (a + b)^2*x - ((a + b)^2*Tanh[c + d*x])/d - (b*(2*a + b)*Tanh[c + d*x]^3)/(
3*d) - (b^2*Tanh[c + d*x]^5)/(5*d)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+bx^2\right)^2}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-(a+b)^2 - b(2a+b)x^2 - b^2x^4 + \frac{a^2+2ab+b^2}{1-x^2}\right)dx, x, \tanh(c+\right.}{d}$$

$$= -\frac{(a+b)^2\tanh(c+dx)}{d} - \frac{b(2a+b)\tanh^3(c+dx)}{3d} - \frac{b^2\tanh^5(c+dx)}{5d}$$

$$= (a+b)^2x - \frac{(a+b)^2\tanh(c+dx)}{d} - \frac{b(2a+b)\tanh^3(c+dx)}{3d} - \frac{b^2\tanh}{5}$$

Mathematica [B] time = 0.0478517, size = 137, normalized size = 2.17

$$\frac{a^2\tanh^{-1}(\tanh(c+dx))}{d} - \frac{a^2\tanh(c+dx)}{d} - \frac{2ab\tanh^3(c+dx)}{3d} + \frac{2ab\tanh^{-1}(\tanh(c+dx))}{d} - \frac{2ab\tanh(c+dx)}{d} -$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (a^2*ArcTanh[Tanh[c + d*x]])/d + (2*a*b*ArcTanh[Tanh[c + d*x]])/d + (b^2*Ar
cTanh[Tanh[c + d*x]])/d - (a^2*Tanh[c + d*x])/d - (2*a*b*Tanh[c + d*x])/d -
 (b^2*Tanh[c + d*x])/d - (2*a*b*Tanh[c + d*x]^3)/(3*d) - (b^2*Tanh[c + d*x]
^3)/(3*d) - (b^2*Tanh[c + d*x]^5)/(5*d)
```

Maple [B] time = 0.005, size = 189, normalized size = 3.

$$-\frac{a^2\ln(\tanh(dx+c)-1)}{2d} - \frac{\ln(\tanh(dx+c)-1)ab}{d} - \frac{\ln(\tanh(dx+c)-1)b^2}{2d} - \frac{b^2(\tanh(dx+c))^3}{3d} - \frac{b^2(\tanh(d}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] -1/2*a^2/d*ln(tanh(d*x+c)-1)-1/d*ln(tanh(d*x+c)-1)*a*b-1/2/d*ln(tanh(d*x+c)
-1)*b^2-1/3*b^2*tanh(d*x+c)^3/d-1/5*b^2*tanh(d*x+c)^5/d-2*a*b*tanh(d*x+c)/d
-2/3*a*b*tanh(d*x+c)^3/d+1/2/d*ln(tanh(d*x+c)+1)*a^2+1/d*ln(tanh(d*x+c)+1)*
a*b+1/2/d*ln(tanh(d*x+c)+1)*b^2-a^2*tanh(d*x+c)/d-b^2*tanh(d*x+c)/d
```

Maxima [B] time = 1.16304, size = 312, normalized size = 4.95

$$\frac{1}{15}b^2\left(15x + \frac{15c}{d} - \frac{2\left(70e^{(-2dx-2c)} + 140e^{(-4dx-4c)} + 90e^{(-6dx-6c)} + 45e^{(-8dx-8c)} + 23\right)}{d\left(5e^{(-2dx-2c)} + 10e^{(-4dx-4c)} + 10e^{(-6dx-6c)} + 5e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1\right)}\right) + \frac{2}{3}ab\left(3x + \frac{3c}{d} -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*b^2*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 9
0*e^(-6*d*x - 6*c) + 45*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*
```

```
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
10*c) + 1))) + 2/3*a*b*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x -
 4*c) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) +
 1))) + a^2*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1)))
```

Fricas [B] time = 2.09198, size = 1245, normalized size = 19.76

$$\frac{\left(15\left(a^2+2ab+b^2\right)dx+15a^2+40ab+23b^2\right)\cosh(dx+c)^5+5\left(15\left(a^2+2ab+b^2\right)dx+15a^2+40ab+23b^2\right)\cosh(d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/15*((15*(a^2 + 2*a*b + b^2)*d*x + 15*a^2 + 40*a*b + 23*b^2)*cosh(d*x + c)
^5 + 5*(15*(a^2 + 2*a*b + b^2)*d*x + 15*a^2 + 40*a*b + 23*b^2)*cosh(d*x + c
)*sinh(d*x + c)^4 - (15*a^2 + 40*a*b + 23*b^2)*sinh(d*x + c)^5 + 5*(15*(a^2
 + 2*a*b + b^2)*d*x + 15*a^2 + 40*a*b + 23*b^2)*cosh(d*x + c)^3 - 5*(2*(15*
a^2 + 40*a*b + 23*b^2)*cosh(d*x + c)^2 + 9*a^2 + 16*a*b + 5*b^2)*sinh(d*x +
 c)^3 + 5*(2*(15*(a^2 + 2*a*b + b^2)*d*x + 15*a^2 + 40*a*b + 23*b^2)*cosh(d
*x + c)^3 + 3*(15*(a^2 + 2*a*b + b^2)*d*x + 15*a^2 + 40*a*b + 23*b^2)*cosh(
d*x + c))*sinh(d*x + c)^2 + 10*(15*(a^2 + 2*a*b + b^2)*d*x + 15*a^2 + 40*a*
b + 23*b^2)*cosh(d*x + c) - 5*((15*a^2 + 40*a*b + 23*b^2)*cosh(d*x + c)^4 +
 3*(9*a^2 + 16*a*b + 5*b^2)*cosh(d*x + c)^2 + 6*a^2 + 8*a*b + 10*b^2)*sinh(
d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cosh
(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 +
 10*d*cosh(d*x + c))
```

Sympy [A] time = 0.919852, size = 117, normalized size = 1.86

$$\begin{cases} a^2x - \frac{a^2\tanh(c+dx)}{d} + 2abx - \frac{2ab\tanh^3(c+dx)}{3d} - \frac{2ab\tanh(c+dx)}{d} + b^2x - \frac{b^2\tanh^5(c+dx)}{5d} - \frac{b^2\tanh^3(c+dx)}{3d} - \frac{b^2\tanh(c+dx)}{d} & \text{for } d \neq (\\ x\left(a+b\tanh^2(c)\right)^2\tanh^2(c) & \text{otherwi} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**2*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((a**2*x - a**2*tanh(c + d*x)/d + 2*a*b*x - 2*a*b*tanh(c + d*x)**3
/(3*d) - 2*a*b*tanh(c + d*x)/d + b**2*x - b**2*tanh(c + d*x)**5/(5*d) - b**
2*tanh(c + d*x)**3/(3*d) - b**2*tanh(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(
c)**2)**2*tanh(c)**2, True))
```

Giac [B] time = 1.2481, size = 294, normalized size = 4.67

$$\frac{\left(a^2+2ab+b^2\right)(dx+c)}{d}+\frac{2\left(15a^2e^{(8dx+8c)}+60abe^{(8dx+8c)}+45b^2e^{(8dx+8c)}+60a^2e^{(6dx+6c)}+180abe^{(6dx+6c)}+90b^2e^{(6}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] (a^2 + 2*a*b + b^2)*(d*x + c)/d + 2/15*(15*a^2*e^(8*d*x + 8*c) + 60*a*b*e^(
8*d*x + 8*c) + 45*b^2*e^(8*d*x + 8*c) + 60*a^2*e^(6*d*x + 6*c) + 180*a*b*e^
(6*d*x + 6*c) + 90*b^2*e^(6*d*x + 6*c) + 90*a^2*e^(4*d*x + 4*c) + 220*a*b*e
^(4*d*x + 4*c) + 140*b^2*e^(4*d*x + 4*c) + 60*a^2*e^(2*d*x + 2*c) + 140*a*b
*e^(2*d*x + 2*c) + 70*b^2*e^(2*d*x + 2*c) + 15*a^2 + 40*a*b + 23*b^2)/(d*(e
^(2*d*x + 2*c) + 1)^5)
```

3.147 $\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$

Optimal. Leaf size=57

$$-\frac{b(a+b)\tanh^2(c+dx)}{2d}-\frac{\left(a+b\tanh^2(c+dx)\right)^2}{4d}+\frac{(a+b)^2\log(\cosh(c+dx))}{d}$$

```
[Out] ((a + b)^2*Log[Cosh[c + d*x]])/d - (b*(a + b)*Tanh[c + d*x]^2)/(2*d) - (a +
 b*Tanh[c + d*x]^2)^2/(4*d)
```

Rubi [A] time = 0.0733014, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3670, 444, 43}

$$-\frac{b(a+b)\tanh^2(c+dx)}{2d}-\frac{\left(a+b\tanh^2(c+dx)\right)^2}{4d}+\frac{(a+b)^2\log(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((a + b)^2*Log[Cosh[c + d*x]])/d - (b*(a + b)*Tanh[c + d*x]^2)/(2*d) - (a +
 b*Tanh[c + d*x]^2)^2/(4*d)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{x(a+bx^2)^2}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{1-x}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(-b(a+b)+\frac{(a+b)^2}{1-x}-b(a+bx)\right)dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{(a+b)^2\log(\cosh(c+dx))}{d} - \frac{b(a+b)\tanh^2(c+dx)}{2d} - \frac{\left(a+b\tanh^2(c+\cdots\right.}{4d}$$

Mathematica [A] time = 0.306984, size = 50, normalized size = 0.88

$$-\frac{2b(2a+b)\tanh^2(c+dx)-4(a+b)^2\log(\cosh(c+dx))+b^2\tanh^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -(-4*(a + b)^2*Log[Cosh[c + d*x]] + 2*b*(2*a + b)*Tanh[c + d*x]^2 + b^2*Tan
h[c + d*x]^4)/(4*d)
```

Maple [B] time = 0.004, size = 149, normalized size = 2.6

$$-\frac{(\tanh(dx+c))^4 b^2}{4d} - \frac{ab(\tanh(dx+c))^2}{d} - \frac{b^2(\tanh(dx+c))^2}{2d} - \frac{a^2\ln(\tanh(dx+c)-1)}{2d} - \frac{\ln(\tanh(dx+c)-1)\cdots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] -1/4/d*tanh(d*x+c)^4*b^2-a*b*tanh(d*x+c)^2/d-1/2*b^2*tanh(d*x+c)^2/d-1/2*a^
2/d*ln(tanh(d*x+c)-1)-1/d*ln(tanh(d*x+c)-1)*a*b-1/2/d*ln(tanh(d*x+c)-1)*b^2
-1/2/d*ln(tanh(d*x+c)+1)*a^2-1/d*ln(tanh(d*x+c)+1)*a*b-1/2/d*ln(tanh(d*x+c)
+1)*b^2
```

Maxima [B] time = 1.72274, size = 251, normalized size = 4.4

$$b^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2dx-2c)}+1\right)}{d}+\frac{4\left(e^{(-2dx-2c)}+e^{(-4dx-4c)}+e^{(-6dx-6c)}\right)}{d\left(4e^{(-2dx-2c)}+6e^{(-4dx-4c)}+4e^{(-6dx-6c)}+e^{(-8dx-8c)}+1\right)}\right)+2ab\left(x+\frac{c}{d}+\frac{\log\left(e^{(\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] b^2*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 4*(e^(-2*d*x - 2*c) + e^(-4*d*
x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) +
4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) + 2*a*b*(x + c/d + log(e^(-2*d
```

```
*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x -
4*c) + 1))) + a^2*log(cosh(d*x + c))/d
```

Fricas [B] time = 2.16579, size = 4165, normalized size = 73.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -((a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*d*x*cosh(
d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^8 + 4*((a^
2 + 2*a*b + b^2)*d*x - a*b - b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 + 2*a*b + b^2
)*d*x*cosh(d*x + c)^2 + (a^2 + 2*a*b + b^2)*d*x - a*b - b^2)*sinh(d*x + c)^
6 + 8*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 + 3*((a^2 + 2*a*b + b^2)*d
*x - a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*(a^2 + 2*a*b + b^2)*d
*x - 4*a*b - 2*b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*d*x*cosh(d*
x + c)^4 + 3*(a^2 + 2*a*b + b^2)*d*x + 30*((a^2 + 2*a*b + b^2)*d*x - a*b -
b^2)*cosh(d*x + c)^2 - 4*a*b - 2*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b +
 b^2)*d*x*cosh(d*x + c)^5 + 10*((a^2 + 2*a*b + b^2)*d*x - a*b - b^2)*cosh(d
*x + c)^3 + (3*(a^2 + 2*a*b + b^2)*d*x - 4*a*b - 2*b^2)*cosh(d*x + c))*sinh
(d*x + c)^3 + (a^2 + 2*a*b + b^2)*d*x + 4*((a^2 + 2*a*b + b^2)*d*x - a*b -
b^2)*cosh(d*x + c)^2 + 4*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^6 + 15*((
a^2 + 2*a*b + b^2)*d*x - a*b - b^2)*cosh(d*x + c)^4 + (a^2 + 2*a*b + b^2)*d
*x + 3*(3*(a^2 + 2*a*b + b^2)*d*x - 4*a*b - 2*b^2)*cosh(d*x + c)^2 - a*b -
b^2)*sinh(d*x + c)^2 - ((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*
b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^
8 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d
*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(
a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x + c
)^4 + 30*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 3*a^2 + 6*a*b + 3*b^2)*sinh(
d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 10*(a^2 + 2*a*b + b
^2)*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3
+ 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x
 + c)^6 + 15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 9*(a^2 + 2*a*b + b^2)*co
sh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 8*
((a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^
5 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*cosh(d*x +
c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 8
*((a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^7 + 3*((a^2 + 2*a*b + b^2)*d*x - a*
b - b^2)*cosh(d*x + c)^5 + (3*(a^2 + 2*a*b + b^2)*d*x - 4*a*b - 2*b^2)*cosh
(d*x + c)^3 + ((a^2 + 2*a*b + b^2)*d*x - a*b - b^2)*cosh(d*x + c))*sinh(d*x
 + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x
+ c)^8 + 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6
+ 8*(7*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*
x + c)^4 + 2*(35*d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x +
 c)^4 + 8*(7*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*
sinh(d*x + c)^3 + 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 15*d*cosh(
d*x + c)^4 + 9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^
7 + 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x +
 c) + d)
```

Sympy [A] time = 0.704204, size = 122, normalized size = 2.14

$$\begin{cases} a^2x - \frac{a^2\log(\tanh(c+dx)+1)}{d} + 2abx - \frac{2ab\log(\tanh(c+dx)+1)}{d} - \frac{ab\tanh^2(c+dx)}{d} + b^2x - \frac{b^2\log(\tanh(c+dx)+1)}{d} - \frac{b^2\tanh^4(c+dx)}{4d} - \frac{b^2 t}{} \\ x\left(a + b\tanh^2(c)\right)^2\tanh(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((a**2*x - a**2*log(tanh(c + d*x) + 1)/d + 2*a*b*x - 2*a*b*log(tan
h(c + d*x) + 1)/d - a*b*tanh(c + d*x)**2/d + b**2*x - b**2*log(tanh(c + d*x
) + 1)/d - b**2*tanh(c + d*x)**4/(4*d) - b**2*tanh(c + d*x)**2/(2*d), Ne(d,
 0)), (x*(a + b*tanh(c)**2)**2*tanh(c), True))
```

Giac [B] time = 1.26775, size = 162, normalized size = 2.84

$$-\frac{(a^2 + 2ab + b^2)(dx + c)}{d} + \frac{(a^2 + 2ab + b^2)\log\left(e^{(2dx+2c)} + 1\right)}{d} + \frac{4\left((ab + b^2)e^{(6dx+6c)} + (2ab + b^2)e^{(4dx+4c)} + (ab + \right.}{d\left(e^{(2dx+2c)} + 1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -(a^2 + 2*a*b + b^2)*(d*x + c)/d + (a^2 + 2*a*b + b^2)*log(e^(2*d*x + 2*c)
+ 1)/d + 4*((a*b + b^2)*e^(6*d*x + 6*c) + (2*a*b + b^2)*e^(4*d*x + 4*c) + (
a*b + b^2)*e^(2*d*x + 2*c))/(d*(e^(2*d*x + 2*c) + 1)^4)
```

3.148 $\int \left(a+b\tanh^2(c+dx)\right)^2 dx$

Optimal. Leaf size=43

$$-\frac{b(2a+b)\tanh(c+dx)}{d}+x(a+b)^2-\frac{b^2\tanh^3(c+dx)}{3d}$$

```
[Out] (a + b)^2*x - (b*(2*a + b)*Tanh[c + d*x])/d - (b^2*Tanh[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0314103, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3661, 390, 206}

$$-\frac{b(2a+b)\tanh(c+dx)}{d}+x(a+b)^2-\frac{b^2\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (a + b)^2*x - (b*(2*a + b)*Tanh[c + d*x])/d - (b^2*Tanh[c + d*x]^3)/(3*d)
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\tanh^2(c+dx)\right)^2 dx &= \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{1-x^2}\,dx,x,\tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(-b(2a+b)-b^2x^2+\frac{(a+b)^2}{1-x^2}\right)dx,x,\tanh(c+dx)\right)}{d}\\
&= -\frac{b(2a+b)\tanh(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}+\frac{(a+b)^2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\tanh(c+dx\right)}{d}\\
&= (a+b)^2x-\frac{b(2a+b)\tanh(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.706642, size = 65, normalized size = 1.51

$$\frac{\tanh(c+dx)\left(\frac{3(a+b)^2\tanh^{-1}\left(\sqrt{\tanh^2(c+dx)}\right)}{\sqrt{\tanh^2(c+dx)}}-b\left(6a+b\left(\tanh^2(c+dx)+3\right)\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (Tanh[c + d*x]*((3*(a + b)^2*ArcTanh[Sqrt[Tanh[c + d*x]^2]])/Sqrt[Tanh[c +
d*x]^2] - b*(6*a + b*(3 + Tanh[c + d*x]^2))))/(3*d)
```

Maple [B] time = 0.006, size = 144, normalized size = 3.4

$$-\frac{b^2(\tanh(dx+c))^3}{3d}-2\frac{ab\tanh(dx+c)}{d}-\frac{b^2\tanh(dx+c)}{d}-\frac{a^2\ln(\tanh(dx+c)-1)}{2d}-\frac{\ln(\tanh(dx+c)-1)ab}{d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] -1/3*b^2*tanh(d*x+c)^3/d-2*a*b*tanh(d*x+c)/d-b^2*tanh(d*x+c)/d-1/2*a^2/d*ln
(tanh(d*x+c)-1)-1/d*ln(tanh(d*x+c)-1)*a*b-1/2/d*ln(tanh(d*x+c)-1)*b^2+1/2/d
*ln(tanh(d*x+c)+1)*a^2+1/d*ln(tanh(d*x+c)+1)*a*b+1/2/d*ln(tanh(d*x+c)+1)*b^
2
```

Maxima [B] time = 1.05768, size = 154, normalized size = 3.58

$$\frac{1}{3}b^2\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+2\right)}{d\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1\right)}\right)+2ab\left(x+\frac{c}{d}-\frac{2}{d\left(e^{(-2dx-2c)}+1\right)}\right)+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(
3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 2*a*b*(
x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a^2*x
```

Fricas [B] time = 1.84934, size = 509, normalized size = 11.84

$$\frac{\left(3\left(a^2+2ab+b^2\right)dx+6ab+4b^2\right)\cosh(dx+c)^3+3\left(3\left(a^2+2ab+b^2\right)dx+6ab+4b^2\right)\cosh(dx+c)\sinh(dx+c)}{3\left(d\cosh(dx+c)^3+3dc\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*((3*(a^2 + 2*a*b + b^2)*d*x + 6*a*b + 4*b^2)*cosh(d*x + c)^3 + 3*(3*(a^
2 + 2*a*b + b^2)*d*x + 6*a*b + 4*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 - 2*(3*
a*b + 2*b^2)*sinh(d*x + c)^3 + 3*(3*(a^2 + 2*a*b + b^2)*d*x + 6*a*b + 4*b^2
)*cosh(d*x + c) - 6*((3*a*b + 2*b^2)*cosh(d*x + c)^2 + a*b)*sinh(d*x + c))/
(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))
```

Sympy [A] time = 0.419502, size = 68, normalized size = 1.58

$$\begin{cases} a^2x + 2abx - \frac{2ab\tanh(c+dx)}{d} + b^2x - \frac{b^2\tanh^3(c+dx)}{3d} - \frac{b^2\tanh(c+dx)}{d} & \text{for } d \neq 0 \\ x\left(a + b\tanh^2(c)\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*x - 2*a*b*tanh(c + d*x)/d + b**2*x - b**2*tanh(c
+ d*x)**3/(3*d) - b**2*tanh(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c)**2)**2
, True))
```

Giac [B] time = 1.19123, size = 139, normalized size = 3.23

$$\frac{\left(a^2 + 2\,ab + b^2\right)(dx + c)}{d} + \frac{4\left(3\,abe^{(4\,dx+4\,c)} + 3\,b^2e^{(4\,dx+4\,c)} + 6\,abe^{(2\,dx+2\,c)} + 3\,b^2e^{(2\,dx+2\,c)} + 3\,ab + 2\,b^2\right)}{3\,d\left(e^{(2\,dx+2\,c)} + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] (a^2 + 2*a*b + b^2)*(d*x + c)/d + 4/3*(3*a*b*e^(4*d*x + 4*c) + 3*b^2*e^(4*d
*x + 4*c) + 6*a*b*e^(2*d*x + 2*c) + 3*b^2*e^(2*d*x + 2*c) + 3*a*b + 2*b^2)/
(d*(e^(2*d*x + 2*c) + 1)^3)
```

3.149 $\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=49

$$\frac{a^2\log(\tanh(c+dx))}{d}+\frac{(a+b)^2\log(\cosh(c+dx))}{d}-\frac{b^2\tanh^2(c+dx)}{2d}$$

```
[Out] ((a + b)^2*Log[Cosh[c + d*x]])/d + (a^2*Log[Tanh[c + d*x]])/d - (b^2*Tanh[c
 + d*x]^2)/(2*d)
```

Rubi [A] time = 0.0769667, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3670, 446, 72}

$$\frac{a^2\log(\tanh(c+dx))}{d}+\frac{(a+b)^2\log(\cosh(c+dx))}{d}-\frac{b^2\tanh^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((a + b)^2*Log[Cosh[c + d*x]])/d + (a^2*Log[Tanh[c + d*x]])/d - (b^2*Tanh[c
 + d*x]^2)/(2*d)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{x\left(1-x^2\right)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{(1-x)x}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(-b^2-\frac{(a+b)^2}{-1+x}+\frac{a^2}{x}\right)dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{(a+b)^2\log(\cosh(c+dx))}{d}+\frac{a^2\log(\tanh(c+dx))}{d}-\frac{b^2\tanh^2(c+dx)}{2d}$$

Mathematica [A] time = 0.125025, size = 48, normalized size = 0.98

$$\frac{2\left(a^2\log(\tanh(c+dx))+(a+b)^2\log(\cosh(c+dx))\right)-b^2\tanh^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (2*((a + b)^2*Log[Cosh[c + d*x]] + a^2*Log[Tanh[c + d*x]]) - b^2*Tanh[c + d
*x]^2)/(2*d)
```

Maple [A] time = 0.053, size = 60, normalized size = 1.2

$$\frac{a^2\ln(\sinh(dx+c))}{d}+2\frac{ab\ln(\cosh(dx+c))}{d}+\frac{b^2\ln(\cosh(dx+c))}{d}-\frac{b^2(\tanh(dx+c))^2}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*a^2*ln(sinh(d*x+c))+2*a*b*ln(cosh(d*x+c))/d+1/d*b^2*ln(cosh(d*x+c))-1/2
*b^2*tanh(d*x+c)^2/d
```

Maxima [B] time = 1.52864, size = 140, normalized size = 2.86

$$b^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1\right)}\right)+\frac{2\,ab\log\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{d}+\frac{a^2\log(\sinh(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] b^2*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2
*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + 2*a*b*log(e^(d*x + c) + e^(-d*x - c
))/d + a^2*log(sinh(d*x + c))/d
```

Fricas [B] time = 2.09311, size = 1715, normalized size = 35.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -((a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*d*x*cosh(
d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^4 + (a^2 +
 2*a*b + b^2)*d*x + 2*((a^2 + 2*a*b + b^2)*d*x - b^2)*cosh(d*x + c)^2 + 2*(
3*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^2 + (a^2 + 2*a*b + b^2)*d*x - b^2)*
sinh(d*x + c)^2 - ((2*a*b + b^2)*cosh(d*x + c)^4 + 4*(2*a*b + b^2)*cosh(d*x
 + c)*sinh(d*x + c)^3 + (2*a*b + b^2)*sinh(d*x + c)^4 + 2*(2*a*b + b^2)*cos
h(d*x + c)^2 + 2*(3*(2*a*b + b^2)*cosh(d*x + c)^2 + 2*a*b + b^2)*sinh(d*x +
 c)^2 + 2*a*b + b^2 + 4*((2*a*b + b^2)*cosh(d*x + c)^3 + (2*a*b + b^2)*cosh
(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c
))) - (a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh
(d*x + c)^4 + 2*a^2*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2)*sinh(
d*x + c)^2 + a^2 + 4*(a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c
))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*((a^2 + 2*a*b +
 b^2)*d*x*cosh(d*x + c)^3 + ((a^2 + 2*a*b + b^2)*d*x - b^2)*cosh(d*x + c))*
sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*s
inh(d*x + c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*sinh(d*x
 + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tanh^2(c + dx)\right)^2 \coth(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*coth(c + d*x), x)
```

Giac [B] time = 1.26452, size = 198, normalized size = 4.04

$$\frac{a^2\log\left(e^{(2\,dx+2\,c)} + e^{(-2\,dx-2\,c)} - 2\right)}{2\,d} + \frac{\left(2\,ab + b^2\right)\log\left(e^{(2\,dx+2\,c)} + e^{(-2\,dx-2\,c)} + 2\right)}{2\,d} - \frac{2\,ab\left(e^{(2\,dx+2\,c)} + e^{(-2\,dx-2\,c)}\right) + b^2\left(e^{(2\,d}\right.}{2\,d\left(e^{(2\,dx+2\,c)} + e^{(\cdot}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*a^2*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2)/d + 1/2*(2*a*b + b^2)*l
og(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2)/d - 1/2*(2*a*b*(e^(2*d*x + 2*c)
+ e^(-2*d*x - 2*c)) + b^2*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 4*a*b - 2*
b^2)/(d*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2))
```

3.150 $\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=36

$$-\frac{a^2\coth(c+dx)}{d}+x(a+b)^2-\frac{b^2\tanh(c+dx)}{d}$$

```
[Out] (a + b)^2*x - (a^2*Coth[c + d*x])/d - (b^2*Tanh[c + d*x])/d
```

Rubi [A] time = 0.0671894, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 461, 207}

$$-\frac{a^2\coth(c+dx)}{d}+x(a+b)^2-\frac{b^2\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (a + b)^2*x - (a^2*Coth[c + d*x])/d - (b^2*Tanh[c + d*x])/d
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{x^2(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-b^2+\frac{a^2}{x^2}-\frac{(a+b)^2}{-1+x^2}\right)dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{a^2\coth(c+dx)}{d}-\frac{b^2\tanh(c+dx)}{d}-\frac{(a+b)^2\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= (a+b)^2x-\frac{a^2\coth(c+dx)}{d}-\frac{b^2\tanh(c+dx)}{d}$$

Mathematica [C] time = 0.0959733, size = 64, normalized size = 1.78

$$-\frac{a^2\coth(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\tanh^2(c+dx)\right)}{d}+2abx+\frac{b^2\tanh^{-1}(\tanh(c+dx))}{d}-\frac{b^2\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] 2*a*b*x + (b^2*ArcTanh[Tanh[c + d*x]])/d - (a^2*Coth[c + d*x]*Hypergeometri
c2F1[-1/2, 1, 1/2, Tanh[c + d*x]^2])/d - (b^2*Tanh[c + d*x])/d
```

Maple [A] time = 0.043, size = 49, normalized size = 1.4

$$\frac{a^2\left(dx+c-\coth\left(dx+c\right)\right)+2\left(dx+c\right)ab+b^2\left(dx+c-\tanh\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*(d*x+c-coth(d*x+c))+2*(d*x+c)*a*b+b^2*(d*x+c-tanh(d*x+c)))
```

Maxima [A] time = 1.07441, size = 86, normalized size = 2.39

$$b^2\left(x+\frac{c}{d}-\frac{2}{d\left(e^{(-2\,dx-2\,c)}+1\right)}\right)+a^2\left(x+\frac{c}{d}+\frac{2}{d\left(e^{(-2\,dx-2\,c)}-1\right)}\right)+2\,abx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] b^2*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a^2*(x + c/d + 2/(d*(e^(-2*d
*x - 2*c) - 1))) + 2*a*b*x
```

Fricas [B] time = 1.90056, size = 243, normalized size = 6.75

$$-\frac{\left(a^{2}+b^{2}\right)\cosh\left(dx+c\right)^{2}-2\left(\left(a^{2}+2\,ab+b^{2}\right)dx+a^{2}+b^{2}\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\left(a^{2}+b^{2}\right)\sinh\left(dx+c\right)^{2}+a^{2}-}{2\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((a^2 + b^2)*cosh(d*x + c)^2 - 2*((a^2 + 2*a*b + b^2)*d*x + a^2 + b^2)
*cosh(d*x + c)*sinh(d*x + c) + (a^2 + b^2)*sinh(d*x + c)^2 + a^2 - b^2)/(d*
cosh(d*x + c)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.30827, size = 99, normalized size = 2.75

$$\frac{\left(a^{2}+2\,ab+b^{2}\right)\left(dx+c\right)}{d}-\frac{2\left(a^{2}e^{\left(2\,dx+2\,c\right)}-b^{2}e^{\left(2\,dx+2\,c\right)}+a^{2}+b^{2}\right)}{d\left(e^{\left(4\,dx+4\,c\right)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] (a^2 + 2*a*b + b^2)*(d*x + c)/d - 2*(a^2*e^(2*d*x + 2*c) - b^2*e^(2*d*x + 2
*c) + a^2 + b^2)/(d*(e^(4*d*x + 4*c) - 1))
```

3.151 $\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$

Optimal. Leaf size=52

$$-\frac{a^2\coth^2(c+dx)}{2d}+\frac{a(a+2b)\log(\tanh(c+dx))}{d}+\frac{(a+b)^2\log(\cosh(c+dx))}{d}$$

```
[Out] -(a^2*Coth[c + d*x]^2)/(2*d) + ((a + b)^2*Log[Cosh[c + d*x]])/d + (a*(a + 2
*b)*Log[Tanh[c + d*x]])/d
```

Rubi [A] time = 0.0947188, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 446, 88}

$$-\frac{a^2\coth^2(c+dx)}{2d}+\frac{a(a+2b)\log(\tanh(c+dx))}{d}+\frac{(a+b)^2\log(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -(a^2*Coth[c + d*x]^2)/(2*d) + ((a + b)^2*Log[Cosh[c + d*x]])/d + (a*(a + 2
*b)*Log[Tanh[c + d*x]])/d
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{x^3\left(1-x^2\right)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{(1-x)x^2}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{(a+b)^2}{-1+x}+\frac{a^2}{x^2}+\frac{a(a+2b)}{x}\right)dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= -\frac{a^2\coth^2(c+dx)}{2d}+\frac{(a+b)^2\log(\cosh(c+dx))}{d}+\frac{a(a+2b)\log(\tanh(c+}{d}$$

Mathematica [A] time = 0.152807, size = 50, normalized size = 0.96

$$\frac{-a^2\coth^2(c+dx)+2a(a+2b)\log(\tanh(c+dx))+2(a+b)^2\log(\cosh(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (-(a^2*Coth[c + d*x]^2) + 2*(a + b)^2*Log[Cosh[c + d*x]] + 2*a*(a + 2*b)*Lo
g[Tanh[c + d*x]])/(2*d)
```

Maple [A] time = 0.058, size = 60, normalized size = 1.2

$$\frac{a^2\ln(\sinh(dx+c))}{d}-\frac{a^2(\coth(dx+c))^2}{2d}+2\frac{ab\ln(\sinh(dx+c))}{d}+\frac{b^2\ln(\cosh(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*a^2*ln(sinh(d*x+c))-1/2*a^2*coth(d*x+c)^2/d+2/d*a*b*ln(sinh(d*x+c))+1/d
*b^2*ln(cosh(d*x+c))
```

Maxima [B] time = 1.01771, size = 181, normalized size = 3.48

$$a^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1\right)}\right)+\frac{b^2\log\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{d}+\frac{2\,ab\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*
d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + b^2*log(e^(d*
x + c) + e^(-d*x - c))/d + 2*a*b*log(e^(d*x + c) - e^(-d*x - c))/d
```

Fricas [B] time = 2.03624, size = 1715, normalized size = 32.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -((a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*d*x*cosh(
d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^4 + (a^2 +
 2*a*b + b^2)*d*x - 2*((a^2 + 2*a*b + b^2)*d*x - a^2)*cosh(d*x + c)^2 + 2*(
3*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^2 - (a^2 + 2*a*b + b^2)*d*x + a^2)*
sinh(d*x + c)^2 - (b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^
3 + b^2*sinh(d*x + c)^4 - 2*b^2*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2
- b^2)*sinh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 - b^2*cosh(d*x + c))*
sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) - ((a^2
 + 2*a*b)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b)*cosh(d*x + c)*sinh(d*x + c)^3 +
 (a^2 + 2*a*b)*sinh(d*x + c)^4 - 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*(a^
2 + 2*a*b)*cosh(d*x + c)^2 - a^2 - 2*a*b)*sinh(d*x + c)^2 + a^2 + 2*a*b + 4
*((a^2 + 2*a*b)*cosh(d*x + c)^3 - (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c
))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*((a^2 + 2*a*b +
 b^2)*d*x*cosh(d*x + c)^3 - ((a^2 + 2*a*b + b^2)*d*x - a^2)*cosh(d*x + c))*
sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*s
inh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 - d)*sinh(d*x
 + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.2946, size = 198, normalized size = 3.81

$$\frac{b^2\log\left(e^{(2dx+2c)}+e^{(-2dx-2c)}+2\right)}{2d}+\frac{\left(a^2+2ab\right)\log\left(e^{(2dx+2c)}+e^{(-2dx-2c)}-2\right)}{2d}-\frac{a^2\left(e^{(2dx+2c)}+e^{(-2dx-2c)}\right)+2ab\left(e^{(2c\ldots}{2d\left(e^{(2dx+2c)}+e^{\ldots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*b^2*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2)/d + 1/2*(a^2 + 2*a*b)*l
og(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2)/d - 1/2*(a^2*(e^(2*d*x + 2*c) +
e^(-2*d*x - 2*c)) + 2*a*b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 2*a^2 - 4*
a*b)/(d*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2))
```

3.152 $\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=43

$$-\frac{a^2\coth^3(c+dx)}{3d}-\frac{a(a+2b)\coth(c+dx)}{d}+x(a+b)^2$$

```
[Out] (a + b)^2*x - (a*(a + 2*b)*Coth[c + d*x])/d - (a^2*Coth[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0714495, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 461, 207}

$$-\frac{a^2\coth^3(c+dx)}{3d}-\frac{a(a+2b)\coth(c+dx)}{d}+x(a+b)^2$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (a + b)^2*x - (a*(a + 2*b)*Coth[c + d*x])/d - (a^2*Coth[c + d*x]^3)/(3*d)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{x^4(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^4}+\frac{a(a+2b)}{x^2}-\frac{(a+b)^2}{-1+x^2}\right)dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{a(a+2b)\coth(c+dx)}{d}-\frac{a^2\coth^3(c+dx)}{3d}-\frac{(a+b)^2\,\text{Subst}\left(\int \frac{1}{-1+x^2}\right.}{d}$$
$$= (a+b)^2x-\frac{a(a+2b)\coth(c+dx)}{d}-\frac{a^2\coth^3(c+dx)}{3d}$$

Mathematica [A] time = 0.571053, size = 65, normalized size = 1.51

$$-\frac{\coth(c+dx)\left(a\left(a\coth^2(c+dx)+3a+6b\right)-3(a+b)^2\tanh^{-1}\left(\sqrt{\tanh^2(c+dx)}\right)\sqrt{\tanh^2(c+dx)}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -(Coth[c + d*x]*(a*(3*a + 6*b + a*Coth[c + d*x]^2) - 3*(a + b)^2*ArcTanh[Sq
rt[Tanh[c + d*x]^2]]*Sqrt[Tanh[c + d*x]^2]))/(3*d)
```

Maple [A] time = 0.05, size = 59, normalized size = 1.4

$$\frac{1}{d}\left(a^2\left(dx+c-\coth(dx+c)-\frac{(\coth(dx+c))^3}{3}\right)+2\,ab\,(dx+c-\coth(dx+c))+b^2\,(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*(d*x+c-coth(d*x+c)-1/3*coth(d*x+c)^3)+2*a*b*(d*x+c-coth(d*x+c))+b^
2*(d*x+c))
```

Maxima [B] time = 1.0667, size = 154, normalized size = 3.58

$$\frac{1}{3}a^2\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}-2\right)}{d\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1\right)}\right)+2ab\left(x+\frac{c}{d}+\frac{2}{d\left(e^{(-2dx-2c)}-1\right)}\right)+b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - 2)/(d*(
3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + 2*a*b*(
x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + b^2*x
```

Fricas [B] time = 2.06659, size = 489, normalized size = 11.37

$$-\frac{2\left(2a^2+3ab\right)\cosh\left(dx+c\right)^3+6\left(2a^2+3ab\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2-\left(3\left(a^2+2ab+b^2\right)dx+4a^2+6ab\right)\sinh\left(\right.}{3\left(d\sinh\left(dx+c\right)^3+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(2*(2*a^2 + 3*a*b)*cosh(d*x + c)^3 + 6*(2*a^2 + 3*a*b)*cosh(d*x + c)*s
inh(d*x + c)^2 - (3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 6*a*b)*sinh(d*x + c)^
3 - 6*a*b*cosh(d*x + c) + 3*(3*(a^2 + 2*a*b + b^2)*d*x - (3*(a^2 + 2*a*b +
b^2)*d*x + 4*a^2 + 6*a*b)*cosh(d*x + c)^2 + 4*a^2 + 6*a*b)*sinh(d*x + c))/(
d*sinh(d*x + c)^3 + 3*(d*cosh(d*x + c)^2 - d)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**4*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27391, size = 139, normalized size = 3.23

$$\frac{\left(a^2+2ab+b^2\right)(dx+c)}{d}-\frac{4\left(3a^2e^{(4dx+4c)}+3abe^{(4dx+4c)}-3a^2e^{(2dx+2c)}-6abe^{(2dx+2c)}+2a^2+3ab\right)}{3d\left(e^{(2dx+2c)}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] (a^2 + 2*a*b + b^2)*(d*x + c)/d - 4/3*(3*a^2*e^(4*d*x + 4*c) + 3*a*b*e^(4*d
*x + 4*c) - 3*a^2*e^(2*d*x + 2*c) - 6*a*b*e^(2*d*x + 2*c) + 2*a^2 + 3*a*b)/
(d*(e^(2*d*x + 2*c) - 1)^3)
```

3.153 $\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=72

$$-\frac{a^2\coth^4(c+dx)}{4d}-\frac{a(a+2b)\coth^2(c+dx)}{2d}+\frac{(a+b)^2\log(\tanh(c+dx))}{d}+\frac{(a+b)^2\log(\cosh(c+dx))}{d}$$

```
[Out] -(a*(a + 2*b)*Coth[c + d*x]^2)/(2*d) - (a^2*Coth[c + d*x]^4)/(4*d) + ((a +
b)^2*Log[Cosh[c + d*x]])/d + ((a + b)^2*Log[Tanh[c + d*x]])/d
```

Rubi [A] time = 0.0988802, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 446, 88}

$$-\frac{a^2\coth^4(c+dx)}{4d}-\frac{a(a+2b)\coth^2(c+dx)}{2d}+\frac{(a+b)^2\log(\tanh(c+dx))}{d}+\frac{(a+b)^2\log(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^5*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -(a*(a + 2*b)*Coth[c + d*x]^2)/(2*d) - (a^2*Coth[c + d*x]^4)/(4*d) + ((a +
b)^2*Log[Cosh[c + d*x]])/d + ((a + b)^2*Log[Tanh[c + d*x]])/d
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{x^5\left(1-x^2\right)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{(1-x)x^3}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{(a+b)^2}{-1+x}+\frac{a^2}{x^3}+\frac{a(a+2b)}{x^2}+\frac{(a+b)^2}{x}\right)dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= -\frac{a(a+2b)\coth^2(c+dx)}{2d}-\frac{a^2\coth^4(c+dx)}{4d}+\frac{(a+b)^2\log(\cosh(c+dx))}{d}$$

Mathematica [A] time = 0.390955, size = 58, normalized size = 0.81

$$-\frac{a^2\coth^4(c+dx)+2a(a+2b)\coth^2(c+dx)-4(a+b)^2(\log(\tanh(c+dx))+\log(\cosh(c+dx)))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^5*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -(2*a*(a + 2*b)*Coth[c + d*x]^2 + a^2*Coth[c + d*x]^4 - 4*(a + b)^2*(Log[Co
sh[c + d*x]] + Log[Tanh[c + d*x]]))/(4*d)
```

Maple [A] time = 0.054, size = 91, normalized size = 1.3

$$\frac{a^2\ln(\sinh(dx+c))}{d}-\frac{a^2(\coth(dx+c))^2}{2d}-\frac{a^2(\coth(dx+c))^4}{4d}-\frac{ab(\coth(dx+c))^2}{d}+2\frac{ab\ln(\sinh(dx+c))}{d}+\frac{b^2\ln(s}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*a^2*ln(sinh(d*x+c))-1/2*a^2*coth(d*x+c)^2/d-1/4*a^2*coth(d*x+c)^4/d-1/d
*a*b*coth(d*x+c)^2+2/d*a*b*ln(sinh(d*x+c))+1/d*b^2*ln(sinh(d*x+c))
```

Maxima [B] time = 1.03643, size = 319, normalized size = 4.43

$$a^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{4\left(e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}\right)}{d\left(4\,e^{(-2\,dx-2\,c)}-6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}-e^{(-8\,dx-8\,c)}-1\right)}\right)+2\,ab\left(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 4*(e^(-2
*d*x - 2*c) - e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) -
 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + 2*a*b*
(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*d*x
- 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + b^2*log(e^(d*x +
```

```
c) - e^(-d*x - c))/d
```

Fricas [B] time = 2.18359, size = 4165, normalized size = 57.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -((a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*d*x*cosh(
d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^8 - 4*((a^
2 + 2*a*b + b^2)*d*x - a^2 - a*b)*cosh(d*x + c)^6 + 4*(7*(a^2 + 2*a*b + b^2
)*d*x*cosh(d*x + c)^2 - (a^2 + 2*a*b + b^2)*d*x + a^2 + a*b)*sinh(d*x + c)^
6 + 8*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 - 3*((a^2 + 2*a*b + b^2)*d
*x - a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*(a^2 + 2*a*b + b^2)*d
*x - 2*a^2 - 4*a*b)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*d*x*cosh(d*
x + c)^4 + 3*(a^2 + 2*a*b + b^2)*d*x - 30*((a^2 + 2*a*b + b^2)*d*x - a^2 -
a*b)*cosh(d*x + c)^2 - 2*a^2 - 4*a*b)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b +
 b^2)*d*x*cosh(d*x + c)^5 - 10*((a^2 + 2*a*b + b^2)*d*x - a^2 - a*b)*cosh(d
*x + c)^3 + (3*(a^2 + 2*a*b + b^2)*d*x - 2*a^2 - 4*a*b)*cosh(d*x + c))*sinh
(d*x + c)^3 + (a^2 + 2*a*b + b^2)*d*x - 4*((a^2 + 2*a*b + b^2)*d*x - a^2 -
a*b)*cosh(d*x + c)^2 + 4*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^6 - 15*((
a^2 + 2*a*b + b^2)*d*x - a^2 - a*b)*cosh(d*x + c)^4 - (a^2 + 2*a*b + b^2)*d
*x + 3*(3*(a^2 + 2*a*b + b^2)*d*x - 2*a^2 - 4*a*b)*cosh(d*x + c)^2 + a^2 +
a*b)*sinh(d*x + c)^2 - ((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*
b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^
8 - 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d
*x + c)^2 - a^2 - 2*a*b - b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)^3 - 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(
a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x + c
)^4 - 30*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 3*a^2 + 6*a*b + 3*b^2)*sinh(
d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 - 10*(a^2 + 2*a*b + b
^2)*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3
- 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x
 + c)^6 - 15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 9*(a^2 + 2*a*b + b^2)*co
sh(d*x + c)^2 - a^2 - 2*a*b - b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 8*
((a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 - 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^
5 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - (a^2 + 2*a*b + b^2)*cosh(d*x +
c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 8
*((a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^7 - 3*((a^2 + 2*a*b + b^2)*d*x - a^
2 - a*b)*cosh(d*x + c)^5 + (3*(a^2 + 2*a*b + b^2)*d*x - 2*a^2 - 4*a*b)*cosh
(d*x + c)^3 - ((a^2 + 2*a*b + b^2)*d*x - a^2 - a*b)*cosh(d*x + c))*sinh(d*x
 + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x
+ c)^8 - 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^6
+ 8*(7*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*
x + c)^4 + 2*(35*d*cosh(d*x + c)^4 - 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x +
 c)^4 + 8*(7*d*cosh(d*x + c)^5 - 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*
sinh(d*x + c)^3 - 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 - 15*d*cosh(
d*x + c)^4 + 9*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^
7 - 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x +
 c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**5*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31524, size = 165, normalized size = 2.29

$$-\frac{(a^2+2\,ab+b^2)(dx+c)}{d}+\frac{(a^2+2\,ab+b^2)\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)}{d}-\frac{4\left((a^2+ab)e^{(6\,dx+6\,c)}-(a^2+2\,ab)e^{(4\,dx+4\,c)}+(a^2+ab\right.}{d\left(e^{(2\,dx+2\,c)}-1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -(a^2 + 2*a*b + b^2)*(d*x + c)/d + (a^2 + 2*a*b + b^2)*log(abs(e^(2*d*x + 2
*c) - 1))/d - 4*((a^2 + a*b)*e^(6*d*x + 6*c) - (a^2 + 2*a*b)*e^(4*d*x + 4*c
) + (a^2 + a*b)*e^(2*d*x + 2*c))/(d*(e^(2*d*x + 2*c) - 1)^4)
```

3.154 $\int \coth^6(c+dx) \left(a+b \tanh^2(c+dx)\right)^2 \, dx$

Optimal. Leaf size=63

$$-\frac{a^2 \coth^5(c+dx)}{5d}-\frac{a(a+2b) \coth^3(c+dx)}{3d}-\frac{(a+b)^2 \coth(c+dx)}{d}+x(a+b)^2$$

```
[Out] (a + b)^2*x - ((a + b)^2*Coth[c + d*x])/d - (a*(a + 2*b)*Coth[c + d*x]^3)/(
3*d) - (a^2*Coth[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0771258, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 461, 207}

$$-\frac{a^2 \coth^5(c+dx)}{5d}-\frac{a(a+2b) \coth^3(c+dx)}{3d}-\frac{(a+b)^2 \coth(c+dx)}{d}+x(a+b)^2$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^6*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (a + b)^2*x - ((a + b)^2*Coth[c + d*x])/d - (a*(a + 2*b)*Coth[c + d*x]^3)/(
3*d) - (a^2*Coth[c + d*x]^5)/(5*d)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{x^6\left(1-x^2\right)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^6}+\frac{a(a+2b)}{x^4}+\frac{(a+b)^2}{x^2}-\frac{(a+b)^2}{-1+x^2}\right)dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{(a+b)^2\coth(c+dx)}{d}-\frac{a(a+2b)\coth^3(c+dx)}{3d}-\frac{a^2\coth^5(c+dx)}{5d}-\frac{(a$$
$$= (a+b)^2x-\frac{(a+b)^2\coth(c+dx)}{d}-\frac{a(a+2b)\coth^3(c+dx)}{3d}-\frac{a^2\coth^5(c$$

Mathematica [C] time = 0.0794081, size = 98, normalized size = 1.56

$$-\frac{a^2\coth^5(c+dx)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\tanh^2(c+dx)\right)}{5d}-\frac{2ab\coth^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\tanh^2(c+dx)\right)}{3d}-\frac{b^2\coth(c+dx)\,{}_2$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^6*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -(a^2*Coth[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, Tanh[c + d*x]^2])/(5
*d) - (2*a*b*Coth[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Tanh[c + d*x]
^2])/(3*d) - (b^2*Coth[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[c + d*
x]^2])/d
```

Maple [A] time = 0.056, size = 87, normalized size = 1.4

$$\frac{1}{d}\left(a^2\left(dx+c-\coth(dx+c)-\frac{(\coth(dx+c))^3}{3}-\frac{(\coth(dx+c))^5}{5}\right)+2\,ab\left(dx+c-\coth(dx+c)-1/3\,(\coth(dx+c))^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*(d*x+c-coth(d*x+c)-1/3*coth(d*x+c)^3-1/5*coth(d*x+c)^5)+2*a*b*(d*x
+c-coth(d*x+c)-1/3*coth(d*x+c)^3)+b^2*(d*x+c-coth(d*x+c)))
```

Maxima [B] time = 1.06904, size = 312, normalized size = 4.95

$$\frac{1}{15}a^2\left(15\,x+\frac{15\,c}{d}-\frac{2\left(70\,e^{(-2\,dx-2\,c)}-140\,e^{(-4\,dx-4\,c)}+90\,e^{(-6\,dx-6\,c)}-45\,e^{(-8\,dx-8\,c)}-23\right)}{d\left(5\,e^{(-2\,dx-2\,c)}-10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}-5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}-1\right)}\right)+\frac{2}{3}\,ab\left(3\,x+\frac{3\,c}{d}-\frac{}{d(3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*a^2*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) - 140*e^(-4*d*x - 4*c) + 9
0*e^(-6*d*x - 6*c) - 45*e^(-8*d*x - 8*c) - 23)/(d*(5*e^(-2*d*x - 2*c) - 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
```

```
10*c) - 1))) + 2/3*a*b*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x -
 4*c) - 2)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) -
 1))) + b^2*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1)))
```

Fricas [B] time = 2.04268, size = 1206, normalized size = 19.14

$$-\frac{\left(23\,a^2+40\,ab+15\,b^2\right)\cosh\left(dx+c\right)^5+5\left(23\,a^2+40\,ab+15\,b^2\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4-\left(15\left(a^2+2\,ab+b^2\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/15*((23*a^2 + 40*a*b + 15*b^2)*cosh(d*x + c)^5 + 5*(23*a^2 + 40*a*b + 15
*b^2)*cosh(d*x + c)*sinh(d*x + c)^4 - (15*(a^2 + 2*a*b + b^2)*d*x + 23*a^2
+ 40*a*b + 15*b^2)*sinh(d*x + c)^5 - 5*(5*a^2 + 16*a*b + 9*b^2)*cosh(d*x +
c)^3 + 5*(15*(a^2 + 2*a*b + b^2)*d*x - 2*(15*(a^2 + 2*a*b + b^2)*d*x + 23*a
^2 + 40*a*b + 15*b^2)*cosh(d*x + c)^2 + 23*a^2 + 40*a*b + 15*b^2)*sinh(d*x
+ c)^3 + 5*(2*(23*a^2 + 40*a*b + 15*b^2)*cosh(d*x + c)^3 - 3*(5*a^2 + 16*a*
b + 9*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(5*a^2 + 4*a*b + 3*b^2)*cosh
(d*x + c) - 5*((15*(a^2 + 2*a*b + b^2)*d*x + 23*a^2 + 40*a*b + 15*b^2)*cosh
(d*x + c)^4 + 30*(a^2 + 2*a*b + b^2)*d*x - 3*(15*(a^2 + 2*a*b + b^2)*d*x +
23*a^2 + 40*a*b + 15*b^2)*cosh(d*x + c)^2 + 46*a^2 + 80*a*b + 30*b^2)*sinh(
d*x + c))/(d*sinh(d*x + c)^5 + 5*(2*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^3
+ 5*(d*cosh(d*x + c)^4 - 3*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**6*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.28937, size = 294, normalized size = 4.67

$$\frac{\left(a^2+2\,ab+b^2\right)(dx+c)}{d}-\frac{2\left(45\,a^2e^{(8\,dx+8\,c)}+60\,abe^{(8\,dx+8\,c)}+15\,b^2e^{(8\,dx+8\,c)}-90\,a^2e^{(6\,dx+6\,c)}-180\,abe^{(6\,dx+6\,c)}-60\,b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] (a^2 + 2*a*b + b^2)*(d*x + c)/d - 2/15*(45*a^2*e^(8*d*x + 8*c) + 60*a*b*e^(
8*d*x + 8*c) + 15*b^2*e^(8*d*x + 8*c) - 90*a^2*e^(6*d*x + 6*c) - 180*a*b*e^
(6*d*x + 6*c) - 60*b^2*e^(6*d*x + 6*c) + 140*a^2*e^(4*d*x + 4*c) + 220*a*b*
e^(4*d*x + 4*c) + 90*b^2*e^(4*d*x + 4*c) - 70*a^2*e^(2*d*x + 2*c) - 140*a*b
*e^(2*d*x + 2*c) - 60*b^2*e^(2*d*x + 2*c) + 23*a^2 + 40*a*b + 15*b^2)/(d*(e
^(2*d*x + 2*c) - 1)^5)
```

3.155 $\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$

Optimal. Leaf size=92

$$-\frac{a^2\coth^6(c+dx)}{6d}-\frac{a(a+2b)\coth^4(c+dx)}{4d}-\frac{(a+b)^2\coth^2(c+dx)}{2d}+\frac{(a+b)^2\log(\tanh(c+dx))}{d}+\frac{(a+b)^2\log(\cosh$$

```
[Out] -((a + b)^2*Coth[c + d*x]^2)/(2*d) - (a*(a + 2*b)*Coth[c + d*x]^4)/(4*d) -
(a^2*Coth[c + d*x]^6)/(6*d) + ((a + b)^2*Log[Cosh[c + d*x]])/d + ((a + b)^2
*Log[Tanh[c + d*x]])/d
```

Rubi [A] time = 0.113083, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 446, 88}

$$-\frac{a^2\coth^6(c+dx)}{6d}-\frac{a(a+2b)\coth^4(c+dx)}{4d}-\frac{(a+b)^2\coth^2(c+dx)}{2d}+\frac{(a+b)^2\log(\tanh(c+dx))}{d}+\frac{(a+b)^2\log(\cosh$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^7*(a + b*Tanh[c + d*x]^2)^2,x]
```


```
[Out] -((a + b)^2*Coth[c + d*x]^2)/(2*d) - (a*(a + 2*b)*Coth[c + d*x]^4)/(4*d) -
(a^2*Coth[c + d*x]^6)/(6*d) + ((a + b)^2*Log[Cosh[c + d*x]])/d + ((a + b)^2
*Log[Tanh[c + d*x]])/d
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{x^7\left(1-x^2\right)} dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{(1-x)x^4} dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{(a+b)^2}{-1+x} + \frac{a^2}{x^4} + \frac{a(a+2b)}{x^3} + \frac{(a+b)^2}{x^2} + \frac{(a+b)^2}{x}\right) dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= -\frac{(a+b)^2\coth^2(c+dx)}{2d} - \frac{a(a+2b)\coth^4(c+dx)}{4d} - \frac{a^2\coth^6(c+dx)}{6d}$$

Mathematica [A] time = 0.484339, size = 74, normalized size = 0.8

$$-\frac{2a^2\coth^6(c+dx) + 3a(a+2b)\coth^4(c+dx) + 6(a+b)^2\coth^2(c+dx) - 12(a+b)^2(\log(\tanh(c+dx)) + \log(\cosh(}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^7*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -(6*(a + b)^2*Coth[c + d*x]^2 + 3*a*(a + 2*b)*Coth[c + d*x]^4 + 2*a^2*Coth[
c + d*x]^6 - 12*(a + b)^2*(Log[Cosh[c + d*x]] + Log[Tanh[c + d*x]]))/(12*d)
```

Maple [A] time = 0.059, size = 138, normalized size = 1.5

$$\frac{a^2\ln(\sinh(dx+c))}{d} - \frac{a^2(\coth(dx+c))^2}{2d} - \frac{a^2(\coth(dx+c))^4}{4d} - \frac{a^2(\coth(dx+c))^6}{6d} + 2\frac{ab\ln(\sinh(dx+c))}{d} - \frac{ab(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*a^2*ln(sinh(d*x+c))-1/2*a^2*coth(d*x+c)^2/d-1/4*a^2*coth(d*x+c)^4/d-1/6
*a^2*coth(d*x+c)^6/d+2/d*a*b*ln(sinh(d*x+c))-1/d*a*b*coth(d*x+c)^2-1/2/d*a*
b*coth(d*x+c)^4+1/d*b^2*ln(sinh(d*x+c))-1/2/d*b^2*coth(d*x+c)^2
```

Maxima [B] time = 1.07226, size = 527, normalized size = 5.73

$$\frac{1}{3}a^2\left(3x + \frac{3c}{d} + \frac{3\log\left(e^{(-dx-c)}+1\right)}{d} + \frac{3\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\left(9e^{(-2dx-2c)} - 18e^{(-4dx-4c)} + 34e^{(-6dx-6c)} - 1\right.}{d\left(6e^{(-2dx-2c)} - 15e^{(-4dx-4c)} + 20e^{(-6dx-6c)} - 15e^{(-8dx-8}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*(3*x + 3*c/d + 3*log(e^(-d*x - c) + 1)/d + 3*log(e^(-d*x - c) - 1)/
d + 2*(9*e^(-2*d*x - 2*c) - 18*e^(-4*d*x - 4*c) + 34*e^(-6*d*x - 6*c) - 18*
e^(-8*d*x - 8*c) + 9*e^(-10*d*x - 10*c))/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*
d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*
c) - e^(-12*d*x - 12*c) - 1))) + 2*a*b*(x + c/d + log(e^(-d*x - c) + 1)/d +
```

```
 log(e^(-d*x - c) - 1)/d + 4*(e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) + e^(-6*d
*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c)
 - e^(-8*d*x - 8*c) - 1))) + b^2*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e
^(-d*x - c) - 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x
- 4*c) - 1)))
```

Fricas [B] time = 2.45536, size = 8982, normalized size = 97.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^12 + 36*(a^2 + 2*a*b + b^2)*d
*x*cosh(d*x + c)*sinh(d*x + c)^11 + 3*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)
^12 - 6*(3*(a^2 + 2*a*b + b^2)*d*x - 3*a^2 - 4*a*b - b^2)*cosh(d*x + c)^10
+ 6*(33*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^2 - 3*(a^2 + 2*a*b + b^2)*d*x
 + 3*a^2 + 4*a*b + b^2)*sinh(d*x + c)^10 + 60*(11*(a^2 + 2*a*b + b^2)*d*x*c
osh(d*x + c)^3 - (3*(a^2 + 2*a*b + b^2)*d*x - 3*a^2 - 4*a*b - b^2)*cosh(d*x
 + c))*sinh(d*x + c)^9 + 3*(15*(a^2 + 2*a*b + b^2)*d*x - 12*a^2 - 24*a*b -
8*b^2)*cosh(d*x + c)^8 + 3*(495*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + 1
5*(a^2 + 2*a*b + b^2)*d*x - 90*(3*(a^2 + 2*a*b + b^2)*d*x - 3*a^2 - 4*a*b -
 b^2)*cosh(d*x + c)^2 - 12*a^2 - 24*a*b - 8*b^2)*sinh(d*x + c)^8 + 24*(99*(
a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^5 - 30*(3*(a^2 + 2*a*b + b^2)*d*x - 3*
a^2 - 4*a*b - b^2)*cosh(d*x + c)^3 + (15*(a^2 + 2*a*b + b^2)*d*x - 12*a^2 -
 24*a*b - 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 4*(15*(a^2 + 2*a*b + b^2)
*d*x - 17*a^2 - 24*a*b - 9*b^2)*cosh(d*x + c)^6 + 4*(693*(a^2 + 2*a*b + b^2
)*d*x*cosh(d*x + c)^6 - 315*(3*(a^2 + 2*a*b + b^2)*d*x - 3*a^2 - 4*a*b - b^
2)*cosh(d*x + c)^4 - 15*(a^2 + 2*a*b + b^2)*d*x + 21*(15*(a^2 + 2*a*b + b^2
)*d*x - 12*a^2 - 24*a*b - 8*b^2)*cosh(d*x + c)^2 + 17*a^2 + 24*a*b + 9*b^2)
*sinh(d*x + c)^6 + 24*(99*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^7 - 63*(3*(
a^2 + 2*a*b + b^2)*d*x - 3*a^2 - 4*a*b - b^2)*cosh(d*x + c)^5 + 7*(15*(a^2
+ 2*a*b + b^2)*d*x - 12*a^2 - 24*a*b - 8*b^2)*cosh(d*x + c)^3 - (15*(a^2 +
2*a*b + b^2)*d*x - 17*a^2 - 24*a*b - 9*b^2)*cosh(d*x + c))*sinh(d*x + c)^5
+ 3*(15*(a^2 + 2*a*b + b^2)*d*x - 12*a^2 - 24*a*b - 8*b^2)*cosh(d*x + c)^4
+ 3*(495*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^8 - 420*(3*(a^2 + 2*a*b + b^
2)*d*x - 3*a^2 - 4*a*b - b^2)*cosh(d*x + c)^6 + 70*(15*(a^2 + 2*a*b + b^2)*
d*x - 12*a^2 - 24*a*b - 8*b^2)*cosh(d*x + c)^4 + 15*(a^2 + 2*a*b + b^2)*d*x
 - 20*(15*(a^2 + 2*a*b + b^2)*d*x - 17*a^2 - 24*a*b - 9*b^2)*cosh(d*x + c)^
2 - 12*a^2 - 24*a*b - 8*b^2)*sinh(d*x + c)^4 + 4*(165*(a^2 + 2*a*b + b^2)*d
*x*cosh(d*x + c)^9 - 180*(3*(a^2 + 2*a*b + b^2)*d*x - 3*a^2 - 4*a*b - b^2)*
cosh(d*x + c)^7 + 42*(15*(a^2 + 2*a*b + b^2)*d*x - 12*a^2 - 24*a*b - 8*b^2)
*cosh(d*x + c)^5 - 20*(15*(a^2 + 2*a*b + b^2)*d*x - 17*a^2 - 24*a*b - 9*b^2
)*cosh(d*x + c)^3 + 3*(15*(a^2 + 2*a*b + b^2)*d*x - 12*a^2 - 24*a*b - 8*b^2
)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*d*x - 6*(3*(a^2 +
2*a*b + b^2)*d*x - 3*a^2 - 4*a*b - b^2)*cosh(d*x + c)^2 + 6*(33*(a^2 + 2*a*
b + b^2)*d*x*cosh(d*x + c)^10 - 45*(3*(a^2 + 2*a*b + b^2)*d*x - 3*a^2 - 4*a
*b - b^2)*cosh(d*x + c)^8 + 14*(15*(a^2 + 2*a*b + b^2)*d*x - 12*a^2 - 24*a*
b - 8*b^2)*cosh(d*x + c)^6 - 10*(15*(a^2 + 2*a*b + b^2)*d*x - 17*a^2 - 24*a
*b - 9*b^2)*cosh(d*x + c)^4 - 3*(a^2 + 2*a*b + b^2)*d*x + 3*(15*(a^2 + 2*a*
b + b^2)*d*x - 12*a^2 - 24*a*b - 8*b^2)*cosh(d*x + c)^2 + 3*a^2 + 4*a*b + b
^2)*sinh(d*x + c)^2 - 3*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^12 + 12*(a^2 + 2
*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^11 + (a^2 + 2*a*b + b^2)*sinh(d*x +
 c)^12 - 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^10 + 6*(11*(a^2 + 2*a*b + b^2)
*cosh(d*x + c)^2 - a^2 - 2*a*b - b^2)*sinh(d*x + c)^10 + 20*(11*(a^2 + 2*a*
b + b^2)*cosh(d*x + c)^3 - 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x +
```

```
c)^9 + 15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 15*(33*(a^2 + 2*a*b + b^2)*
cosh(d*x + c)^4 - 18*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^
2)*sinh(d*x + c)^8 + 24*(33*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 - 30*(a^2 +
 2*a*b + b^2)*cosh(d*x + c)^3 + 5*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d
*x + c)^7 - 20*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 4*(231*(a^2 + 2*a*b +
b^2)*cosh(d*x + c)^6 - 315*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 105*(a^2 +
 2*a*b + b^2)*cosh(d*x + c)^2 - 5*a^2 - 10*a*b - 5*b^2)*sinh(d*x + c)^6 + 2
4*(33*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 - 63*(a^2 + 2*a*b + b^2)*cosh(d*x
 + c)^5 + 35*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - 5*(a^2 + 2*a*b + b^2)*co
sh(d*x + c))*sinh(d*x + c)^5 + 15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 15*
(33*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 - 84*(a^2 + 2*a*b + b^2)*cosh(d*x +
 c)^6 + 70*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 - 20*(a^2 + 2*a*b + b^2)*cos
h(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 20*(11*(a^2 + 2*a*b + b
^2)*cosh(d*x + c)^9 - 36*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 42*(a^2 + 2*
a*b + b^2)*cosh(d*x + c)^5 - 20*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^
2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 6*(a^2 + 2*a*b + b^2)*cos
h(d*x + c)^2 + 6*(11*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^10 - 45*(a^2 + 2*a*b
 + b^2)*cosh(d*x + c)^8 + 70*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 - 50*(a^2
+ 2*a*b + b^2)*cosh(d*x + c)^4 + 15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 - a
^2 - 2*a*b - b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 12*((a^2 + 2*a*b +
b^2)*cosh(d*x + c)^11 - 5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^9 + 10*(a^2 + 2
*a*b + b^2)*cosh(d*x + c)^7 - 10*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 5*(a
^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - (a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh
(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 12*(3*(a^
2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^11 - 5*(3*(a^2 + 2*a*b + b^2)*d*x - 3*a^
2 - 4*a*b - b^2)*cosh(d*x + c)^9 + 2*(15*(a^2 + 2*a*b + b^2)*d*x - 12*a^2 -
 24*a*b - 8*b^2)*cosh(d*x + c)^7 - 2*(15*(a^2 + 2*a*b + b^2)*d*x - 17*a^2 -
 24*a*b - 9*b^2)*cosh(d*x + c)^5 + (15*(a^2 + 2*a*b + b^2)*d*x - 12*a^2 - 2
4*a*b - 8*b^2)*cosh(d*x + c)^3 - (3*(a^2 + 2*a*b + b^2)*d*x - 3*a^2 - 4*a*b
 - b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^12 + 12*d*cosh(d*x +
 c)*sinh(d*x + c)^11 + d*sinh(d*x + c)^12 - 6*d*cosh(d*x + c)^10 + 6*(11*d*
cosh(d*x + c)^2 - d)*sinh(d*x + c)^10 + 20*(11*d*cosh(d*x + c)^3 - 3*d*cosh
(d*x + c))*sinh(d*x + c)^9 + 15*d*cosh(d*x + c)^8 + 15*(33*d*cosh(d*x + c)^
4 - 18*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 24*(33*d*cosh(d*x + c)^5 -
30*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^7 - 20*d*cosh(d*x +
 c)^6 + 4*(231*d*cosh(d*x + c)^6 - 315*d*cosh(d*x + c)^4 + 105*d*cosh(d*x +
 c)^2 - 5*d)*sinh(d*x + c)^6 + 24*(33*d*cosh(d*x + c)^7 - 63*d*cosh(d*x + c
)^5 + 35*d*cosh(d*x + c)^3 - 5*d*cosh(d*x + c))*sinh(d*x + c)^5 + 15*d*cosh
(d*x + c)^4 + 15*(33*d*cosh(d*x + c)^8 - 84*d*cosh(d*x + c)^6 + 70*d*cosh(d
*x + c)^4 - 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 20*(11*d*cosh(d*x +
 c)^9 - 36*d*cosh(d*x + c)^7 + 42*d*cosh(d*x + c)^5 - 20*d*cosh(d*x + c)^3
+ 3*d*cosh(d*x + c))*sinh(d*x + c)^3 - 6*d*cosh(d*x + c)^2 + 6*(11*d*cosh(d
*x + c)^10 - 45*d*cosh(d*x + c)^8 + 70*d*cosh(d*x + c)^6 - 50*d*cosh(d*x +
c)^4 + 15*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 12*(d*cosh(d*x + c)^11 -
 5*d*cosh(d*x + c)^9 + 10*d*cosh(d*x + c)^7 - 10*d*cosh(d*x + c)^5 + 5*d*co
sh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**7*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.30747, size = 263, normalized size = 2.86

$$-\frac{(a^2+2\,ab+b^2)(dx+c)}{d}+\frac{(a^2+2\,ab+b^2)\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)}{d}-\frac{2\left(3\left(3\,a^2+4\,ab+b^2\right)e^{(10\,dx+10\,c)}-6\left(3\,a^2+6\,ab+2\,b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -(a^2 + 2*a*b + b^2)*(d*x + c)/d + (a^2 + 2*a*b + b^2)*log(abs(e^(2*d*x + 2
*c) - 1))/d - 2/3*(3*(3*a^2 + 4*a*b + b^2)*e^(10*d*x + 10*c) - 6*(3*a^2 + 6
*a*b + 2*b^2)*e^(8*d*x + 8*c) + 2*(17*a^2 + 24*a*b + 9*b^2)*e^(6*d*x + 6*c)
 - 6*(3*a^2 + 6*a*b + 2*b^2)*e^(4*d*x + 4*c) + 3*(3*a^2 + 4*a*b + b^2)*e^(2
*d*x + 2*c))/(d*(e^(2*d*x + 2*c) - 1)^6)
```

3.156 $\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx$

Optimal. Leaf size=114

$$-\frac{b\left(3a^2+3ab+b^2\right)\tanh^5(c+dx)}{5d}-\frac{b^2(3a+b)\tanh^7(c+dx)}{7d}-\frac{(a+b)^3\tanh^3(c+dx)}{3d}-\frac{(a+b)^3\tanh(c+dx)}{d}+x($$

```
[Out] (a + b)^3*x - ((a + b)^3*Tanh[c + d*x])/d - ((a + b)^3*Tanh[c + d*x]^3)/(3*
d) - (b*(3*a^2 + 3*a*b + b^2)*Tanh[c + d*x]^5)/(5*d) - (b^2*(3*a + b)*Tanh[
c + d*x]^7)/(7*d) - (b^3*Tanh[c + d*x]^9)/(9*d)
```

Rubi [A] time = 0.100523, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 461, 206}

$$-\frac{b\left(3a^2+3ab+b^2\right)\tanh^5(c+dx)}{5d}-\frac{b^2(3a+b)\tanh^7(c+dx)}{7d}-\frac{(a+b)^3\tanh^3(c+dx)}{3d}-\frac{(a+b)^3\tanh(c+dx)}{d}+x($$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (a + b)^3*x - ((a + b)^3*Tanh[c + d*x])/d - ((a + b)^3*Tanh[c + d*x]^3)/(3*
d) - (b*(3*a^2 + 3*a*b + b^2)*Tanh[c + d*x]^5)/(5*d) - (b^2*(3*a + b)*Tanh[
c + d*x]^7)/(7*d) - (b^3*Tanh[c + d*x]^9)/(9*d)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int \frac{x^4(a+bx^2)^3}{1-x^2}dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-(a+b)^3-(a+b)^3x^2-b\left(3a^2+3ab+b^2\right)x^4-b^2(3a+b)x^6-\right.\right.}{d}$$
$$= -\frac{(a+b)^3\tanh(c+dx)}{d}-\frac{(a+b)^3\tanh^3(c+dx)}{3d}-\frac{b\left(3a^2+3ab+b^2\right)\tan}{5d}$$
$$= (a+b)^3x-\frac{(a+b)^3\tanh(c+dx)}{d}-\frac{(a+b)^3\tanh^3(c+dx)}{3d}-\frac{b\left(3a^2+3ab\right.}{}$$

Mathematica [A] time = 1.43208, size = 123, normalized size = 1.08

$$\frac{\tanh(c+dx)\left(-63b\left(3a^2+3ab+b^2\right)\tanh^4(c+dx)-45b^2(3a+b)\tanh^6(c+dx)-105(a+b)^3\tanh^2(c+dx)+\frac{315(a+b)^3}{}\right.}{315d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (Tanh[c + d*x]*(-315*(a + b)^3 - 105*(a + b)^3*Tanh[c + d*x]^2 - 63*b*(3*a^
2 + 3*a*b + b^2)*Tanh[c + d*x]^4 - 45*b^2*(3*a + b)*Tanh[c + d*x]^6 - 35*b^
3*Tanh[c + d*x]^8 + (315*(a + b)^3*ArcTanh[Sqrt[Tanh[c + d*x]^2]])/Sqrt[Tan
h[c + d*x]^2]))/(315*d)
```

Maple [B] time = 0.007, size = 365, normalized size = 3.2

$$-\frac{a^3\ln(\tanh(dx+c)-1)}{2d}-\frac{3\ln(\tanh(dx+c)-1)a^2b}{2d}-\frac{3\ln(\tanh(dx+c)-1)ab^2}{2d}-\frac{\ln(\tanh(dx+c)-1)b^3}{2d}-\frac{b^3(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] -1/2/d*a^3*ln(tanh(d*x+c)-1)-3/2/d*ln(tanh(d*x+c)-1)*a^2*b-3/2/d*ln(tanh(d*
x+c)-1)*a*b^2-1/2/d*ln(tanh(d*x+c)-1)*b^3-1/7*b^3*tanh(d*x+c)^7/d-1/5*b^3*t
anh(d*x+c)^5/d-1/3/d*tanh(d*x+c)^3*a^3-1/3*b^3*tanh(d*x+c)^3/d-b^3*tanh(d*x
+c)/d-1/9*b^3*tanh(d*x+c)^9/d-a^2*b*tanh(d*x+c)^3/d-a*b^2*tanh(d*x+c)^3/d-3
/7/d*tanh(d*x+c)^7*a*b^2-3/5/d*tanh(d*x+c)^5*a^2*b-3/5*a*b^2*tanh(d*x+c)^5/
d-3*a^2*b*tanh(d*x+c)/d+1/2/d*ln(tanh(d*x+c)+1)*a^3+3/2/d*ln(tanh(d*x+c)+1)
*a^2*b+3/2/d*ln(tanh(d*x+c)+1)*a*b^2+1/2/d*ln(tanh(d*x+c)+1)*b^3-a^3*tanh(d
*x+c)/d-3*a*b^2*tanh(d*x+c)/d
```

Maxima [B] time = 1.15145, size = 787, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/315*b^3*(315*x + 315*c/d - 2*(3492*e^(-2*d*x - 2*c) + 13968*e^(-4*d*x - 4
*c) + 26292*e^(-6*d*x - 6*c) + 39438*e^(-8*d*x - 8*c) + 31500*e^(-10*d*x -
10*c) + 21000*e^(-12*d*x - 12*c) + 6300*e^(-14*d*x - 14*c) + 1575*e^(-16*d*
x - 16*c) + 563)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*
x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x -
12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) +
 1))) + 1/35*a*b^2*(105*x + 105*c/d - 8*(203*e^(-2*d*x - 2*c) + 609*e^(-4*d
*x - 4*c) + 770*e^(-6*d*x - 6*c) + 770*e^(-8*d*x - 8*c) + 315*e^(-10*d*x -
10*c) + 105*e^(-12*d*x - 12*c) + 44)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x
- 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c)
+ 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 1/5*a^2*b*(15*x + 15*c
/d - 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 90*e^(-6*d*x - 6*c) +
45*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10
*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 1/3*a^
3*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(3*e^(-
2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))
```

Fricas [B] time = 2.19007, size = 4166, normalized size = 36.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/315*((420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^9 + 9*(420*a^3 + 1449*a^2*b + 1584*a*b^2
+ 563*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)*sinh(d*x
 + c)^8 - (420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3)*sinh(d*x + c)^9 + 9
*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*d*x)*cosh(d*x + c)^7 - 9*(280*a^3 + 819*a^2*b + 744*a*b^2 + 213*b^
3 + 4*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3)*cosh(d*x + c)^2)*sinh(d
*x + c)^7 + 21*(4*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 + 3*(420*a^3 + 1449*a^2*b +
1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x +
c))*sinh(d*x + c)^6 + 36*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315
*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 - 9*(14*(420*a^3 + 14
49*a^2*b + 1584*a*b^2 + 563*b^3)*cosh(d*x + c)^4 + 700*a^3 + 2016*a^2*b + 2
136*a*b^2 + 852*b^3 + 21*(280*a^3 + 819*a^2*b + 744*a*b^2 + 213*b^3)*cosh(d
*x + c)^2)*sinh(d*x + c)^5 + 9*(14*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563
*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 + 35*(420*a
^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3
)*d*x)*cosh(d*x + c)^3 + 20*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 +
315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^4 + 8
4*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*d*x)*cosh(d*x + c)^3 - 3*(28*(420*a^3 + 1449*a^2*b + 1584*a*b^2 +
 563*b^3)*cosh(d*x + c)^6 + 105*(280*a^3 + 819*a^2*b + 744*a*b^2 + 213*b^3)
*cosh(d*x + c)^4 + 2660*a^3 + 8232*a^2*b + 8232*a*b^2 + 1764*b^3 + 120*(175
*a^3 + 504*a^2*b + 534*a*b^2 + 213*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 +
9*(4*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*d*x)*cosh(d*x + c)^7 + 21*(420*a^3 + 1449*a^2*b + 1584*a*b^2 +
 563*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 + 40*(4
20*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*d*x)*cosh(d*x + c)^3 + 28*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^
3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2
```

```
 + 126*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*d*x)*cosh(d*x + c) - 9*((420*a^3 + 1449*a^2*b + 1584*a*b^2 +
 563*b^3)*cosh(d*x + c)^8 + 7*(280*a^3 + 819*a^2*b + 744*a*b^2 + 213*b^3)*c
osh(d*x + c)^6 + 20*(175*a^3 + 504*a^2*b + 534*a*b^2 + 213*b^3)*cosh(d*x +
c)^4 + 420*a^3 + 1386*a^2*b + 1176*a*b^2 + 882*b^3 + 28*(95*a^3 + 294*a^2*b
 + 294*a*b^2 + 63*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^9 +
 9*d*cosh(d*x + c)*sinh(d*x + c)^8 + 9*d*cosh(d*x + c)^7 + 21*(4*d*cosh(d*x
 + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^6 + 36*d*cosh(d*x + c)^5 + 9*(14
*d*cosh(d*x + c)^5 + 35*d*cosh(d*x + c)^3 + 20*d*cosh(d*x + c))*sinh(d*x +
c)^4 + 84*d*cosh(d*x + c)^3 + 9*(4*d*cosh(d*x + c)^7 + 21*d*cosh(d*x + c)^5
 + 40*d*cosh(d*x + c)^3 + 28*d*cosh(d*x + c))*sinh(d*x + c)^2 + 126*d*cosh(
d*x + c))
```

Sympy [A] time = 2.8266, size = 260, normalized size = 2.28

$$\begin{cases} a^3x - \frac{a^3\tanh^3(c+dx)}{3d} - \frac{a^3\tanh(c+dx)}{d} + 3a^2bx - \frac{3a^2b\tanh^5(c+dx)}{5d} - \frac{a^2b\tanh^3(c+dx)}{d} - \frac{3a^2b\tanh(c+dx)}{d} + 3ab^2x - \frac{3ab^2\tanh^7(c+dx)}{7d} - \\ x\left(a+b\tanh^2(c)\right)^3\tanh^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**4*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((a**3*x - a**3*tanh(c + d*x)**3/(3*d) - a**3*tanh(c + d*x)/d + 3*
a**2*b*x - 3*a**2*b*tanh(c + d*x)**5/(5*d) - a**2*b*tanh(c + d*x)**3/d - 3*
a**2*b*tanh(c + d*x)/d + 3*a*b**2*x - 3*a*b**2*tanh(c + d*x)**7/(7*d) - 3*a
*b**2*tanh(c + d*x)**5/(5*d) - a*b**2*tanh(c + d*x)**3/d - 3*a*b**2*tanh(c
+ d*x)/d + b**3*x - b**3*tanh(c + d*x)**9/(9*d) - b**3*tanh(c + d*x)**7/(7*
d) - b**3*tanh(c + d*x)**5/(5*d) - b**3*tanh(c + d*x)**3/(3*d) - b**3*tanh(
c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c)**2)**3*tanh(c)**4, True))
```

Giac [B] time = 1.45266, size = 721, normalized size = 6.32

$$\frac{\left(a^3+3a^2b+3ab^2+b^3\right)(dx+c)}{d} + \frac{2\left(630a^3e^{(16dx+16c)} + 2835a^2be^{(16dx+16c)} + 3780ab^2e^{(16dx+16c)} + 1575b^3e^{(16dx+16c)} \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c)/d + 2/315*(630*a^3*e^(16*d*x + 16
*c) + 2835*a^2*b*e^(16*d*x + 16*c) + 3780*a*b^2*e^(16*d*x + 16*c) + 1575*b^
3*e^(16*d*x + 16*c) + 4410*a^3*e^(14*d*x + 14*c) + 17010*a^2*b*e^(14*d*x +
14*c) + 18900*a*b^2*e^(14*d*x + 14*c) + 6300*b^3*e^(14*d*x + 14*c) + 13650*
a^3*e^(12*d*x + 12*c) + 48510*a^2*b*e^(12*d*x + 12*c) + 54180*a*b^2*e^(12*d
*x + 12*c) + 21000*b^3*e^(12*d*x + 12*c) + 24570*a^3*e^(10*d*x + 10*c) + 85
050*a^2*b*e^(10*d*x + 10*c) + 94500*a*b^2*e^(10*d*x + 10*c) + 31500*b^3*e^(
10*d*x + 10*c) + 28350*a^3*e^(8*d*x + 8*c) + 97524*a^2*b*e^(8*d*x + 8*c) +
105084*a*b^2*e^(8*d*x + 8*c) + 39438*b^3*e^(8*d*x + 8*c) + 21630*a^3*e^(6*d
*x + 6*c) + 73206*a^2*b*e^(6*d*x + 6*c) + 78876*a*b^2*e^(6*d*x + 6*c) + 262
92*b^3*e^(6*d*x + 6*c) + 10710*a^3*e^(4*d*x + 4*c) + 35154*a^2*b*e^(4*d*x +
 4*c) + 38124*a*b^2*e^(4*d*x + 4*c) + 13968*b^3*e^(4*d*x + 4*c) + 3150*a^3*
e^(2*d*x + 2*c) + 10206*a^2*b*e^(2*d*x + 2*c) + 10476*a*b^2*e^(2*d*x + 2*c)
 + 3492*b^3*e^(2*d*x + 2*c) + 420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3)/
```

```
(d*(e^(2*d*x + 2*c) + 1)^9)
```

3.157 $\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx$

Optimal. Leaf size=107

$$-\frac{b\left(3a^2+3ab+b^2\right)\tanh^4(c+dx)}{4d}-\frac{b^2(3a+b)\tanh^6(c+dx)}{6d}-\frac{(a+b)^3\tanh^2(c+dx)}{2d}+\frac{(a+b)^3\log(\cosh(c+dx))}{d}-$$

```
[Out] ((a + b)^3*Log[Cosh[c + d*x]])/d - ((a + b)^3*Tanh[c + d*x]^2)/(2*d) - (b*(
3*a^2 + 3*a*b + b^2)*Tanh[c + d*x]^4)/(4*d) - (b^2*(3*a + b)*Tanh[c + d*x]^
6)/(6*d) - (b^3*Tanh[c + d*x]^8)/(8*d)
```

Rubi [A] time = 0.152296, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 446, 77}

$$-\frac{b\left(3a^2+3ab+b^2\right)\tanh^4(c+dx)}{4d}-\frac{b^2(3a+b)\tanh^6(c+dx)}{6d}-\frac{(a+b)^3\tanh^2(c+dx)}{2d}+\frac{(a+b)^3\log(\cosh(c+dx))}{d}-$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((a + b)^3*Log[Cosh[c + d*x]])/d - ((a + b)^3*Tanh[c + d*x]^2)/(2*d) - (b*(
3*a^2 + 3*a*b + b^2)*Tanh[c + d*x]^4)/(4*d) - (b^2*(3*a + b)*Tanh[c + d*x]^
6)/(6*d) - (b^3*Tanh[c + d*x]^8)/(8*d)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{x^3(a+bx^2)^3}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{x(a+bx)^3}{1-x}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(-(a+b)^3 - \frac{(a+b)^3}{-1+x} - b\left(3a^2+3ab+b^2\right)x - b^2(3a+b)x^2 - b^3\right.\right.}{2d}$$
$$= \frac{(a+b)^3\log(\cosh(c+dx))}{d} - \frac{(a+b)^3\tanh^2(c+dx)}{2d} - \frac{b\left(3a^2+3ab+b\right.}{4}$$

Mathematica [A] time = 0.288521, size = 98, normalized size = 0.92

$$\frac{-\frac{1}{2}b\left(3a^2+3ab+b^2\right)\tanh^4(c+dx) - \frac{1}{3}b^2(3a+b)\tanh^6(c+dx) - (a+b)^3\tanh^2(c+dx) + 2(a+b)^3\log(\cosh(c+d}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (2*(a + b)^3*Log[Cosh[c + d*x]] - (a + b)^3*Tanh[c + d*x]^2 - (b*(3*a^2 + 3
*a*b + b^2)*Tanh[c + d*x]^4)/2 - (b^2*(3*a + b)*Tanh[c + d*x]^6)/3 - (b^3*T
anh[c + d*x]^8)/4)/(2*d)
```

Maple [B] time = 0.007, size = 307, normalized size = 2.9

$$-\frac{a^3\ln(\tanh(dx+c)-1)}{2d} - \frac{3\ln(\tanh(dx+c)-1)a^2b}{2d} - \frac{3\ln(\tanh(dx+c)-1)ab^2}{2d} - \frac{\ln(\tanh(dx+c)-1)b^3}{2d} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] -1/2/d*a^3*ln(tanh(d*x+c)-1)-3/2/d*ln(tanh(d*x+c)-1)*a^2*b-3/2/d*ln(tanh(d*
x+c)-1)*a*b^2-1/2/d*ln(tanh(d*x+c)-1)*b^3-1/6*b^3*tanh(d*x+c)^6/d-1/4*b^3*t
anh(d*x+c)^4/d-1/2/d*a^3*tanh(d*x+c)^2-1/2/d*b^3*tanh(d*x+c)^2-1/8*b^3*tanh
(d*x+c)^8/d-3/4/d*tanh(d*x+c)^4*a^2*b-3/4/d*tanh(d*x+c)^4*a*b^2-3/2*a^2*b*t
anh(d*x+c)^2/d-3/2/d*tanh(d*x+c)^2*a*b^2-1/2/d*tanh(d*x+c)^6*a*b^2-1/2/d*ln
(tanh(d*x+c)+1)*a^3-3/2/d*ln(tanh(d*x+c)+1)*a^2*b-3/2/d*ln(tanh(d*x+c)+1)*a
*b^2-1/2/d*ln(tanh(d*x+c)+1)*b^3
```

Maxima [B] time = 1.59218, size = 729, normalized size = 6.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] a*b^2*(3*x + 3*c/d + 3*log(e^(-2*d*x - 2*c) + 1)/d + 2*(9*e^(-2*d*x - 2*c)
+ 18*e^(-4*d*x - 4*c) + 34*e^(-6*d*x - 6*c) + 18*e^(-8*d*x - 8*c) + 9*e^(-1
```

```
0*d*x - 10*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x
- 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c) +
1))) + 1/3*b^3*(3*x + 3*c/d + 3*log(e^(-2*d*x - 2*c) + 1)/d + 8*(3*e^(-2*d*
x - 2*c) + 9*e^(-4*d*x - 4*c) + 25*e^(-6*d*x - 6*c) + 26*e^(-8*d*x - 8*c) +
 25*e^(-10*d*x - 10*c) + 9*e^(-12*d*x - 12*c) + 3*e^(-14*d*x - 14*c))/(d*(8
*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*
x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 1
4*c) + e^(-16*d*x - 16*c) + 1))) + 3*a^2*b*(x + c/d + log(e^(-2*d*x - 2*c)
+ 1)/d + 4*(e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e
^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c
) + 1))) + a^3*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/
(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1)))
```

Fricas [B] time = 3.11791, size = 19047, normalized size = 178.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^16 + 48*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^15 + 3*(a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*d*x*sinh(d*x + c)^16 - 6*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^
3 - 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^14 + 6*(60*(a^3 +
3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^2 - a^3 - 6*a^2*b - 9*a*b^2 - 4*
b^3 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*sinh(d*x + c)^14 + 84*(20*(a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^3 - (a^3 + 6*a^2*b + 9*a*b^2
+ 4*b^3 - 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x +
c)^13 - 12*(3*a^3 + 15*a^2*b + 18*a*b^2 + 6*b^3 - 7*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*d*x)*cosh(d*x + c)^12 + 6*(910*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x
*cosh(d*x + c)^4 - 6*a^3 - 30*a^2*b - 36*a*b^2 - 12*b^3 + 14*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*d*x - 91*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3 - 4*(a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^12 + 24*(546*(a^3
+ 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^5 - 91*(a^3 + 6*a^2*b + 9*a*b^
2 + 4*b^3 - 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 - 6*(3*a
^3 + 15*a^2*b + 18*a*b^2 + 6*b^3 - 7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*c
osh(d*x + c))*sinh(d*x + c)^11 - 2*(45*a^3 + 198*a^2*b + 237*a*b^2 + 100*b^
3 - 84*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^10 + 2*(12012*(a^
3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^6 - 3003*(a^3 + 6*a^2*b + 9*
a*b^2 + 4*b^3 - 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 - 45
*a^3 - 198*a^2*b - 237*a*b^2 - 100*b^3 + 84*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
*d*x - 396*(3*a^3 + 15*a^2*b + 18*a*b^2 + 6*b^3 - 7*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 4*(8580*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*d*x*cosh(d*x + c)^7 - 3003*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3
- 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 - 660*(3*a^3 + 15*
a^2*b + 18*a*b^2 + 6*b^3 - 7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x
+ c)^3 - 5*(45*a^3 + 198*a^2*b + 237*a*b^2 + 100*b^3 - 84*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^9 - 2*(60*a^3 + 252*a^2*b
+ 312*a*b^2 + 104*b^3 - 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x +
 c)^8 + 2*(19305*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^8 - 9009
*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3 - 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*
cosh(d*x + c)^6 - 2970*(3*a^3 + 15*a^2*b + 18*a*b^2 + 6*b^3 - 7*(a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 - 60*a^3 - 252*a^2*b - 312*a*b^2
- 104*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 45*(45*a^3 + 198*a^2*
b + 237*a*b^2 + 100*b^3 - 84*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x
+ c)^2)*sinh(d*x + c)^8 + 16*(2145*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh
```

(d*x + c)^9 - 1287*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3 - 4*(a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*d*x)*cosh(d*x + c)^7 - 594*(3*a^3 + 15*a^2*b + 18*a*b^2 + 6*b^
3 - 7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 - 15*(45*a^3 + 1
98*a^2*b + 237*a*b^2 + 100*b^3 - 84*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*co
sh(d*x + c)^3 - (60*a^3 + 252*a^2*b + 312*a*b^2 + 104*b^3 - 105*(a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(45*a^3 + 198*
a^2*b + 237*a*b^2 + 100*b^3 - 84*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(
d*x + c)^6 + 2*(12012*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^10
- 9009*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3 - 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
*d*x)*cosh(d*x + c)^8 - 5544*(3*a^3 + 15*a^2*b + 18*a*b^2 + 6*b^3 - 7*(a^3
+ 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^6 - 210*(45*a^3 + 198*a^2*b +
237*a*b^2 + 100*b^3 - 84*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c
)^4 - 45*a^3 - 198*a^2*b - 237*a*b^2 - 100*b^3 + 84*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*d*x - 28*(60*a^3 + 252*a^2*b + 312*a*b^2 + 104*b^3 - 105*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(3276*(a^3
+ 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^11 - 3003*(a^3 + 6*a^2*b + 9*
a*b^2 + 4*b^3 - 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^9 - 23
76*(3*a^3 + 15*a^2*b + 18*a*b^2 + 6*b^3 - 7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
*d*x)*cosh(d*x + c)^7 - 126*(45*a^3 + 198*a^2*b + 237*a*b^2 + 100*b^3 - 84*
(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 - 28*(60*a^3 + 252*a^2
*b + 312*a*b^2 + 104*b^3 - 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*
x + c)^3 - 3*(45*a^3 + 198*a^2*b + 237*a*b^2 + 100*b^3 - 84*(a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 12*(3*a^3 + 15*a^2*b
+ 18*a*b^2 + 6*b^3 - 7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^
4 + 2*(2730*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^12 - 3003*(a^
3 + 6*a^2*b + 9*a*b^2 + 4*b^3 - 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh
(d*x + c)^10 - 2970*(3*a^3 + 15*a^2*b + 18*a*b^2 + 6*b^3 - 7*(a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^8 - 210*(45*a^3 + 198*a^2*b + 237*a*b^
2 + 100*b^3 - 84*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^6 - 70*
(60*a^3 + 252*a^2*b + 312*a*b^2 + 104*b^3 - 105*(a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*d*x)*cosh(d*x + c)^4 - 18*a^3 - 90*a^2*b - 108*a*b^2 - 36*b^3 + 42*(a^
3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 15*(45*a^3 + 198*a^2*b + 237*a*b^2 + 100
b^3 - 84(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x +
c)^4 + 8*(210*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^13 - 273*(a
^3 + 6*a^2*b + 9*a*b^2 + 4*b^3 - 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cos
h(d*x + c)^11 - 330*(3*a^3 + 15*a^2*b + 18*a*b^2 + 6*b^3 - 7*(a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^9 - 30*(45*a^3 + 198*a^2*b + 237*a*b^2
+ 100*b^3 - 84*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^7 - 14*(
60*a^3 + 252*a^2*b + 312*a*b^2 + 104*b^3 - 105*(a^3 + 3*a^2*b + 3*a*b^2 + b
^3)*d*x)*cosh(d*x + c)^5 - 5*(45*a^3 + 198*a^2*b + 237*a*b^2 + 100*b^3 - 84
*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 - 6*(3*a^3 + 15*a^2*b
+ 18*a*b^2 + 6*b^3 - 7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))
*sinh(d*x + c)^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 6*(a^3 + 6*a^2*b
+ 9*a*b^2 + 4*b^3 - 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2
+ 2*(180*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^14 - 273*(a^3 +
6*a^2*b + 9*a*b^2 + 4*b^3 - 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*
x + c)^12 - 396*(3*a^3 + 15*a^2*b + 18*a*b^2 + 6*b^3 - 7*(a^3 + 3*a^2*b + 3
*a*b^2 + b^3)*d*x)*cosh(d*x + c)^10 - 45*(45*a^3 + 198*a^2*b + 237*a*b^2 +
100*b^3 - 84*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^8 - 28*(60*
a^3 + 252*a^2*b + 312*a*b^2 + 104*b^3 - 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
*d*x)*cosh(d*x + c)^6 - 15*(45*a^3 + 198*a^2*b + 237*a*b^2 + 100*b^3 - 84*(
a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 - 3*a^3 - 18*a^2*b - 27
*a*b^2 - 12*b^3 + 12*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 36*(3*a^3 + 15*a
^2*b + 18*a*b^2 + 6*b^3 - 7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x +
c)^2)*sinh(d*x + c)^2 - 3*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^1
6 + 16*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^15 + (a^
3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^16 + 8*(a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*cosh(d*x + c)^14 + 8*(a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 15*(a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^14 + 112*(5*(a^3 + 3*a^

2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh
(d*x + c))*sinh(d*x + c)^13 + 28*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x +
 c)^12 + 28*(65*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + a^3 + 3*a
^2*b + 3*a*b^2 + b^3 + 26*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*
sinh(d*x + c)^12 + 112*(39*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5
+ 26*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + 3*(a^3 + 3*a^2*b + 3
*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^11 + 56*(a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*cosh(d*x + c)^10 + 56*(143*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x
 + c)^6 + 143*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + a^3 + 3*a^2
*b + 3*a*b^2 + b^3 + 33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*si
nh(d*x + c)^10 + 16*(715*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 +
1001*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 385*(a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*cosh(d*x + c)^3 + 35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d
*x + c))*sinh(d*x + c)^9 + 70*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)
^8 + 2*(6435*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 12012*(a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 6930*(a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*cosh(d*x + c)^4 + 35*a^3 + 105*a^2*b + 105*a*b^2 + 35*b^3 + 1260*(a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 16*(715*(a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^9 + 1716*(a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*cosh(d*x + c)^7 + 1386*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c
)^5 + 420*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + 35*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 56*(a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*cosh(d*x + c)^6 + 56*(143*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh
(d*x + c)^10 + 429*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 462*(a
^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 210*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*cosh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 35*(a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 112*(39*(a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*cosh(d*x + c)^11 + 143*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c
osh(d*x + c)^9 + 198*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 126*
(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 35*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*cosh(d*x + c)^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)
)*sinh(d*x + c)^5 + 28*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 28
(65(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^12 + 286*(a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*cosh(d*x + c)^10 + 495*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos
h(d*x + c)^8 + 420*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 175*(a
^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b
^3 + 30*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 +
112*(5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^13 + 26*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*cosh(d*x + c)^11 + 55*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos
h(d*x + c)^9 + 60*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 35*(a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 10*(a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*cosh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh
(d*x + c)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 8*(a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*cosh(d*x + c)^2 + 8*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^
14 + 91*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^12 + 231*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*cosh(d*x + c)^10 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*
cosh(d*x + c)^8 + 245*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 105
*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2
+ b^3 + 21*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2
 + 16*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^15 + 7*(a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*cosh(d*x + c)^13 + 21*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh
(d*x + c)^11 + 35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^9 + 35*(a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 21*(a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*cosh(d*x + c)^5 + 7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 +
 (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d
*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(12*(a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*d*x*cosh(d*x + c)^15 - 21*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3 - 4*(a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^13 - 36*(3*a^3 + 15*a^2*b + 18
*a*b^2 + 6*b^3 - 7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^11 -

```
5*(45*a^3 + 198*a^2*b + 237*a*b^2 + 100*b^3 - 84*(a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*d*x)*cosh(d*x + c)^9 - 4*(60*a^3 + 252*a^2*b + 312*a*b^2 + 104*b^3 -
105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^7 - 3*(45*a^3 + 198*
a^2*b + 237*a*b^2 + 100*b^3 - 84*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(
d*x + c)^5 - 12*(3*a^3 + 15*a^2*b + 18*a*b^2 + 6*b^3 - 7*(a^3 + 3*a^2*b + 3
*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 - 3*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3 - 4
*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh
(d*x + c)^16 + 16*d*cosh(d*x + c)*sinh(d*x + c)^15 + d*sinh(d*x + c)^16 + 8
*d*cosh(d*x + c)^14 + 8*(15*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^14 + 112*(
5*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^13 + 28*d*cosh(d*x + c
)^12 + 28*(65*d*cosh(d*x + c)^4 + 26*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^1
2 + 112*(39*d*cosh(d*x + c)^5 + 26*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*s
inh(d*x + c)^11 + 56*d*cosh(d*x + c)^10 + 56*(143*d*cosh(d*x + c)^6 + 143*d
*cosh(d*x + c)^4 + 33*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^10 + 16*(715*d*c
osh(d*x + c)^7 + 1001*d*cosh(d*x + c)^5 + 385*d*cosh(d*x + c)^3 + 35*d*cosh
(d*x + c))*sinh(d*x + c)^9 + 70*d*cosh(d*x + c)^8 + 2*(6435*d*cosh(d*x + c)
^8 + 12012*d*cosh(d*x + c)^6 + 6930*d*cosh(d*x + c)^4 + 1260*d*cosh(d*x + c
)^2 + 35*d)*sinh(d*x + c)^8 + 16*(715*d*cosh(d*x + c)^9 + 1716*d*cosh(d*x +
 c)^7 + 1386*d*cosh(d*x + c)^5 + 420*d*cosh(d*x + c)^3 + 35*d*cosh(d*x + c)
)*sinh(d*x + c)^7 + 56*d*cosh(d*x + c)^6 + 56*(143*d*cosh(d*x + c)^10 + 429
*d*cosh(d*x + c)^8 + 462*d*cosh(d*x + c)^6 + 210*d*cosh(d*x + c)^4 + 35*d*c
osh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 112*(39*d*cosh(d*x + c)^11 + 143*d*co
sh(d*x + c)^9 + 198*d*cosh(d*x + c)^7 + 126*d*cosh(d*x + c)^5 + 35*d*cosh(d
*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 28*d*cosh(d*x + c)^4 + 28*
(65*d*cosh(d*x + c)^12 + 286*d*cosh(d*x + c)^10 + 495*d*cosh(d*x + c)^8 + 4
20*d*cosh(d*x + c)^6 + 175*d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 + d)*si
nh(d*x + c)^4 + 112*(5*d*cosh(d*x + c)^13 + 26*d*cosh(d*x + c)^11 + 55*d*co
sh(d*x + c)^9 + 60*d*cosh(d*x + c)^7 + 35*d*cosh(d*x + c)^5 + 10*d*cosh(d*x
 + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^3 + 8*d*cosh(d*x + c)^2 + 8*(15*d*
cosh(d*x + c)^14 + 91*d*cosh(d*x + c)^12 + 231*d*cosh(d*x + c)^10 + 315*d*c
osh(d*x + c)^8 + 245*d*cosh(d*x + c)^6 + 105*d*cosh(d*x + c)^4 + 21*d*cosh(
d*x + c)^2 + d)*sinh(d*x + c)^2 + 16*(d*cosh(d*x + c)^15 + 7*d*cosh(d*x + c
)^13 + 21*d*cosh(d*x + c)^11 + 35*d*cosh(d*x + c)^9 + 35*d*cosh(d*x + c)^7
+ 21*d*cosh(d*x + c)^5 + 7*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x +
c) + d)
```

Sympy [A] time = 2.40044, size = 279, normalized size = 2.61

$$\begin{cases} a^3x - \frac{a^3\log(\tanh(c+dx)+1)}{d} - \frac{a^3\tanh^2(c+dx)}{2d} + 3a^2bx - \frac{3a^2b\log(\tanh(c+dx)+1)}{d} - \frac{3a^2b\tanh^4(c+dx)}{4d} - \frac{3a^2b\tanh^2(c+dx)}{2d} + 3ab^2x - \\ x\left(a+b\tanh^2(c)\right)^3\tanh^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**3*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((a**3*x - a**3*log(tanh(c + d*x) + 1)/d - a**3*tanh(c + d*x)**2/(
2*d) + 3*a**2*b*x - 3*a**2*b*log(tanh(c + d*x) + 1)/d - 3*a**2*b*tanh(c + d
*x)**4/(4*d) - 3*a**2*b*tanh(c + d*x)**2/(2*d) + 3*a*b**2*x - 3*a*b**2*log(
tanh(c + d*x) + 1)/d - a*b**2*tanh(c + d*x)**6/(2*d) - 3*a*b**2*tanh(c + d*
x)**4/(4*d) - 3*a*b**2*tanh(c + d*x)**2/(2*d) + b**3*x - b**3*log(tanh(c +
d*x) + 1)/d - b**3*tanh(c + d*x)**8/(8*d) - b**3*tanh(c + d*x)**6/(6*d) - b
**3*tanh(c + d*x)**4/(4*d) - b**3*tanh(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a
+ b*tanh(c)**2)**3*tanh(c)**3, True))
```

Giac [B] time = 1.46414, size = 421, normalized size = 3.93

$$-\frac{\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)(dx+c)}{d}+\frac{\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)\log\left(e^{(2\,dx+2\,c)}+1\right)}{d}+\frac{2\left(3\left(a^{3}+6\,a^{2}b+9\,ab^{2}+4\,b^{3}\right)e^{(14\,dx}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c)/d + (a^3 + 3*a^2*b + 3*a*b^2 + b
^3)*log(e^(2*d*x + 2*c) + 1)/d + 2/3*(3*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*e
^(14*d*x + 14*c) + 18*(a^3 + 5*a^2*b + 6*a*b^2 + 2*b^3)*e^(12*d*x + 12*c) +
 (45*a^3 + 198*a^2*b + 237*a*b^2 + 100*b^3)*e^(10*d*x + 10*c) + 4*(15*a^3 +
 63*a^2*b + 78*a*b^2 + 26*b^3)*e^(8*d*x + 8*c) + (45*a^3 + 198*a^2*b + 237*
a*b^2 + 100*b^3)*e^(6*d*x + 6*c) + 18*(a^3 + 5*a^2*b + 6*a*b^2 + 2*b^3)*e^(
4*d*x + 4*c) + 3*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*e^(2*d*x + 2*c))/(d*(e^(
2*d*x + 2*c) + 1)^8)
```

3.158 $\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=94

$$-\frac{b\left(3a^2+3ab+b^2\right)\tanh^3(c+dx)}{3d}-\frac{b^2(3a+b)\tanh^5(c+dx)}{5d}-\frac{(a+b)^3\tanh(c+dx)}{d}+x(a+b)^3-\frac{b^3\tanh^7(c+dx)}{7d}$$

```
[Out] (a + b)^3*x - ((a + b)^3*Tanh[c + d*x])/d - (b*(3*a^2 + 3*a*b + b^2)*Tanh[c
 + d*x]^3)/(3*d) - (b^2*(3*a + b)*Tanh[c + d*x]^5)/(5*d) - (b^3*Tanh[c + d*
x]^7)/(7*d)
```

Rubi [A] time = 0.0927085, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 461, 206}

$$-\frac{b\left(3a^2+3ab+b^2\right)\tanh^3(c+dx)}{3d}-\frac{b^2(3a+b)\tanh^5(c+dx)}{5d}-\frac{(a+b)^3\tanh(c+dx)}{d}+x(a+b)^3-\frac{b^3\tanh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (a + b)^3*x - ((a + b)^3*Tanh[c + d*x])/d - (b*(3*a^2 + 3*a*b + b^2)*Tanh[c
 + d*x]^3)/(3*d) - (b^2*(3*a + b)*Tanh[c + d*x]^5)/(5*d) - (b^3*Tanh[c + d*
x]^7)/(7*d)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+bx^2\right)^3}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int\left(-(a+b)^3 - b\left(3a^2+3ab+b^2\right)x^2 - b^2(3a+b)x^4 - b^3x^6 + \frac{a^3+3a}{\ldots}\right.\right.}{d}$$

$$= -\frac{(a+b)^3\tanh(c+dx)}{d} - \frac{b\left(3a^2+3ab+b^2\right)\tanh^3(c+dx)}{3d} - \frac{b^2(3a+b)\,\mathrm{t}}{5}$$

$$= (a+b)^3x - \frac{(a+b)^3\tanh(c+dx)}{d} - \frac{b\left(3a^2+3ab+b^2\right)\tanh^3(c+dx)}{3d} - \frac{b^2}{}$$

Mathematica [A] time = 1.61293, size = 108, normalized size = 1.15

$$\frac{\tanh(c+dx)\left(-35b\left(3a^2+3ab+b^2\right)\tanh^2(c+dx) - 21b^2(3a+b)\tanh^4(c+dx) + \frac{105(a+b)^3\tanh^{-1}\left(\sqrt{\tanh^2(c+dx)}\right)}{\sqrt{\tanh^2(c+dx)}} - 105(a\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (Tanh[c + d*x]*(-105*(a + b)^3 - 35*b*(3*a^2 + 3*a*b + b^2)*Tanh[c + d*x]^2
 - 21*b^2*(3*a + b)*Tanh[c + d*x]^4 - 15*b^3*Tanh[c + d*x]^6 + (105*(a + b)
^3*ArcTanh[Sqrt[Tanh[c + d*x]^2]])/Sqrt[Tanh[c + d*x]^2]))/(105*d)
```

Maple [B] time = 0.005, size = 299, normalized size = 3.2

$$-\frac{a^3\ln(\tanh(dx+c)-1)}{2d} - \frac{3\ln(\tanh(dx+c)-1)a^2b}{2d} - \frac{3\ln(\tanh(dx+c)-1)ab^2}{2d} - \frac{\ln(\tanh(dx+c)-1)b^3}{2d} - \frac{b^3(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] -1/2/d*a^3*ln(tanh(d*x+c)-1)-3/2/d*ln(tanh(d*x+c)-1)*a^2*b-3/2/d*ln(tanh(d*
x+c)-1)*a*b^2-1/2/d*ln(tanh(d*x+c)-1)*b^3-1/7*b^3*tanh(d*x+c)^7/d-1/5*b^3*t
anh(d*x+c)^5/d-1/3*b^3*tanh(d*x+c)^3/d-b^3*tanh(d*x+c)/d-a^2*b*tanh(d*x+c)^
3/d-a*b^2*tanh(d*x+c)^3/d-3/5*a*b^2*tanh(d*x+c)^5/d-3*a^2*b*tanh(d*x+c)/d+1
/2/d*ln(tanh(d*x+c)+1)*a^3+3/2/d*ln(tanh(d*x+c)+1)*a^2*b+3/2/d*ln(tanh(d*x+
c)+1)*a*b^2+1/2/d*ln(tanh(d*x+c)+1)*b^3-a^3*tanh(d*x+c)/d-3*a*b^2*tanh(d*x+
c)/d
```

Maxima [B] time = 1.06923, size = 540, normalized size = 5.74

$$\frac{1}{105}b^3\left(105x + \frac{105c}{d} - \frac{8\left(203e^{(-2dx-2c)} + 609e^{(-4dx-4c)} + 770e^{(-6dx-6c)} + 770e^{(-8dx-8c)} + 315e^{(-10dx-10c)} + 105e^{(-12}\right.}{d\left(7e^{(-2dx-2c)} + 21e^{(-4dx-4c)} + 35e^{(-6dx-6c)} + 35e^{(-8dx-8c)} + 21e^{(-10dx-10c)} + 7e^{(-12dx-12c)} + e^{(-}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/105*b^3*(105*x + 105*c/d - 8*(203*e^(-2*d*x - 2*c) + 609*e^(-4*d*x - 4*c)
 + 770*e^(-6*d*x - 6*c) + 770*e^(-8*d*x - 8*c) + 315*e^(-10*d*x - 10*c) + 1
05*e^(-12*d*x - 12*c) + 44)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) +
35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-1
2*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 1/5*a*b^2*(15*x + 15*c/d - 2*(7
0*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 90*e^(-6*d*x - 6*c) + 45*e^(-8*
d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*
x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + a^2*b*(3*x + 3*
c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(3*e^(-2*d*x - 2*c
) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + a^3*(x + c/d - 2/(d*(e^(
-2*d*x - 2*c) + 1)))
```

Fricas [B] time = 1.97666, size = 2693, normalized size = 28.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/105*((105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*d*x)*cosh(d*x + c)^7 + 7*(105*a^3 + 420*a^2*b + 483*a*b^2 + 17
6*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)*sinh(d*x + c
)^6 - (105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3)*sinh(d*x + c)^7 + 7*(105*
a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
*d*x)*cosh(d*x + c)^5 - 7*(75*a^3 + 240*a^2*b + 213*a*b^2 + 56*b^3 + 3*(105
*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 +
35*((105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*d*x)*cosh(d*x + c)^3 + (105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3
 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^4
+ 21*(105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b + 3*a*
b^2 + b^3)*d*x)*cosh(d*x + c)^3 - 7*(5*(105*a^3 + 420*a^2*b + 483*a*b^2 + 1
76*b^3)*cosh(d*x + c)^4 + 135*a^3 + 360*a^2*b + 369*a*b^2 + 168*b^3 + 10*(7
5*a^3 + 240*a^2*b + 213*a*b^2 + 56*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 +
7*(3*(105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b + 3*a*
b^2 + b^3)*d*x)*cosh(d*x + c)^5 + 10*(105*a^3 + 420*a^2*b + 483*a*b^2 + 176
*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 + 9*(105*a^
3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d
*x)*cosh(d*x + c))*sinh(d*x + c)^2 + 35*(105*a^3 + 420*a^2*b + 483*a*b^2 +
176*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c) - 7*((105*
a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3)*cosh(d*x + c)^6 + 5*(75*a^3 + 240*a^
2*b + 213*a*b^2 + 56*b^3)*cosh(d*x + c)^4 + 75*a^3 + 180*a^2*b + 225*a*b^2
+ 9*(45*a^3 + 120*a^2*b + 123*a*b^2 + 56*b^3)*cosh(d*x + c)^2)*sinh(d*x + c
))/(d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + 7*d*cosh(d*x +
c)^5 + 35*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^4 + 21*d*cosh
(d*x + c)^3 + 7*(3*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 9*d*cosh(d*x
+ c))*sinh(d*x + c)^2 + 35*d*cosh(d*x + c))
```

Sympy [A] time = 1.69827, size = 192, normalized size = 2.04

$$\begin{cases} a^3x - \frac{a^3\tanh(c+dx)}{d} + 3a^2bx - \frac{a^2b\tanh^3(c+dx)}{d} - \frac{3a^2b\tanh(c+dx)}{d} + 3ab^2x - \frac{3ab^2\tanh^5(c+dx)}{5d} - \frac{ab^2\tanh^3(c+dx)}{d} - \frac{3ab^2\tanh(c+dx)}{d} \\ x\left(a+b\tanh^2(c)\right)^3\tanh^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**2*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((a**3*x - a**3*tanh(c + d*x)/d + 3*a**2*b*x - a**2*b*tanh(c + d*x
)**3/d - 3*a**2*b*tanh(c + d*x)/d + 3*a*b**2*x - 3*a*b**2*tanh(c + d*x)**5/
(5*d) - a*b**2*tanh(c + d*x)**3/d - 3*a*b**2*tanh(c + d*x)/d + b**3*x - b**
3*tanh(c + d*x)**7/(7*d) - b**3*tanh(c + d*x)**5/(5*d) - b**3*tanh(c + d*x)
**3/(3*d) - b**3*tanh(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c)**2)**3*tanh(
c)**2, True))
```

Giac [B] time = 1.46132, size = 564, normalized size = 6.

$$\frac{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)(dx+c)}{d}+\frac{2\left(105\,a^3e^{(12\,dx+12\,c)}+630\,a^2be^{(12\,dx+12\,c)}+945\,ab^2e^{(12\,dx+12\,c)}+420\,b^3e^{(12\,dx+12\,c)}+63\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c)/d + 2/105*(105*a^3*e^(12*d*x + 12
*c) + 630*a^2*b*e^(12*d*x + 12*c) + 945*a*b^2*e^(12*d*x + 12*c) + 420*b^3*e
^(12*d*x + 12*c) + 630*a^3*e^(10*d*x + 10*c) + 3150*a^2*b*e^(10*d*x + 10*c)
 + 3780*a*b^2*e^(10*d*x + 10*c) + 1260*b^3*e^(10*d*x + 10*c) + 1575*a^3*e^(
8*d*x + 8*c) + 6720*a^2*b*e^(8*d*x + 8*c) + 7665*a*b^2*e^(8*d*x + 8*c) + 30
80*b^3*e^(8*d*x + 8*c) + 2100*a^3*e^(6*d*x + 6*c) + 7980*a^2*b*e^(6*d*x + 6
*c) + 9240*a*b^2*e^(6*d*x + 6*c) + 3080*b^3*e^(6*d*x + 6*c) + 1575*a^3*e^(4
*d*x + 4*c) + 5670*a^2*b*e^(4*d*x + 4*c) + 6363*a*b^2*e^(4*d*x + 4*c) + 243
6*b^3*e^(4*d*x + 4*c) + 630*a^3*e^(2*d*x + 2*c) + 2310*a^2*b*e^(2*d*x + 2*c
) + 2436*a*b^2*e^(2*d*x + 2*c) + 812*b^3*e^(2*d*x + 2*c) + 105*a^3 + 420*a^
2*b + 483*a*b^2 + 176*b^3)/(d*(e^(2*d*x + 2*c) + 1)^7)
```

3.159 $\int \tanh(c + dx) \left(a + b \tanh^2(c + dx)\right)^3 \, dx$

Optimal. Leaf size=83

$$-\frac{b(a+b)^2 \tanh^2(c+dx)}{2d} - \frac{(a+b)\left(a+b\tanh^2(c+dx)\right)^2}{4d} - \frac{\left(a+b\tanh^2(c+dx)\right)^3}{6d} + \frac{(a+b)^3 \log(\cosh(c+dx))}{d}$$

```
[Out] ((a + b)^3*Log[Cosh[c + d*x]])/d - (b*(a + b)^2*Tanh[c + d*x]^2)/(2*d) - ((
a + b)*(a + b*Tanh[c + d*x]^2)^2)/(4*d) - (a + b*Tanh[c + d*x]^2)^3/(6*d)
```

Rubi [A] time = 0.101441, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3670, 444, 43}

$$-\frac{b(a+b)^2 \tanh^2(c+dx)}{2d} - \frac{(a+b)\left(a+b\tanh^2(c+dx)\right)^2}{4d} - \frac{\left(a+b\tanh^2(c+dx)\right)^3}{6d} + \frac{(a+b)^3 \log(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((a + b)^3*Log[Cosh[c + d*x]])/d - (b*(a + b)^2*Tanh[c + d*x]^2)/(2*d) - ((
a + b)*(a + b*Tanh[c + d*x]^2)^2)/(4*d) - (a + b*Tanh[c + d*x]^2)^3/(6*d)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int \frac{x(a+bx^2)^3}{1-x^2}dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^3}{1-x}dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(-b(a+b)^2+\frac{(a+b)^3}{1-x}-b(a+b)(a+bx)-b(a+bx)^2\right)dx, x, \tanh^2\right.}{2d}$$
$$= \frac{(a+b)^3\log(\cosh(c+dx))}{d} - \frac{b(a+b)^2\tanh^2(c+dx)}{2d} - \frac{(a+b)\left(a+b\tanh\right.}{4d}$$

Mathematica [A] time = 0.21378, size = 76, normalized size = 0.92

$$-\frac{b(a+b)^2\tanh^2(c+dx)+\frac{1}{2}(a+b)\left(a+b\tanh^2(c+dx)\right)^2+\frac{1}{3}\left(a+b\tanh^2(c+dx)\right)^3-2(a+b)^3\log(\cosh(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -(-2*(a + b)^3*Log[Cosh[c + d*x]] + b*(a + b)^2*Tanh[c + d*x]^2 + ((a + b)*
(a + b*Tanh[c + d*x]^2)^2)/2 + (a + b*Tanh[c + d*x]^2)^3/3)/(2*d)
```

Maple [B] time = 0.006, size = 241, normalized size = 2.9

$$-\frac{a^3\ln(\tanh(dx+c)-1)}{2d} - \frac{3\ln(\tanh(dx+c)-1)a^2b}{2d} - \frac{3\ln(\tanh(dx+c)-1)ab^2}{2d} - \frac{\ln(\tanh(dx+c)-1)b^3}{2d} - \frac{b^3(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] -1/2/d*a^3*ln(tanh(d*x+c)-1)-3/2/d*ln(tanh(d*x+c)-1)*a^2*b-3/2/d*ln(tanh(d*
x+c)-1)*a*b^2-1/2/d*ln(tanh(d*x+c)-1)*b^3-1/6*b^3*tanh(d*x+c)^6/d-1/4*b^3*t
anh(d*x+c)^4/d-1/2/d*b^3*tanh(d*x+c)^2-3/4/d*tanh(d*x+c)^4*a*b^2-3/2*a^2*b*
tanh(d*x+c)^2/d-3/2/d*tanh(d*x+c)^2*a*b^2-1/2/d*ln(tanh(d*x+c)+1)*a^3-3/2/d
*ln(tanh(d*x+c)+1)*a^2*b-3/2/d*ln(tanh(d*x+c)+1)*a*b^2-1/2/d*ln(tanh(d*x+c)
+1)*b^3
```

Maxima [B] time = 1.5997, size = 474, normalized size = 5.71

$$\frac{1}{3}b^3\left(3x+\frac{3c}{d}+\frac{3\log\left(e^{(-2dx-2c)}+1\right)}{d}+\frac{2\left(9e^{(-2dx-2c)}+18e^{(-4dx-4c)}+34e^{(-6dx-6c)}+18e^{(-8dx-8c)}+9e^{(-10dx}\right.}{d\left(6e^{(-2dx-2c)}+15e^{(-4dx-4c)}+20e^{(-6dx-6c)}+15e^{(-8dx-8c)}+6e^{(-10dx-10c)}+e^{(}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*(3*x + 3*c/d + 3*log(e^(-2*d*x - 2*c) + 1)/d + 2*(9*e^(-2*d*x - 2*c
) + 18*e^(-4*d*x - 4*c) + 34*e^(-6*d*x - 6*c) + 18*e^(-8*d*x - 8*c) + 9*e^(
```

```
-10*d*x - 10*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*
x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c)
+ 1))) + 3*a*b^2*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 4*(e^(-2*d*x - 2*
c) + e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*
d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) + 3*a^2*b*(x + c/
d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c)
 + e^(-4*d*x - 4*c) + 1))) + a^3*log(cosh(d*x + c))/d
```

Fricas [B] time = 2.62292, size = 10689, normalized size = 128.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^12 + 36*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^11 + 3*(a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*d*x*sinh(d*x + c)^12 - 18*(a^2*b + 2*a*b^2 + b^3 - (a^3
+ 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^10 + 18*(11*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*d*x*cosh(d*x + c)^2 - a^2*b - 2*a*b^2 - b^3 + (a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*d*x)*sinh(d*x + c)^10 + 60*(11*(a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*d*x*cosh(d*x + c)^3 - 3*(a^2*b + 2*a*b^2 + b^3 - (a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^9 - 9*(8*a^2*b + 12*a*b^2 + 4
*b^3 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^8 + 9*(165*(a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^4 - 8*a^2*b - 12*a*b^2 - 4*b^
3 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 90*(a^2*b + 2*a*b^2 + b^3 - (a^
3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 72*(33
*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^5 - 30*(a^2*b + 2*a*b^2
+ b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 - (8*a^2*b + 1
2*a*b^2 + 4*b^3 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sin
h(d*x + c)^7 - 4*(27*a^2*b + 36*a*b^2 + 17*b^3 - 15*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*d*x)*cosh(d*x + c)^6 + 4*(693*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*
cosh(d*x + c)^6 - 945*(a^2*b + 2*a*b^2 + b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b
^3)*d*x)*cosh(d*x + c)^4 - 27*a^2*b - 36*a*b^2 - 17*b^3 + 15*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*d*x - 63*(8*a^2*b + 12*a*b^2 + 4*b^3 - 5*(a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 24*(99*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^7 - 189*(a^2*b + 2*a*b^2 + b^3 - (a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 - 21*(8*a^2*b + 12*a*b^2 +
 4*b^3 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 - (27*a^2*b
 + 36*a*b^2 + 17*b^3 - 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c
))*sinh(d*x + c)^5 - 9*(8*a^2*b + 12*a*b^2 + 4*b^3 - 5*(a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*d*x)*cosh(d*x + c)^4 + 3*(495*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d
*x*cosh(d*x + c)^8 - 1260*(a^2*b + 2*a*b^2 + b^3 - (a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*d*x)*cosh(d*x + c)^6 - 210*(8*a^2*b + 12*a*b^2 + 4*b^3 - 5*(a^3 + 3
*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 - 24*a^2*b - 36*a*b^2 - 12*b^3
 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 20*(27*a^2*b + 36*a*b^2 + 17*b^
3 - 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^
4 + 4*(165*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^9 - 540*(a^2*b
 + 2*a*b^2 + b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^7 - 1
26*(8*a^2*b + 12*a*b^2 + 4*b^3 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cos
h(d*x + c)^5 - 20*(27*a^2*b + 36*a*b^2 + 17*b^3 - 15*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*d*x)*cosh(d*x + c)^3 - 9*(8*a^2*b + 12*a*b^2 + 4*b^3 - 5*(a^3 + 3
*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*d*x - 18*(a^2*b + 2*a*b^2 + b^3 - (a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*d*x)*cosh(d*x + c)^2 + 6*(33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*
x*cosh(d*x + c)^10 - 135*(a^2*b + 2*a*b^2 + b^3 - (a^3 + 3*a^2*b + 3*a*b^2
```

+ b^3)*d*x)*cosh(d*x + c)^8 - 42*(8*a^2*b + 12*a*b^2 + 4*b^3 - 5*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^6 - 10*(27*a^2*b + 36*a*b^2 + 17*b
^3 - 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 - 3*a^2*b - 6*
a*b^2 - 3*b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 9*(8*a^2*b + 12*a*b
^2 + 4*b^3 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d
x + c)^2 - 3((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^12 + 12*(a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^11 + (a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*sinh(d*x + c)^12 + 6*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d
x + c)^10 + 6(a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 11*(a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 20*(11*(a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*cosh(d*x + c)^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*
sinh(d*x + c)^9 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 15*(
33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^
2 + b^3 + 18*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)
^8 + 24*(33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 30*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c
osh(d*x + c))*sinh(d*x + c)^7 + 20*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x
 + c)^6 + 4*(231*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 315*(a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 5*a^3 + 15*a^2*b + 15*a*b^2 +
 5*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)
^6 + 24*(33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 63*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*
cosh(d*x + c)^3 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x
 + c)^5 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 15*(33*(a^3
+ 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 84*(a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*cosh(d*x + c)^6 + 70*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 +
 a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 20*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*
x + c)^2)*sinh(d*x + c)^4 + 20*(11*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x
 + c)^9 + 36*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 42*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 20*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
*cosh(d*x + c)^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*
x + c)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 6*(a^3 + 3*a^2*b + 3*a*b^2 + b^3
)*cosh(d*x + c)^2 + 6*(11*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^10
+ 45*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 70*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*cosh(d*x + c)^6 + 50*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*
x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^
3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 12*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c
osh(d*x + c)^11 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^9 + 10*(a
^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 10*(a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*cosh(d*x + c)^5 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3
 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh
(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 12*(3*(a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*d*x*cosh(d*x + c)^11 - 15*(a^2*b + 2*a*b^2 + b^3 - (a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^9 - 6*(8*a^2*b + 12*a*b^2 + 4*b^3 - 5*(a
^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^7 - 2*(27*a^2*b + 36*a*b^2
 + 17*b^3 - 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 - 3*(8*
a^2*b + 12*a*b^2 + 4*b^3 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x
+ c)^3 - 3*(a^2*b + 2*a*b^2 + b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*co
sh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^12 + 12*d*cosh(d*x + c)*sinh(d
*x + c)^11 + d*sinh(d*x + c)^12 + 6*d*cosh(d*x + c)^10 + 6*(11*d*cosh(d*x +
 c)^2 + d)*sinh(d*x + c)^10 + 20*(11*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))
*sinh(d*x + c)^9 + 15*d*cosh(d*x + c)^8 + 15*(33*d*cosh(d*x + c)^4 + 18*d*c
osh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 24*(33*d*cosh(d*x + c)^5 + 30*d*cosh(
d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^7 + 20*d*cosh(d*x + c)^6 + 4*
(231*d*cosh(d*x + c)^6 + 315*d*cosh(d*x + c)^4 + 105*d*cosh(d*x + c)^2 + 5*
d)*sinh(d*x + c)^6 + 24*(33*d*cosh(d*x + c)^7 + 63*d*cosh(d*x + c)^5 + 35*d
*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^5 + 15*d*cosh(d*x + c)^
4 + 15*(33*d*cosh(d*x + c)^8 + 84*d*cosh(d*x + c)^6 + 70*d*cosh(d*x + c)^4
+ 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 20*(11*d*cosh(d*x + c)^9 + 36

```
*d*cosh(d*x + c)^7 + 42*d*cosh(d*x + c)^5 + 20*d*cosh(d*x + c)^3 + 3*d*cosh
(d*x + c))*sinh(d*x + c)^3 + 6*d*cosh(d*x + c)^2 + 6*(11*d*cosh(d*x + c)^10
 + 45*d*cosh(d*x + c)^8 + 70*d*cosh(d*x + c)^6 + 50*d*cosh(d*x + c)^4 + 15*
d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 12*(d*cosh(d*x + c)^11 + 5*d*cosh(
d*x + c)^9 + 10*d*cosh(d*x + c)^7 + 10*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c
)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [A] time = 1.40382, size = 211, normalized size = 2.54

$$\begin{cases} a^3x - \frac{a^3\log(\tanh(c+dx)+1)}{d} + 3a^2bx - \frac{3a^2b\log(\tanh(c+dx)+1)}{d} - \frac{3a^2b\tanh^2(c+dx)}{2d} + 3ab^2x - \frac{3ab^2\log(\tanh(c+dx)+1)}{d} - \frac{3ab^2\tanh^4(c}{4d} \\ x\left(a+b\tanh^2(c)\right)^3\tanh(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((a**3*x - a**3*log(tanh(c + d*x) + 1)/d + 3*a**2*b*x - 3*a**2*b*l
og(tanh(c + d*x) + 1)/d - 3*a**2*b*tanh(c + d*x)**2/(2*d) + 3*a*b**2*x - 3*
a*b**2*log(tanh(c + d*x) + 1)/d - 3*a*b**2*tanh(c + d*x)**4/(4*d) - 3*a*b**
2*tanh(c + d*x)**2/(2*d) + b**3*x - b**3*log(tanh(c + d*x) + 1)/d - b**3*ta
nh(c + d*x)**6/(6*d) - b**3*tanh(c + d*x)**4/(4*d) - b**3*tanh(c + d*x)**2/
(2*d), Ne(d, 0)), (x*(a + b*tanh(c)**2)**3*tanh(c), True))
```

Giac [B] time = 1.38566, size = 296, normalized size = 3.57

$$-\frac{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)(dx+c)}{d}+\frac{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\log\left(e^{(2\,dx+2\,c)}+1\right)}{d}+\frac{2\left(9\left(a^2b+2\,ab^2+b^3\right)e^{(10\,dx+10\,c)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c)/d + (a^3 + 3*a^2*b + 3*a*b^2 + b
^3)*log(e^(2*d*x + 2*c) + 1)/d + 2/3*(9*(a^2*b + 2*a*b^2 + b^3)*e^(10*d*x +
 10*c) + 18*(2*a^2*b + 3*a*b^2 + b^3)*e^(8*d*x + 8*c) + 2*(27*a^2*b + 36*a*
b^2 + 17*b^3)*e^(6*d*x + 6*c) + 18*(2*a^2*b + 3*a*b^2 + b^3)*e^(4*d*x + 4*c
) + 9*(a^2*b + 2*a*b^2 + b^3)*e^(2*d*x + 2*c))/(d*(e^(2*d*x + 2*c) + 1)^6)
```

3.160 $\int \left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=74

$$-\frac{b\left(3a^2+3ab+b^2\right)\tanh(c+dx)}{d}-\frac{b^2(3a+b)\tanh^3(c+dx)}{3d}+x(a+b)^3-\frac{b^3\tanh^5(c+dx)}{5d}$$

```
[Out] (a + b)^3*x - (b*(3*a^2 + 3*a*b + b^2)*Tanh[c + d*x])/d - (b^2*(3*a + b)*Ta
nh[c + d*x]^3)/(3*d) - (b^3*Tanh[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0458704, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3661, 390, 206}

$$-\frac{b\left(3a^2+3ab+b^2\right)\tanh(c+dx)}{d}-\frac{b^2(3a+b)\tanh^3(c+dx)}{3d}+x(a+b)^3-\frac{b^3\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tanh[c + d*x]^2)^3,x]
```


```
[Out] (a + b)^3*x - (b*(3*a^2 + 3*a*b + b^2)*Tanh[c + d*x])/d - (b^2*(3*a + b)*Ta
nh[c + d*x]^3)/(3*d) - (b^3*Tanh[c + d*x]^5)/(5*d)
```


Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\tanh^2(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^3}{1-x^2} dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-b\left(3a^2+3ab+b^2\right)-b^2(3a+b)x^2-b^3x^4+\frac{(a+b)^3}{1-x^2}\right) dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{b\left(3a^2+3ab+b^2\right)\tanh(c+dx)}{d}-\frac{b^2(3a+b)\tanh^3(c+dx)}{3d}-\frac{b^3\tanh^5(c+dx)}{5d}+\cdots$$
$$= (a+b)^3x-\frac{b\left(3a^2+3ab+b^2\right)\tanh(c+dx)}{d}-\frac{b^2(3a+b)\tanh^3(c+dx)}{3d}-\frac{b^3\tanh^5(c}{5d}$$

Mathematica [A] time = 0.570454, size = 95, normalized size = 1.28

$$\frac{\tanh(c+dx)\left(\frac{15(a+b)^3\tanh^{-1}\left(\sqrt{\tanh^2(c+dx)}\right)}{\sqrt{\tanh^2(c+dx)}}-b\left(45a^2+15ab\left(\tanh^2(c+dx)+3\right)+b^2\left(3\tanh^4(c+dx)+5\tanh^2(c+d\right.\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (Tanh[c + d*x]*((15*(a + b)^3*ArcTanh[Sqrt[Tanh[c + d*x]^2]])/Sqrt[Tanh[c +
 d*x]^2] - b*(45*a^2 + 15*a*b*(3 + Tanh[c + d*x]^2) + b^2*(15 + 5*Tanh[c +
d*x]^2 + 3*Tanh[c + d*x]^4))))/(15*d)
```

Maple [B] time = 0.004, size = 235, normalized size = 3.2

$$-\frac{a^3\ln(\tanh(dx+c)-1)}{2d}-\frac{3\ln(\tanh(dx+c)-1)a^2b}{2d}-\frac{3\ln(\tanh(dx+c)-1)ab^2}{2d}-\frac{\ln(\tanh(dx+c)-1)b^3}{2d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] -1/2/d*a^3*ln(tanh(d*x+c)-1)-3/2/d*ln(tanh(d*x+c)-1)*a^2*b-3/2/d*ln(tanh(d*
x+c)-1)*a*b^2-1/2/d*ln(tanh(d*x+c)-1)*b^3-1/5*b^3*tanh(d*x+c)^5/d-1/3*b^3*t
anh(d*x+c)^3/d-b^3*tanh(d*x+c)/d-a*b^2*tanh(d*x+c)^3/d-3*a^2*b*tanh(d*x+c)/
d+1/2/d*ln(tanh(d*x+c)+1)*a^3+3/2/d*ln(tanh(d*x+c)+1)*a^2*b+3/2/d*ln(tanh(d
*x+c)+1)*a*b^2+1/2/d*ln(tanh(d*x+c)+1)*b^3-3*a*b^2*tanh(d*x+c)/d
```

Maxima [B] time = 1.10733, size = 323, normalized size = 4.36

$$\frac{1}{15}b^3\left(15x+\frac{15c}{d}-\frac{2\left(70e^{(-2dx-2c)}+140e^{(-4dx-4c)}+90e^{(-6dx-6c)}+45e^{(-8dx-8c)}+23\right)}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1\right)}\right)+ab^2\left(3x+\frac{3c}{d}-\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/15*b^3*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 9
0*e^(-6*d*x - 6*c) + 45*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
10*c) + 1))) + a*b^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4
*c) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1
))) + 3*a^2*b*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a^3*x
```

Fricas [B] time = 2.0443, size = 1412, normalized size = 19.08

$$\frac{\left(45\,a^2b+60\,ab^2+23\,b^3+15\left(a^3+3\,a^2b+3\,ab^2+b^3\right)dx\right)\cosh\left(dx+c\right)^5+5\left(45\,a^2b+60\,ab^2+23\,b^3+15\left(a^3+3\,a^2b+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*((45*a^2*b + 60*a*b^2 + 23*b^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*
x)*cosh(d*x + c)^5 + 5*(45*a^2*b + 60*a*b^2 + 23*b^3 + 15*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*d*x)*cosh(d*x + c)*sinh(d*x + c)^4 - (45*a^2*b + 60*a*b^2 +
23*b^3)*sinh(d*x + c)^5 + 5*(45*a^2*b + 60*a*b^2 + 23*b^3 + 15*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 - 5*(27*a^2*b + 24*a*b^2 + 5*b^3 +
 2*(45*a^2*b + 60*a*b^2 + 23*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 5*(2*(
45*a^2*b + 60*a*b^2 + 23*b^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh
(d*x + c)^3 + 3*(45*a^2*b + 60*a*b^2 + 23*b^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(45*a^2*b + 60*a*b^2 + 23*
b^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c) - 5*((45*a^2*b
+ 60*a*b^2 + 23*b^3)*cosh(d*x + c)^4 + 18*a^2*b + 12*a*b^2 + 10*b^3 + 3*(27
*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)
^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(
d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))
```

Sympy [A] time = 0.904005, size = 126, normalized size = 1.7

$$\begin{cases} a^3x+3a^2bx-\frac{3a^2b\tanh\left(c+dx\right)}{d}+3ab^2x-\frac{ab^2\tanh^3\left(c+dx\right)}{d}-\frac{3ab^2\tanh\left(c+dx\right)}{d}+b^3x-\frac{b^3\tanh^5\left(c+dx\right)}{5d}-\frac{b^3\tanh^3\left(c+dx\right)}{3d}-\frac{b^3\tanh\left(c+dx\right)}{d} \\ x\left(a+b\tanh^2\left(c\right)\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*x - 3*a**2*b*tanh(c + d*x)/d + 3*a*b**2*x - a*
b**2*tanh(c + d*x)**3/d - 3*a*b**2*tanh(c + d*x)/d + b**3*x - b**3*tanh(c +
 d*x)**5/(5*d) - b**3*tanh(c + d*x)**3/(3*d) - b**3*tanh(c + d*x)/d, Ne(d,
0)), (x*(a + b*tanh(c)**2)**3, True))
```

Giac [B] time = 1.18731, size = 325, normalized size = 4.39

$$\frac{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\left(dx+c\right)}{d}+\frac{2\left(45\,a^2be^{(8\,dx+8\,c)}+90\,ab^2e^{(8\,dx+8\,c)}+45\,b^3e^{(8\,dx+8\,c)}+180\,a^2be^{(6\,dx+6\,c)}+270\,ab^2e^{(6\,dx}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c)/d + 2/15*(45*a^2*b*e^(8*d*x + 8*c
) + 90*a*b^2*e^(8*d*x + 8*c) + 45*b^3*e^(8*d*x + 8*c) + 180*a^2*b*e^(6*d*x
+ 6*c) + 270*a*b^2*e^(6*d*x + 6*c) + 90*b^3*e^(6*d*x + 6*c) + 270*a^2*b*e^(
4*d*x + 4*c) + 330*a*b^2*e^(4*d*x + 4*c) + 140*b^3*e^(4*d*x + 4*c) + 180*a^
2*b*e^(2*d*x + 2*c) + 210*a*b^2*e^(2*d*x + 2*c) + 70*b^3*e^(2*d*x + 2*c) +
45*a^2*b + 60*a*b^2 + 23*b^3)/(d*(e^(2*d*x + 2*c) + 1)^5)
```

3.161 $\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx$

Optimal. Leaf size=72

$$\frac{a^3\log(\tanh(c+dx))}{d}-\frac{b^2(3a+b)\tanh^2(c+dx)}{2d}+\frac{(a+b)^3\log(\cosh(c+dx))}{d}-\frac{b^3\tanh^4(c+dx)}{4d}$$

```
[Out] ((a + b)^3*Log[Cosh[c + d*x]])/d + (a^3*Log[Tanh[c + d*x]])/d - (b^2*(3*a +
 b)*Tanh[c + d*x]^2)/(2*d) - (b^3*Tanh[c + d*x]^4)/(4*d)
```

Rubi [A] time = 0.0984251, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3670, 446, 72}

$$\frac{a^3\log(\tanh(c+dx))}{d}-\frac{b^2(3a+b)\tanh^2(c+dx)}{2d}+\frac{(a+b)^3\log(\cosh(c+dx))}{d}-\frac{b^3\tanh^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((a + b)^3*Log[Cosh[c + d*x]])/d + (a^3*Log[Tanh[c + d*x]])/d - (b^2*(3*a +
 b)*Tanh[c + d*x]^2)/(2*d) - (b^3*Tanh[c + d*x]^4)/(4*d)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^3}{x\left(1-x^2\right)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^3}{(1-x)x}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(-b^2(3a+b) - \frac{(a+b)^3}{-1+x} + \frac{a^3}{x} - b^3x\right)dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{(a+b)^3\log(\cosh(c+dx))}{d} + \frac{a^3\log(\tanh(c+dx))}{d} - \frac{b^2(3a+b)\tanh^2(c}{2d}$$

Mathematica [A] time = 0.518212, size = 67, normalized size = 0.93

$$\frac{2a^3\log(\tanh(c+dx)) - b^2(3a+b)\tanh^2(c+dx) + 2(a+b)^3\log(\cosh(c+dx)) - \frac{1}{2}b^3\tanh^4(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (2*(a + b)^3*Log[Cosh[c + d*x]] + 2*a^3*Log[Tanh[c + d*x]] - b^2*(3*a + b)*
Tanh[c + d*x]^2 - (b^3*Tanh[c + d*x]^4)/2)/(2*d)
```

Maple [A] time = 0.055, size = 111, normalized size = 1.5

$$\frac{a^3\ln(\sinh(dx+c))}{d} + 3\frac{a^2b\ln(\cosh(dx+c))}{d} + 3\frac{ab^2\ln(\cosh(dx+c))}{d} - \frac{3(\tanh(dx+c))^2ab^2}{2d} + \frac{b^3\ln(\cosh(dx+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*a^3*ln(sinh(d*x+c))+3/d*a^2*b*ln(cosh(d*x+c))+3/d*a*b^2*ln(cosh(d*x+c))
-3/2/d*tanh(d*x+c)^2*a*b^2+1/d*b^3*ln(cosh(d*x+c))-1/2/d*b^3*tanh(d*x+c)^2-
1/4*b^3*tanh(d*x+c)^4/d
```

Maxima [B] time = 1.70616, size = 289, normalized size = 4.01

$$b^3\left(x + \frac{c}{d} + \frac{\log\left(e^{(-2dx-2c)}+1\right)}{d} + \frac{4\left(e^{(-2dx-2c)} + e^{(-4dx-4c)} + e^{(-6dx-6c)}\right)}{d\left(4e^{(-2dx-2c)} + 6e^{(-4dx-4c)} + 4e^{(-6dx-6c)} + e^{(-8dx-8c)} + 1\right)}\right) + 3ab^2\left(x + \frac{c}{d} + \frac{\log\left(e\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] b^3*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 4*(e^(-2*d*x - 2*c) + e^(-4*d*
x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) +
4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) + 3*a*b^2*(x + c/d + log(e^(-2
*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x
- 4*c) + 1))) + 3*a^2*b*log(e^(d*x + c) + e^(-d*x - c))/d + a^3*log(sinh(d*
```

```
x + c))/d
```

Fricas [B] time = 2.22105, size = 5854, normalized size = 81.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^8 + 8*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*d*x*sinh(d*x + c)^8 - 2*(3*a*b^2 + 2*b^3 - 2*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*d*x)*cosh(d*x + c)^6 + 2*(14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*
cosh(d*x + c)^2 - 3*a*b^2 - 2*b^3 + 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*
sinh(d*x + c)^6 + 4*(14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^3
 - 3*(3*a*b^2 + 2*b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c
))*sinh(d*x + c)^5 - 2*(6*a*b^2 + 2*b^3 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
*d*x)*cosh(d*x + c)^4 + 2*(35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x
+ c)^4 - 6*a*b^2 - 2*b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 15*(3*a*
b^2 + 2*b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(
d*x + c)^4 + 8*(7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^5 - 5*(
3*a*b^2 + 2*b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 -
(6*a*b^2 + 2*b^3 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*si
nh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 2*(3*a*b^2 + 2*b^3 -
2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2 + 2*(14*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^6 - 15*(3*a*b^2 + 2*b^3 - 2*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 - 3*a*b^2 - 2*b^3 + 2*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d*x - 6*(6*a*b^2 + 2*b^3 - 3*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((3*a^2*b + 3*a*b^2 + b^3)
*cosh(d*x + c)^8 + 8*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^
7 + (3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^8 + 4*(3*a^2*b + 3*a*b^2 + b^3)
*cosh(d*x + c)^6 + 4*(3*a^2*b + 3*a*b^2 + b^3 + 7*(3*a^2*b + 3*a*b^2 + b^3)
*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x
 + c)^3 + 3*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(3
*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 2*(35*(3*a^2*b + 3*a*b^2 + b^3)*c
osh(d*x + c)^4 + 9*a^2*b + 9*a*b^2 + 3*b^3 + 30*(3*a^2*b + 3*a*b^2 + b^3)*c
osh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x +
 c)^5 + 10*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + 3*(3*a^2*b + 3*a*b^2
 + b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*a^2*b + 3*a*b^2 + b^3 + 4*(3*a^2
*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2 + 4*(7*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d
*x + c)^6 + 15*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 3*a^2*b + 3*a*b^
2 + b^3 + 9*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*
((3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 3*(3*a^2*b + 3*a*b^2 + b^3)*co
sh(d*x + c)^5 + 3*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (3*a^2*b + 3*
a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x +
c) - sinh(d*x + c))) - (a^3*cosh(d*x + c)^8 + 8*a^3*cosh(d*x + c)*sinh(d*x
+ c)^7 + a^3*sinh(d*x + c)^8 + 4*a^3*cosh(d*x + c)^6 + 6*a^3*cosh(d*x + c)^
4 + 4*(7*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^6 + 8*(7*a^3*cosh(d*x + c
)^3 + 3*a^3*cosh(d*x + c))*sinh(d*x + c)^5 + 4*a^3*cosh(d*x + c)^2 + 2*(35*
a^3*cosh(d*x + c)^4 + 30*a^3*cosh(d*x + c)^2 + 3*a^3)*sinh(d*x + c)^4 + 8*(
7*a^3*cosh(d*x + c)^5 + 10*a^3*cosh(d*x + c)^3 + 3*a^3*cosh(d*x + c))*sinh(
d*x + c)^3 + a^3 + 4*(7*a^3*cosh(d*x + c)^6 + 15*a^3*cosh(d*x + c)^4 + 9*a^
3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^2 + 8*(a^3*cosh(d*x + c)^7 + 3*a^3*c
osh(d*x + c)^5 + 3*a^3*cosh(d*x + c)^3 + a^3*cosh(d*x + c))*sinh(d*x + c))*
log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(2*(a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^7 - 3*(3*a*b^2 + 2*b^3 - 2*(a^3 + 3*a^2*
```

```
b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 - 2*(6*a*b^2 + 2*b^3 - 3*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 - (3*a*b^2 + 2*b^3 - 2*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^
8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 + 4*d*cosh(d*x +
c)^6 + 4*(7*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3
 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(
d*x + c)^4 + 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x
+ c)^5 + 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*d*co
sh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*
x + c)^2 + d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 + 3*d*cosh(d*x + c)^5
+ 3*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.41302, size = 366, normalized size = 5.08

$$\frac{a^3\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}-2\right)}{2\,d}+\frac{\left(3\,a^2b+3\,ab^2+b^3\right)\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}+2\right)}{2\,d}-\frac{9\,a^2b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*a^3*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2)/d + 1/2*(3*a^2*b + 3*a*
b^2 + b^3)*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2)/d - 1/4*(9*a^2*b*(e^
(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + 9*a*b^2*(e^(2*d*x + 2*c) + e^(-2*d*x
- 2*c))^2 + 3*b^3*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + 36*a^2*b*(e^(2*d
*x + 2*c) + e^(-2*d*x - 2*c)) + 12*a*b^2*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c
)) - 4*b^3*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 36*a^2*b - 12*a*b^2 - 4*b
^3)/(d*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2)^2)
```

3.162 $\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=59

$$-\frac{a^3\coth(c+dx)}{d}-\frac{b^2(3a+b)\tanh(c+dx)}{d}+x(a+b)^3-\frac{b^3\tanh^3(c+dx)}{3d}$$

```
[Out] (a + b)^3*x - (a^3*Coth[c + d*x])/d - (b^2*(3*a + b)*Tanh[c + d*x])/d - (b^
3*Tanh[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0800658, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 461, 207}

$$-\frac{a^3\coth(c+dx)}{d}-\frac{b^2(3a+b)\tanh(c+dx)}{d}+x(a+b)^3-\frac{b^3\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (a + b)^3*x - (a^3*Coth[c + d*x])/d - (b^2*(3*a + b)*Tanh[c + d*x])/d - (b^
3*Tanh[c + d*x]^3)/(3*d)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^3}{x^2(1-x^2)} dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-b^2(3a+b)+\frac{a^3}{x^2}-b^3x^2-\frac{(a+b)^3}{-1+x^2}\right) dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{a^3\coth(c+dx)}{d}-\frac{b^2(3a+b)\tanh(c+dx)}{d}-\frac{b^3\tanh^3(c+dx)}{3d}-\frac{(a+}{}$$
$$= (a+b)^3x-\frac{a^3\coth(c+dx)}{d}-\frac{b^2(3a+b)\tanh(c+dx)}{d}-\frac{b^3\tanh^3(c+}{3d}$$

Mathematica [A] time = 2.18137, size = 81, normalized size = 1.37

$$\frac{\tanh(c+dx)\left(-3a^3\coth^2(c+dx)-b^2\left(9a+b\tanh^2(c+dx)+3b\right)+3(a+b)^3\sqrt{\coth^2(c+dx)}\tanh^{-1}\left(\sqrt{\coth^2(c+\right.\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (Tanh[c + d*x]*(-3*a^3*Coth[c + d*x]^2 + 3*(a + b)^3*ArcTanh[Sqrt[Coth[c +
d*x]^2]]*Sqrt[Coth[c + d*x]^2] - b^2*(9*a + 3*b + b*Tanh[c + d*x]^2)))/(3*d
)
```

Maple [A] time = 0.049, size = 80, normalized size = 1.4

$$\frac{1}{d}\left(a^3(dx+c-\coth(dx+c))+3a^2b(dx+c)+3ab^2(dx+c-\tanh(dx+c))+b^3\left(dx+c-\tanh(dx+c)-\frac{(\tanh(}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*(d*x+c-coth(d*x+c))+3*a^2*b*(d*x+c)+3*a*b^2*(d*x+c-tanh(d*x+c))+b^
3*(d*x+c-tanh(d*x+c)-1/3*tanh(d*x+c)^3))
```

Maxima [B] time = 1.15813, size = 198, normalized size = 3.36

$$\frac{1}{3}b^3\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+2\right)}{d\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1\right)}\right)+3ab^2\left(x+\frac{c}{d}-\frac{2}{d\left(e^{(-2dx-2c)}+1\right)}\right)+a^3\left(x+\frac{c}{d}+\frac{}{d\left(e^{(\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(
3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 3*a*b^2
*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a^3*(x + c/d + 2/(d*(e^(-2*d*x
```

```
- 2*c) - 1))) + 3*a^2*b*x
```

Fricas [B] time = 1.951, size = 811, normalized size = 13.75

$$-\frac{\left(3\,a^{3}+9\,ab^{2}+4\,b^{3}\right)\cosh\left(dx+c\right)^{4}-4\left(3\,a^{3}+9\,ab^{2}+4\,b^{3}+3\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)dx\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/12*((3*a^3 + 9*a*b^2 + 4*b^3)*cosh(d*x + c)^4 - 4*(3*a^3 + 9*a*b^2 + 4*b
^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)*sinh(d*x + c)^3 +
 (3*a^3 + 9*a*b^2 + 4*b^3)*sinh(d*x + c)^4 + 9*a^3 - 9*a*b^2 + 4*(3*a^3 - b
^3)*cosh(d*x + c)^2 + 2*(6*a^3 - 2*b^3 + 3*(3*a^3 + 9*a*b^2 + 4*b^3)*cosh(d
*x + c)^2)*sinh(d*x + c)^2 - 4*((3*a^3 + 9*a*b^2 + 4*b^3 + 3*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 + (3*a^3 + 9*a*b^2 + 4*b^3 + 3*(a^3
+ 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x +
 c)*sinh(d*x + c)^3 + (d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.42829, size = 186, normalized size = 3.15

$$\frac{\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)\left(dx+c\right)}{d}-\frac{2\,a^{3}}{d\left(e^{\left(2\,dx+2\,c\right)}-1\right)}+\frac{2\left(9\,ab^{2}e^{\left(4\,dx+4\,c\right)}+6\,b^{3}e^{\left(4\,dx+4\,c\right)}+18\,ab^{2}e^{\left(2\,dx+2\,c\right)}+6\,b^{3}e^{\left(2\,dx+2\,c\right)}+\right.}{3\,d\left(e^{\left(2\,dx+2\,c\right)}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c)/d - 2*a^3/(d*(e^(2*d*x + 2*c) - 1
)) + 2/3*(9*a*b^2*e^(4*d*x + 4*c) + 6*b^3*e^(4*d*x + 4*c) + 18*a*b^2*e^(2*d
*x + 2*c) + 6*b^3*e^(2*d*x + 2*c) + 9*a*b^2 + 4*b^3)/(d*(e^(2*d*x + 2*c) +
1)^3)
```

3.163 $\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=72

$$\frac{a^2(a+3b)\log(\tanh(c+dx))}{d}-\frac{a^3\coth^2(c+dx)}{2d}+\frac{(a+b)^3\log(\cosh(c+dx))}{d}-\frac{b^3\tanh^2(c+dx)}{2d}$$

```
[Out] -(a^3*Coth[c + d*x]^2)/(2*d) + ((a + b)^3*Log[Cosh[c + d*x]])/d + (a^2*(a +
 3*b)*Log[Tanh[c + d*x]])/d - (b^3*Tanh[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.108481, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 446, 88}

$$\frac{a^2(a+3b)\log(\tanh(c+dx))}{d}-\frac{a^3\coth^2(c+dx)}{2d}+\frac{(a+b)^3\log(\cosh(c+dx))}{d}-\frac{b^3\tanh^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -(a^3*Coth[c + d*x]^2)/(2*d) + ((a + b)^3*Log[Cosh[c + d*x]])/d + (a^2*(a +
 3*b)*Log[Tanh[c + d*x]])/d - (b^3*Tanh[c + d*x]^2)/(2*d)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^3}{x^3\left(1-x^2\right)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^3}{(1-x)x^2}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(-b^3-\frac{(a+b)^3}{-1+x}+\frac{a^3}{x^2}+\frac{a^2(a+3b)}{x}\right)dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= -\frac{a^3\coth^2(c+dx)}{2d}+\frac{(a+b)^3\log(\cosh(c+dx))}{d}+\frac{a^2(a+3b)\log(\tanh(c+$$

Mathematica [A] time = 0.436675, size = 63, normalized size = 0.88

$$-\frac{-2a^2(a+3b)\log(\tanh(c+dx))+a^3\coth^2(c+dx)-2(a+b)^3\log(\cosh(c+dx))+b^3\tanh^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -(a^3*Coth[c + d*x]^2 - 2*(a + b)^3*Log[Cosh[c + d*x]] - 2*a^2*(a + 3*b)*Lo
g[Tanh[c + d*x]] + b^3*Tanh[c + d*x]^2)/(2*d)
```

Maple [A] time = 0.06, size = 94, normalized size = 1.3

$$\frac{a^3\ln(\sinh(dx+c))}{d}-\frac{a^3(\coth(dx+c))^2}{2d}+3\frac{a^2b\ln(\sinh(dx+c))}{d}+3\frac{ab^2\ln(\cosh(dx+c))}{d}+\frac{b^3\ln(\cosh(dx+c))}{d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*a^3*ln(sinh(d*x+c))-1/2*a^3*coth(d*x+c)^2/d+3/d*a^2*b*ln(sinh(d*x+c))+3
/d*a*b^2*ln(cosh(d*x+c))+1/d*b^3*ln(cosh(d*x+c))-1/2/d*b^3*tanh(d*x+c)^2
```

Maxima [B] time = 1.57342, size = 274, normalized size = 3.81

$$a^3\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2e^{(-2dx-2c)}}{d\left(2e^{(-2dx-2c)}-e^{(-4dx-4c)}-1\right)}\right)+b^3\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2dx-2c)}+1\right)}{d}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] a^3*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*
d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + b^3*(x + c/d
+ log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) +
 e^(-4*d*x - 4*c) + 1))) + 3*a*b^2*log(e^(d*x + c) + e^(-d*x - c))/d + 3*a^
2*b*log(e^(d*x + c) - e^(-d*x - c))/d
```

Fricas [B] time = 2.17306, size = 4113, normalized size = 57.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^8 + 8*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*d*x*sinh(d*x + c)^8 + 2*(a^3 - b^3)*cosh(d*x + c)^6 + 2*(14*(a^3 +
3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^2 + a^3 - b^3)*sinh(d*x + c)^6 +
 4*(14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^3 + 3*(a^3 - b^3)*
cosh(d*x + c))*sinh(d*x + c)^5 + 2*(2*a^3 + 2*b^3 - (a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*d*x)*cosh(d*x + c)^4 + 2*(35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*c
osh(d*x + c)^4 + 2*a^3 + 2*b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x + 15*(
a^3 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*d*x*cosh(d*x + c)^5 + 5*(a^3 - b^3)*cosh(d*x + c)^3 + (2*a^3 + 2*b^
3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + (
a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x + 2*(a^3 - b^3)*cosh(d*x + c)^2 + 2*(14*
(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^6 + 15*(a^3 - b^3)*cosh(d
*x + c)^4 + a^3 - b^3 + 6*(2*a^3 + 2*b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*
d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((3*a*b^2 + b^3)*cosh(d*x + c)^8 +
56*(3*a*b^2 + b^3)*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*(3*a*b^2 + b^3)*cos
h(d*x + c)^2*sinh(d*x + c)^6 + 8*(3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c
)^7 + (3*a*b^2 + b^3)*sinh(d*x + c)^8 - 2*(3*a*b^2 + b^3)*cosh(d*x + c)^4 +
 2*(35*(3*a*b^2 + b^3)*cosh(d*x + c)^4 - 3*a*b^2 - b^3)*sinh(d*x + c)^4 + 8
*(7*(3*a*b^2 + b^3)*cosh(d*x + c)^5 - (3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d
*x + c)^3 + 3*a*b^2 + b^3 + 4*(7*(3*a*b^2 + b^3)*cosh(d*x + c)^6 - 3*(3*a*b
^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((3*a*b^2 + b^3)*cosh(d*x +
c)^7 - (3*a*b^2 + b^3)*cosh(d*x + c)^3)*sinh(d*x + c))*log(2*cosh(d*x + c)/
(cosh(d*x + c) - sinh(d*x + c))) - ((a^3 + 3*a^2*b)*cosh(d*x + c)^8 + 56*(a
^3 + 3*a^2*b)*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*(a^3 + 3*a^2*b)*cosh(d*x
 + c)^2*sinh(d*x + c)^6 + 8*(a^3 + 3*a^2*b)*cosh(d*x + c)*sinh(d*x + c)^7 +
 (a^3 + 3*a^2*b)*sinh(d*x + c)^8 - 2*(a^3 + 3*a^2*b)*cosh(d*x + c)^4 + 2*(3
5*(a^3 + 3*a^2*b)*cosh(d*x + c)^4 - a^3 - 3*a^2*b)*sinh(d*x + c)^4 + 8*(7*(
a^3 + 3*a^2*b)*cosh(d*x + c)^5 - (a^3 + 3*a^2*b)*cosh(d*x + c))*sinh(d*x +
c)^3 + a^3 + 3*a^2*b + 4*(7*(a^3 + 3*a^2*b)*cosh(d*x + c)^6 - 3*(a^3 + 3*a^
2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^3 + 3*a^2*b)*cosh(d*x + c)^7
- (a^3 + 3*a^2*b)*cosh(d*x + c)^3)*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh
(d*x + c) - sinh(d*x + c))) + 4*(2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh
(d*x + c)^7 + 3*(a^3 - b^3)*cosh(d*x + c)^5 + 2*(2*a^3 + 2*b^3 - (a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 + (a^3 - b^3)*cosh(d*x + c))*sin
h(d*x + c))/(d*cosh(d*x + c)^8 + 56*d*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*
d*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*s
inh(d*x + c)^8 - 2*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 - d)*sinh(d*
x + c)^4 + 8*(7*d*cosh(d*x + c)^5 - d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7
*d*cosh(d*x + c)^6 - 3*d*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(d*cosh(d*x +
 c)^7 - d*cosh(d*x + c)^3)*sinh(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.42108, size = 375, normalized size = 5.21

$$\frac{(3\,ab^2+b^3)\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}+2\right)}{2\,d}+\frac{(a^3+3\,a^2b)\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}-2\right)}{2\,d}-\frac{a^3\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)^2+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*(3*a*b^2 + b^3)*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2)/d + 1/2*(a^
3 + 3*a^2*b)*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2)/d - 1/4*(a^3*(e^(2
*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + 3*a^2*b*(e^(2*d*x + 2*c) + e^(-2*d*x -
2*c))^2 + 3*a*b^2*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + b^3*(e^(2*d*x +
2*c) + e^(-2*d*x - 2*c))^2 + 8*a^3*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) - 8
*b^3*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 12*a^3 - 12*a^2*b - 12*a*b^2 +
12*b^3)/(((e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 - 4)*d)
```

3.164 $\int \coth^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx$

Optimal. Leaf size=59

$$-\frac{a^2(a+3b)\coth(c+dx)}{d}-\frac{a^3\coth^3(c+dx)}{3d}+x(a+b)^3-\frac{b^3\tanh(c+dx)}{d}$$

```
[Out] (a + b)^3*x - (a^2*(a + 3*b)*Coth[c + d*x])/d - (a^3*Coth[c + d*x]^3)/(3*d)
 - (b^3*Tanh[c + d*x])/d
```

Rubi [A] time = 0.0812287, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 461, 207}

$$-\frac{a^2(a+3b)\coth(c+dx)}{d}-\frac{a^3\coth^3(c+dx)}{3d}+x(a+b)^3-\frac{b^3\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (a + b)^3*x - (a^2*(a + 3*b)*Coth[c + d*x])/d - (a^3*Coth[c + d*x]^3)/(3*d)
 - (b^3*Tanh[c + d*x])/d
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^3}{x^4(1-x^2)}dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-b^3+\frac{a^3}{x^4}+\frac{a^2(a+3b)}{x^2}-\frac{(a+b)^3}{-1+x^2}\right)dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{a^2(a+3b)\coth(c+dx)}{d}-\frac{a^3\coth^3(c+dx)}{3d}-\frac{b^3\tanh(c+dx)}{d}-\frac{(a+b)^3}{}$$
$$= (a+b)^3x-\frac{a^2(a+3b)\coth(c+dx)}{d}-\frac{a^3\coth^3(c+dx)}{3d}-\frac{b^3\tanh(c+dx)}{d}$$

Mathematica [A] time = 1.18732, size = 82, normalized size = 1.39

$$\frac{\tanh(c+dx)\left(-3a^2(a+3b)\coth^2(c+dx)-a^3\coth^4(c+dx)+3(a+b)^3\sqrt{\coth^2(c+dx)}\tanh^{-1}\left(\sqrt{\coth^2(c+dx)}\right)-3b\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((-3*b^3 - 3*a^2*(a + 3*b)*Coth[c + d*x]^2 - a^3*Coth[c + d*x]^4 + 3*(a + b
)^3*ArcTanh[Sqrt[Coth[c + d*x]^2]]*Sqrt[Coth[c + d*x]^2])*Tanh[c + d*x])/(3
*d)
```

Maple [A] time = 0.05, size = 80, normalized size = 1.4

$$\frac{1}{d}\left(a^3\left(dx+c-\coth(dx+c)-\frac{(\coth(dx+c))^3}{3}\right)+3a^2b(dx+c-\coth(dx+c))+3ab^2(dx+c)+b^3(dx+c-\tanh(d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*(d*x+c-coth(d*x+c)-1/3*coth(d*x+c)^3)+3*a^2*b*(d*x+c-coth(d*x+c))+
3*a*b^2*(d*x+c)+b^3*(d*x+c-tanh(d*x+c)))
```

Maxima [B] time = 1.07444, size = 198, normalized size = 3.36

$$\frac{1}{3}a^3\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}-2\right)}{d\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1\right)}\right)+b^3\left(x+\frac{c}{d}-\frac{2}{d\left(e^{(-2dx-2c)}+1\right)}\right)+3a^2b\left(x+\frac{c}{d}+\frac{2}{d\left(e^{(-2dx}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - 2)/(d*(
3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + b^3*(x
+ c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + 3*a^2*b*(x + c/d + 2/(d*(e^(-2*d*x
```

```
- 2*c) - 1))) + 3*a*b^2*x
```

Fricas [B] time = 2.00627, size = 811, normalized size = 13.75

$$-\frac{\left(4\,a^3+9\,a^2b+3\,b^3\right)\cosh\left(dx+c\right)^4-4\left(4\,a^3+9\,a^2b+3\,b^3+3\left(a^3+3\,a^2b+3\,ab^2+b^3\right)dx\right)\cosh\left(dx+c\right)\sinh\left(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/12*((4*a^3 + 9*a^2*b + 3*b^3)*cosh(d*x + c)^4 - 4*(4*a^3 + 9*a^2*b + 3*b
^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)*sinh(d*x + c)^3 +
 (4*a^3 + 9*a^2*b + 3*b^3)*sinh(d*x + c)^4 - 9*a^2*b + 9*b^3 + 4*(a^3 - 3*b
^3)*cosh(d*x + c)^2 + 2*(2*a^3 - 6*b^3 + 3*(4*a^3 + 9*a^2*b + 3*b^3)*cosh(d
*x + c)^2)*sinh(d*x + c)^2 - 4*((4*a^3 + 9*a^2*b + 3*b^3 + 3*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 - (4*a^3 + 9*a^2*b + 3*b^3 + 3*(a^3
+ 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x +
 c)*sinh(d*x + c)^3 + (d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**4*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.4275, size = 186, normalized size = 3.15

$$\frac{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\left(dx+c\right)}{d}+\frac{2\,b^3}{d\left(e^{(2\,dx+2\,c)}+1\right)}-\frac{2\left(6\,a^3e^{(4\,dx+4\,c)}+9\,a^2be^{(4\,dx+4\,c)}-6\,a^3e^{(2\,dx+2\,c)}-18\,a^2be^{(2\,dx+2\,c}\right.}{3\,d\left(e^{(2\,dx+2\,c)}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c)/d + 2*b^3/(d*(e^(2*d*x + 2*c) + 1
)) - 2/3*(6*a^3*e^(4*d*x + 4*c) + 9*a^2*b*e^(4*d*x + 4*c) - 6*a^3*e^(2*d*x
+ 2*c) - 18*a^2*b*e^(2*d*x + 2*c) + 4*a^3 + 9*a^2*b)/(d*(e^(2*d*x + 2*c) -
1)^3)
```

3.165 $\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=83

$$\frac{a\left(a^2+3ab+3b^2\right)\log(\tanh(c+dx))}{d}-\frac{a^2(a+3b)\coth^2(c+dx)}{2d}-\frac{a^3\coth^4(c+dx)}{4d}+\frac{(a+b)^3\log(\cosh(c+dx))}{d}$$

```
[Out] -(a^2*(a + 3*b)*Coth[c + d*x]^2)/(2*d) - (a^3*Coth[c + d*x]^4)/(4*d) + ((a
+ b)^3*Log[Cosh[c + d*x]])/d + (a*(a^2 + 3*a*b + 3*b^2)*Log[Tanh[c + d*x]])
/d
```

Rubi [A] time = 0.117336, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 446, 88}

$$\frac{a\left(a^2+3ab+3b^2\right)\log(\tanh(c+dx))}{d}-\frac{a^2(a+3b)\coth^2(c+dx)}{2d}-\frac{a^3\coth^4(c+dx)}{4d}+\frac{(a+b)^3\log(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^5*(a + b*Tanh[c + d*x]^2)^3,x]
```



```
[Out] -(a^2*(a + 3*b)*Coth[c + d*x]^2)/(2*d) - (a^3*Coth[c + d*x]^4)/(4*d) + ((a
+ b)^3*Log[Cosh[c + d*x]])/d + (a*(a^2 + 3*a*b + 3*b^2)*Log[Tanh[c + d*x]])
/d
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^3}{x^5\left(1-x^2\right)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^3}{(1-x)x^3}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{(a+b)^3}{-1+x}+\frac{a^3}{x^3}+\frac{a^2(a+3b)}{x^2}+\frac{a\left(a^2+3ab+3b^2\right)}{x}\right)dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= -\frac{a^2(a+3b)\coth^2(c+dx)}{2d} - \frac{a^3\coth^4(c+dx)}{4d} + \frac{(a+b)^3\log(\cosh(c+$$

Mathematica [A] time = 0.555253, size = 67, normalized size = 0.81

$$\frac{-a^2(a+3b)\coth^2(c+dx)-\frac{1}{2}a^3\coth^4(c+dx)+2(a+b)^3\log(\sinh(c+dx))-2b^3\log(\tanh(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^5*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (-(a^2*(a + 3*b)*Coth[c + d*x]^2) - (a^3*Coth[c + d*x]^4)/2 + 2*(a + b)^3*L
og[Sinh[c + d*x]] - 2*b^3*Log[Tanh[c + d*x]])/(2*d)
```

Maple [A] time = 0.062, size = 111, normalized size = 1.3

$$\frac{a^3\ln(\sinh(dx+c))}{d} - \frac{a^3(\coth(dx+c))^2}{2d} - \frac{a^3(\coth(dx+c))^4}{4d} + 3\frac{a^2b\ln(\sinh(dx+c))}{d} - \frac{3a^2b(\coth(dx+c))^2}{2d} + 3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*a^3*ln(sinh(d*x+c))-1/2*a^3*coth(d*x+c)^2/d-1/4*a^3*coth(d*x+c)^4/d+3/d
*a^2*b*ln(sinh(d*x+c))-3/2/d*a^2*b*coth(d*x+c)^2+3/d*a*b^2*ln(sinh(d*x+c))+
1/d*b^3*ln(cosh(d*x+c))
```

Maxima [B] time = 1.07568, size = 356, normalized size = 4.29

$$a^3\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{4\left(e^{(-2dx-2c)}-e^{(-4dx-4c)}+e^{(-6dx-6c)}\right)}{d\left(4e^{(-2dx-2c)}-6e^{(-4dx-4c)}+4e^{(-6dx-6c)}-e^{(-8dx-8c)}-1\right)}\right)+3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] a^3*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 4*(e^(-2
*d*x - 2*c) - e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) -
 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + 3*a^2*
b*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*d*
```

```
x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + b^3*log(e^(d*x
+ c) + e^(-d*x - c))/d + 3*a*b^2*log(e^(d*x + c) - e^(-d*x - c))/d
```

Fricas [B] time = 2.36982, size = 5854, normalized size = 70.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^8 + 8*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*d*x*sinh(d*x + c)^8 + 2*(2*a^3 + 3*a^2*b - 2*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*d*x)*cosh(d*x + c)^6 + 2*(14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*
cosh(d*x + c)^2 + 2*a^3 + 3*a^2*b - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*
sinh(d*x + c)^6 + 4*(14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^3
 + 3*(2*a^3 + 3*a^2*b - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c
))*sinh(d*x + c)^5 - 2*(2*a^3 + 6*a^2*b - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
*d*x)*cosh(d*x + c)^4 + 2*(35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x
+ c)^4 - 2*a^3 - 6*a^2*b + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x + 15*(2*a^
3 + 3*a^2*b - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(
d*x + c)^4 + 8*(7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^5 + 5*(
2*a^3 + 3*a^2*b - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 -
(2*a^3 + 6*a^2*b - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*si
nh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x + 2*(2*a^3 + 3*a^2*b -
2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2 + 2*(14*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^6 + 15*(2*a^3 + 3*a^2*b - 2*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 + 2*a^3 + 3*a^2*b - 2*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d*x - 6*(2*a^3 + 6*a^2*b - 3*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - (b^3*cosh(d*x + c)^8 + 8*b
^3*cosh(d*x + c)*sinh(d*x + c)^7 + b^3*sinh(d*x + c)^8 - 4*b^3*cosh(d*x + c
)^6 + 6*b^3*cosh(d*x + c)^4 + 4*(7*b^3*cosh(d*x + c)^2 - b^3)*sinh(d*x + c)
^6 + 8*(7*b^3*cosh(d*x + c)^3 - 3*b^3*cosh(d*x + c))*sinh(d*x + c)^5 - 4*b^
3*cosh(d*x + c)^2 + 2*(35*b^3*cosh(d*x + c)^4 - 30*b^3*cosh(d*x + c)^2 + 3*
b^3)*sinh(d*x + c)^4 + 8*(7*b^3*cosh(d*x + c)^5 - 10*b^3*cosh(d*x + c)^3 +
3*b^3*cosh(d*x + c))*sinh(d*x + c)^3 + b^3 + 4*(7*b^3*cosh(d*x + c)^6 - 15*
b^3*cosh(d*x + c)^4 + 9*b^3*cosh(d*x + c)^2 - b^3)*sinh(d*x + c)^2 + 8*(b^3
*cosh(d*x + c)^7 - 3*b^3*cosh(d*x + c)^5 + 3*b^3*cosh(d*x + c)^3 - b^3*cosh
(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c
))) - ((a^3 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c)^8 + 8*(a^3 + 3*a^2*b + 3*a*b
^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3 + 3*a^2*b + 3*a*b^2)*sinh(d*x + c)
^8 - 4*(a^3 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c)^6 - 4*(a^3 + 3*a^2*b + 3*a*b
^2 - 7*(a^3 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a
^3 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c)^3 - 3*(a^3 + 3*a^2*b + 3*a*b^2)*cosh(
d*x + c))*sinh(d*x + c)^5 + 6*(a^3 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c)^4 + 2
*(35*(a^3 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c)^4 + 3*a^3 + 9*a^2*b + 9*a*b^2
- 30*(a^3 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^3
 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c)^5 - 10*(a^3 + 3*a^2*b + 3*a*b^2)*cosh(d
*x + c)^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + a^
3 + 3*a^2*b + 3*a*b^2 - 4*(a^3 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c)^2 + 4*(7*
(a^3 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c)^6 - 15*(a^3 + 3*a^2*b + 3*a*b^2)*co
sh(d*x + c)^4 - a^3 - 3*a^2*b - 3*a*b^2 + 9*(a^3 + 3*a^2*b + 3*a*b^2)*cosh(
d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^3 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c)^7
- 3*(a^3 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c)^5 + 3*(a^3 + 3*a^2*b + 3*a*b^2)
*cosh(d*x + c)^3 - (a^3 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c))*sinh(d*x + c))*
log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(2*(a^3 + 3*a^2*b
```

```
+ 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^7 + 3*(2*a^3 + 3*a^2*b - 2*(a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 - 2*(2*a^3 + 6*a^2*b - 3*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 + (2*a^3 + 3*a^2*b - 2*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^
8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 - 4*d*cosh(d*x +
c)^6 + 4*(7*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3
 - 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(
d*x + c)^4 - 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x
+ c)^5 - 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 - 4*d*co
sh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 - 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*
x + c)^2 - d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 - 3*d*cosh(d*x + c)^5
+ 3*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**5*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.50226, size = 366, normalized size = 4.41

$$\frac{b^3 \log\left(e^{(2\,dx+2\,c)} + e^{(-2\,dx-2\,c)} + 2\right)}{2\,d} + \frac{\left(a^3 + 3\,a^2b + 3\,ab^2\right)\log\left(e^{(2\,dx+2\,c)} + e^{(-2\,dx-2\,c)} - 2\right)}{2\,d} - \frac{3\,a^3\left(e^{(2\,dx+2\,c)} + e^{(-2\,dx-2\,c)}\right)^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*b^3*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2)/d + 1/2*(a^3 + 3*a^2*b
+ 3*a*b^2)*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2)/d - 1/4*(3*a^3*(e^(2
*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + 9*a^2*b*(e^(2*d*x + 2*c) + e^(-2*d*x -
2*c))^2 + 9*a*b^2*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + 4*a^3*(e^(2*d*x
+ 2*c) + e^(-2*d*x - 2*c)) - 12*a^2*b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))
- 36*a*b^2*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) - 4*a^3 - 12*a^2*b + 36*a*b
^2)/(d*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2)^2)
```

3.166 $\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=74

$$-\frac{a\left(a^2+3ab+3b^2\right)\coth(c+dx)}{d}-\frac{a^2(a+3b)\coth^3(c+dx)}{3d}-\frac{a^3\coth^5(c+dx)}{5d}+x(a+b)^3$$

```
[Out] (a + b)^3*x - (a*(a^2 + 3*a*b + 3*b^2)*Coth[c + d*x])/d - (a^2*(a + 3*b)*Co
th[c + d*x]^3)/(3*d) - (a^3*Coth[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.088843, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 461, 207}

$$-\frac{a\left(a^2+3ab+3b^2\right)\coth(c+dx)}{d}-\frac{a^2(a+3b)\coth^3(c+dx)}{3d}-\frac{a^3\coth^5(c+dx)}{5d}+x(a+b)^3$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^6*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (a + b)^3*x - (a*(a^2 + 3*a*b + 3*b^2)*Coth[c + d*x])/d - (a^2*(a + 3*b)*Co
th[c + d*x]^3)/(3*d) - (a^3*Coth[c + d*x]^5)/(5*d)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^3}{x^6(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^3}{x^6}+\frac{a^2(a+3b)}{x^4}+\frac{a(a^2+3ab+3b^2)}{x^2}-\frac{(a+b)^3}{-1+x^2}\right)dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{a\left(a^2+3ab+3b^2\right)\coth(c+dx)}{d}-\frac{a^2(a+3b)\coth^3(c+dx)}{3d}-\frac{a^3\coth$$
$$= (a+b)^3x-\frac{a\left(a^2+3ab+3b^2\right)\coth(c+dx)}{d}-\frac{a^2(a+3b)\coth^3(c+dx)}{3d}$$

Mathematica [A] time = 1.67193, size = 100, normalized size = 1.35

$$\frac{(a+b)^3\tanh^{-1}\left(\sqrt{\tanh^2(c+dx)}\right)\tanh(c+dx)}{d\sqrt{\tanh^2(c+dx)}}-\frac{a\coth(c+dx)\left(15\left(a^2+3ab+3b^2\right)+3a^2\coth^4(c+dx)+5a(a+}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^6*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -(a*Coth[c + d*x]*(15*(a^2 + 3*a*b + 3*b^2) + 5*a*(a + 3*b)*Coth[c + d*x]^2
 + 3*a^2*Coth[c + d*x]^4))/(15*d) + ((a + b)^3*ArcTanh[Sqrt[Tanh[c + d*x]^2
]]*Tanh[c + d*x])/(d*Sqrt[Tanh[c + d*x]^2])
```

Maple [A] time = 0.065, size = 100, normalized size = 1.4

$$\frac{1}{d}\left(a^3\left(dx+c-\coth(dx+c)-\frac{(\coth(dx+c))^3}{3}-\frac{(\coth(dx+c))^5}{5}\right)+3a^2b\left(dx+c-\coth(dx+c)-1/3(\coth(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*(d*x+c-coth(d*x+c)-1/3*coth(d*x+c)^3-1/5*coth(d*x+c)^5)+3*a^2*b*(d
*x+c-coth(d*x+c)-1/3*coth(d*x+c)^3)+3*a*b^2*(d*x+c-coth(d*x+c))+(d*x+c)*b^3
)
```

Maxima [B] time = 1.07012, size = 323, normalized size = 4.36

$$\frac{1}{15}a^3\left(15x+\frac{15c}{d}-\frac{2\left(70e^{(-2dx-2c)}-140e^{(-4dx-4c)}+90e^{(-6dx-6c)}-45e^{(-8dx-8c)}-23\right)}{d\left(5e^{(-2dx-2c)}-10e^{(-4dx-4c)}+10e^{(-6dx-6c)}-5e^{(-8dx-8c)}+e^{(-10dx-10c)}-1\right)}\right)+a^2b\left(3x+\frac{3c}{d}-\frac{}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/15*a^3*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) - 140*e^(-4*d*x - 4*c) + 9
0*e^(-6*d*x - 6*c) - 45*e^(-8*d*x - 8*c) - 23)/(d*(5*e^(-2*d*x - 2*c) - 10*
```

```
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
10*c) - 1))) + a^2*b*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4
*c) - 2)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1
))) + 3*a*b^2*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + b^3*x
```

Fricas [B] time = 2.10097, size = 1372, normalized size = 18.54

$$-\frac{\left(23\,a^3+60\,a^2b+45\,ab^2\right)\cosh\left(dx+c\right)^5+5\left(23\,a^3+60\,a^2b+45\,ab^2\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4-\left(23\,a^3+60\,a^2b+4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/15*((23*a^3 + 60*a^2*b + 45*a*b^2)*cosh(d*x + c)^5 + 5*(23*a^3 + 60*a^2*
b + 45*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^4 - (23*a^3 + 60*a^2*b + 45*a*b^2
 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*sinh(d*x + c)^5 - 5*(5*a^3 + 24*
a^2*b + 27*a*b^2)*cosh(d*x + c)^3 + 5*(23*a^3 + 60*a^2*b + 45*a*b^2 + 15*(a
^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 2*(23*a^3 + 60*a^2*b + 45*a*b^2 + 15*(a
^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 5*(2*
(23*a^3 + 60*a^2*b + 45*a*b^2)*cosh(d*x + c)^3 - 3*(5*a^3 + 24*a^2*b + 27*a
*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(5*a^3 + 6*a^2*b + 9*a*b^2)*cosh(
d*x + c) - 5*((23*a^3 + 60*a^2*b + 45*a*b^2 + 15*(a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*d*x)*cosh(d*x + c)^4 + 46*a^3 + 120*a^2*b + 90*a*b^2 + 30*(a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*d*x - 3*(23*a^3 + 60*a^2*b + 45*a*b^2 + 15*(a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*sinh(d*x + c)^
5 + 5*(2*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^3 + 5*(d*cosh(d*x + c)^4 - 3*
d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**6*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.51101, size = 325, normalized size = 4.39

$$\frac{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\left(dx+c\right)}{d}-\frac{2\left(45\,a^3e^{(8\,dx+8\,c)}+90\,a^2be^{(8\,dx+8\,c)}+45\,ab^2e^{(8\,dx+8\,c)}-90\,a^3e^{(6\,dx+6\,c)}-270\,a^2be^{(6\,dx+6}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c)/d - 2/15*(45*a^3*e^(8*d*x + 8*c)
+ 90*a^2*b*e^(8*d*x + 8*c) + 45*a*b^2*e^(8*d*x + 8*c) - 90*a^3*e^(6*d*x + 6
*c) - 270*a^2*b*e^(6*d*x + 6*c) - 180*a*b^2*e^(6*d*x + 6*c) + 140*a^3*e^(4*
```

```
d*x + 4*c) + 330*a^2*b*e^(4*d*x + 4*c) + 270*a*b^2*e^(4*d*x + 4*c) - 70*a^3
*e^(2*d*x + 2*c) - 210*a^2*b*e^(2*d*x + 2*c) - 180*a*b^2*e^(2*d*x + 2*c) +
23*a^3 + 60*a^2*b + 45*a*b^2)/(d*(e^(2*d*x + 2*c) - 1)^5)
```

3.167 $\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=103

$$-\frac{a\left(a^2+3ab+3b^2\right)\coth^2(c+dx)}{2d}-\frac{a^2(a+3b)\coth^4(c+dx)}{4d}-\frac{a^3\coth^6(c+dx)}{6d}+\frac{(a+b)^3\log(\tanh(c+dx))}{d}+\frac{(a+b}{}$$

```
[Out] -(a*(a^2 + 3*a*b + 3*b^2)*Coth[c + d*x]^2)/(2*d) - (a^2*(a + 3*b)*Coth[c +
d*x]^4)/(4*d) - (a^3*Coth[c + d*x]^6)/(6*d) + ((a + b)^3*Log[Cosh[c + d*x]]
)/d + ((a + b)^3*Log[Tanh[c + d*x]])/d
```

Rubi [A] time = 0.128969, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 446, 88}

$$-\frac{a\left(a^2+3ab+3b^2\right)\coth^2(c+dx)}{2d}-\frac{a^2(a+3b)\coth^4(c+dx)}{4d}-\frac{a^3\coth^6(c+dx)}{6d}+\frac{(a+b)^3\log(\tanh(c+dx))}{d}+\frac{(a+b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^7*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -(a*(a^2 + 3*a*b + 3*b^2)*Coth[c + d*x]^2)/(2*d) - (a^2*(a + 3*b)*Coth[c +
d*x]^4)/(4*d) - (a^3*Coth[c + d*x]^6)/(6*d) + ((a + b)^3*Log[Cosh[c + d*x]]
)/d + ((a + b)^3*Log[Tanh[c + d*x]])/d
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^3}{x^7(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^3}{(1-x)x^4}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{(a+b)^3}{-1+x} + \frac{a^3}{x^4} + \frac{a^2(a+3b)}{x^3} + \frac{a\left(a^2+3ab+3b^2\right)}{x^2} + \frac{(a+b)^3}{x}\right) dx, x, \tanh^2(\right.}{2d}$$
$$= -\frac{a\left(a^2+3ab+3b^2\right)\coth^2(c+dx)}{2d} - \frac{a^2(a+3b)\coth^4(c+dx)}{4d} - \frac{a^3\cot}{}$$

Mathematica [A] time = 0.237595, size = 76, normalized size = 0.74

$$-\frac{a(a+b)^2\coth^2(c+dx) + \frac{1}{2}(a+b)\left(a\coth^2(c+dx)+b\right)^2 + \frac{1}{3}\left(a\coth^2(c+dx)+b\right)^3 - 2(a+b)^3\log(\sinh(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^7*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -(a*(a + b)^2*Coth[c + d*x]^2 + ((a + b)*(b + a*Coth[c + d*x]^2)^2)/2 + (b
+ a*Coth[c + d*x]^2)^3/3 - 2*(a + b)^3*Log[Sinh[c + d*x]])/(2*d)
```

Maple [A] time = 0.062, size = 161, normalized size = 1.6

$$\frac{a^3\ln(\sinh(dx+c))}{d} - \frac{a^3(\coth(dx+c))^2}{2d} - \frac{a^3(\coth(dx+c))^4}{4d} - \frac{a^3(\coth(dx+c))^6}{6d} + 3\frac{a^2b\ln(\sinh(dx+c))}{d} - \frac{3a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*a^3*ln(sinh(d*x+c))-1/2*a^3*coth(d*x+c)^2/d-1/4*a^3*coth(d*x+c)^4/d-1/6
*a^3*coth(d*x+c)^6/d+3/d*a^2*b*ln(sinh(d*x+c))-3/2/d*a^2*b*coth(d*x+c)^2-3/
4/d*a^2*b*coth(d*x+c)^4+3/d*a*b^2*ln(sinh(d*x+c))-3/2/d*a*b^2*coth(d*x+c)^2
+1/d*b^3*ln(sinh(d*x+c))
```

Maxima [B] time = 1.07398, size = 567, normalized size = 5.5

$$\frac{1}{3}a^3\left(3x + \frac{3c}{d} + \frac{3\log\left(e^{(-dx-c)}+1\right)}{d} + \frac{3\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\left(9e^{(-2dx-2c)} - 18e^{(-4dx-4c)} + 34e^{(-6dx-6c)} - 1\right.}{d\left(6e^{(-2dx-2c)} - 15e^{(-4dx-4c)} + 20e^{(-6dx-6c)} - 15e^{(-8dx-8}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*(3*x + 3*c/d + 3*log(e^(-d*x - c) + 1)/d + 3*log(e^(-d*x - c) - 1)/
d + 2*(9*e^(-2*d*x - 2*c) - 18*e^(-4*d*x - 4*c) + 34*e^(-6*d*x - 6*c) - 18*
e^(-8*d*x - 8*c) + 9*e^(-10*d*x - 10*c))/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*
```

```
d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*
c) - e^(-12*d*x - 12*c) - 1))) + 3*a^2*b*(x + c/d + log(e^(-d*x - c) + 1)/d
 + log(e^(-d*x - c) - 1)/d + 4*(e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) + e^(-6
*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*
c) - e^(-8*d*x - 8*c) - 1))) + 3*a*b^2*(x + c/d + log(e^(-d*x - c) + 1)/d +
 log(e^(-d*x - c) - 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-
4*d*x - 4*c) - 1))) + b^3*log(e^(d*x + c) - e^(-d*x - c))/d
```

Fricas [B] time = 2.87791, size = 10689, normalized size = 103.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^12 + 36*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^11 + 3*(a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*d*x*sinh(d*x + c)^12 + 18*(a^3 + 2*a^2*b + a*b^2 - (a^3
+ 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^10 + 18*(11*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*d*x*cosh(d*x + c)^2 + a^3 + 2*a^2*b + a*b^2 - (a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*d*x)*sinh(d*x + c)^10 + 60*(11*(a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*d*x*cosh(d*x + c)^3 + 3*(a^3 + 2*a^2*b + a*b^2 - (a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^9 - 9*(4*a^3 + 12*a^2*b + 8*a
*b^2 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^8 + 9*(165*(a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^4 - 4*a^3 - 12*a^2*b - 8*a*b^
2 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x + 90*(a^3 + 2*a^2*b + a*b^2 - (a^
3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 72*(33
*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^5 + 30*(a^3 + 2*a^2*b +
a*b^2 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 - (4*a^3 + 12*
a^2*b + 8*a*b^2 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sin
h(d*x + c)^7 + 4*(17*a^3 + 36*a^2*b + 27*a*b^2 - 15*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*d*x)*cosh(d*x + c)^6 + 4*(693*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*
cosh(d*x + c)^6 + 945*(a^3 + 2*a^2*b + a*b^2 - (a^3 + 3*a^2*b + 3*a*b^2 + b
^3)*d*x)*cosh(d*x + c)^4 + 17*a^3 + 36*a^2*b + 27*a*b^2 - 15*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*d*x - 63*(4*a^3 + 12*a^2*b + 8*a*b^2 - 5*(a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 24*(99*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^7 + 189*(a^3 + 2*a^2*b + a*b^2 - (a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 - 21*(4*a^3 + 12*a^2*b + 8
*a*b^2 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 + (17*a^3 +
 36*a^2*b + 27*a*b^2 - 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c
))*sinh(d*x + c)^5 - 9*(4*a^3 + 12*a^2*b + 8*a*b^2 - 5*(a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*d*x)*cosh(d*x + c)^4 + 3*(495*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d
*x*cosh(d*x + c)^8 + 1260*(a^3 + 2*a^2*b + a*b^2 - (a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*d*x)*cosh(d*x + c)^6 - 210*(4*a^3 + 12*a^2*b + 8*a*b^2 - 5*(a^3 + 3
*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 - 12*a^3 - 36*a^2*b - 24*a*b^2
 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x + 20*(17*a^3 + 36*a^2*b + 27*a*b^
2 - 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^
4 + 4*(165*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^9 + 540*(a^3 +
 2*a^2*b + a*b^2 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^7 - 1
26*(4*a^3 + 12*a^2*b + 8*a*b^2 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cos
h(d*x + c)^5 + 20*(17*a^3 + 36*a^2*b + 27*a*b^2 - 15*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*d*x)*cosh(d*x + c)^3 - 9*(4*a^3 + 12*a^2*b + 8*a*b^2 - 5*(a^3 + 3
*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*d*x + 18*(a^3 + 2*a^2*b + a*b^2 - (a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*d*x)*cosh(d*x + c)^2 + 6*(33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*
x*cosh(d*x + c)^10 + 135*(a^3 + 2*a^2*b + a*b^2 - (a^3 + 3*a^2*b + 3*a*b^2
```

```
+ b^3)*d*x)*cosh(d*x + c)^8 - 42*(4*a^3 + 12*a^2*b + 8*a*b^2 - 5*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^6 + 10*(17*a^3 + 36*a^2*b + 27*a*b
^2 - 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 + 3*a^3 + 6*a^
2*b + 3*a*b^2 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 9*(4*a^3 + 12*a^2*b
 + 8*a*b^2 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d
*x + c)^2 - 3*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^12 + 12*(a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^11 + (a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*sinh(d*x + c)^12 - 6*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d
*x + c)^10 - 6*(a^3 + 3*a^2*b + 3*a*b^2 + b^3 - 11*(a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 20*(11*(a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*cosh(d*x + c)^3 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*
sinh(d*x + c)^9 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 15*(
33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^
2 + b^3 - 18*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)
^8 + 24*(33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 - 30*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c
osh(d*x + c))*sinh(d*x + c)^7 - 20*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x
 + c)^6 + 4*(231*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 - 315*(a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 - 5*a^3 - 15*a^2*b - 15*a*b^2 -
 5*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)
^6 + 24*(33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 - 63*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*
cosh(d*x + c)^3 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x
 + c)^5 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 15*(33*(a^3
+ 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 - 84*(a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*cosh(d*x + c)^6 + 70*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 +
 a^3 + 3*a^2*b + 3*a*b^2 + b^3 - 20*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*
x + c)^2)*sinh(d*x + c)^4 + 20*(11*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x
 + c)^9 - 36*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 42*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 - 20*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
*cosh(d*x + c)^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*
x + c)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 - 6*(a^3 + 3*a^2*b + 3*a*b^2 + b^3
)*cosh(d*x + c)^2 + 6*(11*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^10
- 45*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 70*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*cosh(d*x + c)^6 - 50*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*
x + c)^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^
3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 12*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c
osh(d*x + c)^11 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^9 + 10*(a
^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 - 10*(a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*cosh(d*x + c)^5 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3
 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh
(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 12*(3*(a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*d*x*cosh(d*x + c)^11 + 15*(a^3 + 2*a^2*b + a*b^2 - (a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^9 - 6*(4*a^3 + 12*a^2*b + 8*a*b^2 - 5*(a
^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^7 + 2*(17*a^3 + 36*a^2*b +
 27*a*b^2 - 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 - 3*(4*
a^3 + 12*a^2*b + 8*a*b^2 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x
+ c)^3 + 3*(a^3 + 2*a^2*b + a*b^2 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*co
sh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^12 + 12*d*cosh(d*x + c)*sinh(d
*x + c)^11 + d*sinh(d*x + c)^12 - 6*d*cosh(d*x + c)^10 + 6*(11*d*cosh(d*x +
 c)^2 - d)*sinh(d*x + c)^10 + 20*(11*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))
*sinh(d*x + c)^9 + 15*d*cosh(d*x + c)^8 + 15*(33*d*cosh(d*x + c)^4 - 18*d*c
osh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 24*(33*d*cosh(d*x + c)^5 - 30*d*cosh(
d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^7 - 20*d*cosh(d*x + c)^6 + 4*
(231*d*cosh(d*x + c)^6 - 315*d*cosh(d*x + c)^4 + 105*d*cosh(d*x + c)^2 - 5*
d)*sinh(d*x + c)^6 + 24*(33*d*cosh(d*x + c)^7 - 63*d*cosh(d*x + c)^5 + 35*d
*cosh(d*x + c)^3 - 5*d*cosh(d*x + c))*sinh(d*x + c)^5 + 15*d*cosh(d*x + c)^
4 + 15*(33*d*cosh(d*x + c)^8 - 84*d*cosh(d*x + c)^6 + 70*d*cosh(d*x + c)^4
- 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 20*(11*d*cosh(d*x + c)^9 - 36
```

```
*d*cosh(d*x + c)^7 + 42*d*cosh(d*x + c)^5 - 20*d*cosh(d*x + c)^3 + 3*d*cosh
(d*x + c))*sinh(d*x + c)^3 - 6*d*cosh(d*x + c)^2 + 6*(11*d*cosh(d*x + c)^10
 - 45*d*cosh(d*x + c)^8 + 70*d*cosh(d*x + c)^6 - 50*d*cosh(d*x + c)^4 + 15*
d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 12*(d*cosh(d*x + c)^11 - 5*d*cosh(
d*x + c)^9 + 10*d*cosh(d*x + c)^7 - 10*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c
)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**7*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.53015, size = 297, normalized size = 2.88

$$-\frac{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)(dx+c)}{d}+\frac{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)}{d}-\frac{2\left(9\left(a^3+2\,a^2b+ab^2\right)e^{(10\,dx+10\,c)}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c)/d + (a^3 + 3*a^2*b + 3*a*b^2 + b
^3)*log(abs(e^(2*d*x + 2*c) - 1))/d - 2/3*(9*(a^3 + 2*a^2*b + a*b^2)*e^(10*
d*x + 10*c) - 18*(a^3 + 3*a^2*b + 2*a*b^2)*e^(8*d*x + 8*c) + 2*(17*a^3 + 36
*a^2*b + 27*a*b^2)*e^(6*d*x + 6*c) - 18*(a^3 + 3*a^2*b + 2*a*b^2)*e^(4*d*x
+ 4*c) + 9*(a^3 + 2*a^2*b + a*b^2)*e^(2*d*x + 2*c))/(d*(e^(2*d*x + 2*c) - 1
)^6)
```

3.168 $\int \left(a+b\tanh^2(c+dx)\right)^4\,dx$

Optimal. Leaf size=110

$$-\frac{b^2\left(6a^2+4ab+b^2\right)\tanh^3(c+dx)}{3d}-\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tanh(c+dx)}{d}-\frac{b^3(4a+b)\tanh^5(c+dx)}{5d}+x(a+b)$$

```
[Out] (a + b)^4*x - (b*(2*a + b)*(2*a^2 + 2*a*b + b^2)*Tanh[c + d*x])/d - (b^2*(6
*a^2 + 4*a*b + b^2)*Tanh[c + d*x]^3)/(3*d) - (b^3*(4*a + b)*Tanh[c + d*x]^5
)/(5*d) - (b^4*Tanh[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.070031, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3661, 390, 206}

$$-\frac{b^2\left(6a^2+4ab+b^2\right)\tanh^3(c+dx)}{3d}-\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tanh(c+dx)}{d}-\frac{b^3(4a+b)\tanh^5(c+dx)}{5d}+x(a+b)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tanh[c + d*x]^2)^4,x]
```


```
[Out] (a + b)^4*x - (b*(2*a + b)*(2*a^2 + 2*a*b + b^2)*Tanh[c + d*x])/d - (b^2*(6
*a^2 + 4*a*b + b^2)*Tanh[c + d*x]^3)/(3*d) - (b^3*(4*a + b)*Tanh[c + d*x]^5
)/(5*d) - (b^4*Tanh[c + d*x]^7)/(7*d)
```


Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\tanh^2(c+dx)\right)^4\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^4}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-b(2a+b)\left(2a^2+2ab+b^2\right)-b^2\left(6a^2+4ab+b^2\right)x^2-b^3(4a+b)x^4-b^4x^6+\right.\right.}{d}$$
$$= -\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tanh(c+dx)}{d}-\frac{b^2\left(6a^2+4ab+b^2\right)\tanh^3(c+dx)}{3d}-\frac{b^3(4a+}{}$$
$$= (a+b)^4x-\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tanh(c+dx)}{d}-\frac{b^2\left(6a^2+4ab+b^2\right)\tanh^3(c+dx)}{3d}$$

Mathematica [A] time = 1.67281, size = 128, normalized size = 1.16

$$\frac{\tanh(c+dx)\left(\frac{105(a+b)^4\tanh^{-1}\left(\sqrt{\tanh^2(c+dx)}\right)}{\sqrt{\tanh^2(c+dx)}}-b\left(35b\left(6a^2+4ab+b^2\right)\tanh^2(c+dx)+105\left(6a^2b+4a^3+4ab^2+b^3\right)+21b\right.\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tanh[c + d*x]^2)^4,x]
```

```
[Out] (Tanh[c + d*x]*((105*(a + b)^4*ArcTanh[Sqrt[Tanh[c + d*x]^2]])/Sqrt[Tanh[c
+ d*x]^2] - b*(105*(4*a^3 + 6*a^2*b + 4*a*b^2 + b^3) + 35*b*(6*a^2 + 4*a*b
+ b^2)*Tanh[c + d*x]^2 + 21*b^2*(4*a + b)*Tanh[c + d*x]^4 + 15*b^3*Tanh[c +
 d*x]^6)))/(105*d)
```

Maple [B] time = 0.006, size = 344, normalized size = 3.1

$$-\frac{4\,(\tanh(dx+c))^5\,ab^3}{5\,d}-2\,\frac{(\tanh(dx+c))^3\,a^2b^2}{d}-\frac{4\,(\tanh(dx+c))^3\,ab^3}{3\,d}-\frac{a^4\ln(\tanh(dx+c)-1)}{2\,d}-2\,\frac{\ln(\tanh(dx-}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tanh(d*x+c)^2)^4,x)
```

```
[Out] -4/5/d*tanh(d*x+c)^5*a*b^3-2/d*tanh(d*x+c)^3*a^2*b^2-4/3/d*tanh(d*x+c)^3*a*
b^3-1/2/d*a^4*ln(tanh(d*x+c)-1)-2/d*ln(tanh(d*x+c)-1)*a^3*b-3/d*ln(tanh(d*x
+c)-1)*a^2*b^2-2/d*ln(tanh(d*x+c)-1)*a*b^3-1/2/d*ln(tanh(d*x+c)-1)*b^4-1/5/
d*tanh(d*x+c)^5*b^4-1/3/d*tanh(d*x+c)^3*b^4-1/d*b^4*tanh(d*x+c)-1/7*b^4*tan
h(d*x+c)^7/d-6/d*a^2*b^2*tanh(d*x+c)-4/d*a*b^3*tanh(d*x+c)-4/d*a^3*b*tanh(d
*x+c)+1/2/d*ln(tanh(d*x+c)+1)*a^4+2/d*ln(tanh(d*x+c)+1)*a^3*b+3/d*ln(tanh(d
*x+c)+1)*a^2*b^2+2/d*ln(tanh(d*x+c)+1)*a*b^3+1/2/d*ln(tanh(d*x+c)+1)*b^4
```

Maxima [B] time = 1.11768, size = 554, normalized size = 5.04

$$\frac{1}{105}\,b^4\left(105\,x+\frac{105\,c}{d}-\frac{8\left(203\,e^{(-2\,dx-2\,c)}+609\,e^{(-4\,dx-4\,c)}+770\,e^{(-6\,dx-6\,c)}+770\,e^{(-8\,dx-8\,c)}+315\,e^{(-10\,dx-10\,c)}+105\,e^{(-12}\right.}{d\left(7\,e^{(-2\,dx-2\,c)}+21\,e^{(-4\,dx-4\,c)}+35\,e^{(-6\,dx-6\,c)}+35\,e^{(-8\,dx-8\,c)}+21\,e^{(-10\,dx-10\,c)}+7\,e^{(-12\,dx-12\,c)}+e^{(-}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)^2)^4,x, algorithm="maxima")
```

```
[Out] 1/105*b^4*(105*x + 105*c/d - 8*(203*e^(-2*d*x - 2*c) + 609*e^(-4*d*x - 4*c)
 + 770*e^(-6*d*x - 6*c) + 770*e^(-8*d*x - 8*c) + 315*e^(-10*d*x - 10*c) + 1
05*e^(-12*d*x - 12*c) + 44)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) +
35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-1
2*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 4/15*a*b^3*(15*x + 15*c/d - 2*(
70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 90*e^(-6*d*x - 6*c) + 45*e^(-8
*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d
*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 2*a^2*b^2*(3*x
 + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(3*e^(-2*d*x
- 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 4*a^3*b*(x + c/d -
2/(d*(e^(-2*d*x - 2*c) + 1))) + a^4*x
```

Fricas [B] time = 2.52543, size = 2954, normalized size = 26.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)^2)^4,x, algorithm="fricas")
```

```
[Out] 1/105*((420*a^3*b + 840*a^2*b^2 + 644*a*b^3 + 176*b^4 + 105*(a^4 + 4*a^3*b
+ 6*a^2*b^2 + 4*a*b^3 + b^4)*d*x)*cosh(d*x + c)^7 + 7*(420*a^3*b + 840*a^2*
b^2 + 644*a*b^3 + 176*b^4 + 105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)
*d*x)*cosh(d*x + c)*sinh(d*x + c)^6 - 4*(105*a^3*b + 210*a^2*b^2 + 161*a*b^
3 + 44*b^4)*sinh(d*x + c)^7 + 7*(420*a^3*b + 840*a^2*b^2 + 644*a*b^3 + 176*
b^4 + 105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*x)*cosh(d*x + c)^5
- 28*(75*a^3*b + 120*a^2*b^2 + 71*a*b^3 + 14*b^4 + 3*(105*a^3*b + 210*a^2*b
^2 + 161*a*b^3 + 44*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 35*((420*a^3*b
+ 840*a^2*b^2 + 644*a*b^3 + 176*b^4 + 105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*
b^3 + b^4)*d*x)*cosh(d*x + c)^3 + (420*a^3*b + 840*a^2*b^2 + 644*a*b^3 + 17
6*b^4 + 105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*x)*cosh(d*x + c))
*sinh(d*x + c)^4 + 21*(420*a^3*b + 840*a^2*b^2 + 644*a*b^3 + 176*b^4 + 105*
(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*x)*cosh(d*x + c)^3 - 28*(5*(1
05*a^3*b + 210*a^2*b^2 + 161*a*b^3 + 44*b^4)*cosh(d*x + c)^4 + 135*a^3*b +
180*a^2*b^2 + 123*a*b^3 + 42*b^4 + 10*(75*a^3*b + 120*a^2*b^2 + 71*a*b^3 +
14*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 7*(3*(420*a^3*b + 840*a^2*b^2 +
644*a*b^3 + 176*b^4 + 105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*x)*
cosh(d*x + c)^5 + 10*(420*a^3*b + 840*a^2*b^2 + 644*a*b^3 + 176*b^4 + 105*(
a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*x)*cosh(d*x + c)^3 + 9*(420*a^
3*b + 840*a^2*b^2 + 644*a*b^3 + 176*b^4 + 105*(a^4 + 4*a^3*b + 6*a^2*b^2 +
4*a*b^3 + b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2 + 35*(420*a^3*b + 840*a^
2*b^2 + 644*a*b^3 + 176*b^4 + 105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^
4)*d*x)*cosh(d*x + c) - 28*((105*a^3*b + 210*a^2*b^2 + 161*a*b^3 + 44*b^4)*
cosh(d*x + c)^6 + 5*(75*a^3*b + 120*a^2*b^2 + 71*a*b^3 + 14*b^4)*cosh(d*x +
 c)^4 + 75*a^3*b + 90*a^2*b^2 + 75*a*b^3 + 9*(45*a^3*b + 60*a^2*b^2 + 41*a*
b^3 + 14*b^4)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*cosh
(d*x + c)*sinh(d*x + c)^6 + 7*d*cosh(d*x + c)^5 + 35*(d*cosh(d*x + c)^3 + d
*cosh(d*x + c))*sinh(d*x + c)^4 + 21*d*cosh(d*x + c)^3 + 7*(3*d*cosh(d*x +
c)^5 + 10*d*cosh(d*x + c)^3 + 9*d*cosh(d*x + c))*sinh(d*x + c)^2 + 35*d*cos
h(d*x + c))
```

Sympy [A] time = 1.60797, size = 209, normalized size = 1.9

$$\begin{cases} a^4x + 4a^3bx - \frac{4a^3b\tanh(c+dx)}{d} + 6a^2b^2x - \frac{2a^2b^2\tanh^3(c+dx)}{d} - \frac{6a^2b^2\tanh(c+dx)}{d} + 4ab^3x - \frac{4ab^3\tanh^5(c+dx)}{5d} - \frac{4ab^3\tanh^3(c+dx)}{3d} - 4 \\ x\left(a + b\tanh^2(c)\right)^4 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)**2)**4,x)
```

```
[Out] Piecewise((a**4*x + 4*a**3*b*x - 4*a**3*b*tanh(c + d*x)/d + 6*a**2*b**2*x -
 2*a**2*b**2*tanh(c + d*x)**3/d - 6*a**2*b**2*tanh(c + d*x)/d + 4*a*b**3*x
- 4*a*b**3*tanh(c + d*x)**5/(5*d) - 4*a*b**3*tanh(c + d*x)**3/(3*d) - 4*a*b
**3*tanh(c + d*x)/d + b**4*x - b**4*tanh(c + d*x)**7/(7*d) - b**4*tanh(c +
d*x)**5/(5*d) - b**4*tanh(c + d*x)**3/(3*d) - b**4*tanh(c + d*x)/d, Ne(d, 0
)), (x*(a + b*tanh(c)**2)**4, True))
```

Giac [B] time = 1.19404, size = 603, normalized size = 5.48

$$\frac{\left(a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4\right)(dx + c)}{d} + \frac{8\left(105a^3be^{(12dx+12c)} + 315a^2b^2e^{(12dx+12c)} + 315ab^3e^{(12dx+12c)} + 105b^4e^{(12d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)^2)^4,x, algorithm="giac")
```

```
[Out] (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(d*x + c)/d + 8/105*(105*a^3*b*
e^(12*d*x + 12*c) + 315*a^2*b^2*e^(12*d*x + 12*c) + 315*a*b^3*e^(12*d*x + 1
2*c) + 105*b^4*e^(12*d*x + 12*c) + 630*a^3*b*e^(10*d*x + 10*c) + 1575*a^2*b
^2*e^(10*d*x + 10*c) + 1260*a*b^3*e^(10*d*x + 10*c) + 315*b^4*e^(10*d*x + 1
0*c) + 1575*a^3*b*e^(8*d*x + 8*c) + 3360*a^2*b^2*e^(8*d*x + 8*c) + 2555*a*b
^3*e^(8*d*x + 8*c) + 770*b^4*e^(8*d*x + 8*c) + 2100*a^3*b*e^(6*d*x + 6*c) +
 3990*a^2*b^2*e^(6*d*x + 6*c) + 3080*a*b^3*e^(6*d*x + 6*c) + 770*b^4*e^(6*d
*x + 6*c) + 1575*a^3*b*e^(4*d*x + 4*c) + 2835*a^2*b^2*e^(4*d*x + 4*c) + 212
1*a*b^3*e^(4*d*x + 4*c) + 609*b^4*e^(4*d*x + 4*c) + 630*a^3*b*e^(2*d*x + 2*
c) + 1155*a^2*b^2*e^(2*d*x + 2*c) + 812*a*b^3*e^(2*d*x + 2*c) + 203*b^4*e^(
2*d*x + 2*c) + 105*a^3*b + 210*a^2*b^2 + 161*a*b^3 + 44*b^4)/(d*(e^(2*d*x +
 2*c) + 1)^7)
```

3.169 $\int \left(a+b\tanh^2(c+dx)\right)^5\,dx$

Optimal. Leaf size=160

$$-\frac{b^3\left(10a^2+5ab+b^2\right)\tanh^5(c+dx)}{5d}-\frac{b^2\left(10a^2b+10a^3+5ab^2+b^3\right)\tanh^3(c+dx)}{3d}-\frac{b\left(10a^2b^2+10a^3b+5a^4+5ab^3\right.}{d}$$

```
[Out] (a + b)^5*x - (b*(5*a^4 + 10*a^3*b + 10*a^2*b^2 + 5*a*b^3 + b^4)*Tanh[c + d
*x])/d - (b^2*(10*a^3 + 10*a^2*b + 5*a*b^2 + b^3)*Tanh[c + d*x]^3)/(3*d) -
(b^3*(10*a^2 + 5*a*b + b^2)*Tanh[c + d*x]^5)/(5*d) - (b^4*(5*a + b)*Tanh[c
+ d*x]^7)/(7*d) - (b^5*Tanh[c + d*x]^9)/(9*d)
```

Rubi [A] time = 0.0942217, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3661, 390, 206}

$$-\frac{b^3\left(10a^2+5ab+b^2\right)\tanh^5(c+dx)}{5d}-\frac{b^2\left(10a^2b+10a^3+5ab^2+b^3\right)\tanh^3(c+dx)}{3d}-\frac{b\left(10a^2b^2+10a^3b+5a^4+5ab^3\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tanh[c + d*x]^2)^5,x]
```

```
[Out] (a + b)^5*x - (b*(5*a^4 + 10*a^3*b + 10*a^2*b^2 + 5*a*b^3 + b^4)*Tanh[c + d
*x])/d - (b^2*(10*a^3 + 10*a^2*b + 5*a*b^2 + b^3)*Tanh[c + d*x]^3)/(3*d) -
(b^3*(10*a^2 + 5*a*b + b^2)*Tanh[c + d*x]^5)/(5*d) - (b^4*(5*a + b)*Tanh[c
+ d*x]^7)/(7*d) - (b^5*Tanh[c + d*x]^9)/(9*d)
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\tanh^2(c+dx)\right)^5 dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^5}{1-x^2}dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-b\left(5a^4+10a^3b+10a^2b^2+5ab^3+b^4\right)-b^2\left(10a^3+10a^2b+5ab^2+b^3\right)x^2-b\right.\right.}{d}$$
$$= -\frac{b\left(5a^4+10a^3b+10a^2b^2+5ab^3+b^4\right)\tanh(c+dx)}{d} - \frac{b^2\left(10a^3+10a^2b+5ab^2+b^3\right)\tan}{3d}$$
$$= (a+b)^5x - \frac{b\left(5a^4+10a^3b+10a^2b^2+5ab^3+b^4\right)\tanh(c+dx)}{d} - \frac{b^2\left(10a^3+10a^2b+5ab\right.}{3d}$$

Mathematica [A] time = 2.14821, size = 170, normalized size = 1.06

$$\frac{\tanh(c+dx)\left(\frac{315(a+b)^5\tanh^{-1}\left(\sqrt{\tanh^2(c+dx)}\right)}{\sqrt{\tanh^2(c+dx)}} - b\left(63b^2\left(10a^2+5ab+b^2\right)\tanh^4(c+dx)+105b\left(10a^2b+10a^3+5ab^2+b^3\right)\right.\right.}{315d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tanh[c + d*x]^2)^5,x]
```

```
[Out] (Tanh[c + d*x]*((315*(a + b)^5*ArcTanh[Sqrt[Tanh[c + d*x]^2]])/Sqrt[Tanh[c
+ d*x]^2] - b*(315*(5*a^4 + 10*a^3*b + 10*a^2*b^2 + 5*a*b^3 + b^4) + 105*b*
(10*a^3 + 10*a^2*b + 5*a*b^2 + b^3)*Tanh[c + d*x]^2 + 63*b^2*(10*a^2 + 5*a*
b + b^2)*Tanh[c + d*x]^4 + 45*b^3*(5*a + b)*Tanh[c + d*x]^6 + 35*b^4*Tanh[c
 + d*x]^8)))/(315*d)
```

Maple [B] time = 0.006, size = 472, normalized size = 3.

$$-\frac{a^5\ln(\tanh(dx+c)-1)}{2d} - \frac{(\tanh(dx+c))^5ab^4}{d} - \frac{10(\tanh(dx+c))^3a^3b^2}{3d} - \frac{10(\tanh(dx+c))^3a^2b^3}{3d} - \frac{5(\tanh(dx+}{7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tanh(d*x+c)^2)^5,x)
```

```
[Out] -1/2/d*a^5*ln(tanh(d*x+c)-1)-1/d*tanh(d*x+c)^5*a*b^4-10/3/d*tanh(d*x+c)^3*a
^3*b^2-10/3/d*tanh(d*x+c)^3*a^2*b^3-5/7/d*tanh(d*x+c)^7*a*b^4-10/d*a^2*b^3*
tanh(d*x+c)-5/d*a^4*b*tanh(d*x+c)-2/d*tanh(d*x+c)^5*a^2*b^3-5/2/d*ln(tanh(d
*x+c)-1)*a*b^4-5/3/d*tanh(d*x+c)^3*a*b^4-5/d*a*b^4*tanh(d*x+c)-10/d*a^3*b^2
*tanh(d*x+c)+5/d*ln(tanh(d*x+c)+1)*a^3*b^2+5/2/d*ln(tanh(d*x+c)+1)*a^4*b-1/
7/d*tanh(d*x+c)^7*b^5-1/5/d*tanh(d*x+c)^5*b^5-1/3/d*tanh(d*x+c)^3*b^5-1/d*b
^5*tanh(d*x+c)-1/9*b^5*tanh(d*x+c)^9/d+1/2/d*ln(tanh(d*x+c)+1)*a^5+1/2/d*ln
(tanh(d*x+c)+1)*b^5-1/2/d*ln(tanh(d*x+c)-1)*b^5+5/d*ln(tanh(d*x+c)+1)*a^2*b
^3-5/2/d*ln(tanh(d*x+c)-1)*a^4*b-5/d*ln(tanh(d*x+c)-1)*a^3*b^2-5/d*ln(tanh(
d*x+c)-1)*a^2*b^3+5/2/d*ln(tanh(d*x+c)+1)*a*b^4
```

Maxima [B] time = 1.20509, size = 842, normalized size = 5.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)^2)^5,x, algorithm="maxima")
```

```
[Out] 1/315*b^5*(315*x + 315*c/d - 2*(3492*e^(-2*d*x - 2*c) + 13968*e^(-4*d*x - 4
*c) + 26292*e^(-6*d*x - 6*c) + 39438*e^(-8*d*x - 8*c) + 31500*e^(-10*d*x -
10*c) + 21000*e^(-12*d*x - 12*c) + 6300*e^(-14*d*x - 14*c) + 1575*e^(-16*d*
x - 16*c) + 563)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*
x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x -
12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) +
 1))) + 1/21*a*b^4*(105*x + 105*c/d - 8*(203*e^(-2*d*x - 2*c) + 609*e^(-4*d
*x - 4*c) + 770*e^(-6*d*x - 6*c) + 770*e^(-8*d*x - 8*c) + 315*e^(-10*d*x -
10*c) + 105*e^(-12*d*x - 12*c) + 44)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x
- 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c)
+ 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 2/3*a^2*b^3*(15*x + 15
*c/d - 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 90*e^(-6*d*x - 6*c)
+ 45*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) +
10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 10/3
*a^3*b^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*
(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 5*a^4*
b*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a^5*x
```

Fricas [B] time = 2.67108, size = 5516, normalized size = 34.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)^2)^5,x, algorithm="fricas")
```

```
[Out] 1/315*((1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5 + 3
15*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*cosh(d*x
+ c)^9 + 9*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5
 + 315*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*cosh(
d*x + c)*sinh(d*x + c)^8 - (1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640
*a*b^4 + 563*b^5)*sinh(d*x + c)^9 + 9*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2
*b^3 + 2640*a*b^4 + 563*b^5 + 315*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3
+ 5*a*b^4 + b^5)*d*x)*cosh(d*x + c)^7 - 9*(1225*a^4*b + 2800*a^3*b^2 + 2730
*a^2*b^3 + 1240*a*b^4 + 213*b^5 + 4*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b
^3 + 2640*a*b^4 + 563*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 21*(4*(1575*a
^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5 + 315*(a^5 + 5*a^
4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*cosh(d*x + c)^3 + 3*(15
75*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5 + 315*(a^5 +
5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*cosh(d*x + c))*sinh
(d*x + c)^6 + 36*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 5
63*b^5 + 315*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)
*cosh(d*x + c)^5 - 9*(3500*a^4*b + 7000*a^3*b^2 + 6720*a^2*b^3 + 3560*a*b^4
 + 852*b^5 + 14*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 56
3*b^5)*cosh(d*x + c)^4 + 21*(1225*a^4*b + 2800*a^3*b^2 + 2730*a^2*b^3 + 124
0*a*b^4 + 213*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 9*(14*(1575*a^4*b + 4
200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5 + 315*(a^5 + 5*a^4*b + 10
*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*cosh(d*x + c)^5 + 35*(1575*a^4*
b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5 + 315*(a^5 + 5*a^4*b
 + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*cosh(d*x + c)^3 + 20*(1575
*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5 + 315*(a^5 + 5*
a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*cosh(d*x + c))*sinh(d
*x + c)^4 + 84*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563
```

```
*b^5 + 315*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*c
osh(d*x + c)^3 - 3*(28*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b
^4 + 563*b^5)*cosh(d*x + c)^6 + 14700*a^4*b + 26600*a^3*b^2 + 27440*a^2*b^3
 + 13720*a*b^4 + 1764*b^5 + 105*(1225*a^4*b + 2800*a^3*b^2 + 2730*a^2*b^3 +
 1240*a*b^4 + 213*b^5)*cosh(d*x + c)^4 + 120*(875*a^4*b + 1750*a^3*b^2 + 16
80*a^2*b^3 + 890*a*b^4 + 213*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 9*(4*(
1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5 + 315*(a^5
+ 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*cosh(d*x + c)^7 +
 21*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5 + 315*
(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*cosh(d*x + c
)^5 + 40*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5 +
 315*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*cosh(d*
x + c)^3 + 28*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*
b^5 + 315*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*co
sh(d*x + c))*sinh(d*x + c)^2 + 126*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^
3 + 2640*a*b^4 + 563*b^5 + 315*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5
*a*b^4 + b^5)*d*x)*cosh(d*x + c) - 9*((1575*a^4*b + 4200*a^3*b^2 + 4830*a^2
*b^3 + 2640*a*b^4 + 563*b^5)*cosh(d*x + c)^8 + 7*(1225*a^4*b + 2800*a^3*b^2
 + 2730*a^2*b^3 + 1240*a*b^4 + 213*b^5)*cosh(d*x + c)^6 + 2450*a^4*b + 4200
*a^3*b^2 + 4620*a^2*b^3 + 1960*a*b^4 + 882*b^5 + 20*(875*a^4*b + 1750*a^3*b
^2 + 1680*a^2*b^3 + 890*a*b^4 + 213*b^5)*cosh(d*x + c)^4 + 28*(525*a^4*b +
950*a^3*b^2 + 980*a^2*b^3 + 490*a*b^4 + 63*b^5)*cosh(d*x + c)^2)*sinh(d*x +
 c))/(d*cosh(d*x + c)^9 + 9*d*cosh(d*x + c)*sinh(d*x + c)^8 + 9*d*cosh(d*x
+ c)^7 + 21*(4*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^6 + 36*
d*cosh(d*x + c)^5 + 9*(14*d*cosh(d*x + c)^5 + 35*d*cosh(d*x + c)^3 + 20*d*c
osh(d*x + c))*sinh(d*x + c)^4 + 84*d*cosh(d*x + c)^3 + 9*(4*d*cosh(d*x + c)
^7 + 21*d*cosh(d*x + c)^5 + 40*d*cosh(d*x + c)^3 + 28*d*cosh(d*x + c))*sinh
(d*x + c)^2 + 126*d*cosh(d*x + c))
```

Sympy [A] time = 2.82775, size = 308, normalized size = 1.92

$$\begin{cases} a^5x + 5a^4bx - \frac{5a^4b\tanh(c+dx)}{d} + 10a^3b^2x - \frac{10a^3b^2\tanh^3(c+dx)}{3d} - \frac{10a^3b^2\tanh(c+dx)}{d} + 10a^2b^3x - \frac{2a^2b^3\tanh^5(c+dx)}{d} - \frac{10a^2b^3\tanh^3(c}{3d} \\ x\left(a + b\tanh^2(c)\right)^5 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)**2)**5,x)
```

```
[Out] Piecewise((a**5*x + 5*a**4*b*x - 5*a**4*b*tanh(c + d*x)/d + 10*a**3*b**2*x
- 10*a**3*b**2*tanh(c + d*x)**3/(3*d) - 10*a**3*b**2*tanh(c + d*x)/d + 10*a
**2*b**3*x - 2*a**2*b**3*tanh(c + d*x)**5/d - 10*a**2*b**3*tanh(c + d*x)**3
/(3*d) - 10*a**2*b**3*tanh(c + d*x)/d + 5*a*b**4*x - 5*a*b**4*tanh(c + d*x)
**7/(7*d) - a*b**4*tanh(c + d*x)**5/d - 5*a*b**4*tanh(c + d*x)**3/(3*d) - 5
*a*b**4*tanh(c + d*x)/d + b**5*x - b**5*tanh(c + d*x)**9/(9*d) - b**5*tanh(
c + d*x)**7/(7*d) - b**5*tanh(c + d*x)**5/(5*d) - b**5*tanh(c + d*x)**3/(3*
d) - b**5*tanh(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c)**2)**5, True))
```

Giac [B] time = 1.22694, size = 973, normalized size = 6.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)^2)^5,x, algorithm="giac")
```

```
[Out] (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*(d*x + c)/d + 2/3
15*(1575*a^4*b*e^(16*d*x + 16*c) + 6300*a^3*b^2*e^(16*d*x + 16*c) + 9450*a^
2*b^3*e^(16*d*x + 16*c) + 6300*a*b^4*e^(16*d*x + 16*c) + 1575*b^5*e^(16*d*x
 + 16*c) + 12600*a^4*b*e^(14*d*x + 14*c) + 44100*a^3*b^2*e^(14*d*x + 14*c)
+ 56700*a^2*b^3*e^(14*d*x + 14*c) + 31500*a*b^4*e^(14*d*x + 14*c) + 6300*b^
5*e^(14*d*x + 14*c) + 44100*a^4*b*e^(12*d*x + 12*c) + 136500*a^3*b^2*e^(12*
d*x + 12*c) + 161700*a^2*b^3*e^(12*d*x + 12*c) + 90300*a*b^4*e^(12*d*x + 12
*c) + 21000*b^5*e^(12*d*x + 12*c) + 88200*a^4*b*e^(10*d*x + 10*c) + 245700*
a^3*b^2*e^(10*d*x + 10*c) + 283500*a^2*b^3*e^(10*d*x + 10*c) + 157500*a*b^4
*e^(10*d*x + 10*c) + 31500*b^5*e^(10*d*x + 10*c) + 110250*a^4*b*e^(8*d*x +
8*c) + 283500*a^3*b^2*e^(8*d*x + 8*c) + 325080*a^2*b^3*e^(8*d*x + 8*c) + 17
5140*a*b^4*e^(8*d*x + 8*c) + 39438*b^5*e^(8*d*x + 8*c) + 88200*a^4*b*e^(6*d
*x + 6*c) + 216300*a^3*b^2*e^(6*d*x + 6*c) + 244020*a^2*b^3*e^(6*d*x + 6*c)
 + 131460*a*b^4*e^(6*d*x + 6*c) + 26292*b^5*e^(6*d*x + 6*c) + 44100*a^4*b*e
^(4*d*x + 4*c) + 107100*a^3*b^2*e^(4*d*x + 4*c) + 117180*a^2*b^3*e^(4*d*x +
 4*c) + 63540*a*b^4*e^(4*d*x + 4*c) + 13968*b^5*e^(4*d*x + 4*c) + 12600*a^4
*b*e^(2*d*x + 2*c) + 31500*a^3*b^2*e^(2*d*x + 2*c) + 34020*a^2*b^3*e^(2*d*x
 + 2*c) + 17460*a*b^4*e^(2*d*x + 2*c) + 3492*b^5*e^(2*d*x + 2*c) + 1575*a^4
*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5)/(d*(e^(2*d*x + 2*c
) + 1)^9)
```

3.170 $\int \frac{\tanh^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=66

$$\frac{a^2\log\left(a+b\tanh^2(c+dx)\right)}{2b^2d(a+b)}+\frac{\log(\cosh(c+dx))}{d(a+b)}-\frac{\tanh^2(c+dx)}{2bd}$$

```
[Out] Log[Cosh[c + d*x]]/((a + b)*d) + (a^2*Log[a + b*Tanh[c + d*x]^2])/(2*b^2*(a
 + b)*d) - Tanh[c + d*x]^2/(2*b*d)
```

Rubi [A] time = 0.11458, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 446, 72}

$$\frac{a^2\log\left(a+b\tanh^2(c+dx)\right)}{2b^2d(a+b)}+\frac{\log(\cosh(c+dx))}{d(a+b)}-\frac{\tanh^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^5/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] Log[Cosh[c + d*x]]/((a + b)*d) + (a^2*Log[a + b*Tanh[c + d*x]^2])/(2*b^2*(a
 + b)*d) - Tanh[c + d*x]^2/(2*b*d)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\tanh^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1-x)(a+bx)}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{b}-\frac{1}{(a+b)(-1+x)}+\frac{a^2}{b(a+b)(a+bx)}\right)dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\log(\cosh(c+dx))}{(a+b)d}+\frac{a^2\log\left(a+b\tanh^2(c+dx)\right)}{2b^2(a+b)d}-\frac{\tanh^2(c+dx)}{2bd}$$

Mathematica [A] time = 0.160528, size = 60, normalized size = 0.91

$$-\frac{-\frac{a^2\log\left(a+b\tanh^2(c+dx)\right)}{b^2(a+b)}-\frac{2\log(\cosh(c+dx))}{a+b}+\frac{\tanh^2(c+dx)}{b}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^5/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -((-2*Log[Cosh[c + d*x]])/(a + b) - (a^2*Log[a + b*Tanh[c + d*x]^2])/(b^2*(
a + b)) + Tanh[c + d*x]^2/b)/(2*d)
```

Maple [A] time = 0.018, size = 93, normalized size = 1.4

$$-\frac{(\tanh(dx+c))^2}{2bd}-\frac{\ln(\tanh(dx+c)+1)}{d(2b+2a)}+\frac{a^2\ln\left(a+b(\tanh(dx+c))^2\right)}{2b^2(a+b)d}-\frac{\ln(\tanh(dx+c)-1)}{d(2b+2a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2),x)
```

```
[Out] -1/2*tanh(d*x+c)^2/b/d-1/d/(2*b+2*a)*ln(tanh(d*x+c)+1)+1/2*a^2*ln(a+b*tanh(
d*x+c)^2)/b^2/(a+b)/d-1/d/(2*b+2*a)*ln(tanh(d*x+c)-1)
```

Maxima [B] time = 1.57601, size = 180, normalized size = 2.73

$$\frac{a^2\log\left(2(a-b)e^{(-2dx-2c)}+(a+b)e^{(-4dx-4c)}+a+b\right)}{2\left(ab^2+b^3\right)d}+\frac{dx+c}{(a+b)d}+\frac{2e^{(-2dx-2c)}}{\left(2be^{(-2dx-2c)}+be^{(-4dx-4c)}+b\right)d}-\frac{(a-b)\log\left(e^{(\cdot}\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/
((a*b^2 + b^3)*d) + (d*x + c)/((a + b)*d) + 2*e^(-2*d*x - 2*c)/((2*b*e^(-2*
d*x - 2*c) + b*e^(-4*d*x - 4*c) + b)*d) - (a - b)*log(e^(-2*d*x - 2*c) + 1)
/(b^2*d)
```

Fricas [B] time = 3.05674, size = 1817, normalized size = 27.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*d*x*cosh(d*x + c)^4 + 8*b^2*d*x*cosh(d*x + c)*sinh(d*x + c)^3 +
 2*b^2*d*x*sinh(d*x + c)^4 + 2*b^2*d*x + 4*(b^2*d*x - a*b - b^2)*cosh(d*x +
 c)^2 + 4*(3*b^2*d*x*cosh(d*x + c)^2 + b^2*d*x - a*b - b^2)*sinh(d*x + c)^2
 - (a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*
x + c)^4 + 2*a^2*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x
 + c)^2 + a^2 + 4*(a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c))*
log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a - b)/(cosh(d*x
 + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 2*((a^2 - b^2
)*cosh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 - b^
2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 - b^2)*cosh(
d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(d
*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(
cosh(d*x + c) - sinh(d*x + c))) + 8*(b^2*d*x*cosh(d*x + c)^3 + (b^2*d*x - a
*b - b^2)*cosh(d*x + c))*sinh(d*x + c))/((a*b^2 + b^3)*d*cosh(d*x + c)^4 +
4*(a*b^2 + b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a*b^2 + b^3)*d*sinh(d*x
+ c)^4 + 2*(a*b^2 + b^3)*d*cosh(d*x + c)^2 + 2*(3*(a*b^2 + b^3)*d*cosh(d*x
+ c)^2 + (a*b^2 + b^3)*d)*sinh(d*x + c)^2 + (a*b^2 + b^3)*d + 4*((a*b^2 + b
^3)*d*cosh(d*x + c)^3 + (a*b^2 + b^3)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [A] time = 39.4481, size = 425, normalized size = 6.44

$$\begin{cases} \tilde{\infty} x \tanh^{3}{\left(c \right)} \\ \frac{x - \frac{\log{\left(\tanh{\left(c + d x \right)} + 1 \right)}}{d} - \frac{\tanh^{4}{\left(c + d x \right)}}{4 d} - \frac{\tanh^{2}{\left(c + d x \right)}}{2 d}}{a} \\ \frac{4 d x \tanh^{2}{\left(c + d x \right)}}{2 b d \tanh^{2}{\left(c + d x \right)} - 2 b d} - \frac{4 d x}{2 b d \tanh^{2}{\left(c + d x \right)} - 2 b d} - \frac{4 \log{\left(\tanh{\left(c + d x \right)} + 1 \right)} \tanh^{2}{\left(c + d x \right)}}{2 b d \tanh^{2}{\left(c + d x \right)} - 2 b d} + \frac{4 \log{\left(\tanh{\left(c + d x \right)} + 1 \right)}}{2 b d \tanh^{2}{\left(c + d x \right)} - 2 b d} - \frac{\tanh^{4}{\left(c + d x \right)}}{2 b d \tanh^{2}{\left(c + d x \right)} - 2 b d} + \frac{2}{2 b d \tanh^{2}{\left(c + d x \right)} - 2 b d} \\ \frac{x \tanh^{5}{\left(c \right)}}{a + b \tanh^{2}{\left(c \right)}} \\ \frac{a^{2} \log{\left(- i \sqrt{a} \sqrt{\frac{1}{b}} + \tanh{\left(c + d x \right)} \right)}}{2 a b^{2} d + 2 b^{3} d} + \frac{a^{2} \log{\left(i \sqrt{a} \sqrt{\frac{1}{b}} + \tanh{\left(c + d x \right)} \right)}}{2 a b^{2} d + 2 b^{3} d} - \frac{a b \tanh^{2}{\left(c + d x \right)}}{2 a b^{2} d + 2 b^{3} d} + \frac{2 b^{2} d x}{2 a b^{2} d + 2 b^{3} d} - \frac{2 b^{2} \log{\left(\tanh{\left(c + d x \right)} + 1 \right)}}{2 a b^{2} d + 2 b^{3} d} - \frac{b^{2} \tanh^{2}{\left(c + d x \right)}}{2 a b^{2} d + 2 b^{3} d} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**5/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Piecewise((zoo*x*tanh(c)**3, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x - log(tan
h(c + d*x) + 1)/d - tanh(c + d*x)**4/(4*d) - tanh(c + d*x)**2/(2*d))/a, Eq(
b, 0)), (4*d*x*tanh(c + d*x)**2/(2*b*d*tanh(c + d*x)**2 - 2*b*d) - 4*d*x/(2
*b*d*tanh(c + d*x)**2 - 2*b*d) - 4*log(tanh(c + d*x) + 1)*tanh(c + d*x)**2/
(2*b*d*tanh(c + d*x)**2 - 2*b*d) + 4*log(tanh(c + d*x) + 1)/(2*b*d*tanh(c +
 d*x)**2 - 2*b*d) - tanh(c + d*x)**4/(2*b*d*tanh(c + d*x)**2 - 2*b*d) + 2/(
2*b*d*tanh(c + d*x)**2 - 2*b*d), Eq(a, -b)), (x*tanh(c)**5/(a + b*tanh(c)**
2), Eq(d, 0)), (a**2*log(-I*sqrt(a)*sqrt(1/b) + tanh(c + d*x))/(2*a*b**2*d
+ 2*b**3*d) + a**2*log(I*sqrt(a)*sqrt(1/b) + tanh(c + d*x))/(2*a*b**2*d + 2
*b**3*d) - a*b*tanh(c + d*x)**2/(2*a*b**2*d + 2*b**3*d) + 2*b**2*d*x/(2*a*b
**2*d + 2*b**3*d) - 2*b**2*log(tanh(c + d*x) + 1)/(2*a*b**2*d + 2*b**3*d) -
 b**2*tanh(c + d*x)**2/(2*a*b**2*d + 2*b**3*d), True))
```

Giac [B] time = 1.26656, size = 190, normalized size = 2.88

$$\frac{a^2 \log\left(ae^{(4\,dx+4\,c)} + be^{(4\,dx+4\,c)} + 2\,ae^{(2\,dx+2\,c)} - 2\,be^{(2\,dx+2\,c)} + a + b\right)}{2\left(ab^2d + b^3d\right)} - \frac{dx + c}{ad + bd} - \frac{(a-b)\log\left(e^{(2\,dx+2\,c)} + 1\right)}{b^2d} + \frac{2\,e^{(2\,}}{bd\left(e^{(2\,dx+}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*a^2*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2
*b*e^(2*d*x + 2*c) + a + b)/(a*b^2*d + b^3*d) - (d*x + c)/(a*d + b*d) - (a
- b)*log(e^(2*d*x + 2*c) + 1)/(b^2*d) + 2*e^(2*d*x + 2*c)/(b*d*(e^(2*d*x +
2*c) + 1)^2)
```

3.171 $\int \frac{\tanh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=59

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{b^{3/2}d(a+b)}+\frac{x}{a+b}-\frac{\tanh(c+dx)}{bd}$$

```
[Out] x/(a + b) + (a^(3/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(b^(3/2)*(a +
 b)*d) - Tanh[c + d*x]/(b*d)
```

Rubi [A] time = 0.10793, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3670, 479, 522, 206, 205}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{b^{3/2}d(a+b)}+\frac{x}{a+b}-\frac{\tanh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] x/(a + b) + (a^(3/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(b^(3/2)*(a +
 b)*d) - Tanh[c + d*x]/(b*d)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\tanh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{\tanh(c+dx)}{bd} + \frac{\text{Subst}\left(\int \frac{a+(-a+b)x^2}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{bd}$$

$$= -\frac{\tanh(c+dx)}{bd} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)d} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{b(a+b)d}$$

$$= \frac{x}{a+b} + \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{b^{3/2}(a+b)d} - \frac{\tanh(c+dx)}{bd}$$

Mathematica [A] time = 0.172031, size = 66, normalized size = 1.12

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{b^{3/2}d(a+b)} + \frac{c+dx}{d(a+b)} - \frac{\tanh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (c + d*x)/((a + b)*d) + (a^(3/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(
b^(3/2)*(a + b)*d) - Tanh[c + d*x]/(b*d)
```

Maple [A] time = 0.02, size = 95, normalized size = 1.6

$$-\frac{\tanh(dx+c)}{bd} + \frac{\ln(\tanh(dx+c)+1)}{d(2b+2a)} + \frac{a^2}{d(a+b)b}\arctan\left(b\tanh(dx+c)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} - \frac{\ln(\tanh(dx+c)-1)}{d(2b+2a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x)
```

```
[Out] -tanh(d*x+c)/b/d+1/d/(2*b+2*a)*ln(tanh(d*x+c)+1)+1/d*a^2/(a+b)/b/(a*b)^(1/2
)*arctan(tanh(d*x+c)*b/(a*b)^(1/2))-1/d/(2*b+2*a)*ln(tanh(d*x+c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.23915, size = 2043, normalized size = 34.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*b*d*x*cosh(d*x + c)^2 + 4*b*d*x*cosh(d*x + c)*sinh(d*x + c) + 2*b*d
*x*sinh(d*x + c)^2 + 2*b*d*x + (a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(
d*x + c) + a*sinh(d*x + c)^2 + a)*sqrt(-a/b)*log(((a^2 + 2*a*b + b^2)*cosh(
d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2
*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2
*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^
2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sin
h(d*x + c) + 4*((a*b + b^2)*cosh(d*x + c)^2 + 2*(a*b + b^2)*cosh(d*x + c)*s
inh(d*x + c) + (a*b + b^2)*sinh(d*x + c)^2 + a*b - b^2)*sqrt(-a/b))/((a + b
)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(
d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a -
 b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*s
inh(d*x + c) + a + b)) + 4*a + 4*b)/((a*b + b^2)*d*cosh(d*x + c)^2 + 2*(a*b
 + b^2)*d*cosh(d*x + c)*sinh(d*x + c) + (a*b + b^2)*d*sinh(d*x + c)^2 + (a*
b + b^2)*d), (b*d*x*cosh(d*x + c)^2 + 2*b*d*x*cosh(d*x + c)*sinh(d*x + c) +
 b*d*x*sinh(d*x + c)^2 + b*d*x + (a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sin
h(d*x + c) + a*sinh(d*x + c)^2 + a)*sqrt(a/b)*arctan(1/2*((a + b)*cosh(d*x
+ c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 +
a - b)*sqrt(a/b)/a) + 2*a + 2*b)/((a*b + b^2)*d*cosh(d*x + c)^2 + 2*(a*b +
b^2)*d*cosh(d*x + c)*sinh(d*x + c) + (a*b + b^2)*d*sinh(d*x + c)^2 + (a*b +
 b^2)*d)]
```

Sympy [A] time = 21.1208, size = 495, normalized size = 8.39

$$\begin{cases} \tilde{\infty} x \tanh^{2}{\left(c \right)} \\ \frac{x - \frac{\tanh^{3}{\left(c + d x \right)}}{3 d} - \frac{\tanh{\left(c + d x \right)}}{d}}{a} \\ \frac{x - \frac{\tanh{\left(c + d x \right)}}{d}}{b} \\ \frac{3 d x \tanh^{2}{\left(c + d x \right)}}{2 b d \tanh^{2}{\left(c + d x \right)} - 2 b d} - \frac{3 d x}{2 b d \tanh^{2}{\left(c + d x \right)} - 2 b d} - \frac{2 \tanh^{3}{\left(c + d x \right)}}{2 b d \tanh^{2}{\left(c + d x \right)} - 2 b d} + \frac{3 \tanh{\left(c + d x \right)}}{2 b d \tanh^{2}{\left(c + d x \right)} - 2 b d} \\ \frac{x \tanh^{4}{\left(c \right)}}{a + b \tanh^{2}{\left(c \right)}} \\ - \frac{2 i a^{\frac{3}{2}} b \sqrt{\frac{1}{b}} \tanh{\left(c + d x \right)}}{2 i a^{\frac{3}{2}} b^{2} d \sqrt{\frac{1}{b}} + 2 i \sqrt{a} b^{3} d \sqrt{\frac{1}{b}}} + \frac{2 i \sqrt{a} b^{2} d x \sqrt{\frac{1}{b}}}{2 i a^{\frac{3}{2}} b^{2} d \sqrt{\frac{1}{b}} + 2 i \sqrt{a} b^{3} d \sqrt{\frac{1}{b}}} - \frac{2 i \sqrt{a} b^{2} \sqrt{\frac{1}{b}} \tanh{\left(c + d x \right)}}{2 i a^{\frac{3}{2}} b^{2} d \sqrt{\frac{1}{b}} + 2 i \sqrt{a} b^{3} d \sqrt{\frac{1}{b}}} + \frac{a^{2} \log{\left(- i \sqrt{a} \sqrt{\frac{1}{b}} + \tanh{\left(c + d x \right)} \right)}}{2 i a^{\frac{3}{2}} b^{2} d \sqrt{\frac{1}{b}} + 2 i \sqrt{a} b^{3} d \sqrt{\frac{1}{b}}} - \frac{a^{2} \log{\left(i \sqrt{a} \sqrt{\frac{1}{b}} + \tanh{\left(c + d x \right)} \right)}}{2 i a^{\frac{3}{2}} b^{2} d \sqrt{\frac{1}{b}} + 2 i \sqrt{a} b^{3} d \sqrt{\frac{1}{b}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**4/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Piecewise((zoo*x*tanh(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x - tanh(c
+ d*x)**3/(3*d) - tanh(c + d*x)/d)/a, Eq(b, 0)), ((x - tanh(c + d*x)/d)/b,
```

```
Eq(a, 0)), (3*d*x*tanh(c + d*x)**2/(2*b*d*tanh(c + d*x)**2 - 2*b*d) - 3*d*x
/(2*b*d*tanh(c + d*x)**2 - 2*b*d) - 2*tanh(c + d*x)**3/(2*b*d*tanh(c + d*x)
**2 - 2*b*d) + 3*tanh(c + d*x)/(2*b*d*tanh(c + d*x)**2 - 2*b*d), Eq(a, -b))
, (x*tanh(c)**4/(a + b*tanh(c)**2), Eq(d, 0)), (-2*I*a**(3/2)*b*sqrt(1/b)*t
anh(c + d*x)/(2*I*a**(3/2)*b**2*d*sqrt(1/b) + 2*I*sqrt(a)*b**3*d*sqrt(1/b))
 + 2*I*sqrt(a)*b**2*d*x*sqrt(1/b)/(2*I*a**(3/2)*b**2*d*sqrt(1/b) + 2*I*sqrt
(a)*b**3*d*sqrt(1/b)) - 2*I*sqrt(a)*b**2*sqrt(1/b)*tanh(c + d*x)/(2*I*a**(3
/2)*b**2*d*sqrt(1/b) + 2*I*sqrt(a)*b**3*d*sqrt(1/b)) + a**2*log(-I*sqrt(a)*
sqrt(1/b) + tanh(c + d*x))/(2*I*a**(3/2)*b**2*d*sqrt(1/b) + 2*I*sqrt(a)*b**
3*d*sqrt(1/b)) - a**2*log(I*sqrt(a)*sqrt(1/b) + tanh(c + d*x))/(2*I*a**(3/2
)*b**2*d*sqrt(1/b) + 2*I*sqrt(a)*b**3*d*sqrt(1/b)), True))
```

Giac [A] time = 1.20782, size = 126, normalized size = 2.14

$$\frac{a^2\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\left(abd+b^2d\right)\sqrt{ab}}+\frac{dx+c}{ad+bd}+\frac{2}{bd\left(e^{(2\,dx+2\,c)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] a^2*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/(
(a*b*d + b^2*d)*sqrt(a*b)) + (d*x + c)/(a*d + b*d) + 2/(b*d*(e^(2*d*x + 2*c
) + 1))
```

3.172 $\int \frac{\tanh^3(c+dx)}{a+b\tanh^2(c+dx)} dx$

Optimal. Leaf size=46

$$\frac{\log(\cosh(c+dx))}{d(a+b)} - \frac{a\log\left(a+b\tanh^2(c+dx)\right)}{2bd(a+b)}$$

[Out] Log[Cosh[c + d*x]]/((a + b)*d) - (a*Log[a + b*Tanh[c + d*x]^2])/(2*b*(a + b
)*d)

Rubi [A] time = 0.103652, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 446, 72}

$$\frac{\log(\cosh(c+dx))}{d(a+b)} - \frac{a\log\left(a+b\tanh^2(c+dx)\right)}{2bd(a+b)}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]

[Out] Log[Cosh[c + d*x]]/((a + b)*d) - (a*Log[a + b*Tanh[c + d*x]^2])/(2*b*(a + b
)*d)

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\tanh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{x}{(1-x)(a+bx)}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)(-1+x)} - \frac{a}{(a+b)(a+bx)}\right)dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\log(\cosh(c+dx))}{(a+b)d} - \frac{a\log\left(a+b\tanh^2(c+dx)\right)}{2b(a+b)d}$$

Mathematica [A] time = 0.0343482, size = 42, normalized size = 0.91

$$\frac{2b\log(\cosh(c+dx)) - a\log\left(a+b\tanh^2(c+dx)\right)}{2abd+2b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (2*b*Log[Cosh[c + d*x]] - a*Log[a + b*Tanh[c + d*x]^2])/(2*a*b*d + 2*b^2*d)
```

Maple [A] time = 0.017, size = 75, normalized size = 1.6

$$-\frac{\ln(\tanh(dx+c)+1)}{d(2b+2a)} - \frac{a\ln\left(a+b(\tanh(dx+c))^2\right)}{2b(a+b)d} - \frac{\ln(\tanh(dx+c)-1)}{d(2b+2a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x)
```

```
[Out] -1/d/(2*b+2*a)*ln(tanh(d*x+c)+1)-1/2*a*ln(a+b*tanh(d*x+c)^2)/b/(a+b)/d-1/d/
(2*b+2*a)*ln(tanh(d*x+c)-1)
```

Maxima [A] time = 1.5482, size = 111, normalized size = 2.41

$$-\frac{a\log\left(2(a-b)e^{(-2dx-2c)} + (a+b)e^{(-4dx-4c)} + a + b\right)}{2\left(ab+b^2\right)d} + \frac{dx+c}{(a+b)d} + \frac{\log\left(e^{(-2dx-2c)}+1\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/2*a*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/(
(a*b + b^2)*d) + (d*x + c)/((a + b)*d) + log(e^(-2*d*x - 2*c) + 1)/(b*d)
```

Fricas [B] time = 2.43452, size = 319, normalized size = 6.93

$$-\frac{2\,bdx + a\log\left(\frac{2\left((a+b)\cosh(dx+c)^2+(a+b)\sinh(dx+c)^2+a-b\right)}{\cosh(dx+c)^2-2\,\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2}\right) - 2\,(a+b)\log\left(\frac{2\,\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{2\left(ab+b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*x + a*log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2
+ a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
)) - 2*(a + b)*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/((a*b
+ b^2)*d)
```

Sympy [A] time = 18.753, size = 316, normalized size = 6.87

$$\begin{cases} \tilde{\infty}x\tanh(c) & \text{for } a = (\\ \frac{x-\frac{\log(\tanh(c+dx)+1)}{d}-\frac{\tanh^2(c+dx)}{2d}}{a} & \text{for } b = (\\ \frac{2dx\tanh^2(c+dx)}{2bd\tanh^2(c+dx)-2bd}-\frac{2dx}{2bd\tanh^2(c+dx)-2bd}-\frac{2\log(\tanh(c+dx)+1)\tanh^2(c+dx)}{2bd\tanh^2(c+dx)-2bd}+\frac{2\log(\tanh(c+dx)+1)}{2bd\tanh^2(c+dx)-2bd}+\frac{1}{2bd\tanh^2(c+dx)-2bd} & \text{for } a = - \\ \frac{x\tanh^3(c)}{a+b\tanh^2(c)} & \text{for } d = (\\ -\frac{a\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\tanh(c+dx)\right)}{2abd+2b^2d}-\frac{a\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\tanh(c+dx)\right)}{2abd+2b^2d}+\frac{2bdx}{2abd+2b^2d}-\frac{2b\log(\tanh(c+dx)+1)}{2abd+2b^2d} & \text{otherwi} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**3/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Piecewise((zoo*x*tanh(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x - log(tanh(c
 + d*x) + 1)/d - tanh(c + d*x)**2/(2*d))/a, Eq(b, 0)), (2*d*x*tanh(c + d*x)
**2/(2*b*d*tanh(c + d*x)**2 - 2*b*d) - 2*d*x/(2*b*d*tanh(c + d*x)**2 - 2*b*
d) - 2*log(tanh(c + d*x) + 1)*tanh(c + d*x)**2/(2*b*d*tanh(c + d*x)**2 - 2*
b*d) + 2*log(tanh(c + d*x) + 1)/(2*b*d*tanh(c + d*x)**2 - 2*b*d) + 1/(2*b*d
*tanh(c + d*x)**2 - 2*b*d), Eq(a, -b)), (x*tanh(c)**3/(a + b*tanh(c)**2), E
q(d, 0)), (-a*log(-I*sqrt(a)*sqrt(1/b) + tanh(c + d*x))/(2*a*b*d + 2*b**2*d
) - a*log(I*sqrt(a)*sqrt(1/b) + tanh(c + d*x))/(2*a*b*d + 2*b**2*d) + 2*b*d
*x/(2*a*b*d + 2*b**2*d) - 2*b*log(tanh(c + d*x) + 1)/(2*a*b*d + 2*b**2*d),
True))
```

Giac [B] time = 1.21236, size = 136, normalized size = 2.96

$$-\frac{a\log\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)}{2\left(abd+b^2d\right)}-\frac{dx+c}{ad+bd}+\frac{\log\left(e^{(2\,dx+2\,c)}+1\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/2*a*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*
b*e^(2*d*x + 2*c) + a + b)/(a*b*d + b^2*d) - (d*x + c)/(a*d + b*d) + log(e^
(2*d*x + 2*c) + 1)/(b*d)
```

3.173 $\int \frac{\tanh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=46

$$\frac{x}{a+b}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}d(a+b)}$$

```
[Out] x/(a + b) - (Sqrt[a]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[b]*(a +
 b)*d)
```

Rubi [A] time = 0.083218, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3670, 481, 206, 205}

$$\frac{x}{a+b}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] x/(a + b) - (Sqrt[a]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[b]*(a +
 b)*d)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\tanh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)d} - \frac{a\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)d}$$
$$= \frac{x}{a+b} - \frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}(a+b)d}$$

Mathematica [A] time = 0.0286573, size = 47, normalized size = 1.02

$$\frac{\tanh^{-1}(\tanh(c+dx)) - \frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}}}{d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (-((Sqrt[a]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/Sqrt[b]) + ArcTanh[Tan
h[c + d*x]])/((a + b)*d)
```

Maple [A] time = 0.016, size = 77, normalized size = 1.7

$$\frac{\ln(\tanh(dx+c)+1)}{d(2b+2a)} - \frac{a}{d(a+b)}\arctan\left(b\tanh(dx+c)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} - \frac{\ln(\tanh(dx+c)-1)}{d(2b+2a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x)
```

```
[Out] 1/d/(2*b+2*a)*ln(tanh(d*x+c)+1)-1/d/(a+b)*a/(a*b)^(1/2)*arctan(tanh(d*x+c)*
b/(a*b)^(1/2))-1/d/(2*b+2*a)*ln(tanh(d*x+c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.91305, size = 1249, normalized size = 27.15

$$\left[2\,dx + \sqrt{-\frac{a}{b}}\log\left(\frac{\left(a^2+2\,ab+b^2\right)\cosh(dx+c)^4+4\left(a^2+2\,ab+b^2\right)\cosh(dx+c)\sinh(dx+c)^3+\left(a^2+2\,ab+b^2\right)\sinh(dx+c)^4+2\left(a^2-b^2\right)\cosh(dx+c)^2+2\left(3\left(a^2+2\,ab\right.\right.}{(a+b)\cosh(dx+c)^4+4\,(a+b)\cosh(dx+c)\sinh(dx+c)^3+(a+b)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*d*x + sqrt(-a/b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2
+ 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x
 + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x
 + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b
+ b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a*b
 + b^2)*cosh(d*x + c)^2 + 2*(a*b + b^2)*cosh(d*x + c)*sinh(d*x + c) + (a*b
+ b^2)*sinh(d*x + c)^2 + a*b - b^2)*sqrt(-a/b))/((a + b)*cosh(d*x + c)^4 +
4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a -
b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2
+ 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b
)))/((a + b)*d), (d*x - sqrt(a/b)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(
a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(
a/b)/a))/((a + b)*d)]
```

Sympy [A] time = 11.6277, size = 294, normalized size = 6.39

$$\begin{cases} \tilde{\infty}x & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{x - \frac{\tanh(c+dx)}{d}}{a} & \text{for } b = 0 \\ \frac{x}{b} & \text{for } a = 0 \\ \frac{dx\tanh^2(c+dx)}{2bd\tanh^2(c+dx)-2bd} - \frac{dx}{2bd\tanh^2(c+dx)-2bd} + \frac{\tanh(c+dx)}{2bd\tanh^2(c+dx)-2bd} & \text{for } a = -b \\ \frac{x\tanh^2(c)}{a+b\tanh^2(c)} & \text{for } d = 0 \\ \frac{2i\sqrt{a}bdx\sqrt{\frac{1}{b}}}{2ia^{\frac{3}{2}}bd\sqrt{\frac{1}{b}}+2i\sqrt{a}b^2d\sqrt{\frac{1}{b}}} - \frac{a\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\tanh(c+dx)\right)}{2ia^{\frac{3}{2}}bd\sqrt{\frac{1}{b}}+2i\sqrt{a}b^2d\sqrt{\frac{1}{b}}} + \frac{a\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\tanh(c+dx)\right)}{2ia^{\frac{3}{2}}bd\sqrt{\frac{1}{b}}+2i\sqrt{a}b^2d\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**2/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x - tanh(c + d*x)/d)/a
, Eq(b, 0)), (x/b, Eq(a, 0)), (d*x*tanh(c + d*x)**2/(2*b*d*tanh(c + d*x)**2
 - 2*b*d) - d*x/(2*b*d*tanh(c + d*x)**2 - 2*b*d) + tanh(c + d*x)/(2*b*d*tan
h(c + d*x)**2 - 2*b*d), Eq(a, -b)), (x*tanh(c)**2/(a + b*tanh(c)**2), Eq(d,
 0)), (2*I*sqrt(a)*b*d*x*sqrt(1/b)/(2*I*a**(3/2)*b*d*sqrt(1/b) + 2*I*sqrt(a
)*b**2*d*sqrt(1/b)) - a*log(-I*sqrt(a)*sqrt(1/b) + tanh(c + d*x))/(2*I*a**(
3/2)*b*d*sqrt(1/b) + 2*I*sqrt(a)*b**2*d*sqrt(1/b)) + a*log(I*sqrt(a)*sqrt(1
/b) + tanh(c + d*x))/(2*I*a**(3/2)*b*d*sqrt(1/b) + 2*I*sqrt(a)*b**2*d*sqrt(
1/b)), True))
```

Giac [A] time = 1.17616, size = 92, normalized size = 2.

$$-\frac{a\arctan\left(\frac{ae^{(2dx+2c)}+be^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}(ad+bd)} + \frac{dx+c}{ad+bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -a*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/(s
qrt(a*b)*(a*d + b*d)) + (d*x + c)/(a*d + b*d)
```

3.174 $\int \frac{\tanh(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=42

$$\frac{\log\left(a+b\tanh^2(c+dx)\right)}{2d(a+b)}+\frac{\log(\cosh(c+dx))}{d(a+b)}$$

```
[Out] Log[Cosh[c + d*x]]/((a + b)*d) + Log[a + b*Tanh[c + d*x]^2]/(2*(a + b)*d)
```

Rubi [A] time = 0.0612942, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3670, 444, 36, 31}

$$\frac{\log\left(a+b\tanh^2(c+dx)\right)}{2d(a+b)}+\frac{\log(\cosh(c+dx))}{d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] Log[Cosh[c + d*x]]/((a + b)*d) + Log[a + b*Tanh[c + d*x]^2]/(2*(a + b)*d)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\tanh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \tanh^2(c+dx)\right)}{2(a+b)d} + \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, \tanh^2(c+dx)\right)}{2(a+b)d}$$
$$= \frac{\log(\cosh(c+dx))}{(a+b)d} + \frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)d}$$

Mathematica [A] time = 0.0237822, size = 35, normalized size = 0.83

$$\frac{\log\left(a+b\tanh^2(c+dx)\right)+2\log(\cosh(c+dx))}{2ad+2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (2*Log[Cosh[c + d*x]] + Log[a + b*Tanh[c + d*x]^2])/(2*a*d + 2*b*d)
```

Maple [A] time = 0.018, size = 71, normalized size = 1.7

$$-\frac{\ln(\tanh(dx+c)+1)}{d(2b+2a)} + \frac{\ln\left(a+b(\tanh(dx+c))^2\right)}{d(2b+2a)} - \frac{\ln(\tanh(dx+c)-1)}{d(2b+2a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)/(a+b*tanh(d*x+c)^2),x)
```

```
[Out] -1/d/(2*b+2*a)*ln(tanh(d*x+c)+1)+1/2*ln(a+b*tanh(d*x+c)^2)/(a+b)/d-1/d/(2*b
+2*a)*ln(tanh(d*x+c)-1)
```

Maxima [A] time = 1.08011, size = 78, normalized size = 1.86

$$\frac{dx+c}{(a+b)d} + \frac{\log\left(2(a-b)e^{(-2dx-2c)}+(a+b)e^{(-4dx-4c)}+a+b\right)}{2(a+b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] (d*x + c)/((a + b)*d) + 1/2*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*
d*x - 4*c) + a + b)/((a + b)*d)
```

Fricas [B] time = 1.75035, size = 220, normalized size = 5.24

$$-\frac{2\,dx - \log\left(\frac{2\left((a+b)\cosh(dx+c)^2+(a+b)\sinh(dx+c)^2+a-b\right)}{\cosh(dx+c)^2-2\,\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2}\right)}{2\,(a+b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*d*x - log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a
- b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)))/
((a + b)*d)
```

Sympy [A] time = 11.3964, size = 156, normalized size = 3.71

$$\begin{cases} \frac{\tilde{\infty}x}{\tanh(c)} & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{x - \frac{\log(\tanh(c+dx)+1)}{d}}{a} & \text{for } b = 0 \\ \frac{1}{2bd\tanh^2(c+dx)-2bd} & \text{for } a = -b \\ \frac{x\tanh(c)}{a+b\tanh^2(c)} & \text{for } d = 0 \\ \frac{2dx}{2ad+2bd} + \frac{\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\tanh(c+dx)\right)}{2ad+2bd} + \frac{\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\tanh(c+dx)\right)}{2ad+2bd} - \frac{2\log(\tanh(c+dx)+1)}{2ad+2bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Piecewise((zoo*x/tanh(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x - log(tanh(c
 + d*x) + 1)/d)/a, Eq(b, 0)), (1/(2*b*d*tanh(c + d*x)**2 - 2*b*d), Eq(a, -b
)), (x*tanh(c)/(a + b*tanh(c)**2), Eq(d, 0)), (2*d*x/(2*a*d + 2*b*d) + log(
-I*sqrt(a)*sqrt(1/b) + tanh(c + d*x))/(2*a*d + 2*b*d) + log(I*sqrt(a)*sqrt(
1/b) + tanh(c + d*x))/(2*a*d + 2*b*d) - 2*log(tanh(c + d*x) + 1)/(2*a*d + 2
*b*d), True))
```

Giac [A] time = 1.18993, size = 84, normalized size = 2.

$$\frac{\log\left(\left|a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a-2\,b\right|\right)}{2\,(ad+bd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c) + e
^(-2*d*x - 2*c)) + 2*a - 2*b))/(a*d + b*d)
```

3.175 $\int \frac{1}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=45

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a+b)}+\frac{x}{a+b}$$

```
[Out] x/(a + b) + (Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a +
 b)*d)
```

Rubi [A] time = 0.0740487, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3660, 3675, 205}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a+b)}+\frac{x}{a+b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tanh[c + d*x]^2)^(-1),x]
```

```
[Out] x/(a + b) + (Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a +
 b)*d)
```

Rule 3660

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/(a -
 b), x] - Dist[b/(a - b), Int[Sec[e + f*x]^2/(a + b*Tan[e + f*x]^2), x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a, b]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\tanh^2(c+dx)}\,dx &= \frac{x}{a+b}+\frac{b\int \frac{\operatorname{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx}{a+b}\\ &= \frac{x}{a+b}+\frac{b\,\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\tanh(c+dx)\right)}{(a+b)d}\\ &= \frac{x}{a+b}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)d}\end{aligned}$$

Mathematica [A] time = 0.0770452, size = 65, normalized size = 1.44

$$\frac{\frac{2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}}-\log(1-\tanh(c+dx))+\log(\tanh(c+dx)+1)}{2ad+2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tanh[c + d*x]^2)^(-1),x]
```

```
[Out] ((2*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/Sqrt[a] - Log[1 - Tanh
[c + d*x]] + Log[1 + Tanh[c + d*x]])/(2*a*d + 2*b*d)
```

Maple [B] time = 0.017, size = 76, normalized size = 1.7

$$\frac{\ln(\tanh(dx+c)+1)}{d(2b+2a)}+\frac{b}{d(a+b)}\arctan\left(b\tanh(dx+c)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{\ln(\tanh(dx+c)-1)}{d(2b+2a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tanh(d*x+c)^2),x)
```

```
[Out] 1/d/(2*b+2*a)*ln(tanh(d*x+c)+1)+1/d*b/(a+b)/(a*b)^(1/2)*arctan(tanh(d*x+c)*
b/(a*b)^(1/2))-1/d/(2*b+2*a)*ln(tanh(d*x+c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.95923, size = 1249, normalized size = 27.76

$$\left[\frac{2\,dx+\sqrt{-\frac{b}{a}}\log\left(\frac{\left(a^2+2\,ab+b^2\right)\cosh(dx+c)^4+4\left(a^2+2\,ab+b^2\right)\cosh(dx+c)\sinh(dx+c)^3+\left(a^2+2\,ab+b^2\right)\sinh(dx+c)^4+2\left(a^2-b^2\right)\cosh(dx+c)^2+2\left(3\left(a^2+\right.\right.}{(a+b)\cosh(dx+c)^4+4\,(a+b)\cosh(dx+c)\sinh(dx+c)^3+(a}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*d*x + sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2
+ 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x
 + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x
 + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b
+ b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2
 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2
+ a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 +
4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a -
b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2
+ 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b
)))/((a + b)*d), (d*x + sqrt(b/a)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(
a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(
b/a)/b))/((a + b)*d)]
```

Sympy [A] time = 11.581, size = 280, normalized size = 6.22

$$\begin{cases} \frac{\tilde{\infty} x}{\tanh^{2}{\left(c \right)}} & \text{for}\: a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{x}{a} & \text{for}\: b = 0 \\ \frac{x - \frac{1}{d \tanh{\left(c + d x \right)}}}{b} & \text{for}\: a = 0 \\ - \frac{d x \tanh^{2}{\left(c + d x \right)}}{2 b d \tanh^{2}{\left(c + d x \right)} - 2 b d} + \frac{d x}{2 b d \tanh^{2}{\left(c + d x \right)} - 2 b d} + \frac{\tanh{\left(c + d x \right)}}{2 b d \tanh^{2}{\left(c + d x \right)} - 2 b d} & \text{for}\: a = - b \\ \frac{x}{a + b \tanh^{2}{\left(c \right)}} & \text{for}\: d = 0 \\ \frac{2 i \sqrt{a} d x \sqrt{\frac{1}{b}}}{2 i a^{\frac{3}{2}} d \sqrt{\frac{1}{b}} + 2 i \sqrt{a} b d \sqrt{\frac{1}{b}}} + \frac{\log{\left(- i \sqrt{a} \sqrt{\frac{1}{b}} + \tanh{\left(c + d x \right)} \right)}}{2 i a^{\frac{3}{2}} d \sqrt{\frac{1}{b}} + 2 i \sqrt{a} b d \sqrt{\frac{1}{b}}} - \frac{\log{\left(i \sqrt{a} \sqrt{\frac{1}{b}} + \tanh{\left(c + d x \right)} \right)}}{2 i a^{\frac{3}{2}} d \sqrt{\frac{1}{b}} + 2 i \sqrt{a} b d \sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Piecewise((zoo*x/tanh(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x/a, Eq(b, 0
)), ((x - 1/(d*tanh(c + d*x)))/b, Eq(a, 0)), (-d*x*tanh(c + d*x)**2/(2*b*d*
tanh(c + d*x)**2 - 2*b*d) + d*x/(2*b*d*tanh(c + d*x)**2 - 2*b*d) + tanh(c +
 d*x)/(2*b*d*tanh(c + d*x)**2 - 2*b*d), Eq(a, -b)), (x/(a + b*tanh(c)**2),
Eq(d, 0)), (2*I*sqrt(a)*d*x*sqrt(1/b)/(2*I*a**(3/2)*d*sqrt(1/b) + 2*I*sqrt(
a)*b*d*sqrt(1/b)) + log(-I*sqrt(a)*sqrt(1/b) + tanh(c + d*x))/(2*I*a**(3/2)
*d*sqrt(1/b) + 2*I*sqrt(a)*b*d*sqrt(1/b)) - log(I*sqrt(a)*sqrt(1/b) + tanh(
c + d*x))/(2*I*a**(3/2)*d*sqrt(1/b) + 2*I*sqrt(a)*b*d*sqrt(1/b)), True))
```

Giac [A] time = 1.16146, size = 90, normalized size = 2.

$$\frac{b \arctan\left(\frac{a e^{(2\,dx+2\,c)} + b e^{(2\,dx+2\,c)} + a - b}{2\,\sqrt{ab}}\right)}{\sqrt{ab}(ad + bd)} + \frac{dx + c}{ad + bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] b*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/(sq
rt(a*b)*(a*d + b*d)) + (d*x + c)/(a*d + b*d)
```

3.176 $\int \frac{\coth(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=60

$$-\frac{b\log\left(a+b\tanh^2(c+dx)\right)}{2ad(a+b)}+\frac{\log(\cosh(c+dx))}{d(a+b)}+\frac{\log(\tanh(c+dx))}{ad}$$

```
[Out] Log[Cosh[c + d*x]]/((a + b)*d) + Log[Tanh[c + d*x]]/(a*d) - (b*Log[a + b*Ta
nh[c + d*x]^2])/(2*a*(a + b)*d)
```

Rubi [A] time = 0.101497, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3670, 446, 72}

$$-\frac{b\log\left(a+b\tanh^2(c+dx)\right)}{2ad(a+b)}+\frac{\log(\cosh(c+dx))}{d(a+b)}+\frac{\log(\tanh(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] Log[Cosh[c + d*x]]/((a + b)*d) + Log[Tanh[c + d*x]]/(a*d) - (b*Log[a + b*Ta
nh[c + d*x]^2])/(2*a*(a + b)*d)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\coth(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{(1-x)x(a+bx)}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)(-1+x)}+\frac{1}{ax}-\frac{b^2}{a(a+b)(a+bx)}\right)dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\log(\cosh(c+dx))}{(a+b)d}+\frac{\log(\tanh(c+dx))}{ad}-\frac{b\log\left(a+b\tanh^2(c+dx)\right)}{2a(a+b)d}$$

Mathematica [A] time = 0.0539794, size = 54, normalized size = 0.9

$$\frac{-b\log\left(a+b\tanh^2(c+dx)\right)+2(a+b)\log(\tanh(c+dx))+2a\log(\cosh(c+dx))}{2ad(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (2*a*Log[Cosh[c + d*x]] + 2*(a + b)*Log[Tanh[c + d*x]] - b*Log[a + b*Tanh[c
 + d*x]^2])/(2*a*(a + b)*d)
```

Maple [B] time = 0.076, size = 121, normalized size = 2.

$$-\frac{1}{d(a+b)}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{b}{2da(a+b)}\ln\left(\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4 a+2\left(\tanh(1/2\,dx+c/2)\right)^2 a+4\left(\tanh(1/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)/(a+b*tanh(d*x+c)^2),x)
```

```
[Out] -1/d/(a+b)*ln(tanh(1/2*d*x+1/2*c)+1)-1/2/d*b/a/(a+b)*ln(tanh(1/2*d*x+1/2*c)
^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)+1/d/a*ln(tanh(1
/2*d*x+1/2*c))-1/d/(a+b)*ln(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [A] time = 1.09107, size = 136, normalized size = 2.27

$$-\frac{b\log\left(2(a-b)e^{(-2dx-2c)}+(a+b)e^{(-4dx-4c)}+a+b\right)}{2\left(a^2+ab\right)d}+\frac{dx+c}{(a+b)d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{ad}+\frac{\log\left(e^{(-dx-c)}-1\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/2*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/(
(a^2 + a*b)*d) + (d*x + c)/((a + b)*d) + log(e^(-d*x - c) + 1)/(a*d) + log(
e^(-d*x - c) - 1)/(a*d)
```

Fricas [B] time = 2.0569, size = 319, normalized size = 5.32

$$-\frac{2\,adx+b\log\left(\frac{2\left((a+b)\cosh(dx+c)^2+(a+b)\sinh(dx+c)^2+a-b\right)}{\cosh(dx+c)^2-2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2}\right)-2\,(a+b)\log\left(\frac{2\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{2\left(a^2+ab\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*d*x + b*log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2
+ a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
)) - 2*(a + b)*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/((a^2
+ a*b)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\coth(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(coth(c + d*x)/(a + b*tanh(c + d*x)**2), x)
```

Giac [A] time = 1.21516, size = 138, normalized size = 2.3

$$-\frac{b\log\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)}{2\left(a^2d+abd\right)}-\frac{dx+c}{ad+bd}+\frac{\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/2*b*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*
b*e^(2*d*x + 2*c) + a + b)/(a^2*d + a*b*d) - (d*x + c)/(a*d + b*d) + log(ab
s(e^(2*d*x + 2*c) - 1))/(a*d)
```

3.177 $\int \frac{\coth^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=60

$$-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d(a+b)}+\frac{x}{a+b}-\frac{\coth(c+dx)}{ad}$$

```
[Out] x/(a + b) - (b^(3/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*(a +
 b)*d) - Coth[c + d*x]/(a*d)
```

Rubi [A] time = 0.106131, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3670, 480, 522, 206, 205}

$$-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d(a+b)}+\frac{x}{a+b}-\frac{\coth(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] x/(a + b) - (b^(3/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*(a +
 b)*d) - Coth[c + d*x]/(a*d)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\coth^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{\coth(c+dx)}{ad} + \frac{\text{Subst}\left(\int \frac{a-b+bx^2}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{ad}$$
$$= -\frac{\coth(c+dx)}{ad} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)d} - \frac{b^2\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx\right)}{a(a+b)d}$$
$$= \frac{x}{a+b} - \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}(a+b)d} - \frac{\coth(c+dx)}{ad}$$

Mathematica [A] time = 0.169774, size = 67, normalized size = 1.12

$$-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d(a+b)} + \frac{c+dx}{d(a+b)} - \frac{\coth(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (c + d*x)/((a + b)*d) - (b^(3/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(
a^(3/2)*(a + b)*d) - Coth[c + d*x]/(a*d)
```

Maple [B] time = 0.084, size = 494, normalized size = 8.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2),x)
```

```
[Out] -1/2/d/a*tanh(1/2*d*x+1/2*c)+1/d/(a+b)*ln(tanh(1/2*d*x+1/2*c)+1)+1/d*b^2/(a
+b)/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*
d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-1/d*b^2/(a+b)/a/((2*(b*(a+b
))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-
a-2*b)*a)^(1/2))+1/d*b^3/(a+b)/(b*(a+b))^(1/2)/a/((2*(b*(a+b))^(1/2)-a-2*b)
*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)
)+1/d*b^2/(a+b)/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(
a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/d*b^2/(a+b)/a/
((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+
b))^(1/2)+a+2*b)*a)^(1/2))+1/d*b^3/(a+b)/(b*(a+b))^(1/2)/a/((2*(b*(a+b))^(1
```

```
/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)
*a)^(1/2))-1/2/d/a/tanh(1/2*d*x+1/2*c)-1/d/(a+b)*ln(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 1.87998, size = 2043, normalized size = 34.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")

[Out] [1/2*(2*a*d*x*cosh(d*x + c)^2 + 4*a*d*x*cosh(d*x + c)*sinh(d*x + c) + 2*a*d
*x*sinh(d*x + c)^2 - 2*a*d*x + (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(
d*x + c) + b*sinh(d*x + c)^2 - b)*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(
d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2
*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2
*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^
2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sin
h(d*x + c) - 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*s
inh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((a + b
)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(
d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a -
 b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*s
inh(d*x + c) + a + b)) - 4*a - 4*b)/((a^2 + a*b)*d*cosh(d*x + c)^2 + 2*(a^2
 + a*b)*d*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*d*sinh(d*x + c)^2 - (a^
2 + a*b)*d), (a*d*x*cosh(d*x + c)^2 + 2*a*d*x*cosh(d*x + c)*sinh(d*x + c) +
 a*d*x*sinh(d*x + c)^2 - a*d*x - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sin
h(d*x + c) + b*sinh(d*x + c)^2 - b)*sqrt(b/a)*arctan(1/2*((a + b)*cosh(d*x
+ c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 +
a - b)*sqrt(b/a)/b) - 2*a - 2*b)/((a^2 + a*b)*d*cosh(d*x + c)^2 + 2*(a^2 +
a*b)*d*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*d*sinh(d*x + c)^2 - (a^2 +
 a*b)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(coth(c + d*x)**2/(a + b*tanh(c + d*x)**2), x)
```

Giac [A] time = 1.24944, size = 127, normalized size = 2.12

$$-\frac{b^2 \arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\left(a^2d+abd\right)\sqrt{ab}}+\frac{dx+c}{ad+bd}-\frac{2}{ad\left(e^{(2\,dx+2\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -b^2*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/
((a^2*d + a*b*d)*sqrt(a*b)) + (d*x + c)/(a*d + b*d) - 2/(a*d*(e^(2*d*x + 2*
c) - 1))
```

3.178 $\int \frac{\coth^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=85

$$\frac{b^2\log\left(a+b\tanh^2(c+dx)\right)}{2a^2d(a+b)}+\frac{(a-b)\log(\tanh(c+dx))}{a^2d}+\frac{\log(\cosh(c+dx))}{d(a+b)}-\frac{\coth^2(c+dx)}{2ad}$$

```
[Out] -Coth[c + d*x]^2/(2*a*d) + Log[Cosh[c + d*x]]/((a + b)*d) + ((a - b)*Log[Ta
nh[c + d*x]])/(a^2*d) + (b^2*Log[a + b*Tanh[c + d*x]^2])/(2*a^2*(a + b)*d)
```

Rubi [A] time = 0.137222, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 446, 72}

$$\frac{b^2\log\left(a+b\tanh^2(c+dx)\right)}{2a^2d(a+b)}+\frac{(a-b)\log(\tanh(c+dx))}{a^2d}+\frac{\log(\cosh(c+dx))}{d(a+b)}-\frac{\coth^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -Coth[c + d*x]^2/(2*a*d) + Log[Cosh[c + d*x]]/((a + b)*d) + ((a - b)*Log[Ta
nh[c + d*x]])/(a^2*d) + (b^2*Log[a + b*Tanh[c + d*x]^2])/(2*a^2*(a + b)*d)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\coth^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{(1-x)x^2(a+bx)}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)(-1+x)} + \frac{1}{ax^2} + \frac{a-b}{a^2x} + \frac{b^3}{a^2(a+b)(a+bx)}\right)dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= -\frac{\coth^2(c+dx)}{2ad} + \frac{\log(\cosh(c+dx))}{(a+b)d} + \frac{(a-b)\log(\tanh(c+dx))}{a^2d} + \frac{b^2\log\left(a+b\tanh^2(c+\right.}{2a^2(a+b)d}$$

Mathematica [A] time = 0.163663, size = 60, normalized size = 0.71

$$-\frac{-\frac{b^2\log\left(a\coth^2(c+dx)+b\right)}{a^2(a+b)} - \frac{2\log(\sinh(c+dx))}{a+b} + \frac{\coth^2(c+dx)}{a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -(Coth[c + d*x]^2/a - (b^2*Log[b + a*Coth[c + d*x]^2])/(a^2*(a + b)) - (2*L
og[Sinh[c + d*x]])/(a + b))/(2*d)
```

Maple [B] time = 0.086, size = 180, normalized size = 2.1

$$-\frac{1}{8da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 - \frac{1}{d(a+b)}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right) + \frac{b^2}{2da^2(a+b)}\ln\left(\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4 a + 2\,(\tanh(1/2\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2),x)
```

```
[Out] -1/8/d/a*tanh(1/2*d*x+1/2*c)^2-1/d/(a+b)*ln(tanh(1/2*d*x+1/2*c)+1)+1/2/d*b^
2/a^2/(a+b)*ln(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2
*d*x+1/2*c)^2*b+a)-1/8/d/a/tanh(1/2*d*x+1/2*c)^2+1/d/a*ln(tanh(1/2*d*x+1/2*
c))-1/d/a^2*b*ln(tanh(1/2*d*x+1/2*c))-1/d/(a+b)*ln(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [A] time = 1.08124, size = 215, normalized size = 2.53

$$\frac{b^2\log\left(2(a-b)e^{(-2dx-2c)} + (a+b)e^{(-4dx-4c)} + a + b\right)}{2(a^3+a^2b)d} + \frac{dx+c}{(a+b)d} + \frac{2e^{(-2dx-2c)}}{\left(2ae^{(-2dx-2c)} - ae^{(-4dx-4c)} - a\right)d} + \frac{(a-b)\log\left(e^{(-dx-}\right.}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*b^2*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/
((a^3 + a^2*b)*d) + (d*x + c)/((a + b)*d) + 2*e^(-2*d*x - 2*c)/((2*a*e^(-2*
```

```
d*x - 2*c) - a*e^(-4*d*x - 4*c) - a)*d) + (a - b)*log(e^(-d*x - c) + 1)/(a^
2*d) + (a - b)*log(e^(-d*x - c) - 1)/(a^2*d)
```

Fricas [B] time = 2.48711, size = 1817, normalized size = 21.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*d*x*cosh(d*x + c)^4 + 8*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^3 +
 2*a^2*d*x*sinh(d*x + c)^4 + 2*a^2*d*x - 4*(a^2*d*x - a^2 - a*b)*cosh(d*x +
 c)^2 + 4*(3*a^2*d*x*cosh(d*x + c)^2 - a^2*d*x + a^2 + a*b)*sinh(d*x + c)^2
 - (b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*
x + c)^4 - 2*b^2*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 - b^2)*sinh(d*x
 + c)^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 - b^2*cosh(d*x + c))*sinh(d*x + c))*
log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a - b)/(cosh(d*x
 + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 2*((a^2 - b^2
)*cosh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 - b^
2)*sinh(d*x + c)^4 - 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 - b^2)*cosh(
d*x + c)^2 - a^2 + b^2)*sinh(d*x + c)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(d
*x + c)^3 - (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(
cosh(d*x + c) - sinh(d*x + c))) + 8*(a^2*d*x*cosh(d*x + c)^3 - (a^2*d*x - a
^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))/((a^3 + a^2*b)*d*cosh(d*x + c)^4 +
4*(a^3 + a^2*b)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 + a^2*b)*d*sinh(d*x
+ c)^4 - 2*(a^3 + a^2*b)*d*cosh(d*x + c)^2 + 2*(3*(a^3 + a^2*b)*d*cosh(d*x
+ c)^2 - (a^3 + a^2*b)*d)*sinh(d*x + c)^2 + (a^3 + a^2*b)*d + 4*((a^3 + a^2
*b)*d*cosh(d*x + c)^3 - (a^3 + a^2*b)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(coth(c + d*x)**3/(a + b*tanh(c + d*x)**2), x)
```

Giac [A] time = 1.19926, size = 190, normalized size = 2.24

$$\frac{b^2\log\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)}{2\left(a^3d+a^2bd\right)}-\frac{dx+c}{ad+bd}+\frac{(a-b)\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)}{a^2d}-\frac{2\,e^{(2}}{ad\left(e^{(2\,d}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*b^2*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2
*b*e^(2*d*x + 2*c) + a + b)/(a^3*d + a^2*b*d) - (d*x + c)/(a*d + b*d) + (a
```

```
- b)*log(abs(e^(2*d*x + 2*c) - 1))/(a^2*d) - 2*e^(2*d*x + 2*c)/(a*d*(e^(2*d
*x + 2*c) - 1)^2)
```

3.179 $\int \frac{\coth^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=82

$$\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d(a+b)}-\frac{(a-b)\coth(c+dx)}{a^2d}+\frac{x}{a+b}-\frac{\coth^3(c+dx)}{3ad}$$

```
[Out] x/(a + b) + (b^(5/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(5/2)*(a +
 b)*d) - ((a - b)*Coth[c + d*x])/(a^2*d) - Coth[c + d*x]^3/(3*a*d)
```

Rubi [A] time = 0.177203, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3670, 480, 583, 522, 206, 205}

$$\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d(a+b)}-\frac{(a-b)\coth(c+dx)}{a^2d}+\frac{x}{a+b}-\frac{\coth^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] x/(a + b) + (b^(5/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(5/2)*(a +
 b)*d) - ((a - b)*Coth[c + d*x])/(a^2*d) - Coth[c + d*x]^3/(3*a*d)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\coth^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{\coth^3(c+dx)}{3ad} + \frac{\text{Subst}\left(\int \frac{3(a-b)+3bx^2}{x^2(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{3ad}$$
$$= -\frac{(a-b)\coth(c+dx)}{a^2d} - \frac{\coth^3(c+dx)}{3ad} - \frac{\text{Subst}\left(\int \frac{-3(a^2-ab+b^2)-3(a-b)bx^2}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{3a^2d}$$
$$= -\frac{(a-b)\coth(c+dx)}{a^2d} - \frac{\coth^3(c+dx)}{3ad} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)d} + \frac{b^3\,\text{Subst}\left(\int\right.}{}$$
$$= \frac{x}{a+b} + \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}(a+b)d} - \frac{(a-b)\coth(c+dx)}{a^2d} - \frac{\coth^3(c+dx)}{3ad}$$

Mathematica [A] time = 0.60517, size = 91, normalized size = 1.11

$$\frac{\frac{6\left(\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}}+c+dx\right)}{a+b} - \frac{\coth(c+dx)\text{csch}^2(c+dx)((4a-3b)\cosh(2(c+dx))-2a+3b)}{a^2}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((6*(c + d*x + (b^(5/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/a^(5/2)))/
(a + b) - ((-2*a + 3*b + (4*a - 3*b)*Cosh[2*(c + d*x)])*Coth[c + d*x]*Csch[
c + d*x]^2)/a^2)/(6*d)
```

Maple [B] time = 0.096, size = 580, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2),x)
```

```
[Out] -1/24/d/a*tanh(1/2*d*x+1/2*c)^3-5/8/d/a*tanh(1/2*d*x+1/2*c)+1/2/d/a^2*tanh(
1/2*d*x+1/2*c)*b+1/d/(a+b)*ln(tanh(1/2*d*x+1/2*c)+1)-1/d*b^3/(a+b)/(b*(a+b)
)^(1/2)/a/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)
/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/d*b^3/(a+b)/a^2/((2*(b*(a+b))^(1/2)
-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a
)^(1/2))-1/d*b^4/(a+b)/a^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1
/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-1/d*
b^3/(a+b)/(b*(a+b))^(1/2)/a/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*ta
nh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/d*b^3/(a+b)/a^2/((
2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b)
)^(1/2)+a+2*b)*a)^(1/2))-1/d*b^4/(a+b)/a^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1
/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)
*a)^(1/2))-1/24/d/a/tanh(1/2*d*x+1/2*c)^3-5/8/d/a/tanh(1/2*d*x+1/2*c)+1/2/d
/a^2/tanh(1/2*d*x+1/2*c)*b-1/d/(a+b)*ln(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.2186, size = 5846, normalized size = 71.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(6*a^2*d*x*cosh(d*x + c)^6 + 36*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^5
+ 6*a^2*d*x*sinh(d*x + c)^6 - 6*(3*a^2*d*x + 4*a^2 + 2*a*b - 2*b^2)*cosh(d*
x + c)^4 + 6*(15*a^2*d*x*cosh(d*x + c)^2 - 3*a^2*d*x - 4*a^2 - 2*a*b + 2*b^
2)*sinh(d*x + c)^4 - 6*a^2*d*x + 24*(5*a^2*d*x*cosh(d*x + c)^3 - (3*a^2*d*x
 + 4*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(3*a^2*d*x + 4
*a^2 - 4*b^2)*cosh(d*x + c)^2 + 6*(15*a^2*d*x*cosh(d*x + c)^4 + 3*a^2*d*x -
 6*(3*a^2*d*x + 4*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c)^2 + 4*a^2 - 4*b^2)*sin
h(d*x + c)^2 + 3*(b^2*cosh(d*x + c)^6 + 6*b^2*cosh(d*x + c)*sinh(d*x + c)^5
 + b^2*sinh(d*x + c)^6 - 3*b^2*cosh(d*x + c)^4 + 3*(5*b^2*cosh(d*x + c)^2 -
 b^2)*sinh(d*x + c)^4 + 3*b^2*cosh(d*x + c)^2 + 4*(5*b^2*cosh(d*x + c)^3 -
3*b^2*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*b^2*cosh(d*x + c)^4 - 6*b^2*cos
h(d*x + c)^2 + b^2)*sinh(d*x + c)^2 - b^2 + 6*(b^2*cosh(d*x + c)^5 - 2*b^2*
cosh(d*x + c)^3 + b^2*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/a)*log(((a^2 +
2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x
 + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)
^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2
+ a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*
cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*
b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*s
```

```
qrt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)
^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cos
h(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b
)*cosh(d*x + c))*sinh(d*x + c) + a + b)) - 16*a^2 - 4*a*b + 12*b^2 + 12*(3*
a^2*d*x*cosh(d*x + c)^5 - 2*(3*a^2*d*x + 4*a^2 + 2*a*b - 2*b^2)*cosh(d*x +
c)^3 + (3*a^2*d*x + 4*a^2 - 4*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^3 + a^
2*b)*d*cosh(d*x + c)^6 + 6*(a^3 + a^2*b)*d*cosh(d*x + c)*sinh(d*x + c)^5 +
(a^3 + a^2*b)*d*sinh(d*x + c)^6 - 3*(a^3 + a^2*b)*d*cosh(d*x + c)^4 + 3*(5*
(a^3 + a^2*b)*d*cosh(d*x + c)^2 - (a^3 + a^2*b)*d)*sinh(d*x + c)^4 + 3*(a^3
 + a^2*b)*d*cosh(d*x + c)^2 + 4*(5*(a^3 + a^2*b)*d*cosh(d*x + c)^3 - 3*(a^3
 + a^2*b)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*(a^3 + a^2*b)*d*cosh(d*x
+ c)^4 - 6*(a^3 + a^2*b)*d*cosh(d*x + c)^2 + (a^3 + a^2*b)*d)*sinh(d*x + c)
^2 - (a^3 + a^2*b)*d + 6*((a^3 + a^2*b)*d*cosh(d*x + c)^5 - 2*(a^3 + a^2*b)
*d*cosh(d*x + c)^3 + (a^3 + a^2*b)*d*cosh(d*x + c))*sinh(d*x + c)), 1/3*(3*
a^2*d*x*cosh(d*x + c)^6 + 18*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^5 + 3*a^2*
d*x*sinh(d*x + c)^6 - 3*(3*a^2*d*x + 4*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c)^4
 + 3*(15*a^2*d*x*cosh(d*x + c)^2 - 3*a^2*d*x - 4*a^2 - 2*a*b + 2*b^2)*sinh(
d*x + c)^4 - 3*a^2*d*x + 12*(5*a^2*d*x*cosh(d*x + c)^3 - (3*a^2*d*x + 4*a^2
 + 2*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(3*a^2*d*x + 4*a^2 - 4
*b^2)*cosh(d*x + c)^2 + 3*(15*a^2*d*x*cosh(d*x + c)^4 + 3*a^2*d*x - 6*(3*a^
2*d*x + 4*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c)^2 + 4*a^2 - 4*b^2)*sinh(d*x +
c)^2 + 3*(b^2*cosh(d*x + c)^6 + 6*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + b^2*s
inh(d*x + c)^6 - 3*b^2*cosh(d*x + c)^4 + 3*(5*b^2*cosh(d*x + c)^2 - b^2)*si
nh(d*x + c)^4 + 3*b^2*cosh(d*x + c)^2 + 4*(5*b^2*cosh(d*x + c)^3 - 3*b^2*co
sh(d*x + c))*sinh(d*x + c)^3 + 3*(5*b^2*cosh(d*x + c)^4 - 6*b^2*cosh(d*x +
c)^2 + b^2)*sinh(d*x + c)^2 - b^2 + 6*(b^2*cosh(d*x + c)^5 - 2*b^2*cosh(d*x
 + c)^3 + b^2*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*((a + b)*c
osh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x +
 c)^2 + a - b)*sqrt(b/a)/b) - 8*a^2 - 2*a*b + 6*b^2 + 6*(3*a^2*d*x*cosh(d*x
 + c)^5 - 2*(3*a^2*d*x + 4*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c)^3 + (3*a^2*d*
x + 4*a^2 - 4*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^3 + a^2*b)*d*cosh(d*x
+ c)^6 + 6*(a^3 + a^2*b)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^3 + a^2*b)*d*
sinh(d*x + c)^6 - 3*(a^3 + a^2*b)*d*cosh(d*x + c)^4 + 3*(5*(a^3 + a^2*b)*d*
cosh(d*x + c)^2 - (a^3 + a^2*b)*d)*sinh(d*x + c)^4 + 3*(a^3 + a^2*b)*d*cosh
(d*x + c)^2 + 4*(5*(a^3 + a^2*b)*d*cosh(d*x + c)^3 - 3*(a^3 + a^2*b)*d*cosh
(d*x + c))*sinh(d*x + c)^3 + 3*(5*(a^3 + a^2*b)*d*cosh(d*x + c)^4 - 6*(a^3
+ a^2*b)*d*cosh(d*x + c)^2 + (a^3 + a^2*b)*d)*sinh(d*x + c)^2 - (a^3 + a^2*
b)*d + 6*((a^3 + a^2*b)*d*cosh(d*x + c)^5 - 2*(a^3 + a^2*b)*d*cosh(d*x + c)
^3 + (a^3 + a^2*b)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**4/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(coth(c + d*x)**4/(a + b*tanh(c + d*x)**2), x)
```

Giac [B] time = 1.20022, size = 203, normalized size = 2.48

$$\frac{b^3\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\left(a^3d+a^2bd\right)\sqrt{ab}}+\frac{dx+c}{ad+bd}-\frac{2\left(6\,ae^{(4\,dx+4\,c)}-3\,be^{(4\,dx+4\,c)}-6\,ae^{(2\,dx+2\,c)}+6\,be^{(2\,dx+2\,c)}+4\,a-3\,b\right)}{3\,a^2d\left(e^{(2\,dx+2\,c)}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] b^3*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/(
(a^3*d + a^2*b*d)*sqrt(a*b)) + (d*x + c)/(a*d + b*d) - 2/3*(6*a*e^(4*d*x +
4*c) - 3*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*c) + 6*b*e^(2*d*x + 2*c) + 4*
a - 3*b)/(a^2*d*(e^(2*d*x + 2*c) - 1)^3)
```

3.180 $\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=83

$$-\frac{a^2}{2b^2d(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{a(a+2b)\log\left(a+b\tanh^2(c+dx)\right)}{2b^2d(a+b)^2}+\frac{\log(\cosh(c+dx))}{d(a+b)^2}$$

```
[Out] Log[Cosh[c + d*x]]/((a + b)^2*d) - (a*(a + 2*b)*Log[a + b*Tanh[c + d*x]^2])
/(2*b^2*(a + b)^2*d) - a^2/(2*b^2*(a + b)*d*(a + b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.150087, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 446, 88}

$$-\frac{a^2}{2b^2d(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{a(a+2b)\log\left(a+b\tanh^2(c+dx)\right)}{2b^2d(a+b)^2}+\frac{\log(\cosh(c+dx))}{d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] Log[Cosh[c + d*x]]/((a + b)^2*d) - (a*(a + 2*b)*Log[a + b*Tanh[c + d*x]^2])
/(2*b^2*(a + b)^2*d) - a^2/(2*b^2*(a + b)*d*(a + b*Tanh[c + d*x]^2))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(1-x^2)\left(a+bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1-x)(a+bx)^2}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^2(-1+x)} + \frac{a^2}{b(a+b)(a+bx)^2} - \frac{a(a+2b)}{b(a+b)^2(a+bx)}\right) dx, x, \tanh^2(c+dx)\right)}{2d}$$

$$= \frac{\log(\cosh(c+dx))}{(a+b)^2d} - \frac{a(a+2b)\log\left(a+b\tanh^2(c+dx)\right)}{2b^2(a+b)^2d} - \frac{a^2}{2b^2(a+b)d\left(a+b\tanh^2(c\right.}$$

Mathematica [A] time = 0.473557, size = 69, normalized size = 0.83

$$-\frac{\frac{a^2(a+b)}{b^2\left(a+b\tanh^2(c+dx)\right)} + \frac{a(a+2b)\log\left(a+b\tanh^2(c+dx)\right)}{b^2} - 2\log(\cosh(c+dx))}{2d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -(-2*Log[Cosh[c + d*x]] + (a*(a + 2*b)*Log[a + b*Tanh[c + d*x]^2])/b^2 + (a
^2*(a + b))/(b^2*(a + b*Tanh[c + d*x]^2)))/(2*(a + b)^2*d)
```

Maple [A] time = 0.027, size = 156, normalized size = 1.9

$$-\frac{\ln(\tanh(dx+c)+1)}{2d(a+b)^2} - \frac{a^3}{2d(a+b)^2b^2\left(a+b(\tanh(dx+c))^2\right)} - \frac{a^2}{2d(a+b)^2b\left(a+b(\tanh(dx+c))^2\right)} - \frac{a^2\ln\left(a+\right.}{2\left.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] -1/2/d/(a+b)^2*ln(tanh(d*x+c)+1)-1/2/d/(a+b)^2*a^3/b^2/(a+b*tanh(d*x+c)^2)-
1/2/d/(a+b)^2*a^2/b/(a+b*tanh(d*x+c)^2)-1/2/d/(a+b)^2*a^2/b^2*ln(a+b*tanh(d
*x+c)^2)-1/d/(a+b)^2*a/b*ln(a+b*tanh(d*x+c)^2)-1/2/d/(a+b)^2*ln(tanh(d*x+c)
-1)
```

Maxima [B] time = 1.63801, size = 293, normalized size = 3.53

$$-\frac{2a^2e^{(-2dx-2c)}}{\left(a^3b+3a^2b^2+3ab^3+b^4+2\left(a^3b+a^2b^2-ab^3-b^4\right)e^{(-2dx-2c)}+\left(a^3b+3a^2b^2+3ab^3+b^4\right)e^{(-4dx-4c)}\right)d} - \frac{\left(a^2+2a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -2*a^2*e^(-2*d*x - 2*c)/((a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4 + 2*(a^3*b + a^
2*b^2 - a*b^3 - b^4)*e^(-2*d*x - 2*c) + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)
*e^(-4*d*x - 4*c))*d) - 1/2*(a^2 + 2*a*b)*log(2*(a - b)*e^(-2*d*x - 2*c) +
(a + b)*e^(-4*d*x - 4*c) + a + b)/((a^2*b^2 + 2*a*b^3 + b^4)*d) + (d*x + c)
/((a^2 + 2*a*b + b^2)*d) + log(e^(-2*d*x - 2*c) + 1)/(b^2*d)
```

Fricas [B] time = 2.61627, size = 2678, normalized size = 32.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a*b^2 + b^3)*d*x*cosh(d*x + c)^4 + 8*(a*b^2 + b^3)*d*x*cosh(d*x +
c)*sinh(d*x + c)^3 + 2*(a*b^2 + b^3)*d*x*sinh(d*x + c)^4 + 2*(a*b^2 + b^3)*
d*x + 4*(a^2*b + (a*b^2 - b^3)*d*x)*cosh(d*x + c)^2 + 4*(3*(a*b^2 + b^3)*d*
x*cosh(d*x + c)^2 + a^2*b + (a*b^2 - b^3)*d*x)*sinh(d*x + c)^2 + ((a^3 + 3*
a^2*b + 2*a*b^2)*cosh(d*x + c)^4 + 4*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(d*x + c
)*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 2*a*b^2)*sinh(d*x + c)^4 + a^3 + 3*a^2
*b + 2*a*b^2 + 2*(a^3 + a^2*b - 2*a*b^2)*cosh(d*x + c)^2 + 2*(a^3 + a^2*b -
 2*a*b^2 + 3*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4
*((a^3 + 3*a^2*b + 2*a*b^2)*cosh(d*x + c)^3 + (a^3 + a^2*b - 2*a*b^2)*cosh(
d*x + c))*sinh(d*x + c))*log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x
+ c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x
 + c)^2)) - 2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 4*(a^3 + 3
*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*sinh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 + a^2
*b - a*b^2 - b^3)*cosh(d*x + c)^2 + 2*(a^3 + a^2*b - a*b^2 - b^3 + 3*(a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (a^3 + a^2*b - a*b^2 - b^3)*cosh(d*x
 + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c)))
+ 8*((a*b^2 + b^3)*d*x*cosh(d*x + c)^3 + (a^2*b + (a*b^2 - b^3)*d*x)*cosh(d
*x + c))*sinh(d*x + c))/((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*d*cosh(d*x +
 c)^4 + 4*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*d*cosh(d*x + c)*sinh(d*x +
c)^3 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*d*sinh(d*x + c)^4 + 2*(a^3*b^2
 + a^2*b^3 - a*b^4 - b^5)*d*cosh(d*x + c)^2 + 2*(3*(a^3*b^2 + 3*a^2*b^3 + 3
*a*b^4 + b^5)*d*cosh(d*x + c)^2 + (a^3*b^2 + a^2*b^3 - a*b^4 - b^5)*d)*sinh
(d*x + c)^2 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*d + 4*((a^3*b^2 + 3*a^2
*b^3 + 3*a*b^4 + b^5)*d*cosh(d*x + c)^3 + (a^3*b^2 + a^2*b^3 - a*b^4 - b^5)
*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**5/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26749, size = 275, normalized size = 3.31

$$-\frac{\left(a^2+2\,ab\right)\log\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)}{2\left(a^2b^2d+2\,ab^3d+b^4d\right)}-\frac{dx+c}{a^2d+2\,abd+b^2d}-\frac{\cdots}{\left(ae^{(4\,dx+4\,c)}+be^{(4\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(a^2 + 2*a*b)*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x
 + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)/(a^2*b^2*d + 2*a*b^3*d + b^4*d) - (d
*x + c)/(a^2*d + 2*a*b*d + b^2*d) - 2*a^2*e^(2*d*x + 2*c)/((a*e^(4*d*x + 4*
c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)
*(a + b)^2*b*d) + log(e^(2*d*x + 2*c) + 1)/(b^2*d)
```

3.181 $\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=89

$$-\frac{\sqrt{a}(a+3b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2b^{3/2}d(a+b)^2}+\frac{a\tanh(c+dx)}{2bd(a+b)\left(a+b\tanh^2(c+dx)\right)}+\frac{x}{(a+b)^2}$$

```
[Out] x/(a + b)^2 - (Sqrt[a]*(a + 3*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(
2*b^(3/2)*(a + b)^2*d) + (a*Tanh[c + d*x])/(2*b*(a + b)*d*(a + b*Tanh[c + d
*x]^2))
```

Rubi [A] time = 0.116831, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3670, 470, 522, 206, 205}

$$-\frac{\sqrt{a}(a+3b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2b^{3/2}d(a+b)^2}+\frac{a\tanh(c+dx)}{2bd(a+b)\left(a+b\tanh^2(c+dx)\right)}+\frac{x}{(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] x/(a + b)^2 - (Sqrt[a]*(a + 3*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(
2*b^(3/2)*(a + b)^2*d) + (a*Tanh[c + d*x])/(2*b*(a + b)*d*(a + b*Tanh[c + d
*x]^2))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)\left(a+bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{a\tanh(c+dx)}{2b(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{a+(-a-2b)x^2}{(1-x^2)\left(a+bx^2\right)}\,dx, x, \tanh(c+dx)\right)}{2b(a+b)d}$$

$$= \frac{a\tanh(c+dx)}{2b(a+b)d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)^2d} - \frac{(a(a+3b))}{}$$

$$= \frac{x}{(a+b)^2} - \frac{\sqrt{a}(a+3b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2b^{3/2}(a+b)^2d} + \frac{a\tanh(c+dx)}{2b(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

Mathematica [A] time = 0.484395, size = 90, normalized size = 1.01

$$\frac{-\frac{\sqrt{a}(a+3b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{b^{3/2}} + \frac{a(a+b)\sinh(2(c+dx))}{b((a+b)\cosh(2(c+dx))+a-b)} + 2(c+dx)}{2d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (2*(c + d*x) - (Sqrt[a]*(a + 3*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x]])/Sqrt[a]])/
b^(3/2) + (a*(a + b)*Sinh[2*(c + d*x)])/(b*(a - b + (a + b)*Cosh[2*(c + d*x
)]))))/(2*(a + b)^2*d)
```

Maple [B] time = 0.026, size = 172, normalized size = 1.9

$$\frac{\ln(\tanh(dx+c)+1)}{2d(a+b)^2} + \frac{a^2\tanh(dx+c)}{2d(a+b)^2b\left(a+b(\tanh(dx+c))^2\right)} + \frac{a\tanh(dx+c)}{2d(a+b)^2\left(a+b(\tanh(dx+c))^2\right)} - \frac{a^2}{2d(a+b)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] 1/2/d/(a+b)^2*ln(tanh(d*x+c)+1)+1/2/d/(a+b)^2*a^2/b*tanh(d*x+c)/(a+b*tanh(d
*x+c)^2)+1/2/d/(a+b)^2*a*tanh(d*x+c)/(a+b*tanh(d*x+c)^2)-1/2/d/(a+b)^2*a^2/
```

```
b/(a*b)^(1/2)*arctan(tanh(d*x+c)*b/(a*b)^(1/2))-3/2/d/(a+b)^2*a/(a*b)^(1/2)
*arctan(tanh(d*x+c)*b/(a*b)^(1/2))-1/2/d/(a+b)^2*ln(tanh(d*x+c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.10094, size = 4703, normalized size = 52.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a*b + b^2)*d*x*cosh(d*x + c)^4 + 16*(a*b + b^2)*d*x*cosh(d*x + c)*
sinh(d*x + c)^3 + 4*(a*b + b^2)*d*x*sinh(d*x + c)^4 + 4*(a*b + b^2)*d*x + 4
*(2*(a*b - b^2)*d*x - a^2 + a*b)*cosh(d*x + c)^2 + 4*(6*(a*b + b^2)*d*x*cos
h(d*x + c)^2 + 2*(a*b - b^2)*d*x - a^2 + a*b)*sinh(d*x + c)^2 + ((a^2 + 4*a
*b + 3*b^2)*cosh(d*x + c)^4 + 4*(a^2 + 4*a*b + 3*b^2)*cosh(d*x + c)*sinh(d*
x + c)^3 + (a^2 + 4*a*b + 3*b^2)*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b - 3*b^2)*
cosh(d*x + c)^2 + 2*(3*(a^2 + 4*a*b + 3*b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b
- 3*b^2)*sinh(d*x + c)^2 + a^2 + 4*a*b + 3*b^2 + 4*((a^2 + 4*a*b + 3*b^2)*c
osh(d*x + c)^3 + (a^2 + 2*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-
a/b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(
d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b
^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)
*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)
^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a*b + b^2)*cosh(d*x + c
)^2 + 2*(a*b + b^2)*cosh(d*x + c)*sinh(d*x + c) + (a*b + b^2)*sinh(d*x + c)
^2 + a*b - b^2)*sqrt(-a/b))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x +
 c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 +
 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*
x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) - 4*a^2 - 4*a*b +
 8*(2*(a*b + b^2)*d*x*cosh(d*x + c)^3 + (2*(a*b - b^2)*d*x - a^2 + a*b)*cos
h(d*x + c))*sinh(d*x + c))/((a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*cosh(d*x
+ c)^4 + 4*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*cosh(d*x + c)*sinh(d*x + c
)^3 + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*sinh(d*x + c)^4 + 2*(a^3*b + a^
2*b^2 - a*b^3 - b^4)*d*cosh(d*x + c)^2 + 2*(3*(a^3*b + 3*a^2*b^2 + 3*a*b^3
+ b^4)*d*cosh(d*x + c)^2 + (a^3*b + a^2*b^2 - a*b^3 - b^4)*d)*sinh(d*x + c)
^2 + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d + 4*((a^3*b + 3*a^2*b^2 + 3*a*b^
3 + b^4)*d*cosh(d*x + c)^3 + (a^3*b + a^2*b^2 - a*b^3 - b^4)*d*cosh(d*x + c
))*sinh(d*x + c)), 1/2*(2*(a*b + b^2)*d*x*cosh(d*x + c)^4 + 8*(a*b + b^2)*d
*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a*b + b^2)*d*x*sinh(d*x + c)^4 + 2*(a
*b + b^2)*d*x + 2*(2*(a*b - b^2)*d*x - a^2 + a*b)*cosh(d*x + c)^2 + 2*(6*(a
*b + b^2)*d*x*cosh(d*x + c)^2 + 2*(a*b - b^2)*d*x - a^2 + a*b)*sinh(d*x + c
)^2 - ((a^2 + 4*a*b + 3*b^2)*cosh(d*x + c)^4 + 4*(a^2 + 4*a*b + 3*b^2)*cosh
(d*x + c)*sinh(d*x + c)^3 + (a^2 + 4*a*b + 3*b^2)*sinh(d*x + c)^4 + 2*(a^2
+ 2*a*b - 3*b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 4*a*b + 3*b^2)*cosh(d*x + c)
```

```
^2 + a^2 + 2*a*b - 3*b^2)*sinh(d*x + c)^2 + a^2 + 4*a*b + 3*b^2 + 4*((a^2 +
 4*a*b + 3*b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b - 3*b^2)*cosh(d*x + c))*sinh
(d*x + c))*sqrt(a/b)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d
*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(a/b)/a) - 2*a
^2 - 2*a*b + 4*(2*(a*b + b^2)*d*x*cosh(d*x + c)^3 + (2*(a*b - b^2)*d*x - a^
2 + a*b)*cosh(d*x + c))*sinh(d*x + c))/((a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)
*d*cosh(d*x + c)^4 + 4*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*cosh(d*x + c)*
sinh(d*x + c)^3 + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*sinh(d*x + c)^4 + 2
*(a^3*b + a^2*b^2 - a*b^3 - b^4)*d*cosh(d*x + c)^2 + 2*(3*(a^3*b + 3*a^2*b^
2 + 3*a*b^3 + b^4)*d*cosh(d*x + c)^2 + (a^3*b + a^2*b^2 - a*b^3 - b^4)*d)*s
inh(d*x + c)^2 + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d + 4*((a^3*b + 3*a^2*
b^2 + 3*a*b^3 + b^4)*d*cosh(d*x + c)^3 + (a^3*b + a^2*b^2 - a*b^3 - b^4)*d*
cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [A] time = 101.773, size = 2179, normalized size = 24.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**4/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x - tanh(c + d*x)**3/(
3*d) - tanh(c + d*x)/d)/a**2, Eq(b, 0)), (x/b**2, Eq(a, 0)), (x*tanh(c)**4/
(a + b*tanh(c)**2)**2, Eq(d, 0)), (2*I*a**(5/2)*b*sqrt(1/b)*tanh(c + d*x)/(
4*I*a**(7/2)*b**2*d*sqrt(1/b) + 4*I*a**(5/2)*b**3*d*sqrt(1/b)*tanh(c + d*x)
**2 + 8*I*a**(5/2)*b**3*d*sqrt(1/b) + 8*I*a**(3/2)*b**4*d*sqrt(1/b)*tanh(c
+ d*x)**2 + 4*I*a**(3/2)*b**4*d*sqrt(1/b) + 4*I*sqrt(a)*b**5*d*sqrt(1/b)*ta
nh(c + d*x)**2) + 4*I*a**(3/2)*b**2*d*x*sqrt(1/b)/(4*I*a**(7/2)*b**2*d*sqrt
(1/b) + 4*I*a**(5/2)*b**3*d*sqrt(1/b)*tanh(c + d*x)**2 + 8*I*a**(5/2)*b**3*
d*sqrt(1/b) + 8*I*a**(3/2)*b**4*d*sqrt(1/b)*tanh(c + d*x)**2 + 4*I*a**(3/2)
*b**4*d*sqrt(1/b) + 4*I*sqrt(a)*b**5*d*sqrt(1/b)*tanh(c + d*x)**2) + 2*I*a*
*(3/2)*b**2*sqrt(1/b)*tanh(c + d*x)/(4*I*a**(7/2)*b**2*d*sqrt(1/b) + 4*I*a*
*(5/2)*b**3*d*sqrt(1/b)*tanh(c + d*x)**2 + 8*I*a**(5/2)*b**3*d*sqrt(1/b) +
8*I*a**(3/2)*b**4*d*sqrt(1/b)*tanh(c + d*x)**2 + 4*I*a**(3/2)*b**4*d*sqrt(1
/b) + 4*I*sqrt(a)*b**5*d*sqrt(1/b)*tanh(c + d*x)**2) + 4*I*sqrt(a)*b**3*d*x
*sqrt(1/b)*tanh(c + d*x)**2/(4*I*a**(7/2)*b**2*d*sqrt(1/b) + 4*I*a**(5/2)*b
**3*d*sqrt(1/b)*tanh(c + d*x)**2 + 8*I*a**(5/2)*b**3*d*sqrt(1/b) + 8*I*a**(
3/2)*b**4*d*sqrt(1/b)*tanh(c + d*x)**2 + 4*I*a**(3/2)*b**4*d*sqrt(1/b) + 4*
I*sqrt(a)*b**5*d*sqrt(1/b)*tanh(c + d*x)**2) - a**3*log(-I*sqrt(a)*sqrt(1/b
) + tanh(c + d*x))/(4*I*a**(7/2)*b**2*d*sqrt(1/b) + 4*I*a**(5/2)*b**3*d*sqr
t(1/b)*tanh(c + d*x)**2 + 8*I*a**(5/2)*b**3*d*sqrt(1/b) + 8*I*a**(3/2)*b**4
*d*sqrt(1/b)*tanh(c + d*x)**2 + 4*I*a**(3/2)*b**4*d*sqrt(1/b) + 4*I*sqrt(a)
*b**5*d*sqrt(1/b)*tanh(c + d*x)**2) + a**3*log(I*sqrt(a)*sqrt(1/b) + tanh(c
 + d*x))/(4*I*a**(7/2)*b**2*d*sqrt(1/b) + 4*I*a**(5/2)*b**3*d*sqrt(1/b)*tan
h(c + d*x)**2 + 8*I*a**(5/2)*b**3*d*sqrt(1/b) + 8*I*a**(3/2)*b**4*d*sqrt(1/
b)*tanh(c + d*x)**2 + 4*I*a**(3/2)*b**4*d*sqrt(1/b) + 4*I*sqrt(a)*b**5*d*sq
rt(1/b)*tanh(c + d*x)**2) - a**2*b*log(-I*sqrt(a)*sqrt(1/b) + tanh(c + d*x)
)*tanh(c + d*x)**2/(4*I*a**(7/2)*b**2*d*sqrt(1/b) + 4*I*a**(5/2)*b**3*d*sqr
t(1/b)*tanh(c + d*x)**2 + 8*I*a**(5/2)*b**3*d*sqrt(1/b) + 8*I*a**(3/2)*b**4
*d*sqrt(1/b)*tanh(c + d*x)**2 + 4*I*a**(3/2)*b**4*d*sqrt(1/b) + 4*I*sqrt(a)
*b**5*d*sqrt(1/b)*tanh(c + d*x)**2) - 3*a**2*b*log(-I*sqrt(a)*sqrt(1/b) + t
anh(c + d*x))/(4*I*a**(7/2)*b**2*d*sqrt(1/b) + 4*I*a**(5/2)*b**3*d*sqrt(1/b
)*tanh(c + d*x)**2 + 8*I*a**(5/2)*b**3*d*sqrt(1/b) + 8*I*a**(3/2)*b**4*d*sq
rt(1/b)*tanh(c + d*x)**2 + 4*I*a**(3/2)*b**4*d*sqrt(1/b) + 4*I*sqrt(a)*b**5
*d*sqrt(1/b)*tanh(c + d*x)**2) + a**2*b*log(I*sqrt(a)*sqrt(1/b) + tanh(c +
d*x))*tanh(c + d*x)**2/(4*I*a**(7/2)*b**2*d*sqrt(1/b) + 4*I*a**(5/2)*b**3*d
```

```
*sqrt(1/b)*tanh(c + d*x)**2 + 8*I*a**(5/2)*b**3*d*sqrt(1/b) + 8*I*a**(3/2)*
b**4*d*sqrt(1/b)*tanh(c + d*x)**2 + 4*I*a**(3/2)*b**4*d*sqrt(1/b) + 4*I*sqr
t(a)*b**5*d*sqrt(1/b)*tanh(c + d*x)**2) + 3*a**2*b*log(I*sqrt(a)*sqrt(1/b)
+ tanh(c + d*x))/(4*I*a**(7/2)*b**2*d*sqrt(1/b) + 4*I*a**(5/2)*b**3*d*sqrt(
1/b)*tanh(c + d*x)**2 + 8*I*a**(5/2)*b**3*d*sqrt(1/b) + 8*I*a**(3/2)*b**4*d
*sqrt(1/b)*tanh(c + d*x)**2 + 4*I*a**(3/2)*b**4*d*sqrt(1/b) + 4*I*sqrt(a)*b
**5*d*sqrt(1/b)*tanh(c + d*x)**2) - 3*a*b**2*log(-I*sqrt(a)*sqrt(1/b) + tan
h(c + d*x))*tanh(c + d*x)**2/(4*I*a**(7/2)*b**2*d*sqrt(1/b) + 4*I*a**(5/2)*
b**3*d*sqrt(1/b)*tanh(c + d*x)**2 + 8*I*a**(5/2)*b**3*d*sqrt(1/b) + 8*I*a**
(3/2)*b**4*d*sqrt(1/b)*tanh(c + d*x)**2 + 4*I*a**(3/2)*b**4*d*sqrt(1/b) + 4
*I*sqrt(a)*b**5*d*sqrt(1/b)*tanh(c + d*x)**2) + 3*a*b**2*log(I*sqrt(a)*sqrt
(1/b) + tanh(c + d*x))*tanh(c + d*x)**2/(4*I*a**(7/2)*b**2*d*sqrt(1/b) + 4*
I*a**(5/2)*b**3*d*sqrt(1/b)*tanh(c + d*x)**2 + 8*I*a**(5/2)*b**3*d*sqrt(1/b
) + 8*I*a**(3/2)*b**4*d*sqrt(1/b)*tanh(c + d*x)**2 + 4*I*a**(3/2)*b**4*d*sq
rt(1/b) + 4*I*sqrt(a)*b**5*d*sqrt(1/b)*tanh(c + d*x)**2), True))
```

Giac [B] time = 1.21292, size = 274, normalized size = 3.08

$$-\frac{\left(a^{2}+3\,ab\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{2\,\left(a^{2}bd+2\,ab^{2}d+b^{3}d\right)\sqrt{ab}}+\frac{dx+c}{a^{2}d+2\,abd+b^{2}d}-\frac{a^{2}e^{(2\,dx+2\,c)}-abe^{(2\,dx+2\,c)}+a^{2}}{\left(a^{2}bd+2\,ab^{2}d+b^{3}d\right)\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(a^2 + 3*a*b)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a -
b)/sqrt(a*b))/((a^2*b*d + 2*a*b^2*d + b^3*d)*sqrt(a*b)) + (d*x + c)/(a^2*d
+ 2*a*b*d + b^2*d) - (a^2*e^(2*d*x + 2*c) - a*b*e^(2*d*x + 2*c) + a^2 + a*b
)/((a^2*b*d + 2*a*b^2*d + b^3*d)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2
*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b))
```

3.182 $\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=72

$$\frac{a}{2bd(a+b)\left(a+b\tanh^2(c+dx)\right)}+\frac{\log\left(a+b\tanh^2(c+dx)\right)}{2d(a+b)^2}+\frac{\log(\cosh(c+dx))}{d(a+b)^2}$$

```
[Out] Log[Cosh[c + d*x]]/((a + b)^2*d) + Log[a + b*Tanh[c + d*x]^2]/(2*(a + b)^2*
d) + a/(2*b*(a + b)*d*(a + b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.118215, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 446, 77}

$$\frac{a}{2bd(a+b)\left(a+b\tanh^2(c+dx)\right)}+\frac{\log\left(a+b\tanh^2(c+dx)\right)}{2d(a+b)^2}+\frac{\log(\cosh(c+dx))}{d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] Log[Cosh[c + d*x]]/((a + b)^2*d) + Log[a + b*Tanh[c + d*x]^2]/(2*(a + b)^2*
d) + a/(2*b*(a + b)*d*(a + b*Tanh[c + d*x]^2))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{x}{(1-x)(a+bx)^2}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^2(-1+x)} - \frac{a}{(a+b)(a+bx)^2} + \frac{b}{(a+b)^2(a+bx)}\right)dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\log(\cosh(c+dx))}{(a+b)^2d} + \frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)^2d} + \frac{a}{2b(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

Mathematica [A] time = 0.438799, size = 57, normalized size = 0.79

$$\frac{\frac{a(a+b)}{b\left(a+b\tanh^2(c+dx)\right)} + \log\left(a+b\tanh^2(c+dx)\right) + 2\log(\cosh(c+dx))}{2d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (2*Log[Cosh[c + d*x]] + Log[a + b*Tanh[c + d*x]^2] + (a*(a + b))/(b*(a + b*
Tanh[c + d*x]^2)))/(2*(a + b)^2*d)
```

Maple [A] time = 0.024, size = 118, normalized size = 1.6

$$-\frac{\ln(\tanh(dx+c)+1)}{2d(a+b)^2} + \frac{a^2}{2d(a+b)^2 b\left(a+b(\tanh(dx+c))^2\right)} + \frac{a}{2d(a+b)^2\left(a+b(\tanh(dx+c))^2\right)} + \frac{\ln\left(a+b(\tanh\right.}{2d(a+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] -1/2/d/(a+b)^2*ln(tanh(d*x+c)+1)+1/2/d/(a+b)^2*a^2/b/(a+b*tanh(d*x+c)^2)+1/
2/d/(a+b)^2*a/(a+b*tanh(d*x+c)^2)+1/2*ln(a+b*tanh(d*x+c)^2)/(a+b)^2/d-1/2/d
/(a+b)^2*ln(tanh(d*x+c)-1)
```

Maxima [B] time = 1.08869, size = 230, normalized size = 3.19

$$\frac{2\,ae^{(-2\,dx-2\,c)}}{\left(a^3+3\,a^2b+3\,ab^2+b^3+2\left(a^3+a^2b-ab^2-b^3\right)e^{(-2\,dx-2\,c)}+\left(a^3+3\,a^2b+3\,ab^2+b^3\right)e^{(-4\,dx-4\,c)}\right)d} + \frac{dx+c}{\left(a^2+2\,ab+b^2\right)d} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 2*a*e^(-2*d*x - 2*c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 + a^2*b - a*b
^2 - b^3)*e^(-2*d*x - 2*c) + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*e^(-4*d*x - 4*
```

```
c))*d) + (d*x + c)/((a^2 + 2*a*b + b^2)*d) + 1/2*log(2*(a - b)*e^(-2*d*x -
2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^2 + 2*a*b + b^2)*d)
```

Fricas [B] time = 1.7972, size = 1611, normalized size = 22.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a + b)*d*x*cosh(d*x + c)^4 + 8*(a + b)*d*x*cosh(d*x + c)*sinh(d*x
+ c)^3 + 2*(a + b)*d*x*sinh(d*x + c)^4 + 2*(a + b)*d*x + 4*((a - b)*d*x - a
)*cosh(d*x + c)^2 + 4*(3*(a + b)*d*x*cosh(d*x + c)^2 + (a - b)*d*x - a)*sin
h(d*x + c)^2 - ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x
+ c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)
*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a
 - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*log(2*((a + b)*cosh(d*x + c)^2
+ (a + b)*sinh(d*x + c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(
d*x + c) + sinh(d*x + c)^2)) + 8*((a + b)*d*x*cosh(d*x + c)^3 + ((a - b)*d*
x - a)*cosh(d*x + c))*sinh(d*x + c))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cos
h(d*x + c)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)*sinh(d*x +
 c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*sinh(d*x + c)^4 + 2*(a^3 + a^2*b
- a*b^2 - b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*c
osh(d*x + c)^2 + (a^3 + a^2*b - a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d + 4*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x +
c)^3 + (a^3 + a^2*b - a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**3/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23256, size = 208, normalized size = 2.89

$$\frac{\log\left(\left|a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a-2\,b\right|\right)}{2\left(a^2d+2\,abd+b^2d\right)}-\frac{e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}}{2\,(ad+bd)\left(a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*log(abs(a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c) + e
^(-2*d*x - 2*c)) + 2*a - 2*b))/(a^2*d + 2*a*b*d + b^2*d) - 1/2*(e^(2*d*x +
2*c) + e^(-2*d*x - 2*c) - 2)/((a*d + b*d)*(a*(e^(2*d*x + 2*c) + e^(-2*d*x -
 2*c)) + b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 2*a - 2*b))
```

3.183 $\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=85

$$-\frac{\tanh(c+dx)}{2d(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d(a+b)^2}+\frac{x}{(a+b)^2}$$

```
[Out] x/(a + b)^2 - ((a - b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(2*Sqrt[a]*
Sqrt[b]*(a + b)^2*d) - Tanh[c + d*x]/(2*(a + b)*d*(a + b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.107958, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3670, 471, 522, 206, 205}

$$-\frac{\tanh(c+dx)}{2d(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d(a+b)^2}+\frac{x}{(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] x/(a + b)^2 - ((a - b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(2*Sqrt[a]*
Sqrt[b]*(a + b)^2*d) - Tanh[c + d*x]/(2*(a + b)*d*(a + b*Tanh[c + d*x]^2))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{\tanh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1+x^2}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{2(a+b)d}$$

$$= -\frac{\tanh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)^2 d} - \frac{(a-b)\,\text{Sub}}{}$$

$$= \frac{x}{(a+b)^2} - \frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}(a+b)^2 d} - \frac{\tanh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

Mathematica [A] time = 0.37579, size = 86, normalized size = 1.01

$$\frac{\frac{(b-a)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}} - \frac{(a+b)\sinh(2(c+dx))}{(a+b)\cosh(2(c+dx))+a-b} + 2(c+dx)}{2d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (2*(c + d*x) + ((-a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*
Sqrt[b]) - ((a + b)*Sinh[2*(c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)]))
/(2*(a + b)^2*d)
```

Maple [B] time = 0.023, size = 162, normalized size = 1.9

$$\frac{\ln(\tanh(dx+c)+1)}{2d(a+b)^2} - \frac{a\tanh(dx+c)}{2d(a+b)^2\left(a+b(\tanh(dx+c))^2\right)} - \frac{b\tanh(dx+c)}{2d(a+b)^2\left(a+b(\tanh(dx+c))^2\right)} - \frac{a}{2d(a+b)^2}\,\text{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] 1/2/d/(a+b)^2*ln(tanh(d*x+c)+1)-1/2/d/(a+b)^2*a*tanh(d*x+c)/(a+b*tanh(d*x+c
)^2)-1/2*b*tanh(d*x+c)/(a+b)^2/d/(a+b*tanh(d*x+c)^2)-1/2/d/(a+b)^2*a/(a*b)^
(1/2)*arctan(tanh(d*x+c)*b/(a*b)^(1/2))+1/2/d/(a+b)^2/(a*b)^(1/2)*arctan(ta
```

```
nh(d*x+c)*b/(a*b)^(1/2))*b-1/2/d/(a+b)^2*ln(tanh(d*x+c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.14726, size = 4747, normalized size = 55.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^2*b + a*b^2)*d*x*cosh(d*x + c)^4 + 16*(a^2*b + a*b^2)*d*x*cosh(d
*x + c)*sinh(d*x + c)^3 + 4*(a^2*b + a*b^2)*d*x*sinh(d*x + c)^4 + 4*a^2*b +
 4*a*b^2 + 4*(a^2*b + a*b^2)*d*x + 4*(a^2*b - a*b^2 + 2*(a^2*b - a*b^2)*d*x
)*cosh(d*x + c)^2 + 4*(6*(a^2*b + a*b^2)*d*x*cosh(d*x + c)^2 + a^2*b - a*b^
2 + 2*(a^2*b - a*b^2)*d*x)*sinh(d*x + c)^2 + ((a^2 - b^2)*cosh(d*x + c)^4 +
 4*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 - b^2)*sinh(d*x + c)^4
+ 2*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 - b^2)*cosh(d*x + c)^2
+ a^2 - 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(d*x
+ c)^3 + (a^2 - 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b)*log((
(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*s
inh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d
*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x
+ c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2
- b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b
)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b)
)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a +
 b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c
)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*
x + c))*sinh(d*x + c) + a + b)) + 8*(2*(a^2*b + a*b^2)*d*x*cosh(d*x + c)^3
+ (a^2*b - a*b^2 + 2*(a^2*b - a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a
^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4 + 4*(a^4*b + 3*a^3*
b^2 + 3*a^2*b^3 + a*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b + 3*a^3*b
^2 + 3*a^2*b^3 + a*b^4)*d*sinh(d*x + c)^4 + 2*(a^4*b + a^3*b^2 - a^2*b^3 -
a*b^4)*d*cosh(d*x + c)^2 + 2*(3*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*d*c
osh(d*x + c)^2 + (a^4*b + a^3*b^2 - a^2*b^3 - a*b^4)*d)*sinh(d*x + c)^2 + (
a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*d + 4*((a^4*b + 3*a^3*b^2 + 3*a^2*b^
3 + a*b^4)*d*cosh(d*x + c)^3 + (a^4*b + a^3*b^2 - a^2*b^3 - a*b^4)*d*cosh(d
*x + c))*sinh(d*x + c)), 1/2*(2*(a^2*b + a*b^2)*d*x*cosh(d*x + c)^4 + 8*(a^
2*b + a*b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a^2*b + a*b^2)*d*x*sinh
(d*x + c)^4 + 2*a^2*b + 2*a*b^2 + 2*(a^2*b + a*b^2)*d*x + 2*(a^2*b - a*b^2
+ 2*(a^2*b - a*b^2)*d*x)*cosh(d*x + c)^2 + 2*(6*(a^2*b + a*b^2)*d*x*cosh(d*
x + c)^2 + a^2*b - a*b^2 + 2*(a^2*b - a*b^2)*d*x)*sinh(d*x + c)^2 - ((a^2 -
 b^2)*cosh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2
- b^2)*sinh(d*x + c)^4 + 2*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(a^2
- b^2)*cosh(d*x + c)^2 + a^2 - 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 - b^2 + 4
```

```
*((a^2 - b^2)*cosh(d*x + c)^3 + (a^2 - 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x
 + c))*sqrt(a*b)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x +
 c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(a*b)/(a*b)) + 4*(
2*(a^2*b + a*b^2)*d*x*cosh(d*x + c)^3 + (a^2*b - a*b^2 + 2*(a^2*b - a*b^2)*
d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)
*d*cosh(d*x + c)^4 + 4*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*d*cosh(d*x +
 c)*sinh(d*x + c)^3 + (a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*d*sinh(d*x +
c)^4 + 2*(a^4*b + a^3*b^2 - a^2*b^3 - a*b^4)*d*cosh(d*x + c)^2 + 2*(3*(a^4*
b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + (a^4*b + a^3*b^2 - a
^2*b^3 - a*b^4)*d)*sinh(d*x + c)^2 + (a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4
)*d + 4*((a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^3 + (a^4*b
 + a^3*b^2 - a^2*b^3 - a*b^4)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [A] time = 83.2681, size = 2144, normalized size = 25.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**2/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((zoo*x/tanh(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x - tanh(c
+ d*x)/d)/a**2, Eq(b, 0)), ((x - 1/(d*tanh(c + d*x)))/b**2, Eq(a, 0)), (x*t
anh(c)**2/(a + b*tanh(c)**2)**2, Eq(d, 0)), (4*I*a**(3/2)*b*d*x*sqrt(1/b)/(
4*I*a**(7/2)*b*d*sqrt(1/b) + 4*I*a**(5/2)*b**2*d*sqrt(1/b)*tanh(c + d*x)**2
 + 8*I*a**(5/2)*b**2*d*sqrt(1/b) + 8*I*a**(3/2)*b**3*d*sqrt(1/b)*tanh(c + d
*x)**2 + 4*I*a**(3/2)*b**3*d*sqrt(1/b) + 4*I*sqrt(a)*b**4*d*sqrt(1/b)*tanh(
c + d*x)**2) - 2*I*a**(3/2)*b*sqrt(1/b)*tanh(c + d*x)/(4*I*a**(7/2)*b*d*sqr
t(1/b) + 4*I*a**(5/2)*b**2*d*sqrt(1/b)*tanh(c + d*x)**2 + 8*I*a**(5/2)*b**2
*d*sqrt(1/b) + 8*I*a**(3/2)*b**3*d*sqrt(1/b)*tanh(c + d*x)**2 + 4*I*a**(3/2
)*b**3*d*sqrt(1/b) + 4*I*sqrt(a)*b**4*d*sqrt(1/b)*tanh(c + d*x)**2) + 4*I*s
qrt(a)*b**2*d*x*sqrt(1/b)*tanh(c + d*x)**2/(4*I*a**(7/2)*b*d*sqrt(1/b) + 4*
I*a**(5/2)*b**2*d*sqrt(1/b)*tanh(c + d*x)**2 + 8*I*a**(5/2)*b**2*d*sqrt(1/b
) + 8*I*a**(3/2)*b**3*d*sqrt(1/b)*tanh(c + d*x)**2 + 4*I*a**(3/2)*b**3*d*sq
rt(1/b) + 4*I*sqrt(a)*b**4*d*sqrt(1/b)*tanh(c + d*x)**2) - 2*I*sqrt(a)*b**2
*sqrt(1/b)*tanh(c + d*x)/(4*I*a**(7/2)*b*d*sqrt(1/b) + 4*I*a**(5/2)*b**2*d*
sqrt(1/b)*tanh(c + d*x)**2 + 8*I*a**(5/2)*b**2*d*sqrt(1/b) + 8*I*a**(3/2)*b
**3*d*sqrt(1/b)*tanh(c + d*x)**2 + 4*I*a**(3/2)*b**3*d*sqrt(1/b) + 4*I*sqrt
(a)*b**4*d*sqrt(1/b)*tanh(c + d*x)**2) - a**2*log(-I*sqrt(a)*sqrt(1/b) + ta
nh(c + d*x))/(4*I*a**(7/2)*b*d*sqrt(1/b) + 4*I*a**(5/2)*b**2*d*sqrt(1/b)*ta
nh(c + d*x)**2 + 8*I*a**(5/2)*b**2*d*sqrt(1/b) + 8*I*a**(3/2)*b**3*d*sqrt(1
/b)*tanh(c + d*x)**2 + 4*I*a**(3/2)*b**3*d*sqrt(1/b) + 4*I*sqrt(a)*b**4*d*s
qrt(1/b)*tanh(c + d*x)**2) + a**2*log(I*sqrt(a)*sqrt(1/b) + tanh(c + d*x))/
(4*I*a**(7/2)*b*d*sqrt(1/b) + 4*I*a**(5/2)*b**2*d*sqrt(1/b)*tanh(c + d*x)**
2 + 8*I*a**(5/2)*b**2*d*sqrt(1/b) + 8*I*a**(3/2)*b**3*d*sqrt(1/b)*tanh(c +
d*x)**2 + 4*I*a**(3/2)*b**3*d*sqrt(1/b) + 4*I*sqrt(a)*b**4*d*sqrt(1/b)*tanh
(c + d*x)**2) - a*b*log(-I*sqrt(a)*sqrt(1/b) + tanh(c + d*x))*tanh(c + d*x)
**2/(4*I*a**(7/2)*b*d*sqrt(1/b) + 4*I*a**(5/2)*b**2*d*sqrt(1/b)*tanh(c + d*
x)**2 + 8*I*a**(5/2)*b**2*d*sqrt(1/b) + 8*I*a**(3/2)*b**3*d*sqrt(1/b)*tanh(
c + d*x)**2 + 4*I*a**(3/2)*b**3*d*sqrt(1/b) + 4*I*sqrt(a)*b**4*d*sqrt(1/b)*
tanh(c + d*x)**2) + a*b*log(-I*sqrt(a)*sqrt(1/b) + tanh(c + d*x))/(4*I*a**(
7/2)*b*d*sqrt(1/b) + 4*I*a**(5/2)*b**2*d*sqrt(1/b)*tanh(c + d*x)**2 + 8*I*a
**(5/2)*b**2*d*sqrt(1/b) + 8*I*a**(3/2)*b**3*d*sqrt(1/b)*tanh(c + d*x)**2 +
 4*I*a**(3/2)*b**3*d*sqrt(1/b) + 4*I*sqrt(a)*b**4*d*sqrt(1/b)*tanh(c + d*x)
**2) + a*b*log(I*sqrt(a)*sqrt(1/b) + tanh(c + d*x))*tanh(c + d*x)**2/(4*I*a
**(7/2)*b*d*sqrt(1/b) + 4*I*a**(5/2)*b**2*d*sqrt(1/b)*tanh(c + d*x)**2 + 8*
I*a**(5/2)*b**2*d*sqrt(1/b) + 8*I*a**(3/2)*b**3*d*sqrt(1/b)*tanh(c + d*x)**
```

```
2 + 4*I*a**(3/2)*b**3*d*sqrt(1/b) + 4*I*sqrt(a)*b**4*d*sqrt(1/b)*tanh(c + d
*x)**2) - a*b*log(I*sqrt(a)*sqrt(1/b) + tanh(c + d*x))/(4*I*a**(7/2)*b*d*sq
rt(1/b) + 4*I*a**(5/2)*b**2*d*sqrt(1/b)*tanh(c + d*x)**2 + 8*I*a**(5/2)*b**
2*d*sqrt(1/b) + 8*I*a**(3/2)*b**3*d*sqrt(1/b)*tanh(c + d*x)**2 + 4*I*a**(3/
2)*b**3*d*sqrt(1/b) + 4*I*sqrt(a)*b**4*d*sqrt(1/b)*tanh(c + d*x)**2) + b**2
*log(-I*sqrt(a)*sqrt(1/b) + tanh(c + d*x))*tanh(c + d*x)**2/(4*I*a**(7/2)*b
*d*sqrt(1/b) + 4*I*a**(5/2)*b**2*d*sqrt(1/b)*tanh(c + d*x)**2 + 8*I*a**(5/2
)*b**2*d*sqrt(1/b) + 8*I*a**(3/2)*b**3*d*sqrt(1/b)*tanh(c + d*x)**2 + 4*I*a
**(3/2)*b**3*d*sqrt(1/b) + 4*I*sqrt(a)*b**4*d*sqrt(1/b)*tanh(c + d*x)**2) -
 b**2*log(I*sqrt(a)*sqrt(1/b) + tanh(c + d*x))*tanh(c + d*x)**2/(4*I*a**(7/
2)*b*d*sqrt(1/b) + 4*I*a**(5/2)*b**2*d*sqrt(1/b)*tanh(c + d*x)**2 + 8*I*a**
(5/2)*b**2*d*sqrt(1/b) + 8*I*a**(3/2)*b**3*d*sqrt(1/b)*tanh(c + d*x)**2 + 4
*I*a**(3/2)*b**3*d*sqrt(1/b) + 4*I*sqrt(a)*b**4*d*sqrt(1/b)*tanh(c + d*x)**
2), True))
```

Giac [B] time = 1.22123, size = 251, normalized size = 2.95

$$-\frac{(a-b)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{2\left(a^2d+2\,abd+b^2d\right)\sqrt{ab}}+\frac{dx+c}{a^2d+2\,abd+b^2d}+\frac{ae^{(2\,dx+2\,c)}-be^{(2\,dx+2\,c)}+a+b}{\left(a^2d+2\,abd+b^2d\right)\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(a - b)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqr
t(a*b))/((a^2*d + 2*a*b*d + b^2*d)*sqrt(a*b)) + (d*x + c)/(a^2*d + 2*a*b*d
+ b^2*d) + (a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) + a + b)/((a^2*d + 2*a*b*
d + b^2*d)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2
*b*e^(2*d*x + 2*c) + a + b))
```

3.184 $\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=68

$$-\frac{1}{2d(a+b)\left(a+b\tanh^2(c+dx)\right)}+\frac{\log\left(a+b\tanh^2(c+dx)\right)}{2d(a+b)^2}+\frac{\log(\cosh(c+dx))}{d(a+b)^2}$$

```
[Out] Log[Cosh[c + d*x]]/((a + b)^2*d) + Log[a + b*Tanh[c + d*x]^2]/(2*(a + b)^2*
d) - 1/(2*(a + b)*d*(a + b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.0857955, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3670, 444, 44}

$$-\frac{1}{2d(a+b)\left(a+b\tanh^2(c+dx)\right)}+\frac{\log\left(a+b\tanh^2(c+dx)\right)}{2d(a+b)^2}+\frac{\log(\cosh(c+dx))}{d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] Log[Cosh[c + d*x]]/((a + b)^2*d) + Log[a + b*Tanh[c + d*x]^2]/(2*(a + b)^2*
d) - 1/(2*(a + b)*d*(a + b*Tanh[c + d*x]^2))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)^2}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^2(-1+x)} + \frac{b}{(a+b)(a+bx)^2} + \frac{b}{(a+b)^2(a+bx)}\right) dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\log(\cosh(c+dx))}{(a+b)^2 d} + \frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)^2 d} - \frac{1}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

Mathematica [A] time = 0.371088, size = 55, normalized size = 0.81

$$-\frac{\frac{a+b}{a+b\tanh^2(c+dx)} - \log\left(a+b\tanh^2(c+dx)\right) - 2\log(\cosh(c+dx))}{2d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -(-2*Log[Cosh[c + d*x]] - Log[a + b*Tanh[c + d*x]^2] + (a + b)/(a + b*Tanh[
c + d*x]^2))/(2*(a + b)^2*d)
```

Maple [A] time = 0.027, size = 113, normalized size = 1.7

$$-\frac{\ln(\tanh(dx+c)+1)}{2d(a+b)^2} - \frac{a}{2d(a+b)^2\left(a+b(\tanh(dx+c))^2\right)} - \frac{b}{2d(a+b)^2\left(a+b(\tanh(dx+c))^2\right)} + \frac{\ln\left(a+b(\tanh(\right.}{2d(a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] -1/2/d/(a+b)^2*ln(tanh(d*x+c)+1)-1/2/d/(a+b)^2*a/(a+b*tanh(d*x+c)^2)-1/2/d*
b/(a+b)^2/(a+b*tanh(d*x+c)^2)+1/2*ln(a+b*tanh(d*x+c)^2)/(a+b)^2/d-1/2/d/(a+
b)^2*ln(tanh(d*x+c)-1)
```

Maxima [B] time = 1.09235, size = 230, normalized size = 3.38

$$-\frac{2\,be^{(-2\,dx-2\,c)}}{\left(a^3+3\,a^2b+3\,ab^2+b^3+2\left(a^3+a^2b-ab^2-b^3\right)e^{(-2\,dx-2\,c)}+\left(a^3+3\,a^2b+3\,ab^2+b^3\right)e^{(-4\,dx-4\,c)}\right)d} + \frac{dx+c}{\left(a^2+2\,ab+b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -2*b*e^(-2*d*x - 2*c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 + a^2*b - a*
b^2 - b^3)*e^(-2*d*x - 2*c) + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*e^(-4*d*x - 4
```

```
*c))*d) + (d*x + c)/((a^2 + 2*a*b + b^2)*d) + 1/2*log(2*(a - b)*e^(-2*d*x -
 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^2 + 2*a*b + b^2)*d)
```

Fricas [B] time = 1.95637, size = 1611, normalized size = 23.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a + b)*d*x*cosh(d*x + c)^4 + 8*(a + b)*d*x*cosh(d*x + c)*sinh(d*x
+ c)^3 + 2*(a + b)*d*x*sinh(d*x + c)^4 + 2*(a + b)*d*x + 4*((a - b)*d*x + b
)*cosh(d*x + c)^2 + 4*(3*(a + b)*d*x*cosh(d*x + c)^2 + (a - b)*d*x + b)*sin
h(d*x + c)^2 - ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x
+ c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)
*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a
 - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*log(2*((a + b)*cosh(d*x + c)^2
+ (a + b)*sinh(d*x + c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(
d*x + c) + sinh(d*x + c)^2)) + 8*((a + b)*d*x*cosh(d*x + c)^3 + ((a - b)*d*
x + b)*cosh(d*x + c))*sinh(d*x + c))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cos
h(d*x + c)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)*sinh(d*x +
 c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*sinh(d*x + c)^4 + 2*(a^3 + a^2*b
- a*b^2 - b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*c
osh(d*x + c)^2 + (a^3 + a^2*b - a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d + 4*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x +
c)^3 + (a^3 + a^2*b - a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.22318, size = 208, normalized size = 3.06

$$\frac{\log\left(\left|a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a-2\,b\right|\right)}{2\left(a^2d+2\,abd+b^2d\right)}-\frac{e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}}{2\,(ad+bd)\left(a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*log(abs(a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c) + e
^(-2*d*x - 2*c)) + 2*a - 2*b))/(a^2*d + 2*a*b*d + b^2*d) - 1/2*(e^(2*d*x +
2*c) + e^(-2*d*x - 2*c) + 2)/((a*d + b*d)*(a*(e^(2*d*x + 2*c) + e^(-2*d*x -
 2*c)) + b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 2*a - 2*b))
```

3.185 $\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=89

$$\frac{\sqrt{b}(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a+b)^2}+\frac{b\tanh(c+dx)}{2ad(a+b)\left(a+b\tanh^2(c+dx)\right)}+\frac{x}{(a+b)^2}$$

```
[Out] x/(a + b)^2 + (Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(
2*a^(3/2)*(a + b)^2*d) + (b*Tanh[c + d*x])/(2*a*(a + b)*d*(a + b*Tanh[c + d
*x]^2))
```

Rubi [A] time = 0.0882202, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3661, 414, 522, 206, 205}

$$\frac{\sqrt{b}(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a+b)^2}+\frac{b\tanh(c+dx)}{2ad(a+b)\left(a+b\tanh^2(c+dx)\right)}+\frac{x}{(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tanh[c + d*x]^2)^(-2),x]
```

```
[Out] x/(a + b)^2 + (Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(
2*a^(3/2)*(a + b)^2*d) + (b*Tanh[c + d*x])/(2*a*(a + b)*d*(a + b*Tanh[c + d
*x]^2))
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\left(a+bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\tanh(c+dx)}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{b-2(a+b)+bx^2}{(1-x^2)\left(a+bx^2\right)}\,dx, x, \tanh(c+dx)\right)}{2a(a+b)d}$$

$$= \frac{b\tanh(c+dx)}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)^2 d} + \frac{(b(3a+b))}{}$$

$$= \frac{x}{(a+b)^2} + \frac{\sqrt{b}(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^2 d} + \frac{b\tanh(c+dx)}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

Mathematica [A] time = 0.495114, size = 97, normalized size = 1.09

$$\frac{\frac{\sqrt{b}(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}} + \frac{b(a+b)\tanh(c+dx)}{a\left(a+b\tanh^2(c+dx)\right)} - \log(1-\tanh(c+dx)) + \log(\tanh(c+dx)+1)}{2d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tanh[c + d*x]^2)^(-2),x]
```

```
[Out] ((Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/a^(3/2) - Log[
1 - Tanh[c + d*x]] + Log[1 + Tanh[c + d*x]] + (b*(a + b)*Tanh[c + d*x])/(a*
(a + b*Tanh[c + d*x]^2)))/(2*(a + b)^2*d)
```

Maple [B] time = 0.024, size = 172, normalized size = 1.9

$$\frac{\ln(\tanh(dx+c)+1)}{2d(a+b)^2} + \frac{b\tanh(dx+c)}{2d(a+b)^2\left(a+b(\tanh(dx+c))^2\right)} + \frac{b^2\tanh(dx+c)}{2d(a+b)^2 a\left(a+b(\tanh(dx+c))^2\right)} + \frac{3b}{2d(a+b)^2}\,\text{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] 1/2/d/(a+b)^2*ln(tanh(d*x+c)+1)+1/2*b*tanh(d*x+c)/(a+b)^2/d/(a+b*tanh(d*x+c
)^2)+1/2/d*b^2/(a+b)^2/a*tanh(d*x+c)/(a+b*tanh(d*x+c)^2)+3/2/d/(a+b)^2/(a*b
)^(1/2)*arctan(tanh(d*x+c)*b/(a*b)^(1/2))*b+1/2/d*b^2/(a+b)^2/a/(a*b)^(1/2)
```

```
*arctan(tanh(d*x+c)*b/(a*b)^(1/2))-1/2/d/(a+b)^2*ln(tanh(d*x+c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 2.09212, size = 4703, normalized size = 52.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [1/4*(4*(a^2 + a*b)*d*x*cosh(d*x + c)^4 + 16*(a^2 + a*b)*d*x*cosh(d*x + c)*
sinh(d*x + c)^3 + 4*(a^2 + a*b)*d*x*sinh(d*x + c)^4 + 4*(a^2 + a*b)*d*x + 4
*(2*(a^2 - a*b)*d*x - a*b + b^2)*cosh(d*x + c)^2 + 4*(6*(a^2 + a*b)*d*x*cos
h(d*x + c)^2 + 2*(a^2 - a*b)*d*x - a*b + b^2)*sinh(d*x + c)^2 + ((3*a^2 + 4
*a*b + b^2)*cosh(d*x + c)^4 + 4*(3*a^2 + 4*a*b + b^2)*cosh(d*x + c)*sinh(d*
x + c)^3 + (3*a^2 + 4*a*b + b^2)*sinh(d*x + c)^4 + 2*(3*a^2 - 2*a*b - b^2)*
cosh(d*x + c)^2 + 2*(3*(3*a^2 + 4*a*b + b^2)*cosh(d*x + c)^2 + 3*a^2 - 2*a*
b - b^2)*sinh(d*x + c)^2 + 3*a^2 + 4*a*b + b^2 + 4*((3*a^2 + 4*a*b + b^2)*c
osh(d*x + c)^3 + (3*a^2 - 2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-
b/a)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(
d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b
^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)
*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)
^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c
)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)
^2 + a^2 - a*b)*sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x +
 c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 +
 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*
x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) - 4*a*b - 4*b^2 +
 8*(2*(a^2 + a*b)*d*x*cosh(d*x + c)^3 + (2*(a^2 - a*b)*d*x - a*b + b^2)*cos
h(d*x + c))*sinh(d*x + c))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x
+ c)^4 + 4*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)*sinh(d*x + c
)^3 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*sinh(d*x + c)^4 + 2*(a^4 + a^3*
b - a^2*b^2 - a*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^4 + 3*a^3*b + 3*a^2*b^2 +
a*b^3)*d*cosh(d*x + c)^2 + (a^4 + a^3*b - a^2*b^2 - a*b^3)*d)*sinh(d*x + c)
^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d + 4*((a^4 + 3*a^3*b + 3*a^2*b^2
+ a*b^3)*d*cosh(d*x + c)^3 + (a^4 + a^3*b - a^2*b^2 - a*b^3)*d*cosh(d*x + c
))*sinh(d*x + c)), 1/2*(2*(a^2 + a*b)*d*x*cosh(d*x + c)^4 + 8*(a^2 + a*b)*d
*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a^2 + a*b)*d*x*sinh(d*x + c)^4 + 2*(a
^2 + a*b)*d*x + 2*(2*(a^2 - a*b)*d*x - a*b + b^2)*cosh(d*x + c)^2 + 2*(6*(a
^2 + a*b)*d*x*cosh(d*x + c)^2 + 2*(a^2 - a*b)*d*x - a*b + b^2)*sinh(d*x + c
)^2 + ((3*a^2 + 4*a*b + b^2)*cosh(d*x + c)^4 + 4*(3*a^2 + 4*a*b + b^2)*cosh
(d*x + c)*sinh(d*x + c)^3 + (3*a^2 + 4*a*b + b^2)*sinh(d*x + c)^4 + 2*(3*a^
2 - 2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*(3*a^2 + 4*a*b + b^2)*cosh(d*x + c)
^2 + 3*a^2 - 2*a*b - b^2)*sinh(d*x + c)^2 + 3*a^2 + 4*a*b + b^2 + 4*((3*a^2
```

```
 + 4*a*b + b^2)*cosh(d*x + c)^3 + (3*a^2 - 2*a*b - b^2)*cosh(d*x + c))*sinh
(d*x + c))*sqrt(b/a)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d
*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(b/a)/b) - 2*a
*b - 2*b^2 + 4*(2*(a^2 + a*b)*d*x*cosh(d*x + c)^3 + (2*(a^2 - a*b)*d*x - a*
b + b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)
*d*cosh(d*x + c)^4 + 4*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)*
sinh(d*x + c)^3 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*sinh(d*x + c)^4 + 2
*(a^4 + a^3*b - a^2*b^2 - a*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^4 + 3*a^3*b +
3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^2 + (a^4 + a^3*b - a^2*b^2 - a*b^3)*d)*s
inh(d*x + c)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d + 4*((a^4 + 3*a^3*b
+ 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^3 + (a^4 + a^3*b - a^2*b^2 - a*b^3)*d*
cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [A] time = 151.002, size = 2086, normalized size = 23.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((zoo*x/tanh(c)**4, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x/a**2, Eq(b
, 0)), ((x - 1/(d*tanh(c + d*x)) - 1/(3*d*tanh(c + d*x)**3))/b**2, Eq(a, 0)
), (x/(a + b*tanh(c)**2)**2, Eq(d, 0)), (4*I*a**(5/2)*d*x*sqrt(1/b)/(4*I*a*
*(9/2)*d*sqrt(1/b) + 4*I*a**(7/2)*b*d*sqrt(1/b)*tanh(c + d*x)**2 + 8*I*a**(
7/2)*b*d*sqrt(1/b) + 8*I*a**(5/2)*b**2*d*sqrt(1/b)*tanh(c + d*x)**2 + 4*I*a
**(5/2)*b**2*d*sqrt(1/b) + 4*I*a**(3/2)*b**3*d*sqrt(1/b)*tanh(c + d*x)**2)
+ 4*I*a**(3/2)*b*d*x*sqrt(1/b)*tanh(c + d*x)**2/(4*I*a**(9/2)*d*sqrt(1/b) +
 4*I*a**(7/2)*b*d*sqrt(1/b)*tanh(c + d*x)**2 + 8*I*a**(7/2)*b*d*sqrt(1/b) +
 8*I*a**(5/2)*b**2*d*sqrt(1/b)*tanh(c + d*x)**2 + 4*I*a**(5/2)*b**2*d*sqrt(
1/b) + 4*I*a**(3/2)*b**3*d*sqrt(1/b)*tanh(c + d*x)**2) + 2*I*a**(3/2)*b*sqr
t(1/b)*tanh(c + d*x)/(4*I*a**(9/2)*d*sqrt(1/b) + 4*I*a**(7/2)*b*d*sqrt(1/b)
*tanh(c + d*x)**2 + 8*I*a**(7/2)*b*d*sqrt(1/b) + 8*I*a**(5/2)*b**2*d*sqrt(1
/b)*tanh(c + d*x)**2 + 4*I*a**(5/2)*b**2*d*sqrt(1/b) + 4*I*a**(3/2)*b**3*d*
sqrt(1/b)*tanh(c + d*x)**2) + 2*I*sqrt(a)*b**2*sqrt(1/b)*tanh(c + d*x)/(4*I
*a**(9/2)*d*sqrt(1/b) + 4*I*a**(7/2)*b*d*sqrt(1/b)*tanh(c + d*x)**2 + 8*I*a
**(7/2)*b*d*sqrt(1/b) + 8*I*a**(5/2)*b**2*d*sqrt(1/b)*tanh(c + d*x)**2 + 4*
I*a**(5/2)*b**2*d*sqrt(1/b) + 4*I*a**(3/2)*b**3*d*sqrt(1/b)*tanh(c + d*x)**
2) + 3*a**2*log(-I*sqrt(a)*sqrt(1/b) + tanh(c + d*x))/(4*I*a**(9/2)*d*sqrt(
1/b) + 4*I*a**(7/2)*b*d*sqrt(1/b)*tanh(c + d*x)**2 + 8*I*a**(7/2)*b*d*sqrt(
1/b) + 8*I*a**(5/2)*b**2*d*sqrt(1/b)*tanh(c + d*x)**2 + 4*I*a**(5/2)*b**2*d
*sqrt(1/b) + 4*I*a**(3/2)*b**3*d*sqrt(1/b)*tanh(c + d*x)**2) - 3*a**2*log(I
*sqrt(a)*sqrt(1/b) + tanh(c + d*x))/(4*I*a**(9/2)*d*sqrt(1/b) + 4*I*a**(7/2
)*b*d*sqrt(1/b)*tanh(c + d*x)**2 + 8*I*a**(7/2)*b*d*sqrt(1/b) + 8*I*a**(5/2
)*b**2*d*sqrt(1/b)*tanh(c + d*x)**2 + 4*I*a**(5/2)*b**2*d*sqrt(1/b) + 4*I*a
**(3/2)*b**3*d*sqrt(1/b)*tanh(c + d*x)**2) + 3*a*b*log(-I*sqrt(a)*sqrt(1/b)
 + tanh(c + d*x))*tanh(c + d*x)**2/(4*I*a**(9/2)*d*sqrt(1/b) + 4*I*a**(7/2)
*b*d*sqrt(1/b)*tanh(c + d*x)**2 + 8*I*a**(7/2)*b*d*sqrt(1/b) + 8*I*a**(5/2)
*b**2*d*sqrt(1/b)*tanh(c + d*x)**2 + 4*I*a**(5/2)*b**2*d*sqrt(1/b) + 4*I*a*
*(3/2)*b**3*d*sqrt(1/b)*tanh(c + d*x)**2) + a*b*log(-I*sqrt(a)*sqrt(1/b) +
tanh(c + d*x))/(4*I*a**(9/2)*d*sqrt(1/b) + 4*I*a**(7/2)*b*d*sqrt(1/b)*tanh(
c + d*x)**2 + 8*I*a**(7/2)*b*d*sqrt(1/b) + 8*I*a**(5/2)*b**2*d*sqrt(1/b)*ta
nh(c + d*x)**2 + 4*I*a**(5/2)*b**2*d*sqrt(1/b) + 4*I*a**(3/2)*b**3*d*sqrt(1
/b)*tanh(c + d*x)**2) - 3*a*b*log(I*sqrt(a)*sqrt(1/b) + tanh(c + d*x))*tanh
(c + d*x)**2/(4*I*a**(9/2)*d*sqrt(1/b) + 4*I*a**(7/2)*b*d*sqrt(1/b)*tanh(c
+ d*x)**2 + 8*I*a**(7/2)*b*d*sqrt(1/b) + 8*I*a**(5/2)*b**2*d*sqrt(1/b)*tanh
(c + d*x)**2 + 4*I*a**(5/2)*b**2*d*sqrt(1/b) + 4*I*a**(3/2)*b**3*d*sqrt(1/b
```

```
)*tanh(c + d*x)**2) - a*b*log(I*sqrt(a)*sqrt(1/b) + tanh(c + d*x))/(4*I*a**
(9/2)*d*sqrt(1/b) + 4*I*a**(7/2)*b*d*sqrt(1/b)*tanh(c + d*x)**2 + 8*I*a**(7
/2)*b*d*sqrt(1/b) + 8*I*a**(5/2)*b**2*d*sqrt(1/b)*tanh(c + d*x)**2 + 4*I*a*
*(5/2)*b**2*d*sqrt(1/b) + 4*I*a**(3/2)*b**3*d*sqrt(1/b)*tanh(c + d*x)**2) +
 b**2*log(-I*sqrt(a)*sqrt(1/b) + tanh(c + d*x))*tanh(c + d*x)**2/(4*I*a**(9
/2)*d*sqrt(1/b) + 4*I*a**(7/2)*b*d*sqrt(1/b)*tanh(c + d*x)**2 + 8*I*a**(7/2
)*b*d*sqrt(1/b) + 8*I*a**(5/2)*b**2*d*sqrt(1/b)*tanh(c + d*x)**2 + 4*I*a**(
5/2)*b**2*d*sqrt(1/b) + 4*I*a**(3/2)*b**3*d*sqrt(1/b)*tanh(c + d*x)**2) - b
**2*log(I*sqrt(a)*sqrt(1/b) + tanh(c + d*x))*tanh(c + d*x)**2/(4*I*a**(9/2)
*d*sqrt(1/b) + 4*I*a**(7/2)*b*d*sqrt(1/b)*tanh(c + d*x)**2 + 8*I*a**(7/2)*b
*d*sqrt(1/b) + 8*I*a**(5/2)*b**2*d*sqrt(1/b)*tanh(c + d*x)**2 + 4*I*a**(5/2
)*b**2*d*sqrt(1/b) + 4*I*a**(3/2)*b**3*d*sqrt(1/b)*tanh(c + d*x)**2), True)
)
```

Giac [B] time = 1.19008, size = 274, normalized size = 3.08

$$\frac{\left(3\,ab+b^2\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{2\left(a^3d+2\,a^2bd+ab^2d\right)\sqrt{ab}}+\frac{dx+c}{a^2d+2\,abd+b^2d}-\frac{abe^{(2\,dx+2\,c)}-b^2e^{(2\,dx+2\,c)}+ab\,\cdots}{\left(a^3d+2\,a^2bd+ab^2d\right)\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,\cdots}\cdots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*a*b + b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b
)/sqrt(a*b))/((a^3*d + 2*a^2*b*d + a*b^2*d)*sqrt(a*b)) + (d*x + c)/(a^2*d +
 2*a*b*d + b^2*d) - (a*b*e^(2*d*x + 2*c) - b^2*e^(2*d*x + 2*c) + a*b + b^2)
/((a^3*d + 2*a^2*b*d + a*b^2*d)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*
a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b))
```

3.186 $\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2} dx$

Optimal. Leaf size=95

$$-\frac{b(2a+b)\log\left(a+b\tanh^2(c+dx)\right)}{2a^2d(a+b)^2}+\frac{\log(\tanh(c+dx))}{a^2d}+\frac{b}{2ad(a+b)\left(a+b\tanh^2(c+dx)\right)}+\frac{\log(\cosh(c+dx))}{d(a+b)^2}$$

```
[Out] Log[Cosh[c + d*x]]/((a + b)^2*d) + Log[Tanh[c + d*x]]/(a^2*d) - (b*(2*a + b
)*Log[a + b*Tanh[c + d*x]^2])/(2*a^2*(a + b)^2*d) + b/(2*a*(a + b)*d*(a + b
*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.149079, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3670, 446, 72}

$$-\frac{b(2a+b)\log\left(a+b\tanh^2(c+dx)\right)}{2a^2d(a+b)^2}+\frac{\log(\tanh(c+dx))}{a^2d}+\frac{b}{2ad(a+b)\left(a+b\tanh^2(c+dx)\right)}+\frac{\log(\cosh(c+dx))}{d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```


```
[Out] Log[Cosh[c + d*x]]/((a + b)^2*d) + Log[Tanh[c + d*x]]/(a^2*d) - (b*(2*a + b
)*Log[a + b*Tanh[c + d*x]^2])/(2*a^2*(a + b)^2*d) + b/(2*a*(a + b)*d*(a + b
*Tanh[c + d*x]^2))
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(1-x)x(a+bx)^2}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^2(-1+x)}+\frac{1}{a^2x}-\frac{b^2}{a(a+b)(a+bx)^2}-\frac{b^2(2a+b)}{a^2(a+b)^2(a+bx)}\right)dx, x, \tanh^2(c+dx)\right)}{2d}$$

$$= \frac{\log(\cosh(c+dx))}{(a+b)^2d}+\frac{\log(\tanh(c+dx))}{a^2d}-\frac{b(2a+b)\log\left(a+b\tanh^2(c+dx)\right)}{2a^2(a+b)^2d}+\frac{}{2a(a+}$$

Mathematica [A] time = 1.8582, size = 83, normalized size = 0.87

$$\frac{\frac{\frac{b\left(\frac{a(a+b)}{a+b\tanh^2(c+dx)}-(2a+b)\log\left(a+b\tanh^2(c+dx)\right)\right)}{(a+b)^2}+2\log(\tanh(c+dx))}{a^2}+\frac{2\log(\cosh(c+dx))}{(a+b)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((2*Log[Cosh[c + d*x]])/(a + b)^2 + (2*Log[Tanh[c + d*x]] + (b*(-((2*a + b)
*Log[a + b*Tanh[c + d*x]^2]) + (a*(a + b))/(a + b*Tanh[c + d*x]^2)))/(a + b
)^2)/a^2)/(2*d)
```

Maple [B] time = 0.102, size = 325, normalized size = 3.4

$$-\frac{1}{d(a+b)^2}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-2\frac{b^2(\tanh(1/2\,dx+c/2))^2}{d(a+b)^2a\left((\tanh(1/2\,dx+c/2))^4a+2(\tanh(1/2\,dx+c/2))^2a+4(\tanh(1/2\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] -1/d/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)+1)-2/d*b^2/(a+b)^2/a*tanh(1/2*d*x+1/2*c
)^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c
)^2*b+a)-2/d*b^3/(a+b)^2/a^2*tanh(1/2*d*x+1/2*c)^2/(tanh(1/2*d*x+1/2*c)^4*a
+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)-1/d*b/(a+b)^2/a*ln(
tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b
+a)-1/2/d*b^2/(a+b)^2/a^2*ln(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^
2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)+1/d/a^2*ln(tanh(1/2*d*x+1/2*c))-1/d/(a+b)^
2*ln(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.12821, size = 317, normalized size = 3.34

$$\frac{2b^2e^{(-2dx-2c)}}{\left(a^4+3a^3b+3a^2b^2+ab^3+2\left(a^4+a^3b-a^2b^2-ab^3\right)e^{(-2dx-2c)}+\left(a^4+3a^3b+3a^2b^2+ab^3\right)e^{(-4dx-4c)}\right)d}-\frac{\left(2ab+b^2\right)\text{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 2*b^2*e^(-2*d*x - 2*c)/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 + 2*(a^4 + a^3*b
 - a^2*b^2 - a*b^3)*e^(-2*d*x - 2*c) + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*
e^(-4*d*x - 4*c))*d) - 1/2*(2*a*b + b^2)*log(2*(a - b)*e^(-2*d*x - 2*c) + (
a + b)*e^(-4*d*x - 4*c) + a + b)/((a^4 + 2*a^3*b + a^2*b^2)*d) + (d*x + c)/
((a^2 + 2*a*b + b^2)*d) + log(e^(-d*x - c) + 1)/(a^2*d) + log(e^(-d*x - c)
- 1)/(a^2*d)
```

Fricas [B] time = 3.14217, size = 2678, normalized size = 28.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^3 + a^2*b)*d*x*cosh(d*x + c)^4 + 8*(a^3 + a^2*b)*d*x*cosh(d*x +
c)*sinh(d*x + c)^3 + 2*(a^3 + a^2*b)*d*x*sinh(d*x + c)^4 + 2*(a^3 + a^2*b)*
d*x - 4*(a*b^2 - (a^3 - a^2*b)*d*x)*cosh(d*x + c)^2 + 4*(3*(a^3 + a^2*b)*d*
x*cosh(d*x + c)^2 - a*b^2 + (a^3 - a^2*b)*d*x)*sinh(d*x + c)^2 + ((2*a^2*b
+ 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 4*(2*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c
)*sinh(d*x + c)^3 + (2*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^4 + 2*a^2*b + 3
*a*b^2 + b^3 + 2*(2*a^2*b - a*b^2 - b^3)*cosh(d*x + c)^2 + 2*(2*a^2*b - a*b
^2 - b^3 + 3*(2*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4
*((2*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (2*a^2*b - a*b^2 - b^3)*cosh(
d*x + c))*sinh(d*x + c))*log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x
+ c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x
 + c)^2)) - 2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 4*(a^3 + 3
*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*sinh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 + a^2
*b - a*b^2 - b^3)*cosh(d*x + c)^2 + 2*(a^3 + a^2*b - a*b^2 - b^3 + 3*(a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (a^3 + a^2*b - a*b^2 - b^3)*cosh(d*x
 + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c)))
+ 8*((a^3 + a^2*b)*d*x*cosh(d*x + c)^3 - (a*b^2 - (a^3 - a^2*b)*d*x)*cosh(d
*x + c))*sinh(d*x + c))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x +
 c)^4 + 4*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)*sinh(d*x +
c)^3 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*sinh(d*x + c)^4 + 2*(a^5 + a
^4*b - a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^5 + 3*a^4*b + 3*a^3*b
^2 + a^2*b^3)*d*cosh(d*x + c)^2 + (a^5 + a^4*b - a^3*b^2 - a^2*b^3)*d)*sinh
(d*x + c)^2 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d + 4*((a^5 + 3*a^4*b +
 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^3 + (a^5 + a^4*b - a^3*b^2 - a^2*b^3)
*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(coth(c + d*x)/(a + b*tanh(c + d*x)**2)**2, x)
```

Giac [B] time = 1.21831, size = 279, normalized size = 2.94

$$-\frac{\left(2\,ab+b^2\right)\log\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)}{2\left(a^4d+2\,a^3bd+a^2b^2d\right)}-\frac{dx+c}{a^2d+2\,abd+b^2d}+\frac{\cdots}{\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*a*b + b^2)*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x
 + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)/(a^4*d + 2*a^3*b*d + a^2*b^2*d) - (d
*x + c)/(a^2*d + 2*a*b*d + b^2*d) + 2*b^2*e^(2*d*x + 2*c)/((a*e^(4*d*x + 4*
c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)
*(a + b)^2*a*d) + log(abs(-e^(2*d*x + 2*c) + 1))/(a^2*d)
```

3.187 $\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=119

$$-\frac{b^{3/2}(5a+3b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}d(a+b)^2}-\frac{(2a+3b)\coth(c+dx)}{2a^2d(a+b)}+\frac{b\coth(c+dx)}{2ad(a+b)\left(a+b\tanh^2(c+dx)\right)}+\frac{x}{(a+b)^2}$$

```
[Out] x/(a + b)^2 - (b^(3/2)*(5*a + 3*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])
/(2*a^(5/2)*(a + b)^2*d) - ((2*a + 3*b)*Coth[c + d*x])/(2*a^2*(a + b)*d) +
(b*Coth[c + d*x])/(2*a*(a + b)*d*(a + b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.194075, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3670, 472, 583, 522, 206, 205}

$$-\frac{b^{3/2}(5a+3b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}d(a+b)^2}-\frac{(2a+3b)\coth(c+dx)}{2a^2d(a+b)}+\frac{b\coth(c+dx)}{2ad(a+b)\left(a+b\tanh^2(c+dx)\right)}+\frac{x}{(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```


```
[Out] x/(a + b)^2 - (b^(3/2)*(5*a + 3*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])
/(2*a^(5/2)*(a + b)^2*d) - ((2*a + 3*b)*Coth[c + d*x])/(2*a^2*(a + b)*d) +
(b*Coth[c + d*x])/(2*a*(a + b)*d*(a + b*Tanh[c + d*x]^2))
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
```

```
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\coth(c+dx)}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-2a-3b+3bx^2}{x^2(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{2a(a+b)d}$$

$$= -\frac{(2a+3b)\coth(c+dx)}{2a^2(a+b)d} + \frac{b\coth(c+dx)}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{2a^2-2ab-3b^2+b(2a\cdots}{(1-x^2)(a+bx^2\cdots}\right.}{2a^2(a\cdots}$$

$$= -\frac{(2a+3b)\coth(c+dx)}{2a^2(a+b)d} + \frac{b\coth(c+dx)}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh\cdots\right.}{(a+b)^2d}$$

$$= \frac{x}{(a+b)^2} - \frac{b^{3/2}(5a+3b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}(a+b)^2d} - \frac{(2a+3b)\coth(c+dx)}{2a^2(a+b)d} + \frac{b\coth\cdots}{2a(a+b)d\left(a+\cdots\right.}$$

Mathematica [A] time = 1.55879, size = 111, normalized size = 0.93

$$-\frac{\frac{b^{3/2}(5a+3b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}(a+b)^2} + \frac{b^2\sinh(2(c+dx))}{a^2(a+b)((a+b)\cosh(2(c+dx))+a-b)} + \frac{2\coth(c+dx)}{a^2} - \frac{2(c+dx)}{(a+b)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -((-2*(c + d*x))/(a + b)^2 + (b^(3/2)*(5*a + 3*b)*ArcTan[(Sqrt[b]*Tanh[c +
d*x])/Sqrt[a]])/(a^(5/2)*(a + b)^2) + (2*Coth[c + d*x])/a^2 + (b^2*Sinh[2*(
c + d*x)])/(a^2*(a + b)*(a - b + (a + b)*Cosh[2*(c + d*x)]))))/(2*d)
```

Maple [B] time = 0.108, size = 1061, normalized size = 8.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] -1/2/d/a^2*tanh(1/2*d*x+1/2*c)+1/d/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)+1)-1/d*b^
2/(a+b)^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x
+1/2*c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)^3-1/d*b^3/(a+b)^2/a^2/(tanh(1/2*d*x+1/
2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*tanh(1/2*d*
x+1/2*c)^3-1/d*b^2/(a+b)^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2
*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)-1/d*b^3/(a+b)^2/a^2/(
tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b
+a)*tanh(1/2*d*x+1/2*c)+5/2/d*b^2/(a+b)^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/
2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)
*a)^(1/2))-5/2/d*b^2/(a+b)^2/a/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(
a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+4/d*b^3/(a+b)^2/
a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*
x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+5/2/d*b^2/(a+b)^2/(b*(a+b))^(
1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(
b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+5/2/d*b^2/(a+b)^2/a/((2*(b*(a+b))^(1/2)+a+2
*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/
2))+4/d*b^3/(a+b)^2/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*a
rctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-3/2/d*b^3/
(a+b)^2/a^2/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*
c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+3/2/d*b^4/(a+b)^2/a^2/(b*(a+b))^(1/
2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b
*(a+b))^(1/2)-a-2*b)*a)^(1/2))+3/2/d*b^3/(a+b)^2/a^2/((2*(b*(a+b))^(1/2)+a+
2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1
/2))+3/2/d*b^4/(a+b)^2/a^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1
/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/2/d
/a^2/tanh(1/2*d*x+1/2*c)-1/d/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.73135, size = 8676, normalized size = 72.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^3 + a^2*b)*d*x*cosh(d*x + c)^6 + 24*(a^3 + a^2*b)*d*x*cosh(d*x +
 c)*sinh(d*x + c)^5 + 4*(a^3 + a^2*b)*d*x*sinh(d*x + c)^6 - 4*(2*a^3 + 6*a^
```

```
2*b + 5*a*b^2 + 3*b^3 - (a^3 - 3*a^2*b)*d*x)*cosh(d*x + c)^4 + 4*(15*(a^3 +
 a^2*b)*d*x*cosh(d*x + c)^2 - 2*a^3 - 6*a^2*b - 5*a*b^2 - 3*b^3 + (a^3 - 3*
a^2*b)*d*x)*sinh(d*x + c)^4 + 16*(5*(a^3 + a^2*b)*d*x*cosh(d*x + c)^3 - (2*
a^3 + 6*a^2*b + 5*a*b^2 + 3*b^3 - (a^3 - 3*a^2*b)*d*x)*cosh(d*x + c))*sinh(
d*x + c)^3 - 8*a^3 - 24*a^2*b - 28*a*b^2 - 12*b^3 - 4*(a^3 + a^2*b)*d*x - 4
*(4*a^3 + 4*a^2*b - 4*a*b^2 - 6*b^3 + (a^3 - 3*a^2*b)*d*x)*cosh(d*x + c)^2
+ 4*(15*(a^3 + a^2*b)*d*x*cosh(d*x + c)^4 - 4*a^3 - 4*a^2*b + 4*a*b^2 + 6*b
^3 - (a^3 - 3*a^2*b)*d*x - 6*(2*a^3 + 6*a^2*b + 5*a*b^2 + 3*b^3 - (a^3 - 3*
a^2*b)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((5*a^2*b + 8*a*b^2 + 3*b^3)
*cosh(d*x + c)^6 + 6*(5*a^2*b + 8*a*b^2 + 3*b^3)*cosh(d*x + c)*sinh(d*x + c
)^5 + (5*a^2*b + 8*a*b^2 + 3*b^3)*sinh(d*x + c)^6 + (5*a^2*b - 12*a*b^2 - 9
*b^3)*cosh(d*x + c)^4 + (5*a^2*b - 12*a*b^2 - 9*b^3 + 15*(5*a^2*b + 8*a*b^2
 + 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(5*a^2*b + 8*a*b^2 + 3*b^
3)*cosh(d*x + c)^3 + (5*a^2*b - 12*a*b^2 - 9*b^3)*cosh(d*x + c))*sinh(d*x +
 c)^3 - 5*a^2*b - 8*a*b^2 - 3*b^3 - (5*a^2*b - 12*a*b^2 - 9*b^3)*cosh(d*x +
 c)^2 + (15*(5*a^2*b + 8*a*b^2 + 3*b^3)*cosh(d*x + c)^4 - 5*a^2*b + 12*a*b^
2 + 9*b^3 + 6*(5*a^2*b - 12*a*b^2 - 9*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2
 + 2*(3*(5*a^2*b + 8*a*b^2 + 3*b^3)*cosh(d*x + c)^5 + 2*(5*a^2*b - 12*a*b^2
 - 9*b^3)*cosh(d*x + c)^3 - (5*a^2*b - 12*a*b^2 - 9*b^3)*cosh(d*x + c))*sin
h(d*x + c))*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 +
2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x +
 c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x +
 c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b +
b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a^2 +
 a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 +
a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*
(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)
*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 +
4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b))
 + 8*(3*(a^3 + a^2*b)*d*x*cosh(d*x + c)^5 - 2*(2*a^3 + 6*a^2*b + 5*a*b^2 +
3*b^3 - (a^3 - 3*a^2*b)*d*x)*cosh(d*x + c)^3 - (4*a^3 + 4*a^2*b - 4*a*b^2 -
 6*b^3 + (a^3 - 3*a^2*b)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^5 + 3*a^4*b
 + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^6 + 6*(a^5 + 3*a^4*b + 3*a^3*b^2 +
a^2*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2
*b^3)*d*sinh(d*x + c)^6 + (a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*d*cosh(d*x
+ c)^4 + (15*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + (a^5
 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*d)*sinh(d*x + c)^4 - (a^5 - a^4*b - 5*a^3
*b^2 - 3*a^2*b^3)*d*cosh(d*x + c)^2 + 4*(5*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2
*b^3)*d*cosh(d*x + c)^3 + (a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*d*cosh(d*x
+ c))*sinh(d*x + c)^3 + (15*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*
x + c)^4 + 6*(a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*d*cosh(d*x + c)^2 - (a^5
 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*d)*sinh(d*x + c)^2 - (a^5 + 3*a^4*b + 3*a
^3*b^2 + a^2*b^3)*d + 2*(3*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x
 + c)^5 + 2*(a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*d*cosh(d*x + c)^3 - (a^5
- a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*(a
^3 + a^2*b)*d*x*cosh(d*x + c)^6 + 12*(a^3 + a^2*b)*d*x*cosh(d*x + c)*sinh(d
*x + c)^5 + 2*(a^3 + a^2*b)*d*x*sinh(d*x + c)^6 - 2*(2*a^3 + 6*a^2*b + 5*a*
b^2 + 3*b^3 - (a^3 - 3*a^2*b)*d*x)*cosh(d*x + c)^4 + 2*(15*(a^3 + a^2*b)*d*
x*cosh(d*x + c)^2 - 2*a^3 - 6*a^2*b - 5*a*b^2 - 3*b^3 + (a^3 - 3*a^2*b)*d*x
)*sinh(d*x + c)^4 + 8*(5*(a^3 + a^2*b)*d*x*cosh(d*x + c)^3 - (2*a^3 + 6*a^2
*b + 5*a*b^2 + 3*b^3 - (a^3 - 3*a^2*b)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3
- 4*a^3 - 12*a^2*b - 14*a*b^2 - 6*b^3 - 2*(a^3 + a^2*b)*d*x - 2*(4*a^3 + 4*
a^2*b - 4*a*b^2 - 6*b^3 + (a^3 - 3*a^2*b)*d*x)*cosh(d*x + c)^2 + 2*(15*(a^3
 + a^2*b)*d*x*cosh(d*x + c)^4 - 4*a^3 - 4*a^2*b + 4*a*b^2 + 6*b^3 - (a^3 -
3*a^2*b)*d*x - 6*(2*a^3 + 6*a^2*b + 5*a*b^2 + 3*b^3 - (a^3 - 3*a^2*b)*d*x)*
cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((5*a^2*b + 8*a*b^2 + 3*b^3)*cosh(d*x +
c)^6 + 6*(5*a^2*b + 8*a*b^2 + 3*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (5*a^2
*b + 8*a*b^2 + 3*b^3)*sinh(d*x + c)^6 + (5*a^2*b - 12*a*b^2 - 9*b^3)*cosh(d
*x + c)^4 + (5*a^2*b - 12*a*b^2 - 9*b^3 + 15*(5*a^2*b + 8*a*b^2 + 3*b^3)*co
```

```
sh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(5*a^2*b + 8*a*b^2 + 3*b^3)*cosh(d*x
+ c)^3 + (5*a^2*b - 12*a*b^2 - 9*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 5*a^
2*b - 8*a*b^2 - 3*b^3 - (5*a^2*b - 12*a*b^2 - 9*b^3)*cosh(d*x + c)^2 + (15*
(5*a^2*b + 8*a*b^2 + 3*b^3)*cosh(d*x + c)^4 - 5*a^2*b + 12*a*b^2 + 9*b^3 +
6*(5*a^2*b - 12*a*b^2 - 9*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(5*a
^2*b + 8*a*b^2 + 3*b^3)*cosh(d*x + c)^5 + 2*(5*a^2*b - 12*a*b^2 - 9*b^3)*co
sh(d*x + c)^3 - (5*a^2*b - 12*a*b^2 - 9*b^3)*cosh(d*x + c))*sinh(d*x + c))*
sqrt(b/a)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sin
h(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(b/a)/b) + 4*(3*(a^3 + a^
2*b)*d*x*cosh(d*x + c)^5 - 2*(2*a^3 + 6*a^2*b + 5*a*b^2 + 3*b^3 - (a^3 - 3*
a^2*b)*d*x)*cosh(d*x + c)^3 - (4*a^3 + 4*a^2*b - 4*a*b^2 - 6*b^3 + (a^3 - 3
*a^2*b)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^
2*b^3)*d*cosh(d*x + c)^6 + 6*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d
*x + c)*sinh(d*x + c)^5 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*sinh(d*x
+ c)^6 + (a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*d*cosh(d*x + c)^4 + (15*(a^5
 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + (a^5 - a^4*b - 5*a^3*
b^2 - 3*a^2*b^3)*d)*sinh(d*x + c)^4 - (a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)
*d*cosh(d*x + c)^2 + 4*(5*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x
+ c)^3 + (a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*d*cosh(d*x + c))*sinh(d*x +
c)^3 + (15*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^4 + 6*(a^5
 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*d*cosh(d*x + c)^2 - (a^5 - a^4*b - 5*a^3*
b^2 - 3*a^2*b^3)*d)*sinh(d*x + c)^2 - (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)
*d + 2*(3*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^5 + 2*(a^5
- a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*d*cosh(d*x + c)^3 - (a^5 - a^4*b - 5*a^3*b
^2 - 3*a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(coth(c + d*x)**2/(a + b*tanh(c + d*x)**2)**2, x)
```

Giac [B] time = 1.25187, size = 464, normalized size = 3.9

$$-\frac{\left(5\,ab^2+3\,b^3\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{2\left(a^4d+2\,a^3bd+a^2b^2d\right)\sqrt{ab}}+\frac{dx+c}{a^2d+2\,abd+b^2d}-\frac{2\,a^3e^{(4\,dx+4\,c)}+6\,a^2be^{(4\,dx+4\,c)}+5\,ab^2e^{(4\,dx+4\,c)}+\cdots}{\left(a^4d+2\,a^3bd+a^2b^2d\right)\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(5*a*b^2 + 3*b^3)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) +
a - b)/sqrt(a*b))/((a^4*d + 2*a^3*b*d + a^2*b^2*d)*sqrt(a*b)) + (d*x + c)/(
a^2*d + 2*a*b*d + b^2*d) - (2*a^3*e^(4*d*x + 4*c) + 6*a^2*b*e^(4*d*x + 4*c)
 + 5*a*b^2*e^(4*d*x + 4*c) + 3*b^3*e^(4*d*x + 4*c) + 4*a^3*e^(2*d*x + 2*c)
+ 4*a^2*b*e^(2*d*x + 2*c) - 4*a*b^2*e^(2*d*x + 2*c) - 6*b^3*e^(2*d*x + 2*c)
 + 2*a^3 + 6*a^2*b + 7*a*b^2 + 3*b^3)/((a^4*d + 2*a^3*b*d + a^2*b^2*d)*(a*e
^(6*d*x + 6*c) + b*e^(6*d*x + 6*c) + a*e^(4*d*x + 4*c) - 3*b*e^(4*d*x + 4*c
```

```
) - a*e^(2*d*x + 2*c) + 3*b*e^(2*d*x + 2*c) - a - b))
```

3.188 $\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=124

$$-\frac{b^2}{2a^2d(a+b)\left(a+b\tanh^2(c+dx)\right)}+\frac{b^2(3a+2b)\log\left(a+b\tanh^2(c+dx)\right)}{2a^3d(a+b)^2}+\frac{(a-2b)\log(\tanh(c+dx))}{a^3d}-\frac{\coth^2(c+dx)}{2a^2d}$$

```
[Out] -Coth[c + d*x]^2/(2*a^2*d) + Log[Cosh[c + d*x]]/((a + b)^2*d) + ((a - 2*b)*
Log[Tanh[c + d*x]])/(a^3*d) + (b^2*(3*a + 2*b)*Log[a + b*Tanh[c + d*x]^2])/
(2*a^3*(a + b)^2*d) - b^2/(2*a^2*(a + b)*d*(a + b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.188638, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 446, 88}

$$-\frac{b^2}{2a^2d(a+b)\left(a+b\tanh^2(c+dx)\right)}+\frac{b^2(3a+2b)\log\left(a+b\tanh^2(c+dx)\right)}{2a^3d(a+b)^2}+\frac{(a-2b)\log(\tanh(c+dx))}{a^3d}-\frac{\coth^2(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -Coth[c + d*x]^2/(2*a^2*d) + Log[Cosh[c + d*x]]/((a + b)^2*d) + ((a - 2*b)*
Log[Tanh[c + d*x]])/(a^3*d) + (b^2*(3*a + 2*b)*Log[a + b*Tanh[c + d*x]^2])/
(2*a^3*(a + b)^2*d) - b^2/(2*a^2*(a + b)*d*(a + b*Tanh[c + d*x]^2))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3(1-x^2)\left(a+bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{(1-x)x^2(a+bx)^2}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^2(-1+x)}+\frac{1}{a^2x^2}+\frac{a-2b}{a^3x}+\frac{b^3}{a^2(a+b)(a+bx)^2}+\frac{b^3(3a+2b)}{a^3(a+b)^2(a+bx)}\right)dx, x, \tanh^2(c+dx\right.}{2d}$$
$$= -\frac{\coth^2(c+dx)}{2a^2d}+\frac{\log(\cosh(c+dx))}{(a+b)^2d}+\frac{(a-2b)\log(\tanh(c+dx))}{a^3d}+\frac{b^2(3a+2b)\log\left(a\right.}{2a^3(a}$$

Mathematica [A] time = 0.799968, size = 93, normalized size = 0.75

$$\frac{\frac{b^3}{a^3(a+b)\left(a\coth^2(c+dx)+b\right)}+\frac{b^2(3a+2b)\log\left(a\coth^2(c+dx)+b\right)}{a^3(a+b)^2}-\frac{\coth^2(c+dx)}{a^2}+\frac{2\log(\sinh(c+dx))}{(a+b)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (-(Coth[c + d*x]^2/a^2) + b^3/(a^3*(a + b)*(b + a*Coth[c + d*x]^2)) + (b^2*
(3*a + 2*b)*Log[b + a*Coth[c + d*x]^2])/(a^3*(a + b)^2) + (2*Log[Sinh[c + d
*x]])/(a + b)^2)/(2*d)
```

Maple [B] time = 0.113, size = 383, normalized size = 3.1

$$-\frac{1}{8da^2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-\frac{1}{d(a+b)^2}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+2\frac{b^3\left(\tanh(1/2\,dx\right.}{d(a+b)^2a^2\left((\tanh(1/2\,dx+c/2))^4a+2\,(\tanh(1/2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] -1/8/d*tanh(1/2*d*x+1/2*c)^2/a^2-1/d/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)+1)+2/d*
b^3/(a+b)^2/a^2*tanh(1/2*d*x+1/2*c)^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d
*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)+2/d*b^4/a^3/(a+b)^2*tanh(1/2*d*x
+1/2*c)^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x
+1/2*c)^2*b+a)+3/2/d*b^2/(a+b)^2/a^2*ln(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*
d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)+1/d*b^3/a^3/(a+b)^2*ln(tanh(1/2
*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)-1/8/
d/a^2/tanh(1/2*d*x+1/2*c)^2+1/d/a^2*ln(tanh(1/2*d*x+1/2*c))-2/d/a^3*ln(tanh
(1/2*d*x+1/2*c))*b-1/d/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.16502, size = 543, normalized size = 4.38

$$\frac{\left(3\,ab^2+2\,b^3\right)\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\left(a^5+2\,a^4b+a^3b^2\right)d}+\frac{dx+c}{\left(a^2+2\,ab+b^2\right)d}-\frac{}{\left(a^5+3\,a^4b+3\,a^3b^2+a^2b^3-4\,(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(3*a*b^2 + 2*b^3)*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x -
4*c) + a + b)/((a^5 + 2*a^4*b + a^3*b^2)*d) + (d*x + c)/((a^2 + 2*a*b + b^2
)*d) - 2*((a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*e^(-2*d*x - 2*c) + 2*(a^3 + a^2
*b - a*b^2 - 2*b^3)*e^(-4*d*x - 4*c) + (a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*e^
(-6*d*x - 6*c))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 - 4*(a^4*b + 2*a^3*b^
2 + a^2*b^3)*e^(-2*d*x - 2*c) - 2*(a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*e^(
-4*d*x - 4*c) - 4*(a^4*b + 2*a^3*b^2 + a^2*b^3)*e^(-6*d*x - 6*c) + (a^5 + 3
*a^4*b + 3*a^3*b^2 + a^2*b^3)*e^(-8*d*x - 8*c))*d) + (a - 2*b)*log(e^(-d*x
- c) + 1)/(a^3*d) + (a - 2*b)*log(e^(-d*x - c) - 1)/(a^3*d)
```

Fricas [B] time = 4.11124, size = 7919, normalized size = 63.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^4 + a^3*b)*d*x*cosh(d*x + c)^8 + 16*(a^4 + a^3*b)*d*x*cosh(d*x +
 c)*sinh(d*x + c)^7 + 2*(a^4 + a^3*b)*d*x*sinh(d*x + c)^8 - 4*(2*a^3*b*d*x
- a^4 - 3*a^3*b - 3*a^2*b^2 - 2*a*b^3)*cosh(d*x + c)^6 - 4*(2*a^3*b*d*x - 1
4*(a^4 + a^3*b)*d*x*cosh(d*x + c)^2 - a^4 - 3*a^3*b - 3*a^2*b^2 - 2*a*b^3)*
sinh(d*x + c)^6 + 8*(14*(a^4 + a^3*b)*d*x*cosh(d*x + c)^3 - 3*(2*a^3*b*d*x
- a^4 - 3*a^3*b - 3*a^2*b^2 - 2*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*(
2*a^4 + 2*a^3*b - 2*a^2*b^2 - 4*a*b^3 - (a^4 - 3*a^3*b)*d*x)*cosh(d*x + c)^
4 + 4*(35*(a^4 + a^3*b)*d*x*cosh(d*x + c)^4 + 2*a^4 + 2*a^3*b - 2*a^2*b^2 -
 4*a*b^3 - (a^4 - 3*a^3*b)*d*x - 15*(2*a^3*b*d*x - a^4 - 3*a^3*b - 3*a^2*b^
2 - 2*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(7*(a^4 + a^3*b)*d*x*cos
h(d*x + c)^5 - 5*(2*a^3*b*d*x - a^4 - 3*a^3*b - 3*a^2*b^2 - 2*a*b^3)*cosh(d
*x + c)^3 + (2*a^4 + 2*a^3*b - 2*a^2*b^2 - 4*a*b^3 - (a^4 - 3*a^3*b)*d*x)*c
osh(d*x + c))*sinh(d*x + c)^3 + 2*(a^4 + a^3*b)*d*x - 4*(2*a^3*b*d*x - a^4
- 3*a^3*b - 3*a^2*b^2 - 2*a*b^3)*cosh(d*x + c)^2 + 4*(14*(a^4 + a^3*b)*d*x*
cosh(d*x + c)^6 - 2*a^3*b*d*x - 15*(2*a^3*b*d*x - a^4 - 3*a^3*b - 3*a^2*b^2
 - 2*a*b^3)*cosh(d*x + c)^4 + a^4 + 3*a^3*b + 3*a^2*b^2 + 2*a*b^3 + 6*(2*a^
4 + 2*a^3*b - 2*a^2*b^2 - 4*a*b^3 - (a^4 - 3*a^3*b)*d*x)*cosh(d*x + c)^2)*s
inh(d*x + c)^2 - ((3*a^2*b^2 + 5*a*b^3 + 2*b^4)*cosh(d*x + c)^8 + 8*(3*a^2*
b^2 + 5*a*b^3 + 2*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^2*b^2 + 5*a*b^3
 + 2*b^4)*sinh(d*x + c)^8 - 4*(3*a*b^3 + 2*b^4)*cosh(d*x + c)^6 - 4*(3*a*b^
3 + 2*b^4 - 7*(3*a^2*b^2 + 5*a*b^3 + 2*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^
6 + 8*(7*(3*a^2*b^2 + 5*a*b^3 + 2*b^4)*cosh(d*x + c)^3 - 3*(3*a*b^3 + 2*b^4
)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(3*a^2*b^2 - 7*a*b^3 - 6*b^4)*cosh(d*x
 + c)^4 + 2*(35*(3*a^2*b^2 + 5*a*b^3 + 2*b^4)*cosh(d*x + c)^4 - 3*a^2*b^2 +
 7*a*b^3 + 6*b^4 - 30*(3*a*b^3 + 2*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 +
3*a^2*b^2 + 5*a*b^3 + 2*b^4 + 8*(7*(3*a^2*b^2 + 5*a*b^3 + 2*b^4)*cosh(d*x +
 c)^5 - 10*(3*a*b^3 + 2*b^4)*cosh(d*x + c)^3 - (3*a^2*b^2 - 7*a*b^3 - 6*b^4
)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(3*a*b^3 + 2*b^4)*cosh(d*x + c)^2 + 4*
(7*(3*a^2*b^2 + 5*a*b^3 + 2*b^4)*cosh(d*x + c)^6 - 15*(3*a*b^3 + 2*b^4)*cos
h(d*x + c)^4 - 3*a*b^3 - 2*b^4 - 3*(3*a^2*b^2 - 7*a*b^3 - 6*b^4)*cosh(d*x +
 c)^2)*sinh(d*x + c)^2 + 8*((3*a^2*b^2 + 5*a*b^3 + 2*b^4)*cosh(d*x + c)^7 -
 3*(3*a*b^3 + 2*b^4)*cosh(d*x + c)^5 - (3*a^2*b^2 - 7*a*b^3 - 6*b^4)*cosh(d
*x + c)^3 - (3*a*b^3 + 2*b^4)*cosh(d*x + c))*sinh(d*x + c))*log(2*((a + b)*
cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cos
h(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 2*((a^4 + a^3*b - 3*a^2*b^2
```

```
- 5*a*b^3 - 2*b^4)*cosh(d*x + c)^8 + 8*(a^4 + a^3*b - 3*a^2*b^2 - 5*a*b^3 -
 2*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^4 + a^3*b - 3*a^2*b^2 - 5*a*b^3
- 2*b^4)*sinh(d*x + c)^8 - 4*(a^3*b - 3*a*b^3 - 2*b^4)*cosh(d*x + c)^6 - 4*
(a^3*b - 3*a*b^3 - 2*b^4 - 7*(a^4 + a^3*b - 3*a^2*b^2 - 5*a*b^3 - 2*b^4)*co
sh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^4 + a^3*b - 3*a^2*b^2 - 5*a*b^3 -
2*b^4)*cosh(d*x + c)^3 - 3*(a^3*b - 3*a*b^3 - 2*b^4)*cosh(d*x + c))*sinh(d*
x + c)^5 - 2*(a^4 - 3*a^3*b - 3*a^2*b^2 + 7*a*b^3 + 6*b^4)*cosh(d*x + c)^4
+ 2*(35*(a^4 + a^3*b - 3*a^2*b^2 - 5*a*b^3 - 2*b^4)*cosh(d*x + c)^4 - a^4 +
 3*a^3*b + 3*a^2*b^2 - 7*a*b^3 - 6*b^4 - 30*(a^3*b - 3*a*b^3 - 2*b^4)*cosh(
d*x + c)^2)*sinh(d*x + c)^4 + a^4 + a^3*b - 3*a^2*b^2 - 5*a*b^3 - 2*b^4 + 8
*(7*(a^4 + a^3*b - 3*a^2*b^2 - 5*a*b^3 - 2*b^4)*cosh(d*x + c)^5 - 10*(a^3*b
 - 3*a*b^3 - 2*b^4)*cosh(d*x + c)^3 - (a^4 - 3*a^3*b - 3*a^2*b^2 + 7*a*b^3
+ 6*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(a^3*b - 3*a*b^3 - 2*b^4)*cosh(
d*x + c)^2 + 4*(7*(a^4 + a^3*b - 3*a^2*b^2 - 5*a*b^3 - 2*b^4)*cosh(d*x + c)
^6 - 15*(a^3*b - 3*a*b^3 - 2*b^4)*cosh(d*x + c)^4 - a^3*b + 3*a*b^3 + 2*b^4
 - 3*(a^4 - 3*a^3*b - 3*a^2*b^2 + 7*a*b^3 + 6*b^4)*cosh(d*x + c)^2)*sinh(d*
x + c)^2 + 8*((a^4 + a^3*b - 3*a^2*b^2 - 5*a*b^3 - 2*b^4)*cosh(d*x + c)^7 -
 3*(a^3*b - 3*a*b^3 - 2*b^4)*cosh(d*x + c)^5 - (a^4 - 3*a^3*b - 3*a^2*b^2 +
 7*a*b^3 + 6*b^4)*cosh(d*x + c)^3 - (a^3*b - 3*a*b^3 - 2*b^4)*cosh(d*x + c)
)*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 8*(
2*(a^4 + a^3*b)*d*x*cosh(d*x + c)^7 - 3*(2*a^3*b*d*x - a^4 - 3*a^3*b - 3*a^
2*b^2 - 2*a*b^3)*cosh(d*x + c)^5 + 2*(2*a^4 + 2*a^3*b - 2*a^2*b^2 - 4*a*b^3
 - (a^4 - 3*a^3*b)*d*x)*cosh(d*x + c)^3 - (2*a^3*b*d*x - a^4 - 3*a^3*b - 3*
a^2*b^2 - 2*a*b^3)*cosh(d*x + c))*sinh(d*x + c))/((a^6 + 3*a^5*b + 3*a^4*b^
2 + a^3*b^3)*d*cosh(d*x + c)^8 + 8*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*
cosh(d*x + c)*sinh(d*x + c)^7 + (a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*sin
h(d*x + c)^8 - 4*(a^5*b + 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^6 + 4*(7*(a^
6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^2 - (a^5*b + 2*a^4*b^2 +
 a^3*b^3)*d)*sinh(d*x + c)^6 - 2*(a^6 - a^5*b - 5*a^4*b^2 - 3*a^3*b^3)*d*co
sh(d*x + c)^4 + 8*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^
3 - 3*(a^5*b + 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3
5*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^4 - 30*(a^5*b + 2*a
^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^2 - (a^6 - a^5*b - 5*a^4*b^2 - 3*a^3*b^3)
*d)*sinh(d*x + c)^4 - 4*(a^5*b + 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^2 + 8
*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^5 - 10*(a^5*b + 2
*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^3 - (a^6 - a^5*b - 5*a^4*b^2 - 3*a^3*b^
3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3
*b^3)*d*cosh(d*x + c)^6 - 15*(a^5*b + 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^
4 - 3*(a^6 - a^5*b - 5*a^4*b^2 - 3*a^3*b^3)*d*cosh(d*x + c)^2 - (a^5*b + 2*
a^4*b^2 + a^3*b^3)*d)*sinh(d*x + c)^2 + (a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^
3)*d + 8*((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^7 - 3*(a^5*
b + 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^5 - (a^6 - a^5*b - 5*a^4*b^2 - 3*a
^3*b^3)*d*cosh(d*x + c)^3 - (a^5*b + 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c))*
sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(coth(c + d*x)**3/(a + b*tanh(c + d*x)**2)**2, x)
```

Giac [B] time = 1.28295, size = 450, normalized size = 3.63

$$\frac{\left(3\,ab^2+2\,b^3\right)\log\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)}{2\left(a^5d+2\,a^4bd+a^3b^2d\right)}-\frac{dx+c}{a^2d+2\,abd+b^2d}+\frac{(a-2\,b)\log\left(\left|e^{(\ldots}\right.\right.}{a^3d\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*a*b^2 + 2*b^3)*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*
d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)/(a^5*d + 2*a^4*b*d + a^3*b^2*d) -
 (d*x + c)/(a^2*d + 2*a*b*d + b^2*d) + (a - 2*b)*log(abs(e^(2*d*x + 2*c) -
1))/(a^3*d) - 2*((a^4 + 3*a^3*b + 3*a^2*b^2 + 2*a*b^3)*e^(6*d*x + 6*c)/(a +
 b) + 2*(a^4 + a^3*b - a^2*b^2 - 2*a*b^3)*e^(4*d*x + 4*c)/(a + b) + (a^4 +
3*a^3*b + 3*a^2*b^2 + 2*a*b^3)*e^(2*d*x + 2*c)/(a + b))/((a*e^(4*d*x + 4*c)
 + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)*(
a + b)*a^3*d*(e^(2*d*x + 2*c) - 1)^2)
```

3.189 $\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=159

$$-\frac{\left(2a^2-2ab-5b^2\right)\coth(c+dx)}{2a^3d(a+b)}+\frac{b^{5/2}(7a+5b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}d(a+b)^2}-\frac{(2a+5b)\coth^3(c+dx)}{6a^2d(a+b)}+\frac{b\coth^3(c+\ldots}{2ad(a+b)\left(a+b\tan\ldots\right.}$$

```
[Out] x/(a + b)^2 + (b^(5/2)*(7*a + 5*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])
/(2*a^(7/2)*(a + b)^2*d) - ((2*a^2 - 2*a*b - 5*b^2)*Coth[c + d*x])/(2*a^3*(
a + b)*d) - ((2*a + 5*b)*Coth[c + d*x]^3)/(6*a^2*(a + b)*d) + (b*Coth[c + d
*x]^3)/(2*a*(a + b)*d*(a + b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.282777, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3670, 472, 583, 522, 206, 205}

$$-\frac{\left(2a^2-2ab-5b^2\right)\coth(c+dx)}{2a^3d(a+b)}+\frac{b^{5/2}(7a+5b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}d(a+b)^2}-\frac{(2a+5b)\coth^3(c+dx)}{6a^2d(a+b)}+\frac{b\coth^3(c+\ldots}{2ad(a+b)\left(a+b\tan\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] x/(a + b)^2 + (b^(5/2)*(7*a + 5*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])
/(2*a^(7/2)*(a + b)^2*d) - ((2*a^2 - 2*a*b - 5*b^2)*Coth[c + d*x])/(2*a^3*(
a + b)*d) - ((2*a + 5*b)*Coth[c + d*x]^3)/(6*a^2*(a + b)*d) + (b*Coth[c + d
*x]^3)/(2*a*(a + b)*d*(a + b*Tanh[c + d*x]^2))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
```

```
e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4\left(1-x^2\right)\left(a+bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\coth^3(c+dx)}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-2a-5b+5bx^2}{x^4\left(1-x^2\right)\left(a+bx^2\right)}\,dx, x, \tanh(c+dx)\right)}{2a(a+b)d}$$

$$= -\frac{(2a+5b)\coth^3(c+dx)}{6a^2(a+b)d} + \frac{b\coth^3(c+dx)}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{3\left(2a^2-2ab-5b\right.}{x^2\left(1-x\right.}\right.}{}$$

$$= -\frac{\left(2a^2-2ab-5b^2\right)\coth(c+dx)}{2a^3(a+b)d} - \frac{(2a+5b)\coth^3(c+dx)}{6a^2(a+b)d} + \frac{b\coth^3(c+d}{2a(a+b)d\left(a+b\tanh\right.}$$

$$= -\frac{\left(2a^2-2ab-5b^2\right)\coth(c+dx)}{2a^3(a+b)d} - \frac{(2a+5b)\coth^3(c+dx)}{6a^2(a+b)d} + \frac{b\coth^3(c+d}{2a(a+b)d\left(a+b\tanh\right.}$$

$$= \frac{x}{(a+b)^2} + \frac{b^{5/2}(7a+5b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}(a+b)^2d} - \frac{\left(2a^2-2ab-5b^2\right)\coth(c+dx)}{2a^3(a+b)d} - (2a$$

Mathematica [A] time = 1.32661, size = 139, normalized size = 0.87

$$\frac{\frac{3b^{5/2}(7a+5b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{7/2}(a+b)^2} + \frac{3b^3\sinh(2(c+dx))}{a^3(a+b)((a+b)\cosh(2(c+dx))+a-b)} + \frac{4(3b-2a)\coth(c+dx)}{a^3} - \frac{2\coth(c+dx)\text{csch}^2(c+dx)}{a^2} + \frac{6(c+dx)}{(a+b)^2}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((6*(c + d*x))/(a + b)^2 + (3*b^(5/2)*(7*a + 5*b)*ArcTan[(Sqrt[b]*Tanh[c +
d*x])/Sqrt[a]])/(a^(7/2)*(a + b)^2) + (4*(-2*a + 3*b)*Coth[c + d*x])/a^3 -
(2*Coth[c + d*x]*Csch[c + d*x]^2)/a^2 + (3*b^3*Sinh[2*(c + d*x)])/(a^3*(a +
 b)*(a - b + (a + b)*Cosh[2*(c + d*x)])))/(6*d)
```

Maple [B] time = 0.122, size = 1137, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] -1/24/d/a^2*tanh(1/2*d*x+1/2*c)^3-5/8/d/a^2*tanh(1/2*d*x+1/2*c)+1/d/a^3*tan
h(1/2*d*x+1/2*c)*b+1/d/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)+1)+1/d*b^3/(a+b)^2/a^
2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^
2*b+a)*tanh(1/2*d*x+1/2*c)^3+1/d*b^4/(a+b)^2/a^3/(tanh(1/2*d*x+1/2*c)^4*a+2
*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*tanh(1/2*d*x+1/2*c)^3
+1/d*b^3/(a+b)^2/a^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*t
anh(1/2*d*x+1/2*c)^2*b+a)*tanh(1/2*d*x+1/2*c)+1/d*b^4/(a+b)^2/a^3/(tanh(1/2
*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*tanh
(1/2*d*x+1/2*c)-7/2/d*b^3/(a+b)^2/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2
*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1
/2))+7/2/d*b^3/(a+b)^2/a^2/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*ta
nh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-6/d*b^4/(a+b)^2/a^2/
(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+
1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-7/2/d*b^3/(a+b)^2/a/(b*(a+b))^(
1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(
b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-7/2/d*b^3/(a+b)^2/a^2/((2*(b*(a+b))^(1/2)+a
+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(
1/2))-6/d*b^4/(a+b)^2/a^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/
2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+5/2/d*
b^4/(a+b)^2/a^3/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+
1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-5/2/d*b^5/(a+b)^2/a^3/(b*(a+b))
^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((
2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-5/2/d*b^4/(a+b)^2/a^3/((2*(b*(a+b))^(1/2
)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a
)^(1/2))-5/2/d*b^5/(a+b)^2/a^3/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a
)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1
/24/d/a^2/tanh(1/2*d*x+1/2*c)^3-5/8/d/a^2/tanh(1/2*d*x+1/2*c)+1/d/a^3/tanh(
1/2*d*x+1/2*c)*b-1/d/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.24653, size = 19919, normalized size = 125.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(12*(a^4 + a^3*b)*d*x*cosh(d*x + c)^10 + 120*(a^4 + a^3*b)*d*x*cosh(d
*x + c)*sinh(d*x + c)^9 + 12*(a^4 + a^3*b)*d*x*sinh(d*x + c)^10 - 12*(4*a^4
 + 8*a^3*b - 7*a*b^3 - 5*b^4 + (a^4 + 5*a^3*b)*d*x)*cosh(d*x + c)^8 + 12*(4
5*(a^4 + a^3*b)*d*x*cosh(d*x + c)^2 - 4*a^4 - 8*a^3*b + 7*a*b^3 + 5*b^4 - (
a^4 + 5*a^3*b)*d*x)*sinh(d*x + c)^8 + 96*(15*(a^4 + a^3*b)*d*x*cosh(d*x + c
)^3 - (4*a^4 + 8*a^3*b - 7*a*b^3 - 5*b^4 + (a^4 + 5*a^3*b)*d*x)*cosh(d*x +
c))*sinh(d*x + c)^7 - 24*(2*a^4 - 2*a^3*b - 2*a^2*b^2 + 9*a*b^3 + 10*b^4 +
(a^4 - 5*a^3*b)*d*x)*cosh(d*x + c)^6 + 24*(105*(a^4 + a^3*b)*d*x*cosh(d*x +
 c)^4 - 2*a^4 + 2*a^3*b + 2*a^2*b^2 - 9*a*b^3 - 10*b^4 - (a^4 - 5*a^3*b)*d*
x - 14*(4*a^4 + 8*a^3*b - 7*a*b^3 - 5*b^4 + (a^4 + 5*a^3*b)*d*x)*cosh(d*x +
 c)^2)*sinh(d*x + c)^6 + 48*(63*(a^4 + a^3*b)*d*x*cosh(d*x + c)^5 - 14*(4*a
^4 + 8*a^3*b - 7*a*b^3 - 5*b^4 + (a^4 + 5*a^3*b)*d*x)*cosh(d*x + c)^3 - 3*(
2*a^4 - 2*a^3*b - 2*a^2*b^2 + 9*a*b^3 + 10*b^4 + (a^4 - 5*a^3*b)*d*x)*cosh(
d*x + c))*sinh(d*x + c)^5 + 8*(2*a^4 - 30*a^3*b - 30*a^2*b^2 + 38*a*b^3 + 4
5*b^4 + 3*(a^4 - 5*a^3*b)*d*x)*cosh(d*x + c)^4 + 8*(315*(a^4 + a^3*b)*d*x*c
osh(d*x + c)^6 - 105*(4*a^4 + 8*a^3*b - 7*a*b^3 - 5*b^4 + (a^4 + 5*a^3*b)*d
*x)*cosh(d*x + c)^4 + 2*a^4 - 30*a^3*b - 30*a^2*b^2 + 38*a*b^3 + 45*b^4 + 3
*(a^4 - 5*a^3*b)*d*x - 45*(2*a^4 - 2*a^3*b - 2*a^2*b^2 + 9*a*b^3 + 10*b^4 +
 (a^4 - 5*a^3*b)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 32*a^4 - 48*a^3*b
+ 48*a^2*b^2 + 124*a*b^3 + 60*b^4 + 32*(45*(a^4 + a^3*b)*d*x*cosh(d*x + c)^
7 - 21*(4*a^4 + 8*a^3*b - 7*a*b^3 - 5*b^4 + (a^4 + 5*a^3*b)*d*x)*cosh(d*x +
 c)^5 - 15*(2*a^4 - 2*a^3*b - 2*a^2*b^2 + 9*a*b^3 + 10*b^4 + (a^4 - 5*a^3*b
)*d*x)*cosh(d*x + c)^3 + (2*a^4 - 30*a^3*b - 30*a^2*b^2 + 38*a*b^3 + 45*b^4
 + 3*(a^4 - 5*a^3*b)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 12*(a^4 + a^3*b)
*d*x - 4*(4*a^4 - 20*a^3*b - 4*a^2*b^2 + 74*a*b^3 + 60*b^4 - 3*(a^4 + 5*a^3
*b)*d*x)*cosh(d*x + c)^2 + 4*(135*(a^4 + a^3*b)*d*x*cosh(d*x + c)^8 - 84*(4
*a^4 + 8*a^3*b - 7*a*b^3 - 5*b^4 + (a^4 + 5*a^3*b)*d*x)*cosh(d*x + c)^6 - 9
0*(2*a^4 - 2*a^3*b - 2*a^2*b^2 + 9*a*b^3 + 10*b^4 + (a^4 - 5*a^3*b)*d*x)*co
sh(d*x + c)^4 - 4*a^4 + 20*a^3*b + 4*a^2*b^2 - 74*a*b^3 - 60*b^4 + 3*(a^4 +
 5*a^3*b)*d*x + 12*(2*a^4 - 30*a^3*b - 30*a^2*b^2 + 38*a*b^3 + 45*b^4 + 3*(
a^4 - 5*a^3*b)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*((7*a^2*b^2 + 12*a
*b^3 + 5*b^4)*cosh(d*x + c)^10 + 10*(7*a^2*b^2 + 12*a*b^3 + 5*b^4)*cosh(d*x
 + c)*sinh(d*x + c)^9 + (7*a^2*b^2 + 12*a*b^3 + 5*b^4)*sinh(d*x + c)^10 - (
7*a^2*b^2 + 40*a*b^3 + 25*b^4)*cosh(d*x + c)^8 - (7*a^2*b^2 + 40*a*b^3 + 25
*b^4 - 45*(7*a^2*b^2 + 12*a*b^3 + 5*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^8 +
 8*(15*(7*a^2*b^2 + 12*a*b^3 + 5*b^4)*cosh(d*x + c)^3 - (7*a^2*b^2 + 40*a*b
^3 + 25*b^4)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(7*a^2*b^2 - 30*a*b^3 - 25*
b^4)*cosh(d*x + c)^6 + 2*(105*(7*a^2*b^2 + 12*a*b^3 + 5*b^4)*cosh(d*x + c)^
4 - 7*a^2*b^2 + 30*a*b^3 + 25*b^4 - 14*(7*a^2*b^2 + 40*a*b^3 + 25*b^4)*cosh
(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(7*a^2*b^2 + 12*a*b^3 + 5*b^4)*cosh(d*
x + c)^5 - 14*(7*a^2*b^2 + 40*a*b^3 + 25*b^4)*cosh(d*x + c)^3 - 3*(7*a^2*b^
2 - 30*a*b^3 - 25*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(7*a^2*b^2 - 30*a
*b^3 - 25*b^4)*cosh(d*x + c)^4 + 2*(105*(7*a^2*b^2 + 12*a*b^3 + 5*b^4)*cosh
(d*x + c)^6 - 35*(7*a^2*b^2 + 40*a*b^3 + 25*b^4)*cosh(d*x + c)^4 + 7*a^2*b^
2 - 30*a*b^3 - 25*b^4 - 15*(7*a^2*b^2 - 30*a*b^3 - 25*b^4)*cosh(d*x + c)^2)
*sinh(d*x + c)^4 - 7*a^2*b^2 - 12*a*b^3 - 5*b^4 + 8*(15*(7*a^2*b^2 + 12*a*b
^3 + 5*b^4)*cosh(d*x + c)^7 - 7*(7*a^2*b^2 + 40*a*b^3 + 25*b^4)*cosh(d*x +
c)^5 - 5*(7*a^2*b^2 - 30*a*b^3 - 25*b^4)*cosh(d*x + c)^3 + (7*a^2*b^2 - 30*
a*b^3 - 25*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + (7*a^2*b^2 + 40*a*b^3 + 25
*b^4)*cosh(d*x + c)^2 + (45*(7*a^2*b^2 + 12*a*b^3 + 5*b^4)*cosh(d*x + c)^8
 - 28*(7*a^2*b^2 + 40*a*b^3 + 25*b^4)*cosh(d*x + c)^6 - 30*(7*a^2*b^2 - 30*a
```

*b^3 - 25*b^4)*cosh(d*x + c)^4 + 7*a^2*b^2 + 40*a*b^3 + 25*b^4 + 12*(7*a^2*
b^2 - 30*a*b^3 - 25*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(5*(7*a^2*b^2
 + 12*a*b^3 + 5*b^4)*cosh(d*x + c)^9 - 4*(7*a^2*b^2 + 40*a*b^3 + 25*b^4)*co
sh(d*x + c)^7 - 6*(7*a^2*b^2 - 30*a*b^3 - 25*b^4)*cosh(d*x + c)^5 + 4*(7*a^
2*b^2 - 30*a*b^3 - 25*b^4)*cosh(d*x + c)^3 + (7*a^2*b^2 + 40*a*b^3 + 25*b^4
)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(d*
x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a
*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a
*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2
 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(
d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sin
h(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((a + b)*
cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*
x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b
)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sin
h(d*x + c) + a + b)) + 8*(15*(a^4 + a^3*b)*d*x*cosh(d*x + c)^9 - 12*(4*a^4
 + 8*a^3*b - 7*a*b^3 - 5*b^4 + (a^4 + 5*a^3*b)*d*x)*cosh(d*x + c)^7 - 18*(2*
a^4 - 2*a^3*b - 2*a^2*b^2 + 9*a*b^3 + 10*b^4 + (a^4 - 5*a^3*b)*d*x)*cosh(d*
x + c)^5 + 4*(2*a^4 - 30*a^3*b - 30*a^2*b^2 + 38*a*b^3 + 45*b^4 + 3*(a^4 -
5*a^3*b)*d*x)*cosh(d*x + c)^3 - (4*a^4 - 20*a^3*b - 4*a^2*b^2 + 74*a*b^3 +
60*b^4 - 3*(a^4 + 5*a^3*b)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^6 + 3*a^5
*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^10 + 10*(a^6 + 3*a^5*b + 3*a^4*b^
2 + a^3*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^6 + 3*a^5*b + 3*a^4*b^2 +
 a^3*b^3)*d*sinh(d*x + c)^10 - (a^6 + 7*a^5*b + 11*a^4*b^2 + 5*a^3*b^3)*d*c
osh(d*x + c)^8 + (45*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^
2 - (a^6 + 7*a^5*b + 11*a^4*b^2 + 5*a^3*b^3)*d)*sinh(d*x + c)^8 - 2*(a^6 -
3*a^5*b - 9*a^4*b^2 - 5*a^3*b^3)*d*cosh(d*x + c)^6 + 8*(15*(a^6 + 3*a^5*b +
 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^3 - (a^6 + 7*a^5*b + 11*a^4*b^2 + 5*a
^3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^6 + 3*a^5*b + 3*a^4*b^
2 + a^3*b^3)*d*cosh(d*x + c)^4 - 14*(a^6 + 7*a^5*b + 11*a^4*b^2 + 5*a^3*b^3
)*d*cosh(d*x + c)^2 - (a^6 - 3*a^5*b - 9*a^4*b^2 - 5*a^3*b^3)*d)*sinh(d*x +
 c)^6 + 2*(a^6 - 3*a^5*b - 9*a^4*b^2 - 5*a^3*b^3)*d*cosh(d*x + c)^4 + 4*(63
*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^5 - 14*(a^6 + 7*a^5*
b + 11*a^4*b^2 + 5*a^3*b^3)*d*cosh(d*x + c)^3 - 3*(a^6 - 3*a^5*b - 9*a^4*b^
2 - 5*a^3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^6 + 3*a^5*b + 3
*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^6 - 35*(a^6 + 7*a^5*b + 11*a^4*b^2 + 5*
a^3*b^3)*d*cosh(d*x + c)^4 - 15*(a^6 - 3*a^5*b - 9*a^4*b^2 - 5*a^3*b^3)*d*c
osh(d*x + c)^2 + (a^6 - 3*a^5*b - 9*a^4*b^2 - 5*a^3*b^3)*d)*sinh(d*x + c)^4
 + (a^6 + 7*a^5*b + 11*a^4*b^2 + 5*a^3*b^3)*d*cosh(d*x + c)^2 + 8*(15*(a^6
 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^7 - 7*(a^6 + 7*a^5*b + 11*
a^4*b^2 + 5*a^3*b^3)*d*cosh(d*x + c)^5 - 5*(a^6 - 3*a^5*b - 9*a^4*b^2 - 5*a
^3*b^3)*d*cosh(d*x + c)^3 + (a^6 - 3*a^5*b - 9*a^4*b^2 - 5*a^3*b^3)*d*cosh(
d*x + c))*sinh(d*x + c)^3 + (45*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cos
h(d*x + c)^8 - 28*(a^6 + 7*a^5*b + 11*a^4*b^2 + 5*a^3*b^3)*d*cosh(d*x + c)^
6 - 30*(a^6 - 3*a^5*b - 9*a^4*b^2 - 5*a^3*b^3)*d*cosh(d*x + c)^4 + 12*(a^6
 - 3*a^5*b - 9*a^4*b^2 - 5*a^3*b^3)*d*cosh(d*x + c)^2 + (a^6 + 7*a^5*b + 11*
a^4*b^2 + 5*a^3*b^3)*d)*sinh(d*x + c)^2 - (a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*
b^3)*d + 2*(5*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^9 - 4*(
a^6 + 7*a^5*b + 11*a^4*b^2 + 5*a^3*b^3)*d*cosh(d*x + c)^7 - 6*(a^6 - 3*a^5*
b - 9*a^4*b^2 - 5*a^3*b^3)*d*cosh(d*x + c)^5 + 4*(a^6 - 3*a^5*b - 9*a^4*b^2
 - 5*a^3*b^3)*d*cosh(d*x + c)^3 + (a^6 + 7*a^5*b + 11*a^4*b^2 + 5*a^3*b^3)*
d*cosh(d*x + c))*sinh(d*x + c)), 1/6*(6*(a^4 + a^3*b)*d*x*cosh(d*x + c)^10
 + 60*(a^4 + a^3*b)*d*x*cosh(d*x + c)*sinh(d*x + c)^9 + 6*(a^4 + a^3*b)*d*x*
sinh(d*x + c)^10 - 6*(4*a^4 + 8*a^3*b - 7*a*b^3 - 5*b^4 + (a^4 + 5*a^3*b)*d
*x)*cosh(d*x + c)^8 + 6*(45*(a^4 + a^3*b)*d*x*cosh(d*x + c)^2 - 4*a^4 - 8*a
^3*b + 7*a*b^3 + 5*b^4 - (a^4 + 5*a^3*b)*d*x)*sinh(d*x + c)^8 + 48*(15*(a^4
 + a^3*b)*d*x*cosh(d*x + c)^3 - (4*a^4 + 8*a^3*b - 7*a*b^3 - 5*b^4 + (a^4 +
 5*a^3*b)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 - 12*(2*a^4 - 2*a^3*b - 2*a^2
*b^2 + 9*a*b^3 + 10*b^4 + (a^4 - 5*a^3*b)*d*x)*cosh(d*x + c)^6 + 12*(105*(a

^4 + a^3*b)*d*x*cosh(d*x + c)^4 - 2*a^4 + 2*a^3*b + 2*a^2*b^2 - 9*a*b^3 - 1
0*b^4 - (a^4 - 5*a^3*b)*d*x - 14*(4*a^4 + 8*a^3*b - 7*a*b^3 - 5*b^4 + (a^4
+ 5*a^3*b)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 24*(63*(a^4 + a^3*b)*d*x
*cosh(d*x + c)^5 - 14*(4*a^4 + 8*a^3*b - 7*a*b^3 - 5*b^4 + (a^4 + 5*a^3*b)*
d*x)*cosh(d*x + c)^3 - 3*(2*a^4 - 2*a^3*b - 2*a^2*b^2 + 9*a*b^3 + 10*b^4 +
(a^4 - 5*a^3*b)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*(2*a^4 - 30*a^3*b -
30*a^2*b^2 + 38*a*b^3 + 45*b^4 + 3*(a^4 - 5*a^3*b)*d*x)*cosh(d*x + c)^4 +
4*(315*(a^4 + a^3*b)*d*x*cosh(d*x + c)^6 - 105*(4*a^4 + 8*a^3*b - 7*a*b^3 -
5*b^4 + (a^4 + 5*a^3*b)*d*x)*cosh(d*x + c)^4 + 2*a^4 - 30*a^3*b - 30*a^2*b
^2 + 38*a*b^3 + 45*b^4 + 3*(a^4 - 5*a^3*b)*d*x - 45*(2*a^4 - 2*a^3*b - 2*a^
2*b^2 + 9*a*b^3 + 10*b^4 + (a^4 - 5*a^3*b)*d*x)*cosh(d*x + c)^2)*sinh(d*x +
c)^4 - 16*a^4 - 24*a^3*b + 24*a^2*b^2 + 62*a*b^3 + 30*b^4 + 16*(45*(a^4 +
a^3*b)*d*x*cosh(d*x + c)^7 - 21*(4*a^4 + 8*a^3*b - 7*a*b^3 - 5*b^4 + (a^4 +
5*a^3*b)*d*x)*cosh(d*x + c)^5 - 15*(2*a^4 - 2*a^3*b - 2*a^2*b^2 + 9*a*b^3
+ 10*b^4 + (a^4 - 5*a^3*b)*d*x)*cosh(d*x + c)^3 + (2*a^4 - 30*a^3*b - 30*a^
2*b^2 + 38*a*b^3 + 45*b^4 + 3*(a^4 - 5*a^3*b)*d*x)*cosh(d*x + c))*sinh(d*x
+ c)^3 - 6*(a^4 + a^3*b)*d*x - 2*(4*a^4 - 20*a^3*b - 4*a^2*b^2 + 74*a*b^3 +
60*b^4 - 3*(a^4 + 5*a^3*b)*d*x)*cosh(d*x + c)^2 + 2*(135*(a^4 + a^3*b)*d*x
*cosh(d*x + c)^8 - 84*(4*a^4 + 8*a^3*b - 7*a*b^3 - 5*b^4 + (a^4 + 5*a^3*b)*
d*x)*cosh(d*x + c)^6 - 90*(2*a^4 - 2*a^3*b - 2*a^2*b^2 + 9*a*b^3 + 10*b^4 +
(a^4 - 5*a^3*b)*d*x)*cosh(d*x + c)^4 - 4*a^4 + 20*a^3*b + 4*a^2*b^2 - 74*a
*b^3 - 60*b^4 + 3*(a^4 + 5*a^3*b)*d*x + 12*(2*a^4 - 30*a^3*b - 30*a^2*b^2 +
38*a*b^3 + 45*b^4 + 3*(a^4 - 5*a^3*b)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^
2 + 3*((7*a^2*b^2 + 12*a*b^3 + 5*b^4)*cosh(d*x + c)^10 + 10*(7*a^2*b^2 + 12
*a*b^3 + 5*b^4)*cosh(d*x + c)*sinh(d*x + c)^9 + (7*a^2*b^2 + 12*a*b^3 + 5*b
^4)*sinh(d*x + c)^10 - (7*a^2*b^2 + 40*a*b^3 + 25*b^4)*cosh(d*x + c)^8 - (7
*a^2*b^2 + 40*a*b^3 + 25*b^4 - 45*(7*a^2*b^2 + 12*a*b^3 + 5*b^4)*cosh(d*x +
c)^2)*sinh(d*x + c)^8 + 8*(15*(7*a^2*b^2 + 12*a*b^3 + 5*b^4)*cosh(d*x + c)
^3 - (7*a^2*b^2 + 40*a*b^3 + 25*b^4)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(7*
a^2*b^2 - 30*a*b^3 - 25*b^4)*cosh(d*x + c)^6 + 2*(105*(7*a^2*b^2 + 12*a*b^3
+ 5*b^4)*cosh(d*x + c)^4 - 7*a^2*b^2 + 30*a*b^3 + 25*b^4 - 14*(7*a^2*b^2 +
40*a*b^3 + 25*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(7*a^2*b^2 + 1
2*a*b^3 + 5*b^4)*cosh(d*x + c)^5 - 14*(7*a^2*b^2 + 40*a*b^3 + 25*b^4)*cosh(
d*x + c)^3 - 3*(7*a^2*b^2 - 30*a*b^3 - 25*b^4)*cosh(d*x + c))*sinh(d*x + c)
^5 + 2*(7*a^2*b^2 - 30*a*b^3 - 25*b^4)*cosh(d*x + c)^4 + 2*(105*(7*a^2*b^2
+ 12*a*b^3 + 5*b^4)*cosh(d*x + c)^6 - 35*(7*a^2*b^2 + 40*a*b^3 + 25*b^4)*co
sh(d*x + c)^4 + 7*a^2*b^2 - 30*a*b^3 - 25*b^4 - 15*(7*a^2*b^2 - 30*a*b^3 -
25*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 7*a^2*b^2 - 12*a*b^3 - 5*b^4 + 8
(15(7*a^2*b^2 + 12*a*b^3 + 5*b^4)*cosh(d*x + c)^7 - 7*(7*a^2*b^2 + 40*a*b
^3 + 25*b^4)*cosh(d*x + c)^5 - 5*(7*a^2*b^2 - 30*a*b^3 - 25*b^4)*cosh(d*x +
c)^3 + (7*a^2*b^2 - 30*a*b^3 - 25*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + (7
*a^2*b^2 + 40*a*b^3 + 25*b^4)*cosh(d*x + c)^2 + (45*(7*a^2*b^2 + 12*a*b^3 +
5*b^4)*cosh(d*x + c)^8 - 28*(7*a^2*b^2 + 40*a*b^3 + 25*b^4)*cosh(d*x + c)^
6 - 30*(7*a^2*b^2 - 30*a*b^3 - 25*b^4)*cosh(d*x + c)^4 + 7*a^2*b^2 + 40*a*b
^3 + 25*b^4 + 12*(7*a^2*b^2 - 30*a*b^3 - 25*b^4)*cosh(d*x + c)^2)*sinh(d*x
+ c)^2 + 2*(5*(7*a^2*b^2 + 12*a*b^3 + 5*b^4)*cosh(d*x + c)^9 - 4*(7*a^2*b^2
+ 40*a*b^3 + 25*b^4)*cosh(d*x + c)^7 - 6*(7*a^2*b^2 - 30*a*b^3 - 25*b^4)*c
osh(d*x + c)^5 + 4*(7*a^2*b^2 - 30*a*b^3 - 25*b^4)*cosh(d*x + c)^3 + (7*a^2
*b^2 + 40*a*b^3 + 25*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/a)*arctan(1/
2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b
)*sinh(d*x + c)^2 + a - b)*sqrt(b/a)/b) + 4*(15*(a^4 + a^3*b)*d*x*cosh(d*x
+ c)^9 - 12*(4*a^4 + 8*a^3*b - 7*a*b^3 - 5*b^4 + (a^4 + 5*a^3*b)*d*x)*cosh(
d*x + c)^7 - 18*(2*a^4 - 2*a^3*b - 2*a^2*b^2 + 9*a*b^3 + 10*b^4 + (a^4 - 5*
a^3*b)*d*x)*cosh(d*x + c)^5 + 4*(2*a^4 - 30*a^3*b - 30*a^2*b^2 + 38*a*b^3 +
45*b^4 + 3*(a^4 - 5*a^3*b)*d*x)*cosh(d*x + c)^3 - (4*a^4 - 20*a^3*b - 4*a^
2*b^2 + 74*a*b^3 + 60*b^4 - 3*(a^4 + 5*a^3*b)*d*x)*cosh(d*x + c))*sinh(d*x
+ c))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^10 + 10*(a^6 +
3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^6 + 3*
a^5*b + 3*a^4*b^2 + a^3*b^3)*d*sinh(d*x + c)^10 - (a^6 + 7*a^5*b + 11*a^4*b

```
^2 + 5*a^3*b^3)*d*cosh(d*x + c)^8 + (45*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^
3)*d*cosh(d*x + c)^2 - (a^6 + 7*a^5*b + 11*a^4*b^2 + 5*a^3*b^3)*d)*sinh(d*x
 + c)^8 - 2*(a^6 - 3*a^5*b - 9*a^4*b^2 - 5*a^3*b^3)*d*cosh(d*x + c)^6 + 8*(
15*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^3 - (a^6 + 7*a^5*b
 + 11*a^4*b^2 + 5*a^3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^6 +
 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^4 - 14*(a^6 + 7*a^5*b + 11*
a^4*b^2 + 5*a^3*b^3)*d*cosh(d*x + c)^2 - (a^6 - 3*a^5*b - 9*a^4*b^2 - 5*a^3
*b^3)*d)*sinh(d*x + c)^6 + 2*(a^6 - 3*a^5*b - 9*a^4*b^2 - 5*a^3*b^3)*d*cosh
(d*x + c)^4 + 4*(63*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^5
 - 14*(a^6 + 7*a^5*b + 11*a^4*b^2 + 5*a^3*b^3)*d*cosh(d*x + c)^3 - 3*(a^6 -
 3*a^5*b - 9*a^4*b^2 - 5*a^3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105
*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^6 - 35*(a^6 + 7*a^5*
b + 11*a^4*b^2 + 5*a^3*b^3)*d*cosh(d*x + c)^4 - 15*(a^6 - 3*a^5*b - 9*a^4*b
^2 - 5*a^3*b^3)*d*cosh(d*x + c)^2 + (a^6 - 3*a^5*b - 9*a^4*b^2 - 5*a^3*b^3)
*d)*sinh(d*x + c)^4 + (a^6 + 7*a^5*b + 11*a^4*b^2 + 5*a^3*b^3)*d*cosh(d*x +
 c)^2 + 8*(15*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^7 - 7*(
a^6 + 7*a^5*b + 11*a^4*b^2 + 5*a^3*b^3)*d*cosh(d*x + c)^5 - 5*(a^6 - 3*a^5*
b - 9*a^4*b^2 - 5*a^3*b^3)*d*cosh(d*x + c)^3 + (a^6 - 3*a^5*b - 9*a^4*b^2 -
 5*a^3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (45*(a^6 + 3*a^5*b + 3*a^4*b
^2 + a^3*b^3)*d*cosh(d*x + c)^8 - 28*(a^6 + 7*a^5*b + 11*a^4*b^2 + 5*a^3*b^
3)*d*cosh(d*x + c)^6 - 30*(a^6 - 3*a^5*b - 9*a^4*b^2 - 5*a^3*b^3)*d*cosh(d*
x + c)^4 + 12*(a^6 - 3*a^5*b - 9*a^4*b^2 - 5*a^3*b^3)*d*cosh(d*x + c)^2 + (
a^6 + 7*a^5*b + 11*a^4*b^2 + 5*a^3*b^3)*d)*sinh(d*x + c)^2 - (a^6 + 3*a^5*b
 + 3*a^4*b^2 + a^3*b^3)*d + 2*(5*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*co
sh(d*x + c)^9 - 4*(a^6 + 7*a^5*b + 11*a^4*b^2 + 5*a^3*b^3)*d*cosh(d*x + c)^
7 - 6*(a^6 - 3*a^5*b - 9*a^4*b^2 - 5*a^3*b^3)*d*cosh(d*x + c)^5 + 4*(a^6 -
3*a^5*b - 9*a^4*b^2 - 5*a^3*b^3)*d*cosh(d*x + c)^3 + (a^6 + 7*a^5*b + 11*a^
4*b^2 + 5*a^3*b^3)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**4/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(coth(c + d*x)**4/(a + b*tanh(c + d*x)**2)**2, x)
```

Giac [B] time = 1.26889, size = 393, normalized size = 2.47

$$\frac{\left(7\,ab^3+5\,b^4\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{2\left(a^5d+2\,a^4bd+a^3b^2d\right)\sqrt{ab}}+\frac{dx+c}{a^2d+2\,abd+b^2d}-\frac{ab^3e^{(2\,dx+2\,c)}-b^4e^{(2\,dx+2\,c)}+}{\left(a^5d+2\,a^4bd+a^3b^2d\right)\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(7*a*b^3 + 5*b^4)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a
 - b)/sqrt(a*b))/((a^5*d + 2*a^4*b*d + a^3*b^2*d)*sqrt(a*b)) + (d*x + c)/(a
^2*d + 2*a*b*d + b^2*d) - (a*b^3*e^(2*d*x + 2*c) - b^4*e^(2*d*x + 2*c) + a*
b^3 + b^4)/((a^5*d + 2*a^4*b*d + a^3*b^2*d)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x
```

```
 + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)) - 4/3*(3*a*e^
(4*d*x + 4*c) - 3*b*e^(4*d*x + 4*c) - 3*a*e^(2*d*x + 2*c) + 6*b*e^(2*d*x +
2*c) + 2*a - 3*b)/(a^3*d*(e^(2*d*x + 2*c) - 1)^3)
```

3.190 $\int \frac{\tanh^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=144

$$-\frac{\sqrt{a}\left(3a^2+10ab+15b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8b^{5/2}d(a+b)^3}+\frac{a(3a+7b)\tanh(c+dx)}{8b^2d(a+b)^2\left(a+b\tanh^2(c+dx)\right)}+\frac{a\tanh^3(c+dx)}{4bd(a+b)\left(a+b\tanh^2(c+dx)\right)^2}$$

```
[Out] x/(a + b)^3 - (Sqrt[a]*(3*a^2 + 10*a*b + 15*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d
*x])/Sqrt[a]])/(8*b^(5/2)*(a + b)^3*d) + (a*Tanh[c + d*x]^3)/(4*b*(a + b)*d
*(a + b*Tanh[c + d*x]^2)^2) + (a*(3*a + 7*b)*Tanh[c + d*x])/(8*b^2*(a + b)^
2*d*(a + b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.208249, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3670, 470, 578, 522, 206, 205}

$$-\frac{\sqrt{a}\left(3a^2+10ab+15b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8b^{5/2}d(a+b)^3}+\frac{a(3a+7b)\tanh(c+dx)}{8b^2d(a+b)^2\left(a+b\tanh^2(c+dx)\right)}+\frac{a\tanh^3(c+dx)}{4bd(a+b)\left(a+b\tanh^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^6/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] x/(a + b)^3 - (Sqrt[a]*(3*a^2 + 10*a*b + 15*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d
*x])/Sqrt[a]])/(8*b^(5/2)*(a + b)^3*d) + (a*Tanh[c + d*x]^3)/(4*b*(a + b)*d
*(a + b*Tanh[c + d*x]^2)^2) + (a*(3*a + 7*b)*Tanh[c + d*x])/(8*b^2*(a + b)^
2*d*(a + b*Tanh[c + d*x]^2))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
```

```
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\tanh^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)\left(a+bx^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{a\tanh^3(c+dx)}{4b(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{x^2\left(3a+(-3a-4b)x^2\right)}{(1-x^2)\left(a+bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{4b(a+b)d}$$

$$= \frac{a\tanh^3(c+dx)}{4b(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{a(3a+7b)\tanh(c+dx)}{8b^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int$$

$$= \frac{a\tanh^3(c+dx)}{4b(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{a(3a+7b)\tanh(c+dx)}{8b^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int$$

$$= \frac{x}{(a+b)^3} - \frac{\sqrt{a}\left(3a^2+10ab+15b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8b^{5/2}(a+b)^3d} + \frac{a\tanh^3(c+dx)}{4b(a+b)d\left(a+b\tanh^2(c\right.}$$

Mathematica [A] time = 1.18233, size = 144, normalized size = 1.

$$\frac{-\frac{\sqrt{a}\left(3a^2+10ab+15b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{b^{5/2}} - \frac{4a^2(a+b)\sinh(2(c+dx))}{b((a+b)\cosh(2(c+dx))+a-b)^2} + \frac{3a(a+b)(a+3b)\sinh(2(c+dx))}{b^2((a+b)\cosh(2(c+dx))+a-b)} + 8(c+dx)}{8d(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^6/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (8*(c + d*x) - (Sqrt[a]*(3*a^2 + 10*a*b + 15*b^2)*ArcTan[(Sqrt[b]*Tanh[c +
d*x])/Sqrt[a]])/b^(5/2) - (4*a^2*(a + b)*Sinh[2*(c + d*x)])/(b*(a - b + (a
+ b)*Cosh[2*(c + d*x)])^2) + (3*a*(a + b)*(a + 3*b)*Sinh[2*(c + d*x)])/(b^2
*(a - b + (a + b)*Cosh[2*(c + d*x)])))/(8*(a + b)^3*d)
```

Maple [B] time = 0.027, size = 352, normalized size = 2.4

$$\frac{\ln(\tanh(dx+c)+1)}{2d(a+b)^3} + \frac{5a^3(\tanh(dx+c))^3}{8d(a+b)^3\left(a+b(\tanh(dx+c))^2\right)^2 b} + \frac{7a^2(\tanh(dx+c))^3}{4d(a+b)^3\left(a+b(\tanh(dx+c))^2\right)^2} + \frac{9ab(\mathrm{ta}}{8d(a+b)^3\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 1/2/d/(a+b)^3*ln(tanh(d*x+c)+1)+5/8/d/(a+b)^3*a^3/(a+b*tanh(d*x+c)^2)^2/b*t
anh(d*x+c)^3+7/4/d/(a+b)^3*a^2/(a+b*tanh(d*x+c)^2)^2*tanh(d*x+c)^3+9/8/d/(a
+b)^3*a/(a+b*tanh(d*x+c)^2)^2*b*tanh(d*x+c)^3+3/8/d/(a+b)^3*a^4/(a+b*tanh(d
*x+c)^2)^2/b^2*tanh(d*x+c)+5/4/d/(a+b)^3*a^3/(a+b*tanh(d*x+c)^2)^2/b*tanh(d
*x+c)+7/8/d/(a+b)^3*a^2/(a+b*tanh(d*x+c)^2)^2*tanh(d*x+c)-3/8/d/(a+b)^3*a^3
/b^2/(a*b)^(1/2)*arctan(tanh(d*x+c)*b/(a*b)^(1/2))-5/4/d/(a+b)^3*a^2/b/(a*b
)^(1/2)*arctan(tanh(d*x+c)*b/(a*b)^(1/2))-15/8/d/(a+b)^3*a/(a*b)^(1/2)*arct
an(tanh(d*x+c)*b/(a*b)^(1/2))-1/2/d/(a+b)^3*ln(tanh(d*x+c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.47518, size = 17204, normalized size = 119.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*(a^2*b^2 + 2*a*b^3 + b^4)*d*x*cosh(d*x + c)^8 + 128*(a^2*b^2 + 2*
a*b^3 + b^4)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 16*(a^2*b^2 + 2*a*b^3 + b^
4)*d*x*sinh(d*x + c)^8 - 4*(3*a^4 + 13*a^3*b + a^2*b^2 - 9*a*b^3 - 16*(a^2*
b^2 - b^4)*d*x)*cosh(d*x + c)^6 + 4*(112*(a^2*b^2 + 2*a*b^3 + b^4)*d*x*cosh
(d*x + c)^2 - 3*a^4 - 13*a^3*b - a^2*b^2 + 9*a*b^3 + 16*(a^2*b^2 - b^4)*d*x
)*sinh(d*x + c)^6 + 8*(112*(a^2*b^2 + 2*a*b^3 + b^4)*d*x*cosh(d*x + c)^3 -
3*(3*a^4 + 13*a^3*b + a^2*b^2 - 9*a*b^3 - 16*(a^2*b^2 - b^4)*d*x)*cosh(d*x
+ c))*sinh(d*x + c)^5 - 4*(9*a^4 + 21*a^3*b - 9*a^2*b^2 + 27*a*b^3 - 8*(3*a
^2*b^2 - 2*a*b^3 + 3*b^4)*d*x)*cosh(d*x + c)^4 + 4*(280*(a^2*b^2 + 2*a*b^3
+ b^4)*d*x*cosh(d*x + c)^4 - 9*a^4 - 21*a^3*b + 9*a^2*b^2 - 27*a*b^3 + 8*(3
```

```
*a^2*b^2 - 2*a*b^3 + 3*b^4)*d*x - 15*(3*a^4 + 13*a^3*b + a^2*b^2 - 9*a*b^3
- 16*(a^2*b^2 - b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 12*a^4 - 60*a^
3*b - 84*a^2*b^2 - 36*a*b^3 + 16*(56*(a^2*b^2 + 2*a*b^3 + b^4)*d*x*cosh(d*x
 + c)^5 - 5*(3*a^4 + 13*a^3*b + a^2*b^2 - 9*a*b^3 - 16*(a^2*b^2 - b^4)*d*x)
*cosh(d*x + c)^3 - (9*a^4 + 21*a^3*b - 9*a^2*b^2 + 27*a*b^3 - 8*(3*a^2*b^2
- 2*a*b^3 + 3*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 16*(a^2*b^2 + 2*a*
b^3 + b^4)*d*x - 4*(9*a^4 + 23*a^3*b - 13*a^2*b^2 - 27*a*b^3 - 16*(a^2*b^2
- b^4)*d*x)*cosh(d*x + c)^2 + 4*(112*(a^2*b^2 + 2*a*b^3 + b^4)*d*x*cosh(d*x
 + c)^6 - 15*(3*a^4 + 13*a^3*b + a^2*b^2 - 9*a*b^3 - 16*(a^2*b^2 - b^4)*d*x
)*cosh(d*x + c)^4 - 9*a^4 - 23*a^3*b + 13*a^2*b^2 + 27*a*b^3 + 16*(a^2*b^2
- b^4)*d*x - 6*(9*a^4 + 21*a^3*b - 9*a^2*b^2 + 27*a*b^3 - 8*(3*a^2*b^2 - 2*
a*b^3 + 3*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((3*a^4 + 16*a^3*b +
 38*a^2*b^2 + 40*a*b^3 + 15*b^4)*cosh(d*x + c)^8 + 8*(3*a^4 + 16*a^3*b + 38
*a^2*b^2 + 40*a*b^3 + 15*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^4 + 16*a
^3*b + 38*a^2*b^2 + 40*a*b^3 + 15*b^4)*sinh(d*x + c)^8 + 4*(3*a^4 + 10*a^3*
b + 12*a^2*b^2 - 10*a*b^3 - 15*b^4)*cosh(d*x + c)^6 + 4*(3*a^4 + 10*a^3*b +
 12*a^2*b^2 - 10*a*b^3 - 15*b^4 + 7*(3*a^4 + 16*a^3*b + 38*a^2*b^2 + 40*a*b
^3 + 15*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^4 + 16*a^3*b + 38
*a^2*b^2 + 40*a*b^3 + 15*b^4)*cosh(d*x + c)^3 + 3*(3*a^4 + 10*a^3*b + 12*a^
2*b^2 - 10*a*b^3 - 15*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(9*a^4 + 24*a
^3*b + 34*a^2*b^2 + 45*b^4)*cosh(d*x + c)^4 + 2*(35*(3*a^4 + 16*a^3*b + 38*
a^2*b^2 + 40*a*b^3 + 15*b^4)*cosh(d*x + c)^4 + 9*a^4 + 24*a^3*b + 34*a^2*b^
2 + 45*b^4 + 30*(3*a^4 + 10*a^3*b + 12*a^2*b^2 - 10*a*b^3 - 15*b^4)*cosh(d*
x + c)^2)*sinh(d*x + c)^4 + 3*a^4 + 16*a^3*b + 38*a^2*b^2 + 40*a*b^3 + 15*b
^4 + 8*(7*(3*a^4 + 16*a^3*b + 38*a^2*b^2 + 40*a*b^3 + 15*b^4)*cosh(d*x + c)
^5 + 10*(3*a^4 + 10*a^3*b + 12*a^2*b^2 - 10*a*b^3 - 15*b^4)*cosh(d*x + c)^3
 + (9*a^4 + 24*a^3*b + 34*a^2*b^2 + 45*b^4)*cosh(d*x + c))*sinh(d*x + c)^3
+ 4*(3*a^4 + 10*a^3*b + 12*a^2*b^2 - 10*a*b^3 - 15*b^4)*cosh(d*x + c)^2 + 4
*(7*(3*a^4 + 16*a^3*b + 38*a^2*b^2 + 40*a*b^3 + 15*b^4)*cosh(d*x + c)^6 + 1
5*(3*a^4 + 10*a^3*b + 12*a^2*b^2 - 10*a*b^3 - 15*b^4)*cosh(d*x + c)^4 + 3*a
^4 + 10*a^3*b + 12*a^2*b^2 - 10*a*b^3 - 15*b^4 + 3*(9*a^4 + 24*a^3*b + 34*a
^2*b^2 + 45*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((3*a^4 + 16*a^3*b +
38*a^2*b^2 + 40*a*b^3 + 15*b^4)*cosh(d*x + c)^7 + 3*(3*a^4 + 10*a^3*b + 12*
a^2*b^2 - 10*a*b^3 - 15*b^4)*cosh(d*x + c)^5 + (9*a^4 + 24*a^3*b + 34*a^2*b
^2 + 45*b^4)*cosh(d*x + c)^3 + (3*a^4 + 10*a^3*b + 12*a^2*b^2 - 10*a*b^3 -
15*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a/b)*log(((a^2 + 2*a*b + b^2)*c
osh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2
 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2
 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b
+ b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))
*sinh(d*x + c) - 4*((a*b + b^2)*cosh(d*x + c)^2 + 2*(a*b + b^2)*cosh(d*x +
c)*sinh(d*x + c) + (a*b + b^2)*sinh(d*x + c)^2 + a*b - b^2)*sqrt(-a/b))/((a
 + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*s
inh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 +
 a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c
))*sinh(d*x + c) + a + b)) + 8*(16*(a^2*b^2 + 2*a*b^3 + b^4)*d*x*cosh(d*x +
 c)^7 - 3*(3*a^4 + 13*a^3*b + a^2*b^2 - 9*a*b^3 - 16*(a^2*b^2 - b^4)*d*x)*c
osh(d*x + c)^5 - 2*(9*a^4 + 21*a^3*b - 9*a^2*b^2 + 27*a*b^3 - 8*(3*a^2*b^2
- 2*a*b^3 + 3*b^4)*d*x)*cosh(d*x + c)^3 - (9*a^4 + 23*a^3*b - 13*a^2*b^2 -
27*a*b^3 - 16*(a^2*b^2 - b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^5*b^2
+ 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*d*cosh(d*x + c)^8 +
8*(a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*d*cosh(d*
x + c)*sinh(d*x + c)^7 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5
*a*b^6 + b^7)*d*sinh(d*x + c)^8 + 4*(a^5*b^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^
2*b^5 - 3*a*b^6 - b^7)*d*cosh(d*x + c)^6 + 4*(7*(a^5*b^2 + 5*a^4*b^3 + 10*a
^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*d*cosh(d*x + c)^2 + (a^5*b^2 + 3*a^4*b
^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7)*d)*sinh(d*x + c)^6 + 2*(3*a^5*b
^2 + 7*a^4*b^3 + 6*a^3*b^4 + 6*a^2*b^5 + 7*a*b^6 + 3*b^7)*d*cosh(d*x + c)^4
 + 8*(7*(a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*d*c
```

osh(d*x + c)^3 + 3*(a^5*b^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*d*cosh(d*x + c)^4 + 30*(a^5*b^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7)*d*cosh(d*x + c)^2 + (3*a^5*b^2 + 7*a^4*b^3 + 6*a^3*b^4 + 6*a^2*b^5 + 7*a*b^6 + 3*b^7)*d)*sinh(d*x + c)^4 + 4*(a^5*b^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7)*d*cosh(d*x + c)^2 + 8*(7*(a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*d*cosh(d*x + c)^5 + 10*(a^5*b^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7)*d*cosh(d*x + c)^3 + (3*a^5*b^2 + 7*a^4*b^3 + 6*a^3*b^4 + 6*a^2*b^5 + 7*a*b^6 + 3*b^7)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*d*cosh(d*x + c)^6 + 15*(a^5*b^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7)*d*cosh(d*x + c)^4 + 3*(3*a^5*b^2 + 7*a^4*b^3 + 6*a^3*b^4 + 6*a^2*b^5 + 7*a*b^6 + 3*b^7)*d*cosh(d*x + c)^2 + (a^5*b^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7)*d)*sinh(d*x + c)^2 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*d + 8*((a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*d*cosh(d*x + c)^7 + 3*(a^5*b^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7)*d*cosh(d*x + c)^5 + (3*a^5*b^2 + 7*a^4*b^3 + 6*a^3*b^4 + 6*a^2*b^5 + 7*a*b^6 + 3*b^7)*d*cosh(d*x + c)^3 + (a^5*b^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7)*d*cosh(d*x + c))*sinh(d*x + c)) , 1/8*(8*(a^2*b^2 + 2*a*b^3 + b^4)*d*x*cosh(d*x + c)^8 + 64*(a^2*b^2 + 2*a*b^3 + b^4)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 8*(a^2*b^2 + 2*a*b^3 + b^4)*d*x*sinh(d*x + c)^8 - 2*(3*a^4 + 13*a^3*b + a^2*b^2 - 9*a*b^3 - 16*(a^2*b^2 - b^4)*d*x)*cosh(d*x + c)^6 + 2*(112*(a^2*b^2 + 2*a*b^3 + b^4)*d*x*cosh(d*x + c)^2 - 3*a^4 - 13*a^3*b - a^2*b^2 + 9*a*b^3 + 16*(a^2*b^2 - b^4)*d*x)*sinh(d*x + c)^6 + 4*(112*(a^2*b^2 + 2*a*b^3 + b^4)*d*x*cosh(d*x + c)^3 - 3*(3*a^4 + 13*a^3*b + a^2*b^2 - 9*a*b^3 - 16*(a^2*b^2 - b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(9*a^4 + 21*a^3*b - 9*a^2*b^2 + 27*a*b^3 - 8*(3*a^2*b^2 - 2*a*b^3 + 3*b^4)*d*x)*cosh(d*x + c)^4 + 2*(280*(a^2*b^2 + 2*a*b^3 + b^4)*d*x*cosh(d*x + c)^4 - 9*a^4 - 21*a^3*b + 9*a^2*b^2 - 27*a*b^3 + 8*(3*a^2*b^2 - 2*a*b^3 + 3*b^4)*d*x - 15*(3*a^4 + 13*a^3*b + a^2*b^2 - 9*a*b^3 - 16*(a^2*b^2 - b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 6*a^4 - 30*a^3*b - 42*a^2*b^2 - 18*a*b^3 + 8*(56*(a^2*b^2 + 2*a*b^3 + b^4)*d*x*cosh(d*x + c)^5 - 5*(3*a^4 + 13*a^3*b + a^2*b^2 - 9*a*b^3 - 16*(a^2*b^2 - b^4)*d*x)*cosh(d*x + c)^3 - (9*a^4 + 21*a^3*b - 9*a^2*b^2 + 27*a*b^3 - 8*(3*a^2*b^2 - 2*a*b^3 + 3*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 8*(a^2*b^2 + 2*a*b^3 + b^4)*d*x - 2*(9*a^4 + 23*a^3*b - 13*a^2*b^2 - 27*a*b^3 - 16*(a^2*b^2 - b^4)*d*x)*cosh(d*x + c)^2 + 2*(112*(a^2*b^2 + 2*a*b^3 + b^4)*d*x*cosh(d*x + c)^6 - 15*(3*a^4 + 13*a^3*b + a^2*b^2 - 9*a*b^3 - 16*(a^2*b^2 - b^4)*d*x)*cosh(d*x + c)^4 - 9*a^4 - 23*a^3*b + 13*a^2*b^2 + 27*a*b^3 + 16*(a^2*b^2 - b^4)*d*x - 6*(9*a^4 + 21*a^3*b - 9*a^2*b^2 + 27*a*b^3 - 8*(3*a^2*b^2 - 2*a*b^3 + 3*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((3*a^4 + 16*a^3*b + 38*a^2*b^2 + 40*a*b^3 + 15*b^4)*cosh(d*x + c)^8 + 8*(3*a^4 + 16*a^3*b + 38*a^2*b^2 + 40*a*b^3 + 15*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^4 + 16*a^3*b + 38*a^2*b^2 + 40*a*b^3 + 15*b^4)*sinh(d*x + c)^8 + 4*(3*a^4 + 10*a^3*b + 12*a^2*b^2 - 10*a*b^3 - 15*b^4)*cosh(d*x + c)^6 + 4*(3*a^4 + 10*a^3*b + 12*a^2*b^2 - 10*a*b^3 - 15*b^4 + 7*(3*a^4 + 16*a^3*b + 38*a^2*b^2 + 40*a*b^3 + 15*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^4 + 16*a^3*b + 38*a^2*b^2 + 40*a*b^3 + 15*b^4)*cosh(d*x + c)^3 + 3*(3*a^4 + 10*a^3*b + 12*a^2*b^2 - 10*a*b^3 - 15*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(9*a^4 + 24*a^3*b + 34*a^2*b^2 + 45*b^4)*cosh(d*x + c)^4 + 2*(35*(3*a^4 + 16*a^3*b + 38*a^2*b^2 + 40*a*b^3 + 15*b^4)*cosh(d*x + c)^4 + 9*a^4 + 24*a^3*b + 34*a^2*b^2 + 45*b^4 + 30*(3*a^4 + 10*a^3*b + 12*a^2*b^2 - 10*a*b^3 - 15*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 3*a^4 + 16*a^3*b + 38*a^2*b^2 + 40*a*b^3 + 15*b^4 + 8*(7*(3*a^4 + 16*a^3*b + 38*a^2*b^2 + 40*a*b^3 + 15*b^4)*cosh(d*x + c)^5 + 10*(3*a^4 + 10*a^3*b + 12*a^2*b^2 - 10*a*b^3 - 15*b^4)*cosh(d*x + c)^3 + (9*a^4 + 24*a^3*b + 34*a^2*b^2 + 45*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(3*a^4 + 10*a^3*b + 12*a^2*b^2 - 10*a*b^3 - 15*b^4)*cosh(d*x + c)^2 + 4*(7*(3*a^4 + 16*a^3*b + 38*a^2*b^2 + 40*a*b^3 + 15*b^4)*cosh(d*x + c)^6 + 15*(3*a

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^4 + 10*a^3*b + 12*a^2*b^2 - 10*a*b^3 - 15*b^4)*cosh(d*x + c)^4 + 3*a^4 + 1
0*a^3*b + 12*a^2*b^2 - 10*a*b^3 - 15*b^4 + 3*(9*a^4 + 24*a^3*b + 34*a^2*b^2
 + 45*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((3*a^4 + 16*a^3*b + 38*a^2
*b^2 + 40*a*b^3 + 15*b^4)*cosh(d*x + c)^7 + 3*(3*a^4 + 10*a^3*b + 12*a^2*b^
2 - 10*a*b^3 - 15*b^4)*cosh(d*x + c)^5 + (9*a^4 + 24*a^3*b + 34*a^2*b^2 + 4
5*b^4)*cosh(d*x + c)^3 + (3*a^4 + 10*a^3*b + 12*a^2*b^2 - 10*a*b^3 - 15*b^4
)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a/b)*arctan(1/2*((a + b)*cosh(d*x + c)
^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a -
b)*sqrt(a/b)/a) + 4*(16*(a^2*b^2 + 2*a*b^3 + b^4)*d*x*cosh(d*x + c)^7 - 3*(
3*a^4 + 13*a^3*b + a^2*b^2 - 9*a*b^3 - 16*(a^2*b^2 - b^4)*d*x)*cosh(d*x + c
)^5 - 2*(9*a^4 + 21*a^3*b - 9*a^2*b^2 + 27*a*b^3 - 8*(3*a^2*b^2 - 2*a*b^3 +
 3*b^4)*d*x)*cosh(d*x + c)^3 - (9*a^4 + 23*a^3*b - 13*a^2*b^2 - 27*a*b^3 -
16*(a^2*b^2 - b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^5*b^2 + 5*a^4*b^3
 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*d*cosh(d*x + c)^8 + 8*(a^5*b^2
+ 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*d*cosh(d*x + c)*sinh
(d*x + c)^7 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^
7)*d*sinh(d*x + c)^8 + 4*(a^5*b^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a
*b^6 - b^7)*d*cosh(d*x + c)^6 + 4*(7*(a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10
*a^2*b^5 + 5*a*b^6 + b^7)*d*cosh(d*x + c)^2 + (a^5*b^2 + 3*a^4*b^3 + 2*a^3*
b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7)*d)*sinh(d*x + c)^6 + 2*(3*a^5*b^2 + 7*a^4*
b^3 + 6*a^3*b^4 + 6*a^2*b^5 + 7*a*b^6 + 3*b^7)*d*cosh(d*x + c)^4 + 8*(7*(a^
5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*d*cosh(d*x + c
)^3 + 3*(a^5*b^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7)*d*cos
h(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*
a^2*b^5 + 5*a*b^6 + b^7)*d*cosh(d*x + c)^4 + 30*(a^5*b^2 + 3*a^4*b^3 + 2*a^
3*b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7)*d*cosh(d*x + c)^2 + (3*a^5*b^2 + 7*a^4*b
^3 + 6*a^3*b^4 + 6*a^2*b^5 + 7*a*b^6 + 3*b^7)*d)*sinh(d*x + c)^4 + 4*(a^5*b
^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7)*d*cosh(d*x + c)^2 +
 8*(7*(a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*d*cos
h(d*x + c)^5 + 10*(a^5*b^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^6 -
b^7)*d*cosh(d*x + c)^3 + (3*a^5*b^2 + 7*a^4*b^3 + 6*a^3*b^4 + 6*a^2*b^5 + 7
*a*b^6 + 3*b^7)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^5*b^2 + 5*a^4*b^
3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*d*cosh(d*x + c)^6 + 15*(a^5*b^
2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7)*d*cosh(d*x + c)^4 +
3*(3*a^5*b^2 + 7*a^4*b^3 + 6*a^3*b^4 + 6*a^2*b^5 + 7*a*b^6 + 3*b^7)*d*cosh(
d*x + c)^2 + (a^5*b^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7)*
d)*sinh(d*x + c)^2 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b
^6 + b^7)*d + 8*((a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 +
 b^7)*d*cosh(d*x + c)^7 + 3*(a^5*b^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 -
3*a*b^6 - b^7)*d*cosh(d*x + c)^5 + (3*a^5*b^2 + 7*a^4*b^3 + 6*a^3*b^4 + 6*a
^2*b^5 + 7*a*b^6 + 3*b^7)*d*cosh(d*x + c)^3 + (a^5*b^2 + 3*a^4*b^3 + 2*a^3*
b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**6/(a+b*tanh(d*x+c)**2)**3,x)

[Out] Timed out
```

Giac [B] time = 1.26636, size = 566, normalized size = 3.93

$$-\frac{\left(3\,a^3+10\,a^2b+15\,ab^2\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{8\,\left(a^3b^2d+3\,a^2b^3d+3\,ab^4d+b^5d\right)\sqrt{ab}}+\frac{dx+c}{a^3d+3\,a^2bd+3\,ab^2d+b^3d}-\frac{3\,a^4e^{(6\,dx+6\,c)}+13\,a^3be^{(6\,dx+6\,c)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*a^3 + 10*a^2*b + 15*a*b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d
*x + 2*c) + a - b)/sqrt(a*b))/((a^3*b^2*d + 3*a^2*b^3*d + 3*a*b^4*d + b^5*d
)*sqrt(a*b)) + (d*x + c)/(a^3*d + 3*a^2*b*d + 3*a*b^2*d + b^3*d) - 1/4*(3*a
^4*e^(6*d*x + 6*c) + 13*a^3*b*e^(6*d*x + 6*c) + a^2*b^2*e^(6*d*x + 6*c) - 9
*a*b^3*e^(6*d*x + 6*c) + 9*a^4*e^(4*d*x + 4*c) + 21*a^3*b*e^(4*d*x + 4*c) -
 9*a^2*b^2*e^(4*d*x + 4*c) + 27*a*b^3*e^(4*d*x + 4*c) + 9*a^4*e^(2*d*x + 2*
c) + 23*a^3*b*e^(2*d*x + 2*c) - 13*a^2*b^2*e^(2*d*x + 2*c) - 27*a*b^3*e^(2*
d*x + 2*c) + 3*a^4 + 15*a^3*b + 21*a^2*b^2 + 9*a*b^3)/((a^3*b^2*d + 3*a^2*b
^3*d + 3*a*b^4*d + b^5*d)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2
*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2)
```

3.191 $\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=109

$$-\frac{a^2}{4b^2d(a+b)\left(a+b\tanh^2(c+dx)\right)^2}+\frac{a(a+2b)}{2b^2d(a+b)^2\left(a+b\tanh^2(c+dx)\right)}+\frac{\log\left(a+b\tanh^2(c+dx)\right)}{2d(a+b)^3}+\frac{\log(\cosh(c}{d(a+b}$$

```
[Out] Log[Cosh[c + d*x]]/((a + b)^3*d) + Log[a + b*Tanh[c + d*x]^2]/(2*(a + b)^3*
d) - a^2/(4*b^2*(a + b)*d*(a + b*Tanh[c + d*x]^2)^2) + (a*(a + 2*b))/(2*b^2
*(a + b)^2*d*(a + b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.17042, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 446, 88}

$$-\frac{a^2}{4b^2d(a+b)\left(a+b\tanh^2(c+dx)\right)^2}+\frac{a(a+2b)}{2b^2d(a+b)^2\left(a+b\tanh^2(c+dx)\right)}+\frac{\log\left(a+b\tanh^2(c+dx)\right)}{2d(a+b)^3}+\frac{\log(\cosh(c}{d(a+b}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] Log[Cosh[c + d*x]]/((a + b)^3*d) + Log[a + b*Tanh[c + d*x]^2]/(2*(a + b)^3*
d) - a^2/(4*b^2*(a + b)*d*(a + b*Tanh[c + d*x]^2)^2) + (a*(a + 2*b))/(2*b^2
*(a + b)^2*d*(a + b*Tanh[c + d*x]^2))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(1-x^2)\left(a+bx^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1-x)(a+bx)^3}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^3(-1+x)} + \frac{a^2}{b(a+b)(a+bx)^3} - \frac{a(a+2b)}{b(a+b)^2(a+bx)^2} + \frac{b}{(a+b)^3(a+bx)}\right) dx, x, \tanh^2(c+dx)\right)}{2d}$$

$$= \frac{\log(\cosh(c+dx))}{(a+b)^3 d} + \frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)^3 d} - \frac{a^2}{4b^2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \ldots$$

Mathematica [A] time = 0.9841, size = 91, normalized size = 0.83

$$-\frac{\frac{a^2(a+b)^2}{b^2\left(a+b\tanh^2(c+dx)\right)^2} - \frac{2a(a+2b)(a+b)}{b^2\left(a+b\tanh^2(c+dx)\right)} - 2\log\left(a+b\tanh^2(c+dx)\right) - 4\log(\cosh(c+dx))}{4d(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -(-4*Log[Cosh[c + d*x]] - 2*Log[a + b*Tanh[c + d*x]^2] + (a^2*(a + b)^2)/(b
^2*(a + b*Tanh[c + d*x]^2)^2) - (2*a*(a + b)*(a + 2*b))/(b^2*(a + b*Tanh[c
+ d*x]^2)))/(4*(a + b)^3*d)
```

Maple [B] time = 0.027, size = 234, normalized size = 2.2

$$-\frac{\ln(\tanh(dx+c)+1)}{2d(a+b)^3} + \frac{a^3}{2d(a+b)^3 b^2\left(a+b(\tanh(dx+c))^2\right)} + \frac{3a^2}{2d(a+b)^3 b\left(a+b(\tanh(dx+c))^2\right)} + \frac{\ldots}{d(a+b)^3\left(a \ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] -1/2/d/(a+b)^3*ln(tanh(d*x+c)+1)+1/2/d/(a+b)^3*a^3/b^2/(a+b*tanh(d*x+c)^2)+
3/2/d/(a+b)^3*a^2/b/(a+b*tanh(d*x+c)^2)+1/d/(a+b)^3*a/(a+b*tanh(d*x+c)^2)-1
/4/d/(a+b)^3*a^4/b^2/(a+b*tanh(d*x+c)^2)^2-1/2/d/(a+b)^3*a^3/b/(a+b*tanh(d*
x+c)^2)^2-1/4/d/(a+b)^3*a^2/(a+b*tanh(d*x+c)^2)^2+1/2*ln(a+b*tanh(d*x+c)^2)
/(a+b)^3/d-1/2/d/(a+b)^3*ln(tanh(d*x+c)-1)
```

Maxima [B] time = 1.24097, size = 508, normalized size = 4.66

$$\frac{dx+c}{\left(a^3+3a^2b+3ab^2+b^3\right)d} + \frac{\ldots}{\left(a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5+4\left(a^5+3a^4b+2a^3b^2-2a^2b^3-3ab^4-b^5\right)e^{\ldots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] (d*x + c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + 4*((a^2 + a*b)*e^(-2*d*x -
2*c) + (a^2 - 2*a*b)*e^(-4*d*x - 4*c) + (a^2 + a*b)*e^(-6*d*x - 6*c))/((a^5
 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5 + 4*(a^5 + 3*a^4*b + 2
*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*e^(-2*d*x - 2*c) + 2*(3*a^5 + 7*a^4*b
 + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*e^(-4*d*x - 4*c) + 4*(a^5 + 3*a
^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*e^(-6*d*x - 6*c) + (a^5 + 5*a
^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*e^(-8*d*x - 8*c))*d) + 1/2*
log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^3 +
3*a^2*b + 3*a*b^2 + b^3)*d)
```

Fricas [B] time = 2.45023, size = 6044, normalized size = 55.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^8 + 16*(a^2 + 2*a*b + b^2)*d*
x*cosh(d*x + c)*sinh(d*x + c)^7 + 2*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^8
 + 8*((a^2 - b^2)*d*x - a^2 - a*b)*cosh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^
2)*d*x*cosh(d*x + c)^2 + (a^2 - b^2)*d*x - a^2 - a*b)*sinh(d*x + c)^6 + 16*
(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 + 3*((a^2 - b^2)*d*x - a^2 - a*b
)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*((3*a^2 - 2*a*b + 3*b^2)*d*x - 2*a^2 +
 4*a*b)*cosh(d*x + c)^4 + 4*(35*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + (
3*a^2 - 2*a*b + 3*b^2)*d*x + 30*((a^2 - b^2)*d*x - a^2 - a*b)*cosh(d*x + c)
^2 - 2*a^2 + 4*a*b)*sinh(d*x + c)^4 + 16*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*
x + c)^5 + 10*((a^2 - b^2)*d*x - a^2 - a*b)*cosh(d*x + c)^3 + ((3*a^2 - 2*a
*b + 3*b^2)*d*x - 2*a^2 + 4*a*b)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(a^2 +
2*a*b + b^2)*d*x + 8*((a^2 - b^2)*d*x - a^2 - a*b)*cosh(d*x + c)^2 + 8*(7*(
a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^6 + 15*((a^2 - b^2)*d*x - a^2 - a*b)*c
osh(d*x + c)^4 + (a^2 - b^2)*d*x + 3*((3*a^2 - 2*a*b + 3*b^2)*d*x - 2*a^2 +
 4*a*b)*cosh(d*x + c)^2 - a^2 - a*b)*sinh(d*x + c)^2 - ((a^2 + 2*a*b + b^2)
*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a
^2 + 2*a*b + b^2)*sinh(d*x + c)^8 + 4*(a^2 - b^2)*cosh(d*x + c)^6 + 4*(7*(a
^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^6 + 8*(7*(a^2
+ 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c)
^5 + 2*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*
cosh(d*x + c)^4 + 30*(a^2 - b^2)*cosh(d*x + c)^2 + 3*a^2 - 2*a*b + 3*b^2)*s
inh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 10*(a^2 - b^2)*
cosh(d*x + c)^3 + (3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 +
4*(a^2 - b^2)*cosh(d*x + c)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 +
15*(a^2 - b^2)*cosh(d*x + c)^4 + 3*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^2
+ a^2 - b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*c
osh(d*x + c)^7 + 3*(a^2 - b^2)*cosh(d*x + c)^5 + (3*a^2 - 2*a*b + 3*b^2)*co
sh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*((a + b)*co
sh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(
d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 16*((a^2 + 2*a*b + b^2)*d*x*co
sh(d*x + c)^7 + 3*((a^2 - b^2)*d*x - a^2 - a*b)*cosh(d*x + c)^5 + ((3*a^2 -
 2*a*b + 3*b^2)*d*x - 2*a^2 + 4*a*b)*cosh(d*x + c)^3 + ((a^2 - b^2)*d*x - a
^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a
^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^8 + 8*(a^5 + 5*a^4*b + 10*a^3*b^2 +
 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^5 + 5*a^4
*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*sinh(d*x + c)^8 + 4*(a^5 +
3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^6 + 4*(7*(
a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^2
+ (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d)*sinh(d*x + c)^
```

```
6 + 2*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d*cosh(d*
x + c)^4 + 8*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d
*cosh(d*x + c)^3 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5
)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10
*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^4 + 30*(a^5 + 3*a^4*b + 2*a^3*b^2
 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^2 + (3*a^5 + 7*a^4*b + 6*a^3*
b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d)*sinh(d*x + c)^4 + 4*(a^5 + 3*a^4*b +
2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^5 + 5*a^
4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^5 + 10*(a^5
+ 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^3 + (3*a
^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d*cosh(d*x + c))*si
nh(d*x + c)^3 + 4*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b
^5)*d*cosh(d*x + c)^6 + 15*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4
 - b^5)*d*cosh(d*x + c)^4 + 3*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*
a*b^4 + 3*b^5)*d*cosh(d*x + c)^2 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 -
 3*a*b^4 - b^5)*d)*sinh(d*x + c)^2 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b
^3 + 5*a*b^4 + b^5)*d + 8*((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b
^4 + b^5)*d*cosh(d*x + c)^7 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*
a*b^4 - b^5)*d*cosh(d*x + c)^5 + (3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 +
 7*a*b^4 + 3*b^5)*d*cosh(d*x + c)^3 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^
3 - 3*a*b^4 - b^5)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**5/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26539, size = 339, normalized size = 3.11

$$\frac{\log\left(\left|a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a-2\,b\right|\right)}{2\left(a^3d+3\,a^2bd+3\,ab^2d+b^3d\right)}-\frac{3\,a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)^2+3\,b\left(e^{(2\,dx+2\,c)}+e^{(-2\,c}\right.}{4\left(a^2d+2\,abd+b^2d\right)\left(a\left(e^{(2\,dx}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*log(abs(a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c) + e
^(-2*d*x - 2*c)) + 2*a - 2*b))/(a^3*d + 3*a^2*b*d + 3*a*b^2*d + b^3*d) - 1/
4*(3*a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + 3*b*(e^(2*d*x + 2*c) + e^(-
2*d*x - 2*c))^2 - 4*a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) - 12*b*(e^(2*d*x
 + 2*c) + e^(-2*d*x - 2*c)) - 4*a + 12*b)/((a^2*d + 2*a*b*d + b^2*d)*(a*(e^
(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))
+ 2*a - 2*b)^2)
```

3.192 $\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=137

$$-\frac{\left(a^2+6ab-3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}b^{3/2}d(a+b)^3}-\frac{(a+5b)\tanh(c+dx)}{8bd(a+b)^2\left(a+b\tanh^2(c+dx)\right)}+\frac{a\tanh(c+dx)}{4bd(a+b)\left(a+b\tanh^2(c+dx)\right)^2}+\frac{}{(a}$$

```
[Out] x/(a + b)^3 - ((a^2 + 6*a*b - 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]
])/(8*Sqrt[a]*b^(3/2)*(a + b)^3*d) + (a*Tanh[c + d*x])/(4*b*(a + b)*d*(a +
b*Tanh[c + d*x]^2)^2) - ((a + 5*b)*Tanh[c + d*x])/(8*b*(a + b)^2*d*(a + b*T
anh[c + d*x]^2))
```

Rubi [A] time = 0.195489, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3670, 470, 527, 522, 206, 205}

$$-\frac{\left(a^2+6ab-3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}b^{3/2}d(a+b)^3}-\frac{(a+5b)\tanh(c+dx)}{8bd(a+b)^2\left(a+b\tanh^2(c+dx)\right)}+\frac{a\tanh(c+dx)}{4bd(a+b)\left(a+b\tanh^2(c+dx)\right)^2}+\frac{}{(a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] x/(a + b)^3 - ((a^2 + 6*a*b - 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]
])/(8*Sqrt[a]*b^(3/2)*(a + b)^3*d) + (a*Tanh[c + d*x])/(4*b*(a + b)*d*(a +
b*Tanh[c + d*x]^2)^2) - ((a + 5*b)*Tanh[c + d*x])/(8*b*(a + b)^2*d*(a + b*T
anh[c + d*x]^2))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)\left(a+bx^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{a\tanh(c+dx)}{4b(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{a+(-a-4b)x^2}{(1-x^2)\left(a+bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{4b(a+b)d}$$

$$= \frac{a\tanh(c+dx)}{4b(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(a+5b)\tanh(c+dx)}{8b(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{-a(a-}{(1-}\right.}{}$$

$$= \frac{a\tanh(c+dx)}{4b(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(a+5b)\tanh(c+dx)}{8b(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\right.}{(a}$$

$$= \frac{x}{(a+b)^3} - \frac{\left(a^2+6ab-3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}b^{3/2}(a+b)^3d} + \frac{a\tanh(c+dx)}{4b(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} -$$

Mathematica [A] time = 1.06016, size = 135, normalized size = 0.99

$$\frac{-\frac{\left(a^2+6ab-3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}} + \frac{(a-5b)(a+b)\sinh(2(c+dx))}{b((a+b)\cosh(2(c+dx))+a-b)} + \frac{4a(a+b)\sinh(2(c+dx))}{((a+b)\cosh(2(c+dx))+a-b)^2} + 8(c+dx)}{8d(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (8*(c + d*x) - ((a^2 + 6*a*b - 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a
]])/(Sqrt[a]*b^(3/2)) + (4*a*(a + b)*Sinh[2*(c + d*x)])/(a - b + (a + b)*Co
```

```
sh[2*(c + d*x)])^2 + ((a - 5*b)*(a + b)*Sinh[2*(c + d*x)])/(b*(a - b + (a +
 b)*Cosh[2*(c + d*x)]))))/(8*(a + b)^3*d)
```

Maple [B] time = 0.029, size = 340, normalized size = 2.5

$$\frac{\ln(\tanh(dx+c)+1)}{2d(a+b)^3} - \frac{a^2(\tanh(dx+c))^3}{8d(a+b)^3\left(a+b(\tanh(dx+c))^2\right)^2} - \frac{3ab(\tanh(dx+c))^3}{4d(a+b)^3\left(a+b(\tanh(dx+c))^2\right)^2} - \frac{5(\tan}{8d(a+b)^3(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 1/2/d/(a+b)^3*ln(tanh(d*x+c)+1)-1/8/d/(a+b)^3*a^2/(a+b*tanh(d*x+c)^2)^2*tan
h(d*x+c)^3-3/4/d/(a+b)^3*a/(a+b*tanh(d*x+c)^2)^2*b*tanh(d*x+c)^3-5/8/d/(a+b
)^3/(a+b*tanh(d*x+c)^2)^2*tanh(d*x+c)^3*b^2+1/8/d/(a+b)^3*a^3/(a+b*tanh(d*x
+c)^2)^2/b*tanh(d*x+c)-1/4/d/(a+b)^3*a^2/(a+b*tanh(d*x+c)^2)^2*tanh(d*x+c)-
3/8/d/(a+b)^3/(a+b*tanh(d*x+c)^2)^2*a*b*tanh(d*x+c)-1/8/d/(a+b)^3*a^2/b/(a*
b)^(1/2)*arctan(tanh(d*x+c)*b/(a*b)^(1/2))-3/4/d/(a+b)^3*a/(a*b)^(1/2)*arct
an(tanh(d*x+c)*b/(a*b)^(1/2))+3/8/d/(a+b)^3*b/(a*b)^(1/2)*arctan(tanh(d*x+c
)*b/(a*b)^(1/2))-1/2/d/(a+b)^3*ln(tanh(d*x+c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.19345, size = 17383, normalized size = 126.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*x*cosh(d*x + c)^8 + 128*(a^3*b^2
+ 2*a^2*b^3 + a*b^4)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 16*(a^3*b^2 + 2*a^
2*b^3 + a*b^4)*d*x*sinh(d*x + c)^8 - 4*(a^4*b - 9*a^3*b^2 - 5*a^2*b^3 + 5*a
*b^4 - 16*(a^3*b^2 - a*b^4)*d*x)*cosh(d*x + c)^6 - 4*(a^4*b - 9*a^3*b^2 - 5
*a^2*b^3 + 5*a*b^4 - 112*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*x*cosh(d*x + c)^2
- 16*(a^3*b^2 - a*b^4)*d*x)*sinh(d*x + c)^6 + 8*(112*(a^3*b^2 + 2*a^2*b^3 +
 a*b^4)*d*x*cosh(d*x + c)^3 - 3*(a^4*b - 9*a^3*b^2 - 5*a^2*b^3 + 5*a*b^4 -
16*(a^3*b^2 - a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 4*a^4*b + 12*a^3
*b^2 + 36*a^2*b^3 + 20*a*b^4 - 4*(3*a^4*b - 17*a^3*b^2 + 13*a^2*b^3 - 15*a*
b^4 - 8*(3*a^3*b^2 - 2*a^2*b^3 + 3*a*b^4)*d*x)*cosh(d*x + c)^4 + 4*(280*(a^
3*b^2 + 2*a^2*b^3 + a*b^4)*d*x*cosh(d*x + c)^4 - 3*a^4*b + 17*a^3*b^2 - 13*
a^2*b^3 + 15*a*b^4 + 8*(3*a^3*b^2 - 2*a^2*b^3 + 3*a*b^4)*d*x - 15*(a^4*b -
```

9*a^3*b^2 - 5*a^2*b^3 + 5*a*b^4 - 16*(a^3*b^2 - a*b^4)*d*x)*cosh(d*x + c)^2
)*sinh(d*x + c)^4 + 16*(56*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*x*cosh(d*x + c)^
5 - 5*(a^4*b - 9*a^3*b^2 - 5*a^2*b^3 + 5*a*b^4 - 16*(a^3*b^2 - a*b^4)*d*x)*
cosh(d*x + c)^3 - (3*a^4*b - 17*a^3*b^2 + 13*a^2*b^3 - 15*a*b^4 - 8*(3*a^3*
b^2 - 2*a^2*b^3 + 3*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 16*(a^3*b^
2 + 2*a^2*b^3 + a*b^4)*d*x - 4*(3*a^4*b - 11*a^3*b^2 + a^2*b^3 + 15*a*b^4 -
16*(a^3*b^2 - a*b^4)*d*x)*cosh(d*x + c)^2 + 4*(112*(a^3*b^2 + 2*a^2*b^3 +
a*b^4)*d*x*cosh(d*x + c)^6 - 3*a^4*b + 11*a^3*b^2 - a^2*b^3 - 15*a*b^4 - 15
*(a^4*b - 9*a^3*b^2 - 5*a^2*b^3 + 5*a*b^4 - 16*(a^3*b^2 - a*b^4)*d*x)*cosh(
d*x + c)^4 + 16*(a^3*b^2 - a*b^4)*d*x - 6*(3*a^4*b - 17*a^3*b^2 + 13*a^2*b^
3 - 15*a*b^4 - 8*(3*a^3*b^2 - 2*a^2*b^3 + 3*a*b^4)*d*x)*cosh(d*x + c)^2)*si
nh(d*x + c)^2 + ((a^4 + 8*a^3*b + 10*a^2*b^2 - 3*b^4)*cosh(d*x + c)^8 + 8*(
a^4 + 8*a^3*b + 10*a^2*b^2 - 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^4 +
8*a^3*b + 10*a^2*b^2 - 3*b^4)*sinh(d*x + c)^8 + 4*(a^4 + 6*a^3*b - 4*a^2*b^
2 - 6*a*b^3 + 3*b^4)*cosh(d*x + c)^6 + 4*(a^4 + 6*a^3*b - 4*a^2*b^2 - 6*a*b
^3 + 3*b^4 + 7*(a^4 + 8*a^3*b + 10*a^2*b^2 - 3*b^4)*cosh(d*x + c)^2)*sinh(d
x + c)^6 + 8(7*(a^4 + 8*a^3*b + 10*a^2*b^2 - 3*b^4)*cosh(d*x + c)^3 + 3*(
a^4 + 6*a^3*b - 4*a^2*b^2 - 6*a*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^5
+ 2*(3*a^4 + 16*a^3*b - 18*a^2*b^2 + 24*a*b^3 - 9*b^4)*cosh(d*x + c)^4 + 2
(35(a^4 + 8*a^3*b + 10*a^2*b^2 - 3*b^4)*cosh(d*x + c)^4 + 3*a^4 + 16*a^3*
b - 18*a^2*b^2 + 24*a*b^3 - 9*b^4 + 30*(a^4 + 6*a^3*b - 4*a^2*b^2 - 6*a*b^3
+ 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + a^4 + 8*a^3*b + 10*a^2*b^2 - 3
b^4 + 8(7*(a^4 + 8*a^3*b + 10*a^2*b^2 - 3*b^4)*cosh(d*x + c)^5 + 10*(a^4
+ 6*a^3*b - 4*a^2*b^2 - 6*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + (3*a^4 + 16*a^3*
b - 18*a^2*b^2 + 24*a*b^3 - 9*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^4
+ 6*a^3*b - 4*a^2*b^2 - 6*a*b^3 + 3*b^4)*cosh(d*x + c)^2 + 4*(7*(a^4 + 8*a^
3*b + 10*a^2*b^2 - 3*b^4)*cosh(d*x + c)^6 + 15*(a^4 + 6*a^3*b - 4*a^2*b^2 -
6*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + a^4 + 6*a^3*b - 4*a^2*b^2 - 6*a*b^3 + 3
b^4 + 3(3*a^4 + 16*a^3*b - 18*a^2*b^2 + 24*a*b^3 - 9*b^4)*cosh(d*x + c)^2
)*sinh(d*x + c)^2 + 8*((a^4 + 8*a^3*b + 10*a^2*b^2 - 3*b^4)*cosh(d*x + c)^7
+ 3*(a^4 + 6*a^3*b - 4*a^2*b^2 - 6*a*b^3 + 3*b^4)*cosh(d*x + c)^5 + (3*a^4
+ 16*a^3*b - 18*a^2*b^2 + 24*a*b^3 - 9*b^4)*cosh(d*x + c)^3 + (a^4 + 6*a^3
*b - 4*a^2*b^2 - 6*a*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b)*
log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x +
c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*c
osh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh
(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 +
(a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(
a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(
-a*b))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 +
(a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*
x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*co
sh(d*x + c))*sinh(d*x + c) + a + b)) + 8*(16*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*
d*x*cosh(d*x + c)^7 - 3*(a^4*b - 9*a^3*b^2 - 5*a^2*b^3 + 5*a*b^4 - 16*(a^3*
b^2 - a*b^4)*d*x)*cosh(d*x + c)^5 - 2*(3*a^4*b - 17*a^3*b^2 + 13*a^2*b^3 -
15*a*b^4 - 8*(3*a^3*b^2 - 2*a^2*b^3 + 3*a*b^4)*d*x)*cosh(d*x + c)^3 - (3*a^
4*b - 11*a^3*b^2 + a^2*b^3 + 15*a*b^4 - 16*(a^3*b^2 - a*b^4)*d*x)*cosh(d*x
+ c))*sinh(d*x + c))/((a^6*b^2 + 5*a^5*b^3 + 10*a^4*b^4 + 10*a^3*b^5 + 5*a^
2*b^6 + a*b^7)*d*cosh(d*x + c)^8 + 8*(a^6*b^2 + 5*a^5*b^3 + 10*a^4*b^4 + 10
*a^3*b^5 + 5*a^2*b^6 + a*b^7)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^6*b^2 +
5*a^5*b^3 + 10*a^4*b^4 + 10*a^3*b^5 + 5*a^2*b^6 + a*b^7)*d*sinh(d*x + c)^8
+ 4*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4 - 2*a^3*b^5 - 3*a^2*b^6 - a*b^7)*d*cos
h(d*x + c)^6 + 4*(7*(a^6*b^2 + 5*a^5*b^3 + 10*a^4*b^4 + 10*a^3*b^5 + 5*a^2*
b^6 + a*b^7)*d*cosh(d*x + c)^2 + (a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4 - 2*a^3*b
^5 - 3*a^2*b^6 - a*b^7)*d)*sinh(d*x + c)^6 + 2*(3*a^6*b^2 + 7*a^5*b^3 + 6*a
^4*b^4 + 6*a^3*b^5 + 7*a^2*b^6 + 3*a*b^7)*d*cosh(d*x + c)^4 + 8*(7*(a^6*b^2
+ 5*a^5*b^3 + 10*a^4*b^4 + 10*a^3*b^5 + 5*a^2*b^6 + a*b^7)*d*cosh(d*x + c)
^3 + 3*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4 - 2*a^3*b^5 - 3*a^2*b^6 - a*b^7)*d*
cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^6*b^2 + 5*a^5*b^3 + 10*a^4*b^4 +

```
10*a^3*b^5 + 5*a^2*b^6 + a*b^7)*d*cosh(d*x + c)^4 + 30*(a^6*b^2 + 3*a^5*b^3
 + 2*a^4*b^4 - 2*a^3*b^5 - 3*a^2*b^6 - a*b^7)*d*cosh(d*x + c)^2 + (3*a^6*b^
2 + 7*a^5*b^3 + 6*a^4*b^4 + 6*a^3*b^5 + 7*a^2*b^6 + 3*a*b^7)*d)*sinh(d*x +
c)^4 + 4*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4 - 2*a^3*b^5 - 3*a^2*b^6 - a*b^7)*
d*cosh(d*x + c)^2 + 8*(7*(a^6*b^2 + 5*a^5*b^3 + 10*a^4*b^4 + 10*a^3*b^5 + 5
*a^2*b^6 + a*b^7)*d*cosh(d*x + c)^5 + 10*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4 -
 2*a^3*b^5 - 3*a^2*b^6 - a*b^7)*d*cosh(d*x + c)^3 + (3*a^6*b^2 + 7*a^5*b^3
+ 6*a^4*b^4 + 6*a^3*b^5 + 7*a^2*b^6 + 3*a*b^7)*d*cosh(d*x + c))*sinh(d*x +
c)^3 + 4*(7*(a^6*b^2 + 5*a^5*b^3 + 10*a^4*b^4 + 10*a^3*b^5 + 5*a^2*b^6 + a*
b^7)*d*cosh(d*x + c)^6 + 15*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4 - 2*a^3*b^5 -
3*a^2*b^6 - a*b^7)*d*cosh(d*x + c)^4 + 3*(3*a^6*b^2 + 7*a^5*b^3 + 6*a^4*b^4
 + 6*a^3*b^5 + 7*a^2*b^6 + 3*a*b^7)*d*cosh(d*x + c)^2 + (a^6*b^2 + 3*a^5*b^
3 + 2*a^4*b^4 - 2*a^3*b^5 - 3*a^2*b^6 - a*b^7)*d)*sinh(d*x + c)^2 + (a^6*b^
2 + 5*a^5*b^3 + 10*a^4*b^4 + 10*a^3*b^5 + 5*a^2*b^6 + a*b^7)*d + 8*((a^6*b^
2 + 5*a^5*b^3 + 10*a^4*b^4 + 10*a^3*b^5 + 5*a^2*b^6 + a*b^7)*d*cosh(d*x + c
)^7 + 3*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4 - 2*a^3*b^5 - 3*a^2*b^6 - a*b^7)*d
*cosh(d*x + c)^5 + (3*a^6*b^2 + 7*a^5*b^3 + 6*a^4*b^4 + 6*a^3*b^5 + 7*a^2*b
^6 + 3*a*b^7)*d*cosh(d*x + c)^3 + (a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4 - 2*a^3*
b^5 - 3*a^2*b^6 - a*b^7)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*(8*(a^3*b^2 +
 2*a^2*b^3 + a*b^4)*d*x*cosh(d*x + c)^8 + 64*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*
d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 8*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*x*sin
h(d*x + c)^8 - 2*(a^4*b - 9*a^3*b^2 - 5*a^2*b^3 + 5*a*b^4 - 16*(a^3*b^2 - a
*b^4)*d*x)*cosh(d*x + c)^6 - 2*(a^4*b - 9*a^3*b^2 - 5*a^2*b^3 + 5*a*b^4 - 1
12*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*x*cosh(d*x + c)^2 - 16*(a^3*b^2 - a*b^4)
*d*x)*sinh(d*x + c)^6 + 4*(112*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*x*cosh(d*x +
 c)^3 - 3*(a^4*b - 9*a^3*b^2 - 5*a^2*b^3 + 5*a*b^4 - 16*(a^3*b^2 - a*b^4)*d
*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*a^4*b + 6*a^3*b^2 + 18*a^2*b^3 + 10*
a*b^4 - 2*(3*a^4*b - 17*a^3*b^2 + 13*a^2*b^3 - 15*a*b^4 - 8*(3*a^3*b^2 - 2*
a^2*b^3 + 3*a*b^4)*d*x)*cosh(d*x + c)^4 + 2*(280*(a^3*b^2 + 2*a^2*b^3 + a*b
^4)*d*x*cosh(d*x + c)^4 - 3*a^4*b + 17*a^3*b^2 - 13*a^2*b^3 + 15*a*b^4 + 8*
(3*a^3*b^2 - 2*a^2*b^3 + 3*a*b^4)*d*x - 15*(a^4*b - 9*a^3*b^2 - 5*a^2*b^3 +
 5*a*b^4 - 16*(a^3*b^2 - a*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(
56*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*x*cosh(d*x + c)^5 - 5*(a^4*b - 9*a^3*b^2
 - 5*a^2*b^3 + 5*a*b^4 - 16*(a^3*b^2 - a*b^4)*d*x)*cosh(d*x + c)^3 - (3*a^4
*b - 17*a^3*b^2 + 13*a^2*b^3 - 15*a*b^4 - 8*(3*a^3*b^2 - 2*a^2*b^3 + 3*a*b^
4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 8*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*
x - 2*(3*a^4*b - 11*a^3*b^2 + a^2*b^3 + 15*a*b^4 - 16*(a^3*b^2 - a*b^4)*d*x
)*cosh(d*x + c)^2 + 2*(112*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*x*cosh(d*x + c)^
6 - 3*a^4*b + 11*a^3*b^2 - a^2*b^3 - 15*a*b^4 - 15*(a^4*b - 9*a^3*b^2 - 5*a
^2*b^3 + 5*a*b^4 - 16*(a^3*b^2 - a*b^4)*d*x)*cosh(d*x + c)^4 + 16*(a^3*b^2
- a*b^4)*d*x - 6*(3*a^4*b - 17*a^3*b^2 + 13*a^2*b^3 - 15*a*b^4 - 8*(3*a^3*b
^2 - 2*a^2*b^3 + 3*a*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((a^4 + 8
*a^3*b + 10*a^2*b^2 - 3*b^4)*cosh(d*x + c)^8 + 8*(a^4 + 8*a^3*b + 10*a^2*b^
2 - 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^4 + 8*a^3*b + 10*a^2*b^2 - 3*
b^4)*sinh(d*x + c)^8 + 4*(a^4 + 6*a^3*b - 4*a^2*b^2 - 6*a*b^3 + 3*b^4)*cosh
(d*x + c)^6 + 4*(a^4 + 6*a^3*b - 4*a^2*b^2 - 6*a*b^3 + 3*b^4 + 7*(a^4 + 8*a
^3*b + 10*a^2*b^2 - 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^4 + 8
*a^3*b + 10*a^2*b^2 - 3*b^4)*cosh(d*x + c)^3 + 3*(a^4 + 6*a^3*b - 4*a^2*b^2
 - 6*a*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^4 + 16*a^3*b -
18*a^2*b^2 + 24*a*b^3 - 9*b^4)*cosh(d*x + c)^4 + 2*(35*(a^4 + 8*a^3*b + 10*
a^2*b^2 - 3*b^4)*cosh(d*x + c)^4 + 3*a^4 + 16*a^3*b - 18*a^2*b^2 + 24*a*b^3
 - 9*b^4 + 30*(a^4 + 6*a^3*b - 4*a^2*b^2 - 6*a*b^3 + 3*b^4)*cosh(d*x + c)^2
)*sinh(d*x + c)^4 + a^4 + 8*a^3*b + 10*a^2*b^2 - 3*b^4 + 8*(7*(a^4 + 8*a^3*
b + 10*a^2*b^2 - 3*b^4)*cosh(d*x + c)^5 + 10*(a^4 + 6*a^3*b - 4*a^2*b^2 - 6
*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + (3*a^4 + 16*a^3*b - 18*a^2*b^2 + 24*a*b^3
 - 9*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^4 + 6*a^3*b - 4*a^2*b^2 - 6
*a*b^3 + 3*b^4)*cosh(d*x + c)^2 + 4*(7*(a^4 + 8*a^3*b + 10*a^2*b^2 - 3*b^4)
*cosh(d*x + c)^6 + 15*(a^4 + 6*a^3*b - 4*a^2*b^2 - 6*a*b^3 + 3*b^4)*cosh(d*
x + c)^4 + a^4 + 6*a^3*b - 4*a^2*b^2 - 6*a*b^3 + 3*b^4 + 3*(3*a^4 + 16*a^3*
```

```
b - 18*a^2*b^2 + 24*a*b^3 - 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a
^4 + 8*a^3*b + 10*a^2*b^2 - 3*b^4)*cosh(d*x + c)^7 + 3*(a^4 + 6*a^3*b - 4*a
^2*b^2 - 6*a*b^3 + 3*b^4)*cosh(d*x + c)^5 + (3*a^4 + 16*a^3*b - 18*a^2*b^2
+ 24*a*b^3 - 9*b^4)*cosh(d*x + c)^3 + (a^4 + 6*a^3*b - 4*a^2*b^2 - 6*a*b^3
+ 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b)*arctan(1/2*((a + b)*cosh(d
*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2
 + a - b)*sqrt(a*b)/(a*b)) + 4*(16*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*x*cosh(d
*x + c)^7 - 3*(a^4*b - 9*a^3*b^2 - 5*a^2*b^3 + 5*a*b^4 - 16*(a^3*b^2 - a*b^
4)*d*x)*cosh(d*x + c)^5 - 2*(3*a^4*b - 17*a^3*b^2 + 13*a^2*b^3 - 15*a*b^4 -
 8*(3*a^3*b^2 - 2*a^2*b^3 + 3*a*b^4)*d*x)*cosh(d*x + c)^3 - (3*a^4*b - 11*a
^3*b^2 + a^2*b^3 + 15*a*b^4 - 16*(a^3*b^2 - a*b^4)*d*x)*cosh(d*x + c))*sinh
(d*x + c))/((a^6*b^2 + 5*a^5*b^3 + 10*a^4*b^4 + 10*a^3*b^5 + 5*a^2*b^6 + a*
b^7)*d*cosh(d*x + c)^8 + 8*(a^6*b^2 + 5*a^5*b^3 + 10*a^4*b^4 + 10*a^3*b^5 +
 5*a^2*b^6 + a*b^7)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^6*b^2 + 5*a^5*b^3
+ 10*a^4*b^4 + 10*a^3*b^5 + 5*a^2*b^6 + a*b^7)*d*sinh(d*x + c)^8 + 4*(a^6*b
^2 + 3*a^5*b^3 + 2*a^4*b^4 - 2*a^3*b^5 - 3*a^2*b^6 - a*b^7)*d*cosh(d*x + c)
^6 + 4*(7*(a^6*b^2 + 5*a^5*b^3 + 10*a^4*b^4 + 10*a^3*b^5 + 5*a^2*b^6 + a*b^
7)*d*cosh(d*x + c)^2 + (a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4 - 2*a^3*b^5 - 3*a^2
*b^6 - a*b^7)*d)*sinh(d*x + c)^6 + 2*(3*a^6*b^2 + 7*a^5*b^3 + 6*a^4*b^4 + 6
*a^3*b^5 + 7*a^2*b^6 + 3*a*b^7)*d*cosh(d*x + c)^4 + 8*(7*(a^6*b^2 + 5*a^5*b
^3 + 10*a^4*b^4 + 10*a^3*b^5 + 5*a^2*b^6 + a*b^7)*d*cosh(d*x + c)^3 + 3*(a^
6*b^2 + 3*a^5*b^3 + 2*a^4*b^4 - 2*a^3*b^5 - 3*a^2*b^6 - a*b^7)*d*cosh(d*x +
 c))*sinh(d*x + c)^5 + 2*(35*(a^6*b^2 + 5*a^5*b^3 + 10*a^4*b^4 + 10*a^3*b^5
 + 5*a^2*b^6 + a*b^7)*d*cosh(d*x + c)^4 + 30*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b
^4 - 2*a^3*b^5 - 3*a^2*b^6 - a*b^7)*d*cosh(d*x + c)^2 + (3*a^6*b^2 + 7*a^5*
b^3 + 6*a^4*b^4 + 6*a^3*b^5 + 7*a^2*b^6 + 3*a*b^7)*d)*sinh(d*x + c)^4 + 4*(
a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4 - 2*a^3*b^5 - 3*a^2*b^6 - a*b^7)*d*cosh(d*x
 + c)^2 + 8*(7*(a^6*b^2 + 5*a^5*b^3 + 10*a^4*b^4 + 10*a^3*b^5 + 5*a^2*b^6 +
 a*b^7)*d*cosh(d*x + c)^5 + 10*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4 - 2*a^3*b^5
 - 3*a^2*b^6 - a*b^7)*d*cosh(d*x + c)^3 + (3*a^6*b^2 + 7*a^5*b^3 + 6*a^4*b^
4 + 6*a^3*b^5 + 7*a^2*b^6 + 3*a*b^7)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(
7*(a^6*b^2 + 5*a^5*b^3 + 10*a^4*b^4 + 10*a^3*b^5 + 5*a^2*b^6 + a*b^7)*d*cos
h(d*x + c)^6 + 15*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4 - 2*a^3*b^5 - 3*a^2*b^6
- a*b^7)*d*cosh(d*x + c)^4 + 3*(3*a^6*b^2 + 7*a^5*b^3 + 6*a^4*b^4 + 6*a^3*b
^5 + 7*a^2*b^6 + 3*a*b^7)*d*cosh(d*x + c)^2 + (a^6*b^2 + 3*a^5*b^3 + 2*a^4*
b^4 - 2*a^3*b^5 - 3*a^2*b^6 - a*b^7)*d)*sinh(d*x + c)^2 + (a^6*b^2 + 5*a^5*
b^3 + 10*a^4*b^4 + 10*a^3*b^5 + 5*a^2*b^6 + a*b^7)*d + 8*((a^6*b^2 + 5*a^5*
b^3 + 10*a^4*b^4 + 10*a^3*b^5 + 5*a^2*b^6 + a*b^7)*d*cosh(d*x + c)^7 + 3*(a
^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4 - 2*a^3*b^5 - 3*a^2*b^6 - a*b^7)*d*cosh(d*x
+ c)^5 + (3*a^6*b^2 + 7*a^5*b^3 + 6*a^4*b^4 + 6*a^3*b^5 + 7*a^2*b^6 + 3*a*b
^7)*d*cosh(d*x + c)^3 + (a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4 - 2*a^3*b^5 - 3*a^
2*b^6 - a*b^7)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**4/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.25725, size = 533, normalized size = 3.89

$$-\frac{\left(a^2+6\,ab-3\,b^2\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{8\left(a^3bd+3\,a^2b^2d+3\,ab^3d+b^4d\right)\sqrt{ab}}+\frac{dx+c}{a^3d+3\,a^2bd+3\,ab^2d+b^3d}-\frac{a^3e^{(6\,dx+6\,c)}-9\,a^2be^{(6\,dx+6\,c)}-5\,al}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(a^2 + 6*a*b - 3*b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c
) + a - b)/sqrt(a*b))/((a^3*b*d + 3*a^2*b^2*d + 3*a*b^3*d + b^4*d)*sqrt(a*b
)) + (d*x + c)/(a^3*d + 3*a^2*b*d + 3*a*b^2*d + b^3*d) - 1/4*(a^3*e^(6*d*x
+ 6*c) - 9*a^2*b*e^(6*d*x + 6*c) - 5*a*b^2*e^(6*d*x + 6*c) + 5*b^3*e^(6*d*x
 + 6*c) + 3*a^3*e^(4*d*x + 4*c) - 17*a^2*b*e^(4*d*x + 4*c) + 13*a*b^2*e^(4*
d*x + 4*c) - 15*b^3*e^(4*d*x + 4*c) + 3*a^3*e^(2*d*x + 2*c) - 11*a^2*b*e^(2
*d*x + 2*c) + a*b^2*e^(2*d*x + 2*c) + 15*b^3*e^(2*d*x + 2*c) + a^3 - 3*a^2*
b - 9*a*b^2 - 5*b^3)/((a^3*b*d + 3*a^2*b^2*d + 3*a*b^3*d + b^4*d)*(a*e^(4*d
*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) +
 a + b)^2)
```

3.193 $\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=98

$$\frac{a}{4bd(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{1}{2d(a+b)^2\left(a+b\tanh^2(c+dx)\right)}+\frac{\log\left(a+b\tanh^2(c+dx)\right)}{2d(a+b)^3}+\frac{\log(\cosh(c+dx))}{d(a+b)^3}$$

```
[Out] Log[Cosh[c + d*x]]/((a + b)^3*d) + Log[a + b*Tanh[c + d*x]^2]/(2*(a + b)^3*
d) + a/(4*b*(a + b)*d*(a + b*Tanh[c + d*x]^2)^2) - 1/(2*(a + b)^2*d*(a + b*
Tanh[c + d*x]^2))
```

Rubi [A] time = 0.138291, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 446, 77}

$$\frac{a}{4bd(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{1}{2d(a+b)^2\left(a+b\tanh^2(c+dx)\right)}+\frac{\log\left(a+b\tanh^2(c+dx)\right)}{2d(a+b)^3}+\frac{\log(\cosh(c+dx))}{d(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] Log[Cosh[c + d*x]]/((a + b)^3*d) + Log[a + b*Tanh[c + d*x]^2]/(2*(a + b)^3*
d) + a/(4*b*(a + b)*d*(a + b*Tanh[c + d*x]^2)^2) - 1/(2*(a + b)^2*d*(a + b*
Tanh[c + d*x]^2))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(1-x^2)(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{(1-x)(a+bx)^3}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^3(-1+x)} - \frac{a}{(a+b)(a+bx)^3} + \frac{b}{(a+b)^2(a+bx)^2} + \frac{b}{(a+b)^3(a+bx)}\right) dx, x, \tanh^2(c+dx\right.}{2d}$$

$$= \frac{\log(\cosh(c+dx))}{(a+b)^3 d} + \frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)^3 d} + \frac{a}{4b(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} -$$

Mathematica [A] time = 0.453482, size = 80, normalized size = 0.82

$$\frac{\frac{a(a+b)^2}{b\left(a+b\tanh^2(c+dx)\right)^2} - \frac{2(a+b)}{a+b\tanh^2(c+dx)} + 2\log\left(a+b\tanh^2(c+dx)\right) + 4\log(\cosh(c+dx))}{4d(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (4*Log[Cosh[c + d*x]] + 2*Log[a + b*Tanh[c + d*x]^2] + (a*(a + b)^2)/(b*(a
+ b*Tanh[c + d*x]^2)^2) - (2*(a + b))/(a + b*Tanh[c + d*x]^2))/(4*(a + b)^3
*d)
```

Maple [B] time = 0.027, size = 196, normalized size = 2.

$$-\frac{\ln(\tanh(dx+c)+1)}{2d(a+b)^3} - \frac{a}{2d(a+b)^3\left(a+b(\tanh(dx+c))^2\right)} - \frac{b}{2d(a+b)^3\left(a+b(\tanh(dx+c))^2\right)} + \frac{}{4d(a+b)^3 b\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] -1/2/d/(a+b)^3*ln(tanh(d*x+c)+1)-1/2/d/(a+b)^3*a/(a+b*tanh(d*x+c)^2)-1/2/d/
(a+b)^3/(a+b*tanh(d*x+c)^2)*b+1/4/d/(a+b)^3*a^3/b/(a+b*tanh(d*x+c)^2)^2+1/2
/d/(a+b)^3*a^2/(a+b*tanh(d*x+c)^2)^2+1/4/d/(a+b)^3*a*b/(a+b*tanh(d*x+c)^2)^
2+1/2*ln(a+b*tanh(d*x+c)^2)/(a+b)^3/d-1/2/d/(a+b)^3*ln(tanh(d*x+c)-1)
```

Maxima [B] time = 1.31291, size = 518, normalized size = 5.29

$$\frac{dx+c}{\left(a^3+3a^2b+3ab^2+b^3\right)d} + \frac{}{\left(a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5+4\left(a^5+3a^4b+2a^3b^2-2a^2b^3-3ab^4-b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] (d*x + c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + 2*((a^2 - b^2)*e^(-2*d*x -
2*c) + 2*(a^2 - a*b + b^2)*e^(-4*d*x - 4*c) + (a^2 - b^2)*e^(-6*d*x - 6*c))
/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5 + 4*(a^5 + 3*a^4
*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*e^(-2*d*x - 2*c) + 2*(3*a^5 + 7
*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*e^(-4*d*x - 4*c) + 4*(a^5
 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*e^(-6*d*x - 6*c) + (a^5
 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*e^(-8*d*x - 8*c))*d)
+ 1/2*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((
a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d)
```

Fricas [B] time = 2.4369, size = 6130, normalized size = 62.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^8 + 16*(a^2 + 2*a*b + b^2)*d*
x*cosh(d*x + c)*sinh(d*x + c)^7 + 2*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^8
 + 4*(2*(a^2 - b^2)*d*x - a^2 + b^2)*cosh(d*x + c)^6 + 4*(14*(a^2 + 2*a*b +
 b^2)*d*x*cosh(d*x + c)^2 + 2*(a^2 - b^2)*d*x - a^2 + b^2)*sinh(d*x + c)^6
+ 8*(14*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 + 3*(2*(a^2 - b^2)*d*x - a^
2 + b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*((3*a^2 - 2*a*b + 3*b^2)*d*x -
2*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c)^4 + 4*(35*(a^2 + 2*a*b + b^2)*d*x*cosh
(d*x + c)^4 + (3*a^2 - 2*a*b + 3*b^2)*d*x + 15*(2*(a^2 - b^2)*d*x - a^2 + b
^2)*cosh(d*x + c)^2 - 2*a^2 + 2*a*b - 2*b^2)*sinh(d*x + c)^4 + 16*(7*(a^2 +
 2*a*b + b^2)*d*x*cosh(d*x + c)^5 + 5*(2*(a^2 - b^2)*d*x - a^2 + b^2)*cosh(
d*x + c)^3 + ((3*a^2 - 2*a*b + 3*b^2)*d*x - 2*a^2 + 2*a*b - 2*b^2)*cosh(d*x
 + c))*sinh(d*x + c)^3 + 2*(a^2 + 2*a*b + b^2)*d*x + 4*(2*(a^2 - b^2)*d*x -
 a^2 + b^2)*cosh(d*x + c)^2 + 4*(14*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^6
 + 15*(2*(a^2 - b^2)*d*x - a^2 + b^2)*cosh(d*x + c)^4 + 2*(a^2 - b^2)*d*x +
 6*((3*a^2 - 2*a*b + 3*b^2)*d*x - 2*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c)^2 -
a^2 + b^2)*sinh(d*x + c)^2 - ((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2
+ 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x
 + c)^8 + 4*(a^2 - b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x
 + c)^2 + a^2 - b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x +
c)^3 + 3*(a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^2 - 2*a*b + 3*
b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 30*(a^2
- b^2)*cosh(d*x + c)^2 + 3*a^2 - 2*a*b + 3*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2
 + 2*a*b + b^2)*cosh(d*x + c)^5 + 10*(a^2 - b^2)*cosh(d*x + c)^3 + (3*a^2 -
 2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^2 - b^2)*cosh(d*x + c
)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 15*(a^2 - b^2)*cosh(d*x +
c)^4 + 3*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)
^2 + a^2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 3*(a^2 -
b^2)*cosh(d*x + c)^5 + (3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^3 + (a^2 - b^2
)*cosh(d*x + c))*sinh(d*x + c))*log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*si
nh(d*x + c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + s
inh(d*x + c)^2)) + 8*(2*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^7 + 3*(2*(a^2
 - b^2)*d*x - a^2 + b^2)*cosh(d*x + c)^5 + 2*((3*a^2 - 2*a*b + 3*b^2)*d*x -
 2*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c)^3 + (2*(a^2 - b^2)*d*x - a^2 + b^2)*c
osh(d*x + c))*sinh(d*x + c))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*
a*b^4 + b^5)*d*cosh(d*x + c)^8 + 8*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3
 + 5*a*b^4 + b^5)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^5 + 5*a^4*b + 10*a^3
*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*sinh(d*x + c)^8 + 4*(a^5 + 3*a^4*b + 2
*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^6 + 4*(7*(a^5 + 5*a^4
*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^2 + (a^5 + 3*
```

```
a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d)*sinh(d*x + c)^6 + 2*(3*a^
5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d*cosh(d*x + c)^4 +
8*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x +
 c)^3 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*
x + c))*sinh(d*x + c)^5 + 2*(35*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 +
5*a*b^4 + b^5)*d*cosh(d*x + c)^4 + 30*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^
3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^2 + (3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2
*b^3 + 7*a*b^4 + 3*b^5)*d)*sinh(d*x + c)^4 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 -
 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^5 + 5*a^4*b + 10*a^
3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^5 + 10*(a^5 + 3*a^4*b +
 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^3 + (3*a^5 + 7*a^4*
b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)
^3 + 4*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(
d*x + c)^6 + 15*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*c
osh(d*x + c)^4 + 3*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b
^5)*d*cosh(d*x + c)^2 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 -
b^5)*d)*sinh(d*x + c)^2 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^
4 + b^5)*d + 8*((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d
*cosh(d*x + c)^7 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5
)*d*cosh(d*x + c)^5 + (3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 +
3*b^5)*d*cosh(d*x + c)^3 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4
 - b^5)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**3/(a+b*tanh(d*x+c)**2)**3,x)

[Out] Timed out
```

Giac [B] time = 1.26798, size = 339, normalized size = 3.46

$$\frac{\log\left(\left|a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a-2\,b\right|\right)}{2\left(a^3d+3\,a^2bd+3\,ab^2d+b^3d\right)}-\frac{3\,a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)^2+3\,b\left(e^{(2\,dx+2\,c)}+e\cdots\right.}{4\left(a^2d+2\,abd+b^2d\right)\left(a\left(e^{(2\cdots}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")

[Out] 1/2*log(abs(a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c) + e
^(-2*d*x - 2*c)) + 2*a - 2*b))/(a^3*d + 3*a^2*b*d + 3*a*b^2*d + b^3*d) - 1/
4*(3*a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + 3*b*(e^(2*d*x + 2*c) + e^(-
2*d*x - 2*c))^2 + 4*a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) - 4*b*(e^(2*d*x
+ 2*c) + e^(-2*d*x - 2*c)) - 4*a - 4*b)/((a^2*d + 2*a*b*d + b^2*d)*(a*(e^(2
*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) +
2*a - 2*b)^2)
```

3.194 $\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=137

$$-\frac{\left(3a^2-6ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}\sqrt{b}d(a+b)^3}-\frac{(3a-b)\tanh(c+dx)}{8ad(a+b)^2\left(a+b\tanh^2(c+dx)\right)}-\frac{\tanh(c+dx)}{4d(a+b)\left(a+b\tanh^2(c+dx)\right)^2}+\frac{x}{(a+b}$$

```
[Out] x/(a + b)^3 - ((3*a^2 - 6*a*b - b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]
])/(8*a^(3/2)*Sqrt[b]*(a + b)^3*d) - Tanh[c + d*x]/(4*(a + b)*d*(a + b*Tanh
[c + d*x]^2)^2) - ((3*a - b)*Tanh[c + d*x])/(8*a*(a + b)^2*d*(a + b*Tanh[c
+ d*x]^2))
```

Rubi [A] time = 0.157022, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3670, 471, 527, 522, 206, 205}

$$-\frac{\left(3a^2-6ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}\sqrt{b}d(a+b)^3}-\frac{(3a-b)\tanh(c+dx)}{8ad(a+b)^2\left(a+b\tanh^2(c+dx)\right)}-\frac{\tanh(c+dx)}{4d(a+b)\left(a+b\tanh^2(c+dx)\right)^2}+\frac{x}{(a+b}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] x/(a + b)^3 - ((3*a^2 - 6*a*b - b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]
])/(8*a^(3/2)*Sqrt[b]*(a + b)^3*d) - Tanh[c + d*x]/(4*(a + b)*d*(a + b*Tanh
[c + d*x]^2)^2) - ((3*a - b)*Tanh[c + d*x])/(8*a*(a + b)^2*d*(a + b*Tanh[c
+ d*x]^2))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
```

```
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{\tanh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{1+3x^2}{(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{4(a+b)d}$$

$$= -\frac{\tanh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(3a-b)\tanh(c+dx)}{8a(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \right.}{}$$

$$= -\frac{\tanh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(3a-b)\tanh(c+dx)}{8a(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \right.}{}$$

$$= \frac{x}{(a+b)^3} - \frac{\left(3a^2-6ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}\sqrt{b}(a+b)^3d} - \frac{\tanh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2}$$

Mathematica [A] time = 1.06931, size = 137, normalized size = 1.

$$\frac{\frac{\left(-3a^2+6ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}} - \frac{(5a-b)(a+b)\sinh(2(c+dx))}{a((a+b)\cosh(2(c+dx))+a-b)} - \frac{4b(a+b)\sinh(2(c+dx))}{((a+b)\cosh(2(c+dx))+a-b)^2} + 8(c+dx)}{8d(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (8*(c + d*x) + ((-3*a^2 + 6*a*b + b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[
a]])/(a^(3/2)*Sqrt[b]) - (4*b*(a + b)*Sinh[2*(c + d*x)])/(a - b + (a + b)*C
```

```
osh[2*(c + d*x)])^2 - ((5*a - b)*(a + b)*Sinh[2*(c + d*x)])/(a*(a - b + (a
+ b)*Cosh[2*(c + d*x)])))/(8*(a + b)^3*d)
```

Maple [B] time = 0.026, size = 340, normalized size = 2.5

$$\frac{\ln(\tanh(dx+c)+1)}{2d(a+b)^3} - \frac{3ab(\tanh(dx+c))^3}{8d(a+b)^3\left(a+b(\tanh(dx+c))^2\right)^2} - \frac{(\tanh(dx+c))^3 b^2}{4d(a+b)^3\left(a+b(\tanh(dx+c))^2\right)^2} + \frac{b^3(\tan}{8d(a+b)^3\left(a+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 1/2/d/(a+b)^3*ln(tanh(d*x+c)+1)-3/8/d/(a+b)^3*a/(a+b*tanh(d*x+c)^2)^2*b*tan
h(d*x+c)^3-1/4/d/(a+b)^3/(a+b*tanh(d*x+c)^2)^2*tanh(d*x+c)^3*b^2+1/8/d/(a+b
)^3/(a+b*tanh(d*x+c)^2)^2*b^3/a*tanh(d*x+c)^3-5/8/d/(a+b)^3*a^2/(a+b*tanh(d
*x+c)^2)^2*tanh(d*x+c)-3/4/d/(a+b)^3/(a+b*tanh(d*x+c)^2)^2*a*b*tanh(d*x+c)-
1/8/d/(a+b)^3/(a+b*tanh(d*x+c)^2)^2*tanh(d*x+c)*b^2-3/8/d/(a+b)^3*a/(a*b)^(
1/2)*arctan(tanh(d*x+c)*b/(a*b)^(1/2))+3/4/d/(a+b)^3*b/(a*b)^(1/2)*arctan(t
anh(d*x+c)*b/(a*b)^(1/2))+1/8/d/(a+b)^3/a/(a*b)^(1/2)*arctan(tanh(d*x+c)*b/
(a*b)^(1/2))*b^2-1/2/d/(a+b)^3*ln(tanh(d*x+c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.33037, size = 17383, normalized size = 126.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^8 + 128*(a^4*b +
2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 16*(a^4*b + 2*a^3*
b^2 + a^2*b^3)*d*x*sinh(d*x + c)^8 + 4*(5*a^4*b - 5*a^3*b^2 - 9*a^2*b^3 + a
*b^4 + 16*(a^4*b - a^2*b^3)*d*x)*cosh(d*x + c)^6 + 4*(5*a^4*b - 5*a^3*b^2 -
 9*a^2*b^3 + a*b^4 + 112*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^2
+ 16*(a^4*b - a^2*b^3)*d*x)*sinh(d*x + c)^6 + 8*(112*(a^4*b + 2*a^3*b^2 + a
^2*b^3)*d*x*cosh(d*x + c)^3 + 3*(5*a^4*b - 5*a^3*b^2 - 9*a^2*b^3 + a*b^4 +
16*(a^4*b - a^2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 20*a^4*b + 36*a^
3*b^2 + 12*a^2*b^3 - 4*a*b^4 + 4*(15*a^4*b - 13*a^3*b^2 + 17*a^2*b^3 - 3*a*
b^4 + 8*(3*a^4*b - 2*a^3*b^2 + 3*a^2*b^3)*d*x)*cosh(d*x + c)^4 + 4*(280*(a^
4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^4 + 15*a^4*b - 13*a^3*b^2 + 17
*a^2*b^3 - 3*a*b^4 + 8*(3*a^4*b - 2*a^3*b^2 + 3*a^2*b^3)*d*x + 15*(5*a^4*b
```

- 5*a^3*b^2 - 9*a^2*b^3 + a*b^4 + 16*(a^4*b - a^2*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(56*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^ 5 + 5*(5*a^4*b - 5*a^3*b^2 - 9*a^2*b^3 + a*b^4 + 16*(a^4*b - a^2*b^3)*d*x)* cosh(d*x + c)^3 + (15*a^4*b - 13*a^3*b^2 + 17*a^2*b^3 - 3*a*b^4 + 8*(3*a^4* b - 2*a^3*b^2 + 3*a^2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 16*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x + 4*(15*a^4*b + a^3*b^2 - 11*a^2*b^3 + 3*a*b^4 + 16*(a^4*b - a^2*b^3)*d*x)*cosh(d*x + c)^2 + 4*(112*(a^4*b + 2*a^3*b^2 + a^ 2*b^3)*d*x*cosh(d*x + c)^6 + 15*a^4*b + a^3*b^2 - 11*a^2*b^3 + 3*a*b^4 + 15 *(5*a^4*b - 5*a^3*b^2 - 9*a^2*b^3 + a*b^4 + 16*(a^4*b - a^2*b^3)*d*x)*cosh(d*x + c)^4 + 16*(a^4*b - a^2*b^3)*d*x + 6*(15*a^4*b - 13*a^3*b^2 + 17*a^2*b ^3 - 3*a*b^4 + 8*(3*a^4*b - 2*a^3*b^2 + 3*a^2*b^3)*d*x)*cosh(d*x + c)^2)*si nh(d*x + c)^2 + ((3*a^4 - 10*a^2*b^2 - 8*a*b^3 - b^4)*cosh(d*x + c)^8 + 8*(3*a^4 - 10*a^2*b^2 - 8*a*b^3 - b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^4 - 10*a^2*b^2 - 8*a*b^3 - b^4)*sinh(d*x + c)^8 + 4*(3*a^4 - 6*a^3*b - 4*a^2* b^2 + 6*a*b^3 + b^4)*cosh(d*x + c)^6 + 4*(3*a^4 - 6*a^3*b - 4*a^2*b^2 + 6*a *b^3 + b^4 + 7*(3*a^4 - 10*a^2*b^2 - 8*a*b^3 - b^4)*cosh(d*x + c)^2)*sinh(d *x + c)^6 + 8*(7*(3*a^4 - 10*a^2*b^2 - 8*a*b^3 - b^4)*cosh(d*x + c)^3 + 3*(3*a^4 - 6*a^3*b - 4*a^2*b^2 + 6*a*b^3 + b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(9*a^4 - 24*a^3*b + 18*a^2*b^2 - 16*a*b^3 - 3*b^4)*cosh(d*x + c)^4 + 2 *(35*(3*a^4 - 10*a^2*b^2 - 8*a*b^3 - b^4)*cosh(d*x + c)^4 + 9*a^4 - 24*a^3* b + 18*a^2*b^2 - 16*a*b^3 - 3*b^4 + 30*(3*a^4 - 6*a^3*b - 4*a^2*b^2 + 6*a*b ^3 + b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 3*a^4 - 10*a^2*b^2 - 8*a*b^3 - b^4 + 8*(7*(3*a^4 - 10*a^2*b^2 - 8*a*b^3 - b^4)*cosh(d*x + c)^5 + 10*(3*a^ 4 - 6*a^3*b - 4*a^2*b^2 + 6*a*b^3 + b^4)*cosh(d*x + c)^3 + (9*a^4 - 24*a^3* b + 18*a^2*b^2 - 16*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(3*a^ 4 - 6*a^3*b - 4*a^2*b^2 + 6*a*b^3 + b^4)*cosh(d*x + c)^2 + 4*(7*(3*a^4 - 10 *a^2*b^2 - 8*a*b^3 - b^4)*cosh(d*x + c)^6 + 15*(3*a^4 - 6*a^3*b - 4*a^2*b^2 + 6*a*b^3 + b^4)*cosh(d*x + c)^4 + 3*a^4 - 6*a^3*b - 4*a^2*b^2 + 6*a*b^3 + b^4 + 3*(9*a^4 - 24*a^3*b + 18*a^2*b^2 - 16*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((3*a^4 - 10*a^2*b^2 - 8*a*b^3 - b^4)*cosh(d*x + c)^7 + 3*(3*a^4 - 6*a^3*b - 4*a^2*b^2 + 6*a*b^3 + b^4)*cosh(d*x + c)^5 + (9*a^4 - 24*a^3*b + 18*a^2*b^2 - 16*a*b^3 - 3*b^4)*cosh(d*x + c)^3 + (3*a^4 - 6*a ^3*b - 4*a^2*b^2 + 6*a*b^3 + b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b)* log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*c osh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh (d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d* x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*co sh(d*x + c))*sinh(d*x + c) + a + b)) + 8*(16*(a^4*b + 2*a^3*b^2 + a^2*b^3)* d*x*cosh(d*x + c)^7 + 3*(5*a^4*b - 5*a^3*b^2 - 9*a^2*b^3 + a*b^4 + 16*(a^4* b - a^2*b^3)*d*x)*cosh(d*x + c)^5 + 2*(15*a^4*b - 13*a^3*b^2 + 17*a^2*b^3 - 3*a*b^4 + 8*(3*a^4*b - 2*a^3*b^2 + 3*a^2*b^3)*d*x)*cosh(d*x + c)^3 + (15*a ^4*b + a^3*b^2 - 11*a^2*b^3 + 3*a*b^4 + 16*(a^4*b - a^2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^7*b + 5*a^6*b^2 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3* b^5 + a^2*b^6)*d*cosh(d*x + c)^8 + 8*(a^7*b + 5*a^6*b^2 + 10*a^5*b^3 + 10*a ^4*b^4 + 5*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^7*b + 5* a^6*b^2 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a^2*b^6)*d*sinh(d*x + c)^8 + 4*(a^7*b + 3*a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4 - 3*a^3*b^5 - a^2*b^6)*d*cos h(d*x + c)^6 + 4*(7*(a^7*b + 5*a^6*b^2 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^ 5 + a^2*b^6)*d*cosh(d*x + c)^2 + (a^7*b + 3*a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4 - 3*a^3*b^5 - a^2*b^6)*d)*sinh(d*x + c)^6 + 2*(3*a^7*b + 7*a^6*b^2 + 6*a^5 *b^3 + 6*a^4*b^4 + 7*a^3*b^5 + 3*a^2*b^6)*d*cosh(d*x + c)^4 + 8*(7*(a^7*b + 5*a^6*b^2 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c) ^3 + 3*(a^7*b + 3*a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4 - 3*a^3*b^5 - a^2*b^6)*d* cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^7*b + 5*a^6*b^2 + 10*a^5*b^3 + 10 *a^4*b^4 + 5*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^4 + 30*(a^7*b + 3*a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4 - 3*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)^2 + (3*a^7*b + 7*a^6*b^2 + 6*a^5*b^3 + 6*a^4*b^4 + 7*a^3*b^5 + 3*a^2*b^6)*d)*sinh(d*x + c)^4 + 4*(a^7*b + 3*a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4 - 3*a^3*b^5 - a^2*b^6)* d*cosh(d*x + c)^2 + 8*(7*(a^7*b + 5*a^6*b^2 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a ^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^5 + 10*(a^7*b + 3*a^6*b^2 + 2*a^5*b^3 - 2 *a^4*b^4 - 3*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)^3 + (3*a^7*b + 7*a^6*b^2 + 6*a^5*b^3 + 6*a^4*b^4 + 7*a^3*b^5 + 3*a^2*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^7*b + 5*a^6*b^2 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a^2* b^6)*d*cosh(d*x + c)^6 + 15*(a^7*b + 3*a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4 - 3* a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)^4 + 3*(3*a^7*b + 7*a^6*b^2 + 6*a^5*b^3 + 6*a^4*b^4 + 7*a^3*b^5 + 3*a^2*b^6)*d*cosh(d*x + c)^2 + (a^7*b + 3*a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4 - 3*a^3*b^5 - a^2*b^6)*d)*sinh(d*x + c)^2 + (a^7*b + 5*a^6*b^2 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a^2*b^6)*d + 8*((a^7*b + 5*a^6*b^2 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^7 + 3*(a^7*b + 3*a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4 - 3*a^3*b^5 - a^2*b^6)*d *cosh(d*x + c)^5 + (3*a^7*b + 7*a^6*b^2 + 6*a^5*b^3 + 6*a^4*b^4 + 7*a^3*b^5 + 3*a^2*b^6)*d*cosh(d*x + c)^3 + (a^7*b + 3*a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^ 4 - 3*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*(8*(a^4*b + 2 *a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^8 + 64*(a^4*b + 2*a^3*b^2 + a^2*b^3)* d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 8*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*sin h(d*x + c)^8 + 2*(5*a^4*b - 5*a^3*b^2 - 9*a^2*b^3 + a*b^4 + 16*(a^4*b - a^2 *b^3)*d*x)*cosh(d*x + c)^6 + 2*(5*a^4*b - 5*a^3*b^2 - 9*a^2*b^3 + a*b^4 + 1 12*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^2 + 16*(a^4*b - a^2*b^3) *d*x)*sinh(d*x + c)^6 + 4*(112*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^3 + 3*(5*a^4*b - 5*a^3*b^2 - 9*a^2*b^3 + a*b^4 + 16*(a^4*b - a^2*b^3)*d *x)*cosh(d*x + c))*sinh(d*x + c)^5 + 10*a^4*b + 18*a^3*b^2 + 6*a^2*b^3 - 2* a*b^4 + 2*(15*a^4*b - 13*a^3*b^2 + 17*a^2*b^3 - 3*a*b^4 + 8*(3*a^4*b - 2*a^ 3*b^2 + 3*a^2*b^3)*d*x)*cosh(d*x + c)^4 + 2*(280*(a^4*b + 2*a^3*b^2 + a^2*b ^3)*d*x*cosh(d*x + c)^4 + 15*a^4*b - 13*a^3*b^2 + 17*a^2*b^3 - 3*a*b^4 + 8* (3*a^4*b - 2*a^3*b^2 + 3*a^2*b^3)*d*x + 15*(5*a^4*b - 5*a^3*b^2 - 9*a^2*b^3 + a*b^4 + 16*(a^4*b - a^2*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(56*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^5 + 5*(5*a^4*b - 5*a^3*b ^2 - 9*a^2*b^3 + a*b^4 + 16*(a^4*b - a^2*b^3)*d*x)*cosh(d*x + c)^3 + (15*a^ 4*b - 13*a^3*b^2 + 17*a^2*b^3 - 3*a*b^4 + 8*(3*a^4*b - 2*a^3*b^2 + 3*a^2*b^ 3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 8*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d* x + 2*(15*a^4*b + a^3*b^2 - 11*a^2*b^3 + 3*a*b^4 + 16*(a^4*b - a^2*b^3)*d*x)*cosh(d*x + c)^2 + 2*(112*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^ 6 + 15*a^4*b + a^3*b^2 - 11*a^2*b^3 + 3*a*b^4 + 15*(5*a^4*b - 5*a^3*b^2 - 9 *a^2*b^3 + a*b^4 + 16*(a^4*b - a^2*b^3)*d*x)*cosh(d*x + c)^4 + 16*(a^4*b - a^2*b^3)*d*x + 6*(15*a^4*b - 13*a^3*b^2 + 17*a^2*b^3 - 3*a*b^4 + 8*(3*a^4*b - 2*a^3*b^2 + 3*a^2*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((3*a^4 - 10*a^2*b^2 - 8*a*b^3 - b^4)*cosh(d*x + c)^8 + 8*(3*a^4 - 10*a^2*b^2 - 8*a* b^3 - b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^4 - 10*a^2*b^2 - 8*a*b^3 - b^4)*sinh(d*x + c)^8 + 4*(3*a^4 - 6*a^3*b - 4*a^2*b^2 + 6*a*b^3 + b^4)*cosh (d*x + c)^6 + 4*(3*a^4 - 6*a^3*b - 4*a^2*b^2 + 6*a*b^3 + b^4 + 7*(3*a^4 - 1 0*a^2*b^2 - 8*a*b^3 - b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^4 - 10*a^2*b^2 - 8*a*b^3 - b^4)*cosh(d*x + c)^3 + 3*(3*a^4 - 6*a^3*b - 4*a^2*b ^2 + 6*a*b^3 + b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(9*a^4 - 24*a^3*b + 18*a^2*b^2 - 16*a*b^3 - 3*b^4)*cosh(d*x + c)^4 + 2*(35*(3*a^4 - 10*a^2*b^2 - 8*a*b^3 - b^4)*cosh(d*x + c)^4 + 9*a^4 - 24*a^3*b + 18*a^2*b^2 - 16*a*b^3 - 3*b^4 + 30*(3*a^4 - 6*a^3*b - 4*a^2*b^2 + 6*a*b^3 + b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 3*a^4 - 10*a^2*b^2 - 8*a*b^3 - b^4 + 8*(7*(3*a^4 - 10*a ^2*b^2 - 8*a*b^3 - b^4)*cosh(d*x + c)^5 + 10*(3*a^4 - 6*a^3*b - 4*a^2*b^2 + 6*a*b^3 + b^4)*cosh(d*x + c)^3 + (9*a^4 - 24*a^3*b + 18*a^2*b^2 - 16*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(3*a^4 - 6*a^3*b - 4*a^2*b^2 + 6*a*b^3 + b^4)*cosh(d*x + c)^2 + 4*(7*(3*a^4 - 10*a^2*b^2 - 8*a*b^3 - b^4) *cosh(d*x + c)^6 + 15*(3*a^4 - 6*a^3*b - 4*a^2*b^2 + 6*a*b^3 + b^4)*cosh(d* x + c)^4 + 3*a^4 - 6*a^3*b - 4*a^2*b^2 + 6*a*b^3 + b^4 + 3*(9*a^4 - 24*a^3*

```
b + 18*a^2*b^2 - 16*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((3
*a^4 - 10*a^2*b^2 - 8*a*b^3 - b^4)*cosh(d*x + c)^7 + 3*(3*a^4 - 6*a^3*b - 4
*a^2*b^2 + 6*a*b^3 + b^4)*cosh(d*x + c)^5 + (9*a^4 - 24*a^3*b + 18*a^2*b^2
 - 16*a*b^3 - 3*b^4)*cosh(d*x + c)^3 + (3*a^4 - 6*a^3*b - 4*a^2*b^2 + 6*a*b^
3 + b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b)*arctan(1/2*((a + b)*cosh(d
*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2
 + a - b)*sqrt(a*b)/(a*b)) + 4*(16*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d
*x + c)^7 + 3*(5*a^4*b - 5*a^3*b^2 - 9*a^2*b^3 + a*b^4 + 16*(a^4*b - a^2*b^
3)*d*x)*cosh(d*x + c)^5 + 2*(15*a^4*b - 13*a^3*b^2 + 17*a^2*b^3 - 3*a*b^4 +
 8*(3*a^4*b - 2*a^3*b^2 + 3*a^2*b^3)*d*x)*cosh(d*x + c)^3 + (15*a^4*b + a^3
*b^2 - 11*a^2*b^3 + 3*a*b^4 + 16*(a^4*b - a^2*b^3)*d*x)*cosh(d*x + c))*sinh
(d*x + c))/((a^7*b + 5*a^6*b^2 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a^2*
b^6)*d*cosh(d*x + c)^8 + 8*(a^7*b + 5*a^6*b^2 + 10*a^5*b^3 + 10*a^4*b^4 + 5
*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^7*b + 5*a^6*b^2 +
10*a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a^2*b^6)*d*sinh(d*x + c)^8 + 4*(a^7*b
 + 3*a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4 - 3*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)
^6 + 4*(7*(a^7*b + 5*a^6*b^2 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a^2*b^
6)*d*cosh(d*x + c)^2 + (a^7*b + 3*a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4 - 3*a^3*b
^5 - a^2*b^6)*d)*sinh(d*x + c)^6 + 2*(3*a^7*b + 7*a^6*b^2 + 6*a^5*b^3 + 6*a
^4*b^4 + 7*a^3*b^5 + 3*a^2*b^6)*d*cosh(d*x + c)^4 + 8*(7*(a^7*b + 5*a^6*b^2
 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^3 + 3*(a^
7*b + 3*a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4 - 3*a^3*b^5 - a^2*b^6)*d*cosh(d*x +
 c))*sinh(d*x + c)^5 + 2*(35*(a^7*b + 5*a^6*b^2 + 10*a^5*b^3 + 10*a^4*b^4 +
 5*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^4 + 30*(a^7*b + 3*a^6*b^2 + 2*a^5*b^3
 - 2*a^4*b^4 - 3*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)^2 + (3*a^7*b + 7*a^6*b^
2 + 6*a^5*b^3 + 6*a^4*b^4 + 7*a^3*b^5 + 3*a^2*b^6)*d)*sinh(d*x + c)^4 + 4*(
a^7*b + 3*a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4 - 3*a^3*b^5 - a^2*b^6)*d*cosh(d*x
 + c)^2 + 8*(7*(a^7*b + 5*a^6*b^2 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a
^2*b^6)*d*cosh(d*x + c)^5 + 10*(a^7*b + 3*a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4 -
 3*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)^3 + (3*a^7*b + 7*a^6*b^2 + 6*a^5*b^3
+ 6*a^4*b^4 + 7*a^3*b^5 + 3*a^2*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(
7*(a^7*b + 5*a^6*b^2 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a^2*b^6)*d*cos
h(d*x + c)^6 + 15*(a^7*b + 3*a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4 - 3*a^3*b^5 -
a^2*b^6)*d*cosh(d*x + c)^4 + 3*(3*a^7*b + 7*a^6*b^2 + 6*a^5*b^3 + 6*a^4*b^4
 + 7*a^3*b^5 + 3*a^2*b^6)*d*cosh(d*x + c)^2 + (a^7*b + 3*a^6*b^2 + 2*a^5*b^
3 - 2*a^4*b^4 - 3*a^3*b^5 - a^2*b^6)*d)*sinh(d*x + c)^2 + (a^7*b + 5*a^6*b^
2 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a^2*b^6)*d + 8*((a^7*b + 5*a^6*b^
2 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^7 + 3*(a
^7*b + 3*a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4 - 3*a^3*b^5 - a^2*b^6)*d*cosh(d*x
+ c)^5 + (3*a^7*b + 7*a^6*b^2 + 6*a^5*b^3 + 6*a^4*b^4 + 7*a^3*b^5 + 3*a^2*b
^6)*d*cosh(d*x + c)^3 + (a^7*b + 3*a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4 - 3*a^3*
b^5 - a^2*b^6)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**2/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26422, size = 539, normalized size = 3.93

$$-\frac{\left(3\,a^2-6\,ab-b^2\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{8\left(a^4d+3\,a^3bd+3\,a^2b^2d+ab^3d\right)\sqrt{ab}}+\frac{dx+c}{a^3d+3\,a^2bd+3\,ab^2d+b^3d}+\frac{5\,a^3e^{(6\,dx+6\,c)}-5\,a^2be^{(6\,dx+6\,c)}-9\,ab^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*a^2 - 6*a*b - b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c
) + a - b)/sqrt(a*b))/((a^4*d + 3*a^3*b*d + 3*a^2*b^2*d + a*b^3*d)*sqrt(a*b
)) + (d*x + c)/(a^3*d + 3*a^2*b*d + 3*a*b^2*d + b^3*d) + 1/4*(5*a^3*e^(6*d*
x + 6*c) - 5*a^2*b*e^(6*d*x + 6*c) - 9*a*b^2*e^(6*d*x + 6*c) + b^3*e^(6*d*x
 + 6*c) + 15*a^3*e^(4*d*x + 4*c) - 13*a^2*b*e^(4*d*x + 4*c) + 17*a*b^2*e^(4
*d*x + 4*c) - 3*b^3*e^(4*d*x + 4*c) + 15*a^3*e^(2*d*x + 2*c) + a^2*b*e^(2*d
*x + 2*c) - 11*a*b^2*e^(2*d*x + 2*c) + 3*b^3*e^(2*d*x + 2*c) + 5*a^3 + 9*a^
2*b + 3*a*b^2 - b^3)/((a^4*d + 3*a^3*b*d + 3*a^2*b^2*d + a*b^3*d)*(a*e^(4*d
*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) +
 a + b)^2)
```

3.195 $\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=94

$$-\frac{1}{2d(a+b)^2\left(a+b\tanh^2(c+dx)\right)}-\frac{1}{4d(a+b)\left(a+b\tanh^2(c+dx)\right)^2}+\frac{\log\left(a+b\tanh^2(c+dx)\right)}{2d(a+b)^3}+\frac{\log(\cosh(c+dx))}{d(a+b)^3}$$

```
[Out] Log[Cosh[c + d*x]]/((a + b)^3*d) + Log[a + b*Tanh[c + d*x]^2]/(2*(a + b)^3*
d) - 1/(4*(a + b)*d*(a + b*Tanh[c + d*x]^2)^2) - 1/(2*(a + b)^2*d*(a + b*Ta
nh[c + d*x]^2))
```

Rubi [A] time = 0.104568, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3670, 444, 44}

$$-\frac{1}{2d(a+b)^2\left(a+b\tanh^2(c+dx)\right)}-\frac{1}{4d(a+b)\left(a+b\tanh^2(c+dx)\right)^2}+\frac{\log\left(a+b\tanh^2(c+dx)\right)}{2d(a+b)^3}+\frac{\log(\cosh(c+dx))}{d(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] Log[Cosh[c + d*x]]/((a + b)^3*d) + Log[a + b*Tanh[c + d*x]^2]/(2*(a + b)^3*
d) - 1/(4*(a + b)*d*(a + b*Tanh[c + d*x]^2)^2) - 1/(2*(a + b)^2*d*(a + b*Ta
nh[c + d*x]^2))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1-x^2)(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)^3}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^3(-1+x)} + \frac{b}{(a+b)(a+bx)^3} + \frac{b}{(a+b)^2(a+bx)^2} + \frac{b}{(a+b)^3(a+bx)}\right) dx, x, \tanh^2(c+dx)\right)}{2d}$$

$$= \frac{\log(\cosh(c+dx))}{(a+b)^3d} + \frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)^3d} - \frac{1}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{}{2(a}$$

Mathematica [A] time = 0.514048, size = 77, normalized size = 0.82

$$\frac{-\frac{(a+b)^2}{\left(a+b\tanh^2(c+dx)\right)^2} - \frac{2(a+b)}{a+b\tanh^2(c+dx)} + 2\log\left(a+b\tanh^2(c+dx)\right) + 4\log(\cosh(c+dx))}{4d(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (4*Log[Cosh[c + d*x]] + 2*Log[a + b*Tanh[c + d*x]^2] - (a + b)^2/(a + b*Tan
h[c + d*x]^2)^2 - (2*(a + b))/(a + b*Tanh[c + d*x]^2))/(4*(a + b)^3*d)
```

Maple [B] time = 0.029, size = 193, normalized size = 2.1

$$-\frac{\ln(\tanh(dx+c)+1)}{2d(a+b)^3} - \frac{a}{2d(a+b)^3\left(a+b(\tanh(dx+c))^2\right)} - \frac{b}{2d(a+b)^3\left(a+b(\tanh(dx+c))^2\right)} - \frac{}{4d(a+b)^3(a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] -1/2/d/(a+b)^3*ln(tanh(d*x+c)+1)-1/2/d/(a+b)^3*a/(a+b*tanh(d*x+c)^2)-1/2/d/
(a+b)^3/(a+b*tanh(d*x+c)^2)*b-1/4/d/(a+b)^3*a^2/(a+b*tanh(d*x+c)^2)^2-1/2/d
/(a+b)^3*a*b/(a+b*tanh(d*x+c)^2)^2-1/4/d/(a+b)^3*b^2/(a+b*tanh(d*x+c)^2)^2+
1/2*ln(a+b*tanh(d*x+c)^2)/(a+b)^3/d-1/2/d/(a+b)^3*ln(tanh(d*x+c)-1)
```

Maxima [B] time = 1.36952, size = 510, normalized size = 5.43

$$\frac{dx+c}{\left(a^3+3a^2b+3ab^2+b^3\right)d} - \frac{}{\left(a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5+4\left(a^5+3a^4b+2a^3b^2-2a^2b^3-3ab^4-b^5\right)e^{(}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] (d*x + c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 4*((a*b + b^2)*e^(-2*d*x -
2*c) + (2*a*b - b^2)*e^(-4*d*x - 4*c) + (a*b + b^2)*e^(-6*d*x - 6*c))/((a^5
 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5 + 4*(a^5 + 3*a^4*b + 2
*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*e^(-2*d*x - 2*c) + 2*(3*a^5 + 7*a^4*b
 + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*e^(-4*d*x - 4*c) + 4*(a^5 + 3*a
^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*e^(-6*d*x - 6*c) + (a^5 + 5*a
^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*e^(-8*d*x - 8*c))*d) + 1/2*
log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^3 +
3*a^2*b + 3*a*b^2 + b^3)*d)
```

Fricas [B] time = 2.54662, size = 6044, normalized size = 64.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^8 + 16*(a^2 + 2*a*b + b^2)*d*
x*cosh(d*x + c)*sinh(d*x + c)^7 + 2*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^8
 + 8*((a^2 - b^2)*d*x + a*b + b^2)*cosh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^
2)*d*x*cosh(d*x + c)^2 + (a^2 - b^2)*d*x + a*b + b^2)*sinh(d*x + c)^6 + 16*
(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 + 3*((a^2 - b^2)*d*x + a*b + b^2
)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*((3*a^2 - 2*a*b + 3*b^2)*d*x + 4*a*b -
 2*b^2)*cosh(d*x + c)^4 + 4*(35*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + (
3*a^2 - 2*a*b + 3*b^2)*d*x + 30*((a^2 - b^2)*d*x + a*b + b^2)*cosh(d*x + c)
^2 + 4*a*b - 2*b^2)*sinh(d*x + c)^4 + 16*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*
x + c)^5 + 10*((a^2 - b^2)*d*x + a*b + b^2)*cosh(d*x + c)^3 + ((3*a^2 - 2*a
*b + 3*b^2)*d*x + 4*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(a^2 +
2*a*b + b^2)*d*x + 8*((a^2 - b^2)*d*x + a*b + b^2)*cosh(d*x + c)^2 + 8*(7*(
a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^6 + 15*((a^2 - b^2)*d*x + a*b + b^2)*c
osh(d*x + c)^4 + (a^2 - b^2)*d*x + 3*((3*a^2 - 2*a*b + 3*b^2)*d*x + 4*a*b -
 2*b^2)*cosh(d*x + c)^2 + a*b + b^2)*sinh(d*x + c)^2 - ((a^2 + 2*a*b + b^2)
*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a
^2 + 2*a*b + b^2)*sinh(d*x + c)^8 + 4*(a^2 - b^2)*cosh(d*x + c)^6 + 4*(7*(a
^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^6 + 8*(7*(a^2
+ 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c)
^5 + 2*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*
cosh(d*x + c)^4 + 30*(a^2 - b^2)*cosh(d*x + c)^2 + 3*a^2 - 2*a*b + 3*b^2)*s
inh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 10*(a^2 - b^2)*
cosh(d*x + c)^3 + (3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 +
4*(a^2 - b^2)*cosh(d*x + c)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 +
15*(a^2 - b^2)*cosh(d*x + c)^4 + 3*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^2
+ a^2 - b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*c
osh(d*x + c)^7 + 3*(a^2 - b^2)*cosh(d*x + c)^5 + (3*a^2 - 2*a*b + 3*b^2)*co
sh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*((a + b)*co
sh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(
d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 16*((a^2 + 2*a*b + b^2)*d*x*co
sh(d*x + c)^7 + 3*((a^2 - b^2)*d*x + a*b + b^2)*cosh(d*x + c)^5 + ((3*a^2 -
 2*a*b + 3*b^2)*d*x + 4*a*b - 2*b^2)*cosh(d*x + c)^3 + ((a^2 - b^2)*d*x + a
*b + b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a
^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^8 + 8*(a^5 + 5*a^4*b + 10*a^3*b^2 +
 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^5 + 5*a^4
*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*sinh(d*x + c)^8 + 4*(a^5 +
3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^6 + 4*(7*(
a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^2
+ (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d)*sinh(d*x + c)^
```

```
6 + 2*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d*cosh(d*
x + c)^4 + 8*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d
*cosh(d*x + c)^3 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5
)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10
*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^4 + 30*(a^5 + 3*a^4*b + 2*a^3*b^2
 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^2 + (3*a^5 + 7*a^4*b + 6*a^3*
b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d)*sinh(d*x + c)^4 + 4*(a^5 + 3*a^4*b +
2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^5 + 5*a^
4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^5 + 10*(a^5
+ 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^3 + (3*a
^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d*cosh(d*x + c))*si
nh(d*x + c)^3 + 4*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b
^5)*d*cosh(d*x + c)^6 + 15*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4
 - b^5)*d*cosh(d*x + c)^4 + 3*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*
a*b^4 + 3*b^5)*d*cosh(d*x + c)^2 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 -
 3*a*b^4 - b^5)*d)*sinh(d*x + c)^2 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b
^3 + 5*a*b^4 + b^5)*d + 8*((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b
^4 + b^5)*d*cosh(d*x + c)^7 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*
a*b^4 - b^5)*d*cosh(d*x + c)^5 + (3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 +
 7*a*b^4 + 3*b^5)*d*cosh(d*x + c)^3 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^
3 - 3*a*b^4 - b^5)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24027, size = 339, normalized size = 3.61

$$\frac{\log\left(\left|a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a-2\,b\right|\right)}{2\left(a^3d+3\,a^2bd+3\,ab^2d+b^3d\right)}-\frac{3\,a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)^2+3\,b\left(e^{(2\,dx+2\,c)}+e^{(-2\,c}\right.}{4\left(a^2d+2\,abd+b^2d\right)\left(a\left(e^{(2\,dx}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*log(abs(a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c) + e
^(-2*d*x - 2*c)) + 2*a - 2*b))/(a^3*d + 3*a^2*b*d + 3*a*b^2*d + b^3*d) - 1/
4*(3*a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + 3*b*(e^(2*d*x + 2*c) + e^(-
2*d*x - 2*c))^2 + 12*a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 4*b*(e^(2*d*x
 + 2*c) + e^(-2*d*x - 2*c)) + 12*a - 4*b)/((a^2*d + 2*a*b*d + b^2*d)*(a*(e^
(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))
+ 2*a - 2*b)^2)
```

3.196 $\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=142

$$\frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a+b)^3}+\frac{b(7a+3b)\tanh(c+dx)}{8a^2d(a+b)^2\left(a+b\tanh^2(c+dx)\right)}+\frac{b\tanh(c+dx)}{4ad(a+b)\left(a+b\tanh^2(c+dx)\right.}$$

```
[Out] x/(a + b)^3 + (Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d
*x])/Sqrt[a]])/(8*a^(5/2)*(a + b)^3*d) + (b*Tanh[c + d*x])/(4*a*(a + b)*d*(
a + b*Tanh[c + d*x]^2)^2) + (b*(7*a + 3*b)*Tanh[c + d*x])/(8*a^2*(a + b)^2*
d*(a + b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.162685, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3661, 414, 527, 522, 206, 205}

$$\frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a+b)^3}+\frac{b(7a+3b)\tanh(c+dx)}{8a^2d(a+b)^2\left(a+b\tanh^2(c+dx)\right)}+\frac{b\tanh(c+dx)}{4ad(a+b)\left(a+b\tanh^2(c+dx)\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tanh[c + d*x]^2)^(-3),x]
```

```
[Out] x/(a + b)^3 + (Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d
*x])/Sqrt[a]])/(8*a^(5/2)*(a + b)^3*d) + (b*Tanh[c + d*x])/(4*a*(a + b)*d*(
a + b*Tanh[c + d*x]^2)^2) + (b*(7*a + 3*b)*Tanh[c + d*x])/(8*a^2*(a + b)^2*
d*(a + b*Tanh[c + d*x]^2))
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\left(a+bx^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\tanh(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{b-4(a+b)+3bx^2}{(1-x^2)\left(a+bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{4a(a+b)d}$$

$$= \frac{b\tanh(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(7a+3b)\tanh(c+dx)}{8a^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{8a^2+\cdots}{\cdots}\right.}{\cdots}$$

$$= \frac{b\tanh(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(7a+3b)\tanh(c+dx)}{8a^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\cdots\right.}{(\cdots}$$

$$= \frac{x}{(a+b)^3} + \frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^3d} + \frac{b\tanh(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+d\cdots\right.}$$

Mathematica [A] time = 0.276913, size = 147, normalized size = 1.04

$$\frac{\frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}} + \frac{b(7a+3b)(a+b)\tanh(c+dx)}{a^2\left(a+b\tanh^2(c+dx)\right)} + \frac{2b(a+b)^2\tanh(c+dx)}{a\left(a+b\tanh^2(c+dx)\right)^2} - 4\log(1-\tanh(c+dx)) + 4\log(\tanh(c+\cdots}{8d(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tanh[c + d*x]^2)^(-3),x]
```

```
[Out] ((Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]]
)/a^(5/2) - 4*Log[1 - Tanh[c + d*x]] + 4*Log[1 + Tanh[c + d*x]] + (2*b*(a +
 b)^2*Tanh[c + d*x])/(a*(a + b*Tanh[c + d*x]^2)^2) + (b*(a + b)*(7*a + 3*b)
```

```
*Tanh[c + d*x])/(a^2*(a + b*Tanh[c + d*x]^2)))/(8*(a + b)^3*d)
```

Maple [B] time = 0.031, size = 352, normalized size = 2.5

$$\frac{\ln(\tanh(dx+c)+1)}{2d(a+b)^3}+\frac{7(\tanh(dx+c))^3 b^2}{8d(a+b)^3\left(a+b(\tanh(dx+c))^2\right)^2}+\frac{5b^3(\tanh(dx+c))^3}{4d(a+b)^3\left(a+b(\tanh(dx+c))^2\right)^2 a}+\frac{3b}{8d(a+b)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 1/2/d/(a+b)^3*ln(tanh(d*x+c)+1)+7/8/d/(a+b)^3/(a+b*tanh(d*x+c)^2)^2*tanh(d*
x+c)^3*b^2+5/4/d/(a+b)^3/(a+b*tanh(d*x+c)^2)^2*b^3/a*tanh(d*x+c)^3+3/8/d/(a
+b)^3*b^4/(a+b*tanh(d*x+c)^2)^2/a^2*tanh(d*x+c)^3+9/8/d/(a+b)^3/(a+b*tanh(d
*x+c)^2)^2*a*b*tanh(d*x+c)+7/4/d/(a+b)^3/(a+b*tanh(d*x+c)^2)^2*tanh(d*x+c)*
b^2+5/8/d/(a+b)^3*b^3/(a+b*tanh(d*x+c)^2)^2/a*tanh(d*x+c)+15/8/d/(a+b)^3*b/
(a*b)^(1/2)*arctan(tanh(d*x+c)*b/(a*b)^(1/2))+5/4/d/(a+b)^3/a/(a*b)^(1/2)*a
rctan(tanh(d*x+c)*b/(a*b)^(1/2))*b^2+3/8/d/(a+b)^3*b^3/a^2/(a*b)^(1/2)*arct
an(tanh(d*x+c)*b/(a*b)^(1/2))-1/2/d/(a+b)^3*ln(tanh(d*x+c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.36689, size = 17204, normalized size = 121.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)^8 + 128*(a^4 + 2*a^3*
b + a^2*b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 16*(a^4 + 2*a^3*b + a^2*b^
2)*d*x*sinh(d*x + c)^8 - 4*(9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4 - 16*(a^4
- a^2*b^2)*d*x)*cosh(d*x + c)^6 + 4*(112*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh
(d*x + c)^2 - 9*a^3*b + a^2*b^2 + 13*a*b^3 + 3*b^4 + 16*(a^4 - a^2*b^2)*d*x
)*sinh(d*x + c)^6 + 8*(112*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)^3 -
3*(9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4 - 16*(a^4 - a^2*b^2)*d*x)*cosh(d*x
+ c))*sinh(d*x + c)^5 - 4*(27*a^3*b - 9*a^2*b^2 + 21*a*b^3 + 9*b^4 - 8*(3*a
^4 - 2*a^3*b + 3*a^2*b^2)*d*x)*cosh(d*x + c)^4 + 4*(280*(a^4 + 2*a^3*b + a^
2*b^2)*d*x*cosh(d*x + c)^4 - 27*a^3*b + 9*a^2*b^2 - 21*a*b^3 - 9*b^4 + 8*(3
*a^4 - 2*a^3*b + 3*a^2*b^2)*d*x - 15*(9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4
- 16*(a^4 - a^2*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 36*a^3*b - 84*
a^2*b^2 - 60*a*b^3 - 12*b^4 + 16*(56*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x
```

```
+ c)^5 - 5*(9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4 - 16*(a^4 - a^2*b^2)*d*x)
*cosh(d*x + c)^3 - (27*a^3*b - 9*a^2*b^2 + 21*a*b^3 + 9*b^4 - 8*(3*a^4 - 2*
a^3*b + 3*a^2*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 16*(a^4 + 2*a^3*b
+ a^2*b^2)*d*x - 4*(27*a^3*b + 13*a^2*b^2 - 23*a*b^3 - 9*b^4 - 16*(a^4 - a^
2*b^2)*d*x)*cosh(d*x + c)^2 + 4*(112*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x
 + c)^6 - 15*(9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4 - 16*(a^4 - a^2*b^2)*d*x
)*cosh(d*x + c)^4 - 27*a^3*b - 13*a^2*b^2 + 23*a*b^3 + 9*b^4 + 16*(a^4 - a^
2*b^2)*d*x - 6*(27*a^3*b - 9*a^2*b^2 + 21*a*b^3 + 9*b^4 - 8*(3*a^4 - 2*a^3*
b + 3*a^2*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((15*a^4 + 40*a^3*b
+ 38*a^2*b^2 + 16*a*b^3 + 3*b^4)*cosh(d*x + c)^8 + 8*(15*a^4 + 40*a^3*b + 3
8*a^2*b^2 + 16*a*b^3 + 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (15*a^4 + 40*
a^3*b + 38*a^2*b^2 + 16*a*b^3 + 3*b^4)*sinh(d*x + c)^8 + 4*(15*a^4 + 10*a^3
*b - 12*a^2*b^2 - 10*a*b^3 - 3*b^4)*cosh(d*x + c)^6 + 4*(15*a^4 + 10*a^3*b
- 12*a^2*b^2 - 10*a*b^3 - 3*b^4 + 7*(15*a^4 + 40*a^3*b + 38*a^2*b^2 + 16*a*
b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(15*a^4 + 40*a^3*b + 3
8*a^2*b^2 + 16*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + 3*(15*a^4 + 10*a^3*b - 12*a
^2*b^2 - 10*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(45*a^4 + 34*
a^2*b^2 + 24*a*b^3 + 9*b^4)*cosh(d*x + c)^4 + 2*(35*(15*a^4 + 40*a^3*b + 38
*a^2*b^2 + 16*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 45*a^4 + 34*a^2*b^2 + 24*a*b
^3 + 9*b^4 + 30*(15*a^4 + 10*a^3*b - 12*a^2*b^2 - 10*a*b^3 - 3*b^4)*cosh(d*
x + c)^2)*sinh(d*x + c)^4 + 15*a^4 + 40*a^3*b + 38*a^2*b^2 + 16*a*b^3 + 3*b
^4 + 8*(7*(15*a^4 + 40*a^3*b + 38*a^2*b^2 + 16*a*b^3 + 3*b^4)*cosh(d*x + c)
^5 + 10*(15*a^4 + 10*a^3*b - 12*a^2*b^2 - 10*a*b^3 - 3*b^4)*cosh(d*x + c)^3
 + (45*a^4 + 34*a^2*b^2 + 24*a*b^3 + 9*b^4)*cosh(d*x + c))*sinh(d*x + c)^3
+ 4*(15*a^4 + 10*a^3*b - 12*a^2*b^2 - 10*a*b^3 - 3*b^4)*cosh(d*x + c)^2 + 4
*(7*(15*a^4 + 40*a^3*b + 38*a^2*b^2 + 16*a*b^3 + 3*b^4)*cosh(d*x + c)^6 + 1
5*(15*a^4 + 10*a^3*b - 12*a^2*b^2 - 10*a*b^3 - 3*b^4)*cosh(d*x + c)^4 + 15*
a^4 + 10*a^3*b - 12*a^2*b^2 - 10*a*b^3 - 3*b^4 + 3*(45*a^4 + 34*a^2*b^2 + 2
4*a*b^3 + 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((15*a^4 + 40*a^3*b +
 38*a^2*b^2 + 16*a*b^3 + 3*b^4)*cosh(d*x + c)^7 + 3*(15*a^4 + 10*a^3*b - 12
*a^2*b^2 - 10*a*b^3 - 3*b^4)*cosh(d*x + c)^5 + (45*a^4 + 34*a^2*b^2 + 24*a*
b^3 + 9*b^4)*cosh(d*x + c)^3 + (15*a^4 + 10*a^3*b - 12*a^2*b^2 - 10*a*b^3 -
 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*c
osh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2
 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2
 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b
+ b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))
*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x +
c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((a
 + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*s
inh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 +
 a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c
))*sinh(d*x + c) + a + b)) + 8*(16*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x +
 c)^7 - 3*(9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4 - 16*(a^4 - a^2*b^2)*d*x)*c
osh(d*x + c)^5 - 2*(27*a^3*b - 9*a^2*b^2 + 21*a*b^3 + 9*b^4 - 8*(3*a^4 - 2*
a^3*b + 3*a^2*b^2)*d*x)*cosh(d*x + c)^3 - (27*a^3*b + 13*a^2*b^2 - 23*a*b^3
 - 9*b^4 - 16*(a^4 - a^2*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^7 + 5*
a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^8 +
8*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*cosh(d*
x + c)*sinh(d*x + c)^7 + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b
^4 + a^2*b^5)*d*sinh(d*x + c)^8 + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3
- 3*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^6 + 4*(7*(a^7 + 5*a^6*b + 10*a^5*b^2
 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^2 + (a^7 + 3*a^6*b + 2
*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*d)*sinh(d*x + c)^6 + 2*(3*a^7 +
 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)*d*cosh(d*x + c)^4
 + 8*(7*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*c
osh(d*x + c)^3 + 3*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2
*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^7 + 5*a^6*b + 10*a^5*b^2
+ 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^4 + 30*(a^7 + 3*a^6*b +
```

2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^2 + (3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)*d)*sinh(d*x + c)^4 + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^5 + 10*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^3 + (3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^6 + 15*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^4 + 3*(3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)*d*cosh(d*x + c)^2 + (a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*d)*sinh(d*x + c)^2 + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d + 8*((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^7 + 3*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^5 + (3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)*d*cosh(d*x + c)^3 + (a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*(8*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)^8 + 64*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 8*(a^4 + 2*a^3*b + a^2*b^2)*d*x*sinh(d*x + c)^8 - 2*(9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4 - 16*(a^4 - a^2*b^2)*d*x)*cosh(d*x + c)^6 + 2*(112*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)^2 - 9*a^3*b + a^2*b^2 + 13*a*b^3 + 3*b^4 + 16*(a^4 - a^2*b^2)*d*x)*sinh(d*x + c)^6 + 4*(112*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)^3 - 3*(9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4 - 16*(a^4 - a^2*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(27*a^3*b - 9*a^2*b^2 + 21*a*b^3 + 9*b^4 - 8*(3*a^4 - 2*a^3*b + 3*a^2*b^2)*d*x)*cosh(d*x + c)^4 + 2*(280*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)^4 - 27*a^3*b + 9*a^2*b^2 - 21*a*b^3 - 9*b^4 + 8*(3*a^4 - 2*a^3*b + 3*a^2*b^2)*d*x - 15*(9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4 - 16*(a^4 - a^2*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 18*a^3*b - 42*a^2*b^2 - 30*a*b^3 - 6*b^4 + 8*(56*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)^5 - 5*(9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4 - 16*(a^4 - a^2*b^2)*d*x)*cosh(d*x + c)^3 - (27*a^3*b - 9*a^2*b^2 + 21*a*b^3 + 9*b^4 - 8*(3*a^4 - 2*a^3*b + 3*a^2*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 8*(a^4 + 2*a^3*b + a^2*b^2)*d*x - 2*(27*a^3*b + 13*a^2*b^2 - 23*a*b^3 - 9*b^4 - 16*(a^4 - a^2*b^2)*d*x)*cosh(d*x + c)^2 + 2*(112*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)^6 - 15*(9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4 - 16*(a^4 - a^2*b^2)*d*x)*cosh(d*x + c)^4 - 27*a^3*b - 13*a^2*b^2 + 23*a*b^3 + 9*b^4 + 16*(a^4 - a^2*b^2)*d*x - 6*(27*a^3*b - 9*a^2*b^2 + 21*a*b^3 + 9*b^4 - 8*(3*a^4 - 2*a^3*b + 3*a^2*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((15*a^4 + 40*a^3*b + 38*a^2*b^2 + 16*a*b^3 + 3*b^4)*cosh(d*x + c)^8 + 8*(15*a^4 + 40*a^3*b + 38*a^2*b^2 + 16*a*b^3 + 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (15*a^4 + 40*a^3*b + 38*a^2*b^2 + 16*a*b^3 + 3*b^4)*sinh(d*x + c)^8 + 4*(15*a^4 + 10*a^3*b - 12*a^2*b^2 - 10*a*b^3 - 3*b^4)*cosh(d*x + c)^6 + 4*(15*a^4 + 10*a^3*b - 12*a^2*b^2 - 10*a*b^3 - 3*b^4 + 7*(15*a^4 + 40*a^3*b + 38*a^2*b^2 + 16*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(15*a^4 + 40*a^3*b + 38*a^2*b^2 + 16*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + 3*(15*a^4 + 10*a^3*b - 12*a^2*b^2 - 10*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(45*a^4 + 34*a^2*b^2 + 24*a*b^3 + 9*b^4)*cosh(d*x + c)^4 + 2*(35*(15*a^4 + 40*a^3*b + 38*a^2*b^2 + 16*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 45*a^4 + 34*a^2*b^2 + 24*a*b^3 + 9*b^4 + 30*(15*a^4 + 10*a^3*b - 12*a^2*b^2 - 10*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 15*a^4 + 40*a^3*b + 38*a^2*b^2 + 16*a*b^3 + 3*b^4 + 8*(7*(15*a^4 + 40*a^3*b + 38*a^2*b^2 + 16*a*b^3 + 3*b^4)*cosh(d*x + c)^5 + 10*(15*a^4 + 10*a^3*b - 12*a^2*b^2 - 10*a*b^3 - 3*b^4)*cosh(d*x + c)^3 + (45*a^4 + 34*a^2*b^2 + 24*a*b^3 + 9*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(15*a^4 + 10*a^3*b - 12*a^2*b^2 - 10*a*b^3 - 3*b^4)*cosh(d*x + c)^2 + 4*(7*(15*a^4 + 40*a^3*b + 38*a^2*b^2 + 16*a*b^3 + 3*b^4)*cosh(d*x + c)^6 + 15*(15*a^4 + 10*a^3*b - 12*a^2*b^2 - 10*a*b^3 - 3*b^4)*cosh(d*x + c)^4 + 15*a^4 + 10*a^3*b - 12*a^2*b^2 - 10*a*b^3 - 3*b^4 + 3*(45*a^4 + 34*a^2*b^2 + 24*a*b^3 + 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((15*a^4 + 40*a^3*b + 38*a^

```
2*b^2 + 16*a*b^3 + 3*b^4)*cosh(d*x + c)^7 + 3*(15*a^4 + 10*a^3*b - 12*a^2*b
^2 - 10*a*b^3 - 3*b^4)*cosh(d*x + c)^5 + (45*a^4 + 34*a^2*b^2 + 24*a*b^3 +
9*b^4)*cosh(d*x + c)^3 + (15*a^4 + 10*a^3*b - 12*a^2*b^2 - 10*a*b^3 - 3*b^4
)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*((a + b)*cosh(d*x + c)
^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a -
b)*sqrt(b/a)/b) + 4*(16*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)^7 - 3*(
9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4 - 16*(a^4 - a^2*b^2)*d*x)*cosh(d*x + c
)^5 - 2*(27*a^3*b - 9*a^2*b^2 + 21*a*b^3 + 9*b^4 - 8*(3*a^4 - 2*a^3*b + 3*a
^2*b^2)*d*x)*cosh(d*x + c)^3 - (27*a^3*b + 13*a^2*b^2 - 23*a*b^3 - 9*b^4 -
16*(a^4 - a^2*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^7 + 5*a^6*b + 10*
a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^8 + 8*(a^7 + 5*
a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)*sinh
(d*x + c)^7 + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^
5)*d*sinh(d*x + c)^8 + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4
 - a^2*b^5)*d*cosh(d*x + c)^6 + 4*(7*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b
^3 + 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^2 + (a^7 + 3*a^6*b + 2*a^5*b^2 -
2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*d)*sinh(d*x + c)^6 + 2*(3*a^7 + 7*a^6*b +
6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^
7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c
)^3 + 3*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*d*cos
h(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^
3 + 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^4 + 30*(a^7 + 3*a^6*b + 2*a^5*b^2
- 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^2 + (3*a^7 + 7*a^6*b + 6
*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)*d)*sinh(d*x + c)^4 + 4*(a^7 +
 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^2 +
 8*(7*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*cos
h(d*x + c)^5 + 10*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*
b^5)*d*cosh(d*x + c)^3 + (3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b
^4 + 3*a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^7 + 5*a^6*b + 10
*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^6 + 15*(a^7 +
3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^4 +
3*(3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)*d*cosh(
d*x + c)^2 + (a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*
d)*sinh(d*x + c)^2 + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 +
 a^2*b^5)*d + 8*((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2
*b^5)*d*cosh(d*x + c)^7 + 3*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*
b^4 - a^2*b^5)*d*cosh(d*x + c)^5 + (3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3
 + 7*a^3*b^4 + 3*a^2*b^5)*d*cosh(d*x + c)^3 + (a^7 + 3*a^6*b + 2*a^5*b^2 -
2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.22079, size = 567, normalized size = 3.99

$$\frac{\left(15\,a^2b+10\,ab^2+3\,b^3\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{8\,\left(a^5d+3\,a^4bd+3\,a^3b^2d+a^2b^3d\right)\sqrt{ab}}+\frac{dx+c}{a^3d+3\,a^2bd+3\,ab^2d+b^3d}-\frac{9\,a^3be^{(6\,dx+6\,c)}-a^2b^2e^{(6\,dx+6\,c)}-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(15*a^2*b + 10*a*b^2 + 3*b^3)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*
x + 2*c) + a - b)/sqrt(a*b))/((a^5*d + 3*a^4*b*d + 3*a^3*b^2*d + a^2*b^3*d)
*sqrt(a*b)) + (d*x + c)/(a^3*d + 3*a^2*b*d + 3*a*b^2*d + b^3*d) - 1/4*(9*a^
3*b*e^(6*d*x + 6*c) - a^2*b^2*e^(6*d*x + 6*c) - 13*a*b^3*e^(6*d*x + 6*c) -
3*b^4*e^(6*d*x + 6*c) + 27*a^3*b*e^(4*d*x + 4*c) - 9*a^2*b^2*e^(4*d*x + 4*c
) + 21*a*b^3*e^(4*d*x + 4*c) + 9*b^4*e^(4*d*x + 4*c) + 27*a^3*b*e^(2*d*x +
2*c) + 13*a^2*b^2*e^(2*d*x + 2*c) - 23*a*b^3*e^(2*d*x + 2*c) - 9*b^4*e^(2*d
*x + 2*c) + 9*a^3*b + 21*a^2*b^2 + 15*a*b^3 + 3*b^4)/((a^5*d + 3*a^4*b*d +
3*a^3*b^2*d + a^2*b^3*d)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*
d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2)
```

3.197 $\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3} dx$

Optimal. Leaf size=138

$$-\frac{b\left(3a^2+3ab+b^2\right)\log\left(a+b\tanh^2(c+dx)\right)}{2a^3d(a+b)^3}+\frac{b(2a+b)}{2a^2d(a+b)^2\left(a+b\tanh^2(c+dx)\right)}+\frac{\log(\tanh(c+dx))}{a^3d}+\frac{}{4ad(a+b)\left(\right.}$$

```
[Out] Log[Cosh[c + d*x]]/((a + b)^3*d) + Log[Tanh[c + d*x]]/(a^3*d) - (b*(3*a^2 +
 3*a*b + b^2)*Log[a + b*Tanh[c + d*x]^2])/(2*a^3*(a + b)^3*d) + b/(4*a*(a +
 b)*d*(a + b*Tanh[c + d*x]^2)^2) + (b*(2*a + b))/(2*a^2*(a + b)^2*d*(a + b*
Tanh[c + d*x]^2))
```

Rubi [A] time = 0.204705, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3670, 446, 72}

$$-\frac{b\left(3a^2+3ab+b^2\right)\log\left(a+b\tanh^2(c+dx)\right)}{2a^3d(a+b)^3}+\frac{b(2a+b)}{2a^2d(a+b)^2\left(a+b\tanh^2(c+dx)\right)}+\frac{\log(\tanh(c+dx))}{a^3d}+\frac{}{4ad(a+b)\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] Log[Cosh[c + d*x]]/((a + b)^3*d) + Log[Tanh[c + d*x]]/(a^3*d) - (b*(3*a^2 +
 3*a*b + b^2)*Log[a + b*Tanh[c + d*x]^2])/(2*a^3*(a + b)^3*d) + b/(4*a*(a +
 b)*d*(a + b*Tanh[c + d*x]^2)^2) + (b*(2*a + b))/(2*a^2*(a + b)^2*d*(a + b*
Tanh[c + d*x]^2))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(1-x^2)\left(a+bx^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(1-x)x(a+bx)^3}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^3(-1+x)} + \frac{1}{a^3x} - \frac{b^2}{a(a+b)(a+bx)^3} - \frac{b^2(2a+b)}{a^2(a+b)^2(a+bx)^2} - \frac{b^2(3a^2+3ab+b^2)}{a^3(a+b)^3(a+bx)}\right) dx, x, \tan\right.}{2d}$$

$$= \frac{\log(\cosh(c+dx))}{(a+b)^3d} + \frac{\log(\tanh(c+dx))}{a^3d} - \frac{b\left(3a^2+3ab+b^2\right)\log\left(a+b\tanh^2(c+dx\right.}{2a^3(a+b)^3d}$$

Mathematica [A] time = 1.56875, size = 117, normalized size = 0.85

$$\frac{\frac{\frac{b\left(\frac{a(a+b)\left(2b(2a+b)\tanh^2(c+dx)+a(5a+3b)\right)}{\left(a+b\tanh^2(c+dx)\right)^2}-2\left(3a^2+3ab+b^2\right)\log\left(a+b\tanh^2(c+dx)\right)\right)}{(a+b)^3}+4\log(\tanh(c+dx))}{a^3}+\frac{4\log(\cosh(c+dx))}{(a+b)^3}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((4*Log[Cosh[c + d*x]])/(a + b)^3 + (4*Log[Tanh[c + d*x]] + (b*(-2*(3*a^2 +
 3*a*b + b^2)*Log[a + b*Tanh[c + d*x]^2] + (a*(a + b)*(a*(5*a + 3*b) + 2*b*
(2*a + b)*Tanh[c + d*x]^2))/(a + b*Tanh[c + d*x]^2)^2))/(a + b)^3)/a^3)/(4*
d)
```

Maple [B] time = 0.117, size = 952, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] -1/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)+1)-6/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)
^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+
1/2*c)^6-10/d*b^3/(a+b)^3/a/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^
2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^6-4/d*b^4/(a+b)^3/a^
2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^
2*b+a)^2*tanh(1/2*d*x+1/2*c)^6-12/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+2*
tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^
4-40/d*b^3/(a+b)^3/a/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*t
anh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^4-40/d*b^4/(a+b)^3/a^2/(tan
h(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)
^2*tanh(1/2*d*x+1/2*c)^4-12/d*b^5/(a+b)^3/a^3/(tanh(1/2*d*x+1/2*c)^4*a+2*ta
nh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^4-
6/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1
/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^2-10/d*b^3/(a+b)^3/a/(tanh(1/2*d
*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh
```

```
(1/2*d*x+1/2*c)^2-4/d*b^4/(a+b)^3/a^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d
*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^2-3/2/d*b/
(a+b)^3/a*ln(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d
*x+1/2*c)^2*b+a)-3/2/d*b^2/(a+b)^3/a^2*ln(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/
2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)-1/2/d*b^3/(a+b)^3/a^3*ln(tanh
(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)+
1/d/a^3*ln(tanh(1/2*d*x+1/2*c))-1/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.30807, size = 672, normalized size = 4.87

$$-\frac{\left(3\,a^{2}b+3\,ab^{2}+b^{3}\right)\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\left(a^{6}+3\,a^{5}b+3\,a^{4}b^{2}+a^{3}b^{3}\right)d}+\frac{dx+c}{\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)d}+\frac{}{\left(a^{7}+5\,a^{6}b+10\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(3*a^2*b + 3*a*b^2 + b^3)*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(
-4*d*x - 4*c) + a + b)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d) + (d*x + c
)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + 2*((3*a^2*b^2 + 4*a*b^3 + b^4)*e^(-
2*d*x - 2*c) + 2*(3*a^2*b^2 - a*b^3 - b^4)*e^(-4*d*x - 4*c) + (3*a^2*b^2 +
4*a*b^3 + b^4)*e^(-6*d*x - 6*c))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3
+ 5*a^3*b^4 + a^2*b^5 + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^
4 - a^2*b^5)*e^(-2*d*x - 2*c) + 2*(3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3
+ 7*a^3*b^4 + 3*a^2*b^5)*e^(-4*d*x - 4*c) + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 -
2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*e^(-6*d*x - 6*c) + (a^7 + 5*a^6*b + 10*a^5
*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*e^(-8*d*x - 8*c))*d) + log(e^(-d*x
 - c) + 1)/(a^3*d) + log(e^(-d*x - c) - 1)/(a^3*d)
```

Fricas [B] time = 5.28184, size = 10714, normalized size = 77.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^8 + 16*(a^5 + 2*a^4*b +
 a^3*b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 2*(a^5 + 2*a^4*b + a^3*b^2)*d
*x*sinh(d*x + c)^8 - 4*(3*a^3*b^2 + 4*a^2*b^3 + a*b^4 - 2*(a^5 - a^3*b^2)*d
*x)*cosh(d*x + c)^6 - 4*(3*a^3*b^2 + 4*a^2*b^3 + a*b^4 - 14*(a^5 + 2*a^4*b
+ a^3*b^2)*d*x*cosh(d*x + c)^2 - 2*(a^5 - a^3*b^2)*d*x)*sinh(d*x + c)^6 + 8
*(14*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^3 - 3*(3*a^3*b^2 + 4*a^2*b
^3 + a*b^4 - 2*(a^5 - a^3*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 4*(6*a
^3*b^2 - 2*a^2*b^3 - 2*a*b^4 - (3*a^5 - 2*a^4*b + 3*a^3*b^2)*d*x)*cosh(d*x
+ c)^4 + 4*(35*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^4 - 6*a^3*b^2 +
2*a^2*b^3 + 2*a*b^4 + (3*a^5 - 2*a^4*b + 3*a^3*b^2)*d*x - 15*(3*a^3*b^2 + 4
*a^2*b^3 + a*b^4 - 2*(a^5 - a^3*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4
+ 16*(7*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^5 - 5*(3*a^3*b^2 + 4*a^
2*b^3 + a*b^4 - 2*(a^5 - a^3*b^2)*d*x)*cosh(d*x + c)^3 - (6*a^3*b^2 - 2*a^2
*b^3 - 2*a*b^4 - (3*a^5 - 2*a^4*b + 3*a^3*b^2)*d*x)*cosh(d*x + c))*sinh(d*x
 + c)^3 + 2*(a^5 + 2*a^4*b + a^3*b^2)*d*x - 4*(3*a^3*b^2 + 4*a^2*b^3 + a*b^
4 - 2*(a^5 - a^3*b^2)*d*x)*cosh(d*x + c)^2 + 4*(14*(a^5 + 2*a^4*b + a^3*b^2
)*d*x*cosh(d*x + c)^6 - 3*a^3*b^2 - 4*a^2*b^3 - a*b^4 - 15*(3*a^3*b^2 + 4*a
```

```
^2*b^3 + a*b^4 - 2*(a^5 - a^3*b^2)*d*x)*cosh(d*x + c)^4 + 2*(a^5 - a^3*b^2)
*d*x - 6*(6*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 - (3*a^5 - 2*a^4*b + 3*a^3*b^2)*d
*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((3*a^4*b + 9*a^3*b^2 + 10*a^2*b^3 +
 5*a*b^4 + b^5)*cosh(d*x + c)^8 + 8*(3*a^4*b + 9*a^3*b^2 + 10*a^2*b^3 + 5*a
*b^4 + b^5)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^4*b + 9*a^3*b^2 + 10*a^2*b
^3 + 5*a*b^4 + b^5)*sinh(d*x + c)^8 + 4*(3*a^4*b + 3*a^3*b^2 - 2*a^2*b^3 -
3*a*b^4 - b^5)*cosh(d*x + c)^6 + 4*(3*a^4*b + 3*a^3*b^2 - 2*a^2*b^3 - 3*a*b
^4 - b^5 + 7*(3*a^4*b + 9*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(d*x +
c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^4*b + 9*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 +
b^5)*cosh(d*x + c)^3 + 3*(3*a^4*b + 3*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*
cosh(d*x + c))*sinh(d*x + c)^5 + 3*a^4*b + 9*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4
 + b^5 + 2*(9*a^4*b + 3*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*cosh(d*x + c
)^4 + 2*(9*a^4*b + 3*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5 + 35*(3*a^4*b +
9*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(d*x + c)^4 + 30*(3*a^4*b + 3*a
^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7
*(3*a^4*b + 9*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(d*x + c)^5 + 10*(3
*a^4*b + 3*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(d*x + c)^3 + (9*a^4*b
+ 3*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 +
 4*(3*a^4*b + 3*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(d*x + c)^2 + 4*(7
*(3*a^4*b + 9*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(d*x + c)^6 + 3*a^4
*b + 3*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5 + 15*(3*a^4*b + 3*a^3*b^2 - 2*a^
2*b^3 - 3*a*b^4 - b^5)*cosh(d*x + c)^4 + 3*(9*a^4*b + 3*a^3*b^2 + 6*a^2*b^3
 + 7*a*b^4 + 3*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((3*a^4*b + 9*a^3*
b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(d*x + c)^7 + 3*(3*a^4*b + 3*a^3*b^2
- 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(d*x + c)^5 + (9*a^4*b + 3*a^3*b^2 + 6*a^2
*b^3 + 7*a*b^4 + 3*b^5)*cosh(d*x + c)^3 + (3*a^4*b + 3*a^3*b^2 - 2*a^2*b^3
- 3*a*b^4 - b^5)*cosh(d*x + c))*sinh(d*x + c))*log(2*((a + b)*cosh(d*x + c)
^2 + (a + b)*sinh(d*x + c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*si
nh(d*x + c) + sinh(d*x + c)^2)) - 2*((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b
^3 + 5*a*b^4 + b^5)*cosh(d*x + c)^8 + 8*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^
2*b^3 + 5*a*b^4 + b^5)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^5 + 5*a^4*b + 10*
a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*sinh(d*x + c)^8 + 4*(a^5 + 3*a^4*b +
2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(d*x + c)^6 + 4*(a^5 + 3*a^4*b +
 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5 + 7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10
*a^2*b^3 + 5*a*b^4 + b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^5 + 5*
a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(d*x + c)^3 + 3*(a^5 +
 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(d*x + c))*sinh(d*x +
 c)^5 + a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5 + 2*(3*a^5
+ 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*cosh(d*x + c)^4 + 2*(3
*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5 + 35*(a^5 + 5*a^4*
b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(d*x + c)^4 + 30*(a^5 + 3*
a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(d*x + c)^2)*sinh(d*x +
c)^4 + 8*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(
d*x + c)^5 + 10*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cos
h(d*x + c)^3 + (3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*
cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 -
 3*a*b^4 - b^5)*cosh(d*x + c)^2 + 4*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2
*b^3 + 5*a*b^4 + b^5)*cosh(d*x + c)^6 + a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b
^3 - 3*a*b^4 - b^5 + 15*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 -
b^5)*cosh(d*x + c)^4 + 3*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4
 + 3*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^5 + 5*a^4*b + 10*a^3*b^2
 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(d*x + c)^7 + 3*(a^5 + 3*a^4*b + 2*a^3*b
^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(d*x + c)^5 + (3*a^5 + 7*a^4*b + 6*a^3*
b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*cosh(d*x + c)^3 + (a^5 + 3*a^4*b + 2*a^3
*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(
d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 8*(2*(a^5 + 2*a^4*b + a^3*b^2)*
d*x*cosh(d*x + c)^7 - 3*(3*a^3*b^2 + 4*a^2*b^3 + a*b^4 - 2*(a^5 - a^3*b^2)*
d*x)*cosh(d*x + c)^5 - 2*(6*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 - (3*a^5 - 2*a^4*
b + 3*a^3*b^2)*d*x)*cosh(d*x + c)^3 - (3*a^3*b^2 + 4*a^2*b^3 + a*b^4 - 2*(a
```

```
^5 - a^3*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^8 + 5*a^7*b + 10*a^6*b
^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^8 + 8*(a^8 + 5*a^7*b
 + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)*sinh(d*x
+ c)^7 + (a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*
sinh(d*x + c)^8 + 4*(a^8 + 3*a^7*b + 2*a^6*b^2 - 2*a^5*b^3 - 3*a^4*b^4 - a^
3*b^5)*d*cosh(d*x + c)^6 + 4*(7*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 +
5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^2 + (a^8 + 3*a^7*b + 2*a^6*b^2 - 2*a^5
*b^3 - 3*a^4*b^4 - a^3*b^5)*d)*sinh(d*x + c)^6 + 2*(3*a^8 + 7*a^7*b + 6*a^6
*b^2 + 6*a^5*b^3 + 7*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^8 + 5
*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^3 +
 3*(a^8 + 3*a^7*b + 2*a^6*b^2 - 2*a^5*b^3 - 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x
 + c))*sinh(d*x + c)^5 + 2*(35*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5
*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^4 + 30*(a^8 + 3*a^7*b + 2*a^6*b^2 - 2*a
^5*b^3 - 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^2 + (3*a^8 + 7*a^7*b + 6*a^6*
b^2 + 6*a^5*b^3 + 7*a^4*b^4 + 3*a^3*b^5)*d)*sinh(d*x + c)^4 + 4*(a^8 + 3*a^
7*b + 2*a^6*b^2 - 2*a^5*b^3 - 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^2 + 8*(7
*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x
 + c)^5 + 10*(a^8 + 3*a^7*b + 2*a^6*b^2 - 2*a^5*b^3 - 3*a^4*b^4 - a^3*b^5)*
d*cosh(d*x + c)^3 + (3*a^8 + 7*a^7*b + 6*a^6*b^2 + 6*a^5*b^3 + 7*a^4*b^4 +
3*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^8 + 5*a^7*b + 10*a^6*
b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^6 + 15*(a^8 + 3*a^7
*b + 2*a^6*b^2 - 2*a^5*b^3 - 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^4 + 3*(3*
a^8 + 7*a^7*b + 6*a^6*b^2 + 6*a^5*b^3 + 7*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x +
 c)^2 + (a^8 + 3*a^7*b + 2*a^6*b^2 - 2*a^5*b^3 - 3*a^4*b^4 - a^3*b^5)*d)*si
nh(d*x + c)^2 + (a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*
b^5)*d + 8*((a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)
*d*cosh(d*x + c)^7 + 3*(a^8 + 3*a^7*b + 2*a^6*b^2 - 2*a^5*b^3 - 3*a^4*b^4 -
 a^3*b^5)*d*cosh(d*x + c)^5 + (3*a^8 + 7*a^7*b + 6*a^6*b^2 + 6*a^5*b^3 + 7*
a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^3 + (a^8 + 3*a^7*b + 2*a^6*b^2 - 2*a^5
*b^3 - 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral(coth(c + d*x)/(a + b*tanh(c + d*x)**2)**3, x)
```

Giac [B] time = 1.28943, size = 417, normalized size = 3.02

$$-\frac{\left(3\,a^2b+3\,ab^2+b^3\right)\log\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)}{2\left(a^6d+3\,a^5bd+3\,a^4b^2d+a^3b^3d\right)}-\frac{dx+c}{a^3d+3\,a^2bd+3\,ab^2d+b^3d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/2*(3*a^2*b + 3*a*b^2 + b^3)*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) +
2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)/(a^6*d + 3*a^5*b*d + 3*a
```

```
^4*b^2*d + a^3*b^3*d) - (d*x + c)/(a^3*d + 3*a^2*b*d + 3*a*b^2*d + b^3*d) +
 log(abs(-e^(2*d*x + 2*c) + 1))/(a^3*d) + 2*((3*a^2*b^2 + a*b^3)*e^(6*d*x +
 6*c) + (3*a^2*b^2 + a*b^3)*e^(2*d*x + 2*c) + 2*(3*a^3*b^2 - a^2*b^3 - a*b^
4)*e^(4*d*x + 4*c)/(a + b))/((a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e
^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2*(a + b)^2*a^3*d)
```

3.198 $\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=178

$$-\frac{\left(8a^2+27ab+15b^2\right)\coth(c+dx)}{8a^3d(a+b)^2}-\frac{b^{3/2}\left(35a^2+42ab+15b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}d(a+b)^3}+\frac{b(9a+5b)\coth(c+dx)}{8a^2d(a+b)^2\left(a+b\tanh^2(c+dx\right.}$$

```
[Out] x/(a + b)^3 - (b^(3/2)*(35*a^2 + 42*a*b + 15*b^2)*ArcTan[(Sqrt[b]*Tanh[c +
d*x])/Sqrt[a]])/(8*a^(7/2)*(a + b)^3*d) - ((8*a^2 + 27*a*b + 15*b^2)*Coth[c
 + d*x])/(8*a^3*(a + b)^2*d) + (b*Coth[c + d*x])/(4*a*(a + b)*d*(a + b*Tanh
[c + d*x]^2)^2) + (b*(9*a + 5*b)*Coth[c + d*x])/(8*a^2*(a + b)^2*d*(a + b*T
anh[c + d*x]^2))
```

Rubi [A] time = 0.290858, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3670, 472, 579, 583, 522, 206, 205}

$$-\frac{\left(8a^2+27ab+15b^2\right)\coth(c+dx)}{8a^3d(a+b)^2}-\frac{b^{3/2}\left(35a^2+42ab+15b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}d(a+b)^3}+\frac{b(9a+5b)\coth(c+dx)}{8a^2d(a+b)^2\left(a+b\tanh^2(c+dx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```


```
[Out] x/(a + b)^3 - (b^(3/2)*(35*a^2 + 42*a*b + 15*b^2)*ArcTan[(Sqrt[b]*Tanh[c +
d*x])/Sqrt[a]])/(8*a^(7/2)*(a + b)^3*d) - ((8*a^2 + 27*a*b + 15*b^2)*Coth[c
 + d*x])/(8*a^3*(a + b)^2*d) + (b*Coth[c + d*x])/(4*a*(a + b)*d*(a + b*Tanh
[c + d*x]^2)^2) + (b*(9*a + 5*b)*Coth[c + d*x])/(8*a^2*(a + b)^2*d*(a + b*T
anh[c + d*x]^2))
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
```

```
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1-x^2)\left(a+bx^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\coth(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-4a-5b+5bx^2}{x^2(1-x^2)\left(a+bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{4a(a+b)d}$$

$$= \frac{b\coth(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(9a+5b)\coth(c+dx)}{8a^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{8a^2+}{}\right.}{}$$

$$= -\frac{\left(8a^2+27ab+15b^2\right)\coth(c+dx)}{8a^3(a+b)^2d} + \frac{b\coth(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(9a+5}{8a^2(a+b)^2d}$$

$$= -\frac{\left(8a^2+27ab+15b^2\right)\coth(c+dx)}{8a^3(a+b)^2d} + \frac{b\coth(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(9a+5}{8a^2(a+b)^2d}$$

$$= \frac{x}{(a+b)^3} - \frac{b^{3/2}\left(35a^2+42ab+15b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}(a+b)^3d} - \frac{\left(8a^2+27ab+15b^2\right)\coth(c}{8a^3(a+b)^2d}$$

Mathematica [A] time = 5.76014, size = 166, normalized size = 0.93

$$-\frac{\frac{b^{3/2}\left(35a^2+42ab+15b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{7/2}(a+b)^3} + \frac{4b^3\sinh(2(c+dx))}{a^2(a+b)^2((a+b)\cosh(2(c+dx))+a-b)^2} + \frac{b^2(13a+7b)\sinh(2(c+dx))}{a^3(a+b)^2((a+b)\cosh(2(c+dx))+a-b)} + \frac{8\coth(c+dx)}{a^3} - \frac{8(c+dx)}{(a+b)^3}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -((-8*(c + d*x))/(a + b)^3 + (b^(3/2)*(35*a^2 + 42*a*b + 15*b^2)*ArcTan[(Sq
rt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(7/2)*(a + b)^3) + (8*Coth[c + d*x])/a^3
+ (4*b^3*Sinh[2*(c + d*x)])/(a^2*(a + b)^2*(a - b + (a + b)*Cosh[2*(c + d*x
)])^2) + (b^2*(13*a + 7*b)*Sinh[2*(c + d*x)])/(a^3*(a + b)^2*(a - b + (a +
b)*Cosh[2*(c + d*x)])))/(8*d)
```

Maple [B] time = 0.125, size = 2045, normalized size = 11.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] -35/8/d*b^2/(a+b)^3/a/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/
2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-21/4/d*b^3/(a+b)^3/a^2/((
2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b
))^(1/2)-a-2*b)*a)^(1/2))+35/8/d*b^2/(a+b)^3/(b*(a+b))^(1/2)/((2*(b*(a+b))^
(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2
*b)*a)^(1/2))+35/8/d*b^2/(a+b)^3/a/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arct
```

```
an(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+21/4/d*b^3/(a
+b)^3/a^2/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/
((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+35/8/d*b^2/(a+b)^3/(b*(a+b))^(1/2)/((2
*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))
^(1/2)+a+2*b)*a)^(1/2))+15/8/d*b^5/(a+b)^3/a^3/(b*(a+b))^(1/2)/((2*(b*(a+b)
)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a
-2*b)*a)^(1/2))+77/8/d*b^3/(a+b)^3/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-
2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(
1/2))+57/8/d*b^4/(a+b)^3/a^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^
(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+77
/8/d*b^3/(a+b)^3/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arct
an(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+57/8/d*b^4/(a
+b)^3/a^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh
(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/d/(a+b)^3*ln(tanh(1/
2*d*x+1/2*c)+1)-1/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)-1)-7/d*b^5/(a+b)^3/a^3/(
tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b
+a)^2*tanh(1/2*d*x+1/2*c)^3-15/8/d*b^4/(a+b)^3/a^3/((2*(b*(a+b))^(1/2)-a-2*
b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/
2))+15/8/d*b^4/(a+b)^3/a^3/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tan
h(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-9/4/d*b^4/(a+b)^3/a^2
/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2
*b+a)^2*tanh(1/2*d*x+1/2*c)^7-39/4/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+2
*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)
^5-39/4/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*
tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^3-13/4/d*b^2/(a+b)^3/(tanh
(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^
2*tanh(1/2*d*x+1/2*c)+15/8/d*b^5/(a+b)^3/a^3/(b*(a+b))^(1/2)/((2*(b*(a+b))^
(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*
b)*a)^(1/2))-1/2/d/a^3*tanh(1/2*d*x+1/2*c)-1/2/d/a^3/tanh(1/2*d*x+1/2*c)-11
/2/d*b^3/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(
1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^7-55/2/d*b^3/(a+b)^3/(tanh(1/
2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a
*tanh(1/2*d*x+1/2*c)^5-99/4/d*b^4/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1
/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2*tanh(1/2*d*x+1/2*c)^5-
55/2/d*b^3/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tan
h(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^3-99/4/d*b^4/(a+b)^3/(tanh(
1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2
/a^2*tanh(1/2*d*x+1/2*c)^3-11/2/d*b^3/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+2*ta
nh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)-
13/4/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tan
h(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^7-9/4/d*b^4/(a+b)^3/a^2/(tanh
(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^
2*tanh(1/2*d*x+1/2*c)-7/d*b^5/(a+b)^3/a^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1
/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 3.82515, size = 27774, normalized size = 156.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^10 + 160*(a^5 + 2*a^4
*b + a^3*b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^9 + 16*(a^5 + 2*a^4*b + a^3*b
^2)*d*x*sinh(d*x + c)^10 - 4*(8*a^5 + 40*a^4*b + 67*a^3*b^2 + 77*a^2*b^3 +
57*a*b^4 + 15*b^5 - 4*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d*x)*cosh(d*x + c)^8 -
4*(8*a^5 + 40*a^4*b + 67*a^3*b^2 + 77*a^2*b^3 + 57*a*b^4 + 15*b^5 - 180*(a^
5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^2 - 4*(3*a^5 - 2*a^4*b - 5*a^3*b^2
)*d*x)*sinh(d*x + c)^8 + 32*(60*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)
^3 - (8*a^5 + 40*a^4*b + 67*a^3*b^2 + 77*a^2*b^3 + 57*a*b^4 + 15*b^5 - 4*(3
*a^5 - 2*a^4*b - 5*a^3*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 - 8*(16*a^5
 + 48*a^4*b + 19*a^3*b^2 - 28*a^2*b^3 - 69*a*b^4 - 30*b^5 - 4*(a^5 - 2*a^4*
b + 5*a^3*b^2)*d*x)*cosh(d*x + c)^6 + 8*(420*(a^5 + 2*a^4*b + a^3*b^2)*d*x*
cosh(d*x + c)^4 - 16*a^5 - 48*a^4*b - 19*a^3*b^2 + 28*a^2*b^3 + 69*a*b^4 +
30*b^5 + 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x - 14*(8*a^5 + 40*a^4*b + 67*a^3*
b^2 + 77*a^2*b^3 + 57*a*b^4 + 15*b^5 - 4*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d*x)
*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 16*(252*(a^5 + 2*a^4*b + a^3*b^2)*d*x*c
osh(d*x + c)^5 - 14*(8*a^5 + 40*a^4*b + 67*a^3*b^2 + 77*a^2*b^3 + 57*a*b^4
+ 15*b^5 - 4*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d*x)*cosh(d*x + c)^3 - 3*(16*a^5
 + 48*a^4*b + 19*a^3*b^2 - 28*a^2*b^3 - 69*a*b^4 - 30*b^5 - 4*(a^5 - 2*a^4*
b + 5*a^3*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 32*a^5 - 160*a^4*b - 3
72*a^3*b^2 - 452*a^2*b^3 - 268*a*b^4 - 60*b^5 - 8*(24*a^5 + 56*a^4*b + 48*a
^3*b^2 + 33*a^2*b^3 + 86*a*b^4 + 45*b^5 + 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x
)*cosh(d*x + c)^4 + 8*(420*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^6 -
24*a^5 - 56*a^4*b - 48*a^3*b^2 - 33*a^2*b^3 - 86*a*b^4 - 45*b^5 - 35*(8*a^5
 + 40*a^4*b + 67*a^3*b^2 + 77*a^2*b^3 + 57*a*b^4 + 15*b^5 - 4*(3*a^5 - 2*a^
4*b - 5*a^3*b^2)*d*x)*cosh(d*x + c)^4 - 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x -
 15*(16*a^5 + 48*a^4*b + 19*a^3*b^2 - 28*a^2*b^3 - 69*a*b^4 - 30*b^5 - 4*(a
^5 - 2*a^4*b + 5*a^3*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 32*(60*(a
^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^7 - 7*(8*a^5 + 40*a^4*b + 67*a^3*
b^2 + 77*a^2*b^3 + 57*a*b^4 + 15*b^5 - 4*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d*x)
*cosh(d*x + c)^5 - 5*(16*a^5 + 48*a^4*b + 19*a^3*b^2 - 28*a^2*b^3 - 69*a*b^
4 - 30*b^5 - 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x)*cosh(d*x + c)^3 - (24*a^5 +
 56*a^4*b + 48*a^3*b^2 + 33*a^2*b^3 + 86*a*b^4 + 45*b^5 + 4*(a^5 - 2*a^4*b
+ 5*a^3*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 16*(a^5 + 2*a^4*b + a^3*
b^2)*d*x - 8*(16*a^5 + 48*a^4*b + 45*a^3*b^2 - 36*a^2*b^3 - 79*a*b^4 - 30*b
^5 + 2*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d*x)*cosh(d*x + c)^2 + 8*(90*(a^5 + 2*
a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^8 - 14*(8*a^5 + 40*a^4*b + 67*a^3*b^2 +
77*a^2*b^3 + 57*a*b^4 + 15*b^5 - 4*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d*x)*cosh(
d*x + c)^6 - 16*a^5 - 48*a^4*b - 45*a^3*b^2 + 36*a^2*b^3 + 79*a*b^4 + 30*b^
5 - 15*(16*a^5 + 48*a^4*b + 19*a^3*b^2 - 28*a^2*b^3 - 69*a*b^4 - 30*b^5 - 4
*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x)*cosh(d*x + c)^4 - 2*(3*a^5 - 2*a^4*b - 5*
a^3*b^2)*d*x - 6*(24*a^5 + 56*a^4*b + 48*a^3*b^2 + 33*a^2*b^3 + 86*a*b^4 +
45*b^5 + 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^
2 + ((35*a^4*b + 112*a^3*b^2 + 134*a^2*b^3 + 72*a*b^4 + 15*b^5)*cosh(d*x +
c)^10 + 10*(35*a^4*b + 112*a^3*b^2 + 134*a^2*b^3 + 72*a*b^4 + 15*b^5)*cosh(
d*x + c)*sinh(d*x + c)^9 + (35*a^4*b + 112*a^3*b^2 + 134*a^2*b^3 + 72*a*b^4
 + 15*b^5)*sinh(d*x + c)^10 + (105*a^4*b + 56*a^3*b^2 - 214*a^2*b^3 - 240*a
*b^4 - 75*b^5)*cosh(d*x + c)^8 + (105*a^4*b + 56*a^3*b^2 - 214*a^2*b^3 - 24
0*a*b^4 - 75*b^5 + 45*(35*a^4*b + 112*a^3*b^2 + 134*a^2*b^3 + 72*a*b^4 + 15
*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(35*a^4*b + 112*a^3*b^2 + 13
4*a^2*b^3 + 72*a*b^4 + 15*b^5)*cosh(d*x + c)^3 + (105*a^4*b + 56*a^3*b^2 -
214*a^2*b^3 - 240*a*b^4 - 75*b^5)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(35*a^
```

4*b - 28*a^3*b^2 + 106*a^2*b^3 + 180*a*b^4 + 75*b^5)*cosh(d*x + c)^6 + 2*(3
5*a^4*b - 28*a^3*b^2 + 106*a^2*b^3 + 180*a*b^4 + 75*b^5 + 105*(35*a^4*b + 1
12*a^3*b^2 + 134*a^2*b^3 + 72*a*b^4 + 15*b^5)*cosh(d*x + c)^4 + 14*(105*a^4
*b + 56*a^3*b^2 - 214*a^2*b^3 - 240*a*b^4 - 75*b^5)*cosh(d*x + c)^2)*sinh(d
x + c)^6 + 4(63*(35*a^4*b + 112*a^3*b^2 + 134*a^2*b^3 + 72*a*b^4 + 15*b^5
)*cosh(d*x + c)^5 + 14*(105*a^4*b + 56*a^3*b^2 - 214*a^2*b^3 - 240*a*b^4 -
75*b^5)*cosh(d*x + c)^3 + 3*(35*a^4*b - 28*a^3*b^2 + 106*a^2*b^3 + 180*a*b^
4 + 75*b^5)*cosh(d*x + c))*sinh(d*x + c)^5 - 35*a^4*b - 112*a^3*b^2 - 134*a
^2*b^3 - 72*a*b^4 - 15*b^5 - 2*(35*a^4*b - 28*a^3*b^2 + 106*a^2*b^3 + 180*a
*b^4 + 75*b^5)*cosh(d*x + c)^4 + 2*(105*(35*a^4*b + 112*a^3*b^2 + 134*a^2*b
^3 + 72*a*b^4 + 15*b^5)*cosh(d*x + c)^6 - 35*a^4*b + 28*a^3*b^2 - 106*a^2*b
^3 - 180*a*b^4 - 75*b^5 + 35*(105*a^4*b + 56*a^3*b^2 - 214*a^2*b^3 - 240*a*
b^4 - 75*b^5)*cosh(d*x + c)^4 + 15*(35*a^4*b - 28*a^3*b^2 + 106*a^2*b^3 + 1
80*a*b^4 + 75*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(15*(35*a^4*b + 112
*a^3*b^2 + 134*a^2*b^3 + 72*a*b^4 + 15*b^5)*cosh(d*x + c)^7 + 7*(105*a^4*b
+ 56*a^3*b^2 - 214*a^2*b^3 - 240*a*b^4 - 75*b^5)*cosh(d*x + c)^5 + 5*(35*a^
4*b - 28*a^3*b^2 + 106*a^2*b^3 + 180*a*b^4 + 75*b^5)*cosh(d*x + c)^3 - (35*
a^4*b - 28*a^3*b^2 + 106*a^2*b^3 + 180*a*b^4 + 75*b^5)*cosh(d*x + c))*sinh(
d*x + c)^3 - (105*a^4*b + 56*a^3*b^2 - 214*a^2*b^3 - 240*a*b^4 - 75*b^5)*co
sh(d*x + c)^2 + (45*(35*a^4*b + 112*a^3*b^2 + 134*a^2*b^3 + 72*a*b^4 + 15*b
^5)*cosh(d*x + c)^8 + 28*(105*a^4*b + 56*a^3*b^2 - 214*a^2*b^3 - 240*a*b^4
- 75*b^5)*cosh(d*x + c)^6 - 105*a^4*b - 56*a^3*b^2 + 214*a^2*b^3 + 240*a*b^
4 + 75*b^5 + 30*(35*a^4*b - 28*a^3*b^2 + 106*a^2*b^3 + 180*a*b^4 + 75*b^5)*
cosh(d*x + c)^4 - 12*(35*a^4*b - 28*a^3*b^2 + 106*a^2*b^3 + 180*a*b^4 + 75*
b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(5*(35*a^4*b + 112*a^3*b^2 + 134*
a^2*b^3 + 72*a*b^4 + 15*b^5)*cosh(d*x + c)^9 + 4*(105*a^4*b + 56*a^3*b^2 -
214*a^2*b^3 - 240*a*b^4 - 75*b^5)*cosh(d*x + c)^7 + 6*(35*a^4*b - 28*a^3*b^
2 + 106*a^2*b^3 + 180*a*b^4 + 75*b^5)*cosh(d*x + c)^5 - 4*(35*a^4*b - 28*a^
3*b^2 + 106*a^2*b^3 + 180*a*b^4 + 75*b^5)*cosh(d*x + c)^3 - (105*a^4*b + 56
*a^3*b^2 - 214*a^2*b^3 - 240*a*b^4 - 75*b^5)*cosh(d*x + c))*sinh(d*x + c))*
sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)
*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a
^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2
- b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*
x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a^2 + a*b)*cosh(d
x + c)^2 + 2(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*
x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh
(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x +
c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*c
osh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 16*(10*(a
^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^9 - 2*(8*a^5 + 40*a^4*b + 67*a^3*
b^2 + 77*a^2*b^3 + 57*a*b^4 + 15*b^5 - 4*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d*x)
*cosh(d*x + c)^7 - 3*(16*a^5 + 48*a^4*b + 19*a^3*b^2 - 28*a^2*b^3 - 69*a*b^
4 - 30*b^5 - 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x)*cosh(d*x + c)^5 - 2*(24*a^5
 + 56*a^4*b + 48*a^3*b^2 + 33*a^2*b^3 + 86*a*b^4 + 45*b^5 + 4*(a^5 - 2*a^4*
b + 5*a^3*b^2)*d*x)*cosh(d*x + c)^3 - (16*a^5 + 48*a^4*b + 45*a^3*b^2 - 36*
a^2*b^3 - 79*a*b^4 - 30*b^5 + 2*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d*x)*cosh(d*x
 + c))*sinh(d*x + c))/((a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4
 + a^3*b^5)*d*cosh(d*x + c)^10 + 10*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^
3 + 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^8 + 5*a^7*b +
 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*sinh(d*x + c)^10 + (3*a^8
 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*d*cosh(d*x +
c)^8 + (45*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*
d*cosh(d*x + c)^2 + (3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4
- 5*a^3*b^5)*d)*sinh(d*x + c)^8 + 2*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 +
 13*a^4*b^4 + 5*a^3*b^5)*d*cosh(d*x + c)^6 + 8*(15*(a^8 + 5*a^7*b + 10*a^6*
b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^3 + (3*a^8 + 7*a^7*
b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*d*cosh(d*x + c))*sinh(
d*x + c)^7 + 2*(105*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 +

a^3*b^5)*d*cosh(d*x + c)^4 + 14*(3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*d*cosh(d*x + c)^2 + (a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*d)*sinh(d*x + c)^6 - 2*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*d*cosh(d*x + c)^4 + 4*(63*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^5 + 14*(3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*d*cosh(d*x + c)^3 + 3*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^6 + 35*(3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*d*cosh(d*x + c)^4 + 15*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*d*cosh(d*x + c)^2 - (a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*d)*sinh(d*x + c)^4 - (3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*d*cosh(d*x + c)^2 + 8*(15*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^7 + 7*(3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*d*cosh(d*x + c)^5 + 5*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*d*cosh(d*x + c)^3 - (a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (45*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^8 + 28*(3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*d*cosh(d*x + c)^6 + 30*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*d*cosh(d*x + c)^4 - 12*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*d*cosh(d*x + c)^2 - (3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*d)*sinh(d*x + c)^2 - (a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d + 2*(5*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^9 + 4*(3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*d*cosh(d*x + c)^7 + 6*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*d*cosh(d*x + c)^5 - 4*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*d*cosh(d*x + c)^3 - (3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*(8*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^10 + 80*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^9 + 8*(a^5 + 2*a^4*b + a^3*b^2)*d*x*sinh(d*x + c)^10 - 2*(8*a^5 + 40*a^4*b + 67*a^3*b^2 + 77*a^2*b^3 + 57*a*b^4 + 15*b^5 - 4*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d*x)*cosh(d*x + c)^8 - 2*(8*a^5 + 40*a^4*b + 67*a^3*b^2 + 77*a^2*b^3 + 57*a*b^4 + 15*b^5 - 180*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^2 - 4*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d*x)*sinh(d*x + c)^8 + 16*(60*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^3 - (8*a^5 + 40*a^4*b + 67*a^3*b^2 + 77*a^2*b^3 + 57*a*b^4 + 15*b^5 - 4*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 - 4*(16*a^5 + 48*a^4*b + 19*a^3*b^2 - 28*a^2*b^3 - 69*a*b^4 - 30*b^5 - 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x)*cosh(d*x + c)^6 + 4*(420*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^4 - 16*a^5 - 48*a^4*b - 19*a^3*b^2 + 28*a^2*b^3 + 69*a*b^4 + 30*b^5 + 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x - 14*(8*a^5 + 40*a^4*b + 67*a^3*b^2 + 77*a^2*b^3 + 57*a*b^4 + 15*b^5 - 4*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(252*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^5 - 14*(8*a^5 + 40*a^4*b + 67*a^3*b^2 + 77*a^2*b^3 + 57*a*b^4 + 15*b^5 - 4*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d*x)*cosh(d*x + c)^3 - 3*(16*a^5 + 48*a^4*b + 19*a^3*b^2 - 28*a^2*b^3 - 69*a*b^4 - 30*b^5 - 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 16*a^5 - 80*a^4*b - 186*a^3*b^2 - 226*a^2*b^3 - 134*a*b^4 - 30*b^5 - 4*(24*a^5 + 56*a^4*b + 48*a^3*b^2 + 33*a^2*b^3 + 86*a*b^4 + 45*b^5 + 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x)*cosh(d*x + c)^4 + 4*(420*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^6 - 24*a^5 - 56*a^4*b - 48*a^3*b^2 - 33*a^2*b^3 - 86*a*b^4 - 45*b^5 - 35*(8*a^5 + 40*a^4*b + 67*a^3*b^2 + 77*a^2*b^3 + 57*a*b^4 + 15*b^5 - 4*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d*x)*cosh(d*x + c)^4 - 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x - 15*(16*a^5 + 48*a^4*b + 19*a^3*b^2 - 28*a^2*b^3 - 69*a*b^4 - 30*b^5 - 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(60*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^7 - 7*(8*a^5 + 40*a^4*b

+ 67*a^3*b^2 + 77*a^2*b^3 + 57*a*b^4 + 15*b^5 - 4*(3*a^5 - 2*a^4*b - 5*a^3
*b^2)*d*x)*cosh(d*x + c)^5 - 5*(16*a^5 + 48*a^4*b + 19*a^3*b^2 - 28*a^2*b^3
- 69*a*b^4 - 30*b^5 - 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x)*cosh(d*x + c)^3 -
(24*a^5 + 56*a^4*b + 48*a^3*b^2 + 33*a^2*b^3 + 86*a*b^4 + 45*b^5 + 4*(a^5
- 2*a^4*b + 5*a^3*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 8*(a^5 + 2*a^4
*b + a^3*b^2)*d*x - 4*(16*a^5 + 48*a^4*b + 45*a^3*b^2 - 36*a^2*b^3 - 79*a*b
^4 - 30*b^5 + 2*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d*x)*cosh(d*x + c)^2 + 4*(90*
(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^8 - 14*(8*a^5 + 40*a^4*b + 67*a
^3*b^2 + 77*a^2*b^3 + 57*a*b^4 + 15*b^5 - 4*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d
*x)*cosh(d*x + c)^6 - 16*a^5 - 48*a^4*b - 45*a^3*b^2 + 36*a^2*b^3 + 79*a*b^
4 + 30*b^5 - 15*(16*a^5 + 48*a^4*b + 19*a^3*b^2 - 28*a^2*b^3 - 69*a*b^4 - 3
0*b^5 - 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x)*cosh(d*x + c)^4 - 2*(3*a^5 - 2*a
^4*b - 5*a^3*b^2)*d*x - 6*(24*a^5 + 56*a^4*b + 48*a^3*b^2 + 33*a^2*b^3 + 86
*a*b^4 + 45*b^5 + 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x)*cosh(d*x + c)^2)*sinh(
d*x + c)^2 - ((35*a^4*b + 112*a^3*b^2 + 134*a^2*b^3 + 72*a*b^4 + 15*b^5)*co
sh(d*x + c)^10 + 10*(35*a^4*b + 112*a^3*b^2 + 134*a^2*b^3 + 72*a*b^4 + 15*b
^5)*cosh(d*x + c)*sinh(d*x + c)^9 + (35*a^4*b + 112*a^3*b^2 + 134*a^2*b^3 +
72*a*b^4 + 15*b^5)*sinh(d*x + c)^10 + (105*a^4*b + 56*a^3*b^2 - 214*a^2*b^
3 - 240*a*b^4 - 75*b^5)*cosh(d*x + c)^8 + (105*a^4*b + 56*a^3*b^2 - 214*a^2
*b^3 - 240*a*b^4 - 75*b^5 + 45*(35*a^4*b + 112*a^3*b^2 + 134*a^2*b^3 + 72*a
*b^4 + 15*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(35*a^4*b + 112*a^3
*b^2 + 134*a^2*b^3 + 72*a*b^4 + 15*b^5)*cosh(d*x + c)^3 + (105*a^4*b + 56*a
^3*b^2 - 214*a^2*b^3 - 240*a*b^4 - 75*b^5)*cosh(d*x + c))*sinh(d*x + c)^7 +
2*(35*a^4*b - 28*a^3*b^2 + 106*a^2*b^3 + 180*a*b^4 + 75*b^5)*cosh(d*x + c)
^6 + 2*(35*a^4*b - 28*a^3*b^2 + 106*a^2*b^3 + 180*a*b^4 + 75*b^5 + 105*(35*
a^4*b + 112*a^3*b^2 + 134*a^2*b^3 + 72*a*b^4 + 15*b^5)*cosh(d*x + c)^4 + 14
*(105*a^4*b + 56*a^3*b^2 - 214*a^2*b^3 - 240*a*b^4 - 75*b^5)*cosh(d*x + c)^
2)*sinh(d*x + c)^6 + 4*(63*(35*a^4*b + 112*a^3*b^2 + 134*a^2*b^3 + 72*a*b^4
+ 15*b^5)*cosh(d*x + c)^5 + 14*(105*a^4*b + 56*a^3*b^2 - 214*a^2*b^3 - 240
*a*b^4 - 75*b^5)*cosh(d*x + c)^3 + 3*(35*a^4*b - 28*a^3*b^2 + 106*a^2*b^3 +
180*a*b^4 + 75*b^5)*cosh(d*x + c))*sinh(d*x + c)^5 - 35*a^4*b - 112*a^3*b^
2 - 134*a^2*b^3 - 72*a*b^4 - 15*b^5 - 2*(35*a^4*b - 28*a^3*b^2 + 106*a^2*b^
3 + 180*a*b^4 + 75*b^5)*cosh(d*x + c)^4 + 2*(105*(35*a^4*b + 112*a^3*b^2 +
134*a^2*b^3 + 72*a*b^4 + 15*b^5)*cosh(d*x + c)^6 - 35*a^4*b + 28*a^3*b^2 -
106*a^2*b^3 - 180*a*b^4 - 75*b^5 + 35*(105*a^4*b + 56*a^3*b^2 - 214*a^2*b^3
- 240*a*b^4 - 75*b^5)*cosh(d*x + c)^4 + 15*(35*a^4*b - 28*a^3*b^2 + 106*a^
2*b^3 + 180*a*b^4 + 75*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(15*(35*a^
4*b + 112*a^3*b^2 + 134*a^2*b^3 + 72*a*b^4 + 15*b^5)*cosh(d*x + c)^7 + 7*(1
05*a^4*b + 56*a^3*b^2 - 214*a^2*b^3 - 240*a*b^4 - 75*b^5)*cosh(d*x + c)^5 +
5*(35*a^4*b - 28*a^3*b^2 + 106*a^2*b^3 + 180*a*b^4 + 75*b^5)*cosh(d*x + c)
^3 - (35*a^4*b - 28*a^3*b^2 + 106*a^2*b^3 + 180*a*b^4 + 75*b^5)*cosh(d*x +
c))*sinh(d*x + c)^3 - (105*a^4*b + 56*a^3*b^2 - 214*a^2*b^3 - 240*a*b^4 - 7
5*b^5)*cosh(d*x + c)^2 + (45*(35*a^4*b + 112*a^3*b^2 + 134*a^2*b^3 + 72*a*b
^4 + 15*b^5)*cosh(d*x + c)^8 + 28*(105*a^4*b + 56*a^3*b^2 - 214*a^2*b^3 - 2
40*a*b^4 - 75*b^5)*cosh(d*x + c)^6 - 105*a^4*b - 56*a^3*b^2 + 214*a^2*b^3 +
240*a*b^4 + 75*b^5 + 30*(35*a^4*b - 28*a^3*b^2 + 106*a^2*b^3 + 180*a*b^4 +
75*b^5)*cosh(d*x + c)^4 - 12*(35*a^4*b - 28*a^3*b^2 + 106*a^2*b^3 + 180*a*
b^4 + 75*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(5*(35*a^4*b + 112*a^3*b
^2 + 134*a^2*b^3 + 72*a*b^4 + 15*b^5)*cosh(d*x + c)^9 + 4*(105*a^4*b + 56*a
^3*b^2 - 214*a^2*b^3 - 240*a*b^4 - 75*b^5)*cosh(d*x + c)^7 + 6*(35*a^4*b -
28*a^3*b^2 + 106*a^2*b^3 + 180*a*b^4 + 75*b^5)*cosh(d*x + c)^5 - 4*(35*a^4*
b - 28*a^3*b^2 + 106*a^2*b^3 + 180*a*b^4 + 75*b^5)*cosh(d*x + c)^3 - (105*a
^4*b + 56*a^3*b^2 - 214*a^2*b^3 - 240*a*b^4 - 75*b^5)*cosh(d*x + c))*sinh(d
*x + c))*sqrt(b/a)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x
+ c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(b/a)/b) + 8*(10
*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^9 - 2*(8*a^5 + 40*a^4*b + 67*a
^3*b^2 + 77*a^2*b^3 + 57*a*b^4 + 15*b^5 - 4*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d
*x)*cosh(d*x + c)^7 - 3*(16*a^5 + 48*a^4*b + 19*a^3*b^2 - 28*a^2*b^3 - 69*a
*b^4 - 30*b^5 - 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x)*cosh(d*x + c)^5 - 2*(24*

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a^5 + 56*a^4*b + 48*a^3*b^2 + 33*a^2*b^3 + 86*a*b^4 + 45*b^5 + 4*(a^5 - 2*a
^4*b + 5*a^3*b^2)*d*x)*cosh(d*x + c)^3 - (16*a^5 + 48*a^4*b + 45*a^3*b^2 -
36*a^2*b^3 - 79*a*b^4 - 30*b^5 + 2*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d*x)*cosh(
d*x + c))*sinh(d*x + c))/((a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*
b^4 + a^3*b^5)*d*cosh(d*x + c)^10 + 10*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5
*b^3 + 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^8 + 5*a^7*
b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*sinh(d*x + c)^10 + (3*
a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*d*cosh(d*x
 + c)^8 + (45*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^
5)*d*cosh(d*x + c)^2 + (3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b
^4 - 5*a^3*b^5)*d)*sinh(d*x + c)^8 + 2*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^
3 + 13*a^4*b^4 + 5*a^3*b^5)*d*cosh(d*x + c)^6 + 8*(15*(a^8 + 5*a^7*b + 10*a
^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^3 + (3*a^8 + 7*a
^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*d*cosh(d*x + c))*si
nh(d*x + c)^7 + 2*(105*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4
 + a^3*b^5)*d*cosh(d*x + c)^4 + 14*(3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^
3 - 17*a^4*b^4 - 5*a^3*b^5)*d*cosh(d*x + c)^2 + (a^8 + a^7*b + 2*a^6*b^2 +
10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*d)*sinh(d*x + c)^6 - 2*(a^8 + a^7*b +
2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*d*cosh(d*x + c)^4 + 4*(63*
(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x
+ c)^5 + 14*(3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*
b^5)*d*cosh(d*x + c)^3 + 3*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b
^4 + 5*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^8 + 5*a^7*b +
10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^6 + 35*(3*a^
8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*d*cosh(d*x +
 c)^4 + 15*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*
d*cosh(d*x + c)^2 - (a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*
a^3*b^5)*d)*sinh(d*x + c)^4 - (3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 1
7*a^4*b^4 - 5*a^3*b^5)*d*cosh(d*x + c)^2 + 8*(15*(a^8 + 5*a^7*b + 10*a^6*b^
2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^7 + 7*(3*a^8 + 7*a^7*
b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*d*cosh(d*x + c)^5 + 5*
(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*d*cosh(d*x
+ c)^3 - (a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*d*
cosh(d*x + c))*sinh(d*x + c)^3 + (45*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b
^3 + 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^8 + 28*(3*a^8 + 7*a^7*b - 2*a^6*b
^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*d*cosh(d*x + c)^6 + 30*(a^8 + a^7
*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*d*cosh(d*x + c)^4 - 1
2*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*d*cosh(d*
x + c)^2 - (3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b
^5)*d)*sinh(d*x + c)^2 - (a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b
^4 + a^3*b^5)*d + 2*(5*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4
 + a^3*b^5)*d*cosh(d*x + c)^9 + 4*(3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3
 - 17*a^4*b^4 - 5*a^3*b^5)*d*cosh(d*x + c)^7 + 6*(a^8 + a^7*b + 2*a^6*b^2 +
 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*d*cosh(d*x + c)^5 - 4*(a^8 + a^7*b +
2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*d*cosh(d*x + c)^3 - (3*a^8
 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*d*cosh(d*x +
c))*sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral(coth(c + d*x)**2/(a + b*tanh(c + d*x)**2)**3, x)
```

Giac [B] time = 1.33385, size = 609, normalized size = 3.42

$$-\frac{\left(35\,a^2b^2+42\,ab^3+15\,b^4\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{8\left(a^6d+3\,a^5bd+3\,a^4b^2d+a^3b^3d\right)\sqrt{ab}}+\frac{dx+c}{a^3d+3\,a^2bd+3\,ab^2d+b^3d}+\frac{13\,a^3b^2e^{(6\,dx+6\,c)}+3\,a^2b^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(35*a^2*b^2 + 42*a*b^3 + 15*b^4)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(
2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^6*d + 3*a^5*b*d + 3*a^4*b^2*d + a^3*b^
3*d)*sqrt(a*b)) + (d*x + c)/(a^3*d + 3*a^2*b*d + 3*a*b^2*d + b^3*d) + 1/4*(
13*a^3*b^2*e^(6*d*x + 6*c) + 3*a^2*b^3*e^(6*d*x + 6*c) - 17*a*b^4*e^(6*d*x
+ 6*c) - 7*b^5*e^(6*d*x + 6*c) + 39*a^3*b^2*e^(4*d*x + 4*c) - 5*a^2*b^3*e^(
4*d*x + 4*c) + 25*a*b^4*e^(4*d*x + 4*c) + 21*b^5*e^(4*d*x + 4*c) + 39*a^3*b
^2*e^(2*d*x + 2*c) + 25*a^2*b^3*e^(2*d*x + 2*c) - 35*a*b^4*e^(2*d*x + 2*c)
- 21*b^5*e^(2*d*x + 2*c) + 13*a^3*b^2 + 33*a^2*b^3 + 27*a*b^4 + 7*b^5)/((a^
6*d + 3*a^5*b*d + 3*a^4*b^2*d + a^3*b^3*d)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x
+ 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2) - 2/(a^3*d*(
e^(2*d*x + 2*c) - 1))
```

3.199 $\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=171

$$-\frac{b^2(3a+2b)}{2a^3d(a+b)^2\left(a+b\tanh^2(c+dx)\right)}-\frac{b^2}{4a^2d(a+b)\left(a+b\tanh^2(c+dx)\right)^2}+\frac{b^2\left(6a^2+8ab+3b^2\right)\log\left(a+b\tanh^2(c+d\right.}{2a^4d(a+b)^3}$$

```
[Out] -Coth[c + d*x]^2/(2*a^3*d) + Log[Cosh[c + d*x]]/((a + b)^3*d) + ((a - 3*b)*
Log[Tanh[c + d*x]])/(a^4*d) + (b^2*(6*a^2 + 8*a*b + 3*b^2)*Log[a + b*Tanh[c
 + d*x]^2])/(2*a^4*(a + b)^3*d) - b^2/(4*a^2*(a + b)*d*(a + b*Tanh[c + d*x]
^2)^2) - (b^2*(3*a + 2*b))/(2*a^3*(a + b)^2*d*(a + b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.261835, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3670, 446, 88}

$$-\frac{b^2(3a+2b)}{2a^3d(a+b)^2\left(a+b\tanh^2(c+dx)\right)}-\frac{b^2}{4a^2d(a+b)\left(a+b\tanh^2(c+dx)\right)^2}+\frac{b^2\left(6a^2+8ab+3b^2\right)\log\left(a+b\tanh^2(c+d\right.}{2a^4d(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```


```
[Out] -Coth[c + d*x]^2/(2*a^3*d) + Log[Cosh[c + d*x]]/((a + b)^3*d) + ((a - 3*b)*
Log[Tanh[c + d*x]])/(a^4*d) + (b^2*(6*a^2 + 8*a*b + 3*b^2)*Log[a + b*Tanh[c
 + d*x]^2])/(2*a^4*(a + b)^3*d) - b^2/(4*a^2*(a + b)*d*(a + b*Tanh[c + d*x]
^2)^2) - (b^2*(3*a + 2*b))/(2*a^3*(a + b)^2*d*(a + b*Tanh[c + d*x]^2))
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3(1-x^2)\left(a+bx^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(1-x)x^2(a+bx)^3}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^3(-1+x)} + \frac{1}{a^3x^2} + \frac{a-3b}{a^4x} + \frac{b^3}{a^2(a+b)(a+bx)^3} + \frac{b^3(3a+2b)}{a^3(a+b)^2(a+bx)^2} + \frac{b^3\left(6a^2+8ab+3b^2\right)}{a^4(a+b)^3(a+bx)}\right.\right.}{2d}$$

$$= -\frac{\coth^2(c+dx)}{2a^3d} + \frac{\log(\cosh(c+dx))}{(a+b)^3d} + \frac{(a-3b)\log(\tanh(c+dx))}{a^4d} + \frac{b^2\left(6a^2+8ab+\right.}{}$$

Mathematica [A] time = 1.71898, size = 138, normalized size = 0.81

$$-\frac{\frac{b^4}{2a^4(a+b)\left(a\coth^2(c+dx)+b\right)^2} - \frac{b^3(4a+3b)}{a^4(a+b)^2\left(a\coth^2(c+dx)+b\right)} - \frac{b^2\left(6a^2+8ab+3b^2\right)\log\left(a\coth^2(c+dx)+b\right)}{a^4(a+b)^3} + \frac{\coth^2(c+dx)}{a^3} - \frac{2\log(\sinh(c+dx))}{(a+b)^3}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -(Coth[c + d*x]^2/a^3 + b^4/(2*a^4*(a + b)*(b + a*Coth[c + d*x]^2)^2) - (b^
3*(4*a + 3*b))/(a^4*(a + b)^2*(b + a*Coth[c + d*x]^2)) - (b^2*(6*a^2 + 8*a*
b + 3*b^2)*Log[b + a*Coth[c + d*x]^2])/(a^4*(a + b)^3) - (2*Log[Sinh[c + d*
x]])/(a + b)^3)/(2*d)
```

Maple [B] time = 0.128, size = 1020, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] -1/8/d*tanh(1/2*d*x+1/2*c)^2/a^3-1/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)+1)+8/d*
b^3/(a+b)^3/a/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2
*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^6+14/d*b^4/(a+b)^3/a^2/(tanh(1/2*d
*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh
(1/2*d*x+1/2*c)^6+6/d*b^5/a^3/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d
*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^6+16/d*b^3
/(a+b)^3/a/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*
x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^4+56/d*b^4/(a+b)^3/a^2/(tanh(1/2*d*x+
1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/
2*d*x+1/2*c)^4+60/d*b^5/(a+b)^3/a^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x
+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^4+20/d*b^6/a
^4/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*
x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^4+8/d*b^3/(a+b)^3/a/(tanh(1/2*d*x+1/2
*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d
*x+1/2*c)^2+14/d*b^4/(a+b)^3/a^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/
2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^2+6/d*b^5/a^3/(
a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/
```

```
2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^2+3/d*b^2/(a+b)^3/a^2*ln(tanh(1/2*d*x+1/2
*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)+4/d*b^3/(a+b
)^3/a^3*ln(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x
+1/2*c)^2*b+a)+3/2/d*b^4/a^4/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*
d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)-1/8/d/a^3/tanh(1/2*d*x+1/2*c)^2
+1/d/a^3*ln(tanh(1/2*d*x+1/2*c))-3/d/a^4*ln(tanh(1/2*d*x+1/2*c))*b-1/d/(a+b
)^3*ln(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.40136, size = 1040, normalized size = 6.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(6*a^2*b^2 + 8*a*b^3 + 3*b^4)*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*
e^(-4*d*x - 4*c) + a + b)/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d) + (d*x
+ c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 2*((a^5 + 5*a^4*b + 10*a^3*b^2 +
 14*a^2*b^3 + 11*a*b^4 + 3*b^5)*e^(-2*d*x - 2*c) + 2*(2*a^5 + 6*a^4*b + 4*a
^3*b^2 - 4*a^2*b^3 - 13*a*b^4 - 6*b^5)*e^(-4*d*x - 4*c) + 2*(3*a^5 + 7*a^4*
b + 6*a^3*b^2 + 2*a^2*b^3 + 15*a*b^4 + 9*b^5)*e^(-6*d*x - 6*c) + 2*(2*a^5 +
 6*a^4*b + 4*a^3*b^2 - 4*a^2*b^3 - 13*a*b^4 - 6*b^5)*e^(-8*d*x - 8*c) + (a^
5 + 5*a^4*b + 10*a^3*b^2 + 14*a^2*b^3 + 11*a*b^4 + 3*b^5)*e^(-10*d*x - 10*c
))/((a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5 + 2*(a^8
 + a^7*b - 6*a^6*b^2 - 14*a^5*b^3 - 11*a^4*b^4 - 3*a^3*b^5)*e^(-2*d*x - 2*c
) - (a^8 + 5*a^7*b - 6*a^6*b^2 - 38*a^5*b^3 - 43*a^4*b^4 - 15*a^3*b^5)*e^(-
4*d*x - 4*c) - 4*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3
*b^5)*e^(-6*d*x - 6*c) - (a^8 + 5*a^7*b - 6*a^6*b^2 - 38*a^5*b^3 - 43*a^4*b
^4 - 15*a^3*b^5)*e^(-8*d*x - 8*c) + 2*(a^8 + a^7*b - 6*a^6*b^2 - 14*a^5*b^3
 - 11*a^4*b^4 - 3*a^3*b^5)*e^(-10*d*x - 10*c) + (a^8 + 5*a^7*b + 10*a^6*b^2
 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*e^(-12*d*x - 12*c))*d) + (a - 3*b)*log
(e^(-d*x - c) + 1)/(a^4*d) + (a - 3*b)*log(e^(-d*x - c) - 1)/(a^4*d)
```

Fricas [B] time = 7.57986, size = 24520, normalized size = 143.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^6 + 2*a^5*b + a^4*b^2)*d*x*cosh(d*x + c)^12 + 24*(a^6 + 2*a^5*b
+ a^4*b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^11 + 2*(a^6 + 2*a^5*b + a^4*b^2)
*d*x*sinh(d*x + c)^12 + 4*(a^6 + 5*a^5*b + 10*a^4*b^2 + 14*a^3*b^3 + 11*a^2
*b^4 + 3*a*b^5 + (a^6 - 2*a^5*b - 3*a^4*b^2)*d*x)*cosh(d*x + c)^10 + 4*(a^6
 + 5*a^5*b + 10*a^4*b^2 + 14*a^3*b^3 + 11*a^2*b^4 + 3*a*b^5 + 33*(a^6 + 2*a
^5*b + a^4*b^2)*d*x*cosh(d*x + c)^2 + (a^6 - 2*a^5*b - 3*a^4*b^2)*d*x)*sinh
(d*x + c)^10 + 40*(11*(a^6 + 2*a^5*b + a^4*b^2)*d*x*cosh(d*x + c)^3 + (a^6
+ 5*a^5*b + 10*a^4*b^2 + 14*a^3*b^3 + 11*a^2*b^4 + 3*a*b^5 + (a^6 - 2*a^5*b
 - 3*a^4*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^9 + 2*(8*a^6 + 24*a^5*b + 1
6*a^4*b^2 - 16*a^3*b^3 - 52*a^2*b^4 - 24*a*b^5 - (a^6 + 2*a^5*b - 15*a^4*b^
2)*d*x)*cosh(d*x + c)^8 + 2*(495*(a^6 + 2*a^5*b + a^4*b^2)*d*x*cosh(d*x + c
)^4 + 8*a^6 + 24*a^5*b + 16*a^4*b^2 - 16*a^3*b^3 - 52*a^2*b^4 - 24*a*b^5 -
(a^6 + 2*a^5*b - 15*a^4*b^2)*d*x + 90*(a^6 + 5*a^5*b + 10*a^4*b^2 + 14*a^3*
```

b^3 + 11*a^2*b^4 + 3*a*b^5 + (a^6 - 2*a^5*b - 3*a^4*b^2)*d*x)*cosh(d*x + c)
^2)*sinh(d*x + c)^8 + 16*(99*(a^6 + 2*a^5*b + a^4*b^2)*d*x*cosh(d*x + c)^5
+ 30*(a^6 + 5*a^5*b + 10*a^4*b^2 + 14*a^3*b^3 + 11*a^2*b^4 + 3*a*b^5 + (a^6
 - 2*a^5*b - 3*a^4*b^2)*d*x)*cosh(d*x + c)^3 + (8*a^6 + 24*a^5*b + 16*a^4*b
^2 - 16*a^3*b^3 - 52*a^2*b^4 - 24*a*b^5 - (a^6 + 2*a^5*b - 15*a^4*b^2)*d*x)
*cosh(d*x + c))*sinh(d*x + c)^7 + 8*(3*a^6 + 7*a^5*b + 6*a^4*b^2 + 2*a^3*b^
3 + 15*a^2*b^4 + 9*a*b^5 - (a^6 - 2*a^5*b + 5*a^4*b^2)*d*x)*cosh(d*x + c)^6
 + 8*(231*(a^6 + 2*a^5*b + a^4*b^2)*d*x*cosh(d*x + c)^6 + 3*a^6 + 7*a^5*b +
 6*a^4*b^2 + 2*a^3*b^3 + 15*a^2*b^4 + 9*a*b^5 + 105*(a^6 + 5*a^5*b + 10*a^4
*b^2 + 14*a^3*b^3 + 11*a^2*b^4 + 3*a*b^5 + (a^6 - 2*a^5*b - 3*a^4*b^2)*d*x)
*cosh(d*x + c)^4 - (a^6 - 2*a^5*b + 5*a^4*b^2)*d*x + 7*(8*a^6 + 24*a^5*b +
16*a^4*b^2 - 16*a^3*b^3 - 52*a^2*b^4 - 24*a*b^5 - (a^6 + 2*a^5*b - 15*a^4*b
^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 16*(99*(a^6 + 2*a^5*b + a^4*b^2
)*d*x*cosh(d*x + c)^7 + 63*(a^6 + 5*a^5*b + 10*a^4*b^2 + 14*a^3*b^3 + 11*a^
2*b^4 + 3*a*b^5 + (a^6 - 2*a^5*b - 3*a^4*b^2)*d*x)*cosh(d*x + c)^5 + 7*(8*a
^6 + 24*a^5*b + 16*a^4*b^2 - 16*a^3*b^3 - 52*a^2*b^4 - 24*a*b^5 - (a^6 + 2*
a^5*b - 15*a^4*b^2)*d*x)*cosh(d*x + c)^3 + 3*(3*a^6 + 7*a^5*b + 6*a^4*b^2 +
 2*a^3*b^3 + 15*a^2*b^4 + 9*a*b^5 - (a^6 - 2*a^5*b + 5*a^4*b^2)*d*x)*cosh(d
*x + c))*sinh(d*x + c)^5 + 2*(8*a^6 + 24*a^5*b + 16*a^4*b^2 - 16*a^3*b^3 -
52*a^2*b^4 - 24*a*b^5 - (a^6 + 2*a^5*b - 15*a^4*b^2)*d*x)*cosh(d*x + c)^4 +
 2*(495*(a^6 + 2*a^5*b + a^4*b^2)*d*x*cosh(d*x + c)^8 + 420*(a^6 + 5*a^5*b
+ 10*a^4*b^2 + 14*a^3*b^3 + 11*a^2*b^4 + 3*a*b^5 + (a^6 - 2*a^5*b - 3*a^4*b
^2)*d*x)*cosh(d*x + c)^6 + 8*a^6 + 24*a^5*b + 16*a^4*b^2 - 16*a^3*b^3 - 52*
a^2*b^4 - 24*a*b^5 + 70*(8*a^6 + 24*a^5*b + 16*a^4*b^2 - 16*a^3*b^3 - 52*a^
2*b^4 - 24*a*b^5 - (a^6 + 2*a^5*b - 15*a^4*b^2)*d*x)*cosh(d*x + c)^4 - (a^6
 + 2*a^5*b - 15*a^4*b^2)*d*x + 60*(3*a^6 + 7*a^5*b + 6*a^4*b^2 + 2*a^3*b^3
+ 15*a^2*b^4 + 9*a*b^5 - (a^6 - 2*a^5*b + 5*a^4*b^2)*d*x)*cosh(d*x + c)^2)*
sinh(d*x + c)^4 + 8*(55*(a^6 + 2*a^5*b + a^4*b^2)*d*x*cosh(d*x + c)^9 + 60*
(a^6 + 5*a^5*b + 10*a^4*b^2 + 14*a^3*b^3 + 11*a^2*b^4 + 3*a*b^5 + (a^6 - 2*
a^5*b - 3*a^4*b^2)*d*x)*cosh(d*x + c)^7 + 14*(8*a^6 + 24*a^5*b + 16*a^4*b^2
 - 16*a^3*b^3 - 52*a^2*b^4 - 24*a*b^5 - (a^6 + 2*a^5*b - 15*a^4*b^2)*d*x)*c
osh(d*x + c)^5 + 20*(3*a^6 + 7*a^5*b + 6*a^4*b^2 + 2*a^3*b^3 + 15*a^2*b^4 +
 9*a*b^5 - (a^6 - 2*a^5*b + 5*a^4*b^2)*d*x)*cosh(d*x + c)^3 + (8*a^6 + 24*a
^5*b + 16*a^4*b^2 - 16*a^3*b^3 - 52*a^2*b^4 - 24*a*b^5 - (a^6 + 2*a^5*b - 1
5*a^4*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(a^6 + 2*a^5*b + a^4*b^2
)*d*x + 4*(a^6 + 5*a^5*b + 10*a^4*b^2 + 14*a^3*b^3 + 11*a^2*b^4 + 3*a*b^5 +
 (a^6 - 2*a^5*b - 3*a^4*b^2)*d*x)*cosh(d*x + c)^2 + 4*(33*(a^6 + 2*a^5*b +
a^4*b^2)*d*x*cosh(d*x + c)^10 + 45*(a^6 + 5*a^5*b + 10*a^4*b^2 + 14*a^3*b^3
 + 11*a^2*b^4 + 3*a*b^5 + (a^6 - 2*a^5*b - 3*a^4*b^2)*d*x)*cosh(d*x + c)^8
+ 14*(8*a^6 + 24*a^5*b + 16*a^4*b^2 - 16*a^3*b^3 - 52*a^2*b^4 - 24*a*b^5 -
(a^6 + 2*a^5*b - 15*a^4*b^2)*d*x)*cosh(d*x + c)^6 + a^6 + 5*a^5*b + 10*a^4*
b^2 + 14*a^3*b^3 + 11*a^2*b^4 + 3*a*b^5 + 30*(3*a^6 + 7*a^5*b + 6*a^4*b^2 +
 2*a^3*b^3 + 15*a^2*b^4 + 9*a*b^5 - (a^6 - 2*a^5*b + 5*a^4*b^2)*d*x)*cosh(d
*x + c)^4 + (a^6 - 2*a^5*b - 3*a^4*b^2)*d*x + 3*(8*a^6 + 24*a^5*b + 16*a^4*
b^2 - 16*a^3*b^3 - 52*a^2*b^4 - 24*a*b^5 - (a^6 + 2*a^5*b - 15*a^4*b^2)*d*x
)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((6*a^4*b^2 + 20*a^3*b^3 + 25*a^2*b^4
+ 14*a*b^5 + 3*b^6)*cosh(d*x + c)^12 + 12*(6*a^4*b^2 + 20*a^3*b^3 + 25*a^2*
b^4 + 14*a*b^5 + 3*b^6)*cosh(d*x + c)*sinh(d*x + c)^11 + (6*a^4*b^2 + 20*a^
3*b^3 + 25*a^2*b^4 + 14*a*b^5 + 3*b^6)*sinh(d*x + c)^12 + 2*(6*a^4*b^2 - 4*
a^3*b^3 - 31*a^2*b^4 - 30*a*b^5 - 9*b^6)*cosh(d*x + c)^10 + 2*(6*a^4*b^2 -
4*a^3*b^3 - 31*a^2*b^4 - 30*a*b^5 - 9*b^6 + 33*(6*a^4*b^2 + 20*a^3*b^3 + 25
*a^2*b^4 + 14*a*b^5 + 3*b^6)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 20*(11*(6*
a^4*b^2 + 20*a^3*b^3 + 25*a^2*b^4 + 14*a*b^5 + 3*b^6)*cosh(d*x + c)^3 + (6*
a^4*b^2 - 4*a^3*b^3 - 31*a^2*b^4 - 30*a*b^5 - 9*b^6)*cosh(d*x + c))*sinh(d*
x + c)^9 - (6*a^4*b^2 + 20*a^3*b^3 - 71*a^2*b^4 - 114*a*b^5 - 45*b^6)*cosh(
d*x + c)^8 - (6*a^4*b^2 + 20*a^3*b^3 - 71*a^2*b^4 - 114*a*b^5 - 45*b^6 - 49
5*(6*a^4*b^2 + 20*a^3*b^3 + 25*a^2*b^4 + 14*a*b^5 + 3*b^6)*cosh(d*x + c)^4
- 90*(6*a^4*b^2 - 4*a^3*b^3 - 31*a^2*b^4 - 30*a*b^5 - 9*b^6)*cosh(d*x + c)^
2)*sinh(d*x + c)^8 + 8*(99*(6*a^4*b^2 + 20*a^3*b^3 + 25*a^2*b^4 + 14*a*b^5

+ 3*b^6)*cosh(d*x + c)^5 + 30*(6*a^4*b^2 - 4*a^3*b^3 - 31*a^2*b^4 - 30*a*b^
5 - 9*b^6)*cosh(d*x + c)^3 - (6*a^4*b^2 + 20*a^3*b^3 - 71*a^2*b^4 - 114*a*b
^5 - 45*b^6)*cosh(d*x + c))*sinh(d*x + c)^7 - 4*(6*a^4*b^2 - 4*a^3*b^3 + 17
*a^2*b^4 + 34*a*b^5 + 15*b^6)*cosh(d*x + c)^6 + 4*(231*(6*a^4*b^2 + 20*a^3*
b^3 + 25*a^2*b^4 + 14*a*b^5 + 3*b^6)*cosh(d*x + c)^6 - 6*a^4*b^2 + 4*a^3*b^
3 - 17*a^2*b^4 - 34*a*b^5 - 15*b^6 + 105*(6*a^4*b^2 - 4*a^3*b^3 - 31*a^2*b^
4 - 30*a*b^5 - 9*b^6)*cosh(d*x + c)^4 - 7*(6*a^4*b^2 + 20*a^3*b^3 - 71*a^2*
b^4 - 114*a*b^5 - 45*b^6)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 6*a^4*b^2 + 20
*a^3*b^3 + 25*a^2*b^4 + 14*a*b^5 + 3*b^6 + 8*(99*(6*a^4*b^2 + 20*a^3*b^3 +
25*a^2*b^4 + 14*a*b^5 + 3*b^6)*cosh(d*x + c)^7 + 63*(6*a^4*b^2 - 4*a^3*b^3
- 31*a^2*b^4 - 30*a*b^5 - 9*b^6)*cosh(d*x + c)^5 - 7*(6*a^4*b^2 + 20*a^3*b^
3 - 71*a^2*b^4 - 114*a*b^5 - 45*b^6)*cosh(d*x + c)^3 - 3*(6*a^4*b^2 - 4*a^3
*b^3 + 17*a^2*b^4 + 34*a*b^5 + 15*b^6)*cosh(d*x + c))*sinh(d*x + c)^5 - (6*
a^4*b^2 + 20*a^3*b^3 - 71*a^2*b^4 - 114*a*b^5 - 45*b^6)*cosh(d*x + c)^4 + (
495*(6*a^4*b^2 + 20*a^3*b^3 + 25*a^2*b^4 + 14*a*b^5 + 3*b^6)*cosh(d*x + c)^
8 + 420*(6*a^4*b^2 - 4*a^3*b^3 - 31*a^2*b^4 - 30*a*b^5 - 9*b^6)*cosh(d*x +
c)^6 - 6*a^4*b^2 - 20*a^3*b^3 + 71*a^2*b^4 + 114*a*b^5 + 45*b^6 - 70*(6*a^4
*b^2 + 20*a^3*b^3 - 71*a^2*b^4 - 114*a*b^5 - 45*b^6)*cosh(d*x + c)^4 - 60*(
6*a^4*b^2 - 4*a^3*b^3 + 17*a^2*b^4 + 34*a*b^5 + 15*b^6)*cosh(d*x + c)^2)*si
nh(d*x + c)^4 + 4*(55*(6*a^4*b^2 + 20*a^3*b^3 + 25*a^2*b^4 + 14*a*b^5 + 3*b
^6)*cosh(d*x + c)^9 + 60*(6*a^4*b^2 - 4*a^3*b^3 - 31*a^2*b^4 - 30*a*b^5 - 9
*b^6)*cosh(d*x + c)^7 - 14*(6*a^4*b^2 + 20*a^3*b^3 - 71*a^2*b^4 - 114*a*b^5
 - 45*b^6)*cosh(d*x + c)^5 - 20*(6*a^4*b^2 - 4*a^3*b^3 + 17*a^2*b^4 + 34*a*
b^5 + 15*b^6)*cosh(d*x + c)^3 - (6*a^4*b^2 + 20*a^3*b^3 - 71*a^2*b^4 - 114*
a*b^5 - 45*b^6)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(6*a^4*b^2 - 4*a^3*b^3 -
 31*a^2*b^4 - 30*a*b^5 - 9*b^6)*cosh(d*x + c)^2 + 2*(33*(6*a^4*b^2 + 20*a^3
*b^3 + 25*a^2*b^4 + 14*a*b^5 + 3*b^6)*cosh(d*x + c)^10 + 45*(6*a^4*b^2 - 4*
a^3*b^3 - 31*a^2*b^4 - 30*a*b^5 - 9*b^6)*cosh(d*x + c)^8 - 14*(6*a^4*b^2 +
20*a^3*b^3 - 71*a^2*b^4 - 114*a*b^5 - 45*b^6)*cosh(d*x + c)^6 + 6*a^4*b^2 -
 4*a^3*b^3 - 31*a^2*b^4 - 30*a*b^5 - 9*b^6 - 30*(6*a^4*b^2 - 4*a^3*b^3 + 17
*a^2*b^4 + 34*a*b^5 + 15*b^6)*cosh(d*x + c)^4 - 3*(6*a^4*b^2 + 20*a^3*b^3 -
 71*a^2*b^4 - 114*a*b^5 - 45*b^6)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(3*(
6*a^4*b^2 + 20*a^3*b^3 + 25*a^2*b^4 + 14*a*b^5 + 3*b^6)*cosh(d*x + c)^11 +
5*(6*a^4*b^2 - 4*a^3*b^3 - 31*a^2*b^4 - 30*a*b^5 - 9*b^6)*cosh(d*x + c)^9 -
 2*(6*a^4*b^2 + 20*a^3*b^3 - 71*a^2*b^4 - 114*a*b^5 - 45*b^6)*cosh(d*x + c)
^7 - 6*(6*a^4*b^2 - 4*a^3*b^3 + 17*a^2*b^4 + 34*a*b^5 + 15*b^6)*cosh(d*x +
c)^5 - (6*a^4*b^2 + 20*a^3*b^3 - 71*a^2*b^4 - 114*a*b^5 - 45*b^6)*cosh(d*x
+ c)^3 + (6*a^4*b^2 - 4*a^3*b^3 - 31*a^2*b^4 - 30*a*b^5 - 9*b^6)*cosh(d*x +
 c))*sinh(d*x + c))*log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^
2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2)) - 2*((a^6 + 2*a^5*b - 5*a^4*b^2 - 20*a^3*b^3 - 25*a^2*b^4 - 14*a*b^5 -
 3*b^6)*cosh(d*x + c)^12 + 12*(a^6 + 2*a^5*b - 5*a^4*b^2 - 20*a^3*b^3 - 25*
a^2*b^4 - 14*a*b^5 - 3*b^6)*cosh(d*x + c)*sinh(d*x + c)^11 + (a^6 + 2*a^5*b
 - 5*a^4*b^2 - 20*a^3*b^3 - 25*a^2*b^4 - 14*a*b^5 - 3*b^6)*sinh(d*x + c)^12
 + 2*(a^6 - 2*a^5*b - 9*a^4*b^2 + 4*a^3*b^3 + 31*a^2*b^4 + 30*a*b^5 + 9*b^6
)*cosh(d*x + c)^10 + 2*(a^6 - 2*a^5*b - 9*a^4*b^2 + 4*a^3*b^3 + 31*a^2*b^4
+ 30*a*b^5 + 9*b^6 + 33*(a^6 + 2*a^5*b - 5*a^4*b^2 - 20*a^3*b^3 - 25*a^2*b^
4 - 14*a*b^5 - 3*b^6)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 20*(11*(a^6 + 2*a
^5*b - 5*a^4*b^2 - 20*a^3*b^3 - 25*a^2*b^4 - 14*a*b^5 - 3*b^6)*cosh(d*x + c
)^3 + (a^6 - 2*a^5*b - 9*a^4*b^2 + 4*a^3*b^3 + 31*a^2*b^4 + 30*a*b^5 + 9*b^
6)*cosh(d*x + c))*sinh(d*x + c)^9 - (a^6 + 2*a^5*b - 21*a^4*b^2 - 20*a^3*b^
3 + 71*a^2*b^4 + 114*a*b^5 + 45*b^6)*cosh(d*x + c)^8 - (a^6 + 2*a^5*b - 21*
a^4*b^2 - 20*a^3*b^3 + 71*a^2*b^4 + 114*a*b^5 + 45*b^6 - 495*(a^6 + 2*a^5*b
 - 5*a^4*b^2 - 20*a^3*b^3 - 25*a^2*b^4 - 14*a*b^5 - 3*b^6)*cosh(d*x + c)^4
- 90*(a^6 - 2*a^5*b - 9*a^4*b^2 + 4*a^3*b^3 + 31*a^2*b^4 + 30*a*b^5 + 9*b^6
)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(99*(a^6 + 2*a^5*b - 5*a^4*b^2 - 20*
a^3*b^3 - 25*a^2*b^4 - 14*a*b^5 - 3*b^6)*cosh(d*x + c)^5 + 30*(a^6 - 2*a^5*
b - 9*a^4*b^2 + 4*a^3*b^3 + 31*a^2*b^4 + 30*a*b^5 + 9*b^6)*cosh(d*x + c)^3
- (a^6 + 2*a^5*b - 21*a^4*b^2 - 20*a^3*b^3 + 71*a^2*b^4 + 114*a*b^5 + 45*b^

6)*cosh(d*x + c))*sinh(d*x + c)^7 - 4*(a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3
- 17*a^2*b^4 - 34*a*b^5 - 15*b^6)*cosh(d*x + c)^6 + 4*(231*(a^6 + 2*a^5*b -
5*a^4*b^2 - 20*a^3*b^3 - 25*a^2*b^4 - 14*a*b^5 - 3*b^6)*cosh(d*x + c)^6 -
a^6 + 2*a^5*b + a^4*b^2 - 4*a^3*b^3 + 17*a^2*b^4 + 34*a*b^5 + 15*b^6 + 105*
(a^6 - 2*a^5*b - 9*a^4*b^2 + 4*a^3*b^3 + 31*a^2*b^4 + 30*a*b^5 + 9*b^6)*cos
h(d*x + c)^4 - 7*(a^6 + 2*a^5*b - 21*a^4*b^2 - 20*a^3*b^3 + 71*a^2*b^4 + 11
4*a*b^5 + 45*b^6)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + a^6 + 2*a^5*b - 5*a^4*
b^2 - 20*a^3*b^3 - 25*a^2*b^4 - 14*a*b^5 - 3*b^6 + 8*(99*(a^6 + 2*a^5*b - 5
*a^4*b^2 - 20*a^3*b^3 - 25*a^2*b^4 - 14*a*b^5 - 3*b^6)*cosh(d*x + c)^7 + 63
*(a^6 - 2*a^5*b - 9*a^4*b^2 + 4*a^3*b^3 + 31*a^2*b^4 + 30*a*b^5 + 9*b^6)*co
sh(d*x + c)^5 - 7*(a^6 + 2*a^5*b - 21*a^4*b^2 - 20*a^3*b^3 + 71*a^2*b^4 + 1
14*a*b^5 + 45*b^6)*cosh(d*x + c)^3 - 3*(a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3
- 17*a^2*b^4 - 34*a*b^5 - 15*b^6)*cosh(d*x + c))*sinh(d*x + c)^5 - (a^6 +
2*a^5*b - 21*a^4*b^2 - 20*a^3*b^3 + 71*a^2*b^4 + 114*a*b^5 + 45*b^6)*cosh(d
x + c)^4 + (495(a^6 + 2*a^5*b - 5*a^4*b^2 - 20*a^3*b^3 - 25*a^2*b^4 - 14*
a*b^5 - 3*b^6)*cosh(d*x + c)^8 + 420*(a^6 - 2*a^5*b - 9*a^4*b^2 + 4*a^3*b^3
+ 31*a^2*b^4 + 30*a*b^5 + 9*b^6)*cosh(d*x + c)^6 - a^6 - 2*a^5*b + 21*a^4*
b^2 + 20*a^3*b^3 - 71*a^2*b^4 - 114*a*b^5 - 45*b^6 - 70*(a^6 + 2*a^5*b - 21
*a^4*b^2 - 20*a^3*b^3 + 71*a^2*b^4 + 114*a*b^5 + 45*b^6)*cosh(d*x + c)^4 -
60*(a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - 17*a^2*b^4 - 34*a*b^5 - 15*b^6)*c
osh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(55*(a^6 + 2*a^5*b - 5*a^4*b^2 - 20*a^3
*b^3 - 25*a^2*b^4 - 14*a*b^5 - 3*b^6)*cosh(d*x + c)^9 + 60*(a^6 - 2*a^5*b -
9*a^4*b^2 + 4*a^3*b^3 + 31*a^2*b^4 + 30*a*b^5 + 9*b^6)*cosh(d*x + c)^7 - 1
4*(a^6 + 2*a^5*b - 21*a^4*b^2 - 20*a^3*b^3 + 71*a^2*b^4 + 114*a*b^5 + 45*b^
6)*cosh(d*x + c)^5 - 20*(a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - 17*a^2*b^4 -
34*a*b^5 - 15*b^6)*cosh(d*x + c)^3 - (a^6 + 2*a^5*b - 21*a^4*b^2 - 20*a^3*
b^3 + 71*a^2*b^4 + 114*a*b^5 + 45*b^6)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(
a^6 - 2*a^5*b - 9*a^4*b^2 + 4*a^3*b^3 + 31*a^2*b^4 + 30*a*b^5 + 9*b^6)*cosh
(d*x + c)^2 + 2*(33*(a^6 + 2*a^5*b - 5*a^4*b^2 - 20*a^3*b^3 - 25*a^2*b^4 -
14*a*b^5 - 3*b^6)*cosh(d*x + c)^10 + 45*(a^6 - 2*a^5*b - 9*a^4*b^2 + 4*a^3*
b^3 + 31*a^2*b^4 + 30*a*b^5 + 9*b^6)*cosh(d*x + c)^8 - 14*(a^6 + 2*a^5*b -
21*a^4*b^2 - 20*a^3*b^3 + 71*a^2*b^4 + 114*a*b^5 + 45*b^6)*cosh(d*x + c)^6
+ a^6 - 2*a^5*b - 9*a^4*b^2 + 4*a^3*b^3 + 31*a^2*b^4 + 30*a*b^5 + 9*b^6 - 3
0*(a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - 17*a^2*b^4 - 34*a*b^5 - 15*b^6)*co
sh(d*x + c)^4 - 3*(a^6 + 2*a^5*b - 21*a^4*b^2 - 20*a^3*b^3 + 71*a^2*b^4 + 1
14*a*b^5 + 45*b^6)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(3*(a^6 + 2*a^5*b -
5*a^4*b^2 - 20*a^3*b^3 - 25*a^2*b^4 - 14*a*b^5 - 3*b^6)*cosh(d*x + c)^11 +
5*(a^6 - 2*a^5*b - 9*a^4*b^2 + 4*a^3*b^3 + 31*a^2*b^4 + 30*a*b^5 + 9*b^6)*
cosh(d*x + c)^9 - 2*(a^6 + 2*a^5*b - 21*a^4*b^2 - 20*a^3*b^3 + 71*a^2*b^4 +
114*a*b^5 + 45*b^6)*cosh(d*x + c)^7 - 6*(a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b
^3 - 17*a^2*b^4 - 34*a*b^5 - 15*b^6)*cosh(d*x + c)^5 - (a^6 + 2*a^5*b - 21*
a^4*b^2 - 20*a^3*b^3 + 71*a^2*b^4 + 114*a*b^5 + 45*b^6)*cosh(d*x + c)^3 + (
a^6 - 2*a^5*b - 9*a^4*b^2 + 4*a^3*b^3 + 31*a^2*b^4 + 30*a*b^5 + 9*b^6)*cosh
(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c
))) + 8*(3*(a^6 + 2*a^5*b + a^4*b^2)*d*x*cosh(d*x + c)^11 + 5*(a^6 + 5*a^5*
b + 10*a^4*b^2 + 14*a^3*b^3 + 11*a^2*b^4 + 3*a*b^5 + (a^6 - 2*a^5*b - 3*a^4
*b^2)*d*x)*cosh(d*x + c)^9 + 2*(8*a^6 + 24*a^5*b + 16*a^4*b^2 - 16*a^3*b^3
- 52*a^2*b^4 - 24*a*b^5 - (a^6 + 2*a^5*b - 15*a^4*b^2)*d*x)*cosh(d*x + c)^7
+ 6*(3*a^6 + 7*a^5*b + 6*a^4*b^2 + 2*a^3*b^3 + 15*a^2*b^4 + 9*a*b^5 - (a^6
- 2*a^5*b + 5*a^4*b^2)*d*x)*cosh(d*x + c)^5 + (8*a^6 + 24*a^5*b + 16*a^4*b
^2 - 16*a^3*b^3 - 52*a^2*b^4 - 24*a*b^5 - (a^6 + 2*a^5*b - 15*a^4*b^2)*d*x)
*cosh(d*x + c)^3 + (a^6 + 5*a^5*b + 10*a^4*b^2 + 14*a^3*b^3 + 11*a^2*b^4 +
3*a*b^5 + (a^6 - 2*a^5*b - 3*a^4*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((
a^9 + 5*a^8*b + 10*a^7*b^2 + 10*a^6*b^3 + 5*a^5*b^4 + a^4*b^5)*d*cosh(d*x +
c)^12 + 12*(a^9 + 5*a^8*b + 10*a^7*b^2 + 10*a^6*b^3 + 5*a^5*b^4 + a^4*b^5)
*d*cosh(d*x + c)*sinh(d*x + c)^11 + (a^9 + 5*a^8*b + 10*a^7*b^2 + 10*a^6*b^
3 + 5*a^5*b^4 + a^4*b^5)*d*sinh(d*x + c)^12 + 2*(a^9 + a^8*b - 6*a^7*b^2 -
14*a^6*b^3 - 11*a^5*b^4 - 3*a^4*b^5)*d*cosh(d*x + c)^10 + 2*(33*(a^9 + 5*a^
8*b + 10*a^7*b^2 + 10*a^6*b^3 + 5*a^5*b^4 + a^4*b^5)*d*cosh(d*x + c)^2 + (a

```
^9 + a^8*b - 6*a^7*b^2 - 14*a^6*b^3 - 11*a^5*b^4 - 3*a^4*b^5)*d)*sinh(d*x +
 c)^10 - (a^9 + 5*a^8*b - 6*a^7*b^2 - 38*a^6*b^3 - 43*a^5*b^4 - 15*a^4*b^5)
*d*cosh(d*x + c)^8 + 20*(11*(a^9 + 5*a^8*b + 10*a^7*b^2 + 10*a^6*b^3 + 5*a^
5*b^4 + a^4*b^5)*d*cosh(d*x + c)^3 + (a^9 + a^8*b - 6*a^7*b^2 - 14*a^6*b^3
- 11*a^5*b^4 - 3*a^4*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^9 + (495*(a^9 + 5*
a^8*b + 10*a^7*b^2 + 10*a^6*b^3 + 5*a^5*b^4 + a^4*b^5)*d*cosh(d*x + c)^4 +
90*(a^9 + a^8*b - 6*a^7*b^2 - 14*a^6*b^3 - 11*a^5*b^4 - 3*a^4*b^5)*d*cosh(d
*x + c)^2 - (a^9 + 5*a^8*b - 6*a^7*b^2 - 38*a^6*b^3 - 43*a^5*b^4 - 15*a^4*b
^5)*d)*sinh(d*x + c)^8 - 4*(a^9 + a^8*b + 2*a^7*b^2 + 10*a^6*b^3 + 13*a^5*b
^4 + 5*a^4*b^5)*d*cosh(d*x + c)^6 + 8*(99*(a^9 + 5*a^8*b + 10*a^7*b^2 + 10*
a^6*b^3 + 5*a^5*b^4 + a^4*b^5)*d*cosh(d*x + c)^5 + 30*(a^9 + a^8*b - 6*a^7*
b^2 - 14*a^6*b^3 - 11*a^5*b^4 - 3*a^4*b^5)*d*cosh(d*x + c)^3 - (a^9 + 5*a^8
*b - 6*a^7*b^2 - 38*a^6*b^3 - 43*a^5*b^4 - 15*a^4*b^5)*d*cosh(d*x + c))*sin
h(d*x + c)^7 + 4*(231*(a^9 + 5*a^8*b + 10*a^7*b^2 + 10*a^6*b^3 + 5*a^5*b^4
+ a^4*b^5)*d*cosh(d*x + c)^6 + 105*(a^9 + a^8*b - 6*a^7*b^2 - 14*a^6*b^3 -
11*a^5*b^4 - 3*a^4*b^5)*d*cosh(d*x + c)^4 - 7*(a^9 + 5*a^8*b - 6*a^7*b^2 -
38*a^6*b^3 - 43*a^5*b^4 - 15*a^4*b^5)*d*cosh(d*x + c)^2 - (a^9 + a^8*b + 2*
a^7*b^2 + 10*a^6*b^3 + 13*a^5*b^4 + 5*a^4*b^5)*d)*sinh(d*x + c)^6 - (a^9 +
5*a^8*b - 6*a^7*b^2 - 38*a^6*b^3 - 43*a^5*b^4 - 15*a^4*b^5)*d*cosh(d*x + c)
^4 + 8*(99*(a^9 + 5*a^8*b + 10*a^7*b^2 + 10*a^6*b^3 + 5*a^5*b^4 + a^4*b^5)*
d*cosh(d*x + c)^7 + 63*(a^9 + a^8*b - 6*a^7*b^2 - 14*a^6*b^3 - 11*a^5*b^4 -
 3*a^4*b^5)*d*cosh(d*x + c)^5 - 7*(a^9 + 5*a^8*b - 6*a^7*b^2 - 38*a^6*b^3 -
 43*a^5*b^4 - 15*a^4*b^5)*d*cosh(d*x + c)^3 - 3*(a^9 + a^8*b + 2*a^7*b^2 +
10*a^6*b^3 + 13*a^5*b^4 + 5*a^4*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + (49
5*(a^9 + 5*a^8*b + 10*a^7*b^2 + 10*a^6*b^3 + 5*a^5*b^4 + a^4*b^5)*d*cosh(d*
x + c)^8 + 420*(a^9 + a^8*b - 6*a^7*b^2 - 14*a^6*b^3 - 11*a^5*b^4 - 3*a^4*b
^5)*d*cosh(d*x + c)^6 - 70*(a^9 + 5*a^8*b - 6*a^7*b^2 - 38*a^6*b^3 - 43*a^5
*b^4 - 15*a^4*b^5)*d*cosh(d*x + c)^4 - 60*(a^9 + a^8*b + 2*a^7*b^2 + 10*a^6
*b^3 + 13*a^5*b^4 + 5*a^4*b^5)*d*cosh(d*x + c)^2 - (a^9 + 5*a^8*b - 6*a^7*b
^2 - 38*a^6*b^3 - 43*a^5*b^4 - 15*a^4*b^5)*d)*sinh(d*x + c)^4 + 2*(a^9 + a^
8*b - 6*a^7*b^2 - 14*a^6*b^3 - 11*a^5*b^4 - 3*a^4*b^5)*d*cosh(d*x + c)^2 +
4*(55*(a^9 + 5*a^8*b + 10*a^7*b^2 + 10*a^6*b^3 + 5*a^5*b^4 + a^4*b^5)*d*cos
h(d*x + c)^9 + 60*(a^9 + a^8*b - 6*a^7*b^2 - 14*a^6*b^3 - 11*a^5*b^4 - 3*a^
4*b^5)*d*cosh(d*x + c)^7 - 14*(a^9 + 5*a^8*b - 6*a^7*b^2 - 38*a^6*b^3 - 43*
a^5*b^4 - 15*a^4*b^5)*d*cosh(d*x + c)^5 - 20*(a^9 + a^8*b + 2*a^7*b^2 + 10*
a^6*b^3 + 13*a^5*b^4 + 5*a^4*b^5)*d*cosh(d*x + c)^3 - (a^9 + 5*a^8*b - 6*a^
7*b^2 - 38*a^6*b^3 - 43*a^5*b^4 - 15*a^4*b^5)*d*cosh(d*x + c))*sinh(d*x + c
)^3 + 2*(33*(a^9 + 5*a^8*b + 10*a^7*b^2 + 10*a^6*b^3 + 5*a^5*b^4 + a^4*b^5)
*d*cosh(d*x + c)^10 + 45*(a^9 + a^8*b - 6*a^7*b^2 - 14*a^6*b^3 - 11*a^5*b^4
 - 3*a^4*b^5)*d*cosh(d*x + c)^8 - 14*(a^9 + 5*a^8*b - 6*a^7*b^2 - 38*a^6*b^
3 - 43*a^5*b^4 - 15*a^4*b^5)*d*cosh(d*x + c)^6 - 30*(a^9 + a^8*b + 2*a^7*b^
2 + 10*a^6*b^3 + 13*a^5*b^4 + 5*a^4*b^5)*d*cosh(d*x + c)^4 - 3*(a^9 + 5*a^8
*b - 6*a^7*b^2 - 38*a^6*b^3 - 43*a^5*b^4 - 15*a^4*b^5)*d*cosh(d*x + c)^2 +
(a^9 + a^8*b - 6*a^7*b^2 - 14*a^6*b^3 - 11*a^5*b^4 - 3*a^4*b^5)*d)*sinh(d*x
 + c)^2 + (a^9 + 5*a^8*b + 10*a^7*b^2 + 10*a^6*b^3 + 5*a^5*b^4 + a^4*b^5)*d
 + 4*(3*(a^9 + 5*a^8*b + 10*a^7*b^2 + 10*a^6*b^3 + 5*a^5*b^4 + a^4*b^5)*d*c
osh(d*x + c)^11 + 5*(a^9 + a^8*b - 6*a^7*b^2 - 14*a^6*b^3 - 11*a^5*b^4 - 3*
a^4*b^5)*d*cosh(d*x + c)^9 - 2*(a^9 + 5*a^8*b - 6*a^7*b^2 - 38*a^6*b^3 - 43
*a^5*b^4 - 15*a^4*b^5)*d*cosh(d*x + c)^7 - 6*(a^9 + a^8*b + 2*a^7*b^2 + 10*
a^6*b^3 + 13*a^5*b^4 + 5*a^4*b^5)*d*cosh(d*x + c)^5 - (a^9 + 5*a^8*b - 6*a^
7*b^2 - 38*a^6*b^3 - 43*a^5*b^4 - 15*a^4*b^5)*d*cosh(d*x + c)^3 + (a^9 + a^
8*b - 6*a^7*b^2 - 14*a^6*b^3 - 11*a^5*b^4 - 3*a^4*b^5)*d*cosh(d*x + c))*sin
h(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral(coth(c + d*x)**3/(a + b*tanh(c + d*x)**2)**3, x)
```

Giac [B] time = 1.41218, size = 656, normalized size = 3.84

$$\frac{\left(6\,a^2b^2+8\,ab^3+3\,b^4\right)\log\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)}{2\left(a^7d+3\,a^6bd+3\,a^5b^2d+a^4b^3d\right)}-\frac{dx+c}{a^3d+3\,a^2bd+3\,ab^2d+b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*(6*a^2*b^2 + 8*a*b^3 + 3*b^4)*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c)
 + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)/(a^7*d + 3*a^6*b*d +
3*a^5*b^2*d + a^4*b^3*d) - (d*x + c)/(a^3*d + 3*a^2*b*d + 3*a*b^2*d + b^3*d
) + (a - 3*b)*log(abs(e^(2*d*x + 2*c) - 1))/(a^4*d) - 2*((a^5 + 5*a^4*b + 1
0*a^3*b^2 + 14*a^2*b^3 + 11*a*b^4 + 3*b^5)*e^(10*d*x + 10*c) + 2*(2*a^5 + 6
*a^4*b + 4*a^3*b^2 - 4*a^2*b^3 - 13*a*b^4 - 6*b^5)*e^(8*d*x + 8*c) + 2*(3*a
^5 + 7*a^4*b + 6*a^3*b^2 + 2*a^2*b^3 + 15*a*b^4 + 9*b^5)*e^(6*d*x + 6*c) +
2*(2*a^5 + 6*a^4*b + 4*a^3*b^2 - 4*a^2*b^3 - 13*a*b^4 - 6*b^5)*e^(4*d*x + 4
*c) + (a^5 + 5*a^4*b + 10*a^3*b^2 + 14*a^2*b^3 + 11*a*b^4 + 3*b^5)*e^(2*d*x
 + 2*c))/((a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*
b*e^(2*d*x + 2*c) + a + b)^2*(a + b)^3*a^3*d*(e^(2*d*x + 2*c) - 1)^2)
```

3.200 $\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=228

$$-\frac{\left(8a^2+55ab+35b^2\right)\coth^3(c+dx)}{24a^3d(a+b)^2}-\frac{\left(-8a^2b+8a^3-55ab^2-35b^3\right)\coth(c+dx)}{8a^4d(a+b)^2}+\frac{b^{5/2}\left(63a^2+90ab+35b^2\right)\tan^{-1}\left(\right.}{8a^{9/2}d(a+b)^3}$$

```
[Out] x/(a + b)^3 + (b^(5/2)*(63*a^2 + 90*a*b + 35*b^2)*ArcTan[(Sqrt[b]*Tanh[c +
d*x])/Sqrt[a]])/(8*a^(9/2)*(a + b)^3*d) - ((8*a^3 - 8*a^2*b - 55*a*b^2 - 35
*b^3)*Coth[c + d*x])/(8*a^4*(a + b)^2*d) - ((8*a^2 + 55*a*b + 35*b^2)*Coth[
c + d*x]^3)/(24*a^3*(a + b)^2*d) + (b*Coth[c + d*x]^3)/(4*a*(a + b)*d*(a +
b*Tanh[c + d*x]^2)^2) + (b*(11*a + 7*b)*Coth[c + d*x]^3)/(8*a^2*(a + b)^2*d
*(a + b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.368472, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3670, 472, 579, 583, 522, 206, 205}

$$-\frac{\left(8a^2+55ab+35b^2\right)\coth^3(c+dx)}{24a^3d(a+b)^2}-\frac{\left(-8a^2b+8a^3-55ab^2-35b^3\right)\coth(c+dx)}{8a^4d(a+b)^2}+\frac{b^{5/2}\left(63a^2+90ab+35b^2\right)\tan^{-1}\left(\right.}{8a^{9/2}d(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] x/(a + b)^3 + (b^(5/2)*(63*a^2 + 90*a*b + 35*b^2)*ArcTan[(Sqrt[b]*Tanh[c +
d*x])/Sqrt[a]])/(8*a^(9/2)*(a + b)^3*d) - ((8*a^3 - 8*a^2*b - 55*a*b^2 - 35
*b^3)*Coth[c + d*x])/(8*a^4*(a + b)^2*d) - ((8*a^2 + 55*a*b + 35*b^2)*Coth[
c + d*x]^3)/(24*a^3*(a + b)^2*d) + (b*Coth[c + d*x]^3)/(4*a*(a + b)*d*(a +
b*Tanh[c + d*x]^2)^2) + (b*(11*a + 7*b)*Coth[c + d*x]^3)/(8*a^2*(a + b)^2*d
*(a + b*Tanh[c + d*x]^2))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1-x^2)(a+bx^2)^3}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$= \frac{b\coth^3(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-4a-7b+7bx^2}{x^4(1-x^2)(a+bx^2)^2}\,dx,x,\tanh(c+dx)\right)}{4a(a+b)d}$$

$$= \frac{b\coth^3(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(11a+7b)\coth^3(c+dx)}{8a^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{8a^2+}{}\right.}{}$$

$$= -\frac{\left(8a^2+55ab+35b^2\right)\coth^3(c+dx)}{24a^3(a+b)^2d} + \frac{b\coth^3(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(11a+}{8a^2(a+b)^2c}$$

$$= -\frac{\left(8a^3-8a^2b-55ab^2-35b^3\right)\coth(c+dx)}{8a^4(a+b)^2d} - \frac{\left(8a^2+55ab+35b^2\right)\coth^3(c+dx)}{24a^3(a+b)^2d} + \frac{}{4a(a}$$

$$= -\frac{\left(8a^3-8a^2b-55ab^2-35b^3\right)\coth(c+dx)}{8a^4(a+b)^2d} - \frac{\left(8a^2+55ab+35b^2\right)\coth^3(c+dx)}{24a^3(a+b)^2d} + \frac{}{4a(a}$$

$$= \frac{x}{(a+b)^3} + \frac{b^{5/2}\left(63a^2+90ab+35b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}(a+b)^3d} - \frac{\left(8a^3-8a^2b-55ab^2-35b^3\right.}{8a^4(a+b)^2d}$$

Mathematica [A] time = 3.41783, size = 194, normalized size = 0.85

$$\frac{\frac{3b^{5/2}\left(63a^2+90ab+35b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{9/2}(a+b)^3} + \frac{12b^4\sinh(2(c+dx))}{a^3(a+b)^2((a+b)\cosh(2(c+dx))+a-b)^2} + \frac{3b^3(17a+11b)\sinh(2(c+dx))}{a^4(a+b)^2((a+b)\cosh(2(c+dx))+a-b)} + \frac{8(9b-4a)\coth(c+dx)}{a^4} - 8}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((24*(c + d*x))/(a + b)^3 + (3*b^(5/2)*(63*a^2 + 90*a*b + 35*b^2)*ArcTan[(S
qrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(9/2)*(a + b)^3) + (8*(-4*a + 9*b)*Coth[
c + d*x])/a^4 - (8*Coth[c + d*x]*Csch[c + d*x]^2)/a^3 + (12*b^4*Sinh[2*(c +
 d*x)])/(a^3*(a + b)^2*(a - b + (a + b)*Cosh[2*(c + d*x)])^2) + (3*b^3*(17*
a + 11*b)*Sinh[2*(c + d*x)])/(a^4*(a + b)^2*(a - b + (a + b)*Cosh[2*(c + d*
x)])))/(24*d)
```

Maple [B] time = 0.14, size = 2139, normalized size = 9.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 63/8/d*b^3/(a+b)^3/a^2/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1
/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-63/8/d*b^3/(a+b)^3/a^2/(
```

```
(2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b
))^(1/2)+a+2*b)*a)^(1/2))-125/8/d*b^5/(a+b)^3/a^3/(b*(a+b))^(1/2)/((2*(b*(a
+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2
)-a-2*b)*a)^(1/2))-63/8/d*b^3/(a+b)^3/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)
-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a
)^(1/2))-153/8/d*b^4/(a+b)^3/a^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)
*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)
)-63/8/d*b^3/(a+b)^3/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*
arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-153/8/d*b
^4/(a+b)^3/a^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a
*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/d/(a+b)^3*ln(ta
nh(1/2*d*x+1/2*c)+1)-1/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)-1)+143/4/d*b^5/(a+b
)^3/a^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1
/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^3+45/4/d*b^4/(a+b)^3/a^3/((2*(b*(a+b))^(
1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*
b)*a)^(1/2))-45/4/d*b^4/(a+b)^3/a^3/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arc
tan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+15/2/d*b^4/(
a+b)^3/a^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*
x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^7-35/8/d*b^5/(a+b)^3/a^4/((2*(b*(a+b)
)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+
2*b)*a)^(1/2))-125/8/d*b^5/(a+b)^3/a^3/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+
a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^
(1/2))+3/2/d/a^4*tanh(1/2*d*x+1/2*c)*b+3/2/d/a^4/tanh(1/2*d*x+1/2*c)*b+11/d
*b^6/(a+b)^3/a^4/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(
1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5+11/d*b^6/(a+b)^3/a^4/(tanh(1/
2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*t
anh(1/2*d*x+1/2*c)^3-5/8/d/a^3*tanh(1/2*d*x+1/2*c)-5/8/d/a^3/tanh(1/2*d*x+1
/2*c)+17/4/d*b^3/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a
+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^7+51/4/d*b^3/(a+b)^3/
(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*
b+a)^2/a*tanh(1/2*d*x+1/2*c)^5+75/2/d*b^4/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+
2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2*tanh(1/2*d*x+1
/2*c)^5+51/4/d*b^3/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2
*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^3+75/2/d*b^4/(a+b)^
3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^
2*b+a)^2/a^2*tanh(1/2*d*x+1/2*c)^3+17/4/d*b^3/(a+b)^3/(tanh(1/2*d*x+1/2*c)^
4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x
+1/2*c)-35/8/d*b^6/(a+b)^3/a^4/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a
)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1
3/4/d*b^5/(a+b)^3/a^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*
tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^7+35/8/d*b^5/(a+b)^3/a^4/(
(2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+
b))^(1/2)-a-2*b)*a)^(1/2))-1/24/d/a^3*tanh(1/2*d*x+1/2*c)^3-1/24/d/a^3/tanh
(1/2*d*x+1/2*c)^3+13/4/d*b^5/(a+b)^3/a^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/
2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)+15/2/d*
b^4/(a+b)^3/a^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1
/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)+143/4/d*b^5/(a+b)^3/a^3/(tanh(1/
2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*t
anh(1/2*d*x+1/2*c)^5-35/8/d*b^6/(a+b)^3/a^4/(b*(a+b))^(1/2)/((2*(b*(a+b))^(
1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*
b)*a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**4/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral(coth(c + d*x)**4/(a + b*tanh(c + d*x)**2)**3, x)
```

Giac [B] time = 1.45776, size = 683, normalized size = 3.

$$\frac{\left(63\,a^2b^3+90\,ab^4+35\,b^5\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{8\left(a^7d+3\,a^6bd+3\,a^5b^2d+a^4b^3d\right)\sqrt{ab}}+\frac{dx+c}{a^3d+3\,a^2bd+3\,ab^2d+b^3d}-\frac{17\,a^3b^3e^{(6\,dx+6\,c)}+7\,a^2b^4e^{(6\,dx}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(63*a^2*b^3 + 90*a*b^4 + 35*b^5)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2
*d*x + 2*c) + a - b)/sqrt(a*b))/((a^7*d + 3*a^6*b*d + 3*a^5*b^2*d + a^4*b^3
*d)*sqrt(a*b)) + (d*x + c)/(a^3*d + 3*a^2*b*d + 3*a*b^2*d + b^3*d) - 1/4*(1
7*a^3*b^3*e^(6*d*x + 6*c) + 7*a^2*b^4*e^(6*d*x + 6*c) - 21*a*b^5*e^(6*d*x +
 6*c) - 11*b^6*e^(6*d*x + 6*c) + 51*a^3*b^3*e^(4*d*x + 4*c) - a^2*b^4*e^(4*
d*x + 4*c) + 29*a*b^5*e^(4*d*x + 4*c) + 33*b^6*e^(4*d*x + 4*c) + 51*a^3*b^3
*e^(2*d*x + 2*c) + 37*a^2*b^4*e^(2*d*x + 2*c) - 47*a*b^5*e^(2*d*x + 2*c) -
33*b^6*e^(2*d*x + 2*c) + 17*a^3*b^3 + 45*a^2*b^4 + 39*a*b^5 + 11*b^6)/((a^7
*d + 3*a^6*b*d + 3*a^5*b^2*d + a^4*b^3*d)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x +
 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2) - 2/3*(6*a*e^
(4*d*x + 4*c) - 9*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*c) + 18*b*e^(2*d*x +
 2*c) + 4*a - 9*b)/(a^4*d*(e^(2*d*x + 2*c) - 1)^3)
```

3.201 $\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^4}\,dx$

Optimal. Leaf size=201

$$\frac{b\left(19a^2+16ab+5b^2\right)\tanh(c+dx)}{16a^3d(a+b)^3\left(a+b\tanh^2(c+dx)\right)}+\frac{\sqrt{b}\left(35a^2b+35a^3+21ab^2+5b^3\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{16a^{7/2}d(a+b)^4}+\frac{b(11a+5b)\tan}{24a^2d(a+b)^2\left(a+b\right.}$$

```
[Out] x/(a + b)^4 + (Sqrt[b]*(35*a^3 + 35*a^2*b + 21*a*b^2 + 5*b^3)*ArcTan[(Sqrt[
b]*Tanh[c + d*x])/Sqrt[a]])/(16*a^(7/2)*(a + b)^4*d) + (b*Tanh[c + d*x])/(6
*a*(a + b)*d*(a + b*Tanh[c + d*x]^2)^3) + (b*(11*a + 5*b)*Tanh[c + d*x])/(2
4*a^2*(a + b)^2*d*(a + b*Tanh[c + d*x]^2)^2) + (b*(19*a^2 + 16*a*b + 5*b^2)
*Tanh[c + d*x])/(16*a^3*(a + b)^3*d*(a + b*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.277135, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3661, 414, 527, 522, 206, 205}

$$\frac{b\left(19a^2+16ab+5b^2\right)\tanh(c+dx)}{16a^3d(a+b)^3\left(a+b\tanh^2(c+dx)\right)}+\frac{\sqrt{b}\left(35a^2b+35a^3+21ab^2+5b^3\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{16a^{7/2}d(a+b)^4}+\frac{b(11a+5b)\tan}{24a^2d(a+b)^2\left(a+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tanh[c + d*x]^2)^(-4),x]
```


```
[Out] x/(a + b)^4 + (Sqrt[b]*(35*a^3 + 35*a^2*b + 21*a*b^2 + 5*b^3)*ArcTan[(Sqrt[
b]*Tanh[c + d*x])/Sqrt[a]])/(16*a^(7/2)*(a + b)^4*d) + (b*Tanh[c + d*x])/(6
*a*(a + b)*d*(a + b*Tanh[c + d*x]^2)^3) + (b*(11*a + 5*b)*Tanh[c + d*x])/(2
4*a^2*(a + b)^2*d*(a + b*Tanh[c + d*x]^2)^2) + (b*(19*a^2 + 16*a*b + 5*b^2)
*Tanh[c + d*x])/(16*a^3*(a + b)^3*d*(a + b*Tanh[c + d*x]^2))
```


Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
```

```
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^4}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\left(a+bx^2\right)^4}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\tanh(c+dx)}{6a(a+b)d\left(a+b\tanh^2(c+dx)\right)^3} - \frac{\text{Subst}\left(\int \frac{b-6(a+b)+5bx^2}{(1-x^2)\left(a+bx^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{6a(a+b)d}$$

$$= \frac{b\tanh(c+dx)}{6a(a+b)d\left(a+b\tanh^2(c+dx)\right)^3} + \frac{b(11a+5b)\tanh(c+dx)}{24a^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{3}{\ldots}\right.}{\ldots}$$

$$= \frac{b\tanh(c+dx)}{6a(a+b)d\left(a+b\tanh^2(c+dx)\right)^3} + \frac{b(11a+5b)\tanh(c+dx)}{24a^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b\left(19a^2+1\ldots\right.}{16a^3(a+b)\ldots}$$

$$= \frac{b\tanh(c+dx)}{6a(a+b)d\left(a+b\tanh^2(c+dx)\right)^3} + \frac{b(11a+5b)\tanh(c+dx)}{24a^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b\left(19a^2+1\ldots\right.}{16a^3(a+b)\ldots}$$

$$= \frac{x}{(a+b)^4} + \frac{\sqrt{b}\left(35a^3+35a^2b+21ab^2+5b^3\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{16a^{7/2}(a+b)^4d} + \frac{b\tanh(c+\ldots}{6a(a+b)d\left(a+b\,\text{ta}\ldots\right.}$$

Mathematica [A] time = 0.546873, size = 203, normalized size = 1.01

$$\frac{\frac{3b\left(19a^2+16ab+5b^2\right)(a+b)\tanh(c+dx)}{a^3\left(a+b\tanh^2(c+dx)\right)} + \frac{3\sqrt{b}\left(35a^2b+35a^3+21ab^2+5b^3\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{7/2}} + \frac{2b(11a+5b)(a+b)^2\tanh(c+dx)}{a^2\left(a+b\tanh^2(c+dx)\right)^2} + \frac{8b(a+b)^3\tanh(c+dx)}{a\left(a+b\tanh^2(c+dx)\right)^3} - \ldots}{48d(a+b)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tanh[c + d*x]^2)^(-4),x]
```

```
[Out] ((3*Sqrt[b]*(35*a^3 + 35*a^2*b + 21*a*b^2 + 5*b^3)*ArcTan[(Sqrt[b]*Tanh[c +
 d*x])/Sqrt[a]])/a^(7/2) - 24*Log[1 - Tanh[c + d*x]] + 24*Log[1 + Tanh[c +
d*x]] + (8*b*(a + b)^3*Tanh[c + d*x])/(a*(a + b*Tanh[c + d*x]^2)^3) + (2*b*
(a + b)^2*(11*a + 5*b)*Tanh[c + d*x])/(a^2*(a + b*Tanh[c + d*x]^2)^2) + (3*
b*(a + b)*(19*a^2 + 16*a*b + 5*b^2)*Tanh[c + d*x])/(a^3*(a + b*Tanh[c + d*x
]^2)))/(48*(a + b)^4*d)
```

Maple [B] time = 0.034, size = 608, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tanh(d*x+c)^2)^4,x)
```

```
[Out] 1/2/d/(a+b)^4*ln(tanh(d*x+c)+1)+19/16/d/(a+b)^4*b^3/(a+b*tanh(d*x+c)^2)^3*t
anh(d*x+c)^5+35/16/d/(a+b)^4*b^4/(a+b*tanh(d*x+c)^2)^3/a*tanh(d*x+c)^5+21/1
6/d/(a+b)^4*b^5/(a+b*tanh(d*x+c)^2)^3/a^2*tanh(d*x+c)^5+5/16/d/(a+b)^4*b^6/
(a+b*tanh(d*x+c)^2)^3/a^3*tanh(d*x+c)^5+17/6/d/(a+b)^4*b^2/(a+b*tanh(d*x+c)
^2)^3*a*tanh(d*x+c)^3+11/2/d/(a+b)^4*b^3/(a+b*tanh(d*x+c)^2)^3*tanh(d*x+c)^
3+7/2/d/(a+b)^4*b^4/(a+b*tanh(d*x+c)^2)^3/a*tanh(d*x+c)^3+5/6/d/(a+b)^4*b^5
/(a+b*tanh(d*x+c)^2)^3/a^2*tanh(d*x+c)^3+29/16/d/(a+b)^4*b/(a+b*tanh(d*x+c)
^2)^3*a^2*tanh(d*x+c)+61/16/d/(a+b)^4*b^2/(a+b*tanh(d*x+c)^2)^3*a*tanh(d*x+
c)+43/16/d/(a+b)^4*b^3/(a+b*tanh(d*x+c)^2)^3*tanh(d*x+c)+11/16/d/(a+b)^4*b^
4/(a+b*tanh(d*x+c)^2)^3/a*tanh(d*x+c)+35/16/d/(a+b)^4*b/(a*b)^(1/2)*arctan(
tanh(d*x+c)*b/(a*b)^(1/2))+35/16/d/(a+b)^4*b^2/a/(a*b)^(1/2)*arctan(tanh(d*
x+c)*b/(a*b)^(1/2))+21/16/d/(a+b)^4*b^3/a^2/(a*b)^(1/2)*arctan(tanh(d*x+c)*
b/(a*b)^(1/2))+5/16/d/(a+b)^4*b^4/a^3/(a*b)^(1/2)*arctan(tanh(d*x+c)*b/(a*b
)^(1/2))-1/2/d/(a+b)^4*ln(tanh(d*x+c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c)^2)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c)^2)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c)**2)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.31529, size = 1030, normalized size = 5.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c)^2)^4,x, algorithm="giac")
```

```
[Out] 1/16*(35*a^3*b + 35*a^2*b^2 + 21*a*b^3 + 5*b^4)*arctan(1/2*(a*e^(2*d*x + 2*
c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^7*d + 4*a^6*b*d + 6*a^5*b^2*
d + 4*a^4*b^3*d + a^3*b^4*d)*sqrt(a*b)) + (d*x + c)/(a^4*d + 4*a^3*b*d + 6*
a^2*b^2*d + 4*a*b^3*d + b^4*d) - 1/24*(87*a^5*b*e^(10*d*x + 10*c) + 69*a^4*
b^2*e^(10*d*x + 10*c) - 186*a^3*b^3*e^(10*d*x + 10*c) - 246*a^2*b^4*e^(10*d
*x + 10*c) - 93*a*b^5*e^(10*d*x + 10*c) - 15*b^6*e^(10*d*x + 10*c) + 435*a^
5*b*e^(8*d*x + 8*c) + 51*a^4*b^2*e^(8*d*x + 8*c) - 174*a^3*b^3*e^(8*d*x + 8
*c) + 450*a^2*b^4*e^(8*d*x + 8*c) + 315*a*b^5*e^(8*d*x + 8*c) + 75*b^6*e^(8
*d*x + 8*c) + 870*a^5*b*e^(6*d*x + 6*c) + 58*a^4*b^2*e^(6*d*x + 6*c) + 324*
a^3*b^3*e^(6*d*x + 6*c) - 612*a^2*b^4*e^(6*d*x + 6*c) - 490*a*b^5*e^(6*d*x
+ 6*c) - 150*b^6*e^(6*d*x + 6*c) + 870*a^5*b*e^(4*d*x + 4*c) + 558*a^4*b^2*
e^(4*d*x + 4*c) - 36*a^3*b^3*e^(4*d*x + 4*c) + 636*a^2*b^4*e^(4*d*x + 4*c)
+ 510*a*b^5*e^(4*d*x + 4*c) + 150*b^6*e^(4*d*x + 4*c) + 435*a^5*b*e^(2*d*x
+ 2*c) + 801*a^4*b^2*e^(2*d*x + 2*c) + 102*a^3*b^3*e^(2*d*x + 2*c) - 534*a^
2*b^4*e^(2*d*x + 2*c) - 345*a*b^5*e^(2*d*x + 2*c) - 75*b^6*e^(2*d*x + 2*c)
+ 87*a^5*b + 319*a^4*b^2 + 450*a^3*b^3 + 306*a^2*b^4 + 103*a*b^5 + 15*b^6)/
((a^7*d + 4*a^6*b*d + 6*a^5*b^2*d + 4*a^4*b^3*d + a^3*b^4*d)*(a*e^(4*d*x +
4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a +
b)^3)
```

3.202 $\int \sqrt{1-\tanh^2(x)}\,dx$

Optimal. Leaf size=3

$$\sin^{-1}(\tanh(x))$$

```
[Out] ArcSin[Tanh[x]]
```

Rubi [A] time = 0.0165158, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3657, 4122, 216}

$$\sin^{-1}(\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Tanh[x]^2],x]
```

```
[Out] ArcSin[Tanh[x]]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1-\tanh^2(x)}\,dx &= \int \sqrt{\operatorname{sech}^2(x)}\,dx\\ &= \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \tanh(x)\right)\\ &= \sin^{-1}(\tanh(x))\end{aligned}$$

Mathematica [B] time = 0.007552, size = 19, normalized size = 6.33

$$2\cosh(x)\sqrt{\operatorname{sech}^2(x)}\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Tanh[x]^2],x]
```

```
[Out] 2*ArcTan[Tanh[x/2]]*Cosh[x]*Sqrt[Sech[x]^2]
```

Maple [A] time = 0.033, size = 4, normalized size = 1.3

$$\arcsin(\tanh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-tanh(x)^2)^(1/2),x)
```

```
[Out] arcsin(tanh(x))
```

Maxima [A] time = 1.68692, size = 7, normalized size = 2.33

$$2\arctan(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*arctan(e^x)
```

Fricas [B] time = 2.16123, size = 39, normalized size = 13.

$$2\arctan(\cosh(x) + \sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*arctan(cosh(x) + sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{1 - \tanh^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(1 - tanh(x)**2), x)
```

Giac [A] time = 1.16934, size = 7, normalized size = 2.33

$$2\arctan(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(e^x)
```

3.203 $\int \sqrt{-1+\tanh^2(x)}\,dx$

Optimal. Leaf size=16

$$-\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}}\right)$$

```
[Out] -ArcTanh[Tanh[x]/Sqrt[-Sech[x]^2]]
```

Rubi [A] time = 0.0205394, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3657, 4122, 217, 206}

$$-\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + Tanh[x]^2],x]
```

```
[Out] -ArcTanh[Tanh[x]/Sqrt[-Sech[x]^2]]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int\sqrt{-1+\tanh^2(x)}\,dx=\int\sqrt{-\text{sech}^2(x)}\,dx$$
$$=-\text{Subst}\left(\int\frac{1}{\sqrt{-1+x^2}}\,dx,x,\tanh(x)\right)$$
$$=-\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{\tanh(x)}{\sqrt{-\text{sech}^2(x)}}\right)$$
$$=-\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{-\text{sech}^2(x)}}\right)$$

Mathematica [A] time = 0.0067079, size = 21, normalized size = 1.31

$$2\cosh(x)\sqrt{-\text{sech}^2(x)}\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + Tanh[x]^2],x]
```

```
[Out] 2*ArcTan[Tanh[x/2]]*Cosh[x]*Sqrt[-Sech[x]^2]
```

Maple [A] time = 0.036, size = 15, normalized size = 0.9

$$-\ln\left(\tanh(x)+\sqrt{-1+(\tanh(x))^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+tanh(x)^2)^(1/2),x)
```

```
[Out] -ln(tanh(x)+(-1+tanh(x)^2)^(1/2))
```

Maxima [C] time = 1.67764, size = 7, normalized size = 0.44

$$2i\arctan(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*I*arctan(e^x)
```

Fricas [A] time = 2.2341, size = 4, normalized size = 0.25

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\tanh^2(x) - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(tanh(x)**2 - 1), x)
```

Giac [C] time = 1.20325, size = 23, normalized size = 1.44

$$-\frac{1}{2}\log\left(e^{(2x)} + 1\right) + \log\left(ie^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(e^(2*x) + 1) + log(I*e^x + 1)
```

3.204 $\int \left(1-\tanh^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=22

$$\frac{1}{2}\sin^{-1}(\tanh(x))+\frac{1}{2}\tanh(x)\sqrt{\text{sech}^2(x)}$$

```
[Out] ArcSin[Tanh[x]]/2 + (Sqrt[Sech[x]^2]*Tanh[x])/2
```

Rubi [A] time = 0.0199402, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3657, 4122, 195, 216}

$$\frac{1}{2}\sin^{-1}(\tanh(x))+\frac{1}{2}\tanh(x)\sqrt{\text{sech}^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Tanh[x]^2)^(3/2),x]
```

```
[Out] ArcSin[Tanh[x]]/2 + (Sqrt[Sech[x]^2]*Tanh[x])/2
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \left(1-\tanh^2(x)\right)^{3/2} dx = \int \text{sech}^2(x)^{3/2} dx$$
$$= \text{Subst}\left(\int \sqrt{1-x^2}\, dx, x, \tanh(x)\right)$$
$$= \frac{1}{2}\sqrt{\text{sech}^2(x)}\tanh(x) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} dx, x, \tanh(x)\right)$$
$$= \frac{1}{2}\sin^{-1}(\tanh(x)) + \frac{1}{2}\sqrt{\text{sech}^2(x)}\tanh(x)$$

Mathematica [A] time = 0.0181831, size = 29, normalized size = 1.32

$$\frac{\text{sech}(x)\left(2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+\tanh(x)\text{sech}(x)\right)}{2\sqrt{\text{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Tanh[x]^2)^(3/2),x]
```

```
[Out] (Sech[x]*(2*ArcTan[Tanh[x/2]] + Sech[x]*Tanh[x]))/(2*Sqrt[Sech[x]^2])
```

Maple [A] time = 0.03, size = 21, normalized size = 1.

$$\frac{\tanh(x)}{2}\sqrt{1-(\tanh(x))^2}+\frac{\arcsin(\tanh(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-tanh(x)^2)^(3/2),x)
```

```
[Out] 1/2*(1-tanh(x)^2)^(1/2)*tanh(x)+1/2*arcsin(tanh(x))
```

Maxima [A] time = 1.68189, size = 38, normalized size = 1.73

$$\frac{e^{(3x)}-e^{x}}{e^{(4x)}+2e^{(2x)}+1}+\arctan(e^{x})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-tanh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (e^(3*x) - e^x)/(e^(4*x) + 2*e^(2*x) + 1) + arctan(e^x)
```

Fricas [B] time = 2.14712, size = 504, normalized size = 22.91

$$\frac{\cosh(x)^3+3\cosh(x)\sinh(x)^2+\sinh(x)^3+\left(\cosh(x)^4+4\cosh(x)\sinh(x)^3+\sinh(x)^4+2\left(3\cosh(x)^2+1\right)\sinh(x\right.}{\cosh(x)^4+4\cosh(x)\sinh(x)^3+\sinh(x)^4+2\left(3\cosh(x)^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (cosh(x)^4 + 4*cosh(x)*sinh(
x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)
^3 + cosh(x))*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + (3*cosh(x)^2 - 1)*si
nh(x) - cosh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x
)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(1-\tanh^2(x)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-tanh(x)**2)**(3/2),x)
```

```
[Out] Integral((1 - tanh(x)**2)**(3/2), x)
```

Giac [B] time = 1.29679, size = 61, normalized size = 2.77

$$\frac{1}{4}\,\pi-\frac{e^{(-x)}-e^{x}}{\left(e^{(-x)}-e^{x}\right)^{2}+4}+\frac{1}{2}\,\arctan\left(\frac{1}{2}\,\left(e^{(2\,x)}-1\right)e^{(-x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*pi - (e^(-x) - e^x)/((e^(-x) - e^x)^2 + 4) + 1/2*arctan(1/2*(e^(2*x) -
1)*e^(-x))
```

3.205 $\int \left(-1+\tanh^2(x)\right)^{3/2} dx$

Optimal. Leaf size=35

$$\frac{1}{2}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}}\right)-\frac{1}{2}\tanh(x)\sqrt{-\operatorname{sech}^2(x)}$$

```
[Out] ArcTanh[Tanh[x]/Sqrt[-Sech[x]^2]]/2 - (Sqrt[-Sech[x]^2]*Tanh[x])/2
```

Rubi [A] time = 0.0235295, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3657, 4122, 195, 217, 206}

$$\frac{1}{2}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}}\right)-\frac{1}{2}\tanh(x)\sqrt{-\operatorname{sech}^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Tanh[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Tanh[x]/Sqrt[-Sech[x]^2]]/2 - (Sqrt[-Sech[x]^2]*Tanh[x])/2
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(-1+\tanh^2(x)\right)^{3/2} dx = \int \left(-\text{sech}^2(x)\right)^{3/2} dx$$
$$= -\text{Subst}\left(\int \sqrt{-1+x^2}\, dx, x, \tanh(x)\right)$$
$$= -\frac{1}{2}\sqrt{-\text{sech}^2(x)}\tanh(x) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}}\, dx, x, \tanh(x)\right)$$
$$= -\frac{1}{2}\sqrt{-\text{sech}^2(x)}\tanh(x) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\, dx, x, \frac{\tanh(x)}{\sqrt{-\text{sech}^2(x)}}\right)$$
$$= \frac{1}{2}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{-\text{sech}^2(x)}}\right) - \frac{1}{2}\sqrt{-\text{sech}^2(x)}\tanh(x)$$

Mathematica [A] time = 0.0193486, size = 28, normalized size = 0.8

$$-\frac{1}{2}\sqrt{-\text{sech}^2(x)}\left(\tanh(x) + 2\cosh(x)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Tanh[x]^2)^(3/2),x]
```

```
[Out] -(Sqrt[-Sech[x]^2]*(2*ArcTan[Tanh[x/2]]*Cosh[x] + Tanh[x]))/2
```

Maple [A] time = 0.035, size = 28, normalized size = 0.8

$$-\frac{\tanh(x)}{2}\sqrt{-1+(\tanh(x))^2} + \frac{1}{2}\ln\left(\tanh(x) + \sqrt{-1+(\tanh(x))^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+tanh(x)^2)^(3/2),x)
```

```
[Out] -1/2*tanh(x)*(-1+tanh(x)^2)^(1/2)+1/2*ln(tanh(x)+(-1+tanh(x)^2)^(1/2))
```

Maxima [C] time = 1.72371, size = 43, normalized size = 1.23

$$\frac{-i e^{(3x)} + i e^x}{e^{(4x)} + 2e^{(2x)} + 1} - i\arctan\left(e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+tanh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (-I*e^(3*x) + I*e^x)/(e^(4*x) + 2*e^(2*x) + 1) - I*arctan(e^x)
```

Fricas [A] time = 2.35453, size = 4, normalized size = 0.11

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\tanh^2(x) - 1\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+tanh(x)**2)**(3/2),x)
```

```
[Out] Integral((tanh(x)**2 - 1)**(3/2), x)
```

Giac [C] time = 1.20675, size = 84, normalized size = 2.4

$$-\frac{i e^{(-x)} - i e^{x}}{\left(-i e^{(-x)} + i e^{x}\right)^2 - 4} + \frac{1}{8} \log\left(\left(e^{(-x)} - e^{x}\right)^2 + 4\right) - \frac{1}{4} \log\left(-i e^{(-x)} + i e^{x} + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(I*e^(-x) - I*e^x)/((-I*e^(-x) + I*e^x)^2 - 4) + 1/8*log((e^(-x) - e^x)^2
+ 4) - 1/4*log(-I*e^(-x) + I*e^x + 2)
```

3.206 $\int \frac{1}{\sqrt{1-\tanh^2(x)}} \, dx$

Optimal. Leaf size=11

$$\frac{\tanh(x)}{\sqrt{\operatorname{sech}^2(x)}}$$

```
[Out] Tanh[x]/Sqrt[Sech[x]^2]
```

Rubi [A] time = 0.0205086, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3657, 4122, 191}

$$\frac{\tanh(x)}{\sqrt{\operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 - Tanh[x]^2],x]

[Out] Tanh[x]/Sqrt[Sech[x]^2]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{1-\tanh^2(x)}} \, dx &= \int \frac{1}{\sqrt{\operatorname{sech}^2(x)}} \, dx \\
&= \operatorname{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{3/2}} \, dx, x, \tanh(x)\right) \\
&= \frac{\tanh(x)}{\sqrt{\operatorname{sech}^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.0069988, size = 11, normalized size = 1.

$$\frac{\tanh(x)}{\sqrt{\operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 - Tanh[x]^2],x]
```

```
[Out] Tanh[x]/Sqrt[Sech[x]^2]
```

Maple [A] time = 0.01, size = 14, normalized size = 1.3

$$\tanh(x)\frac{1}{\sqrt{1-(\tanh(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-tanh(x)^2)^(1/2),x)
```

```
[Out] 1/(1-tanh(x)^2)^(1/2)*tanh(x)
```

Maxima [A] time = 1.54002, size = 15, normalized size = 1.36

$$-\frac{1}{2}e^{(-x)}+\frac{1}{2}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*e^(-x) + 1/2*e^x
```

Fricas [A] time = 2.22183, size = 12, normalized size = 1.09

$$\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sinh(x)
```

Sympy [A] time = 0.592725, size = 12, normalized size = 1.09

$$\frac{\tanh(x)}{\sqrt{1-\tanh^2(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-tanh(x)**2)**(1/2),x)
```

```
[Out] tanh(x)/sqrt(1 - tanh(x)**2)
```

Giac [A] time = 1.13941, size = 15, normalized size = 1.36

$$-\frac{1}{2}\,e^{(-x)}+\frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*e^(-x) + 1/2*e^x
```

3.207 $\int \frac{1}{\sqrt{-1+\tanh^2(x)}}\,dx$

Optimal. Leaf size=13

$$\frac{\tanh(x)}{\sqrt{-\text{sech}^2(x)}}$$

```
[Out] Tanh[x]/Sqrt[-Sech[x]^2]
```

Rubi [A] time = 0.0210973, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3657, 4122, 191}

$$\frac{\tanh(x)}{\sqrt{-\text{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-1 + Tanh[x]^2],x]
```

```
[Out] Tanh[x]/Sqrt[-Sech[x]^2]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-1+\tanh^2(x)}}\,dx &= \int \frac{1}{\sqrt{-\text{sech}^2(x)}}\,dx\\
&= -\text{Subst}\left(\int \frac{1}{\left(-1+x^2\right)^{3/2}}\,dx, x, \tanh(x)\right)\\
&= \frac{\tanh(x)}{\sqrt{-\text{sech}^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.0067899, size = 13, normalized size = 1.

$$\frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 + Tanh[x]^2],x]
```

```
[Out] Tanh[x]/Sqrt[-Sech[x]^2]
```

Maple [A] time = 0.014, size = 12, normalized size = 0.9

$$\tanh(x)\frac{1}{\sqrt{-1+(\tanh(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+tanh(x)^2)^(1/2),x)
```

```
[Out] tanh(x)/(-1+tanh(x)^2)^(1/2)
```

Maxima [B] time = 1.5597, size = 34, normalized size = 2.62

$$-\frac{e^{(-2x)}}{2\sqrt{-e^{(-2x)}}}+\frac{1}{2\sqrt{-e^{(-2x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*e^(-2*x)/sqrt(-e^(-2*x)) + 1/2/sqrt(-e^(-2*x))
```

Fricas [A] time = 2.28971, size = 4, normalized size = 0.31

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{\tanh^2(x)-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(tanh(x)**2 - 1), x)
```

Giac [C] time = 1.18516, size = 15, normalized size = 1.15

$$\frac{1}{2} i e^{(-x)} - \frac{1}{2} i e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*I*e^(-x) - 1/2*I*e^x
```

3.208 $\int \tanh^5(x)\sqrt{a+b\tanh^2(x)}\,dx$

Optimal. Leaf size=87

$$-\frac{\left(a+b\tanh^2(x)\right)^{5/2}}{5b^2}+\frac{(a-b)\left(a+b\tanh^2(x)\right)^{3/2}}{3b^2}-\sqrt{a+b\tanh^2(x)}+\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)$$

```
[Out] Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - Sqrt[a + b*Tanh[x]
^2] + ((a - b)*(a + b*Tanh[x]^2)^(3/2))/(3*b^2) - (a + b*Tanh[x]^2)^(5/2)/(
5*b^2)
```

Rubi [A] time = 0.160944, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3670, 446, 88, 50, 63, 208}

$$-\frac{\left(a+b\tanh^2(x)\right)^{5/2}}{5b^2}+\frac{(a-b)\left(a+b\tanh^2(x)\right)^{3/2}}{3b^2}-\sqrt{a+b\tanh^2(x)}+\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^5*Sqrt[a + b*Tanh[x]^2],x]
```


```
[Out] Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - Sqrt[a + b*Tanh[x]
^2] + ((a - b)*(a + b*Tanh[x]^2)^(3/2))/(3*b^2) - (a + b*Tanh[x]^2)^(5/2)/(
5*b^2)
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \tanh^5(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Subst}\left(\int \frac{x^5\sqrt{a+bx^2}}{1-x^2}\,dx, x, \tanh(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{x^2\sqrt{a+bx}}{1-x}\,dx, x, \tanh^2(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(a-b)\sqrt{a+bx}}{b} + \frac{\sqrt{a+bx}}{1-x} - \frac{(a+bx)^{3/2}}{b}\right)dx, x, \tanh^2(x)\right)$$
$$= \frac{(a-b)\left(a+b\tanh^2(x)\right)^{3/2}}{3b^2} - \frac{\left(a+b\tanh^2(x)\right)^{5/2}}{5b^2} + \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1-x}\,dx, x, \tanh^2\right.$$
$$= -\sqrt{a+b\tanh^2(x)} + \frac{(a-b)\left(a+b\tanh^2(x)\right)^{3/2}}{3b^2} - \frac{\left(a+b\tanh^2(x)\right)^{5/2}}{5b^2} + \frac{1}{2}(a+b)\,\text{Sub}$$
$$= -\sqrt{a+b\tanh^2(x)} + \frac{(a-b)\left(a+b\tanh^2(x)\right)^{3/2}}{3b^2} - \frac{\left(a+b\tanh^2(x)\right)^{5/2}}{5b^2} + \frac{(a+b)\,\text{Subs}}{}$$
$$= \sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \sqrt{a+b\tanh^2(x)} + \frac{(a-b)\left(a+b\tanh^2(x)\right)^{3/2}}{3b^2} -$$

Mathematica [A] time = 0.466071, size = 85, normalized size = 0.98

$$\frac{\sqrt{a+b\tanh^2(x)}\left(2a^2-b(a+5b)\tanh^2(x)-5ab-3b^2\tanh^4(x)-15b^2\right)}{15b^2} + \sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^5*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] + (Sqrt[a + b*Tanh[x
]^2]*(2*a^2 - 5*a*b - 15*b^2 - b*(a + 5*b)*Tanh[x]^2 - 3*b^2*Tanh[x]^4))/(1
5*b^2)
```

Maple [B] time = 0.071, size = 288, normalized size = 3.3

$$-\frac{(\tanh(x))^2}{5b}\left(a+b(\tanh(x))^2\right)^{\frac{3}{2}}+\frac{2a}{15b^2}\left(a+b(\tanh(x))^2\right)^{\frac{3}{2}}-\frac{1}{3b}\left(a+b(\tanh(x))^2\right)^{\frac{3}{2}}-\frac{1}{2}\sqrt{(1+\tanh(x))^2b-2\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tanh(x)^2)^(1/2)*tanh(x)^5,x)
```

```
[Out] -1/5*tanh(x)^2*(a+b*tanh(x)^2)^(3/2)/b+2/15*a/b^2*(a+b*tanh(x)^2)^(3/2)-1/3
*(a+b*tanh(x)^2)^(3/2)/b-1/2*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)+1/
2*b^(1/2)*ln(((1+tanh(x))*b-b)/b^(1/2)+((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b
)^(1/2))+1/2*(a+b)^(1/2)*ln((2*a+2*b-2*(1+tanh(x))*b+2*(a+b)^(1/2)*((1+tanh
(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))/(1+tanh(x)))-1/2*((tanh(x)-1)^2*b+2*(t
anh(x)-1)*b+a+b)^(1/2)-1/2*b^(1/2)*ln(((tanh(x)-1)*b+b)/b^(1/2)+((tanh(x)-1
)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))+1/2*(a+b)^(1/2)*ln((2*a+2*b+2*(tanh(x)-1)
*b+2*(a+b)^(1/2)*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))/(tanh(x)-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\tanh(x)^2+a}\tanh(x)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tanh(x)^2 + a)*tanh(x)^5, x)
```

Fricas [B] time = 6.51131, size = 12513, normalized size = 143.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^5,x, algorithm="fricas")
```

```
[Out] [1/60*(15*(b^2*cosh(x)^10 + 10*b^2*cosh(x)*sinh(x)^9 + b^2*sinh(x)^10 + 5*b
^2*cosh(x)^8 + 5*(9*b^2*cosh(x)^2 + b^2)*sinh(x)^8 + 10*b^2*cosh(x)^6 + 40*
(3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^7 + 10*(21*b^2*cosh(x)^4 + 14*b^2*c
osh(x)^2 + b^2)*sinh(x)^6 + 10*b^2*cosh(x)^4 + 4*(63*b^2*cosh(x)^5 + 70*b^2
*cosh(x)^3 + 15*b^2*cosh(x))*sinh(x)^5 + 10*(21*b^2*cosh(x)^6 + 35*b^2*cosh
(x)^4 + 15*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 5*b^2*cosh(x)^2 + 40*(3*b^2*cos
h(x)^7 + 7*b^2*cosh(x)^5 + 5*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^3 + 5*(9*
b^2*cosh(x)^8 + 28*b^2*cosh(x)^6 + 30*b^2*cosh(x)^4 + 12*b^2*cosh(x)^2 + b^
2)*sinh(x)^2 + b^2 + 10*(b^2*cosh(x)^9 + 4*b^2*cosh(x)^7 + 6*b^2*cosh(x)^5
+ 4*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a + b)*log(((a^3 + a^2*b)*co
sh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(
2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*s
inh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(
x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x
)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)
^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3
+ 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3
```

```
 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15
*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a
*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*s
inh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh
(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^
2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*
sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a
^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (
a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*
(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b
 - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh
(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)
^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 15*(b^2*c
osh(x)^10 + 10*b^2*cosh(x)*sinh(x)^9 + b^2*sinh(x)^10 + 5*b^2*cosh(x)^8 + 5
*(9*b^2*cosh(x)^2 + b^2)*sinh(x)^8 + 10*b^2*cosh(x)^6 + 40*(3*b^2*cosh(x)^3
 + b^2*cosh(x))*sinh(x)^7 + 10*(21*b^2*cosh(x)^4 + 14*b^2*cosh(x)^2 + b^2)*
sinh(x)^6 + 10*b^2*cosh(x)^4 + 4*(63*b^2*cosh(x)^5 + 70*b^2*cosh(x)^3 + 15*
b^2*cosh(x))*sinh(x)^5 + 10*(21*b^2*cosh(x)^6 + 35*b^2*cosh(x)^4 + 15*b^2*c
osh(x)^2 + b^2)*sinh(x)^4 + 5*b^2*cosh(x)^2 + 40*(3*b^2*cosh(x)^7 + 7*b^2*c
osh(x)^5 + 5*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^3 + 5*(9*b^2*cosh(x)^8 +
28*b^2*cosh(x)^6 + 30*b^2*cosh(x)^4 + 12*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b
^2 + 10*(b^2*cosh(x)^9 + 4*b^2*cosh(x)^7 + 6*b^2*cosh(x)^5 + 4*b^2*cosh(x)^
3 + b^2*cosh(x))*sinh(x))*sqrt(a + b)*log(-((a + b)*cosh(x)^4 + 4*(a + b)*c
osh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(a + b)*cosh(x)
^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^
2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 - b*cosh(x))*sin
h(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*sqrt(2)*((2*
a^2 - 6*a*b - 23*b^2)*cosh(x)^8 + 8*(2*a^2 - 6*a*b - 23*b^2)*cosh(x)*sinh(x
)^7 + (2*a^2 - 6*a*b - 23*b^2)*sinh(x)^8 + 4*(2*a^2 - 5*a*b - 12*b^2)*cosh(
x)^6 + 4*(7*(2*a^2 - 6*a*b - 23*b^2)*cosh(x)^2 + 2*a^2 - 5*a*b - 12*b^2)*si
nh(x)^6 + 8*(7*(2*a^2 - 6*a*b - 23*b^2)*cosh(x)^3 + 3*(2*a^2 - 5*a*b - 12*b
^2)*cosh(x))*sinh(x)^5 + 2*(6*a^2 - 14*a*b - 49*b^2)*cosh(x)^4 + 2*(35*(2*a
^2 - 6*a*b - 23*b^2)*cosh(x)^4 + 30*(2*a^2 - 5*a*b - 12*b^2)*cosh(x)^2 + 6*
a^2 - 14*a*b - 49*b^2)*sinh(x)^4 + 8*(7*(2*a^2 - 6*a*b - 23*b^2)*cosh(x)^5
 + 10*(2*a^2 - 5*a*b - 12*b^2)*cosh(x)^3 + (6*a^2 - 14*a*b - 49*b^2)*cosh(x)
)*sinh(x)^3 + 4*(2*a^2 - 5*a*b - 12*b^2)*cosh(x)^2 + 4*(7*(2*a^2 - 6*a*b -
23*b^2)*cosh(x)^6 + 15*(2*a^2 - 5*a*b - 12*b^2)*cosh(x)^4 + 3*(6*a^2 - 14*a
*b - 49*b^2)*cosh(x)^2 + 2*a^2 - 5*a*b - 12*b^2)*sinh(x)^2 + 2*a^2 - 6*a*b
 - 23*b^2 + 8*((2*a^2 - 6*a*b - 23*b^2)*cosh(x)^7 + 3*(2*a^2 - 5*a*b - 12*b^
2)*cosh(x)^5 + (6*a^2 - 14*a*b - 49*b^2)*cosh(x)^3 + (2*a^2 - 5*a*b - 12*b^
2)*cosh(x))*sinh(x))*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(
cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b^2*cosh(x)^10 + 10*b^2*cosh(
x)*sinh(x)^9 + b^2*sinh(x)^10 + 5*b^2*cosh(x)^8 + 5*(9*b^2*cosh(x)^2 + b^2)
*sinh(x)^8 + 10*b^2*cosh(x)^6 + 40*(3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^
7 + 10*(21*b^2*cosh(x)^4 + 14*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 10*b^2*cosh(
x)^4 + 4*(63*b^2*cosh(x)^5 + 70*b^2*cosh(x)^3 + 15*b^2*cosh(x))*sinh(x)^5 +
 10*(21*b^2*cosh(x)^6 + 35*b^2*cosh(x)^4 + 15*b^2*cosh(x)^2 + b^2)*sinh(x)^
4 + 5*b^2*cosh(x)^2 + 40*(3*b^2*cosh(x)^7 + 7*b^2*cosh(x)^5 + 5*b^2*cosh(x)
^3 + b^2*cosh(x))*sinh(x)^3 + 5*(9*b^2*cosh(x)^8 + 28*b^2*cosh(x)^6 + 30*b^
2*cosh(x)^4 + 12*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 10*(b^2*cosh(x)^9 +
 4*b^2*cosh(x)^7 + 6*b^2*cosh(x)^5 + 4*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)
), -1/30*(15*(b^2*cosh(x)^10 + 10*b^2*cosh(x)*sinh(x)^9 + b^2*sinh(x)^10 +
5*b^2*cosh(x)^8 + 5*(9*b^2*cosh(x)^2 + b^2)*sinh(x)^8 + 10*b^2*cosh(x)^6 +
40*(3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^7 + 10*(21*b^2*cosh(x)^4 + 14*b^
2*cosh(x)^2 + b^2)*sinh(x)^6 + 10*b^2*cosh(x)^4 + 4*(63*b^2*cosh(x)^5 + 70*
b^2*cosh(x)^3 + 15*b^2*cosh(x))*sinh(x)^5 + 10*(21*b^2*cosh(x)^6 + 35*b^2*c
osh(x)^4 + 15*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 5*b^2*cosh(x)^2 + 40*(3*b^2*
cosh(x)^7 + 7*b^2*cosh(x)^5 + 5*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^3 + 5*
```

```
(9*b^2*cosh(x)^8 + 28*b^2*cosh(x)^6 + 30*b^2*cosh(x)^4 + 12*b^2*cosh(x)^2 +
 b^2)*sinh(x)^2 + b^2 + 10*(b^2*cosh(x)^9 + 4*b^2*cosh(x)^7 + 6*b^2*cosh(x)
^5 + 4*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(a
*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a - b)*sqrt((
(a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(
x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 +
 (a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2 + a*b)*cos
h(x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a*b
)*cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh(x))) + 15*(b^2*cosh(x)^10 +
 10*b^2*cosh(x)*sinh(x)^9 + b^2*sinh(x)^10 + 5*b^2*cosh(x)^8 + 5*(9*b^2*cos
h(x)^2 + b^2)*sinh(x)^8 + 10*b^2*cosh(x)^6 + 40*(3*b^2*cosh(x)^3 + b^2*cosh
(x))*sinh(x)^7 + 10*(21*b^2*cosh(x)^4 + 14*b^2*cosh(x)^2 + b^2)*sinh(x)^6 +
 10*b^2*cosh(x)^4 + 4*(63*b^2*cosh(x)^5 + 70*b^2*cosh(x)^3 + 15*b^2*cosh(x)
)*sinh(x)^5 + 10*(21*b^2*cosh(x)^6 + 35*b^2*cosh(x)^4 + 15*b^2*cosh(x)^2 +
b^2)*sinh(x)^4 + 5*b^2*cosh(x)^2 + 40*(3*b^2*cosh(x)^7 + 7*b^2*cosh(x)^5 +
5*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^3 + 5*(9*b^2*cosh(x)^8 + 28*b^2*cosh
(x)^6 + 30*b^2*cosh(x)^4 + 12*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 10*(b^
2*cosh(x)^9 + 4*b^2*cosh(x)^7 + 6*b^2*cosh(x)^5 + 4*b^2*cosh(x)^3 + b^2*cos
h(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*
(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*
(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(
x))*sinh(x) + a + b)) - 2*sqrt(2)*((2*a^2 - 6*a*b - 23*b^2)*cosh(x)^8 + 8*(
2*a^2 - 6*a*b - 23*b^2)*cosh(x)*sinh(x)^7 + (2*a^2 - 6*a*b - 23*b^2)*sinh(x
)^8 + 4*(2*a^2 - 5*a*b - 12*b^2)*cosh(x)^6 + 4*(7*(2*a^2 - 6*a*b - 23*b^2)*
cosh(x)^2 + 2*a^2 - 5*a*b - 12*b^2)*sinh(x)^6 + 8*(7*(2*a^2 - 6*a*b - 23*b^
2)*cosh(x)^3 + 3*(2*a^2 - 5*a*b - 12*b^2)*cosh(x))*sinh(x)^5 + 2*(6*a^2 - 1
4*a*b - 49*b^2)*cosh(x)^4 + 2*(35*(2*a^2 - 6*a*b - 23*b^2)*cosh(x)^4 + 30*(
2*a^2 - 5*a*b - 12*b^2)*cosh(x)^2 + 6*a^2 - 14*a*b - 49*b^2)*sinh(x)^4 + 8*
(7*(2*a^2 - 6*a*b - 23*b^2)*cosh(x)^5 + 10*(2*a^2 - 5*a*b - 12*b^2)*cosh(x)
^3 + (6*a^2 - 14*a*b - 49*b^2)*cosh(x))*sinh(x)^3 + 4*(2*a^2 - 5*a*b - 12*b
^2)*cosh(x)^2 + 4*(7*(2*a^2 - 6*a*b - 23*b^2)*cosh(x)^6 + 15*(2*a^2 - 5*a*b
 - 12*b^2)*cosh(x)^4 + 3*(6*a^2 - 14*a*b - 49*b^2)*cosh(x)^2 + 2*a^2 - 5*a*
b - 12*b^2)*sinh(x)^2 + 2*a^2 - 6*a*b - 23*b^2 + 8*((2*a^2 - 6*a*b - 23*b^2
)*cosh(x)^7 + 3*(2*a^2 - 5*a*b - 12*b^2)*cosh(x)^5 + (6*a^2 - 14*a*b - 49*b
^2)*cosh(x)^3 + (2*a^2 - 5*a*b - 12*b^2)*cosh(x))*sinh(x))*sqrt(((a + b)*co
sh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(
x)^2)))/(b^2*cosh(x)^10 + 10*b^2*cosh(x)*sinh(x)^9 + b^2*sinh(x)^10 + 5*b^2
*cosh(x)^8 + 5*(9*b^2*cosh(x)^2 + b^2)*sinh(x)^8 + 10*b^2*cosh(x)^6 + 40*(3
*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^7 + 10*(21*b^2*cosh(x)^4 + 14*b^2*cos
h(x)^2 + b^2)*sinh(x)^6 + 10*b^2*cosh(x)^4 + 4*(63*b^2*cosh(x)^5 + 70*b^2*c
osh(x)^3 + 15*b^2*cosh(x))*sinh(x)^5 + 10*(21*b^2*cosh(x)^6 + 35*b^2*cosh(x
)^4 + 15*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 5*b^2*cosh(x)^2 + 40*(3*b^2*cosh(
x)^7 + 7*b^2*cosh(x)^5 + 5*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^3 + 5*(9*b^
2*cosh(x)^8 + 28*b^2*cosh(x)^6 + 30*b^2*cosh(x)^4 + 12*b^2*cosh(x)^2 + b^2)
*sinh(x)^2 + b^2 + 10*(b^2*cosh(x)^9 + 4*b^2*cosh(x)^7 + 6*b^2*cosh(x)^5 +
4*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))]
```

Sympy [A] time = 7.92675, size = 97, normalized size = 1.11

$$-\frac{2\left(\frac{b^3\sqrt{a+b\tanh^2(x)}}{2}+\frac{b^3(a+b)\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}}+\frac{b\left(a+b\tanh^2(x)\right)^{\frac{5}{2}}}{10}+\frac{\left(a+b\tanh^2(x)\right)^{\frac{3}{2}}\left(-\frac{ab}{2}+\frac{b^2}{2}\right)}{3}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)**2)**(1/2)*tanh(x)**5,x)
```

```
[Out] -2*(b**3*sqrt(a + b*tanh(x)**2)/2 + b**3*(a + b)*atan(sqrt(a + b*tanh(x)**2
)/sqrt(-a - b))/(2*sqrt(-a - b)) + b*(a + b*tanh(x)**2)**(5/2)/10 + (a + b*
tanh(x)**2)**(3/2)*(-a*b/2 + b**2/2)/3)/b**3
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^5,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.209 $\int \tanh^4(x)\sqrt{a+b\tanh^2(x)}\,dx$

Optimal. Leaf size=121

$$\frac{(a^2-4ab-8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{8b^{3/2}}-\frac{1}{4}\tanh^3(x)\sqrt{a+b\tanh^2(x)}-\frac{(a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)}}{8b}+\sqrt{a+b}\,t$$

```
[Out] ((a^2 - 4*a*b - 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/(8
*b^(3/2)) + Sqrt[a + b]*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]
] - ((a + 4*b)*Tanh[x]*Sqrt[a + b*Tanh[x]^2])/(8*b) - (Tanh[x]^3*Sqrt[a + b
*Tanh[x]^2])/4
```

Rubi [A] time = 0.200288, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {3670, 478, 582, 523, 217, 206, 377}

$$\frac{(a^2-4ab-8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{8b^{3/2}}-\frac{1}{4}\tanh^3(x)\sqrt{a+b\tanh^2(x)}-\frac{(a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)}}{8b}+\sqrt{a+b}\,t$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ((a^2 - 4*a*b - 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/(8
*b^(3/2)) + Sqrt[a + b]*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]
] - ((a + 4*b)*Tanh[x]*Sqrt[a + b*Tanh[x]^2])/(8*b) - (Tanh[x]^3*Sqrt[a + b
*Tanh[x]^2])/4
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
```

```
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\int \tanh^4(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Subst}\left(\int \frac{x^4\sqrt{a+bx^2}}{1-x^2}\,dx, x, \tanh(x)\right)$$

$$= -\frac{1}{4}\tanh^3(x)\sqrt{a+b\tanh^2(x)} + \frac{1}{4}\text{Subst}\left(\int \frac{x^2\left(3a+(a+4b)x^2\right)}{\left(1-x^2\right)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)$$

$$= -\frac{(a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)}}{8b} - \frac{1}{4}\tanh^3(x)\sqrt{a+b\tanh^2(x)} + \frac{\text{Subst}\left(\int \frac{a(a+4b)\cdot}{(1}\right.}{}$$

$$= -\frac{(a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)}}{8b} - \frac{1}{4}\tanh^3(x)\sqrt{a+b\tanh^2(x)} + (a+b)\,\text{Subst}\left(\int\right.$$

$$= -\frac{(a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)}}{8b} - \frac{1}{4}\tanh^3(x)\sqrt{a+b\tanh^2(x)} + (a+b)\,\text{Subst}\left(\int\right.$$

$$= \frac{\left(a^2-4ab-8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{8b^{3/2}} + \sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) - \frac{(a+4}{}$$

Mathematica [C] time = 6.16749, size = 580, normalized size = 4.79

$$\sqrt{\frac{a\cosh(2x)+a+b\cosh(2x)-b}{\cosh(2x)+1}}\left(\frac{\text{sech}(x)(-a\sinh(x)-6b\sinh(x))}{8b}+\frac{1}{4}\tanh(x)\text{sech}^2(x)\right)+\frac{-\frac{b(a^2-4b^2)\sinh^4(x)\text{csch}(2}{\ }}{\ }$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] (-((b*(a^2 - 4*b^2)*Sqrt[(a - b + (a + b)*Cosh[2*x])/(1 + Cosh[2*x])]*Sqrt[
-((a*Coth[x]^2)/b)]*Sqrt[-((a*(1 + Cosh[2*x])*Csch[x]^2)/b)]*Sqrt[((a - b +
 (a + b)*Cosh[2*x])*Csch[x]^2)/b]*Csch[2*x]*EllipticF[ArcSin[Sqrt[((a - b +
 (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1]*Sinh[x]^4)/(a*(a - b + (a +
b)*Cosh[2*x]))) - ((4*I)*b*(4*a*b + 4*b^2)*Sqrt[1 + Cosh[2*x]]*Sqrt[(a - b
+ (a + b)*Cosh[2*x])/(1 + Cosh[2*x])]*(((-I/4)*Sqrt[-((a*Coth[x]^2)/b)]*Sqr
t[-((a*(1 + Cosh[2*x])*Csch[x]^2)/b)]*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csc
h[x]^2)/b]*Csch[2*x]*EllipticF[ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csc
h[x]^2)/b]/Sqrt[2]], 1]*Sinh[x]^4)/(a*Sqrt[1 + Cosh[2*x]]*Sqrt[a - b + (a +
 b)*Cosh[2*x]]) + ((I/2)*Sqrt[-((a*Coth[x]^2)/b)]*Sqrt[-((a*(1 + Cosh[2*x])
*Csch[x]^2)/b)]*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*Csch[2*x]*E
llipticPi[b/(a + b), ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]
/Sqrt[2]], 1]*Sinh[x]^4)/((a + b)*Sqrt[1 + Cosh[2*x]]*Sqrt[a - b + (a + b)*
Cosh[2*x]])))/Sqrt[a - b + (a + b)*Cosh[2*x]])/(4*b) + Sqrt[(a - b + a*Cosh
[2*x] + b*Cosh[2*x])/(1 + Cosh[2*x])]*((Sech[x]*(-(a*Sinh[x]) - 6*b*Sinh[x]
))/(8*b) + (Sech[x]^2*Tanh[x])/4)
```

Maple [B] time = 0.049, size = 337, normalized size = 2.8

$$-\frac{\tanh(x)}{4b}\left(a+b(\tanh(x))^2\right)^{\frac{3}{2}}+\frac{a\tanh(x)}{8b}\sqrt{a+b(\tanh(x))^2}+\frac{a^2}{8}\ln\left(\tanh(x)\sqrt{b}+\sqrt{a+b(\tanh(x))^2}\right)b^{-\frac{3}{2}}-\frac{\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tanh(x)^2)^(1/2)*tanh(x)^4,x)
```

```
[Out] -1/4*tanh(x)*(a+b*tanh(x)^2)^(3/2)/b+1/8*a/b*tanh(x)*(a+b*tanh(x)^2)^(1/2)+
1/8*a^2/b^(3/2)*ln(tanh(x)*b^(1/2)+(a+b*tanh(x)^2)^(1/2))-1/2*(a+b*tanh(x)^
2)^(1/2)*tanh(x)-1/2*a/b^(1/2)*ln(tanh(x)*b^(1/2)+(a+b*tanh(x)^2)^(1/2))+1/
2*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)-1/2*b^(1/2)*ln(((1+tanh(x))*b
-b)/b^(1/2)+((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))-1/2*(a+b)^(1/2)*ln
((2*a+2*b-2*(1+tanh(x))*b+2*(a+b)^(1/2)*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+
b)^(1/2))/(1+tanh(x)))-1/2*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)-1/2*
b^(1/2)*ln(((tanh(x)-1)*b+b)/b^(1/2)+((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^
(1/2))+1/2*(a+b)^(1/2)*ln((2*a+2*b+2*(tanh(x)-1)*b+2*(a+b)^(1/2)*((tanh(x)-
1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))/(tanh(x)-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\tanh(x)^2+a}\tanh(x)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tanh(x)^2 + a)*tanh(x)^4, x)
```

Fricas [B] time = 7.35727, size = 25867, normalized size = 213.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^4,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*c
osh(x)^6 + 4*(7*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2
*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x
)^2 + 3*b^2)*sinh(x)^4 + 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh
(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 +
9*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5
 + 3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a + b)*log(-((a*b^2 + b^3)*
cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2
*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)
*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sin
h(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh
(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(
x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3
- a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b
^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 -
15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a
*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)
*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*si
nh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*
b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2
)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a
^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 +
 (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) +
4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b +
 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(co
sh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(
x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) - ((a^2 -
 4*a*b - 8*b^2)*cosh(x)^8 + 8*(a^2 - 4*a*b - 8*b^2)*cosh(x)*sinh(x)^7 + (a^
2 - 4*a*b - 8*b^2)*sinh(x)^8 + 4*(a^2 - 4*a*b - 8*b^2)*cosh(x)^6 + 4*(7*(a^
2 - 4*a*b - 8*b^2)*cosh(x)^2 + a^2 - 4*a*b - 8*b^2)*sinh(x)^6 + 8*(7*(a^2 -
 4*a*b - 8*b^2)*cosh(x)^3 + 3*(a^2 - 4*a*b - 8*b^2)*cosh(x))*sinh(x)^5 + 6*
(a^2 - 4*a*b - 8*b^2)*cosh(x)^4 + 2*(35*(a^2 - 4*a*b - 8*b^2)*cosh(x)^4 + 3
0*(a^2 - 4*a*b - 8*b^2)*cosh(x)^2 + 3*a^2 - 12*a*b - 24*b^2)*sinh(x)^4 + 8*
(7*(a^2 - 4*a*b - 8*b^2)*cosh(x)^5 + 10*(a^2 - 4*a*b - 8*b^2)*cosh(x)^3 + 3
*(a^2 - 4*a*b - 8*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 - 4*a*b - 8*b^2)*cosh(x)
^2 + 4*(7*(a^2 - 4*a*b - 8*b^2)*cosh(x)^6 + 15*(a^2 - 4*a*b - 8*b^2)*cosh(x
)^4 + 9*(a^2 - 4*a*b - 8*b^2)*cosh(x)^2 + a^2 - 4*a*b - 8*b^2)*sinh(x)^2 +
a^2 - 4*a*b - 8*b^2 + 8*((a^2 - 4*a*b - 8*b^2)*cosh(x)^7 + 3*(a^2 - 4*a*b -
 8*b^2)*cosh(x)^5 + 3*(a^2 - 4*a*b - 8*b^2)*cosh(x)^3 + (a^2 - 4*a*b - 8*b^
2)*cosh(x))*sinh(x))*sqrt(b)*log(-((a + 2*b)*cosh(x)^4 + 4*(a + 2*b)*cosh(x
)*sinh(x)^3 + (a + 2*b)*sinh(x)^4 + 2*(a - 2*b)*cosh(x)^2 + 2*(3*(a + 2*b)*
cosh(x)^2 + a - 2*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2 - 1)*sqrt(b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b
```

```
)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + 2*b)*cosh(x)^3 + (
a - 2*b)*cosh(x))*sinh(x) + a + 2*b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sin
h(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(
x))*sinh(x) + 1)) + 4*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x
)^8 + 4*b^2*cosh(x)^6 + 4*(7*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)
^4 + 8*(7*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 +
30*b^2*cosh(x)^2 + 3*b^2)*sinh(x)^4 + 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5
+ 10*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2
*cosh(x)^4 + 9*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^7 + 3*
b^2*cosh(x)^5 + 3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a + b)*log(((a
 + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cos
h(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)
*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a +
b)*cosh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2)) - 2*sqrt(2)*((a*b + 6*b^2)*cosh(x)^6 + 6*(a*b + 6*b^2)*cosh(x)
*sinh(x)^5 + (a*b + 6*b^2)*sinh(x)^6 + (a*b - 2*b^2)*cosh(x)^4 + (15*(a*b +
 6*b^2)*cosh(x)^2 + a*b - 2*b^2)*sinh(x)^4 + 4*(5*(a*b + 6*b^2)*cosh(x)^3 +
 (a*b - 2*b^2)*cosh(x))*sinh(x)^3 - (a*b - 2*b^2)*cosh(x)^2 + (15*(a*b + 6*
b^2)*cosh(x)^4 + 6*(a*b - 2*b^2)*cosh(x)^2 - a*b + 2*b^2)*sinh(x)^2 - a*b -
 6*b^2 + 2*(3*(a*b + 6*b^2)*cosh(x)^5 + 2*(a*b - 2*b^2)*cosh(x)^3 - (a*b -
2*b^2)*cosh(x))*sinh(x))*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a -
b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b^2*cosh(x)^8 + 8*b^2*cos
h(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*cosh(x)^6 + 4*(7*b^2*cosh(x)^2 + b^2
)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)
^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x)^2 + 3*b^2)*sinh(x)^4 + 4*b^2*cosh
(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 +
4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 + 9*b^2*cosh(x)^2 + b^2)*sinh(x)^2 +
b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5 + 3*b^2*cosh(x)^3 + b^2*cosh(x))*s
inh(x)), -1/8*(((a^2 - 4*a*b - 8*b^2)*cosh(x)^8 + 8*(a^2 - 4*a*b - 8*b^2)*c
osh(x)*sinh(x)^7 + (a^2 - 4*a*b - 8*b^2)*sinh(x)^8 + 4*(a^2 - 4*a*b - 8*b^2
)*cosh(x)^6 + 4*(7*(a^2 - 4*a*b - 8*b^2)*cosh(x)^2 + a^2 - 4*a*b - 8*b^2)*s
inh(x)^6 + 8*(7*(a^2 - 4*a*b - 8*b^2)*cosh(x)^3 + 3*(a^2 - 4*a*b - 8*b^2)*c
osh(x))*sinh(x)^5 + 6*(a^2 - 4*a*b - 8*b^2)*cosh(x)^4 + 2*(35*(a^2 - 4*a*b
- 8*b^2)*cosh(x)^4 + 30*(a^2 - 4*a*b - 8*b^2)*cosh(x)^2 + 3*a^2 - 12*a*b -
24*b^2)*sinh(x)^4 + 8*(7*(a^2 - 4*a*b - 8*b^2)*cosh(x)^5 + 10*(a^2 - 4*a*b
- 8*b^2)*cosh(x)^3 + 3*(a^2 - 4*a*b - 8*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 -
4*a*b - 8*b^2)*cosh(x)^2 + 4*(7*(a^2 - 4*a*b - 8*b^2)*cosh(x)^6 + 15*(a^2 -
 4*a*b - 8*b^2)*cosh(x)^4 + 9*(a^2 - 4*a*b - 8*b^2)*cosh(x)^2 + a^2 - 4*a*b
 - 8*b^2)*sinh(x)^2 + a^2 - 4*a*b - 8*b^2 + 8*((a^2 - 4*a*b - 8*b^2)*cosh(x
)^7 + 3*(a^2 - 4*a*b - 8*b^2)*cosh(x)^5 + 3*(a^2 - 4*a*b - 8*b^2)*cosh(x)^3
 + (a^2 - 4*a*b - 8*b^2)*cosh(x))*sinh(x))*sqrt(-b)*arctan(sqrt(2)*(cosh(x)
^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt(((a + b)*cosh(x)^2 +
(a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a
 + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a -
b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(
x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) - 2*(b^2*cosh(x)^8 + 8*b^2*cosh(x
)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*cosh(x)^6 + 4*(7*b^2*cosh(x)^2 + b^2)*s
inh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^5
+ 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x)^2 + 3*b^2)*sinh(x)^4 + 4*b^2*cosh(x)
^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 4*(
7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 + 9*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2
 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5 + 3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh
(x))*sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*si
nh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2
+ 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(
x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^
3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3
- 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 -
```

```
10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*si
nh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)
^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3
*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + s
qrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cos
h(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*c
osh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 1
8*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3
*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*
sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2
 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^
2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3
*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cos
h(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh
(x)*sinh(x)^5 + sinh(x)^6)) - 2*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 +
b^2*sinh(x)^8 + 4*b^2*cosh(x)^6 + 4*(7*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 6*b
^2*cosh(x)^4 + 8*(7*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*co
sh(x)^4 + 30*b^2*cosh(x)^2 + 3*b^2)*sinh(x)^4 + 4*b^2*cosh(x)^2 + 8*(7*b^2*
cosh(x)^5 + 10*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^
6 + 15*b^2*cosh(x)^4 + 9*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh
(x)^7 + 3*b^2*cosh(x)^5 + 3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a +
b)*log(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4
 + 2*a*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)
^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2
 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))
 + 4*((a + b)*cosh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)
*sinh(x) + sinh(x)^2)) + sqrt(2)*((a*b + 6*b^2)*cosh(x)^6 + 6*(a*b + 6*b^2)
*cosh(x)*sinh(x)^5 + (a*b + 6*b^2)*sinh(x)^6 + (a*b - 2*b^2)*cosh(x)^4 + (1
5*(a*b + 6*b^2)*cosh(x)^2 + a*b - 2*b^2)*sinh(x)^4 + 4*(5*(a*b + 6*b^2)*cos
h(x)^3 + (a*b - 2*b^2)*cosh(x))*sinh(x)^3 - (a*b - 2*b^2)*cosh(x)^2 + (15*(
a*b + 6*b^2)*cosh(x)^4 + 6*(a*b - 2*b^2)*cosh(x)^2 - a*b + 2*b^2)*sinh(x)^2
 - a*b - 6*b^2 + 2*(3*(a*b + 6*b^2)*cosh(x)^5 + 2*(a*b - 2*b^2)*cosh(x)^3 -
 (a*b - 2*b^2)*cosh(x))*sinh(x))*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^
2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b^2*cosh(x)^8 + 8
*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*cosh(x)^6 + 4*(7*b^2*cosh(x)
^2 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2*cosh(x)^3 + 3*b^2*cosh(x))
*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x)^2 + 3*b^2)*sinh(x)^4 + 4*
b^2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh
(x)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 + 9*b^2*cosh(x)^2 + b^2)*sinh
(x)^2 + b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5 + 3*b^2*cosh(x)^3 + b^2*co
sh(x))*sinh(x)), -1/16*(8*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*si
nh(x)^8 + 4*b^2*cosh(x)^6 + 4*(7*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 6*b^2*cos
h(x)^4 + 8*(7*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^
4 + 30*b^2*cosh(x)^2 + 3*b^2)*sinh(x)^4 + 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x
)^5 + 10*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15
*b^2*cosh(x)^4 + 9*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^7
+ 3*b^2*cosh(x)^5 + 3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(-a - b)*ar
ctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - a - b)*sqrt
(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 -
2*cosh(x)*sinh(x) + sinh(x)^2))/((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh
(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^2)*cosh(x)^2 + (6*
(a*b + b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 +
2*(2*(a*b + b^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x))) + 8*(b^
2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*cosh(x)^6 + 4
*(7*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2*cosh(x)^3 +
 3*b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x)^2 + 3*b^2)
*sinh(x)^4 + 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh(x)^3 + 3*b^
2*cosh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 + 9*b^2*cosh(x
)^2 + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5 + 3*b^2*cos
```

```
h(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*c
osh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a +
 b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b
)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*c
osh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3
 + (a - b)*cosh(x))*sinh(x) + a + b)) + ((a^2 - 4*a*b - 8*b^2)*cosh(x)^8 +
8*(a^2 - 4*a*b - 8*b^2)*cosh(x)*sinh(x)^7 + (a^2 - 4*a*b - 8*b^2)*sinh(x)^8
 + 4*(a^2 - 4*a*b - 8*b^2)*cosh(x)^6 + 4*(7*(a^2 - 4*a*b - 8*b^2)*cosh(x)^2
 + a^2 - 4*a*b - 8*b^2)*sinh(x)^6 + 8*(7*(a^2 - 4*a*b - 8*b^2)*cosh(x)^3 +
3*(a^2 - 4*a*b - 8*b^2)*cosh(x))*sinh(x)^5 + 6*(a^2 - 4*a*b - 8*b^2)*cosh(x
)^4 + 2*(35*(a^2 - 4*a*b - 8*b^2)*cosh(x)^4 + 30*(a^2 - 4*a*b - 8*b^2)*cosh
(x)^2 + 3*a^2 - 12*a*b - 24*b^2)*sinh(x)^4 + 8*(7*(a^2 - 4*a*b - 8*b^2)*cos
h(x)^5 + 10*(a^2 - 4*a*b - 8*b^2)*cosh(x)^3 + 3*(a^2 - 4*a*b - 8*b^2)*cosh(
x))*sinh(x)^3 + 4*(a^2 - 4*a*b - 8*b^2)*cosh(x)^2 + 4*(7*(a^2 - 4*a*b - 8*b
^2)*cosh(x)^6 + 15*(a^2 - 4*a*b - 8*b^2)*cosh(x)^4 + 9*(a^2 - 4*a*b - 8*b^2
)*cosh(x)^2 + a^2 - 4*a*b - 8*b^2)*sinh(x)^2 + a^2 - 4*a*b - 8*b^2 + 8*((a^
2 - 4*a*b - 8*b^2)*cosh(x)^7 + 3*(a^2 - 4*a*b - 8*b^2)*cosh(x)^5 + 3*(a^2 -
 4*a*b - 8*b^2)*cosh(x)^3 + (a^2 - 4*a*b - 8*b^2)*cosh(x))*sinh(x))*sqrt(b)
*log(-((a + 2*b)*cosh(x)^4 + 4*(a + 2*b)*cosh(x)*sinh(x)^3 + (a + 2*b)*sinh
(x)^4 + 2*(a - 2*b)*cosh(x)^2 + 2*(3*(a + 2*b)*cosh(x)^2 + a - 2*b)*sinh(x)
^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt
(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sin
h(x) + sinh(x)^2)) + 4*((a + 2*b)*cosh(x)^3 + (a - 2*b)*cosh(x))*sinh(x) +
a + 2*b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)
*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + 2*sqrt(2
)*((a*b + 6*b^2)*cosh(x)^6 + 6*(a*b + 6*b^2)*cosh(x)*sinh(x)^5 + (a*b + 6*b
^2)*sinh(x)^6 + (a*b - 2*b^2)*cosh(x)^4 + (15*(a*b + 6*b^2)*cosh(x)^2 + a*b
 - 2*b^2)*sinh(x)^4 + 4*(5*(a*b + 6*b^2)*cosh(x)^3 + (a*b - 2*b^2)*cosh(x))
*sinh(x)^3 - (a*b - 2*b^2)*cosh(x)^2 + (15*(a*b + 6*b^2)*cosh(x)^4 + 6*(a*b
 - 2*b^2)*cosh(x)^2 - a*b + 2*b^2)*sinh(x)^2 - a*b - 6*b^2 + 2*(3*(a*b + 6*
b^2)*cosh(x)^5 + 2*(a*b - 2*b^2)*cosh(x)^3 - (a*b - 2*b^2)*cosh(x))*sinh(x)
)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(
x)*sinh(x) + sinh(x)^2)))/(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*si
nh(x)^8 + 4*b^2*cosh(x)^6 + 4*(7*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 6*b^2*cos
h(x)^4 + 8*(7*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^
4 + 30*b^2*cosh(x)^2 + 3*b^2)*sinh(x)^4 + 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x
)^5 + 10*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15
*b^2*cosh(x)^4 + 9*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^7
 + 3*b^2*cosh(x)^5 + 3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)), -1/8*(4*(b^2*c
osh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*cosh(x)^6 + 4*(7
*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2*cosh(x)^3 + 3*
b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x)^2 + 3*b^2)*si
nh(x)^4 + 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh(x)^3 + 3*b^2*c
osh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 + 9*b^2*cosh(x)^2
 + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5 + 3*b^2*cosh(x
)^3 + b^2*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*
cosh(x)*sinh(x) + b*sinh(x)^2 - a - b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2
 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/
((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh(x)*sinh(x)^3 + (a*b + b^2)*sinh
(x)^4 + (a^2 - a*b - 2*b^2)*cosh(x)^2 + (6*(a*b + b^2)*cosh(x)^2 + a^2 - a*
b - 2*b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a*b + b^2)*cosh(x)^3 + (a^
2 - a*b - 2*b^2)*cosh(x))*sinh(x))) + 4*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh
(x)^7 + b^2*sinh(x)^8 + 4*b^2*cosh(x)^6 + 4*(7*b^2*cosh(x)^2 + b^2)*sinh(x)
^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^5 + 2*(3
5*b^2*cosh(x)^4 + 30*b^2*cosh(x)^2 + 3*b^2)*sinh(x)^4 + 4*b^2*cosh(x)^2 + 8
*(7*b^2*cosh(x)^5 + 10*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2*
cosh(x)^6 + 15*b^2*cosh(x)^4 + 9*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 8*(
b^2*cosh(x)^7 + 3*b^2*cosh(x)^5 + 3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*s
qrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*
```

```
sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^
2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*
sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^
2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a
+ b)) + ((a^2 - 4*a*b - 8*b^2)*cosh(x)^8 + 8*(a^2 - 4*a*b - 8*b^2)*cosh(x)*
sinh(x)^7 + (a^2 - 4*a*b - 8*b^2)*sinh(x)^8 + 4*(a^2 - 4*a*b - 8*b^2)*cosh(
x)^6 + 4*(7*(a^2 - 4*a*b - 8*b^2)*cosh(x)^2 + a^2 - 4*a*b - 8*b^2)*sinh(x)^
6 + 8*(7*(a^2 - 4*a*b - 8*b^2)*cosh(x)^3 + 3*(a^2 - 4*a*b - 8*b^2)*cosh(x))
*sinh(x)^5 + 6*(a^2 - 4*a*b - 8*b^2)*cosh(x)^4 + 2*(35*(a^2 - 4*a*b - 8*b^2
)*cosh(x)^4 + 30*(a^2 - 4*a*b - 8*b^2)*cosh(x)^2 + 3*a^2 - 12*a*b - 24*b^2)
*sinh(x)^4 + 8*(7*(a^2 - 4*a*b - 8*b^2)*cosh(x)^5 + 10*(a^2 - 4*a*b - 8*b^2
)*cosh(x)^3 + 3*(a^2 - 4*a*b - 8*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 - 4*a*b -
 8*b^2)*cosh(x)^2 + 4*(7*(a^2 - 4*a*b - 8*b^2)*cosh(x)^6 + 15*(a^2 - 4*a*b
- 8*b^2)*cosh(x)^4 + 9*(a^2 - 4*a*b - 8*b^2)*cosh(x)^2 + a^2 - 4*a*b - 8*b^
2)*sinh(x)^2 + a^2 - 4*a*b - 8*b^2 + 8*((a^2 - 4*a*b - 8*b^2)*cosh(x)^7 + 3
*(a^2 - 4*a*b - 8*b^2)*cosh(x)^5 + 3*(a^2 - 4*a*b - 8*b^2)*cosh(x)^3 + (a^2
 - 4*a*b - 8*b^2)*cosh(x))*sinh(x))*sqrt(-b)*arctan(sqrt(2)*(cosh(x)^2 + 2*
cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt(((a + b)*cosh(x)^2 + (a + b)
*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*c
osh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh
(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 +
(a - b)*cosh(x))*sinh(x) + a + b)) + sqrt(2)*((a*b + 6*b^2)*cosh(x)^6 + 6*(
a*b + 6*b^2)*cosh(x)*sinh(x)^5 + (a*b + 6*b^2)*sinh(x)^6 + (a*b - 2*b^2)*co
sh(x)^4 + (15*(a*b + 6*b^2)*cosh(x)^2 + a*b - 2*b^2)*sinh(x)^4 + 4*(5*(a*b
+ 6*b^2)*cosh(x)^3 + (a*b - 2*b^2)*cosh(x))*sinh(x)^3 - (a*b - 2*b^2)*cosh(
x)^2 + (15*(a*b + 6*b^2)*cosh(x)^4 + 6*(a*b - 2*b^2)*cosh(x)^2 - a*b + 2*b^
2)*sinh(x)^2 - a*b - 6*b^2 + 2*(3*(a*b + 6*b^2)*cosh(x)^5 + 2*(a*b - 2*b^2)
*cosh(x)^3 - (a*b - 2*b^2)*cosh(x))*sinh(x))*sqrt(((a + b)*cosh(x)^2 + (a +
 b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b^2*c
osh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*cosh(x)^6 + 4*(7
*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2*cosh(x)^3 + 3*
b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x)^2 + 3*b^2)*si
nh(x)^4 + 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh(x)^3 + 3*b^2*c
osh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 + 9*b^2*cosh(x)^2
 + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5 + 3*b^2*cosh(x
)^3 + b^2*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\tanh^2(x)}\tanh^4(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)**2)**(1/2)*tanh(x)**4,x)
```

```
[Out] Integral(sqrt(a + b*tanh(x)**2)*tanh(x)**4, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.210 $\int \tanh^3(x)\sqrt{a+b\tanh^2(x)}\,dx$

Optimal. Leaf size=63

$$-\frac{\left(a+b\tanh^2(x)\right)^{3/2}}{3b}-\sqrt{a+b\tanh^2(x)}+\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)$$

```
[Out] Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - Sqrt[a + b*Tanh[x]
^2] - (a + b*Tanh[x]^2)^(3/2)/(3*b)
```

Rubi [A] time = 0.119459, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3670, 446, 80, 50, 63, 208}

$$-\frac{\left(a+b\tanh^2(x)\right)^{3/2}}{3b}-\sqrt{a+b\tanh^2(x)}+\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - Sqrt[a + b*Tanh[x]
^2] - (a + b*Tanh[x]^2)^(3/2)/(3*b)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \tanh^3(x)\sqrt{a+b\tanh^2(x)}\,dx &= \text{Subst}\left(\int \frac{x^3\sqrt{a+bx^2}}{1-x^2}\,dx, x, \tanh(x)\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{x\sqrt{a+bx}}{1-x}\,dx, x, \tanh^2(x)\right) \\
&= -\frac{\left(a+b\tanh^2(x)\right)^{3/2}}{3b} + \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1-x}\,dx, x, \tanh^2(x)\right) \\
&= -\sqrt{a+b\tanh^2(x)} - \frac{\left(a+b\tanh^2(x)\right)^{3/2}}{3b} + \frac{1}{2}(a+b)\,\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, \ldots\right. \\
&= -\sqrt{a+b\tanh^2(x)} - \frac{\left(a+b\tanh^2(x)\right)^{3/2}}{3b} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\,\text{t}\ldots}\right.}{b} \\
&= \sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \sqrt{a+b\tanh^2(x)} - \frac{\left(a+b\tanh^2(x)\right)^{3/2}}{3b}
\end{aligned}
$$

Mathematica [A] time = 0.160509, size = 60, normalized size = 0.95

$$\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \frac{\sqrt{a+b\tanh^2(x)}\left(a+b\tanh^2(x)+3b\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (Sqrt[a + b*Tanh[x
]^2]*(a + 3*b + b*Tanh[x]^2))/(3*b)
```

Maple [B] time = 0.045, size = 253, normalized size = 4.

$$-\frac{1}{3\,b}\left(a+b\left(\tanh(x)\right)^2\right)^{\frac{3}{2}} - \frac{1}{2}\sqrt{\left(1+\tanh(x)\right)^2 b - 2\,\left(1+\tanh(x)\right)b + a + b} + \frac{1}{2}\sqrt{b}\ln\left(\left(\left(1+\tanh(x)\right)b - b\right)\frac{1}{\sqrt{b}} + \ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tanh(x)^2)^(1/2)*tanh(x)^3,x)
```

```
[Out] -1/3*(a+b*tanh(x)^2)^(3/2)/b-1/2*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2
)+1/2*b^(1/2)*ln(((1+tanh(x))*b-b)/b^(1/2)+((1+tanh(x))^2*b-2*(1+tanh(x))*b
+a+b)^(1/2))+1/2*(a+b)^(1/2)*ln((2*a+2*b-2*(1+tanh(x))*b+2*(a+b)^(1/2)*((1+
tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))/(1+tanh(x)))-1/2*((tanh(x)-1)^2*b+
2*(tanh(x)-1)*b+a+b)^(1/2)-1/2*b^(1/2)*ln(((tanh(x)-1)*b+b)/b^(1/2)+((tanh(
x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))+1/2*(a+b)^(1/2)*ln((2*a+2*b+2*(tanh(x
)-1)*b+2*(a+b)^(1/2)*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))/(tanh(x)-
1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \tanh(x)^2 + a} \tanh(x)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tanh(x)^2 + a)*tanh(x)^3, x)
```

Fricas [B] time = 3.91679, size = 6839, normalized size = 108.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^3,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b*cosh(x)^6 + 6*b*cosh(x)*sinh(x)^5 + b*sinh(x)^6 + 3*b*cosh(x)^4
 + 3*(5*b*cosh(x)^2 + b)*sinh(x)^4 + 4*(5*b*cosh(x)^3 + 3*b*cosh(x))*sinh(x
)^3 + 3*b*cosh(x)^2 + 3*(5*b*cosh(x)^4 + 6*b*cosh(x)^2 + b)*sinh(x)^2 + 6*(
b*cosh(x)^5 + 2*b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(a + b)*log(((a^3
 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sin
h(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*
cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*co
sh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a
^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x
)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)
^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3
*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*co
sh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 +
 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a
^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2
 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2
+ 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2
*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh
(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*c
osh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh
(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a
^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*si
```

```
nh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(
x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6))
 + 3*(b*cosh(x)^6 + 6*b*cosh(x)*sinh(x)^5 + b*sinh(x)^6 + 3*b*cosh(x)^4 + 3
*(5*b*cosh(x)^2 + b)*sinh(x)^4 + 4*(5*b*cosh(x)^3 + 3*b*cosh(x))*sinh(x)^3
+ 3*b*cosh(x)^2 + 3*(5*b*cosh(x)^4 + 6*b*cosh(x)^2 + b)*sinh(x)^2 + 6*(b*co
sh(x)^5 + 2*b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(a + b)*log(-((a + b)
*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^
2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*
sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh
(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*co
sh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh
(x)^2)) - 4*sqrt(2)*((a + 4*b)*cosh(x)^4 + 4*(a + 4*b)*cosh(x)*sinh(x)^3 +
(a + 4*b)*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*(a + 4*b)*cosh(x)^2 + a
+ 2*b)*sinh(x)^2 + 4*((a + 4*b)*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a
+ 4*b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*
cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)^6 + 6*b*cosh(x)*sinh(x)^5 + b*sin
h(x)^6 + 3*b*cosh(x)^4 + 3*(5*b*cosh(x)^2 + b)*sinh(x)^4 + 4*(5*b*cosh(x)^3
 + 3*b*cosh(x))*sinh(x)^3 + 3*b*cosh(x)^2 + 3*(5*b*cosh(x)^4 + 6*b*cosh(x)^
2 + b)*sinh(x)^2 + 6*(b*cosh(x)^5 + 2*b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)
, -1/6*(3*(b*cosh(x)^6 + 6*b*cosh(x)*sinh(x)^5 + b*sinh(x)^6 + 3*b*cosh(x)^
4 + 3*(5*b*cosh(x)^2 + b)*sinh(x)^4 + 4*(5*b*cosh(x)^3 + 3*b*cosh(x))*sinh(
x)^3 + 3*b*cosh(x)^2 + 3*(5*b*cosh(x)^4 + 6*b*cosh(x)^2 + b)*sinh(x)^2 + 6*
(b*cosh(x)^5 + 2*b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(-a - b)*arctan(
sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a -
 b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cos
h(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*s
inh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2
+ a*b)*cosh(x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*
(a^2 + a*b)*cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh(x))) + 3*(b*cosh(
x)^6 + 6*b*cosh(x)*sinh(x)^5 + b*sinh(x)^6 + 3*b*cosh(x)^4 + 3*(5*b*cosh(x)
^2 + b)*sinh(x)^4 + 4*(5*b*cosh(x)^3 + 3*b*cosh(x))*sinh(x)^3 + 3*b*cosh(x)
^2 + 3*(5*b*cosh(x)^4 + 6*b*cosh(x)^2 + b)*sinh(x)^2 + 6*(b*cosh(x)^5 + 2*b
*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2
 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2
+ (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(
(a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a
- b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cos
h(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + 2*sqrt(2)*((a + 4*b)*cosh(x)^
4 + 4*(a + 4*b)*cosh(x)*sinh(x)^3 + (a + 4*b)*sinh(x)^4 + 2*(a + 2*b)*cosh(
x)^2 + 2*(3*(a + 4*b)*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*((a + 4*b)*cosh(x)
^3 + (a + 2*b)*cosh(x))*sinh(x) + a + 4*b)*sqrt(((a + b)*cosh(x)^2 + (a + b
)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(
x)^6 + 6*b*cosh(x)*sinh(x)^5 + b*sinh(x)^6 + 3*b*cosh(x)^4 + 3*(5*b*cosh(x)
^2 + b)*sinh(x)^4 + 4*(5*b*cosh(x)^3 + 3*b*cosh(x))*sinh(x)^3 + 3*b*cosh(x)
^2 + 3*(5*b*cosh(x)^4 + 6*b*cosh(x)^2 + b)*sinh(x)^2 + 6*(b*cosh(x)^5 + 2*b
*cosh(x)^3 + b*cosh(x))*sinh(x) + b)]
```

Sympy [A] time = 4.42507, size = 71, normalized size = 1.13

$$-\frac{2\left(\frac{b^2\sqrt{a+b\tanh^2(x)}}{2}+\frac{b^2(a+b)\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}}+\frac{b\left(a+b\tanh^2(x)\right)^{\frac{3}{2}}}{6}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)**2)**(1/2)*tanh(x)**3,x)
```

```
[Out] -2*(b**2*sqrt(a + b*tanh(x)**2)/2 + b**2*(a + b)*atan(sqrt(a + b*tanh(x)**2
)/sqrt(-a - b))/(2*sqrt(-a - b)) + b*(a + b*tanh(x)**2)**(3/2)/6)/b**2
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.211 $\int \tanh^2(x)\sqrt{a+b\tanh^2(x)}\,dx$

Optimal. Leaf size=85

$$-\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{2\sqrt{b}}+\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\frac{1}{2}\tanh(x)\sqrt{a+b\tanh^2(x)}$$

```
[Out] -((a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/(2*Sqrt[b]) +
 Sqrt[a + b]*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]] - (Tanh[x
]*Sqrt[a + b*Tanh[x]^2])/2
```

Rubi [A] time = 0.124518, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3670, 478, 523, 217, 206, 377}

$$-\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{2\sqrt{b}}+\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\frac{1}{2}\tanh(x)\sqrt{a+b\tanh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] -((a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/(2*Sqrt[b]) +
 Sqrt[a + b]*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]] - (Tanh[x
]*Sqrt[a + b*Tanh[x]^2])/2
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \tanh^2(x)\sqrt{a+b\tanh^2(x)}\,dx &= \text{Subst}\left(\int \frac{x^2\sqrt{a+bx^2}}{1-x^2}\,dx, x, \tanh(x)\right)\\
&= -\frac{1}{2}\tanh(x)\sqrt{a+b\tanh^2(x)}+\frac{1}{2}\text{Subst}\left(\int \frac{a+(a+2b)x^2}{\left(1-x^2\right)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)\\
&= -\frac{1}{2}\tanh(x)\sqrt{a+b\tanh^2(x)}+\frac{1}{2}(-a-2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)+(a+\\
&= -\frac{1}{2}\tanh(x)\sqrt{a+b\tanh^2(x)}+\frac{1}{2}(-a-2b)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right.\\
&= -\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{2\sqrt{b}}+\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\frac{1}{2}\tanh(x)\sqrt{a}
\end{aligned}$$

Mathematica [C] time = 3.26176, size = 193, normalized size = 2.27

$$\frac{\tanh(x)\left(\sqrt{2}a\sqrt{\frac{\text{csch}^2(x)((a+b)\cosh(2x)+a-b)}{b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\text{csch}^2(x)((a+b)\cosh(2x)+a-b)}{b}}}{\sqrt{2}}\right),1\right)+\text{sech}^2(x)(-((a+b)\cosh(2x)+a\right.}{2\sqrt{2}\sqrt{\text{sech}^2(x)((a+b)\cosh(2x)+a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ((Sqrt[2]*a*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*EllipticF[ArcSi
n[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1] - 2*Sqrt[2]*
a*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*EllipticPi[b/(a + b), Arc
Sin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1] - (a - b +
 (a + b)*Cosh[2*x])*Sech[x]^2)*Tanh[x])/(2*Sqrt[2]*Sqrt[(a - b + (a + b)*Co
sh[2*x])*Sech[x]^2])
```

Maple [B] time = 0.042, size = 276, normalized size = 3.3

$$-\frac{\tanh(x)}{2}\sqrt{a+b(\tanh(x))^2}-\frac{a}{2}\ln\left(\tanh(x)\sqrt{b}+\sqrt{a+b(\tanh(x))^2}\right)\frac{1}{\sqrt{b}}+\frac{1}{2}\sqrt{(1+\tanh(x))^2b-2\,(1+\tanh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tanh(x)^2)^(1/2)*tanh(x)^2,x)
```

```
[Out] -1/2*(a+b*tanh(x)^2)^(1/2)*tanh(x)-1/2*a/b^(1/2)*ln(tanh(x)*b^(1/2)+(a+b*ta
nh(x)^2)^(1/2))+1/2*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)-1/2*b^(1/2)
*ln(((1+tanh(x))*b-b)/b^(1/2)+((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))-
1/2*(a+b)^(1/2)*ln((2*a+2*b-2*(1+tanh(x))*b+2*(a+b)^(1/2)*((1+tanh(x))^2*b-
2*(1+tanh(x))*b+a+b)^(1/2))/(1+tanh(x)))-1/2*((tanh(x)-1)^2*b+2*(tanh(x)-1)
*b+a+b)^(1/2)-1/2*b^(1/2)*ln(((tanh(x)-1)*b+b)/b^(1/2)+((tanh(x)-1)^2*b+2*(
tanh(x)-1)*b+a+b)^(1/2))+1/2*(a+b)^(1/2)*ln((2*a+2*b+2*(tanh(x)-1)*b+2*(a+b
)^(1/2)*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))/(tanh(x)-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\tanh(x)^2+a}\tanh(x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tanh(x)^2 + a)*tanh(x)^2, x)
```

Fricas [B] time = 4.62177, size = 14340, normalized size = 168.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^2,x, algorithm="fricas")
```

```
[Out] [1/4*((b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 +
2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*
sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)
^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b
^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3
- 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*co
sh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*
(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(
a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)
^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 +
2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^
2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2
)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^
4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x
))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2
*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*
cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(
```

a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*
cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2
*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^
2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^
4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*s
inh(x)^5 + sinh(x)^6)) + ((a + 2*b)*cosh(x)^4 + 4*(a + 2*b)*cosh(x)*sinh(x)
^3 + (a + 2*b)*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*(a + 2*b)*cosh(x)^2
 + a + 2*b)*sinh(x)^2 + 4*((a + 2*b)*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x)
 + a + 2*b)*sqrt(b)*log(-((a + 2*b)*cosh(x)^4 + 4*(a + 2*b)*cosh(x)*sinh(x)
^3 + (a + 2*b)*sinh(x)^4 + 2*(a - 2*b)*cosh(x)^2 + 2*(3*(a + 2*b)*cosh(x)^2
 + a - 2*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2 - 1)*sqrt(b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x
)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + 2*b)*cosh(x)^3 + (a - 2*b)*
cosh(x))*sinh(x) + a + 2*b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 +
2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(
x) + 1)) + (b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)
^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x)
+ b)*sqrt(a + b)*log(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a
+ b)*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sq
rt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a
+ b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
+ sinh(x)^2)) + 4*((a + b)*cosh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)
^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 2*sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)
*sinh(x) + b*sinh(x)^2 - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
 - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)^4 + 4*b*cosh
(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)
^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b), 1/4*(2*((a + 2*b)*cosh(x)^4
+ 4*(a + 2*b)*cosh(x)*sinh(x)^3 + (a + 2*b)*sinh(x)^4 + 2*(a + 2*b)*cosh(x)
^2 + 2*(3*(a + 2*b)*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*((a + 2*b)*cosh(x)^3
 + (a + 2*b)*cosh(x))*sinh(x) + a + 2*b)*sqrt(-b)*arctan(sqrt(2)*(cosh(x)^2
 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt(((a + b)*cosh(x)^2 + (a
 + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a +
 b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)
cosh(x)^2 + 2(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)
^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + (b*cosh(x)^4 + 4*b*cosh(x)*sinh(x
)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*
cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)
^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2
 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x
)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5
+ (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 +
 a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 +
4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b
 + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*
(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b
^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 +
6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x
)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4
 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*co
sh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(
x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*
a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b
)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a
*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^
2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6
 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 +
15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + (b*cosh(x)^4 +
 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b
)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(a + b)*log(((a

```
+ b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh
(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh
(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*
sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b
)*cosh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
sinh(x)^2)) - 2*sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 -
b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh
(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)
^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*c
osh(x))*sinh(x) + b), -1/4*(2*(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh
(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 +
b*cosh(x))*sinh(x) + b)*sqrt(-a - b)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh
(x)*sinh(x) + b*sinh(x)^2 - a - b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (
a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a*
b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^
4 + (a^2 - a*b - 2*b^2)*cosh(x)^2 + (6*(a*b + b^2)*cosh(x)^2 + a^2 - a*b -
2*b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a*b + b^2)*cosh(x)^3 + (a^2 -
a*b - 2*b^2)*cosh(x))*sinh(x))) + 2*(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 +
b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x
)^3 + b*cosh(x))*sinh(x) + b)*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*co
sh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a +
b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)
*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*co
sh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3
+ (a - b)*cosh(x))*sinh(x) + a + b)) - ((a + 2*b)*cosh(x)^4 + 4*(a + 2*b)*c
osh(x)*sinh(x)^3 + (a + 2*b)*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*(a +
2*b)*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*((a + 2*b)*cosh(x)^3 + (a + 2*b)*co
sh(x))*sinh(x) + a + 2*b)*sqrt(b)*log(-((a + 2*b)*cosh(x)^4 + 4*(a + 2*b)*c
osh(x)*sinh(x)^3 + (a + 2*b)*sinh(x)^4 + 2*(a - 2*b)*cosh(x)^2 + 2*(3*(a +
2*b)*cosh(x)^2 + a - 2*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh
(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + 2*b)*cosh(x)^
3 + (a - 2*b)*cosh(x))*sinh(x) + a + 2*b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3
+ sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 +
cosh(x))*sinh(x) + 1)) + 2*sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*s
inh(x)^2 - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3
+ b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh
(x)^3 + b*cosh(x))*sinh(x) + b), -1/2*((b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3
 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cos
h(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(-a - b)*arctan(sqrt(2)*(b*cosh(x)^2 +
 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - a - b)*sqrt(-a - b)*sqrt(((a + b)*cosh
(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)
^2))/((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh(x)*sinh(x)^3 + (a*b + b^2)
*sinh(x)^4 + (a^2 - a*b - 2*b^2)*cosh(x)^2 + (6*(a*b + b^2)*cosh(x)^2 + a^2
 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a*b + b^2)*cosh(x)^3
+ (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x))) + (b*cosh(x)^4 + 4*b*cosh(x)*sinh(
x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b
*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2
 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2
+ (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(
(a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a
- b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cos
h(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) - ((a + 2*b)*cosh(x)^4 + 4*(a +
 2*b)*cosh(x)*sinh(x)^3 + (a + 2*b)*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(
3*(a + 2*b)*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*((a + 2*b)*cosh(x)^3 + (a +
2*b)*cosh(x))*sinh(x) + a + 2*b)*sqrt(-b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*si
nh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh
```

```
(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)
^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a
- b)*cosh(x))*sinh(x) + a + b)) + sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x
) + b*sinh(x)^2 - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(
cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)^4 + 4*b*cosh(x)*sin
h(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*
(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \tanh^{2}(x)} \tanh^{2}(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)**2)**(1/2)*tanh(x)**2,x)
```

```
[Out] Integral(sqrt(a + b*tanh(x)**2)*tanh(x)**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.212 $\int \tanh(x)\sqrt{a+b\tanh^2(x)}\,dx$

Optimal. Leaf size=44

$$\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\sqrt{a+b\tanh^2(x)}$$

```
[Out] Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - Sqrt[a + b*Tanh[x]
^2]
```

Rubi [A] time = 0.0770792, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3670, 444, 50, 63, 208}

$$\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\sqrt{a+b\tanh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - Sqrt[a + b*Tanh[x]
^2]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \tanh(x)\sqrt{a+b\tanh^2(x)}\,dx &= \text{Subst}\left(\int \frac{x\sqrt{a+bx^2}}{1-x^2}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1-x}\,dx, x, \tanh^2(x)\right)\\
&= -\sqrt{a+b\tanh^2(x)} + \frac{1}{2}(a+b)\,\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)\\
&= -\sqrt{a+b\tanh^2(x)} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b}\\
&= \sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \sqrt{a+b\tanh^2(x)}
\end{aligned}$$

Mathematica [A] time = 0.0265664, size = 44, normalized size = 1.

$$\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \sqrt{a+b\tanh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - Sqrt[a + b*Tanh[x]
^2]
```

Maple [B] time = 0.049, size = 238, normalized size = 5.4

$$-\frac{1}{2}\sqrt{(1+\tanh(x))^2 b - 2\,(1+\tanh(x))\,b + a + b} + \frac{1}{2}\sqrt{b}\ln\left(((1+\tanh(x))\,b - b)\frac{1}{\sqrt{b}} + \sqrt{(1+\tanh(x))^2 b - 2\,(1+\tan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tanh(x)^2)^(1/2)*tanh(x),x)
```

```
[Out] -1/2*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)+1/2*b^(1/2)*ln(((1+tanh(x)
)*b-b)/b^(1/2)+((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))+1/2*(a+b)^(1/2)
*ln((2*a+2*b-2*(1+tanh(x))*b+2*(a+b)^(1/2)*((1+tanh(x))^2*b-2*(1+tanh(x))*b
+a+b)^(1/2))/(1+tanh(x)))-1/2*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)-1
/2*b^(1/2)*ln(((tanh(x)-1)*b+b)/b^(1/2)+((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+
b)^(1/2))+1/2*(a+b)^(1/2)*ln((2*a+2*b+2*(tanh(x)-1)*b+2*(a+b)^(1/2)*((tanh(
```

```
x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))/(tanh(x)-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tanh(x)^2 + a}\tanh(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tanh(x)^2 + a)*tanh(x), x)
```

Fricas [B] time = 2.85727, size = 4456, normalized size = 101.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x),x, algorithm="fricas")
```

```
[Out] [1/4*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*log(((a^3
 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sin
h(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*
cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*co
sh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a
^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x
)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)
^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3
*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*co
sh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 +
 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a
^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2
 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2
+ 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2
*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh
(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*c
osh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh
(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a
^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*si
nh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(
x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6))
 + (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*log(-((a + b
)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)
^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)
*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sin
h(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*c
osh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sin
h(x)^2)) - 4*sqrt(2)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(
cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x)
 + sinh(x)^2 + 1), -1/2*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sq
rt(-a - b)*arctan(sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2
+ a + b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^
2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*co
```

```
sh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2
*a*b + b^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh
(x))) + (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*arctan
(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(
((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh
(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b
)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)
^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + 2*sqrt(2)*
sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)
*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)]
```

Sympy [A] time = 2.28149, size = 51, normalized size = 1.16

$$-\frac{2\left(\frac{b\sqrt{a+b\tanh^2(x)}}{2}+\frac{b(a+b)\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)**2)**(1/2)*tanh(x),x)
```

```
[Out] -2*(b*sqrt(a + b*tanh(x)**2)/2 + b*(a + b)*atan(sqrt(a + b*tanh(x)**2)/sqrt
(-a - b))/(2*sqrt(-a - b)))/b
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.213 $\int \sqrt{a+b\tanh^2(x)}\,dx$

Optimal. Leaf size=60

$$\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)$$

```
[Out] -(Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]) + Sqrt[a + b]*A
rcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]
```

Rubi [A] time = 0.047025, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3661, 402, 217, 206, 377}

$$\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] -(Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]) + Sqrt[a + b]*A
rcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\int \sqrt{a+b\tanh^2(x)}\,dx = \text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{1-x^2}\,dx, x, \tanh(x)\right)$$
$$= -\left((-a-b)\,\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)\right) - b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tanh(x$$
$$= -\left((-a-b)\,\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)\right) - b\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{}{\sqrt{a-}}\right.$$
$$= -\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) + \sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)$$

Mathematica [B] time = 0.213137, size = 137, normalized size = 2.28

$$\frac{1}{2}\left(-2\sqrt{b}\log\left(\sqrt{b}\sqrt{a+b\tanh^2(x)}+b\tanh(x)\right)-\sqrt{a+b}\log\left(\sqrt{a+b}\sqrt{a+b\tanh^2(x)}+a-b\tanh(x)\right)+\sqrt{a+b}\log\left(\sqrt{\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] (-(Sqrt[a + b]*Log[1 - Tanh[x]]) + Sqrt[a + b]*Log[1 + Tanh[x]] - 2*Sqrt[b]
*Log[b*Tanh[x] + Sqrt[b]*Sqrt[a + b*Tanh[x]^2]] - Sqrt[a + b]*Log[a - b*Tan
h[x] + Sqrt[a + b]*Sqrt[a + b*Tanh[x]^2]] + Sqrt[a + b]*Log[a + b*Tanh[x] +
 Sqrt[a + b]*Sqrt[a + b*Tanh[x]^2]])/2
```

Maple [B] time = 0.046, size = 238, normalized size = 4.

$$\frac{1}{2}\sqrt{(1+\tanh(x))^2 b-2(1+\tanh(x))b+a+b}-\frac{1}{2}\sqrt{b}\ln\left(((1+\tanh(x))b-b)\frac{1}{\sqrt{b}}+\sqrt{(1+\tanh(x))^2 b-2(1+\tanh\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tanh(x)^2)^(1/2),x)
```

```
[Out] 1/2*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)-1/2*b^(1/2)*ln(((1+tanh(x))
*b-b)/b^(1/2)+((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))-1/2*(a+b)^(1/2)*
ln((2*a+2*b-2*(1+tanh(x))*b+2*(a+b)^(1/2)*((1+tanh(x))^2*b-2*(1+tanh(x))*b+
a+b)^(1/2))/(1+tanh(x)))-1/2*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)-1/
2*b^(1/2)*ln(((tanh(x)-1)*b+b)/b^(1/2)+((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b
)^(1/2))+1/2*(a+b)^(1/2)*ln((2*a+2*b+2*(tanh(x)-1)*b+2*(a+b)^(1/2)*((tanh(x
)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))/(tanh(x)-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tanh(x)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tanh(x)^2 + a), x)
```

Fricas [B] time = 3.68879, size = 9993, normalized size = 166.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*si
nh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2
+ 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(
x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^
3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3
- 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 -
 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*si
nh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)
^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3
*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + s
qrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cos
h(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*c
osh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 1
8*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3
*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*
sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2
 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^
2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3
*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cos
h(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh
(x)*sinh(x)^5 + sinh(x)^6)) + 1/2*sqrt(b)*log(-((a + 2*b)*cosh(x)^4 + 4*(a
+ 2*b)*cosh(x)*sinh(x)^3 + (a + 2*b)*sinh(x)^4 + 2*(a - 2*b)*cosh(x)^2 + 2*
(3*(a + 2*b)*cosh(x)^2 + a - 2*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh
(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh
(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + 2*b)*
cosh(x)^3 + (a - 2*b)*cosh(x))*sinh(x) + a + 2*b)/(cosh(x)^4 + 4*cosh(x)*si
nh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh
(x)^3 + cosh(x))*sinh(x) + 1)) + 1/4*sqrt(a + b)*log(((a + b)*cosh(x)^4 + 4
*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*(a +
b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh
(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)
/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 + a*co
sh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)), sqrt(
-b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)
*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x
)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 +
(a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*s
inh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + 1/4*
sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)
^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b
^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3
- 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*co
sh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*
```

(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 1/4*sqrt(a + b)*log(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)), -1/2*sqrt(-a - b)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - a - b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^2)*cosh(x)^2 + (6*(a*b + b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a*b + b^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x))) - 1/2*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + 1/2*sqrt(b)*log(-((a + 2*b)*cosh(x)^4 + 4*(a + 2*b)*cosh(x)*sinh(x)^3 + (a + 2*b)*sinh(x)^4 + 2*(a - 2*b)*cosh(x)^2 + 2*(3*(a + 2*b)*cosh(x)^2 + a - 2*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + 2*b)*cosh(x)^3 + (a - 2*b)*cosh(x))*sinh(x) + a + 2*b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)), -1/2*sqrt(-a - b)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - a - b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^2)*cosh(x)^2 + (6*(a*b + b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a*b + b^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x))) - 1/2*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + sqrt(-b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b))]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\tanh^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tanh(x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.214 $\int \coth(x)\sqrt{a+b\tanh^2(x)}\,dx$

Optimal. Leaf size=56

$$\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)$$

```
[Out] -(Sqrt[a]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]]) + Sqrt[a + b]*ArcTanh[Sqr
t[a + b*Tanh[x]^2]/Sqrt[a + b]]
```

Rubi [A] time = 0.111944, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3670, 446, 83, 63, 208}

$$\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] -(Sqrt[a]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]]) + Sqrt[a + b]*ArcTanh[Sqr
t[a + b*Tanh[x]^2]/Sqrt[a + b]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \coth(x)\sqrt{a+b\tanh^2(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x\left(1-x^2\right)}\,dx, x, \tanh(x)\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{(1-x)x}\,dx, x, \tanh^2(x)\right)\\ &= \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)+\frac{1}{2}(a+b)\,\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x\right.\\ &= \frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b}+\frac{(a+b)\,\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\right.}{b}\\ &= -\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)+\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)\end{aligned}$$

Mathematica [A] time = 0.0277698, size = 56, normalized size = 1.

$$\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] -(Sqrt[a]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]]) + Sqrt[a + b]*ArcTanh[Sqr
t[a + b*Tanh[x]^2]/Sqrt[a + b]]
```

Maple [F] time = 0.168, size = 0, normalized size = 0.

$$\int \coth(x)\sqrt{a+b(\tanh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(a+b*tanh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)*(a+b*tanh(x)^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tanh(x)^2+a}\coth(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tanh(x)^2 + a)*coth(x), x)
```

Fricas [B] time = 3.59514, size = 9991, normalized size = 178.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sin
h(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 +
 a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x
)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3
)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 +
 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 +
10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sin
h(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^
2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 +
3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sq
rt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh
(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*co
sh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18
*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*
a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*s
qrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2
- 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3
 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 +
3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh
(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(
x)*sinh(x)^5 + sinh(x)^6)) + 1/2*sqrt(a)*log(-((2*a + b)*cosh(x)^4 + 4*(2*a
 + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a - b)*cosh(x)^2 + 2*(
3*(2*a + b)*cosh(x)^2 + 2*a - b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(
x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(
x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a + b)*c
osh(x)^3 + (2*a - b)*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sin
h(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(
x)^3 - cosh(x))*sinh(x) + 1)) + 1/4*sqrt(a + b)*log(-((a + b)*cosh(x)^4 + 4
*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(a +
b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh
(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)
/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 - b*co
sh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)), sqrt(
-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)
*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x
)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 +
(a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*s
inh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + 1/4*
sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^
7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*
b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 +
 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cos
h(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(
```

```
2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2
*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^
3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2
*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2
*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)
*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4
 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x)
)*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*
cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*c
osh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a
 + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^
2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2
*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4
*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*si
nh(x)^5 + sinh(x)^6)) + 1/4*sqrt(a + b)*log(-((a + b)*cosh(x)^4 + 4*(a + b)
*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(a + b)*cosh(
x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 -
1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x
)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 - b*cosh(x))*s
inh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)), -1/2*sqrt(-a
- b)*arctan(sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a +
b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(
x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*
b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^
2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b +
 b^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh(x)))
- 1/2*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(c
osh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*c
osh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*c
osh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(
x) + a + b)) + 1/2*sqrt(a)*log(-((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*
sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a - b)*cosh(x)^2 + 2*(3*(2*a + b)*co
sh(x)^2 + 2*a - b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + s
inh(x)^2 + 1)*sqrt(a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a + b)*cosh(x)^3 + (2*
a - b)*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(
x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x)
)*sinh(x) + 1)), sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + s
inh(x)^2 + 1)*sqrt(-a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)
/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b
)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b
)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*si
nh(x) + a + b)) - 1/2*sqrt(-a - b)*arctan(sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x
)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a
+ b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2
+ a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4
+ (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + a*b -
b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 +
a*b - b^2)*cosh(x))*sinh(x))) - 1/2*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2
+ 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 +
 (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((
a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a -
 b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh
(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\tanh^2(x)}\coth(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tanh(x)**2)*coth(x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.215 $\int \coth^2(x)\sqrt{a+b\tanh^2(x)}\,dx$

Optimal. Leaf size=48

$$\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\coth(x)\sqrt{a+b\tanh^2(x)}$$

```
[Out] Sqrt[a + b]*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]] - Coth[x]*
Sqrt[a + b*Tanh[x]^2]
```

Rubi [A] time = 0.0926206, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3670, 475, 12, 377, 206}

$$\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\coth(x)\sqrt{a+b\tanh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] Sqrt[a + b]*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]] - Coth[x]*
Sqrt[a + b*Tanh[x]^2]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \coth^2(x)\sqrt{a+b\tanh^2(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2\left(1-x^2\right)}\,dx, x, \tanh(x)\right)\\ &= -\coth(x)\sqrt{a+b\tanh^2(x)}+\text{Subst}\left(\int \frac{a+b}{\left(1-x^2\right)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)\\ &= -\coth(x)\sqrt{a+b\tanh^2(x)}+(a+b)\,\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)\\ &= -\coth(x)\sqrt{a+b\tanh^2(x)}+(a+b)\,\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)\\ &= \sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\coth(x)\sqrt{a+b\tanh^2(x)}\end{aligned}$$

Mathematica [C] time = 0.105273, size = 42, normalized size = 0.88

$$-\coth(x)\sqrt{a+b\tanh^2(x)}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] -(Coth[x]*Hypergeometric2F1[-1/2, 1, 1/2, ((a + b)*Tanh[x]^2)/(a + b*Tanh[x
]^2)]*Sqrt[a + b*Tanh[x]^2])
```

Maple [F] time = 0.153, size = 0, normalized size = 0.

$$\int (\coth(x))^2\sqrt{a+b(\tanh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2*(a+b*tanh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^2*(a+b*tanh(x)^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tanh(x)^2+a}\coth(x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*tanh(x)^2 + a)*coth(x)^2, x)
```

Fricas [B] time = 2.88513, size = 4456, normalized size = 92.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")

[Out] [1/4*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*log(-((a*
b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*si
nh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)
*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*c
osh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2
+ b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(
x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x
)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b +
3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*c
osh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 -
a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*
b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^
2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 -
 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*
a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cos
h(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*
cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sin
h(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^
3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*s
inh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh
(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)
) + (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*log(((a + b
)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)
^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)
*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sin
h(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*c
osh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sin
h(x)^2)) - 4*sqrt(2)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(
cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x)
 + sinh(x)^2 - 1), -1/2*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sq
rt(-a - b)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2
- a - b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a*b + b^2)*cosh(x)^4 + 4*(a*
b + b^2)*cosh(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^2)*co
sh(x)^2 + (6*(a*b + b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2
*a*b + b^2 + 2*(2*(a*b + b^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh
(x))) + (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*arctan
(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt(
((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh
(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b
)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)
^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + 2*sqrt(2)*
sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)
*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \tanh^2 (x)} \coth^2 (x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2*(a+b*tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tanh(x)**2)*coth(x)**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.216 $\int \coth^3(x)\sqrt{a+b\tanh^2(x)}\,dx$

Optimal. Leaf size=83

$$-\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{2\sqrt{a}}+\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\frac{1}{2}\coth^2(x)\sqrt{a+b\tanh^2(x)}$$

```
[Out] -((2*a + b)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]])/(2*Sqrt[a]) + Sqrt[a +
b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (Coth[x]^2*Sqrt[a + b*Tanh[
x]^2])/2
```

Rubi [A] time = 0.155208, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3670, 446, 99, 156, 63, 208}

$$-\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{2\sqrt{a}}+\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\frac{1}{2}\coth^2(x)\sqrt{a+b\tanh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] -((2*a + b)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]])/(2*Sqrt[a]) + Sqrt[a +
b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (Coth[x]^2*Sqrt[a + b*Tanh[
x]^2])/2
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \coth^3(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^3\left(1-x^2\right)}\,dx, x, \tanh(x)\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{(1-x)x^2}\,dx, x, \tanh^2(x)\right)$$

$$= -\frac{1}{2}\coth^2(x)\sqrt{a+b\tanh^2(x)} + \frac{1}{2}\,\text{Subst}\left(\int \frac{\frac{1}{2}(2a+b)+\frac{bx}{2}}{(1-x)x\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)$$

$$= -\frac{1}{2}\coth^2(x)\sqrt{a+b\tanh^2(x)} + \frac{1}{2}(a+b)\,\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right) + $$

$$= -\frac{1}{2}\coth^2(x)\sqrt{a+b\tanh^2(x)} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b} + \quad (2$$

$$= -\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{2\sqrt{a}} + \sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \frac{1}{2}\coth^2(x)\sqrt{}$$

Mathematica [A] time = 0.205619, size = 83, normalized size = 1.

$$-\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{2\sqrt{a}} + \sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \frac{1}{2}\coth^2(x)\sqrt{a+b\tanh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] -((2*a + b)*ArcTanh[Sqrt[a + b*Tanh[x]^2/Sqrt[a]])/(2*Sqrt[a]) + Sqrt[a +
b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (Coth[x]^2*Sqrt[a + b*Tanh[
x]^2])/2
```

Maple [F] time = 0.155, size = 0, normalized size = 0.

$$\int (\coth(x))^3 \sqrt{a+b(\tanh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3*(a+b*tanh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^3*(a+b*tanh(x)^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tanh(x)^2+a}\coth(x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3*(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tanh(x)^2 + a)*coth(x)^3, x)
```

Fricas [B] time = 4.44697, size = 14338, normalized size = 172.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3*(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 - 2*a*cosh(x)^2 +
2*(3*a*cosh(x)^2 - a)*sinh(x)^2 + 4*(a*cosh(x)^3 - a*cosh(x))*sinh(x) + a)*
sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^
7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*
b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 +
 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cos
h(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(
2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2
*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^
3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2
*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2
*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)
*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4
 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x)
)*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*
cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*c
osh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a
 + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^
2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2
*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4
*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*si
nh(x)^5 + sinh(x)^6)) + ((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^
```

```
3 + (2*a + b)*sinh(x)^4 - 2*(2*a + b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2
 - 2*a - b)*sinh(x)^2 + 4*((2*a + b)*cosh(x)^3 - (2*a + b)*cosh(x))*sinh(x)
 + 2*a + b)*sqrt(a)*log(-((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^
3 + (2*a + b)*sinh(x)^4 + 2*(2*a - b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2
 + 2*a - b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2
 + 1)*sqrt(a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a + b)*cosh(x)^3 + (2*a - b)*c
osh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2
*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x
) + 1)) + (a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 - 2*a*cosh(x)^
2 + 2*(3*a*cosh(x)^2 - a)*sinh(x)^2 + 4*(a*cosh(x)^3 - a*cosh(x))*sinh(x) +
 a)*sqrt(a + b)*log(-((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a
 + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sq
rt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a
 + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
 + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)
^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 2*sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)
*sinh(x) + a*sinh(x)^2 + a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
 - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*cosh(x)^4 + 4*a*cosh
(x)*sinh(x)^3 + a*sinh(x)^4 - 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 - a)*sinh(x)
^2 + 4*(a*cosh(x)^3 - a*cosh(x))*sinh(x) + a), 1/4*(2*((2*a + b)*cosh(x)^4
 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 - 2*(2*a + b)*cosh(x)
^2 + 2*(3*(2*a + b)*cosh(x)^2 - 2*a - b)*sinh(x)^2 + 4*((2*a + b)*cosh(x)^3
 - (2*a + b)*cosh(x))*sinh(x) + 2*a + b)*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2
 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt(((a + b)*cosh(x)^2 + (a
 + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a +
 b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)
*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)
^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + (a*cosh(x)^4 + 4*a*cosh(x)*sinh(x
)^3 + a*sinh(x)^4 - 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 - a)*sinh(x)^2 + 4*(a*
cosh(x)^3 - a*cosh(x))*sinh(x) + a)*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^
8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3
 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)
^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 +
 (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 +
6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4
*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^
2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(
2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^
3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 +
 b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)
^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4
 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cos
h(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x
)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2
*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)
*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^
3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b
^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6
 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 1
5*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + (a*cosh(x)^4 +
4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 - 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 - a)
*sinh(x)^2 + 4*(a*cosh(x)^3 - a*cosh(x))*sinh(x) + a)*sqrt(a + b)*log(-((a
 + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh
(x)^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh
(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*
sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b
)*cosh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
sinh(x)^2)) - 2*sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 +
```

```
a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh
(x)*sinh(x) + sinh(x)^2)))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)
^4 - 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 - a)*sinh(x)^2 + 4*(a*cosh(x)^3 - a*c
osh(x))*sinh(x) + a), -1/4*(2*(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh
(x)^4 - 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 - a)*sinh(x)^2 + 4*(a*cosh(x)^3 -
a*cosh(x))*sinh(x) + a)*sqrt(-a - b)*arctan(sqrt(2)*(a*cosh(x)^2 + 2*a*cosh
(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (
a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^
2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^
4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + a*b
- b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2
+ a*b - b^2)*cosh(x))*sinh(x))) + 2*(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 +
a*sinh(x)^4 - 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 - a)*sinh(x)^2 + 4*(a*cosh(x
)^3 - a*cosh(x))*sinh(x) + a)*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*co
sh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a +
b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)
*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*co
sh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3
+ (a - b)*cosh(x))*sinh(x) + a + b)) - ((2*a + b)*cosh(x)^4 + 4*(2*a + b)*c
osh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 - 2*(2*a + b)*cosh(x)^2 + 2*(3*(2*a
+ b)*cosh(x)^2 - 2*a - b)*sinh(x)^2 + 4*((2*a + b)*cosh(x)^3 - (2*a + b)*co
sh(x))*sinh(x) + 2*a + b)*sqrt(a)*log(-((2*a + b)*cosh(x)^4 + 4*(2*a + b)*c
osh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a - b)*cosh(x)^2 + 2*(3*(2*a
+ b)*cosh(x)^2 + 2*a - b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh
(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a + b)*cosh(x)^
3 + (2*a - b)*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3
+ sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 -
cosh(x))*sinh(x) + 1)) + 2*sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*s
inh(x)^2 + a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3
+ a*sinh(x)^4 - 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 - a)*sinh(x)^2 + 4*(a*cosh
(x)^3 - a*cosh(x))*sinh(x) + a), 1/2*(((2*a + b)*cosh(x)^4 + 4*(2*a + b)*co
sh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 - 2*(2*a + b)*cosh(x)^2 + 2*(3*(2*a +
 b)*cosh(x)^2 - 2*a - b)*sinh(x)^2 + 4*((2*a + b)*cosh(x)^3 - (2*a + b)*cos
h(x))*sinh(x) + 2*a + b)*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sin
h(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2
+ a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 +
4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(
3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cos
h(x))*sinh(x) + a + b)) - (a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^
4 - 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 - a)*sinh(x)^2 + 4*(a*cosh(x)^3 - a*co
sh(x))*sinh(x) + a)*sqrt(-a - b)*arctan(sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*
sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a +
b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 +
a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 +
(2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + a*b - b^
2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + a*
b - b^2)*cosh(x))*sinh(x))) - (a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh
(x)^4 - 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 - a)*sinh(x)^2 + 4*(a*cosh(x)^3 -
a*cosh(x))*sinh(x) + a)*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*
sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sin
h(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(
x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^
2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a -
 b)*cosh(x))*sinh(x) + a + b)) - sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x)
 + a*sinh(x)^2 + a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(c
osh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh
(x)^3 + a*sinh(x)^4 - 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 - a)*sinh(x)^2 + 4*(
```

```
a*cosh(x)^3 - a*cosh(x))*sinh(x) + a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\tanh^2(x)}\coth^3(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3*(a+b*tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tanh(x)**2)*coth(x)**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3*(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.217 $\int \coth^4(x)\sqrt{a+b\tanh^2(x)}\,dx$

Optimal. Leaf size=78

$$\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\frac{1}{3}\coth^3(x)\sqrt{a+b\tanh^2(x)}-\frac{(3a+b)\coth(x)\sqrt{a+b\tanh^2(x)}}{3a}$$

```
[Out] Sqrt[a + b]*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]] - ((3*a +
b)*Coth[x]*Sqrt[a + b*Tanh[x]^2])/(3*a) - (Coth[x]^3*Sqrt[a + b*Tanh[x]^2])
/3
```

Rubi [A] time = 0.145969, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3670, 475, 583, 12, 377, 206}

$$\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\frac{1}{3}\coth^3(x)\sqrt{a+b\tanh^2(x)}-\frac{(3a+b)\coth(x)\sqrt{a+b\tanh^2(x)}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] Sqrt[a + b]*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]] - ((3*a +
b)*Coth[x]*Sqrt[a + b*Tanh[x]^2])/(3*a) - (Coth[x]^3*Sqrt[a + b*Tanh[x]^2])
/3
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
```

```
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \coth^4(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^4\left(1-x^2\right)}\,dx, x, \tanh(x)\right)$$

$$= -\frac{1}{3}\coth^3(x)\sqrt{a+b\tanh^2(x)} + \frac{1}{3}\text{Subst}\left(\int \frac{3a+b+2bx^2}{x^2\left(1-x^2\right)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)$$

$$= -\frac{(3a+b)\coth(x)\sqrt{a+b\tanh^2(x)}}{3a} - \frac{1}{3}\coth^3(x)\sqrt{a+b\tanh^2(x)} - \frac{\text{Subst}\left(\int -\frac{3a(a}{(1-x^2)\ldots}\right.}{\ldots}$$

$$= -\frac{(3a+b)\coth(x)\sqrt{a+b\tanh^2(x)}}{3a} - \frac{1}{3}\coth^3(x)\sqrt{a+b\tanh^2(x)} - (-a-b)\,\text{Subst}\left(\int\right.$$

$$= -\frac{(3a+b)\coth(x)\sqrt{a+b\tanh^2(x)}}{3a} - \frac{1}{3}\coth^3(x)\sqrt{a+b\tanh^2(x)} - (-a-b)\,\text{Subst}\left(\int\right.$$

$$= \sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) - \frac{(3a+b)\coth(x)\sqrt{a+b\tanh^2(x)}}{3a} - \frac{1}{3}\coth^3(x)\sqrt{\ldots}$$

Mathematica [C] time = 8.83605, size = 235, normalized size = 3.01

$$-\frac{\tanh(x)\left(-12\sqrt{2}a(a+b)\sqrt{\frac{\text{csch}^2(x)((a+b)\cosh(2x)+a-b)}{b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\text{csch}^2(x)((a+b)\cosh(2x)+a-b)}{b}}}{\sqrt{2}}\right),1\right)+\text{csch}^4(x)\left(4\left(a^2-3ab\ldots\right.\right.\right.}{12\sqrt{2}a\sqrt{\text{sec}\ldots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] -(((b*(7*a + 3*b) + 4*(a^2 - 3*a*b - b^2)*Cosh[2*x] + (4*a^2 + 5*a*b + b^2)
*Cosh[4*x])*Csch[x]^4 - 12*Sqrt[2]*a*(a + b)*Sqrt[((a - b + (a + b)*Cosh[2*
x])*Csch[x]^2)/b]*EllipticF[ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x
```

```
]^2)/b]/Sqrt[2]], 1] + 12*Sqrt[2]*a^2*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csc
h[x]^2)/b]*EllipticPi[b/(a + b), ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*C
sch[x]^2)/b]/Sqrt[2]], 1])*Tanh[x])/(12*Sqrt[2]*a*Sqrt[(a - b + (a + b)*Cos
h[2*x])*Sech[x]^2])
```

Maple [F] time = 0.152, size = 0, normalized size = 0.

$$\int (\coth (x))^4 \sqrt{a + b (\tanh (x))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4*(a+b*tanh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^4*(a+b*tanh(x)^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \tanh (x)^2 + a} \coth (x)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4*(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tanh(x)^2 + a)*coth(x)^4, x)
```

Fricas [B] time = 3.91429, size = 6839, normalized size = 87.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4*(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 - 3*a*cosh(x)^4
 + 3*(5*a*cosh(x)^2 - a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 - 3*a*cosh(x))*sinh(x
)^3 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 - 6*a*cosh(x)^2 + a)*sinh(x)^2 + 6*(
a*cosh(x)^5 - 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) - a)*sqrt(a + b)*log(-((a*
b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*si
nh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)
*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*c
osh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2
+ b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(
x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x
)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b +
3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*c
osh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 -
a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*
b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^
2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 -
 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*
```

```
a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cos
h(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*
cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sin
h(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^
3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*s
inh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh
(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)
) + 3*(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 - 3*a*cosh(x)^4 +
3*(5*a*cosh(x)^2 - a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 - 3*a*cosh(x))*sinh(x)^3
 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 - 6*a*cosh(x)^2 + a)*sinh(x)^2 + 6*(a*c
osh(x)^5 - 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) - a)*sqrt(a + b)*log(((a + b)
*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)^
2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*
sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh
(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*co
sh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh
(x)^2)) - 4*sqrt(2)*((4*a + b)*cosh(x)^4 + 4*(4*a + b)*cosh(x)*sinh(x)^3 +
(4*a + b)*sinh(x)^4 - 2*(2*a + b)*cosh(x)^2 + 2*(3*(4*a + b)*cosh(x)^2 - 2*
a - b)*sinh(x)^2 + 4*((4*a + b)*cosh(x)^3 - (2*a + b)*cosh(x))*sinh(x) + 4*
a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*
cosh(x)*sinh(x) + sinh(x)^2)))/(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sin
h(x)^6 - 3*a*cosh(x)^4 + 3*(5*a*cosh(x)^2 - a)*sinh(x)^4 + 4*(5*a*cosh(x)^3
 - 3*a*cosh(x))*sinh(x)^3 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 - 6*a*cosh(x)^
2 + a)*sinh(x)^2 + 6*(a*cosh(x)^5 - 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) - a)
, -1/6*(3*(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 - 3*a*cosh(x)^
4 + 3*(5*a*cosh(x)^2 - a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 - 3*a*cosh(x))*sinh(
x)^3 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 - 6*a*cosh(x)^2 + a)*sinh(x)^2 + 6*
(a*cosh(x)^5 - 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) - a)*sqrt(-a - b)*arctan(
sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - a - b)*sqrt(-a -
 b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cos
h(x)*sinh(x) + sinh(x)^2))/((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh(x)*s
inh(x)^3 + (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^2)*cosh(x)^2 + (6*(a*b
+ b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*
(a*b + b^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x))) + 3*(a*cosh(
x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 - 3*a*cosh(x)^4 + 3*(5*a*cosh(x)
^2 - a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 - 3*a*cosh(x))*sinh(x)^3 + 3*a*cosh(x)
^2 + 3*(5*a*cosh(x)^4 - 6*a*cosh(x)^2 + a)*sinh(x)^2 + 6*(a*cosh(x)^5 - 2*a
*cosh(x)^3 + a*cosh(x))*sinh(x) - a)*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2
 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2
+ (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(
(a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a
- b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cos
h(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + 2*sqrt(2)*((4*a + b)*cosh(x)^
4 + 4*(4*a + b)*cosh(x)*sinh(x)^3 + (4*a + b)*sinh(x)^4 - 2*(2*a + b)*cosh(
x)^2 + 2*(3*(4*a + b)*cosh(x)^2 - 2*a - b)*sinh(x)^2 + 4*((4*a + b)*cosh(x)
^3 - (2*a + b)*cosh(x))*sinh(x) + 4*a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b
)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*cosh(
x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 - 3*a*cosh(x)^4 + 3*(5*a*cosh(x)
^2 - a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 - 3*a*cosh(x))*sinh(x)^3 + 3*a*cosh(x)
^2 + 3*(5*a*cosh(x)^4 - 6*a*cosh(x)^2 + a)*sinh(x)^2 + 6*(a*cosh(x)^5 - 2*a
*cosh(x)^3 + a*cosh(x))*sinh(x) - a)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4*(a+b*tanh(x)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4*(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.218 $\int \coth^5(x)\sqrt{a+b\tanh^2(x)}\,dx$

Optimal. Leaf size=121

$$-\frac{(8a^2+4ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{8a^{3/2}}+\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\frac{1}{4}\coth^4(x)\sqrt{a+b\tanh^2(x)}-\frac{(4a+b)}{}$$

```
[Out] -((8*a^2 + 4*a*b - b^2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]])/(8*a^(3/2))
 + Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - ((4*a + b)*Coth
[x]^2*Sqrt[a + b*Tanh[x]^2])/(8*a) - (Coth[x]^4*Sqrt[a + b*Tanh[x]^2])/4
```

Rubi [A] time = 0.21298, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {3670, 446, 99, 151, 156, 63, 208}

$$-\frac{(8a^2+4ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{8a^{3/2}}+\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\frac{1}{4}\coth^4(x)\sqrt{a+b\tanh^2(x)}-\frac{(4a+b)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^5*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] -((8*a^2 + 4*a*b - b^2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]])/(8*a^(3/2))
 + Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - ((4*a + b)*Coth
[x]^2*Sqrt[a + b*Tanh[x]^2])/(8*a) - (Coth[x]^4*Sqrt[a + b*Tanh[x]^2])/4
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \coth^5(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^5\left(1-x^2\right)}\,dx, x, \tanh(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{(1-x)x^3}\,dx, x, \tanh^2(x)\right)$$

$$= -\frac{1}{4}\coth^4(x)\sqrt{a+b\tanh^2(x)} + \frac{1}{4}\text{Subst}\left(\int \frac{\frac{1}{2}(4a+b)+\frac{3bx}{2}}{(1-x)x^2\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)$$

$$= -\frac{(4a+b)\coth^2(x)\sqrt{a+b\tanh^2(x)}}{8a} - \frac{1}{4}\coth^4(x)\sqrt{a+b\tanh^2(x)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{4}(-\cdots}{\cdots}\right.}{\cdots}$$

$$= -\frac{(4a+b)\coth^2(x)\sqrt{a+b\tanh^2(x)}}{8a} - \frac{1}{4}\coth^4(x)\sqrt{a+b\tanh^2(x)} - \frac{1}{2}(-a-b)\,\text{Sub}\cdots$$

$$= -\frac{(4a+b)\coth^2(x)\sqrt{a+b\tanh^2(x)}}{8a} - \frac{1}{4}\coth^4(x)\sqrt{a+b\tanh^2(x)} + \frac{(a+b)\,\text{Subst}\cdots}{\cdots}$$

$$= -\frac{\left(8a^2+4ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{8a^{3/2}} + \sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \frac{(4\cdots}{\cdots}$$

Mathematica [A] time = 0.577567, size = 111, normalized size = 0.92

$$\frac{\left(-8a^2-4ab+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)+\sqrt{a}\left(8a\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\coth^2(x)\sqrt{a+b\tanh^2(x)}\left(2a\coth^2(x\right.\right.}{8a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^5*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ((-8*a^2 - 4*a*b + b^2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]] + Sqrt[a]*(8
*a*Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - Coth[x]^2*(4*a
+ b + 2*a*Coth[x]^2)*Sqrt[a + b*Tanh[x]^2]))/(8*a^(3/2))
```

Maple [F] time = 0.153, size = 0, normalized size = 0.

$$\int (\coth(x))^5\sqrt{a+b(\tanh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5*(a+b*tanh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^5*(a+b*tanh(x)^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tanh(x)^2+a}\coth(x)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5*(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tanh(x)^2 + a)*coth(x)^5, x)
```

Fricas [B] time = 8.32815, size = 25865, normalized size = 213.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5*(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*(a^2*cosh(x)^8 + 8*a^2*cosh(x)*sinh(x)^7 + a^2*sinh(x)^8 - 4*a^2*c
osh(x)^6 + 4*(7*a^2*cosh(x)^2 - a^2)*sinh(x)^6 + 6*a^2*cosh(x)^4 + 8*(7*a^2
*cosh(x)^3 - 3*a^2*cosh(x))*sinh(x)^5 + 2*(35*a^2*cosh(x)^4 - 30*a^2*cosh(x
)^2 + 3*a^2)*sinh(x)^4 - 4*a^2*cosh(x)^2 + 8*(7*a^2*cosh(x)^5 - 10*a^2*cosh
(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + 4*(7*a^2*cosh(x)^6 - 15*a^2*cosh(x)^4 +
9*a^2*cosh(x)^2 - a^2)*sinh(x)^2 + a^2 + 8*(a^2*cosh(x)^7 - 3*a^2*cosh(x)^5
 + 3*a^2*cosh(x)^3 - a^2*cosh(x))*sinh(x))*sqrt(a + b)*log(((a^3 + a^2*b)*c
osh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*
```

```
(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*
sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh
(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(
x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x
)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3
 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^
3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 1
5*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b -
a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*
sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sin
h(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b
^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)
*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*
a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 +
(a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4
*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*
b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cos
h(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x
)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) - ((8*a^2
+ 4*a*b - b^2)*cosh(x)^8 + 8*(8*a^2 + 4*a*b - b^2)*cosh(x)*sinh(x)^7 + (8*a
^2 + 4*a*b - b^2)*sinh(x)^8 - 4*(8*a^2 + 4*a*b - b^2)*cosh(x)^6 + 4*(7*(8*a
^2 + 4*a*b - b^2)*cosh(x)^2 - 8*a^2 - 4*a*b + b^2)*sinh(x)^6 + 8*(7*(8*a^2
+ 4*a*b - b^2)*cosh(x)^3 - 3*(8*a^2 + 4*a*b - b^2)*cosh(x))*sinh(x)^5 + 6*(
8*a^2 + 4*a*b - b^2)*cosh(x)^4 + 2*(35*(8*a^2 + 4*a*b - b^2)*cosh(x)^4 - 30
*(8*a^2 + 4*a*b - b^2)*cosh(x)^2 + 24*a^2 + 12*a*b - 3*b^2)*sinh(x)^4 + 8*(
7*(8*a^2 + 4*a*b - b^2)*cosh(x)^5 - 10*(8*a^2 + 4*a*b - b^2)*cosh(x)^3 + 3*
(8*a^2 + 4*a*b - b^2)*cosh(x))*sinh(x)^3 - 4*(8*a^2 + 4*a*b - b^2)*cosh(x)^
2 + 4*(7*(8*a^2 + 4*a*b - b^2)*cosh(x)^6 - 15*(8*a^2 + 4*a*b - b^2)*cosh(x)
^4 + 9*(8*a^2 + 4*a*b - b^2)*cosh(x)^2 - 8*a^2 - 4*a*b + b^2)*sinh(x)^2 + 8
*a^2 + 4*a*b - b^2 + 8*((8*a^2 + 4*a*b - b^2)*cosh(x)^7 - 3*(8*a^2 + 4*a*b
- b^2)*cosh(x)^5 + 3*(8*a^2 + 4*a*b - b^2)*cosh(x)^3 - (8*a^2 + 4*a*b - b^2
)*cosh(x))*sinh(x))*sqrt(a)*log(-((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)
*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a - b)*cosh(x)^2 + 2*(3*(2*a + b)*c
osh(x)^2 + 2*a - b)*sinh(x)^2 + 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
sinh(x)^2 + 1)*sqrt(a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)
/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a + b)*cosh(x)^3 + (2
*a - b)*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh
(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x
))*sinh(x) + 1)) + 4*(a^2*cosh(x)^8 + 8*a^2*cosh(x)*sinh(x)^7 + a^2*sinh(x)
^8 - 4*a^2*cosh(x)^6 + 4*(7*a^2*cosh(x)^2 - a^2)*sinh(x)^6 + 6*a^2*cosh(x)^
4 + 8*(7*a^2*cosh(x)^3 - 3*a^2*cosh(x))*sinh(x)^5 + 2*(35*a^2*cosh(x)^4 - 3
0*a^2*cosh(x)^2 + 3*a^2)*sinh(x)^4 - 4*a^2*cosh(x)^2 + 8*(7*a^2*cosh(x)^5 -
 10*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + 4*(7*a^2*cosh(x)^6 - 15*a^2*
cosh(x)^4 + 9*a^2*cosh(x)^2 - a^2)*sinh(x)^2 + a^2 + 8*(a^2*cosh(x)^7 - 3*a
^2*cosh(x)^5 + 3*a^2*cosh(x)^3 - a^2*cosh(x))*sinh(x))*sqrt(a + b)*log(-((a
 + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cos
h(x)^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)
*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a +
b)*cosh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2)) - 2*sqrt(2)*((6*a^2 + a*b)*cosh(x)^6 + 6*(6*a^2 + a*b)*cosh(x)
*sinh(x)^5 + (6*a^2 + a*b)*sinh(x)^6 + (2*a^2 - a*b)*cosh(x)^4 + (15*(6*a^2
 + a*b)*cosh(x)^2 + 2*a^2 - a*b)*sinh(x)^4 + 4*(5*(6*a^2 + a*b)*cosh(x)^3 +
 (2*a^2 - a*b)*cosh(x))*sinh(x)^3 + (2*a^2 - a*b)*cosh(x)^2 + (15*(6*a^2 +
a*b)*cosh(x)^4 + 6*(2*a^2 - a*b)*cosh(x)^2 + 2*a^2 - a*b)*sinh(x)^2 + 6*a^2
 + a*b + 2*(3*(6*a^2 + a*b)*cosh(x)^5 + 2*(2*a^2 - a*b)*cosh(x)^3 + (2*a^2
- a*b)*cosh(x))*sinh(x))*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a -
b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^2*cosh(x)^8 + 8*a^2*cos
h(x)*sinh(x)^7 + a^2*sinh(x)^8 - 4*a^2*cosh(x)^6 + 4*(7*a^2*cosh(x)^2 - a^2
```

)*sinh(x)^6 + 6*a^2*cosh(x)^4 + 8*(7*a^2*cosh(x)^3 - 3*a^2*cosh(x))*sinh(x)
^5 + 2*(35*a^2*cosh(x)^4 - 30*a^2*cosh(x)^2 + 3*a^2)*sinh(x)^4 - 4*a^2*cosh
(x)^2 + 8*(7*a^2*cosh(x)^5 - 10*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 +
4*(7*a^2*cosh(x)^6 - 15*a^2*cosh(x)^4 + 9*a^2*cosh(x)^2 - a^2)*sinh(x)^2 +
a^2 + 8*(a^2*cosh(x)^7 - 3*a^2*cosh(x)^5 + 3*a^2*cosh(x)^3 - a^2*cosh(x))*s
inh(x)), 1/8*(((8*a^2 + 4*a*b - b^2)*cosh(x)^8 + 8*(8*a^2 + 4*a*b - b^2)*co
sh(x)*sinh(x)^7 + (8*a^2 + 4*a*b - b^2)*sinh(x)^8 - 4*(8*a^2 + 4*a*b - b^2)
cosh(x)^6 + 4(7*(8*a^2 + 4*a*b - b^2)*cosh(x)^2 - 8*a^2 - 4*a*b + b^2)*si
nh(x)^6 + 8*(7*(8*a^2 + 4*a*b - b^2)*cosh(x)^3 - 3*(8*a^2 + 4*a*b - b^2)*co
sh(x))*sinh(x)^5 + 6*(8*a^2 + 4*a*b - b^2)*cosh(x)^4 + 2*(35*(8*a^2 + 4*a*b
 - b^2)*cosh(x)^4 - 30*(8*a^2 + 4*a*b - b^2)*cosh(x)^2 + 24*a^2 + 12*a*b -
3*b^2)*sinh(x)^4 + 8*(7*(8*a^2 + 4*a*b - b^2)*cosh(x)^5 - 10*(8*a^2 + 4*a*b
 - b^2)*cosh(x)^3 + 3*(8*a^2 + 4*a*b - b^2)*cosh(x))*sinh(x)^3 - 4*(8*a^2 +
 4*a*b - b^2)*cosh(x)^2 + 4*(7*(8*a^2 + 4*a*b - b^2)*cosh(x)^6 - 15*(8*a^2
+ 4*a*b - b^2)*cosh(x)^4 + 9*(8*a^2 + 4*a*b - b^2)*cosh(x)^2 - 8*a^2 - 4*a*
b + b^2)*sinh(x)^2 + 8*a^2 + 4*a*b - b^2 + 8*((8*a^2 + 4*a*b - b^2)*cosh(x)
^7 - 3*(8*a^2 + 4*a*b - b^2)*cosh(x)^5 + 3*(8*a^2 + 4*a*b - b^2)*cosh(x)^3
- (8*a^2 + 4*a*b - b^2)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^
2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt(((a + b)*cosh(x)^2 + (
a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a
+ b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b
)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x
)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + 2*(a^2*cosh(x)^8 + 8*a^2*cosh(x)
*sinh(x)^7 + a^2*sinh(x)^8 - 4*a^2*cosh(x)^6 + 4*(7*a^2*cosh(x)^2 - a^2)*si
nh(x)^6 + 6*a^2*cosh(x)^4 + 8*(7*a^2*cosh(x)^3 - 3*a^2*cosh(x))*sinh(x)^5 +
 2*(35*a^2*cosh(x)^4 - 30*a^2*cosh(x)^2 + 3*a^2)*sinh(x)^4 - 4*a^2*cosh(x)^
2 + 8*(7*a^2*cosh(x)^5 - 10*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + 4*(7
*a^2*cosh(x)^6 - 15*a^2*cosh(x)^4 + 9*a^2*cosh(x)^2 - a^2)*sinh(x)^2 + a^2
+ 8*(a^2*cosh(x)^7 - 3*a^2*cosh(x)^5 + 3*a^2*cosh(x)^3 - a^2*cosh(x))*sinh(
x))*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh
(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 +
a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)
^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)
cosh(x)^4 + (70(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 +
30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 1
0*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh
(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2
 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3
*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqr
t(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(
x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cos
h(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*
a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a
^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sq
rt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 -
 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3
+ a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3
*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(
x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x
)*sinh(x)^5 + sinh(x)^6)) + 2*(a^2*cosh(x)^8 + 8*a^2*cosh(x)*sinh(x)^7 + a^
2*sinh(x)^8 - 4*a^2*cosh(x)^6 + 4*(7*a^2*cosh(x)^2 - a^2)*sinh(x)^6 + 6*a^2
cosh(x)^4 + 8(7*a^2*cosh(x)^3 - 3*a^2*cosh(x))*sinh(x)^5 + 2*(35*a^2*cosh
(x)^4 - 30*a^2*cosh(x)^2 + 3*a^2)*sinh(x)^4 - 4*a^2*cosh(x)^2 + 8*(7*a^2*co
sh(x)^5 - 10*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + 4*(7*a^2*cosh(x)^6
- 15*a^2*cosh(x)^4 + 9*a^2*cosh(x)^2 - a^2)*sinh(x)^2 + a^2 + 8*(a^2*cosh(x
)^7 - 3*a^2*cosh(x)^5 + 3*a^2*cosh(x)^3 - a^2*cosh(x))*sinh(x))*sqrt(a + b)
*log(-((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4
- 2*b*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^
2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2

```
+ (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) +
 4*((a + b)*cosh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*
sinh(x) + sinh(x)^2)) - sqrt(2)*((6*a^2 + a*b)*cosh(x)^6 + 6*(6*a^2 + a*b)*
cosh(x)*sinh(x)^5 + (6*a^2 + a*b)*sinh(x)^6 + (2*a^2 - a*b)*cosh(x)^4 + (15
*(6*a^2 + a*b)*cosh(x)^2 + 2*a^2 - a*b)*sinh(x)^4 + 4*(5*(6*a^2 + a*b)*cosh
(x)^3 + (2*a^2 - a*b)*cosh(x))*sinh(x)^3 + (2*a^2 - a*b)*cosh(x)^2 + (15*(6
*a^2 + a*b)*cosh(x)^4 + 6*(2*a^2 - a*b)*cosh(x)^2 + 2*a^2 - a*b)*sinh(x)^2
+ 6*a^2 + a*b + 2*(3*(6*a^2 + a*b)*cosh(x)^5 + 2*(2*a^2 - a*b)*cosh(x)^3 +
(2*a^2 - a*b)*cosh(x))*sinh(x))*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2
 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^2*cosh(x)^8 + 8*
a^2*cosh(x)*sinh(x)^7 + a^2*sinh(x)^8 - 4*a^2*cosh(x)^6 + 4*(7*a^2*cosh(x)^
2 - a^2)*sinh(x)^6 + 6*a^2*cosh(x)^4 + 8*(7*a^2*cosh(x)^3 - 3*a^2*cosh(x))*
sinh(x)^5 + 2*(35*a^2*cosh(x)^4 - 30*a^2*cosh(x)^2 + 3*a^2)*sinh(x)^4 - 4*a
^2*cosh(x)^2 + 8*(7*a^2*cosh(x)^5 - 10*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(
x)^3 + 4*(7*a^2*cosh(x)^6 - 15*a^2*cosh(x)^4 + 9*a^2*cosh(x)^2 - a^2)*sinh(
x)^2 + a^2 + 8*(a^2*cosh(x)^7 - 3*a^2*cosh(x)^5 + 3*a^2*cosh(x)^3 - a^2*cos
h(x))*sinh(x)), -1/16*(8*(a^2*cosh(x)^8 + 8*a^2*cosh(x)*sinh(x)^7 + a^2*sin
h(x)^8 - 4*a^2*cosh(x)^6 + 4*(7*a^2*cosh(x)^2 - a^2)*sinh(x)^6 + 6*a^2*cosh
(x)^4 + 8*(7*a^2*cosh(x)^3 - 3*a^2*cosh(x))*sinh(x)^5 + 2*(35*a^2*cosh(x)^4
 - 30*a^2*cosh(x)^2 + 3*a^2)*sinh(x)^4 - 4*a^2*cosh(x)^2 + 8*(7*a^2*cosh(x)
^5 - 10*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + 4*(7*a^2*cosh(x)^6 - 15*
a^2*cosh(x)^4 + 9*a^2*cosh(x)^2 - a^2)*sinh(x)^2 + a^2 + 8*(a^2*cosh(x)^7 -
 3*a^2*cosh(x)^5 + 3*a^2*cosh(x)^3 - a^2*cosh(x))*sinh(x))*sqrt(-a - b)*arc
tan(sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt(
-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2
*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(
x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(
a^2 + a*b)*cosh(x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2
*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh(x))) + 8*(a^2
*cosh(x)^8 + 8*a^2*cosh(x)*sinh(x)^7 + a^2*sinh(x)^8 - 4*a^2*cosh(x)^6 + 4*
(7*a^2*cosh(x)^2 - a^2)*sinh(x)^6 + 6*a^2*cosh(x)^4 + 8*(7*a^2*cosh(x)^3 -
3*a^2*cosh(x))*sinh(x)^5 + 2*(35*a^2*cosh(x)^4 - 30*a^2*cosh(x)^2 + 3*a^2)*
sinh(x)^4 - 4*a^2*cosh(x)^2 + 8*(7*a^2*cosh(x)^5 - 10*a^2*cosh(x)^3 + 3*a^2
*cosh(x))*sinh(x)^3 + 4*(7*a^2*cosh(x)^6 - 15*a^2*cosh(x)^4 + 9*a^2*cosh(x)
^2 - a^2)*sinh(x)^2 + a^2 + 8*(a^2*cosh(x)^7 - 3*a^2*cosh(x)^5 + 3*a^2*cosh
(x)^3 - a^2*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*co
sh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a +
b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)
*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*co
sh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3
+ (a - b)*cosh(x))*sinh(x) + a + b)) + ((8*a^2 + 4*a*b - b^2)*cosh(x)^8 + 8
*(8*a^2 + 4*a*b - b^2)*cosh(x)*sinh(x)^7 + (8*a^2 + 4*a*b - b^2)*sinh(x)^8
- 4*(8*a^2 + 4*a*b - b^2)*cosh(x)^6 + 4*(7*(8*a^2 + 4*a*b - b^2)*cosh(x)^2
- 8*a^2 - 4*a*b + b^2)*sinh(x)^6 + 8*(7*(8*a^2 + 4*a*b - b^2)*cosh(x)^3 - 3
*(8*a^2 + 4*a*b - b^2)*cosh(x))*sinh(x)^5 + 6*(8*a^2 + 4*a*b - b^2)*cosh(x)
^4 + 2*(35*(8*a^2 + 4*a*b - b^2)*cosh(x)^4 - 30*(8*a^2 + 4*a*b - b^2)*cosh(
x)^2 + 24*a^2 + 12*a*b - 3*b^2)*sinh(x)^4 + 8*(7*(8*a^2 + 4*a*b - b^2)*cosh
(x)^5 - 10*(8*a^2 + 4*a*b - b^2)*cosh(x)^3 + 3*(8*a^2 + 4*a*b - b^2)*cosh(x
))*sinh(x)^3 - 4*(8*a^2 + 4*a*b - b^2)*cosh(x)^2 + 4*(7*(8*a^2 + 4*a*b - b^
2)*cosh(x)^6 - 15*(8*a^2 + 4*a*b - b^2)*cosh(x)^4 + 9*(8*a^2 + 4*a*b - b^2)
*cosh(x)^2 - 8*a^2 - 4*a*b + b^2)*sinh(x)^2 + 8*a^2 + 4*a*b - b^2 + 8*((8*a
^2 + 4*a*b - b^2)*cosh(x)^7 - 3*(8*a^2 + 4*a*b - b^2)*cosh(x)^5 + 3*(8*a^2
+ 4*a*b - b^2)*cosh(x)^3 - (8*a^2 + 4*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a)*
log(-((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(
x)^4 + 2*(2*a - b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 + 2*a - b)*sinh(x)^
2 + 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt(
((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh
(x) + sinh(x)^2)) + 4*((2*a + b)*cosh(x)^3 + (2*a - b)*cosh(x))*sinh(x) + 2
*a + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*
```

sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)) + 2*sqrt(2)
*((6*a^2 + a*b)*cosh(x)^6 + 6*(6*a^2 + a*b)*cosh(x)*sinh(x)^5 + (6*a^2 + a*
b)*sinh(x)^6 + (2*a^2 - a*b)*cosh(x)^4 + (15*(6*a^2 + a*b)*cosh(x)^2 + 2*a^
2 - a*b)*sinh(x)^4 + 4*(5*(6*a^2 + a*b)*cosh(x)^3 + (2*a^2 - a*b)*cosh(x))*
sinh(x)^3 + (2*a^2 - a*b)*cosh(x)^2 + (15*(6*a^2 + a*b)*cosh(x)^4 + 6*(2*a^
2 - a*b)*cosh(x)^2 + 2*a^2 - a*b)*sinh(x)^2 + 6*a^2 + a*b + 2*(3*(6*a^2 + a
*b)*cosh(x)^5 + 2*(2*a^2 - a*b)*cosh(x)^3 + (2*a^2 - a*b)*cosh(x))*sinh(x))
*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x
)*sinh(x) + sinh(x)^2)))/(a^2*cosh(x)^8 + 8*a^2*cosh(x)*sinh(x)^7 + a^2*sin
h(x)^8 - 4*a^2*cosh(x)^6 + 4*(7*a^2*cosh(x)^2 - a^2)*sinh(x)^6 + 6*a^2*cosh
(x)^4 + 8*(7*a^2*cosh(x)^3 - 3*a^2*cosh(x))*sinh(x)^5 + 2*(35*a^2*cosh(x)^4
 - 30*a^2*cosh(x)^2 + 3*a^2)*sinh(x)^4 - 4*a^2*cosh(x)^2 + 8*(7*a^2*cosh(x)
^5 - 10*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + 4*(7*a^2*cosh(x)^6 - 15*
a^2*cosh(x)^4 + 9*a^2*cosh(x)^2 - a^2)*sinh(x)^2 + a^2 + 8*(a^2*cosh(x)^7 -
 3*a^2*cosh(x)^5 + 3*a^2*cosh(x)^3 - a^2*cosh(x))*sinh(x)), 1/8*(((8*a^2 +
4*a*b - b^2)*cosh(x)^8 + 8*(8*a^2 + 4*a*b - b^2)*cosh(x)*sinh(x)^7 + (8*a^2
 + 4*a*b - b^2)*sinh(x)^8 - 4*(8*a^2 + 4*a*b - b^2)*cosh(x)^6 + 4*(7*(8*a^2
 + 4*a*b - b^2)*cosh(x)^2 - 8*a^2 - 4*a*b + b^2)*sinh(x)^6 + 8*(7*(8*a^2 +
4*a*b - b^2)*cosh(x)^3 - 3*(8*a^2 + 4*a*b - b^2)*cosh(x))*sinh(x)^5 + 6*(8*
a^2 + 4*a*b - b^2)*cosh(x)^4 + 2*(35*(8*a^2 + 4*a*b - b^2)*cosh(x)^4 - 30*(
8*a^2 + 4*a*b - b^2)*cosh(x)^2 + 24*a^2 + 12*a*b - 3*b^2)*sinh(x)^4 + 8*(7*
(8*a^2 + 4*a*b - b^2)*cosh(x)^5 - 10*(8*a^2 + 4*a*b - b^2)*cosh(x)^3 + 3*(8
*a^2 + 4*a*b - b^2)*cosh(x))*sinh(x)^3 - 4*(8*a^2 + 4*a*b - b^2)*cosh(x)^2
+ 4*(7*(8*a^2 + 4*a*b - b^2)*cosh(x)^6 - 15*(8*a^2 + 4*a*b - b^2)*cosh(x)^4
 + 9*(8*a^2 + 4*a*b - b^2)*cosh(x)^2 - 8*a^2 - 4*a*b + b^2)*sinh(x)^2 + 8*a
^2 + 4*a*b - b^2 + 8*((8*a^2 + 4*a*b - b^2)*cosh(x)^7 - 3*(8*a^2 + 4*a*b -
b^2)*cosh(x)^5 + 3*(8*a^2 + 4*a*b - b^2)*cosh(x)^3 - (8*a^2 + 4*a*b - b^2)*
cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
sinh(x)^2 + 1)*sqrt(-a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b
)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a +
b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a +
b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*s
inh(x) + a + b)) - 4*(a^2*cosh(x)^8 + 8*a^2*cosh(x)*sinh(x)^7 + a^2*sinh(x)
^8 - 4*a^2*cosh(x)^6 + 4*(7*a^2*cosh(x)^2 - a^2)*sinh(x)^6 + 6*a^2*cosh(x)^
4 + 8*(7*a^2*cosh(x)^3 - 3*a^2*cosh(x))*sinh(x)^5 + 2*(35*a^2*cosh(x)^4 - 3
0*a^2*cosh(x)^2 + 3*a^2)*sinh(x)^4 - 4*a^2*cosh(x)^2 + 8*(7*a^2*cosh(x)^5 -
 10*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + 4*(7*a^2*cosh(x)^6 - 15*a^2*
cosh(x)^4 + 9*a^2*cosh(x)^2 - a^2)*sinh(x)^2 + a^2 + 8*(a^2*cosh(x)^7 - 3*a
^2*cosh(x)^5 + 3*a^2*cosh(x)^3 - a^2*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(
sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a -
 b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cos
h(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*s
inh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2
+ a*b)*cosh(x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*
(a^2 + a*b)*cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh(x))) - 4*(a^2*cos
h(x)^8 + 8*a^2*cosh(x)*sinh(x)^7 + a^2*sinh(x)^8 - 4*a^2*cosh(x)^6 + 4*(7*a
^2*cosh(x)^2 - a^2)*sinh(x)^6 + 6*a^2*cosh(x)^4 + 8*(7*a^2*cosh(x)^3 - 3*a^
2*cosh(x))*sinh(x)^5 + 2*(35*a^2*cosh(x)^4 - 30*a^2*cosh(x)^2 + 3*a^2)*sinh
(x)^4 - 4*a^2*cosh(x)^2 + 8*(7*a^2*cosh(x)^5 - 10*a^2*cosh(x)^3 + 3*a^2*cos
h(x))*sinh(x)^3 + 4*(7*a^2*cosh(x)^6 - 15*a^2*cosh(x)^4 + 9*a^2*cosh(x)^2 -
 a^2)*sinh(x)^2 + a^2 + 8*(a^2*cosh(x)^7 - 3*a^2*cosh(x)^5 + 3*a^2*cosh(x)^
3 - a^2*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x
)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*s
inh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cos
h(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x
)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a
 - b)*cosh(x))*sinh(x) + a + b)) - sqrt(2)*((6*a^2 + a*b)*cosh(x)^6 + 6*(6*
a^2 + a*b)*cosh(x)*sinh(x)^5 + (6*a^2 + a*b)*sinh(x)^6 + (2*a^2 - a*b)*cosh
(x)^4 + (15*(6*a^2 + a*b)*cosh(x)^2 + 2*a^2 - a*b)*sinh(x)^4 + 4*(5*(6*a^2

```
+ a*b)*cosh(x)^3 + (2*a^2 - a*b)*cosh(x))*sinh(x)^3 + (2*a^2 - a*b)*cosh(x)
^2 + (15*(6*a^2 + a*b)*cosh(x)^4 + 6*(2*a^2 - a*b)*cosh(x)^2 + 2*a^2 - a*b)
*sinh(x)^2 + 6*a^2 + a*b + 2*(3*(6*a^2 + a*b)*cosh(x)^5 + 2*(2*a^2 - a*b)*c
osh(x)^3 + (2*a^2 - a*b)*cosh(x))*sinh(x))*sqrt(((a + b)*cosh(x)^2 + (a + b
)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^2*cos
h(x)^8 + 8*a^2*cosh(x)*sinh(x)^7 + a^2*sinh(x)^8 - 4*a^2*cosh(x)^6 + 4*(7*a
^2*cosh(x)^2 - a^2)*sinh(x)^6 + 6*a^2*cosh(x)^4 + 8*(7*a^2*cosh(x)^3 - 3*a^
2*cosh(x))*sinh(x)^5 + 2*(35*a^2*cosh(x)^4 - 30*a^2*cosh(x)^2 + 3*a^2)*sinh
(x)^4 - 4*a^2*cosh(x)^2 + 8*(7*a^2*cosh(x)^5 - 10*a^2*cosh(x)^3 + 3*a^2*cos
h(x))*sinh(x)^3 + 4*(7*a^2*cosh(x)^6 - 15*a^2*cosh(x)^4 + 9*a^2*cosh(x)^2 -
 a^2)*sinh(x)^2 + a^2 + 8*(a^2*cosh(x)^7 - 3*a^2*cosh(x)^5 + 3*a^2*cosh(x)^
3 - a^2*cosh(x))*sinh(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**5*(a+b*tanh(x)**2)**(1/2),x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5*(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError
```

3.219 $\int \tanh^3(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=82

$$-\frac{\left(a+b\tanh^2(x)\right)^{5/2}}{5b}-\frac{1}{3}\left(a+b\tanh^2(x)\right)^{3/2}-(a+b)\sqrt{a+b\tanh^2(x)}+(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)$$

```
[Out] (a + b)^(3/2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (a + b)*Sqrt[a +
 b*Tanh[x]^2] - (a + b*Tanh[x]^2)^(3/2)/3 - (a + b*Tanh[x]^2)^(5/2)/(5*b)
```

Rubi [A] time = 0.152083, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3670, 446, 80, 50, 63, 208}

$$-\frac{\left(a+b\tanh^2(x)\right)^{5/2}}{5b}-\frac{1}{3}\left(a+b\tanh^2(x)\right)^{3/2}-(a+b)\sqrt{a+b\tanh^2(x)}+(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3*(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] (a + b)^(3/2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (a + b)*Sqrt[a +
 b*Tanh[x]^2] - (a + b*Tanh[x]^2)^(3/2)/3 - (a + b*Tanh[x]^2)^(5/2)/(5*b)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \tanh^3(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx=\text{Subst}\left(\int \frac{x^3\left(a+bx^2\right)^{3/2}}{1-x^2}\,dx,x,\tanh(x)\right)$$

$$=\frac{1}{2}\,\text{Subst}\left(\int \frac{x(a+bx)^{3/2}}{1-x}\,dx,x,\tanh^2(x)\right)$$

$$=-\frac{\left(a+b\tanh^2(x)\right)^{5/2}}{5b}+\frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{1-x}\,dx,x,\tanh^2(x)\right)$$

$$=-\frac{1}{3}\left(a+b\tanh^2(x)\right)^{3/2}-\frac{\left(a+b\tanh^2(x)\right)^{5/2}}{5b}+\frac{1}{2}(a+b)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1-x}\,dx,\right.$$

$$=-(a+b)\sqrt{a+b\tanh^2(x)}-\frac{1}{3}\left(a+b\tanh^2(x)\right)^{3/2}-\frac{\left(a+b\tanh^2(x)\right)^{5/2}}{5b}+\frac{1}{2}(a+$$

$$=-(a+b)\sqrt{a+b\tanh^2(x)}-\frac{1}{3}\left(a+b\tanh^2(x)\right)^{3/2}-\frac{\left(a+b\tanh^2(x)\right)^{5/2}}{5b}+\frac{(a+b}{}$$

$$=(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-(a+b)\sqrt{a+b\tanh^2(x)}-\frac{1}{3}\left(a+b\tanh\right.$$

Mathematica [A] time = 0.411811, size = 86, normalized size = 1.05

$$(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\frac{\sqrt{a+b\tanh^2(x)}\left(3a^2+b(6a+5b)\tanh^2(x)+20ab+3b^2\tanh^4(x)+15b^2\right)}{15b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3*(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] (a + b)^(3/2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (Sqrt[a + b*Tanh
[x]^2]*(3*a^2 + 20*a*b + 15*b^2 + b*(6*a + 5*b)*Tanh[x]^2 + 3*b^2*Tanh[x]^4
))/(15*b)
```

Maple [B] time = 0.027, size = 593, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3*(a+b*tanh(x)^2)^(3/2),x)
```

```
[Out] -1/5*(a+b*tanh(x)^2)^(5/2)/b-1/6*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(3/2
)+1/4*b*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)*tanh(x)+3/4*b^(1/2)*ln(
((1+tanh(x))*b-b)/b^(1/2)+((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))*a+1/
2*(a+b)^(1/2)*ln((2*a+2*b-2*(1+tanh(x))*b+2*(a+b)^(1/2)*((1+tanh(x))^2*b-2*
(1+tanh(x))*b+a+b)^(1/2))/(1+tanh(x)))*a-1/2*((1+tanh(x))^2*b-2*(1+tanh(x))
*b+a+b)^(1/2)*a+1/2*(a+b)^(1/2)*ln((2*a+2*b-2*(1+tanh(x))*b+2*(a+b)^(1/2)*(
(1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))/(1+tanh(x)))*b+1/2*b^(3/2)*ln((
(1+tanh(x))*b-b)/b^(1/2)+((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))-1/2*(
(1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)*b-1/6*((tanh(x)-1)^2*b+2*(tanh(x
)-1)*b+a+b)^(3/2)-1/4*b*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)*tanh(x)
-3/4*b^(1/2)*ln(((tanh(x)-1)*b+b)/b^(1/2)+((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+
a+b)^(1/2))*a+1/2*ln((2*a+2*b+2*(tanh(x)-1)*b+2*(a+b)^(1/2)*((tanh(x)-1)^2*
b+2*(tanh(x)-1)*b+a+b)^(1/2))/(tanh(x)-1))*(a+b)^(1/2)*a-1/2*((tanh(x)-1)^2
*b+2*(tanh(x)-1)*b+a+b)^(1/2)*a+1/2*ln((2*a+2*b+2*(tanh(x)-1)*b+2*(a+b)^(1/
2)*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))/(tanh(x)-1))*(a+b)^(1/2)*b-
1/2*b^(3/2)*ln(((tanh(x)-1)*b+b)/b^(1/2)+((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a
+b)^(1/2))-1/2*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tanh(x)^2 + a\right)^{\frac{3}{2}} \tanh(x)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3*(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tanh(x)^2 + a)^(3/2)*tanh(x)^3, x)
```

Fricas [B] time = 7.37412, size = 13874, normalized size = 169.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3*(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/60*(15*((a*b + b^2)*cosh(x)^10 + 10*(a*b + b^2)*cosh(x)*sinh(x)^9 + (a*b
 + b^2)*sinh(x)^10 + 5*(a*b + b^2)*cosh(x)^8 + 5*(9*(a*b + b^2)*cosh(x)^2 +
 a*b + b^2)*sinh(x)^8 + 40*(3*(a*b + b^2)*cosh(x)^3 + (a*b + b^2)*cosh(x))*
sinh(x)^7 + 10*(a*b + b^2)*cosh(x)^6 + 10*(21*(a*b + b^2)*cosh(x)^4 + 14*(a
*b + b^2)*cosh(x)^2 + a*b + b^2)*sinh(x)^6 + 4*(63*(a*b + b^2)*cosh(x)^5 +
70*(a*b + b^2)*cosh(x)^3 + 15*(a*b + b^2)*cosh(x))*sinh(x)^5 + 10*(a*b + b^
2)*cosh(x)^4 + 10*(21*(a*b + b^2)*cosh(x)^6 + 35*(a*b + b^2)*cosh(x)^4 + 15
*(a*b + b^2)*cosh(x)^2 + a*b + b^2)*sinh(x)^4 + 40*(3*(a*b + b^2)*cosh(x)^7
 + 7*(a*b + b^2)*cosh(x)^5 + 5*(a*b + b^2)*cosh(x)^3 + (a*b + b^2)*cosh(x))
```

```
*sinh(x)^3 + 5*(a*b + b^2)*cosh(x)^2 + 5*(9*(a*b + b^2)*cosh(x)^8 + 28*(a*b
 + b^2)*cosh(x)^6 + 30*(a*b + b^2)*cosh(x)^4 + 12*(a*b + b^2)*cosh(x)^2 + a
*b + b^2)*sinh(x)^2 + a*b + b^2 + 10*((a*b + b^2)*cosh(x)^9 + 4*(a*b + b^2)
*cosh(x)^7 + 6*(a*b + b^2)*cosh(x)^5 + 4*(a*b + b^2)*cosh(x)^3 + (a*b + b^2
)*cosh(x))*sinh(x))*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2
*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)
^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3
+ a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*
b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b
- a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*
b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^
3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b
- b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(
x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)
*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)
^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(
x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2
*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*
b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh
(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a -
 b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(
x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x
)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*si
nh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh
(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 15*((a*b + b^2)*cosh(x)^10 + 10
*(a*b + b^2)*cosh(x)*sinh(x)^9 + (a*b + b^2)*sinh(x)^10 + 5*(a*b + b^2)*cos
h(x)^8 + 5*(9*(a*b + b^2)*cosh(x)^2 + a*b + b^2)*sinh(x)^8 + 40*(3*(a*b + b
^2)*cosh(x)^3 + (a*b + b^2)*cosh(x))*sinh(x)^7 + 10*(a*b + b^2)*cosh(x)^6 +
 10*(21*(a*b + b^2)*cosh(x)^4 + 14*(a*b + b^2)*cosh(x)^2 + a*b + b^2)*sinh(
x)^6 + 4*(63*(a*b + b^2)*cosh(x)^5 + 70*(a*b + b^2)*cosh(x)^3 + 15*(a*b + b
^2)*cosh(x))*sinh(x)^5 + 10*(a*b + b^2)*cosh(x)^4 + 10*(21*(a*b + b^2)*cosh
(x)^6 + 35*(a*b + b^2)*cosh(x)^4 + 15*(a*b + b^2)*cosh(x)^2 + a*b + b^2)*si
nh(x)^4 + 40*(3*(a*b + b^2)*cosh(x)^7 + 7*(a*b + b^2)*cosh(x)^5 + 5*(a*b +
b^2)*cosh(x)^3 + (a*b + b^2)*cosh(x))*sinh(x)^3 + 5*(a*b + b^2)*cosh(x)^2 +
 5*(9*(a*b + b^2)*cosh(x)^8 + 28*(a*b + b^2)*cosh(x)^6 + 30*(a*b + b^2)*cos
h(x)^4 + 12*(a*b + b^2)*cosh(x)^2 + a*b + b^2)*sinh(x)^2 + a*b + b^2 + 10*(
(a*b + b^2)*cosh(x)^9 + 4*(a*b + b^2)*cosh(x)^7 + 6*(a*b + b^2)*cosh(x)^5 +
 4*(a*b + b^2)*cosh(x)^3 + (a*b + b^2)*cosh(x))*sinh(x))*sqrt(a + b)*log(-(
(a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*c
osh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*c
osh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a +
b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a
+ b)*cosh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x)
 + sinh(x)^2)) - 4*sqrt(2)*((3*a^2 + 26*a*b + 23*b^2)*cosh(x)^8 + 8*(3*a^2
+ 26*a*b + 23*b^2)*cosh(x)*sinh(x)^7 + (3*a^2 + 26*a*b + 23*b^2)*sinh(x)^8
+ 4*(3*a^2 + 20*a*b + 12*b^2)*cosh(x)^6 + 4*(7*(3*a^2 + 26*a*b + 23*b^2)*co
sh(x)^2 + 3*a^2 + 20*a*b + 12*b^2)*sinh(x)^6 + 8*(7*(3*a^2 + 26*a*b + 23*b^
2)*cosh(x)^3 + 3*(3*a^2 + 20*a*b + 12*b^2)*cosh(x))*sinh(x)^5 + 2*(9*a^2 +
54*a*b + 49*b^2)*cosh(x)^4 + 2*(35*(3*a^2 + 26*a*b + 23*b^2)*cosh(x)^4 + 30
*(3*a^2 + 20*a*b + 12*b^2)*cosh(x)^2 + 9*a^2 + 54*a*b + 49*b^2)*sinh(x)^4 +
 8*(7*(3*a^2 + 26*a*b + 23*b^2)*cosh(x)^5 + 10*(3*a^2 + 20*a*b + 12*b^2)*co
sh(x)^3 + (9*a^2 + 54*a*b + 49*b^2)*cosh(x))*sinh(x)^3 + 4*(3*a^2 + 20*a*b
+ 12*b^2)*cosh(x)^2 + 4*(7*(3*a^2 + 26*a*b + 23*b^2)*cosh(x)^6 + 15*(3*a^2
+ 20*a*b + 12*b^2)*cosh(x)^4 + 3*(9*a^2 + 54*a*b + 49*b^2)*cosh(x)^2 + 3*a^
2 + 20*a*b + 12*b^2)*sinh(x)^2 + 3*a^2 + 26*a*b + 23*b^2 + 8*((3*a^2 + 26*a
*b + 23*b^2)*cosh(x)^7 + 3*(3*a^2 + 20*a*b + 12*b^2)*cosh(x)^5 + (9*a^2 + 5
4*a*b + 49*b^2)*cosh(x)^3 + (3*a^2 + 20*a*b + 12*b^2)*cosh(x))*sinh(x))*sqr
t(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*si
nh(x) + sinh(x)^2)))/(b*cosh(x)^10 + 10*b*cosh(x)*sinh(x)^9 + b*sinh(x)^10
```

```
+ 5*b*cosh(x)^8 + 5*(9*b*cosh(x)^2 + b)*sinh(x)^8 + 40*(3*b*cosh(x)^3 + b*c
osh(x))*sinh(x)^7 + 10*b*cosh(x)^6 + 10*(21*b*cosh(x)^4 + 14*b*cosh(x)^2 +
b)*sinh(x)^6 + 4*(63*b*cosh(x)^5 + 70*b*cosh(x)^3 + 15*b*cosh(x))*sinh(x)^5
 + 10*b*cosh(x)^4 + 10*(21*b*cosh(x)^6 + 35*b*cosh(x)^4 + 15*b*cosh(x)^2 +
b)*sinh(x)^4 + 40*(3*b*cosh(x)^7 + 7*b*cosh(x)^5 + 5*b*cosh(x)^3 + b*cosh(x
))*sinh(x)^3 + 5*b*cosh(x)^2 + 5*(9*b*cosh(x)^8 + 28*b*cosh(x)^6 + 30*b*cos
h(x)^4 + 12*b*cosh(x)^2 + b)*sinh(x)^2 + 10*(b*cosh(x)^9 + 4*b*cosh(x)^7 +
6*b*cosh(x)^5 + 4*b*cosh(x)^3 + b*cosh(x))*sinh(x) + b), -1/30*(15*((a*b +
b^2)*cosh(x)^10 + 10*(a*b + b^2)*cosh(x)*sinh(x)^9 + (a*b + b^2)*sinh(x)^10
 + 5*(a*b + b^2)*cosh(x)^8 + 5*(9*(a*b + b^2)*cosh(x)^2 + a*b + b^2)*sinh(x
)^8 + 40*(3*(a*b + b^2)*cosh(x)^3 + (a*b + b^2)*cosh(x))*sinh(x)^7 + 10*(a*
b + b^2)*cosh(x)^6 + 10*(21*(a*b + b^2)*cosh(x)^4 + 14*(a*b + b^2)*cosh(x)^
2 + a*b + b^2)*sinh(x)^6 + 4*(63*(a*b + b^2)*cosh(x)^5 + 70*(a*b + b^2)*cos
h(x)^3 + 15*(a*b + b^2)*cosh(x))*sinh(x)^5 + 10*(a*b + b^2)*cosh(x)^4 + 10*
(21*(a*b + b^2)*cosh(x)^6 + 35*(a*b + b^2)*cosh(x)^4 + 15*(a*b + b^2)*cosh(
x)^2 + a*b + b^2)*sinh(x)^4 + 40*(3*(a*b + b^2)*cosh(x)^7 + 7*(a*b + b^2)*c
osh(x)^5 + 5*(a*b + b^2)*cosh(x)^3 + (a*b + b^2)*cosh(x))*sinh(x)^3 + 5*(a*
b + b^2)*cosh(x)^2 + 5*(9*(a*b + b^2)*cosh(x)^8 + 28*(a*b + b^2)*cosh(x)^6
+ 30*(a*b + b^2)*cosh(x)^4 + 12*(a*b + b^2)*cosh(x)^2 + a*b + b^2)*sinh(x)^
2 + a*b + b^2 + 10*((a*b + b^2)*cosh(x)^9 + 4*(a*b + b^2)*cosh(x)^7 + 6*(a*
b + b^2)*cosh(x)^5 + 4*(a*b + b^2)*cosh(x)^3 + (a*b + b^2)*cosh(x))*sinh(x)
)*sqrt(-a - b)*arctan(sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x
)^2 + a + b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a -
 b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4
*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2
)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2
 + 2*a*b + b^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*
sinh(x))) + 15*((a*b + b^2)*cosh(x)^10 + 10*(a*b + b^2)*cosh(x)*sinh(x)^9 +
 (a*b + b^2)*sinh(x)^10 + 5*(a*b + b^2)*cosh(x)^8 + 5*(9*(a*b + b^2)*cosh(x
)^2 + a*b + b^2)*sinh(x)^8 + 40*(3*(a*b + b^2)*cosh(x)^3 + (a*b + b^2)*cosh
(x))*sinh(x)^7 + 10*(a*b + b^2)*cosh(x)^6 + 10*(21*(a*b + b^2)*cosh(x)^4 +
14*(a*b + b^2)*cosh(x)^2 + a*b + b^2)*sinh(x)^6 + 4*(63*(a*b + b^2)*cosh(x)
^5 + 70*(a*b + b^2)*cosh(x)^3 + 15*(a*b + b^2)*cosh(x))*sinh(x)^5 + 10*(a*b
 + b^2)*cosh(x)^4 + 10*(21*(a*b + b^2)*cosh(x)^6 + 35*(a*b + b^2)*cosh(x)^4
 + 15*(a*b + b^2)*cosh(x)^2 + a*b + b^2)*sinh(x)^4 + 40*(3*(a*b + b^2)*cosh
(x)^7 + 7*(a*b + b^2)*cosh(x)^5 + 5*(a*b + b^2)*cosh(x)^3 + (a*b + b^2)*cos
h(x))*sinh(x)^3 + 5*(a*b + b^2)*cosh(x)^2 + 5*(9*(a*b + b^2)*cosh(x)^8 + 28
*(a*b + b^2)*cosh(x)^6 + 30*(a*b + b^2)*cosh(x)^4 + 12*(a*b + b^2)*cosh(x)^
2 + a*b + b^2)*sinh(x)^2 + a*b + b^2 + 10*((a*b + b^2)*cosh(x)^9 + 4*(a*b +
 b^2)*cosh(x)^7 + 6*(a*b + b^2)*cosh(x)^5 + 4*(a*b + b^2)*cosh(x)^3 + (a*b
+ b^2)*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)
*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*si
nh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh
(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)
^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a
- b)*cosh(x))*sinh(x) + a + b)) + 2*sqrt(2)*((3*a^2 + 26*a*b + 23*b^2)*cosh
(x)^8 + 8*(3*a^2 + 26*a*b + 23*b^2)*cosh(x)*sinh(x)^7 + (3*a^2 + 26*a*b + 2
3*b^2)*sinh(x)^8 + 4*(3*a^2 + 20*a*b + 12*b^2)*cosh(x)^6 + 4*(7*(3*a^2 + 26
*a*b + 23*b^2)*cosh(x)^2 + 3*a^2 + 20*a*b + 12*b^2)*sinh(x)^6 + 8*(7*(3*a^2
 + 26*a*b + 23*b^2)*cosh(x)^3 + 3*(3*a^2 + 20*a*b + 12*b^2)*cosh(x))*sinh(x
)^5 + 2*(9*a^2 + 54*a*b + 49*b^2)*cosh(x)^4 + 2*(35*(3*a^2 + 26*a*b + 23*b^
2)*cosh(x)^4 + 30*(3*a^2 + 20*a*b + 12*b^2)*cosh(x)^2 + 9*a^2 + 54*a*b + 49
*b^2)*sinh(x)^4 + 8*(7*(3*a^2 + 26*a*b + 23*b^2)*cosh(x)^5 + 10*(3*a^2 + 20
*a*b + 12*b^2)*cosh(x)^3 + (9*a^2 + 54*a*b + 49*b^2)*cosh(x))*sinh(x)^3 + 4
*(3*a^2 + 20*a*b + 12*b^2)*cosh(x)^2 + 4*(7*(3*a^2 + 26*a*b + 23*b^2)*cosh(
x)^6 + 15*(3*a^2 + 20*a*b + 12*b^2)*cosh(x)^4 + 3*(9*a^2 + 54*a*b + 49*b^2)
*cosh(x)^2 + 3*a^2 + 20*a*b + 12*b^2)*sinh(x)^2 + 3*a^2 + 26*a*b + 23*b^2 +
 8*((3*a^2 + 26*a*b + 23*b^2)*cosh(x)^7 + 3*(3*a^2 + 20*a*b + 12*b^2)*cosh(
x)^5 + (9*a^2 + 54*a*b + 49*b^2)*cosh(x)^3 + (3*a^2 + 20*a*b + 12*b^2)*cosh
```

```
(x))*sinh(x))*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)^10 + 10*b*cosh(x)*sinh(x)^
9 + b*sinh(x)^10 + 5*b*cosh(x)^8 + 5*(9*b*cosh(x)^2 + b)*sinh(x)^8 + 40*(3*
b*cosh(x)^3 + b*cosh(x))*sinh(x)^7 + 10*b*cosh(x)^6 + 10*(21*b*cosh(x)^4 +
14*b*cosh(x)^2 + b)*sinh(x)^6 + 4*(63*b*cosh(x)^5 + 70*b*cosh(x)^3 + 15*b*c
osh(x))*sinh(x)^5 + 10*b*cosh(x)^4 + 10*(21*b*cosh(x)^6 + 35*b*cosh(x)^4 +
15*b*cosh(x)^2 + b)*sinh(x)^4 + 40*(3*b*cosh(x)^7 + 7*b*cosh(x)^5 + 5*b*cos
h(x)^3 + b*cosh(x))*sinh(x)^3 + 5*b*cosh(x)^2 + 5*(9*b*cosh(x)^8 + 28*b*cos
h(x)^6 + 30*b*cosh(x)^4 + 12*b*cosh(x)^2 + b)*sinh(x)^2 + 10*(b*cosh(x)^9 +
 4*b*cosh(x)^7 + 6*b*cosh(x)^5 + 4*b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)]
```

Sympy [B] time = 31.2633, size = 175, normalized size = 2.13

$$-\frac{2a\left(\frac{b^2\sqrt{a+b\tanh^2(x)}}{2}+\frac{b^2(a+b)\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}}+\frac{b\left(a+b\tanh^2(x)\right)^{\frac{3}{2}}}{6}\right)}{b^2}-\frac{2\left(\frac{b^3\sqrt{a+b\tanh^2(x)}}{2}+\frac{b^3(a+b)\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}}+\frac{b(a+\ldots}{}\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3*(a+b*tanh(x)**2)**(3/2),x)
```

```
[Out] -2*a*(b**2*sqrt(a + b*tanh(x)**2)/2 + b**2*(a + b)*atan(sqrt(a + b*tanh(x)*
*2)/sqrt(-a - b))/(2*sqrt(-a - b)) + b*(a + b*tanh(x)**2)**(3/2)/6)/b**2 -
2*(b**3*sqrt(a + b*tanh(x)**2)/2 + b**3*(a + b)*atan(sqrt(a + b*tanh(x)**2)
/sqrt(-a - b))/(2*sqrt(-a - b)) + b*(a + b*tanh(x)**2)**(5/2)/10 + (a + b*t
anh(x)**2)**(3/2)*(-a*b/2 + b**2/2)/3)/b**2
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3*(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.220 $\int \tanh^2(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=123

$$-\frac{\left(3a^2+12ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{8\sqrt{b}}-\frac{1}{4}b\tanh^3(x)\sqrt{a+b\tanh^2(x)}-\frac{1}{8}(5a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)}+(a$$

```
[Out] -((3*a^2 + 12*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]
)/(8*Sqrt[b]) + (a + b)^(3/2)*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh
[x]^2]] - ((5*a + 4*b)*Tanh[x]*Sqrt[a + b*Tanh[x]^2])/8 - (b*Tanh[x]^3*Sqrt
[a + b*Tanh[x]^2])/4
```

Rubi [A] time = 0.244414, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {3670, 477, 582, 523, 217, 206, 377}

$$-\frac{\left(3a^2+12ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{8\sqrt{b}}-\frac{1}{4}b\tanh^3(x)\sqrt{a+b\tanh^2(x)}-\frac{1}{8}(5a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)}+(a$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2*(a + b*Tanh[x]^2)^(3/2),x]
```


```
[Out] -((3*a^2 + 12*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]
)/(8*Sqrt[b]) + (a + b)^(3/2)*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh
[x]^2]] - ((5*a + 4*b)*Tanh[x]*Sqrt[a + b*Tanh[x]^2])/8 - (b*Tanh[x]^3*Sqrt
[a + b*Tanh[x]^2])/4
```


Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
```

```
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\int \tanh^2(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx = \text{Subst}\left(\int \frac{x^2\left(a+bx^2\right)^{3/2}}{1-x^2}\,dx, x, \tanh(x)\right)$$

$$= -\frac{1}{4}b\tanh^3(x)\sqrt{a+b\tanh^2(x)} - \frac{1}{4}\text{Subst}\left(\int \frac{x^2\left(-a(4a+3b)-b(5a+4b)x^2\right)}{\left(1-x^2\right)\sqrt{a+bx^2}}\right.$$

$$= -\frac{1}{8}(5a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)} - \frac{1}{4}b\tanh^3(x)\sqrt{a+b\tanh^2(x)} - \frac{\text{Subst}}{}$$

$$= -\frac{1}{8}(5a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)} - \frac{1}{4}b\tanh^3(x)\sqrt{a+b\tanh^2(x)} + (a+b)$$

$$= -\frac{1}{8}(5a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)} - \frac{1}{4}b\tanh^3(x)\sqrt{a+b\tanh^2(x)} + (a+b)$$

$$= -\frac{\left(3a^2+12ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{8\sqrt{b}} + (a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(}{\sqrt{a+b\tanh^2(}}\right.$$

Mathematica [C] time = 6.20248, size = 584, normalized size = 4.75

$$\sqrt{\frac{a\cosh(2x)+a+b\cosh(2x)-b}{\cosh(2x)+1}}\left(\frac{1}{8}\text{sech}(x)(-5a\sinh(x)-6b\sinh(x))+\frac{1}{4}b\tanh(x)\text{sech}^2(x)\right)+\frac{1}{4}\left(-\frac{b\left(a^2-4ab-4b\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2*(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] (-((b*(a^2 - 4*a*b - 4*b^2)*Sqrt[(a - b + (a + b)*Cosh[2*x])/(1 + Cosh[2*x]
)]*Sqrt[-((a*Coth[x]^2)/b)]*Sqrt[-((a*(1 + Cosh[2*x])*Csch[x]^2)/b)]*Sqrt[(
(a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*Csch[2*x]*EllipticF[ArcSin[Sqrt[(
(a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1]*Sinh[x]^4)/(a*(a - b
 + (a + b)*Cosh[2*x]))) - ((4*I)*b*(4*a^2 + 8*a*b + 4*b^2)*Sqrt[1 + Cosh[2*
x]]*Sqrt[(a - b + (a + b)*Cosh[2*x])/(1 + Cosh[2*x])]*(((-I/4)*Sqrt[-((a*Co
th[x]^2)/b)]*Sqrt[-((a*(1 + Cosh[2*x])*Csch[x]^2)/b)]*Sqrt[((a - b + (a + b
)*Cosh[2*x])*Csch[x]^2)/b]*Csch[2*x]*EllipticF[ArcSin[Sqrt[((a - b + (a + b
)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1]*Sinh[x]^4)/(a*Sqrt[1 + Cosh[2*x]]*S
qrt[a - b + (a + b)*Cosh[2*x]]) + ((I/2)*Sqrt[-((a*Coth[x]^2)/b)]*Sqrt[-((a
*(1 + Cosh[2*x])*Csch[x]^2)/b)]*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2
)/b]*Csch[2*x]*EllipticPi[b/(a + b), ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x
])*Csch[x]^2)/b]/Sqrt[2]], 1]*Sinh[x]^4)/((a + b)*Sqrt[1 + Cosh[2*x]]*Sqrt[
a - b + (a + b)*Cosh[2*x]])))/Sqrt[a - b + (a + b)*Cosh[2*x]])/4 + Sqrt[(a
- b + a*Cosh[2*x] + b*Cosh[2*x])/(1 + Cosh[2*x])]*((Sech[x]*(-5*a*Sinh[x] -
 6*b*Sinh[x]))/8 + (b*Sech[x]^2*Tanh[x])/4)
```

Maple [B] time = 0.022, size = 633, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2*(a+b*tanh(x)^2)^(3/2),x)
```

```
[Out] -1/4*tanh(x)*(a+b*tanh(x)^2)^(3/2)-3/8*a*tanh(x)*(a+b*tanh(x)^2)^(1/2)-3/8*
a^2/b^(1/2)*ln(tanh(x)*b^(1/2)+(a+b*tanh(x)^2)^(1/2))+1/6*((1+tanh(x))^2*b-
2*(1+tanh(x))*b+a+b)^(3/2)-1/4*b*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2
)*tanh(x)-3/4*b^(1/2)*ln(((1+tanh(x))*b-b)/b^(1/2)+((1+tanh(x))^2*b-2*(1+ta
nh(x))*b+a+b)^(1/2))*a-1/2*(a+b)^(1/2)*ln((2*a+2*b-2*(1+tanh(x))*b+2*(a+b)^
(1/2)*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))/(1+tanh(x)))*a+1/2*((1+t
anh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)*a-1/2*b^(3/2)*ln(((1+tanh(x))*b-b)/b
^(1/2)+((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))-1/2*(a+b)^(1/2)*ln((2*a
+2*b-2*(1+tanh(x))*b+2*(a+b)^(1/2)*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1
/2))/(1+tanh(x)))*b+1/2*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)*b-1/6*(
(tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(3/2)-1/4*b*((tanh(x)-1)^2*b+2*(tanh(x
)-1)*b+a+b)^(1/2)*tanh(x)-3/4*b^(1/2)*ln(((tanh(x)-1)*b+b)/b^(1/2)+((tanh(x
```

```
)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))*a+1/2*ln((2*a+2*b+2*(tanh(x)-1)*b+2*(a
+b)^(1/2)*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))/(tanh(x)-1))*(a+b)^(
1/2)*a-1/2*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)*a-1/2*b^(3/2)*ln(((t
anh(x)-1)*b+b)/b^(1/2)+((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))+1/2*ln(
(2*a+2*b+2*(tanh(x)-1)*b+2*(a+b)^(1/2)*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b
)^(1/2))/(tanh(x)-1))*(a+b)^(1/2)*b-1/2*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+
b)^(1/2)*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tanh(x)^2 + a\right)^{\frac{3}{2}} \tanh(x)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2*(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tanh(x)^2 + a)^(3/2)*tanh(x)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2*(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tanh^2(x)\right)^{\frac{3}{2}} \tanh^2(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2*(a+b*tanh(x)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tanh(x)**2)**(3/2)*tanh(x)**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2*(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.221 $\int \tanh(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=63

$$(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-(a+b)\sqrt{a+b\tanh^2(x)}-\frac{1}{3}\left(a+b\tanh^2(x)\right)^{3/2}$$

```
[Out] (a + b)^(3/2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (a + b)*Sqrt[a +
 b*Tanh[x]^2] - (a + b*Tanh[x]^2)^(3/2)/3
```

Rubi [A] time = 0.101468, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3670, 444, 50, 63, 208}

$$(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-(a+b)\sqrt{a+b\tanh^2(x)}-\frac{1}{3}\left(a+b\tanh^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]*(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] (a + b)^(3/2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (a + b)*Sqrt[a +
 b*Tanh[x]^2] - (a + b*Tanh[x]^2)^(3/2)/3
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \tanh(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx &= \text{Subst}\left(\int \frac{x\left(a+bx^2\right)^{3/2}}{1-x^2}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{1-x}\,dx, x, \tanh^2(x)\right)\\
&= -\frac{1}{3}\left(a+b\tanh^2(x)\right)^{3/2}+\frac{1}{2}(a+b)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1-x}\,dx, x, \tanh^2(x)\right)\\
&= -(a+b)\sqrt{a+b\tanh^2(x)}-\frac{1}{3}\left(a+b\tanh^2(x)\right)^{3/2}+\frac{1}{2}(a+b)^2\,\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{\ldots}}\right.\\
&= -(a+b)\sqrt{a+b\tanh^2(x)}-\frac{1}{3}\left(a+b\tanh^2(x)\right)^{3/2}+\frac{(a+b)^2\,\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx,\ldots\right.}{b}\\
&= (a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-(a+b)\sqrt{a+b\tanh^2(x)}-\frac{1}{3}\left(a+b\tanh^2\ldots\right.
\end{aligned}$$

Mathematica [A] time = 0.159523, size = 59, normalized size = 0.94

$$(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\frac{1}{3}\sqrt{a+b\tanh^2(x)}\left(4a+b\tanh^2(x)+3b\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]*(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] (a + b)^(3/2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (Sqrt[a + b*Tanh
[x]^2]*(4*a + 3*b + b*Tanh[x]^2))/3
```

Maple [B] time = 0.023, size = 578, normalized size = 9.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)*(a+b*tanh(x)^2)^(3/2),x)
```

```
[Out] -1/6*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(3/2)+1/4*b*((1+tanh(x))^2*b-2*(
1+tanh(x))*b+a+b)^(1/2)*tanh(x)+3/4*b^(1/2)*ln(((1+tanh(x))*b-b)/b^(1/2)+((
1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))*a+1/2*(a+b)^(1/2)*ln((2*a+2*b-2*
(1+tanh(x))*b+2*(a+b)^(1/2)*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))/(1
```

```
+tanh(x)))*a-1/2*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)*a+1/2*(a+b)^(1
/2)*ln((2*a+2*b-2*(1+tanh(x))*b+2*(a+b)^(1/2)*((1+tanh(x))^2*b-2*(1+tanh(x)
)*b+a+b)^(1/2))/(1+tanh(x)))*b+1/2*b^(3/2)*ln(((1+tanh(x))*b-b)/b^(1/2)+((1
+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))-1/2*((1+tanh(x))^2*b-2*(1+tanh(x)
)*b+a+b)^(1/2)*b-1/6*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(3/2)-1/4*b*((ta
nh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)*tanh(x)-3/4*b^(1/2)*ln(((tanh(x)-1)
*b+b)/b^(1/2)+((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))*a+1/2*ln((2*a+2*
b+2*(tanh(x)-1)*b+2*(a+b)^(1/2)*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)
)/(tanh(x)-1))*(a+b)^(1/2)*a-1/2*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2
)*a+1/2*ln((2*a+2*b+2*(tanh(x)-1)*b+2*(a+b)^(1/2)*((tanh(x)-1)^2*b+2*(tanh(
x)-1)*b+a+b)^(1/2))/(tanh(x)-1))*(a+b)^(1/2)*b-1/2*b^(3/2)*ln(((tanh(x)-1)*
b+b)/b^(1/2)+((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))-1/2*((tanh(x)-1)^
2*b+2*(tanh(x)-1)*b+a+b)^(1/2)*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tanh\left(x\right)^{2}+a\right)^{\frac{3}{2}}\tanh\left(x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)*(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tanh(x)^2 + a)^(3/2)*tanh(x), x)
```

Fricas [B] time = 3.24136, size = 7218, normalized size = 114.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)*(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a + b)*cosh(x)^6 + 6*(a + b)*cosh(x)*sinh(x)^5 + (a + b)*sinh(x)
^6 + 3*(a + b)*cosh(x)^4 + 3*(5*(a + b)*cosh(x)^2 + a + b)*sinh(x)^4 + 4*(5
*(a + b)*cosh(x)^3 + 3*(a + b)*cosh(x))*sinh(x)^3 + 3*(a + b)*cosh(x)^2 + 3
*(5*(a + b)*cosh(x)^4 + 6*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 + 6*((a + b)
*cosh(x)^5 + 2*(a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(a
 + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a
^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14
*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*
a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4
 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3
+ a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 +
 a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^
3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(
a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b
^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*
cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(
5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh
(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x
)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)
^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*
sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)
*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*c
```

```
osh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b
^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(
x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^
5 + sinh(x)^6)) + 3*((a + b)*cosh(x)^6 + 6*(a + b)*cosh(x)*sinh(x)^5 + (a +
 b)*sinh(x)^6 + 3*(a + b)*cosh(x)^4 + 3*(5*(a + b)*cosh(x)^2 + a + b)*sinh(
x)^4 + 4*(5*(a + b)*cosh(x)^3 + 3*(a + b)*cosh(x))*sinh(x)^3 + 3*(a + b)*co
sh(x)^2 + 3*(5*(a + b)*cosh(x)^4 + 6*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 +
 6*((a + b)*cosh(x)^5 + 2*(a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a
+ b)*sqrt(a + b)*log(-((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a
 + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + s
qrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a
 + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
 + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x
)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 16*sqrt(2)*((a + b)*cosh(x)^4 + 4*(
a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + (2*a + b)*cosh(x)^2 + (6*(a
+ b)*cosh(x)^2 + 2*a + b)*sinh(x)^2 + 2*(2*(a + b)*cosh(x)^3 + (2*a + b)*co
sh(x))*sinh(x) + a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b
)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^6 + 6*cosh(x)*sinh
(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(5*cosh
(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2
+ 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*cosh(x)^3 + cosh(x))*sinh(x) + 1), -1/6*(3
*((a + b)*cosh(x)^6 + 6*(a + b)*cosh(x)*sinh(x)^5 + (a + b)*sinh(x)^6 + 3*(
a + b)*cosh(x)^4 + 3*(5*(a + b)*cosh(x)^2 + a + b)*sinh(x)^4 + 4*(5*(a + b)
*cosh(x)^3 + 3*(a + b)*cosh(x))*sinh(x)^3 + 3*(a + b)*cosh(x)^2 + 3*(5*(a +
 b)*cosh(x)^4 + 6*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 + 6*((a + b)*cosh(x)
^5 + 2*(a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(-a - b)*a
rctan(sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqr
t(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 -
 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cos
h(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6
*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 +
 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh(x))) + 3*((
a + b)*cosh(x)^6 + 6*(a + b)*cosh(x)*sinh(x)^5 + (a + b)*sinh(x)^6 + 3*(a +
 b)*cosh(x)^4 + 3*(5*(a + b)*cosh(x)^2 + a + b)*sinh(x)^4 + 4*(5*(a + b)*co
sh(x)^3 + 3*(a + b)*cosh(x))*sinh(x)^3 + 3*(a + b)*cosh(x)^2 + 3*(5*(a + b)
*cosh(x)^4 + 6*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 + 6*((a + b)*cosh(x)^5
+ 2*(a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(-a - b)*arct
an(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqr
t(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*si
nh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a +
 b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(
x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + 8*sqrt(2
)*((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + (2
*a + b)*cosh(x)^2 + (6*(a + b)*cosh(x)^2 + 2*a + b)*sinh(x)^2 + 2*(2*(a + b
)*cosh(x)^3 + (2*a + b)*cosh(x))*sinh(x) + a + b)*sqrt(((a + b)*cosh(x)^2 +
 (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(
cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 + 1)*sinh(x)^4
 + 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 + 6
*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*cosh(x)^3 + cosh
(x))*sinh(x) + 1)]
```

Sympy [B] time = 16.3262, size = 128, normalized size = 2.03

$$-\frac{2a\left(\frac{b\sqrt{a+b\tanh^{2}(x)}}{2}+\frac{b(a+b)\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^{2}(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}}\right)}{b}-\frac{2\left(\frac{b^{2}\sqrt{a+b\tanh^{2}(x)}}{2}+\frac{b^{2}(a+b)\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^{2}(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}}+\frac{b\left(a+b\tanh^{2}(x)\right)^{\frac{3}{2}}}{6}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)*(a+b*tanh(x)**2)**(3/2),x)
```

```
[Out] -2*a*(b*sqrt(a + b*tanh(x)**2)/2 + b*(a + b)*atan(sqrt(a + b*tanh(x)**2)/sq
rt(-a - b))/(2*sqrt(-a - b)))/b - 2*(b**2*sqrt(a + b*tanh(x)**2)/2 + b**2*(
a + b)*atan(sqrt(a + b*tanh(x)**2)/sqrt(-a - b))/(2*sqrt(-a - b)) + b*(a +
b*tanh(x)**2)**(3/2)/6)/b
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)*(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.222 $\int \left(a+b\tanh^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=88

$$(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\frac{1}{2}\sqrt{b}(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\frac{1}{2}b\tanh(x)\sqrt{a+b\tanh^2(x)}$$

```
[Out] -(Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/2 +
 (a + b)^(3/2)*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]] - (b*Ta
nh[x]*Sqrt[a + b*Tanh[x]^2])/2
```

Rubi [A] time = 0.0852639, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3661, 416, 523, 217, 206, 377}

$$(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\frac{1}{2}\sqrt{b}(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\frac{1}{2}b\tanh(x)\sqrt{a+b\tanh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] -(Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/2 +
 (a + b)^(3/2)*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]] - (b*Ta
nh[x]*Sqrt[a + b*Tanh[x]^2])/2
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\int \left(a+b\tanh^2(x)\right)^{3/2}\,dx = \text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{1-x^2}\,dx, x, \tanh(x)\right)$$

$$= -\frac{1}{2}b\tanh(x)\sqrt{a+b\tanh^2(x)} - \frac{1}{2}\text{Subst}\left(\int \frac{-a(2a+b)-b(3a+2b)x^2}{\left(1-x^2\right)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)$$

$$= -\frac{1}{2}b\tanh(x)\sqrt{a+b\tanh^2(x)} + (a+b)^2\,\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right) - \frac{1}{2}(b($$

$$= -\frac{1}{2}b\tanh(x)\sqrt{a+b\tanh^2(x)} + (a+b)^2\,\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) -$$

$$= -\frac{1}{2}\sqrt{b}(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) + (a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) - \frac{1}{2}b\tan$$

Mathematica [A] time = 0.337716, size = 161, normalized size = 1.83

$$\frac{1}{2}\left(-b\tanh(x)\sqrt{a+b\tanh^2(x)} - (a+b)^{3/2}\log\left(\sqrt{a+b}\sqrt{a+b\tanh^2(x)} + a - b\tanh(x)\right) + (a+b)^{3/2}\log\left(\sqrt{a+b}\sqrt{a+b}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] (-((a + b)^(3/2)*Log[1 - Tanh[x]]) + (a + b)^(3/2)*Log[1 + Tanh[x]] - Sqrt[
b]*(3*a + 2*b)*Log[b*Tanh[x] + Sqrt[b]*Sqrt[a + b*Tanh[x]^2]] - (a + b)^(3/
2)*Log[a - b*Tanh[x] + Sqrt[a + b]*Sqrt[a + b*Tanh[x]^2]] + (a + b)^(3/2)*L
og[a + b*Tanh[x] + Sqrt[a + b]*Sqrt[a + b*Tanh[x]^2]] - b*Tanh[x]*Sqrt[a +
b*Tanh[x]^2])/2
```

Maple [B] time = 0.023, size = 578, normalized size = 6.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tanh(x)^2)^(3/2),x)
```

```
[Out] 1/6*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(3/2)-1/4*b*((1+tanh(x))^2*b-2*(1
+tanh(x))*b+a+b)^(1/2)*tanh(x)-3/4*b^(1/2)*ln(((1+tanh(x))*b-b)/b^(1/2)+((1
+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))*a-1/2*(a+b)^(1/2)*ln((2*a+2*b-2*(
1+tanh(x))*b+2*(a+b)^(1/2)*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))/(1+
tanh(x)))*a+1/2*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)*a-1/2*b^(3/2)*l
n(((1+tanh(x))*b-b)/b^(1/2)+((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))-1/
2*(a+b)^(1/2)*ln((2*a+2*b-2*(1+tanh(x))*b+2*(a+b)^(1/2)*((1+tanh(x))^2*b-2*
(1+tanh(x))*b+a+b)^(1/2))/(1+tanh(x)))*b+1/2*((1+tanh(x))^2*b-2*(1+tanh(x))
*b+a+b)^(1/2)*b-1/6*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(3/2)-1/4*b*((tan
h(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)*tanh(x)-3/4*b^(1/2)*ln(((tanh(x)-1)*
b+b)/b^(1/2)+((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))*a+1/2*ln((2*a+2*b
+2*(tanh(x)-1)*b+2*(a+b)^(1/2)*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))
/(tanh(x)-1))*(a+b)^(1/2)*a-1/2*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)
*a-1/2*b^(3/2)*ln(((tanh(x)-1)*b+b)/b^(1/2)+((tanh(x)-1)^2*b+2*(tanh(x)-1)*
b+a+b)^(1/2))+1/2*ln((2*a+2*b+2*(tanh(x)-1)*b+2*(a+b)^(1/2)*((tanh(x)-1)^2*
b+2*(tanh(x)-1)*b+a+b)^(1/2))/(tanh(x)-1))*(a+b)^(1/2)*b-1/2*((tanh(x)-1)^2
*b+2*(tanh(x)-1)*b+a+b)^(1/2)*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tanh(x)^2 + a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tanh(x)^2 + a)^(3/2), x)
```

Fricas [B] time = 4.08731, size = 14453, normalized size = 164.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4
+ 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 + 4*((a +
 b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*log(-((a*b^2
+ b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x
)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cos
h(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(
x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^
3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2
)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3
+ (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*
b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(
x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*
b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*
cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 -
b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a
```

```
*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b
+ 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)
^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh
(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)
^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 -
a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(
x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^
3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) +
((3*a + 2*b)*cosh(x)^4 + 4*(3*a + 2*b)*cosh(x)*sinh(x)^3 + (3*a + 2*b)*sinh
(x)^4 + 2*(3*a + 2*b)*cosh(x)^2 + 2*(3*(3*a + 2*b)*cosh(x)^2 + 3*a + 2*b)*s
inh(x)^2 + 4*((3*a + 2*b)*cosh(x)^3 + (3*a + 2*b)*cosh(x))*sinh(x) + 3*a +
2*b)*sqrt(b)*log(-((a + 2*b)*cosh(x)^4 + 4*(a + 2*b)*cosh(x)*sinh(x)^3 + (a
 + 2*b)*sinh(x)^4 + 2*(a - 2*b)*cosh(x)^2 + 2*(3*(a + 2*b)*cosh(x)^2 + a -
2*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*
sqrt(b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2
*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + 2*b)*cosh(x)^3 + (a - 2*b)*cosh(x)
)*sinh(x) + a + 2*b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*co
sh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)
) + ((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 +
2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 + 4*((a + b
)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*log(((a + b)*co
sh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)^2 +
 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sin
h(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)
^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(
x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
^2)) - 2*sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*sqrt
(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sin
h(x) + sinh(x)^2)))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cos
h(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1),
 1/4*(2*((3*a + 2*b)*cosh(x)^4 + 4*(3*a + 2*b)*cosh(x)*sinh(x)^3 + (3*a + 2
*b)*sinh(x)^4 + 2*(3*a + 2*b)*cosh(x)^2 + 2*(3*(3*a + 2*b)*cosh(x)^2 + 3*a
+ 2*b)*sinh(x)^2 + 4*((3*a + 2*b)*cosh(x)^3 + (3*a + 2*b)*cosh(x))*sinh(x)
+ 3*a + 2*b)*sqrt(-b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(
x)^2 - 1)*sqrt(-b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(co
sh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*co
sh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*co
sh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x
) + a + b)) + ((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*si
nh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 +
 4*((a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*log(-
((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3
)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 +
b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^
3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*
b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*c
osh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*co
sh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*
b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^
3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^
3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6
+ 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh
(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a
^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2
+ 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2
*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a +
 b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
 sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 +
 (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x
```

```
))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*
cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x
)^6)) + ((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^
4 + 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 + 4*((a
 + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*log(((a + b
)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)
^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)
*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sin
h(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*c
osh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sin
h(x)^2)) - 2*sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*
sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)
*sinh(x) + sinh(x)^2)))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3
*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) +
 1), -1/4*(2*((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sin
h(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 +
4*((a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(-a - b)*arcta
n(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - a - b)*sqrt(-a
 - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2))/((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh(x)
*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^2)*cosh(x)^2 + (6*(a*
b + b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(
2*(a*b + b^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x))) + 2*((a +
b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a + b)*
cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 + 4*((a + b)*cosh(x)^
3 + (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(-a - b)*arctan(sqrt(2)*sqrt(-a -
 b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cos
h(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) +
 (a + b)*sinh(x)^2 + a + b)) - ((3*a + 2*b)*cosh(x)^4 + 4*(3*a + 2*b)*cosh(
x)*sinh(x)^3 + (3*a + 2*b)*sinh(x)^4 + 2*(3*a + 2*b)*cosh(x)^2 + 2*(3*(3*a
+ 2*b)*cosh(x)^2 + 3*a + 2*b)*sinh(x)^2 + 4*((3*a + 2*b)*cosh(x)^3 + (3*a +
 2*b)*cosh(x))*sinh(x) + 3*a + 2*b)*sqrt(b)*log(-((a + 2*b)*cosh(x)^4 + 4*(
a + 2*b)*cosh(x)*sinh(x)^3 + (a + 2*b)*sinh(x)^4 + 2*(a - 2*b)*cosh(x)^2 +
2*(3*(a + 2*b)*cosh(x)^2 + a - 2*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*co
sh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*si
nh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + 2*b
)*cosh(x)^3 + (a - 2*b)*cosh(x))*sinh(x) + a + 2*b)/(cosh(x)^4 + 4*cosh(x)*
sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(co
sh(x)^3 + cosh(x))*sinh(x) + 1)) + 2*sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sin
h(x) + b*sinh(x)^2 - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b
)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^4 + 4*cosh(x)*sinh
(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x
)^3 + cosh(x))*sinh(x) + 1), -1/2*(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*s
inh(x)^3 + (a + b)*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2
 + a + b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a +
 b)*sqrt(-a - b)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh
(x)^2 - a - b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
 - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a*b + b^2)*cosh(x)^4 +
 4*(a*b + b^2)*cosh(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b
^2)*cosh(x)^2 + (6*(a*b + b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a
^2 + 2*a*b + b^2 + 2*(2*(a*b + b^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x)
)*sinh(x))) - ((3*a + 2*b)*cosh(x)^4 + 4*(3*a + 2*b)*cosh(x)*sinh(x)^3 + (3
*a + 2*b)*sinh(x)^4 + 2*(3*a + 2*b)*cosh(x)^2 + 2*(3*(3*a + 2*b)*cosh(x)^2
+ 3*a + 2*b)*sinh(x)^2 + 4*((3*a + 2*b)*cosh(x)^3 + (3*a + 2*b)*cosh(x))*si
nh(x) + 3*a + 2*b)*sqrt(-b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2 - 1)*sqrt(-b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a -
b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a +
 b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a +
 b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*
```

```
sinh(x) + a + b)) + ((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a +
 b)*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a + b)*sinh(
x)^2 + 4*((a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(-a - b
)*arctan(sqrt(2)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
 a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^2 + 2
*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)) + sqrt(2)*(b*cosh(x)
^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*sqrt(((a + b)*cosh(x)^2 + (a +
b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x
)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*c
osh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b \tanh^2(x)\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tanh(x)**2)**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.223 $\int \coth(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=71

$$a^{3/2}\left(-\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)\right)+(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-b\sqrt{a+b\tanh^2(x)}$$

```
[Out] -(a^(3/2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]]) + (a + b)^(3/2)*ArcTanh[S
qrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - b*Sqrt[a + b*Tanh[x]^2]
```

Rubi [A] time = 0.136251, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3670, 446, 84, 156, 63, 208}

$$a^{3/2}\left(-\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)\right)+(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-b\sqrt{a+b\tanh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]*(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] -(a^(3/2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]]) + (a + b)^(3/2)*ArcTanh[S
qrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - b*Sqrt[a + b*Tanh[x]^2]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), I
nt[((b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*(e + f*x)^(p - 2))/
((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \coth(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx &= \text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x\left(1-x^2\right)}\,dx,x,\tanh(x)\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{(1-x)x}\,dx,x,\tanh^2(x)\right)\\
&= -b\sqrt{a+b\tanh^2(x)}-\frac{1}{2}\,\text{Subst}\left(\int \frac{-a^2+(-2a-b)bx}{(1-x)x\sqrt{a+bx}}\,dx,x,\tanh^2(x)\right)\\
&= -b\sqrt{a+b\tanh^2(x)}+\frac{1}{2}a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,\tanh^2(x)\right)+\frac{1}{2}(a+b)^2\,\text{Subst}\,\ldots\\
&= -b\sqrt{a+b\tanh^2(x)}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\tanh^2(x)}\right)}{b}+\frac{(a+b)^2\,\text{Subst}\,\ldots}{\ldots}\\
&= -a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)+(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-b\sqrt{a+b\,\ldots}
\end{aligned}$$

Mathematica [A] time = 0.0598874, size = 71, normalized size = 1.

$$a^{3/2}\left(-\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)\right)+(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-b\sqrt{a+b\tanh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]*(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] -(a^(3/2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]]) + (a + b)^(3/2)*ArcTanh[S
qrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - b*Sqrt[a + b*Tanh[x]^2]
```

Maple [F] time = 0.116, size = 0, normalized size = 0.

$$\int \coth(x)\left(a+b(\tanh(x))^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(a+b*tanh(x)^2)^(3/2),x)
```

[Out] int(coth(x)*(a+b*tanh(x)^2)^(3/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tanh\left(x\right)^2+a\right)^{\frac{3}{2}}\coth\left(x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)*(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")

[Out] integrate((b*tanh(x)^2 + a)^(3/2)*coth(x), x)

Fricas [B] time = 3.56931, size = 11954, normalized size = 168.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)*(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")

[Out] [1/4*(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 2*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)*sqrt(a)*log(-((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a - b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 + 2*a - b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a + b)*cosh(x)^3 + (2*a - b)*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)) + ((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)*sqrt(a + b)*log(-((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)* sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*b*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1), 1/4*(4*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + ((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)*sqrt(a + b)*log(-((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*b*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1), -1/2*(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)*sqrt(-a - b)*arctan(sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh(x))) + ((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) - (a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)*sqrt(a)*log(-((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a - b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 + 2*a - b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a + b)*cosh(x)^3 + (2*a - b)*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 +

```
4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)
^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)) + 2*sqrt(2)*b*sqrt(((a + b)*cosh
(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)
^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1), 1/2*(2*(a*cosh(x)^2
+ 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2
 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt(((a + b)*cosh(x)^2 + (a
 + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a +
 b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)
*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)
^3 + (a - b)*cosh(x))*sinh(x) + a + b)) - ((a + b)*cosh(x)^2 + 2*(a + b)*co
sh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)*sqrt(-a - b)*arctan(sqrt(2)*(a*c
osh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a - b)*sqrt(((a
 + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
 + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (
a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(
x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a*b)*
cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh(x))) - ((a + b)*cosh(x)^2 + 2
*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)*sqrt(-a - b)*arctan(s
qrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((
a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x
) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*
sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2
 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) - 2*sqrt(2)*b*
sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)
*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*tanh(x)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.224 $\int \coth^2(x) \left(a + b \tanh^2(x)\right)^{3/2} dx$

Optimal. Leaf size=77

$$b^{3/2}\left(-\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)\right)+(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-a\coth(x)\sqrt{a+b\tanh^2(x)}$$

```
[Out] -(b^(3/2)*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]) + (a + b)^(3/2)
*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]] - a*Coth[x]*Sqrt[a +
b*Tanh[x]^2]
```

Rubi [A] time = 0.127127, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3670, 474, 523, 217, 206, 377}

$$b^{3/2}\left(-\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)\right)+(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-a\coth(x)\sqrt{a+b\tanh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2*(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] -(b^(3/2)*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]) + (a + b)^(3/2)
*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]] - a*Coth[x]*Sqrt[a +
b*Tanh[x]^2]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \coth^2(x)\left(a+b\tanh^2(x)\right)^{3/2}dx &= \text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x^2\left(1-x^2\right)}dx, x, \tanh(x)\right)\\ &= -a\coth(x)\sqrt{a+b\tanh^2(x)}+\text{Subst}\left(\int \frac{a(a+2b)+b^2x^2}{\left(1-x^2\right)\sqrt{a+bx^2}}dx, x, \tanh(x)\right)\\ &= -a\coth(x)\sqrt{a+b\tanh^2(x)}-b^2\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}dx, x, \tanh(x)\right)+(a+b)^2\\ &= -a\coth(x)\sqrt{a+b\tanh^2(x)}-b^2\,\text{Subst}\left(\int \frac{1}{1-bx^2}dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)+(\\ &= -b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)+(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-a\cot\end{aligned}$$

Mathematica [C] time = 2.76287, size = 197, normalized size = 2.56

$$-\frac{a\tanh(x)\left(-\sqrt{2}(a+2b)\sqrt{\frac{\text{csch}^2(x)((a+b)\cosh(2x)+a-b)}{b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\text{csch}^2(x)((a+b)\cosh(2x)+a-b)}{b}}}{\sqrt{2}}\right),1\right)+\text{csch}^2(x)((a+b)\,\text{c}\right.}{\sqrt{2}\sqrt{\text{sech}^2(x)((a+b)\cosh(2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2*(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] -((a*((a - b + (a + b)*Cosh[2*x])*Csch[x]^2 - Sqrt[2]*(a + 2*b)*Sqrt[((a -
b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*EllipticF[ArcSin[Sqrt[((a - b + (a + b
)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1] + Sqrt[2]*(a + b)*Sqrt[((a - b + (a
 + b)*Cosh[2*x])*Csch[x]^2)/b]*EllipticPi[b/(a + b), ArcSin[Sqrt[((a - b +
(a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1])*Tanh[x])/(Sqrt[2]*Sqrt[(a -
b + (a + b)*Cosh[2*x])*Sech[x]^2]))
```

Maple [F] time = 0.115, size = 0, normalized size = 0.

$$\int (\coth(x))^2\left(a+b(\tanh(x))^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2*(a+b*tanh(x)^2)^(3/2),x)
```

```
[Out] int(coth(x)^2*(a+b*tanh(x)^2)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \tanh (x)^2 + a\right)^{\frac{3}{2}} \coth (x)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tanh(x)^2 + a)^(3/2)*coth(x)^2, x)
```

Fricas [B] time = 3.63884, size = 11507, normalized size = 149.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 -
a - b)*sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*
sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^
2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cos
h(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*
b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^
3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5
 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*
sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(
x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 -
 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 +
 sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*c
osh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2
*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 -
 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*
(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x)
)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*
b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 -
 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*c
osh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*co
sh(x)*sinh(x)^5 + sinh(x)^6)) + 2*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*si
nh(x)^2 - b)*sqrt(b)*log(-((a + 2*b)*cosh(x)^4 + 4*(a + 2*b)*cosh(x)*sinh(x
)^3 + (a + 2*b)*sinh(x)^4 + 2*(a - 2*b)*cosh(x)^2 + 2*(3*(a + 2*b)*cosh(x)^
2 + a - 2*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
^2 - 1)*sqrt(b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(
x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + 2*b)*cosh(x)^3 + (a - 2*b)
*cosh(x))*sinh(x) + a + 2*b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 +
 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh
```

(x) + 1)) + ((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x
)^2 - a - b)*sqrt(a + b)*log(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)
^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x
)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*s
qrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*
sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 + a*cosh(x))*sinh(x) + a + b)/
(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*a*sqrt(((a + b)*co
sh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(
x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1), 1/4*(4*(b*cosh(x)^
2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*sqrt(-b)*arctan(sqrt(2)*(cosh(x)
^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt(((a + b)*cosh(x)^2 +
(a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a
 + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a -
b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(
x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + ((a + b)*cosh(x)^2 + 2*(a + b)*
cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a - b)*sqrt(a + b)*log(-((a*b^2 + b^3
)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 -
 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^
2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*s
inh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*co
sh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sin
h(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^
3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 +
 b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6
- 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4
*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(
x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*
sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b -
3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b
^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 -
(a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2
 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))
+ 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b
 + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(
cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sin
h(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a +
 b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a - b)*sqrt
(a + b)*log(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh
(x)^4 + 2*a*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(co
sh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh
(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)
^2)) + 4*((a + b)*cosh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*co
sh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*a*sqrt(((a + b)*cosh(x)^2 + (a + b)
*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^
2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1), -1/2*(((a + b)*cosh(x)^2 + 2*(a + b
)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a - b)*sqrt(-a - b)*arctan(sqrt(2)*
(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - a - b)*sqrt(-a - b)*sqrt
(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sin
h(x) + sinh(x)^2))/((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh(x)*sinh(x)^3
 + (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^2)*cosh(x)^2 + (6*(a*b + b^2)*c
osh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a*b + b
^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x))) + ((a + b)*cosh(x)^2
 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a - b)*sqrt(-a - b)*arct
an(sqrt(2)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b
)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^2 + 2*(a +
b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)) - (b*cosh(x)^2 + 2*b*cosh(
x)*sinh(x) + b*sinh(x)^2 - b)*sqrt(b)*log(-((a + 2*b)*cosh(x)^4 + 4*(a + 2*
b)*cosh(x)*sinh(x)^3 + (a + 2*b)*sinh(x)^4 + 2*(a - 2*b)*cosh(x)^2 + 2*(3*(
a + 2*b)*cosh(x)^2 + a - 2*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*

```
sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^
2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + 2*b)*cosh
(x)^3 + (a - 2*b)*cosh(x))*sinh(x) + a + 2*b)/(cosh(x)^4 + 4*cosh(x)*sinh(x
)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^
3 + cosh(x))*sinh(x) + 1)) + 2*sqrt(2)*a*sqrt(((a + b)*cosh(x)^2 + (a + b)*
sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^2
 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1), -1/2*(((a + b)*cosh(x)^2 + 2*(a + b)
*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a - b)*sqrt(-a - b)*arctan(sqrt(2)*(
b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - a - b)*sqrt(-a - b)*sqrt(
((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh
(x) + sinh(x)^2))/((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh(x)*sinh(x)^3
+ (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^2)*cosh(x)^2 + (6*(a*b + b^2)*co
sh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a*b + b^
2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x))) - 2*(b*cosh(x)^2 + 2*
b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*sqrt(-b)*arctan(sqrt(2)*(cosh(x)^2 + 2
*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt(((a + b)*cosh(x)^2 + (a + b
)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*
cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cos
h(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 +
 (a - b)*cosh(x))*sinh(x) + a + b)) + ((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x
)*sinh(x) + (a + b)*sinh(x)^2 - a - b)*sqrt(-a - b)*arctan(sqrt(2)*sqrt(-a
- b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*co
sh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x)
+ (a + b)*sinh(x)^2 + a + b)) + 2*sqrt(2)*a*sqrt(((a + b)*cosh(x)^2 + (a +
b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x
)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2*(a+b*tanh(x)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.225 $\int \sqrt{1+\tanh^2(x)}\,dx$

Optimal. Leaf size=31

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{\tanh^2(x)+1}}\right)-\sinh^{-1}(\tanh(x))$$

```
[Out] -ArcSinh[Tanh[x]] + Sqrt[2]*ArcTanh[(Sqrt[2]*Tanh[x])/Sqrt[1 + Tanh[x]^2]]
```

Rubi [A] time = 0.0269331, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3661, 402, 215, 377, 206}

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{\tanh^2(x)+1}}\right)-\sinh^{-1}(\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Tanh[x]^2],x]
```

```
[Out] -ArcSinh[Tanh[x]] + Sqrt[2]*ArcTanh[(Sqrt[2]*Tanh[x])/Sqrt[1 + Tanh[x]^2]]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{1+\tanh^2(x)}\,dx = \text{Subst}\left(\int \frac{\sqrt{1+x^2}}{1-x^2}\,dx, x, \tanh(x)\right)$$
$$= 2\,\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{1+x^2}}\,dx, x, \tanh(x)\right) - \text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \tanh(x)\right)$$
$$= -\sinh^{-1}(\tanh(x)) + 2\,\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{1+\tanh^2(x)}}\right)$$
$$= -\sinh^{-1}(\tanh(x)) + \sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{1+\tanh^2(x)}}\right)$$

Mathematica [A] time = 0.0580652, size = 51, normalized size = 1.65

$$\frac{\cosh(x)\sqrt{\tanh^2(x)+1}\left(\sqrt{2}\sinh^{-1}\left(\sqrt{2}\sinh(x)\right)-\tanh^{-1}\left(\frac{\sinh(x)}{\sqrt{\cosh(2x)}}\right)\right)}{\sqrt{\cosh(2x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Tanh[x]^2],x]
```

```
[Out] ((Sqrt[2]*ArcSinh[Sqrt[2]*Sinh[x]] - ArcTanh[Sinh[x]/Sqrt[Cosh[2*x]]])*Cosh
[x]*Sqrt[1 + Tanh[x]^2])/Sqrt[Cosh[2*x]]
```

Maple [B] time = 0.059, size = 97, normalized size = 3.1

$$\frac{1}{2}\sqrt{(1+\tanh(x))^2-2\tanh(x)} - \text{Arcsinh}(\tanh(x)) - \frac{\sqrt{2}}{2}\text{Artanh}\left(\frac{(2-2\tanh(x))\sqrt{2}}{4}\frac{1}{\sqrt{(1+\tanh(x))^2-2\tanh(x)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+tanh(x)^2)^(1/2),x)
```

```
[Out] 1/2*((1+tanh(x))^2-2*tanh(x))^(1/2)-arcsinh(tanh(x))-1/2*2^(1/2)*arctanh(1/
4*(2-2*tanh(x))*2^(1/2)/((1+tanh(x))^2-2*tanh(x))^(1/2))-1/2*((tanh(x)-1)^2
+2*tanh(x))^(1/2)+1/2*2^(1/2)*arctanh(1/4*(2*tanh(x)+2)*2^(1/2)/((tanh(x)-1
)^2+2*tanh(x))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\tanh(x)^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(tanh(x)^2 + 1), x)
```

Fricas [B] time = 1.90972, size = 2319, normalized size = 74.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-2*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + (28*cosh(
x)^2 - 3)*sinh(x)^6 - 3*cosh(x)^6 + 2*(28*cosh(x)^3 - 9*cosh(x))*sinh(x)^5
+ 5*(14*cosh(x)^4 - 9*cosh(x)^2 + 1)*sinh(x)^4 + 5*cosh(x)^4 + 4*(14*cosh(x
)^5 - 15*cosh(x)^3 + 5*cosh(x))*sinh(x)^3 + (28*cosh(x)^6 - 45*cosh(x)^4 +
30*cosh(x)^2 - 4)*sinh(x)^2 - 4*cosh(x)^2 + 2*(4*cosh(x)^7 - 9*cosh(x)^5 +
10*cosh(x)^3 - 4*cosh(x))*sinh(x) + (sqrt(2)*cosh(x)^6 + 6*sqrt(2)*cosh(x)*
sinh(x)^5 + sqrt(2)*sinh(x)^6 + 3*(5*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x)^4
 - 3*sqrt(2)*cosh(x)^4 + 4*(5*sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x))*sinh(x
)^3 + (15*sqrt(2)*cosh(x)^4 - 18*sqrt(2)*cosh(x)^2 + 4*sqrt(2))*sinh(x)^2 +
 4*sqrt(2)*cosh(x)^2 + 2*(3*sqrt(2)*cosh(x)^5 - 6*sqrt(2)*cosh(x)^3 + 4*sqr
t(2)*cosh(x))*sinh(x) - 4*sqrt(2))*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2
- 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4)/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 1
5*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6
*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 1/4*sqrt(2)*log(2*(cosh(x)^4 + 4*cosh(x)
*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(2*cos
h(x)^3 + cosh(x))*sinh(x) + (sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x)
+ sqrt(2)*sinh(x)^2 + sqrt(2))*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*
cosh(x)*sinh(x) + sinh(x)^2)) + 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
^2)) - 1/2*log((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 2*sqrt((cosh(x)
^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 1)/(cosh(x)^
2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 1/2*log((cosh(x)^2 + 2*cosh(x)*sinh(x
) + sinh(x)^2 - 2*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(
x) + sinh(x)^2)) - 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\tanh^2(x) + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(tanh(x)**2 + 1), x)
```

Giac [B] time = 1.21573, size = 140, normalized size = 4.52

$$-\frac{1}{2}\sqrt{2}\left(\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{e^{(4x)}+1}+e^{(2x)}+1}{\sqrt{2}+\sqrt{e^{(4x)}+1}-e^{(2x)}-1}\right)+\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}+1\right)+\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)-\log\left(-\sqrt{e^{(4x)}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(sqrt(2)*log((sqrt(2) - sqrt(e^(4*x) + 1) + e^(2*x) + 1)/(sqrt
(2) + sqrt(e^(4*x) + 1) - e^(2*x) - 1)) + log(sqrt(e^(4*x) + 1) - e^(2*x) +
 1) + log(sqrt(e^(4*x) + 1) - e^(2*x)) - log(-sqrt(e^(4*x) + 1) + e^(2*x) +
 1))
```

3.226 $\int \sqrt{-1-\tanh^2(x)}\,dx$

Optimal. Leaf size=45

$$\tan^{-1}\left(\frac{\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)$$

```
[Out] ArcTan[Tanh[x]/Sqrt[-1 - Tanh[x]^2]] - Sqrt[2]*ArcTan[(Sqrt[2]*Tanh[x])/Sqr
t[-1 - Tanh[x]^2]]
```

Rubi [A] time = 0.0345847, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3661, 402, 217, 203, 377}

$$\tan^{-1}\left(\frac{\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 - Tanh[x]^2],x]
```

```
[Out] ArcTan[Tanh[x]/Sqrt[-1 - Tanh[x]^2]] - Sqrt[2]*ArcTan[(Sqrt[2]*Tanh[x])/Sqr
t[-1 - Tanh[x]^2]]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{-1-\tanh^2(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt{-1-x^2}}{1-x^2}\,dx, x, \tanh(x)\right)\\
&= -\left(2\,\text{Subst}\left(\int \frac{1}{\sqrt{-1-x^2}\left(1-x^2\right)}\,dx, x, \tanh(x)\right)\right) + \text{Subst}\left(\int \frac{1}{\sqrt{-1-x^2}}\,dx, x, \tanh(x)\right)\\
&= -\left(2\,\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right)\right) + \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right)\\
&= \tan^{-1}\left(\frac{\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right) - \sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.033177, size = 53, normalized size = 1.18

$$
\frac{\cosh(x)\sqrt{-\tanh^2(x)-1}\left(\sqrt{2}\sinh^{-1}\left(\sqrt{2}\sinh(x)\right)-\tanh^{-1}\left(\frac{\sinh(x)}{\sqrt{\cosh(2x)}}\right)\right)}{\sqrt{\cosh(2x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 - Tanh[x]^2],x]
```

```
[Out] ((Sqrt[2]*ArcSinh[Sqrt[2]*Sinh[x]] - ArcTanh[Sinh[x]/Sqrt[Cosh[2*x]]])*Cosh
[x]*Sqrt[-1 - Tanh[x]^2])/Sqrt[Cosh[2*x]]
```

Maple [B] time = 0.056, size = 142, normalized size = 3.2

$$
\frac{1}{2}\sqrt{-(1+\tanh(x))^2+2\tanh(x)}+\frac{1}{2}\arctan\left(\tanh(x)\frac{1}{\sqrt{-(1+\tanh(x))^2+2\tanh(x)}}\right)-\frac{\sqrt{2}}{2}\arctan\left(\frac{(2\tanh(x)-}{4}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1-tanh(x)^2)^(1/2),x)
```

```
[Out] 1/2*(-(1+tanh(x))^2+2*tanh(x))^(1/2)+1/2*arctan(tanh(x)/(-(1+tanh(x))^2+2*t
anh(x))^(1/2))-1/2*2^(1/2)*arctan(1/4*(2*tanh(x)-2)*2^(1/2)/(-(1+tanh(x))^2
+2*tanh(x))^(1/2))-1/2*(-(tanh(x)-1)^2-2*tanh(x))^(1/2)+1/2*arctan(tanh(x)/
(-(tanh(x)-1)^2-2*tanh(x))^(1/2))+1/2*2^(1/2)*arctan(1/4*(-2-2*tanh(x))*2^(
1/2)/(-(tanh(x)-1)^2-2*tanh(x))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \sqrt{-\tanh(x)^2-1}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tanh(x)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-tanh(x)^2 - 1), x)
```

Fricas [C] time = 1.87808, size = 710, normalized size = 15.78

$$-\frac{1}{4}\sqrt{-2}\log\left(-\left(\sqrt{-2}\sqrt{-2\,e^{(4\,x)}-2}+2\,e^{(2\,x)}+2\right)e^{(-2\,x)}\right)+\frac{1}{4}\sqrt{-2}\log\left(\left(\sqrt{-2}\sqrt{-2\,e^{(4\,x)}-2}-2\,e^{(2\,x)}-2\right)e^{(-2\,x)}\right)+\frac{1}{4}\sqrt{-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tanh(x)^2)^(1/2),x, algorithm="fricas")

[Out] -1/4*sqrt(-2)*log(-(sqrt(-2)*sqrt(-2*e^(4*x) - 2) + 2*e^(2*x) + 2)*e^(-2*x)
) + 1/4*sqrt(-2)*log((sqrt(-2)*sqrt(-2*e^(4*x) - 2) - 2*e^(2*x) - 2)*e^(-2*
x)) + 1/4*sqrt(-2)*log(-2*(sqrt(-2*e^(4*x) - 2)*(e^(2*x) - 2) + sqrt(-2)*e^
(4*x) - sqrt(-2)*e^(2*x) + 2*sqrt(-2))*e^(-4*x)) - 1/4*sqrt(-2)*log(-2*(sqr
t(-2*e^(4*x) - 2)*(e^(2*x) - 2) - sqrt(-2)*e^(4*x) + sqrt(-2)*e^(2*x) - 2*s
qrt(-2))*e^(-4*x)) - 1/2*I*log((4*I*sqrt(-2*e^(4*x) - 2) - 4*e^(2*x) + 4)*e
^(-2*x)) + 1/2*I*log((-4*I*sqrt(-2*e^(4*x) - 2) - 4*e^(2*x) + 4)*e^(-2*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-\tanh^2(x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tanh(x)**2)**(1/2),x)

[Out] Integral(sqrt(-tanh(x)**2 - 1), x)
```

Giac [C] time = 1.2704, size = 136, normalized size = 3.02

$$-\frac{1}{2}\sqrt{2}\left(2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\left(-i\sqrt{e^{(4\,x)}+1}-i\right)e^{(-2\,x)}-i\right)\right)+i\log\left(-\left(\sqrt{e^{(4\,x)}+1}+1\right)e^{(-2\,x)}\right)-i\log\left(-\left(-i\sqrt{e^{(4\,x)}+1}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tanh(x)^2)^(1/2),x, algorithm="giac")

[Out] -1/2*sqrt(2)*(2*sqrt(2)*arctan(-1/2*sqrt(2)*((-I*sqrt(e^(4*x) + 1) - I)*e^(
-2*x) - I)) + I*log(-(sqrt(e^(4*x) + 1) + 1)*e^(-2*x)) - I*log(-(-I*sqrt(e^
(4*x) + 1) - I)*e^(-2*x) + I) + I*log(-(-I*sqrt(e^(4*x) + 1) - I)*e^(-2*x)
- I))
```

3.227 $\int \left(1+\tanh^2(x)\right)^{3/2} dx$

Optimal. Leaf size=50

$$2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{\tanh^2(x)+1}}\right)-\frac{1}{2}\tanh(x)\sqrt{\tanh^2(x)+1}-\frac{5}{2}\sinh^{-1}(\tanh(x))$$

```
[Out] (-5*ArcSinh[Tanh[x]])/2 + 2*Sqrt[2]*ArcTanh[(Sqrt[2]*Tanh[x])/Sqrt[1 + Tanh
[x]^2]] - (Tanh[x]*Sqrt[1 + Tanh[x]^2])/2
```

Rubi [A] time = 0.0388703, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {3661, 416, 523, 215, 377, 206}

$$2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{\tanh^2(x)+1}}\right)-\frac{1}{2}\tanh(x)\sqrt{\tanh^2(x)+1}-\frac{5}{2}\sinh^{-1}(\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Tanh[x]^2)^(3/2),x]
```

```
[Out] (-5*ArcSinh[Tanh[x]])/2 + 2*Sqrt[2]*ArcTanh[(Sqrt[2]*Tanh[x])/Sqrt[1 + Tanh
[x]^2]] - (Tanh[x]*Sqrt[1 + Tanh[x]^2])/2
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(1+\tanh^2(x)\right)^{3/2} dx = \text{Subst}\left(\int \frac{\left(1+x^2\right)^{3/2}}{1-x^2} dx, x, \tanh(x)\right)$$

$$= -\frac{1}{2}\tanh(x)\sqrt{1+\tanh^2(x)} - \frac{1}{2}\text{Subst}\left(\int \frac{-3-5x^2}{\left(1-x^2\right)\sqrt{1+x^2}} dx, x, \tanh(x)\right)$$

$$= -\frac{1}{2}\tanh(x)\sqrt{1+\tanh^2(x)} - \frac{5}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}} dx, x, \tanh(x)\right) + 4\,\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\cdots}\right.$$

$$= -\frac{5}{2}\sinh^{-1}(\tanh(x)) - \frac{1}{2}\tanh(x)\sqrt{1+\tanh^2(x)} + 4\,\text{Subst}\left(\int \frac{1}{1-2x^2} dx, x, \frac{\tanh(x)}{\sqrt{1+\tanh^2(\cdots}}\right.$$

$$= -\frac{5}{2}\sinh^{-1}(\tanh(x)) + 2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{1+\tanh^2(x)}}\right) - \frac{1}{2}\tanh(x)\sqrt{1+\tanh^2(x)}$$

Mathematica [A] time = 0.133756, size = 74, normalized size = 1.48

$$-\frac{\left(\tanh^2(x)+1\right)^{3/2}\left(-4\sqrt{2}\sinh^{-1}\left(\sqrt{2}\sinh(x)\right)\cosh^3(x)+\sinh(x)\sqrt{\cosh(2x)}\cosh(x)+5\cosh^3(x)\tanh^{-1}\left(\frac{\sinh(x)}{\sqrt{\cosh(2x)}}\cdots\right.\right.}{2\cosh^{\frac{3}{2}}(2x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Tanh[x]^2)^(3/2),x]
```

```
[Out] -((-4*Sqrt[2]*ArcSinh[Sqrt[2]*Sinh[x]]*Cosh[x]^3 + 5*ArcTanh[Sinh[x]/Sqrt[C
osh[2*x]]]*Cosh[x]^3 + Cosh[x]*Sqrt[Cosh[2*x]]*Sinh[x])*(1 + Tanh[x]^2)^(3/
2))/(2*Cosh[2*x]^(3/2))
```

Maple [B] time = 0.025, size = 158, normalized size = 3.2

$$\frac{1}{6}\left((1+\tanh(x))^2-2\tanh(x)\right)^{\frac{3}{2}} - \frac{\tanh(x)}{4}\sqrt{(1+\tanh(x))^2-2\tanh(x)} - \frac{5\,\text{Arcsinh}(\tanh(x))}{2} + \sqrt{(1+\tanh(x\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+tanh(x)^2)^(3/2),x)
```

```
[Out] 1/6*((1+tanh(x))^2-2*tanh(x))^(3/2)-1/4*tanh(x)*((1+tanh(x))^2-2*tanh(x))^(
1/2)-5/2*arcsinh(tanh(x))+((1+tanh(x))^2-2*tanh(x))^(1/2)-2^(1/2)*arctanh(1
/4*(2-2*tanh(x))*2^(1/2)/((1+tanh(x))^2-2*tanh(x))^(1/2))-1/6*((tanh(x)-1)^
2+2*tanh(x))^(3/2)-1/4*tanh(x)*((tanh(x)-1)^2+2*tanh(x))^(1/2)-((tanh(x)-1)
^2+2*tanh(x))^(1/2)+2^(1/2)*arctanh(1/4*(2*tanh(x)+2)*2^(1/2)/((tanh(x)-1)^
2+2*tanh(x))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\tanh(x)^2 + 1\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((tanh(x)^2 + 1)^(3/2), x)
```

Fricas [B] time = 2.0444, size = 3526, normalized size = 70.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(sqrt(2)*cosh(x)^4 + 4*sqrt(2)*cosh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4
 + 2*(3*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^2 + 2*sqrt(2)*cosh(x)^2 + 4*(s
qrt(2)*cosh(x)^3 + sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*log(-2*(cosh(x)^8 +
8*cosh(x)*sinh(x)^7 + sinh(x)^8 + (28*cosh(x)^2 - 3)*sinh(x)^6 - 3*cosh(x)^
6 + 2*(28*cosh(x)^3 - 9*cosh(x))*sinh(x)^5 + 5*(14*cosh(x)^4 - 9*cosh(x)^2
+ 1)*sinh(x)^4 + 5*cosh(x)^4 + 4*(14*cosh(x)^5 - 15*cosh(x)^3 + 5*cosh(x))*
sinh(x)^3 + (28*cosh(x)^6 - 45*cosh(x)^4 + 30*cosh(x)^2 - 4)*sinh(x)^2 - 4*
cosh(x)^2 + 2*(4*cosh(x)^7 - 9*cosh(x)^5 + 10*cosh(x)^3 - 4*cosh(x))*sinh(x
) + (sqrt(2)*cosh(x)^6 + 6*sqrt(2)*cosh(x)*sinh(x)^5 + sqrt(2)*sinh(x)^6 +
3*(5*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x)^4 - 3*sqrt(2)*cosh(x)^4 + 4*(5*sq
rt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x))*sinh(x)^3 + (15*sqrt(2)*cosh(x)^4 - 18
*sqrt(2)*cosh(x)^2 + 4*sqrt(2))*sinh(x)^2 + 4*sqrt(2)*cosh(x)^2 + 2*(3*sqrt
(2)*cosh(x)^5 - 6*sqrt(2)*cosh(x)^3 + 4*sqrt(2)*cosh(x))*sinh(x) - 4*sqrt(2
))*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)
) + 4)/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(
x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6))
 + 2*(sqrt(2)*cosh(x)^4 + 4*sqrt(2)*cosh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4 +
 2*(3*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^2 + 2*sqrt(2)*cosh(x)^2 + 4*(sqr
t(2)*cosh(x)^3 + sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*log(2*(cosh(x)^4 + 4*c
osh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*
(2*cosh(x)^3 + cosh(x))*sinh(x) + (sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*si
nh(x) + sqrt(2)*sinh(x)^2 + sqrt(2))*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^
2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + s
inh(x)^2)) - 5*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^
2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)*log((
cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 2*sqrt((cosh(x)^2 + sinh(x)^2)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 1)/(cosh(x)^2 + 2*cosh(x)*si
nh(x) + sinh(x)^2)) + 5*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3
*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) +
```

```
 1)*log((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 2*sqrt((cosh(x)^2 + si
nh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 1)/(cosh(x)^2 + 2*c
osh(x)*sinh(x) + sinh(x)^2)) - 4*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2
 - 1)*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)
^2)))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*si
nh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\tanh^2(x)+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x)**2)**(3/2),x)
```

```
[Out] Integral((tanh(x)**2 + 1)**(3/2), x)
```

Giac [B] time = 1.25641, size = 273, normalized size = 5.46

$$-\frac{1}{4}\sqrt{2}\Bigg(5\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{e^{(4x)}+1}+e^{(2x)}+1}{\sqrt{2}+\sqrt{e^{(4x)}+1}-e^{(2x)}-1}\right)-\frac{4\Big(3\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^3-\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^2-\sqrt{e^{(4x)}+1}+e^{(2x)}}{\left(\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^2-2\sqrt{e^{(4x)}+1}+2e^{(2x)}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(5*sqrt(2)*log((sqrt(2) - sqrt(e^(4*x) + 1) + e^(2*x) + 1)/(sq
rt(2) + sqrt(e^(4*x) + 1) - e^(2*x) - 1)) - 4*(3*(sqrt(e^(4*x) + 1) - e^(2*
x))^3 - (sqrt(e^(4*x) + 1) - e^(2*x))^2 - sqrt(e^(4*x) + 1) + e^(2*x) - 1)/
((sqrt(e^(4*x) + 1) - e^(2*x))^2 - 2*sqrt(e^(4*x) + 1) + 2*e^(2*x) - 1)^2 +
 4*log(sqrt(e^(4*x) + 1) - e^(2*x) + 1) + 4*log(sqrt(e^(4*x) + 1) - e^(2*x)
) - 4*log(-sqrt(e^(4*x) + 1) + e^(2*x) + 1))
```

3.228 $\int \left(-1-\tanh^2(x)\right)^{3/2} dx$

Optimal. Leaf size=67

$$\frac{1}{2}\tanh(x)\sqrt{-\tanh^2(x)-1}-\frac{5}{2}\tan^{-1}\left(\frac{\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)+2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)$$

```
[Out] (-5*ArcTan[Tanh[x]/Sqrt[-1 - Tanh[x]^2]])/2 + 2*Sqrt[2]*ArcTan[(Sqrt[2]*Tan
h[x])/Sqrt[-1 - Tanh[x]^2]] + (Tanh[x]*Sqrt[-1 - Tanh[x]^2])/2
```

Rubi [A] time = 0.0473687, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3661, 416, 523, 217, 203, 377}

$$\frac{1}{2}\tanh(x)\sqrt{-\tanh^2(x)-1}-\frac{5}{2}\tan^{-1}\left(\frac{\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)+2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - Tanh[x]^2)^(3/2),x]
```

```
[Out] (-5*ArcTan[Tanh[x]/Sqrt[-1 - Tanh[x]^2]])/2 + 2*Sqrt[2]*ArcTan[(Sqrt[2]*Tan
h[x])/Sqrt[-1 - Tanh[x]^2]] + (Tanh[x]*Sqrt[-1 - Tanh[x]^2])/2
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\int \left(-1-\tanh^2(x)\right)^{3/2} dx = \text{Subst}\left(\int \frac{\left(-1-x^2\right)^{3/2}}{1-x^2} dx, x, \tanh(x)\right)$$
$$= \frac{1}{2}\tanh(x)\sqrt{-1-\tanh^2(x)} - \frac{1}{2}\text{Subst}\left(\int \frac{-3-5x^2}{\sqrt{-1-x^2}\left(1-x^2\right)} dx, x, \tanh(x)\right)$$
$$= \frac{1}{2}\tanh(x)\sqrt{-1-\tanh^2(x)} - \frac{5}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-1-x^2}} dx, x, \tanh(x)\right) + 4\,\text{Subst}\left(\int \frac{}{\sqrt{-1-}}\right.$$
$$= \frac{1}{2}\tanh(x)\sqrt{-1-\tanh^2(x)} - \frac{5}{2}\text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \frac{\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right) + 4\,\text{Subst}\left(\int \frac{}{1}\right.$$
$$= -\frac{5}{2}\tan^{-1}\left(\frac{\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right) + 2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right) + \frac{1}{2}\tanh(x)\sqrt{-1-\tanh^2(}$$

Mathematica [A] time = 0.0671395, size = 76, normalized size = 1.13

$$-\frac{\left(-\tanh^2(x)-1\right)^{3/2}\left(-4\sqrt{2}\sinh^{-1}\left(\sqrt{2}\sinh(x)\right)\cosh^3(x)+\sinh(x)\sqrt{\cosh(2x)}\cosh(x)+5\cosh^3(x)\tanh^{-1}\left(\frac{\sinh(x)}{\sqrt{\cosh(2}}\right.\right.}{2\cosh^{\frac{3}{2}}(2x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - Tanh[x]^2)^(3/2),x]
```

```
[Out] -((-4*Sqrt[2]*ArcSinh[Sqrt[2]*Sinh[x]]*Cosh[x]^3 + 5*ArcTanh[Sinh[x]/Sqrt[C
osh[2*x]]]*Cosh[x]^3 + Cosh[x]*Sqrt[Cosh[2*x]]*Sinh[x])*(-1 - Tanh[x]^2)^(3
/2))/(2*Cosh[2*x]^(3/2))
```

Maple [B] time = 0.027, size = 211, normalized size = 3.2

$$\frac{1}{6}\left(-(1+\tanh(x))^2+2\tanh(x)\right)^{\frac{3}{2}}+\frac{\tanh(x)}{4}\sqrt{-(1+\tanh(x))^2+2\tanh(x)}-\frac{5}{4}\arctan\left(\tanh(x)\frac{}{\sqrt{-(1+\tanh}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1-tanh(x)^2)^(3/2),x)
```

```
[Out] 1/6*(-(1+tanh(x))^2+2*tanh(x))^(3/2)+1/4*tanh(x)*(-(1+tanh(x))^2+2*tanh(x))
^(1/2)-5/4*arctan(tanh(x)/(-(1+tanh(x))^2+2*tanh(x))^(1/2))-(-(1+tanh(x))^2
+2*tanh(x))^(1/2)+2^(1/2)*arctan(1/4*(2*tanh(x)-2)*2^(1/2)/(-(1+tanh(x))^2+
2*tanh(x))^(1/2))-1/6*(-(tanh(x)-1)^2-2*tanh(x))^(3/2)+1/4*tanh(x)*(-(tanh(
x)-1)^2-2*tanh(x))^(1/2)-5/4*arctan(tanh(x)/(-(tanh(x)-1)^2-2*tanh(x))^(1/2
))+(-(tanh(x)-1)^2-2*tanh(x))^(1/2)-2^(1/2)*arctan(1/4*(-2-2*tanh(x))*2^(1/
2)/(-(tanh(x)-1)^2-2*tanh(x))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\tanh(x)^2 - 1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tanh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-tanh(x)^2 - 1)^(3/2), x)
```

Fricas [C] time = 1.84119, size = 1103, normalized size = 16.46

$$\frac{2\left(\sqrt{-2}e^{(4x)} + 2\sqrt{-2}e^{(2x)} + \sqrt{-2}\right)\log\left(2\left(\sqrt{-2}\sqrt{-2e^{(4x)}-2} + 2e^{(2x)} + 2\right)e^{(-2x)}\right) - 2\left(\sqrt{-2}e^{(4x)} + 2\sqrt{-2}e^{(2x)} + \sqrt{-2}\right)\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(sqrt(-2)*e^(4*x) + 2*sqrt(-2)*e^(2*x) + sqrt(-2))*log(2*(sqrt(-2)*s
qrt(-2*e^(4*x) - 2) + 2*e^(2*x) + 2)*e^(-2*x)) - 2*(sqrt(-2)*e^(4*x) + 2*sq
rt(-2)*e^(2*x) + sqrt(-2))*log(-2*(sqrt(-2)*sqrt(-2*e^(4*x) - 2) - 2*e^(2*x
) - 2)*e^(-2*x)) + (5*I*e^(4*x) + 10*I*e^(2*x) + 5*I)*log((4*I*sqrt(-2*e^(4
*x) - 2) - 4*e^(2*x) + 4)*e^(-2*x)) + (-5*I*e^(4*x) - 10*I*e^(2*x) - 5*I)*l
og((-4*I*sqrt(-2*e^(4*x) - 2) - 4*e^(2*x) + 4)*e^(-2*x)) - 2*(sqrt(-2)*e^(4
*x) + 2*sqrt(-2)*e^(2*x) + sqrt(-2))*log(4*(sqrt(-2*e^(4*x) - 2)*(e^(2*x) -
 2) + sqrt(-2)*e^(4*x) - sqrt(-2)*e^(2*x) + 2*sqrt(-2))*e^(-4*x)) + 2*(sqrt
(-2)*e^(4*x) + 2*sqrt(-2)*e^(2*x) + sqrt(-2))*log(4*(sqrt(-2*e^(4*x) - 2)*(
e^(2*x) - 2) - sqrt(-2)*e^(4*x) + sqrt(-2)*e^(2*x) - 2*sqrt(-2))*e^(-4*x))
+ 2*sqrt(-2*e^(4*x) - 2)*(e^(2*x) - 1))/(e^(4*x) + 2*e^(2*x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\tanh^2(x) - 1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tanh(x)**2)**(3/2),x)
```

```
[Out] Integral((-tanh(x)**2 - 1)**(3/2), x)
```

Giac [C] time = 1.21645, size = 273, normalized size = 4.07

$$\frac{1}{2}\sqrt{2}\left(5\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\left(-i\sqrt{e^{(4x)}+1}-i\right)e^{(-2x)}-i\right)\right)+\frac{2\left(3\left(i\sqrt{e^{(4x)}+1}+i\right)^{3}e^{(-6x)}+i\left(i\sqrt{e^{(4x)}+1}+i\right)^{2}e^{(-4x)}+\right.}{\left(i\left(i\sqrt{e^{(4x)}+1}+i\right)^{2}e^{(-4x)}+\left(-2i\sqrt{e^{(4x)}+1}\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(5*sqrt(2)*arctan(-1/2*sqrt(2)*((-I*sqrt(e^(4*x) + 1) - I)*e^(-
2*x) - I)) + 2*(3*(I*sqrt(e^(4*x) + 1) + I)^3*e^(-6*x) + I*(I*sqrt(e^(4*x)
+ 1) + I)^2*e^(-4*x) + I*(sqrt(e^(4*x) + 1) + 1)*e^(-2*x) - I)/(I*(I*sqrt(e
^(4*x) + 1) + I)^2*e^(-4*x) + (-2*I*sqrt(e^(4*x) + 1) - 2*I)*e^(-2*x) + I)^
2 + 2*I*log(-(sqrt(e^(4*x) + 1) + 1)*e^(-2*x)) - 2*I*log(-(-I*sqrt(e^(4*x)
+ 1) - I)*e^(-2*x) + I) + 2*I*log(-(-I*sqrt(e^(4*x) + 1) - I)*e^(-2*x) - I)
)
```

3.229 $\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)}} dx$

Optimal. Leaf size=70

$$-\frac{\left(a+b\tanh^2(x)\right)^{3/2}}{3b^2}+\frac{(a-b)\sqrt{a+b\tanh^2(x)}}{b^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/Sqrt[a + b] + ((a - b)*Sqrt[a +
b*Tanh[x]^2])/b^2 - (a + b*Tanh[x]^2)^(3/2)/(3*b^2)
```

Rubi [A] time = 0.136063, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3670, 446, 88, 63, 208}

$$-\frac{\left(a+b\tanh^2(x)\right)^{3/2}}{3b^2}+\frac{(a-b)\sqrt{a+b\tanh^2(x)}}{b^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^5/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/Sqrt[a + b] + ((a - b)*Sqrt[a +
b*Tanh[x]^2])/b^2 - (a + b*Tanh[x]^2)^(3/2)/(3*b^2)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)}}\,dx &= \text{Subst}\left(\int \frac{x^5}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(1-x)\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a-b}{b\sqrt{a+bx}}+\frac{1}{(1-x)\sqrt{a+bx}}-\frac{\sqrt{a+bx}}{b}\right)dx, x, \tanh^2(x)\right)\\
&= \frac{(a-b)\sqrt{a+b\tanh^2(x)}}{b^2}-\frac{\left(a+b\tanh^2(x)\right)^{3/2}}{3b^2}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \tanh^2(x\right.\\
&= \frac{(a-b)\sqrt{a+b\tanh^2(x)}}{b^2}-\frac{\left(a+b\tanh^2(x)\right)^{3/2}}{3b^2}+\frac{\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}+\frac{(a-b)\sqrt{a+b\tanh^2(x)}}{b^2}-\frac{\left(a+b\tanh^2(x)\right)^{3/2}}{3b^2}
\end{aligned}$$

Mathematica [A] time = 0.482292, size = 68, normalized size = 0.97

$$\frac{\text{sech}^2(x)((a-2b)\cosh(2x)+a-b)\sqrt{a+b\tanh^2(x)}}{3b^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^5/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/Sqrt[a + b] + ((a - b + (a - 2*b
)*Cosh[2*x])*Sech[x]^2*Sqrt[a + b*Tanh[x]^2])/(3*b^2)
```

Maple [B] time = 0.053, size = 164, normalized size = 2.3

$$-\frac{(\tanh(x))^2}{3b}\sqrt{a+b(\tanh(x))^2}+\frac{2a}{3b^2}\sqrt{a+b(\tanh(x))^2}-\frac{1}{b}\sqrt{a+b(\tanh(x))^2}+\frac{1}{2}\ln\left(\frac{1}{1+\tanh(x)}\left(2a+2b-2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a+b*tanh(x)^2)^(1/2),x)
```

```
[Out] -1/3*tanh(x)^2/b*(a+b*tanh(x)^2)^(1/2)+2/3*a/b^2*(a+b*tanh(x)^2)^(1/2)-(a+b
*tanh(x)^2)^(1/2)/b+1/2/(a+b)^(1/2)*ln((2*a+2*b-2*(1+tanh(x))*b+2*(a+b)^(1/
```

```
2)*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))/(1+tanh(x)))+1/2/(a+b)^(1/2
)*ln((2*a+2*b+2*(tanh(x)-1)*b+2*(a+b)^(1/2)*((tanh(x)-1)^2*b+2*(tanh(x)-1)*
b+a+b)^(1/2))/(tanh(x)-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^5}{\sqrt{b\tanh(x)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(tanh(x)^5/sqrt(b*tanh(x)^2 + a), x)
```

Fricas [B] time = 4.37119, size = 7692, normalized size = 109.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")

[Out] [1/12*(3*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 + 3*b^2*c
osh(x)^4 + 3*(5*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 3*b^2*cosh(x)^2 + 4*(5*b^2
*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 3*(5*b^2*cosh(x)^4 + 6*b^2*cosh(x)^
2 + b^2)*sinh(x)^2 + b^2 + 6*(b^2*cosh(x)^5 + 2*b^2*cosh(x)^3 + b^2*cosh(x)
)*sinh(x))*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(
x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2
*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*
cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2
 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 +
 b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x
)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x
))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*co
sh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*
a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^
2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^
2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*
a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^
4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 +
 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh
(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*
(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*
a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 1
5*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6
*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 3*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)
^5 + b^2*sinh(x)^6 + 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 + b^2)*sinh(x)^4
+ 3*b^2*cosh(x)^2 + 4*(5*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 3*(5*b^
2*cosh(x)^4 + 6*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 6*(b^2*cosh(x)^5 + 2
*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a + b)*log(-((a + b)*cosh(x)^4
+ 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(a
 + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + s
```

```
inh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a -
 b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 - b
*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 8
*sqrt(2)*((a^2 - a*b - 2*b^2)*cosh(x)^4 + 4*(a^2 - a*b - 2*b^2)*cosh(x)*sin
h(x)^3 + (a^2 - a*b - 2*b^2)*sinh(x)^4 + 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^
2 - a*b - 2*b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 - a*b - 2*b^2 + 4*(
(a^2 - a*b - 2*b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))*sqrt(((a + b)
*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + si
nh(x)^2)))/((a*b^2 + b^3)*cosh(x)^6 + 6*(a*b^2 + b^3)*cosh(x)*sinh(x)^5 + (
a*b^2 + b^3)*sinh(x)^6 + 3*(a*b^2 + b^3)*cosh(x)^4 + 3*(a*b^2 + b^3 + 5*(a*
b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a*b^2 + b^3)*cosh(x)^3 + 3*(a*b^2 +
 b^3)*cosh(x))*sinh(x)^3 + a*b^2 + b^3 + 3*(a*b^2 + b^3)*cosh(x)^2 + 3*(5*(
a*b^2 + b^3)*cosh(x)^4 + a*b^2 + b^3 + 6*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2
 + 6*((a*b^2 + b^3)*cosh(x)^5 + 2*(a*b^2 + b^3)*cosh(x)^3 + (a*b^2 + b^3)*c
osh(x))*sinh(x)), -1/6*(3*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*si
nh(x)^6 + 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 3*b^2*cos
h(x)^2 + 4*(5*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 3*(5*b^2*cosh(x)^4
 + 6*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 6*(b^2*cosh(x)^5 + 2*b^2*cosh(x
)^3 + b^2*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(a*cosh(x)^2 + 2*a*
cosh(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2
 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/
((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh
(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 +
a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*
a^2 + a*b - b^2)*cosh(x))*sinh(x))) + 3*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh
(x)^5 + b^2*sinh(x)^6 + 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 + b^2)*sinh(x)
^4 + 3*b^2*cosh(x)^2 + 4*(5*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 3*(5
*b^2*cosh(x)^4 + 6*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 6*(b^2*cosh(x)^5
+ 2*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(cosh
(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(
x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^
2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 +
2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b
)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) - 4*sqrt(2)*((a^2 - a*b -
2*b^2)*cosh(x)^4 + 4*(a^2 - a*b - 2*b^2)*cosh(x)*sinh(x)^3 + (a^2 - a*b - 2
*b^2)*sinh(x)^4 + 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 - a*b - 2*b^2)*cosh(x
)^2 + a^2 - b^2)*sinh(x)^2 + a^2 - a*b - 2*b^2 + 4*((a^2 - a*b - 2*b^2)*cos
h(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))*sqrt(((a + b)*cosh(x)^2 + (a + b)*si
nh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a*b^2 + b^
3)*cosh(x)^6 + 6*(a*b^2 + b^3)*cosh(x)*sinh(x)^5 + (a*b^2 + b^3)*sinh(x)^6
+ 3*(a*b^2 + b^3)*cosh(x)^4 + 3*(a*b^2 + b^3 + 5*(a*b^2 + b^3)*cosh(x)^2)*s
inh(x)^4 + 4*(5*(a*b^2 + b^3)*cosh(x)^3 + 3*(a*b^2 + b^3)*cosh(x))*sinh(x)^
3 + a*b^2 + b^3 + 3*(a*b^2 + b^3)*cosh(x)^2 + 3*(5*(a*b^2 + b^3)*cosh(x)^4
+ a*b^2 + b^3 + 6*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 6*((a*b^2 + b^3)*cos
h(x)^5 + 2*(a*b^2 + b^3)*cosh(x)^3 + (a*b^2 + b^3)*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^5(x)}{\sqrt{a + b\tanh^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**5/(a+b*tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(tanh(x)**5/sqrt(a + b*tanh(x)**2), x)
```

<u>**Giac**</u> **[F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.230 $\int \frac{\tanh^4(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$

Optimal. Leaf size=88

$$\frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{2b^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}-\frac{\tanh(x)\sqrt{a+b\tanh^2(x)}}{2b}$$

```
[Out] ((a - 2*b)*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/(2*b^(3/2)) +
ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/Sqrt[a + b] - (Tanh[x]
*Sqrt[a + b*Tanh[x]^2])/(2*b)
```

Rubi [A] time = 0.123222, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3670, 479, 523, 217, 206, 377}

$$\frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{2b^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}-\frac{\tanh(x)\sqrt{a+b\tanh^2(x)}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ((a - 2*b)*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/(2*b^(3/2)) +
ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/Sqrt[a + b] - (Tanh[x]
*Sqrt[a + b*Tanh[x]^2])/(2*b)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)$$

$$= -\frac{\tanh(x)\sqrt{a+b\tanh^2(x)}}{2b} + \frac{\text{Subst}\left(\int \frac{a+(-a+2b)x^2}{\left(1-x^2\right)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{2b}$$

$$= -\frac{\tanh(x)\sqrt{a+b\tanh^2(x)}}{2b} + \frac{(a-2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{2b} + \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{\ }}\right.$$

$$= -\frac{\tanh(x)\sqrt{a+b\tanh^2(x)}}{2b} + \frac{(a-2b)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{2b} + \text{Subst}\left(\int \frac{1}{1-(a+}\right.$$

$$= \frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{2b^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}} - \frac{\tanh(x)\sqrt{a+b\tanh^2(x)}}{2b}$$

Mathematica [C] time = 4.43375, size = 208, normalized size = 2.36

$$\frac{\tanh(x)\left(\sqrt{2}a(a+b)\sqrt{\frac{\text{csch}^2(x)((a+b)\cosh(2x)+a-b)}{b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\text{csch}^2(x)((a+b)\cosh(2x)+a-b)}{b}}}{\sqrt{2}}\right),1\right) - (a+b)\text{sech}^2(x)((a+b)\text{co}\right.}{2\sqrt{2}b(a+b)\sqrt{\text{sech}^2(x)((a+b)\cosh(2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ((Sqrt[2]*a*(a + b)*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*Ellipti
cF[ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1] - 2*
Sqrt[2]*a*b*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*EllipticPi[b/(a
 + b), ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1]
- (a + b)*(a - b + (a + b)*Cosh[2*x])*Sech[x]^2)*Tanh[x])/(2*Sqrt[2]*b*(a +
 b)*Sqrt[(a - b + (a + b)*Cosh[2*x])*Sech[x]^2])
```

Maple [B] time = 0.048, size = 178, normalized size = 2.

$$-\frac{\tanh (x)}{2b}\sqrt{a+b(\tanh (x))^2}+\frac{a}{2}\ln \left(\tanh (x)\sqrt{b}+\sqrt{a+b(\tanh (x))^2}\right)b^{-\frac{3}{2}}-\ln \left(\tanh (x)\sqrt{b}+\sqrt{a+b(\tanh (x))^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+b*tanh(x)^2)^(1/2),x)
```

```
[Out] -1/2*(a+b*tanh(x)^2)^(1/2)*tanh(x)/b+1/2*a/b^(3/2)*ln(tanh(x)*b^(1/2)+(a+b*
tanh(x)^2)^(1/2))-ln(tanh(x)*b^(1/2)+(a+b*tanh(x)^2)^(1/2))/b^(1/2)-1/2/(a+
b)^(1/2)*ln((2*a+2*b-2*(1+tanh(x))*b+2*(a+b)^(1/2)*((1+tanh(x))^2*b-2*(1+ta
nh(x))*b+a+b)^(1/2))/(1+tanh(x)))+1/2/(a+b)^(1/2)*ln((2*a+2*b+2*(tanh(x)-1)
*b+2*(a+b)^(1/2)*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))/(tanh(x)-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh (x)^4}{\sqrt{b\tanh (x)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^4/sqrt(b*tanh(x)^2 + a), x)
```

Fricas [B] time = 4.5804, size = 15340, normalized size = 174.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*b^2*cosh
(x)^2 + 2*(3*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 + b^2*
cosh(x))*sinh(x))*sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^
3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^
6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2
 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4
*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a
*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3
)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3
)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2
*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x
)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*
sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^
6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x
)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*
cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b
 - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(
x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a -
```

```
b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x
)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)
^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sin
h(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(
x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) - ((a^2 - a*b - 2*b^2)*cosh(x)^4 +
 4*(a^2 - a*b - 2*b^2)*cosh(x)*sinh(x)^3 + (a^2 - a*b - 2*b^2)*sinh(x)^4 +
2*(a^2 - a*b - 2*b^2)*cosh(x)^2 + 2*(3*(a^2 - a*b - 2*b^2)*cosh(x)^2 + a^2
- a*b - 2*b^2)*sinh(x)^2 + a^2 - a*b - 2*b^2 + 4*((a^2 - a*b - 2*b^2)*cosh(
x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x))*sqrt(b)*log(-((a + 2*b)*cosh(x
)^4 + 4*(a + 2*b)*cosh(x)*sinh(x)^3 + (a + 2*b)*sinh(x)^4 + 2*(a - 2*b)*cos
h(x)^2 + 2*(3*(a + 2*b)*cosh(x)^2 + a - 2*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)
^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt(((a + b)*cosh(x)^2 + (
a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*
((a + 2*b)*cosh(x)^3 + (a - 2*b)*cosh(x))*sinh(x) + a + 2*b)/(cosh(x)^4 + 4
*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^
2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + (b^2*cosh(x)^4 + 4*b^2*cosh(x)*
sinh(x)^3 + b^2*sinh(x)^4 + 2*b^2*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + b^2)*sin
h(x)^2 + b^2 + 4*(b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a + b)*log(((a
 + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cos
h(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)
*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a +
b)*cosh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2)) - 2*sqrt(2)*((a*b + b^2)*cosh(x)^2 + 2*(a*b + b^2)*cosh(x)*sin
h(x) + (a*b + b^2)*sinh(x)^2 - a*b - b^2)*sqrt(((a + b)*cosh(x)^2 + (a + b)
*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a*b^2 +
 b^3)*cosh(x)^4 + 4*(a*b^2 + b^3)*cosh(x)*sinh(x)^3 + (a*b^2 + b^3)*sinh(x)
^4 + a*b^2 + b^3 + 2*(a*b^2 + b^3)*cosh(x)^2 + 2*(a*b^2 + b^3 + 3*(a*b^2 +
b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a*b^2 + b^3)*cosh(x)^3 + (a*b^2 + b^3)*cosh
(x))*sinh(x)), -1/4*(2*((a^2 - a*b - 2*b^2)*cosh(x)^4 + 4*(a^2 - a*b - 2*b^
2)*cosh(x)*sinh(x)^3 + (a^2 - a*b - 2*b^2)*sinh(x)^4 + 2*(a^2 - a*b - 2*b^2
)*cosh(x)^2 + 2*(3*(a^2 - a*b - 2*b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*sinh(
x)^2 + a^2 - a*b - 2*b^2 + 4*((a^2 - a*b - 2*b^2)*cosh(x)^3 + (a^2 - a*b -
2*b^2)*cosh(x))*sinh(x))*sqrt(-b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sin
h(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2
+ a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 +
4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(
3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cos
h(x))*sinh(x) + a + b)) - (b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*si
nh(x)^4 + 2*b^2*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 4*(
b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(
x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b
^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh
(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^
5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4
 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4
+ 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2
*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 +
2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a
*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2
+ 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh
(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)
^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*
cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sin
h(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 -
2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a +
 b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*
(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*
b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)
```

```
^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3
+ 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) - (b^2*cosh(x)
^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*b^2*cosh(x)^2 + 2*(3*b^2*c
osh(x)^2 + b^2)*sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*
sqrt(a + b)*log(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*
sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)
*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*
cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sin
h(x)^2)) + 4*((a + b)*cosh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 +
2*cosh(x)*sinh(x) + sinh(x)^2)) + 2*sqrt(2)*((a*b + b^2)*cosh(x)^2 + 2*(a*b
 + b^2)*cosh(x)*sinh(x) + (a*b + b^2)*sinh(x)^2 - a*b - b^2)*sqrt(((a + b)*
cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sin
h(x)^2)))/((a*b^2 + b^3)*cosh(x)^4 + 4*(a*b^2 + b^3)*cosh(x)*sinh(x)^3 + (a
*b^2 + b^3)*sinh(x)^4 + a*b^2 + b^3 + 2*(a*b^2 + b^3)*cosh(x)^2 + 2*(a*b^2
+ b^3 + 3*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a*b^2 + b^3)*cosh(x)^3 +
 (a*b^2 + b^3)*cosh(x))*sinh(x)), -1/4*(2*(b^2*cosh(x)^4 + 4*b^2*cosh(x)*si
nh(x)^3 + b^2*sinh(x)^4 + 2*b^2*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + b^2)*sinh(
x)^2 + b^2 + 4*(b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(s
qrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - a - b)*sqrt(-a -
b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh
(x)*sinh(x) + sinh(x)^2))/((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh(x)*si
nh(x)^3 + (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^2)*cosh(x)^2 + (6*(a*b +
 b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(
a*b + b^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x))) + 2*(b^2*cosh
(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*b^2*cosh(x)^2 + 2*(3*b^
2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x
))*sqrt(-a - b)*arctan(sqrt(2)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a +
b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)
*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)) + ((a^
2 - a*b - 2*b^2)*cosh(x)^4 + 4*(a^2 - a*b - 2*b^2)*cosh(x)*sinh(x)^3 + (a^2
 - a*b - 2*b^2)*sinh(x)^4 + 2*(a^2 - a*b - 2*b^2)*cosh(x)^2 + 2*(3*(a^2 - a
*b - 2*b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2 - a*b - 2*b^2 +
4*((a^2 - a*b - 2*b^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x))*sq
rt(b)*log(-((a + 2*b)*cosh(x)^4 + 4*(a + 2*b)*cosh(x)*sinh(x)^3 + (a + 2*b)
*sinh(x)^4 + 2*(a - 2*b)*cosh(x)^2 + 2*(3*(a + 2*b)*cosh(x)^2 + a - 2*b)*si
nh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)
*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x
)*sinh(x) + sinh(x)^2)) + 4*((a + 2*b)*cosh(x)^3 + (a - 2*b)*cosh(x))*sinh(
x) + a + 2*b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2
 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + 2*s
qrt(2)*((a*b + b^2)*cosh(x)^2 + 2*(a*b + b^2)*cosh(x)*sinh(x) + (a*b + b^2)
*sinh(x)^2 - a*b - b^2)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b
)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a*b^2 + b^3)*cosh(x)^4 +
4*(a*b^2 + b^3)*cosh(x)*sinh(x)^3 + (a*b^2 + b^3)*sinh(x)^4 + a*b^2 + b^3 +
 2*(a*b^2 + b^3)*cosh(x)^2 + 2*(a*b^2 + b^3 + 3*(a*b^2 + b^3)*cosh(x)^2)*si
nh(x)^2 + 4*((a*b^2 + b^3)*cosh(x)^3 + (a*b^2 + b^3)*cosh(x))*sinh(x)), -1/
2*((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*b^2*cosh(x)
^2 + 2*(3*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 + b^2*cos
h(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(
x) + b*sinh(x)^2 - a - b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*si
nh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a*b + b^2)*
cosh(x)^4 + 4*(a*b + b^2)*cosh(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + (a^2
- a*b - 2*b^2)*cosh(x)^2 + (6*(a*b + b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*si
nh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a*b + b^2)*cosh(x)^3 + (a^2 - a*b - 2*b
^2)*cosh(x))*sinh(x))) + ((a^2 - a*b - 2*b^2)*cosh(x)^4 + 4*(a^2 - a*b - 2*
b^2)*cosh(x)*sinh(x)^3 + (a^2 - a*b - 2*b^2)*sinh(x)^4 + 2*(a^2 - a*b - 2*b
^2)*cosh(x)^2 + 2*(3*(a^2 - a*b - 2*b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*sin
h(x)^2 + a^2 - a*b - 2*b^2 + 4*((a^2 - a*b - 2*b^2)*cosh(x)^3 + (a^2 - a*b
- 2*b^2)*cosh(x))*sinh(x))*sqrt(-b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*s
```

```
inh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^
2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4
+ 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2
*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*c
osh(x))*sinh(x) + a + b)) + (b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*
sinh(x)^4 + 2*b^2*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 4
*(b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*sqrt(-a
 - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x)
 + (a + b)*sinh(x)^2 + a + b)) + sqrt(2)*((a*b + b^2)*cosh(x)^2 + 2*(a*b +
b^2)*cosh(x)*sinh(x) + (a*b + b^2)*sinh(x)^2 - a*b - b^2)*sqrt(((a + b)*cos
h(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x
)^2)))/((a*b^2 + b^3)*cosh(x)^4 + 4*(a*b^2 + b^3)*cosh(x)*sinh(x)^3 + (a*b^
2 + b^3)*sinh(x)^4 + a*b^2 + b^3 + 2*(a*b^2 + b^3)*cosh(x)^2 + 2*(a*b^2 + b
^3 + 3*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a*b^2 + b^3)*cosh(x)^3 + (a
*b^2 + b^3)*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+b*tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(tanh(x)**4/sqrt(a + b*tanh(x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.231 $\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$

Optimal. Leaf size=47

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{\sqrt{a+b\tanh^2(x)}}{b}$$

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/Sqrt[a + b] - Sqrt[a + b*Tanh[x]
^2]/b
```

Rubi [A] time = 0.105658, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3670, 446, 80, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{\sqrt{a+b\tanh^2(x)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/Sqrt[a + b] - Sqrt[a + b*Tanh[x]
^2]/b
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx &= \text{Subst}\left(\int \frac{x^3}{\left(1-x^2\right)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(1-x)\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)\\
&= -\frac{\sqrt{a+b\tanh^2(x)}}{b}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)\\
&= -\frac{\sqrt{a+b\tanh^2(x)}}{b}+\frac{\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{\sqrt{a+b\tanh^2(x)}}{b}
\end{aligned}$$

Mathematica [A] time = 0.0920205, size = 47, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{\sqrt{a+b\tanh^2(x)}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/Sqrt[a + b] - Sqrt[a + b*Tanh[x]
^2]/b
```

Maple [B] time = 0.045, size = 129, normalized size = 2.7

$$-\frac{1}{b}\sqrt{a+b\left(\tanh(x)\right)^2}+\frac{1}{2}\ln\left(\frac{1}{1+\tanh(x)}\left(2a+2b-2\left(1+\tanh(x)\right)b+2\sqrt{a+b}\sqrt{\left(1+\tanh(x)\right)^2b-2\left(1+\tanh(x\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+b*tanh(x)^2)^(1/2),x)
```

```
[Out] -(a+b*tanh(x)^2)^(1/2)/b+1/2/(a+b)^(1/2)*ln((2*a+2*b-2*(1+tanh(x))*b+2*(a+b
)^(1/2)*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))/(1+tanh(x)))+1/2/(a+b)
^(1/2)*ln((2*a+2*b+2*(tanh(x)-1)*b+2*(a+b)^(1/2)*((tanh(x)-1)^2*b+2*(tanh(x
```

```
)-1)*b+a+b)^(1/2))/(tanh(x)-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^3}{\sqrt{b\tanh(x)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^3/sqrt(b*tanh(x)^2 + a), x)
```

Fricas [B] time = 2.68573, size = 4629, normalized size = 98.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*sqrt(a + b)*log
(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*
b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a
^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2
*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a
^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*
cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*c
osh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2
*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2
*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6
*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6
 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cos
h(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (
3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a
^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^
2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a
+ b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
+ sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5
+ (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(
x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20
*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(
x)^6)) + (b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*sqrt(a + b)*
log(-((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 -
 2*b*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2
 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 +
 (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) +
4*((a + b)*cosh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*s
inh(x) + sinh(x)^2)) - 4*sqrt(2)*(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*
sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a*b + b^
2)*cosh(x)^2 + 2*(a*b + b^2)*cosh(x)*sinh(x) + (a*b + b^2)*sinh(x)^2 + a*b
+ b^2), -1/2*((b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*sqrt(-a
 - b)*arctan(sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a +
 b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh
```

```
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a
*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)
^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b
+ b^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh(x)))
 + (b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*sqrt(-a - b)*arcta
n(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqrt
(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sin
h(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a +
b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x
)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + 2*sqrt(2)
*(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 -
2*cosh(x)*sinh(x) + sinh(x)^2)))/((a*b + b^2)*cosh(x)^2 + 2*(a*b + b^2)*cos
h(x)*sinh(x) + (a*b + b^2)*sinh(x)^2 + a*b + b^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^3(x)}{\sqrt{a + b\tanh^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+b*tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(tanh(x)**3/sqrt(a + b*tanh(x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.232 $\int \frac{\tanh^2(x)}{\sqrt{a+b\tanh^2(x)}} dx$

Optimal. Leaf size=60

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}$$

```
[Out] -(ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/Sqrt[b]) + ArcTanh[(Sqrt
[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/Sqrt[a + b]
```

Rubi [A] time = 0.0932622, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3670, 483, 217, 206, 377}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] -(ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/Sqrt[b]) + ArcTanh[(Sqrt
[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/Sqrt[a + b]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 483

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx &= \text{Subst}\left(\int \frac{x^2}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)\\ &= -\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)+\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)\\ &= -\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)+\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}\end{aligned}$$

Mathematica [C] time = 0.376258, size = 101, normalized size = 1.68

$$-\frac{a\coth(x)\sqrt{\operatorname{sech}^2(x)((a+b)\cosh(2x)+a-b)}\,\Pi\left(\frac{b}{a+b};\sin^{-1}\left(\frac{\sqrt{\frac{(a-b+(a+b)\cosh(2x))\operatorname{csch}^2(x)}{b}}}{\sqrt{2}}\right)\middle|1\right)}{b(a+b)\sqrt{\frac{\operatorname{csch}^2(x)((a+b)\cosh(2x)+a-b)}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] -((a*Coth[x]*EllipticPi[b/(a + b), ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])
*Csch[x]^2)/b]/Sqrt[2]], 1]*Sqrt[(a - b + (a + b)*Cosh[2*x])*Sech[x]^2])/(b
*(a + b)*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]))
```

Maple [B] time = 0.044, size = 137, normalized size = 2.3

$$-\ln\left(\tanh(x)\sqrt{b}+\sqrt{a+b(\tanh(x))^2}\right)\frac{1}{\sqrt{b}}-\frac{1}{2}\ln\left(\frac{1}{1+\tanh(x)}\left(2a+2b-2(1+\tanh(x))b+2\sqrt{a+b}\sqrt{(1+\tanh(}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a+b*tanh(x)^2)^(1/2),x)
```

```
[Out] -ln(tanh(x)*b^(1/2)+(a+b*tanh(x)^2)^(1/2))/b^(1/2)-1/2/(a+b)^(1/2)*ln((2*a+
2*b-2*(1+tanh(x))*b+2*(a+b)^(1/2)*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/
2))/(1+tanh(x)))+1/2/(a+b)^(1/2)*ln((2*a+2*b+2*(tanh(x)-1)*b+2*(a+b)^(1/2)*
((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))/(tanh(x)-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^2}{\sqrt{b\tanh(x)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(tanh(x)^2/sqrt(b*tanh(x)^2 + a), x)
```

Fricas [B] time = 3.37956, size = 9661, normalized size = 161.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")

[Out] [1/4*(sqrt(a + b)*b*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)
*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b
^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*co
sh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6
*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b
^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^
5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))
*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh
(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3
- 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2
+ sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*
cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^
2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4
- 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2
*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x
))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x
)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a
*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3
- 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*
cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*c
osh(x)*sinh(x)^5 + sinh(x)^6)) + 2*(a + b)*sqrt(b)*log(-((a + 2*b)*cosh(x)^
4 + 4*(a + 2*b)*cosh(x)*sinh(x)^3 + (a + 2*b)*sinh(x)^4 + 2*(a - 2*b)*cosh(
x)^2 + 2*(3*(a + 2*b)*cosh(x)^2 + a - 2*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2
 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt(((a + b)*cosh(x)^2 + (a
+ b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((
a + 2*b)*cosh(x)^3 + (a - 2*b)*cosh(x))*sinh(x) + a + 2*b)/(cosh(x)^4 + 4*c
osh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2
+ 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + sqrt(a + b)*b*log(((a + b)*cosh(x
)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(
3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x)
 + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
 a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3
 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2))
)/(a*b + b^2), 1/4*(4*(a + b)*sqrt(-b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x
)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(
x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)
^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2
```

```
+ 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b
)*cosh(x))*sinh(x) + a + b)) + sqrt(a + b)*b*log(-((a*b^2 + b^3)*cosh(x)^8
+ 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 +
2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6
 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (
a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^
3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(
14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b +
4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^
3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2
+ 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b
^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5
 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 +
4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(
x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^
2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b
 - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*s
inh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^
2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 +
 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 +
6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*
cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + sqrt(a + b)*b*log
(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a
*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2
*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a
+ b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((
a + b)*cosh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(
x) + sinh(x)^2)))/(a*b + b^2), -1/2*(sqrt(-a - b)*b*arctan(sqrt(2)*(b*cosh(
x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - a - b)*sqrt(-a - b)*sqrt(((a + b
)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + s
inh(x)^2))/((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh(x)*sinh(x)^3 + (a*b
+ b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^2)*cosh(x)^2 + (6*(a*b + b^2)*cosh(x)^2
 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a*b + b^2)*cosh
(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x))) + sqrt(-a - b)*b*arctan(sqrt
(2)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^2 + 2*(a + b)*cosh
(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)) - (a + b)*sqrt(b)*log(-((a + 2*b)
*cosh(x)^4 + 4*(a + 2*b)*cosh(x)*sinh(x)^3 + (a + 2*b)*sinh(x)^4 + 2*(a - 2
*b)*cosh(x)^2 + 2*(3*(a + 2*b)*cosh(x)^2 + a - 2*b)*sinh(x)^2 - 2*sqrt(2)*(
cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt(((a + b)*cosh(x
)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2
)) + 4*((a + 2*b)*cosh(x)^3 + (a - 2*b)*cosh(x))*sinh(x) + a + 2*b)/(cosh(x
)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*c
osh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)))/(a*b + b^2), -1/2*(sqrt(-
a - b)*b*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 -
a - b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(c
osh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a*b + b^2)*cosh(x)^4 + 4*(a*b
+ b^2)*cosh(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^2)*cosh
(x)^2 + (6*(a*b + b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2*a
*b + b^2 + 2*(2*(a*b + b^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x
))) - 2*(a + b)*sqrt(-b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + si
nh(x)^2 - 1)*sqrt(-b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)
*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)
*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sin
h(x) + a + b)) + sqrt(-a - b)*b*arctan(sqrt(2)*sqrt(-a - b)*sqrt(((a + b)*c
osh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh
(x)^2))/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2
+ a + b)))/(a*b + b^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^2(x)}{\sqrt{a + b\tanh^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+b*tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(tanh(x)**2/sqrt(a + b*tanh(x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.233 $\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$

Optimal. Leaf size=29

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/Sqrt[a + b]
```

Rubi [A] time = 0.0643284, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3670, 444, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/Sqrt[a + b]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \text{Subst}\left(\int \frac{x}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

Mathematica [A] time = 0.0142, size = 29, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/Sqrt[a + b]
```

Maple [B] time = 0.05, size = 114, normalized size = 3.9

$$\frac{1}{2}\ln\left(\frac{1}{1+\tanh(x)}\left(2a+2b-2(1+\tanh(x))b+2\sqrt{a+b}\sqrt{(1+\tanh(x))^2b-2(1+\tanh(x))b+a+b}\right)\right)\frac{1}{\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*tanh(x)^2)^(1/2),x)
```

```
[Out] 1/2/(a+b)^(1/2)*ln((2*a+2*b-2*(1+tanh(x))*b+2*(a+b)^(1/2)*((1+tanh(x))^2*b-
2*(1+tanh(x))*b+a+b)^(1/2))/(1+tanh(x)))+1/2/(a+b)^(1/2)*ln((2*a+2*b+2*(tan
h(x)-1)*b+2*(a+b)^(1/2)*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))/(tanh(
x)-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)}{\sqrt{b\tanh(x)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)/sqrt(b*tanh(x)^2 + a), x)
```

Fricas [B] time = 2.14557, size = 3792, normalized size = 130.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*si
nh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3
+ a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(
x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^
3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3
+ 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 +
 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*si
nh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)
^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 +
 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + s
qrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cos
h(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*c
osh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 1
8*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3
*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*
sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2
 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^
3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 +
 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cos
h(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh
(x)*sinh(x)^5 + sinh(x)^6)) + sqrt(a + b)*log(-((a + b)*cosh(x)^4 + 4*(a +
b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(a + b)*cos
h(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2
- 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 - b*cosh(x))
*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a + b), -1
/2*(sqrt(-a - b)*arctan(sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh
(x)^2 + a + b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
 - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 +
 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b
^2)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a
^2 + 2*a*b + b^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x)
)*sinh(x))) + sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
 - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(
a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(
a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x
))*sinh(x) + a + b)))/(a + b)]
```

Sympy [A] time = 1.03482, size = 31, normalized size = 1.07

$$-\frac{\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^{2}(x)}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)**2)**(1/2),x)
```

```
[Out] -atan(sqrt(a + b*tanh(x)**2)/sqrt(-a - b))/sqrt(-a - b)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.234 $\int \frac{1}{\sqrt{a+b\tanh^2(x)}}\,dx$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}$$

```
[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/Sqrt[a + b]
```

Rubi [A] time = 0.0269137, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3661, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Tanh[x]^2],x]

[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/Sqrt[a + b]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\tanh^2(x)}}\,dx = \text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)$$
$$= \text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}$$

Mathematica [A] time = 0.0213452, size = 31, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/Sqrt[a + b]
```

Maple [B] time = 0.046, size = 114, normalized size = 3.7

$$-\frac{1}{2}\ln\left(\frac{1}{1+\tanh(x)}\left(2a+2b-2(1+\tanh(x))b+2\sqrt{a+b}\sqrt{(1+\tanh(x))^2 b-2(1+\tanh(x))b+a+b}\right)\right)\frac{1}{\sqrt{a+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tanh(x)^2)^(1/2),x)
```

```
[Out] -1/2/(a+b)^(1/2)*ln((2*a+2*b-2*(1+tanh(x))*b+2*(a+b)^(1/2)*((1+tanh(x))^2*b
-2*(1+tanh(x))*b+a+b)^(1/2))/(1+tanh(x)))+1/2/(a+b)^(1/2)*ln((2*a+2*b+2*(ta
nh(x)-1)*b+2*(a+b)^(1/2)*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))/(tanh
(x)-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\tanh(x)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*tanh(x)^2 + a), x)
```

Fricas [B] time = 2.22997, size = 3568, normalized size = 115.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*s
inh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2
 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh
(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b
^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3
 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5
- 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*s
inh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x
)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 -
3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 +
sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*co
sh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*
cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 -
18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(
3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))
*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^
2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b
^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 -
3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*co
sh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cos
h(x)*sinh(x)^5 + sinh(x)^6)) + sqrt(a + b)*log(((a + b)*cosh(x)^4 + 4*(a +
b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*(a + b)*cos
h(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2
+ 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 + a*cosh(x))
*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a + b), -1
/2*(sqrt(-a - b)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh
(x)^2 - a - b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
 - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a*b + b^2)*cosh(x)^4 +
 4*(a*b + b^2)*cosh(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b
^2)*cosh(x)^2 + (6*(a*b + b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a
^2 + 2*a*b + b^2 + 2*(2*(a*b + b^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x)
)*sinh(x))) + sqrt(-a - b)*arctan(sqrt(2)*sqrt(-a - b)*sqrt(((a + b)*cosh(x
)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2
))/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a +
 b)))/(a + b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b \tanh^2(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*tanh(x)**2), x)
```

<u>Giac</u> [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.235 $\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$

Optimal. Leaf size=56

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -(ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]]/Sqrt[a]) + ArcTanh[Sqrt[a + b*Tanh
[x]^2]/Sqrt[a + b]]/Sqrt[a + b]
```

Rubi [A] time = 0.112608, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3670, 446, 86, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]]/Sqrt[a]) + ArcTanh[Sqrt[a + b*Tanh
[x]^2]/Sqrt[a + b]]/Sqrt[a + b]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{x\left(1-x^2\right)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(1-x)x\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b}+\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}
\end{aligned}$$

Mathematica [A] time = 0.039176, size = 56, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]]/Sqrt[a]) + ArcTanh[Sqrt[a + b*Tanh
[x]^2]/Sqrt[a + b]]/Sqrt[a + b]
```

Maple [F] time = 0.15, size = 0, normalized size = 0.

$$\int \coth(x)\frac{1}{\sqrt{a+b(\tanh(x))^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*tanh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)/(a+b*tanh(x)^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{\sqrt{b\tanh(x)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)/sqrt(b*tanh(x)^2 + a), x)
```

Fricas [B] time = 3.47749, size = 10107, normalized size = 180.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a + b)*a*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*
sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^
3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cos
h(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 +
b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^
3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5
 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*
sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(
x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3
 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 +
 sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*c
osh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2
*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 +
 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*
(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x)
)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*
a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3
 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*c
osh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*co
sh(x)*sinh(x)^5 + sinh(x)^6)) + 2*(a + b)*sqrt(a)*log(-((2*a + b)*cosh(x)^4
 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a - b)*cosh(x
)^2 + 2*(3*(2*a + b)*cosh(x)^2 + 2*a - b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2
+ 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt(((a + b)*cosh(x)^2 + (a +
 b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2
*a + b)*cosh(x)^3 + (2*a - b)*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*co
sh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 +
 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)) + sqrt(a + b)*a*log(-((a + b)*cosh(x
)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(
3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x)
 + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
 a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3
 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2))
)/(a^2 + a*b), 1/4*(4*sqrt(-a)*(a + b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x
)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(
x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)
^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2
```

```
+ 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b
)*cosh(x))*sinh(x) + a + b)) + sqrt(a + b)*a*log(((a^3 + a^2*b)*cosh(x)^8 +
 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a
^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6
+ 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6
*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a
^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(1
4*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b
 - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a
^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 +
 a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^
3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5
+ a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4
*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x
)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2
 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*
b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*si
nh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 +
 a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2
+ b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6
*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*c
osh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + sqrt(a + b)*a*log(
-((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b
*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2
*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a
+ b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((
a + b)*cosh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(
x) + sinh(x)^2)))/(a^2 + a*b), -1/2*(a*sqrt(-a - b)*arctan(sqrt(2)*(a*cosh(
x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a - b)*sqrt(((a + b
)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + s
inh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2
+ a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2
 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a*b)*cosh
(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh(x))) + a*sqrt(-a - b)*arctan(sqrt
(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((a +
 b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
 sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sin
h(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 +
4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) - (a + b)*sqrt(a)
*log(-((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh
(x)^4 + 2*(2*a - b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 + 2*a - b)*sinh(x)
^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt
(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sin
h(x) + sinh(x)^2)) + 4*((2*a + b)*cosh(x)^3 + (2*a - b)*cosh(x))*sinh(x) +
2*a + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)
*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)))/(a^2 + a*
b), 1/2*(2*sqrt(-a)*(a + b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2 + 1)*sqrt(-a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a -
b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a +
 b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a +
 b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*
sinh(x) + a + b)) - a*sqrt(-a - b)*arctan(sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x
)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a
+ b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2
+ a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4
+ (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + a*b -
b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 +
a*b - b^2)*cosh(x))*sinh(x))) - a*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 +
2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (
```

```
a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a
+ b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b
)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x
)^3 + (a - b)*cosh(x))*sinh(x) + a + b)))/(a^2 + a*b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{\sqrt{a + b\tanh^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(coth(x)/sqrt(a + b*tanh(x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.236 $\int \frac{\coth^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$

Optimal. Leaf size=51

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}-\frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{a}$$

```
[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/Sqrt[a + b] - (Coth[x]
*Sqrt[a + b*Tanh[x]^2])/a
```

Rubi [A] time = 0.0948034, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3670, 480, 12, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}-\frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/Sqrt[a + b] - (Coth[x]
*Sqrt[a + b*Tanh[x]^2])/a
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{x^2\left(1-x^2\right)\sqrt{a+bx^2}}\,dx,x,\tanh(x)\right)\\
&= -\frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{a}+\frac{\text{Subst}\left(\int \frac{a}{\left(1-x^2\right)\sqrt{a+bx^2}}\,dx,x,\tanh(x)\right)}{a}\\
&= -\frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{a}+\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+bx^2}}\,dx,x,\tanh(x)\right)\\
&= -\frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{a}+\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx,x,\frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}-\frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{a}
\end{aligned}
$$

Mathematica [C] time = 5.57917, size = 123, normalized size = 2.41

$$
\frac{\tanh(x)\left(\frac{(a+b)^2((a+b)\cosh(2x)+a-b)^2\,{}_2F_1\left(2,2;\frac{5}{2};-\frac{(a+b)\sinh^2(x)}{a}\right)}{a^3}+3\operatorname{csch}^2(x)\left(a\coth^2(x)+2b\right)\sin^{-1}\left(\sqrt{-\frac{(a+b)\sinh^2(x)}{a}}\right)\sqrt{-\frac{(a+b)\sinh^4}{}}\right.}{3(a+b)\sqrt{a+b\tanh^2(x)}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[x]^2/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ((((a + b)^2*(a - b + (a + b)*Cosh[2*x])^2*Hypergeometric2F1[2, 2, 5/2, -((
(a + b)*Sinh[x]^2)/a)])/a^3 + 3*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]]*(2*b
 + a*Coth[x]^2)*Csch[x]^2*Sqrt[-(((a + b)*(b + a*Coth[x]^2)*Sinh[x]^4)/a^2)
])*Tanh[x])/(3*(a + b)*Sqrt[a + b*Tanh[x]^2])
```

Maple [F] time = 0.151, size = 0, normalized size = 0.

$$
\int (\coth(x))^2 \frac{1}{\sqrt{a+b(\tanh(x))^2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+b*tanh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^2/(a+b*tanh(x)^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)^2}{\sqrt{b\tanh(x)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)^2/sqrt(b*tanh(x)^2 + a), x)
```

Fricas [B] time = 2.51858, size = 4405, normalized size = 86.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*sqrt(a + b)*log
(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b
^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2
+ b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*
b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(
a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)
*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*
cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^
2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 +
b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(
a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^
6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*co
sh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 -
(a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^
2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b
^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a
 + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
 + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5
 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh
(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 2
0*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh
(x)^6)) + (a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*sqrt(a + b)
*log(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 +
 2*a*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2
 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 +
 (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) +
4*((a + b)*cosh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*s
inh(x) + sinh(x)^2)) - 4*sqrt(2)*(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*
sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^2 + a*
b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (a^2 + a*b)*sinh(x)^2 - a^2
- a*b), -1/2*((a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*sqrt(-a
 - b)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - a -
 b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh
```

```
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a*b + b^2)*cosh(x)^4 + 4*(a*b + b
^2)*cosh(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^2)*cosh(x)
^2 + (6*(a*b + b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2*a*b
+ b^2 + 2*(2*(a*b + b^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x)))
 + (a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*sqrt(-a - b)*arcta
n(sqrt(2)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)
/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^2 + 2*(a + b
)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)) + 2*sqrt(2)*(a + b)*sqrt(((
a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x
) + sinh(x)^2)))/((a^2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (
a^2 + a*b)*sinh(x)^2 - a^2 - a*b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2(x)}{\sqrt{a + b\tanh^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+b*tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(coth(x)**2/sqrt(a + b*tanh(x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.237 $\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$

Optimal. Leaf size=88

$$-\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{\coth^2(x)\sqrt{a+b\tanh^2(x)}}{2a}$$

```
[Out] -((2*a - b)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]])/(2*a^(3/2)) + ArcTanh[S
qrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/Sqrt[a + b] - (Coth[x]^2*Sqrt[a + b*Tanh[
x]^2])/(2*a)
```

Rubi [A] time = 0.165949, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3670, 446, 103, 156, 63, 208}

$$-\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{\coth^2(x)\sqrt{a+b\tanh^2(x)}}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] -((2*a - b)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]])/(2*a^(3/2)) + ArcTanh[S
qrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/Sqrt[a + b] - (Coth[x]^2*Sqrt[a + b*Tanh[
x]^2])/(2*a)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{x^3\left(1-x^2\right)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(1-x)x^2\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)\\
&= -\frac{\coth^2(x)\sqrt{a+b\tanh^2(x)}}{2a} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-2a+b)-\frac{bx}{2}}{(1-x)x\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)}{2a}\\
&= -\frac{\coth^2(x)\sqrt{a+b\tanh^2(x)}}{2a} + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right) + \frac{(2a-b)\,\text{Subst}\left(\right.}{}\\
&= -\frac{\coth^2(x)\sqrt{a+b\tanh^2(x)}}{2a} + \frac{\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b} + \frac{(2a-b)\,\text{Subst}\left(\int \right.}{}\\
&= -\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{2a^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} - \frac{\coth^2(x)\sqrt{a+b\tanh^2(x)}}{2a}
\end{aligned}
$$

Mathematica [A] time = 0.422721, size = 107, normalized size = 1.22

$$
\frac{\left(-2a^2-ab+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)+\sqrt{a}\left(2a\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-(a+b)\coth^2(x)\sqrt{a+b\tanh^2(x)}\right)}{2a^{3/2}(a+b)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ((-2*a^2 - a*b + b^2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]] + Sqrt[a]*(2*a
*Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (a + b)*Coth[x]^2
*Sqrt[a + b*Tanh[x]^2]))/(2*a^(3/2)*(a + b))
```

Maple [F] time = 0.154, size = 0, normalized size = 0.

$$\int (\coth(x))^3 \frac{1}{\sqrt{a + b(\tanh(x))^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a+b*tanh(x)^2)^(1/2),x)

[Out] int(coth(x)^3/(a+b*tanh(x)^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)^3}{\sqrt{b\tanh(x)^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(coth(x)^3/sqrt(b*tanh(x)^2 + a), x)
```

Fricas [B] time = 4.62634, size = 15786, normalized size = 179.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")

[Out] [1/4*((a^2*cosh(x)^4 + 4*a^2*cosh(x)*sinh(x)^3 + a^2*sinh(x)^4 - 2*a^2*cosh
(x)^2 + 2*(3*a^2*cosh(x)^2 - a^2)*sinh(x)^2 + a^2 + 4*(a^2*cosh(x)^3 - a^2*
cosh(x))*sinh(x))*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b
)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6
 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 +
a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b
- a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b -
a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)
*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)
*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b -
b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)
^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*s
inh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6
 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)
^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*c
osh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b
+ b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x
))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b
)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)
^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^
3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh
```

```
(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x
)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) - ((2*a^2 + a*b - b^2)*cosh(x)^4 +
4*(2*a^2 + a*b - b^2)*cosh(x)*sinh(x)^3 + (2*a^2 + a*b - b^2)*sinh(x)^4 - 2
*(2*a^2 + a*b - b^2)*cosh(x)^2 + 2*(3*(2*a^2 + a*b - b^2)*cosh(x)^2 - 2*a^2
 - a*b + b^2)*sinh(x)^2 + 2*a^2 + a*b - b^2 + 4*((2*a^2 + a*b - b^2)*cosh(x
)^3 - (2*a^2 + a*b - b^2)*cosh(x))*sinh(x))*sqrt(a)*log(-((2*a + b)*cosh(x)
^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a - b)*cosh
(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 + 2*a - b)*sinh(x)^2 + 2*sqrt(2)*(cosh(x)^
2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt(((a + b)*cosh(x)^2 + (a
 + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(
(2*a + b)*cosh(x)^3 + (2*a - b)*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*
cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2
 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)) + (a^2*cosh(x)^4 + 4*a^2*cosh(x)*s
inh(x)^3 + a^2*sinh(x)^4 - 2*a^2*cosh(x)^2 + 2*(3*a^2*cosh(x)^2 - a^2)*sinh
(x)^2 + a^2 + 4*(a^2*cosh(x)^3 - a^2*cosh(x))*sinh(x))*sqrt(a + b)*log(-((a
 + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cos
h(x)^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)
*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a +
b)*cosh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2)) - 2*sqrt(2)*((a^2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sin
h(x) + (a^2 + a*b)*sinh(x)^2 + a^2 + a*b)*sqrt(((a + b)*cosh(x)^2 + (a + b)
*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^3 + a
^2*b)*cosh(x)^4 + 4*(a^3 + a^2*b)*cosh(x)*sinh(x)^3 + (a^3 + a^2*b)*sinh(x)
^4 + a^3 + a^2*b - 2*(a^3 + a^2*b)*cosh(x)^2 - 2*(a^3 + a^2*b - 3*(a^3 + a^
2*b)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + a^2*b)*cosh(x)^3 - (a^3 + a^2*b)*cosh
(x))*sinh(x)), 1/4*(2*((2*a^2 + a*b - b^2)*cosh(x)^4 + 4*(2*a^2 + a*b - b^2
)*cosh(x)*sinh(x)^3 + (2*a^2 + a*b - b^2)*sinh(x)^4 - 2*(2*a^2 + a*b - b^2)
*cosh(x)^2 + 2*(3*(2*a^2 + a*b - b^2)*cosh(x)^2 - 2*a^2 - a*b + b^2)*sinh(x
)^2 + 2*a^2 + a*b - b^2 + 4*((2*a^2 + a*b - b^2)*cosh(x)^3 - (2*a^2 + a*b -
 b^2)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh
(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
 a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4
*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3
*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh
(x))*sinh(x) + a + b)) + (a^2*cosh(x)^4 + 4*a^2*cosh(x)*sinh(x)^3 + a^2*sin
h(x)^4 - 2*a^2*cosh(x)^2 + 2*(3*a^2*cosh(x)^2 - a^2)*sinh(x)^2 + a^2 + 4*(a
^2*cosh(x)^3 - a^2*cosh(x))*sinh(x))*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)
^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3
 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x
)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5
+ (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 +
 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 +
4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a
^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*
(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a
^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2
+ b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x
)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4
 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*co
sh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(
x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 +
2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b
)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a
^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*
b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6
 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 +
15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + (a^2*cosh(x)^4
 + 4*a^2*cosh(x)*sinh(x)^3 + a^2*sinh(x)^4 - 2*a^2*cosh(x)^2 + 2*(3*a^2*cos
```

```
h(x)^2 - a^2)*sinh(x)^2 + a^2 + 4*(a^2*cosh(x)^3 - a^2*cosh(x))*sinh(x))*sq
rt(a + b)*log(-((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*s
inh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*
(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*c
osh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh
(x)^2)) + 4*((a + b)*cosh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2
*cosh(x)*sinh(x) + sinh(x)^2)) - 2*sqrt(2)*((a^2 + a*b)*cosh(x)^2 + 2*(a^2
+ a*b)*cosh(x)*sinh(x) + (a^2 + a*b)*sinh(x)^2 + a^2 + a*b)*sqrt(((a + b)*c
osh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh
(x)^2)))/((a^3 + a^2*b)*cosh(x)^4 + 4*(a^3 + a^2*b)*cosh(x)*sinh(x)^3 + (a^
3 + a^2*b)*sinh(x)^4 + a^3 + a^2*b - 2*(a^3 + a^2*b)*cosh(x)^2 - 2*(a^3 + a
^2*b - 3*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + a^2*b)*cosh(x)^3 -
(a^3 + a^2*b)*cosh(x))*sinh(x)), -1/4*(2*(a^2*cosh(x)^4 + 4*a^2*cosh(x)*sin
h(x)^3 + a^2*sinh(x)^4 - 2*a^2*cosh(x)^2 + 2*(3*a^2*cosh(x)^2 - a^2)*sinh(x
)^2 + a^2 + 4*(a^2*cosh(x)^3 - a^2*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sq
rt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a - b
)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(
x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sin
h(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2 +
a*b)*cosh(x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a
^2 + a*b)*cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh(x))) + 2*(a^2*cosh(
x)^4 + 4*a^2*cosh(x)*sinh(x)^3 + a^2*sinh(x)^4 - 2*a^2*cosh(x)^2 + 2*(3*a^2
*cosh(x)^2 - a^2)*sinh(x)^2 + a^2 + 4*(a^2*cosh(x)^3 - a^2*cosh(x))*sinh(x)
)*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 -
1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(
x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(
x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(
x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) +
 a + b)) + ((2*a^2 + a*b - b^2)*cosh(x)^4 + 4*(2*a^2 + a*b - b^2)*cosh(x)*s
inh(x)^3 + (2*a^2 + a*b - b^2)*sinh(x)^4 - 2*(2*a^2 + a*b - b^2)*cosh(x)^2
+ 2*(3*(2*a^2 + a*b - b^2)*cosh(x)^2 - 2*a^2 - a*b + b^2)*sinh(x)^2 + 2*a^2
 + a*b - b^2 + 4*((2*a^2 + a*b - b^2)*cosh(x)^3 - (2*a^2 + a*b - b^2)*cosh(
x))*sinh(x))*sqrt(a)*log(-((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x
)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a - b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^
2 + 2*a - b)*sinh(x)^2 + 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
^2 + 1)*sqrt(a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(
x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a + b)*cosh(x)^3 + (2*a - b)
*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 +
 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh
(x) + 1)) + 2*sqrt(2)*((a^2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x
) + (a^2 + a*b)*sinh(x)^2 + a^2 + a*b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*si
nh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^3 + a^2*
b)*cosh(x)^4 + 4*(a^3 + a^2*b)*cosh(x)*sinh(x)^3 + (a^3 + a^2*b)*sinh(x)^4
+ a^3 + a^2*b - 2*(a^3 + a^2*b)*cosh(x)^2 - 2*(a^3 + a^2*b - 3*(a^3 + a^2*b
)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + a^2*b)*cosh(x)^3 - (a^3 + a^2*b)*cosh(x)
)*sinh(x)), 1/2*(((2*a^2 + a*b - b^2)*cosh(x)^4 + 4*(2*a^2 + a*b - b^2)*cos
h(x)*sinh(x)^3 + (2*a^2 + a*b - b^2)*sinh(x)^4 - 2*(2*a^2 + a*b - b^2)*cosh
(x)^2 + 2*(3*(2*a^2 + a*b - b^2)*cosh(x)^2 - 2*a^2 - a*b + b^2)*sinh(x)^2 +
 2*a^2 + a*b - b^2 + 4*((2*a^2 + a*b - b^2)*cosh(x)^3 - (2*a^2 + a*b - b^2)
*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2 + 1)*sqrt(-a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a -
b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a +
 b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a +
 b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*
sinh(x) + a + b)) - (a^2*cosh(x)^4 + 4*a^2*cosh(x)*sinh(x)^3 + a^2*sinh(x)^
4 - 2*a^2*cosh(x)^2 + 2*(3*a^2*cosh(x)^2 - a^2)*sinh(x)^2 + a^2 + 4*(a^2*co
sh(x)^3 - a^2*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(a*cosh(x)^2 +
2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a - b)*sqrt(((a + b)*cosh(
x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^
```

```
2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*
sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^
2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 +
 (2*a^2 + a*b - b^2)*cosh(x))*sinh(x))) - (a^2*cosh(x)^4 + 4*a^2*cosh(x)*si
nh(x)^3 + a^2*sinh(x)^4 - 2*a^2*cosh(x)^2 + 2*(3*a^2*cosh(x)^2 - a^2)*sinh(
x)^2 + a^2 + 4*(a^2*cosh(x)^3 - a^2*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(s
qrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((
a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x
) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*
sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2
 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) - sqrt(2)*((a^
2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (a^2 + a*b)*sinh(x)^2
+ a^2 + a*b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^
2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^3 + a^2*b)*cosh(x)^4 + 4*(a^3 + a^
2*b)*cosh(x)*sinh(x)^3 + (a^3 + a^2*b)*sinh(x)^4 + a^3 + a^2*b - 2*(a^3 + a
^2*b)*cosh(x)^2 - 2*(a^3 + a^2*b - 3*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^2 + 4
*((a^3 + a^2*b)*cosh(x)^3 - (a^3 + a^2*b)*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^3(x)}{\sqrt{a + b\tanh^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(a+b*tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(coth(x)**3/sqrt(a + b*tanh(x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.238 $\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=72

$$-\frac{a^2}{b^2(a+b)\sqrt{a+b\tanh^2(x)}}-\frac{\sqrt{a+b\tanh^2(x)}}{b^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}$$

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/(a + b)^(3/2) - a^2/(b^2*(a + b)
*Sqrt[a + b*Tanh[x]^2]) - Sqrt[a + b*Tanh[x]^2]/b^2
```

Rubi [A] time = 0.16333, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3670, 446, 87, 63, 208}

$$-\frac{a^2}{b^2(a+b)\sqrt{a+b\tanh^2(x)}}-\frac{\sqrt{a+b\tanh^2(x)}}{b^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^5/(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/(a + b)^(3/2) - a^2/(b^2*(a + b)
*Sqrt[a + b*Tanh[x]^2]) - Sqrt[a + b*Tanh[x]^2]/b^2
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx &= \text{Subst}\left(\int \frac{x^5}{\left(1-x^2\right)\left(a+bx^2\right)^{3/2}}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(1-x)(a+bx)^{3/2}}\,dx, x, \tanh^2(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{b(a+b)(a+bx)^{3/2}}-\frac{1}{b\sqrt{a+bx}}-\frac{1}{(a+b)(-1+x)\sqrt{a+bx}}\right)dx, x, \tanh^2(x)\right)\\
&= -\frac{a^2}{b^2(a+b)\sqrt{a+b\tanh^2(x)}}-\frac{\sqrt{a+b\tanh^2(x)}}{b^2}-\frac{\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)}{2(a+b)}\\
&= -\frac{a^2}{b^2(a+b)\sqrt{a+b\tanh^2(x)}}-\frac{\sqrt{a+b\tanh^2(x)}}{b^2}-\frac{\text{Subst}\left(\int \frac{1}{-1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b(a+b)}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{a^2}{b^2(a+b)\sqrt{a+b\tanh^2(x)}}-\frac{\sqrt{a+b\tanh^2(x)}}{b^2}
\end{aligned}$$

Mathematica [C] time = 0.102017, size = 67, normalized size = 0.93

$$\frac{b^2\left(-{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\tanh^2(x)+a}{a+b}\right)\right)-(a+b)\left(2a+b\tanh^2(x)-b\right)}{b^2(a+b)\sqrt{a+b\tanh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^5/(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] (-(b^2*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tanh[x]^2)/(a + b)]) - (a + b
)*(2*a - b + b*Tanh[x]^2))/(b^2*(a + b)*Sqrt[a + b*Tanh[x]^2])
```

Maple [B] time = 0.029, size = 322, normalized size = 4.5

$$-\frac{(\tanh(x))^2}{b}\frac{1}{\sqrt{a+b(\tanh(x))^2}}-2\frac{a}{b^2\sqrt{a+b(\tanh(x))^2}}+\frac{1}{b}\frac{1}{\sqrt{a+b(\tanh(x))^2}}-\frac{1}{2b+2a}\frac{1}{\sqrt{(1+\tanh(x))^2b-2(1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a+b*tanh(x)^2)^(3/2),x)
```

```
[Out] -tanh(x)^2/b/(a+b*tanh(x)^2)^(1/2)-2*a/b^2/(a+b*tanh(x)^2)^(1/2)+1/b/(a+b*t
anh(x)^2)^(1/2)-1/2/(a+b)/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)-b/(a+
b)*(2*(1+tanh(x))*b-2*b)/(4*b*(a+b)-4*b^2)/((1+tanh(x))^2*b-2*(1+tanh(x))*b
+a+b)^(1/2)+1/2/(a+b)^(3/2)*ln((2*a+2*b-2*(1+tanh(x))*b+2*(a+b)^(1/2)*((1+t
anh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))/(1+tanh(x)))-1/2/(a+b)/((tanh(x)-1)
^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)+b/(a+b)*(2*(tanh(x)-1)*b+2*b)/(4*b*(a+b)-4*
b^2)/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)+1/2/(a+b)^(3/2)*ln((2*a+2*
b+2*(tanh(x)-1)*b+2*(a+b)^(1/2)*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)
)/(tanh(x)-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^5}{\left(b\tanh(x)^2 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^5/(b*tanh(x)^2 + a)^(3/2), x)
```

Fricas [B] time = 5.04083, size = 9975, normalized size = 138.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a*b^2 + b^3)*cosh(x)^6 + 6*(a*b^2 + b^3)*cosh(x)*sinh(x)^5 + (a*b^2
 + b^3)*sinh(x)^6 + (3*a*b^2 - b^3)*cosh(x)^4 + (3*a*b^2 - b^3 + 15*(a*b^2
+ b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a*b^2 + b^3)*cosh(x)^3 + (3*a*b^2 - b^3
)*cosh(x))*sinh(x)^3 + a*b^2 + b^3 + (3*a*b^2 - b^3)*cosh(x)^2 + (15*(a*b^2
 + b^3)*cosh(x)^4 + 3*a*b^2 - b^3 + 6*(3*a*b^2 - b^3)*cosh(x)^2)*sinh(x)^2
+ 2*(3*(a*b^2 + b^3)*cosh(x)^5 + 2*(3*a*b^2 - b^3)*cosh(x)^3 + (3*a*b^2 - b
^3)*cosh(x))*sinh(x))*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a
^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(
x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^
3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^
2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*
b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^
2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 +
b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*
b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cos
h(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^
2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(
x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cos
h(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a
^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*
a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*co
sh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
 - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cos
h(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh
(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*
```

```
sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*si
nh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a*b^2 + b^3)*cosh(x)^6 + 6*
(a*b^2 + b^3)*cosh(x)*sinh(x)^5 + (a*b^2 + b^3)*sinh(x)^6 + (3*a*b^2 - b^3)
*cosh(x)^4 + (3*a*b^2 - b^3 + 15*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*
(a*b^2 + b^3)*cosh(x)^3 + (3*a*b^2 - b^3)*cosh(x))*sinh(x)^3 + a*b^2 + b^3
+ (3*a*b^2 - b^3)*cosh(x)^2 + (15*(a*b^2 + b^3)*cosh(x)^4 + 3*a*b^2 - b^3 +
 6*(3*a*b^2 - b^3)*cosh(x)^2)*sinh(x)^2 + 2*(3*(a*b^2 + b^3)*cosh(x)^5 + 2*
(3*a*b^2 - b^3)*cosh(x)^3 + (3*a*b^2 - b^3)*cosh(x))*sinh(x))*sqrt(a + b)*l
og(-((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 -
2*b*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2
+ 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 +
(a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4
*((a + b)*cosh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*si
nh(x) + sinh(x)^2)) - 4*sqrt(2)*((2*a^3 + 4*a^2*b + 3*a*b^2 + b^3)*cosh(x)^
4 + 4*(2*a^3 + 4*a^2*b + 3*a*b^2 + b^3)*cosh(x)*sinh(x)^3 + (2*a^3 + 4*a^2*
b + 3*a*b^2 + b^3)*sinh(x)^4 + 2*a^3 + 4*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 +
 2*a^2*b - a*b^2 - b^3)*cosh(x)^2 + 2*(2*a^3 + 2*a^2*b - a*b^2 - b^3 + 3*(2
*a^3 + 4*a^2*b + 3*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^3 + 4*a^2*b
+ 3*a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 2*a^2*b - a*b^2 - b^3)*cosh(x))*sinh(
x))*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cos
h(x)*sinh(x) + sinh(x)^2)))/((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)^
6 + 6*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)*sinh(x)^5 + (a^3*b^2 +
3*a^2*b^3 + 3*a*b^4 + b^5)*sinh(x)^6 + a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5
+ (3*a^3*b^2 + 5*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + (3*a^3*b^2 + 5*a^2*b^3
+ a*b^4 - b^5 + 15*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)^2)*sinh(x)
^4 + 4*(5*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)^3 + (3*a^3*b^2 + 5*
a^2*b^3 + a*b^4 - b^5)*cosh(x))*sinh(x)^3 + (3*a^3*b^2 + 5*a^2*b^3 + a*b^4
- b^5)*cosh(x)^2 + (3*a^3*b^2 + 5*a^2*b^3 + a*b^4 - b^5 + 15*(a^3*b^2 + 3*a
^2*b^3 + 3*a*b^4 + b^5)*cosh(x)^4 + 6*(3*a^3*b^2 + 5*a^2*b^3 + a*b^4 - b^5)
*cosh(x)^2)*sinh(x)^2 + 2*(3*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)^
5 + 2*(3*a^3*b^2 + 5*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 + (3*a^3*b^2 + 5*a^2*
b^3 + a*b^4 - b^5)*cosh(x))*sinh(x)), -1/2*(((a*b^2 + b^3)*cosh(x)^6 + 6*(a
*b^2 + b^3)*cosh(x)*sinh(x)^5 + (a*b^2 + b^3)*sinh(x)^6 + (3*a*b^2 - b^3)*c
osh(x)^4 + (3*a*b^2 - b^3 + 15*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a
*b^2 + b^3)*cosh(x)^3 + (3*a*b^2 - b^3)*cosh(x))*sinh(x)^3 + a*b^2 + b^3 +
(3*a*b^2 - b^3)*cosh(x)^2 + (15*(a*b^2 + b^3)*cosh(x)^4 + 3*a*b^2 - b^3 + 6
*(3*a*b^2 - b^3)*cosh(x)^2)*sinh(x)^2 + 2*(3*(a*b^2 + b^3)*cosh(x)^5 + 2*(3
*a*b^2 - b^3)*cosh(x)^3 + (3*a*b^2 - b^3)*cosh(x))*sinh(x))*sqrt(-a - b)*ar
ctan(sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt
(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 -
2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh
(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*
(a^2 + a*b)*cosh(x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 +
2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh(x))) + ((a*b
^2 + b^3)*cosh(x)^6 + 6*(a*b^2 + b^3)*cosh(x)*sinh(x)^5 + (a*b^2 + b^3)*sin
h(x)^6 + (3*a*b^2 - b^3)*cosh(x)^4 + (3*a*b^2 - b^3 + 15*(a*b^2 + b^3)*cosh
(x)^2)*sinh(x)^4 + 4*(5*(a*b^2 + b^3)*cosh(x)^3 + (3*a*b^2 - b^3)*cosh(x))*
sinh(x)^3 + a*b^2 + b^3 + (3*a*b^2 - b^3)*cosh(x)^2 + (15*(a*b^2 + b^3)*cos
h(x)^4 + 3*a*b^2 - b^3 + 6*(3*a*b^2 - b^3)*cosh(x)^2)*sinh(x)^2 + 2*(3*(a*b
^2 + b^3)*cosh(x)^5 + 2*(3*a*b^2 - b^3)*cosh(x)^3 + (3*a*b^2 - b^3)*cosh(x)
)*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sin
h(x)^2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a -
b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a +
 b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a +
 b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*
sinh(x) + a + b)) + 2*sqrt(2)*((2*a^3 + 4*a^2*b + 3*a*b^2 + b^3)*cosh(x)^4
+ 4*(2*a^3 + 4*a^2*b + 3*a*b^2 + b^3)*cosh(x)*sinh(x)^3 + (2*a^3 + 4*a^2*b
+ 3*a*b^2 + b^3)*sinh(x)^4 + 2*a^3 + 4*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 2
*a^2*b - a*b^2 - b^3)*cosh(x)^2 + 2*(2*a^3 + 2*a^2*b - a*b^2 - b^3 + 3*(2*a
```

```
^3 + 4*a^2*b + 3*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^3 + 4*a^2*b +
3*a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 2*a^2*b - a*b^2 - b^3)*cosh(x))*sinh(x)
)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(
x)*sinh(x) + sinh(x)^2)))/((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)^6
+ 6*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)*sinh(x)^5 + (a^3*b^2 + 3*
a^2*b^3 + 3*a*b^4 + b^5)*sinh(x)^6 + a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5 +
(3*a^3*b^2 + 5*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + (3*a^3*b^2 + 5*a^2*b^3 +
a*b^4 - b^5 + 15*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)^2)*sinh(x)^4
 + 4*(5*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)^3 + (3*a^3*b^2 + 5*a^
2*b^3 + a*b^4 - b^5)*cosh(x))*sinh(x)^3 + (3*a^3*b^2 + 5*a^2*b^3 + a*b^4 -
b^5)*cosh(x)^2 + (3*a^3*b^2 + 5*a^2*b^3 + a*b^4 - b^5 + 15*(a^3*b^2 + 3*a^2
*b^3 + 3*a*b^4 + b^5)*cosh(x)^4 + 6*(3*a^3*b^2 + 5*a^2*b^3 + a*b^4 - b^5)*c
osh(x)^2)*sinh(x)^2 + 2*(3*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)^5
+ 2*(3*a^3*b^2 + 5*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 + (3*a^3*b^2 + 5*a^2*b^
3 + a*b^4 - b^5)*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**5/(a+b*tanh(x)**2)**(3/2),x)
```

```
[Out] Integral(tanh(x)**5/(a + b*tanh(x)**2)**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.239 $\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=84

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{b^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}}+\frac{a\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}}$$

```
[Out] -(ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/b^(3/2)) + ArcTanh[(Sqrt
[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(3/2) + (a*Tanh[x])/(b*(a +
 b)*Sqrt[a + b*Tanh[x]^2])
```

Rubi [A] time = 0.129904, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3670, 470, 523, 217, 206, 377}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{b^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}}+\frac{a\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/b^(3/2)) + ArcTanh[(Sqrt
[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(3/2) + (a*Tanh[x])/(b*(a +
 b)*Sqrt[a + b*Tanh[x]^2])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
```

```
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)\left(a+bx^2\right)^{3/2}}\,dx, x, \tanh(x)\right)$$

$$= \frac{a\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{\text{Subst}\left(\int \frac{a+(-a-b)x^2}{\left(1-x^2\right)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{b(a+b)}$$

$$= \frac{a\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{b} + \frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+bx^2}}\,dx,\right.}{a+b}$$

$$= \frac{a\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{b} + \frac{\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx,\right.}{a+b}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{b^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}} + \frac{a\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}}$$

Mathematica [C] time = 2.25302, size = 188, normalized size = 2.24

$$-\frac{a\tanh(x)\left(\sqrt{2}(a+b)\sqrt{\frac{\text{csch}^2(x)((a+b)\cosh(2x)+a-b)}{b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\text{csch}^2(x)((a+b)\cosh(2x)+a-b)}{b}}}{\sqrt{2}}\right),1\right)+\sqrt{2}b\sqrt{\frac{\text{csch}^2(x)((a+b)\cos}{b}}\right.}{\sqrt{2}b(a+b)^2\sqrt{\text{sech}^2(x)((a+b)\cosh(2x)+a-b)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] -((a*(-2*a - 2*b + Sqrt[2]*(a + b)*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x
]^2)/b]*EllipticF[ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sq
```

```
rt[2]], 1] + Sqrt[2]*b*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*Elli
pticPi[b/(a + b), ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sq
rt[2]], 1])*Tanh[x])/(Sqrt[2]*b*(a + b)^2*Sqrt[(a - b + (a + b)*Cosh[2*x])*
Sech[x]^2]))
```

Maple [B] time = 0.027, size = 328, normalized size = 3.9

$$\frac{\tanh(x)}{b}\frac{1}{\sqrt{a+b(\tanh(x))^2}}-\ln\left(\tanh(x)\sqrt{b}+\sqrt{a+b(\tanh(x))^2}\right)b^{-\frac{3}{2}}-\frac{\tanh(x)}{a}\frac{1}{\sqrt{a+b(\tanh(x))^2}}+\frac{1}{2b+2a}\frac{1}{\sqrt{(1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+b*tanh(x)^2)^(3/2),x)
```

```
[Out] tanh(x)/b/(a+b*tanh(x)^2)^(1/2)-1/b^(3/2)*ln(tanh(x)*b^(1/2)+(a+b*tanh(x)^2
)^(1/2))-tanh(x)/a/(a+b*tanh(x)^2)^(1/2)+1/2/(a+b)/((1+tanh(x))^2*b-2*(1+ta
nh(x))*b+a+b)^(1/2)+b/(a+b)*(2*(1+tanh(x))*b-2*b)/(4*b*(a+b)-4*b^2)/((1+tan
h(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)-1/2/(a+b)^(3/2)*ln((2*a+2*b-2*(1+tanh(
x))*b+2*(a+b)^(1/2)*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))/(1+tanh(x)
))-1/2/(a+b)/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)+b/(a+b)*(2*(tanh(x
)-1)*b+2*b)/(4*b*(a+b)-4*b^2)/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)+1
/2/(a+b)^(3/2)*ln((2*a+2*b+2*(tanh(x)-1)*b+2*(a+b)^(1/2)*((tanh(x)-1)^2*b+2
*(tanh(x)-1)*b+a+b)^(1/2))/(tanh(x)-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^4}{\left(b\tanh(x)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^4/(b*tanh(x)^2 + a)^(3/2), x)
```

Fricas [B] time = 5.32513, size = 18425, normalized size = 219.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a*b^2 + b^3)*cosh(x)^4 + 4*(a*b^2 + b^3)*cosh(x)*sinh(x)^3 + (a*b^2
 + b^3)*sinh(x)^4 + a*b^2 + b^3 + 2*(a*b^2 - b^3)*cosh(x)^2 + 2*(a*b^2 - b^
3 + 3*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a*b^2 + b^3)*cosh(x)^3 + (a*
b^2 - b^3)*cosh(x))*sinh(x))*sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*
(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^
3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4
*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3
```

```
- a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 -
a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(
a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*
b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 -
3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*
b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*
cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b
^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5
*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2
 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 -
a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3
*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(
x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 +
b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b
^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*co
sh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh
(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 2*((a^3 + 3*a^2*b + 3
*a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)*sinh(x)
^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(x)^4 + a^3 + 3*a^2*b + 3*a*b^2 +
b^3 + 2*(a^3 + a^2*b - a*b^2 - b^3)*cosh(x)^2 + 2*(a^3 + a^2*b - a*b^2 - b^
3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*cosh(x)^3 + (a^3 + a^2*b - a*b^2 - b^3)*cosh(x))*sinh(
x))*sqrt(b)*log(-((a + 2*b)*cosh(x)^4 + 4*(a + 2*b)*cosh(x)*sinh(x)^3 + (a
+ 2*b)*sinh(x)^4 + 2*(a - 2*b)*cosh(x)^2 + 2*(3*(a + 2*b)*cosh(x)^2 + a - 2
*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*s
qrt(b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*
cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + 2*b)*cosh(x)^3 + (a - 2*b)*cosh(x))
*sinh(x) + a + 2*b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cos
h(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1))
 + ((a*b^2 + b^3)*cosh(x)^4 + 4*(a*b^2 + b^3)*cosh(x)*sinh(x)^3 + (a*b^2 +
b^3)*sinh(x)^4 + a*b^2 + b^3 + 2*(a*b^2 - b^3)*cosh(x)^2 + 2*(a*b^2 - b^3 +
 3*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a*b^2 + b^3)*cosh(x)^3 + (a*b^2
 - b^3)*cosh(x))*sinh(x))*sqrt(a + b)*log(((a + b)*cosh(x)^4 + 4*(a + b)*co
sh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^
2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*
sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2
 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 + a*cosh(x))*sinh
(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*(a^2*
b + a*b^2 - (a^2*b + a*b^2)*cosh(x)^2 - 2*(a^2*b + a*b^2)*cosh(x)*sinh(x) -
 (a^2*b + a*b^2)*sinh(x)^2)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
 - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^3*b^2 + 3*a^2*b^3 +
3*a*b^4 + b^5 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)^4 + 4*(a^3*b^
2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)*sinh(x)^3 + (a^3*b^2 + 3*a^2*b^3 + 3
*a*b^4 + b^5)*sinh(x)^4 + 2*(a^3*b^2 + a^2*b^3 - a*b^4 - b^5)*cosh(x)^2 + 2
*(a^3*b^2 + a^2*b^3 - a*b^4 - b^5 + 3*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)
*cosh(x)^2)*sinh(x)^2 + 4*((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)^3
+ (a^3*b^2 + a^2*b^3 - a*b^4 - b^5)*cosh(x))*sinh(x)), 1/4*(4*((a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)*s
inh(x)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(x)^4 + a^3 + 3*a^2*b + 3*a*
b^2 + b^3 + 2*(a^3 + a^2*b - a*b^2 - b^3)*cosh(x)^2 + 2*(a^3 + a^2*b - a*b^
2 - b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3
+ 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^3 + (a^3 + a^2*b - a*b^2 - b^3)*cosh(x))
*sinh(x))*sqrt(-b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2 - 1)*sqrt(-b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(
x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(
x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(
x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) +
 a + b)) + ((a*b^2 + b^3)*cosh(x)^4 + 4*(a*b^2 + b^3)*cosh(x)*sinh(x)^3 + (
a*b^2 + b^3)*sinh(x)^4 + a*b^2 + b^3 + 2*(a*b^2 - b^3)*cosh(x)^2 + 2*(a*b^2
```

- b^3 + 3*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a*b^2 + b^3)*cosh(x)^3 + (a*b^2 - b^3)*cosh(x))*sinh(x))*sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a*b^2 + b^3)*cosh(x)^4 + 4*(a*b^2 + b^3)*cosh(x)*sinh(x)^3 + (a*b^2 + b^3)*sinh(x)^4 + a*b^2 + b^3 + 2*(a*b^2 - b^3)*cosh(x)^2 + 2*(a*b^2 - b^3 + 3*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a*b^2 + b^3)*cosh(x)^3 + (a*b^2 - b^3)*cosh(x))*sinh(x))*sqrt(a + b)*log(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*(a^2*b + a*b^2 - (a^2*b + a*b^2)*cosh(x)^2 - 2*(a^2*b + a*b^2)*cosh(x)*sinh(x) - (a^2*b + a*b^2)*sinh(x)^2)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)^4 + 4*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)*sinh(x)^3 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*sinh(x)^4 + 2*(a^3*b^2 + a^2*b^3 - a*b^4 - b^5)*cosh(x)^2 + 2*(a^3*b^2 + a^2*b^3 - a*b^4 - b^5 + 3*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)^2)*sinh(x)^2 + 4*((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)^3 + (a^3*b^2 + a^2*b^3 - a*b^4 - b^5)*cosh(x))*sinh(x)), -1/2*(((a*b^2 + b^3)*cosh(x)^4 + 4*(a*b^2 + b^3)*cosh(x)*sinh(x)^3 + (a*b^2 + b^3)*sinh(x)^4 + a*b^2 + b^3 + 2*(a*b^2 - b^3)*cosh(x)^2 + 2*(a*b^2 - b^3 + 3*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a*b^2 + b^3)*cosh(x)^3 + (a*b^2 - b^3)*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - a - b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^2)*cosh(x)^2 + (6*(a*b + b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a*b + b^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x))) + ((a*b^2 + b^3)*cosh(x)^4 + 4*(a*b^2 + b^3)*cosh(x)*sinh(x)^3 + (a*b^2 + b^3)*sinh(x)^4 + a*b^2 + b^3 + 2*(a*b^2 - b^3)*cosh(x)^2 + 2*(a*b^2 - b^3 + 3*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a*b^2 + b^3)*cosh(x)^3 + (a*b^2 - b^3)*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) - ((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)*sinh(x)^3 + (a^3 + 3*a^2*b + 3*

```
a*b^2 + b^3)*sinh(x)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 + a^2*b - a
*b^2 - b^3)*cosh(x)^2 + 2*(a^3 + a^2*b - a*b^2 - b^3 + 3*(a^3 + 3*a^2*b + 3
*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos
h(x)^3 + (a^3 + a^2*b - a*b^2 - b^3)*cosh(x))*sinh(x))*sqrt(b)*log(-((a + 2
*b)*cosh(x)^4 + 4*(a + 2*b)*cosh(x)*sinh(x)^3 + (a + 2*b)*sinh(x)^4 + 2*(a
- 2*b)*cosh(x)^2 + 2*(3*(a + 2*b)*cosh(x)^2 + a - 2*b)*sinh(x)^2 - 2*sqrt(2
)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt(((a + b)*cos
h(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x
)^2)) + 4*((a + 2*b)*cosh(x)^3 + (a - 2*b)*cosh(x))*sinh(x) + a + 2*b)/(cos
h(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 +
2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + 2*sqrt(2)*(a^2*b + a*
b^2 - (a^2*b + a*b^2)*cosh(x)^2 - 2*(a^2*b + a*b^2)*cosh(x)*sinh(x) - (a^2*
b + a*b^2)*sinh(x)^2)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^3*b^2 + 3*a^2*b^3 + 3*a*b^
4 + b^5 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)^4 + 4*(a^3*b^2 + 3*
a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)*sinh(x)^3 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4
 + b^5)*sinh(x)^4 + 2*(a^3*b^2 + a^2*b^3 - a*b^4 - b^5)*cosh(x)^2 + 2*(a^3*
b^2 + a^2*b^3 - a*b^4 - b^5 + 3*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(
x)^2)*sinh(x)^2 + 4*((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)^3 + (a^3
*b^2 + a^2*b^3 - a*b^4 - b^5)*cosh(x))*sinh(x)), -1/2*(((a*b^2 + b^3)*cosh(
x)^4 + 4*(a*b^2 + b^3)*cosh(x)*sinh(x)^3 + (a*b^2 + b^3)*sinh(x)^4 + a*b^2
+ b^3 + 2*(a*b^2 - b^3)*cosh(x)^2 + 2*(a*b^2 - b^3 + 3*(a*b^2 + b^3)*cosh(x
)^2)*sinh(x)^2 + 4*((a*b^2 + b^3)*cosh(x)^3 + (a*b^2 - b^3)*cosh(x))*sinh(x
))*sqrt(-a - b)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(
x)^2 - a - b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
- b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a*b + b^2)*cosh(x)^4 +
4*(a*b + b^2)*cosh(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^
2)*cosh(x)^2 + (6*(a*b + b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a^
2 + 2*a*b + b^2 + 2*(2*(a*b + b^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))
*sinh(x))) + ((a*b^2 + b^3)*cosh(x)^4 + 4*(a*b^2 + b^3)*cosh(x)*sinh(x)^3 +
 (a*b^2 + b^3)*sinh(x)^4 + a*b^2 + b^3 + 2*(a*b^2 - b^3)*cosh(x)^2 + 2*(a*b
^2 - b^3 + 3*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a*b^2 + b^3)*cosh(x)^
3 + (a*b^2 - b^3)*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2
+ 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 +
 (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((
a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a -
 b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh
(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) - 2*((a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*cosh(x)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)*sinh(x)^3 + (a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(x)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(
a^3 + a^2*b - a*b^2 - b^3)*cosh(x)^2 + 2*(a^3 + a^2*b - a*b^2 - b^3 + 3*(a^
3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*cosh(x)^3 + (a^3 + a^2*b - a*b^2 - b^3)*cosh(x))*sinh(x))*sqrt(
-b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)
*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x
)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 +
(a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*s
inh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + 2*sq
rt(2)*(a^2*b + a*b^2 - (a^2*b + a*b^2)*cosh(x)^2 - 2*(a^2*b + a*b^2)*cosh(x
)*sinh(x) - (a^2*b + a*b^2)*sinh(x)^2)*sqrt(((a + b)*cosh(x)^2 + (a + b)*si
nh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^3*b^2 + 3
*a^2*b^3 + 3*a*b^4 + b^5 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)^4
+ 4*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)*sinh(x)^3 + (a^3*b^2 + 3*
a^2*b^3 + 3*a*b^4 + b^5)*sinh(x)^4 + 2*(a^3*b^2 + a^2*b^3 - a*b^4 - b^5)*co
sh(x)^2 + 2*(a^3*b^2 + a^2*b^3 - a*b^4 - b^5 + 3*(a^3*b^2 + 3*a^2*b^3 + 3*a
*b^4 + b^5)*cosh(x)^2)*sinh(x)^2 + 4*((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)
*cosh(x)^3 + (a^3*b^2 + a^2*b^3 - a*b^4 - b^5)*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^4(x)}{\left(a + b\tanh^2(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+b*tanh(x)**2)**(3/2),x)
```

```
[Out] Integral(tanh(x)**4/(a + b*tanh(x)**2)**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.240 $\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=52

$$\frac{a}{b(a+b)\sqrt{a+b\tanh^2(x)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}$$

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/(a + b)^(3/2) + a/(b*(a + b)*Sqr
t[a + b*Tanh[x]^2])
```

Rubi [A] time = 0.123291, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3670, 446, 78, 63, 208}

$$\frac{a}{b(a+b)\sqrt{a+b\tanh^2(x)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/(a + b)^(3/2) + a/(b*(a + b)*Sqr
t[a + b*Tanh[x]^2])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx &= \text{Subst}\left(\int \frac{x^3}{\left(1-x^2\right)\left(a+bx^2\right)^{3/2}}\,dx,x,\tanh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(1-x)(a+bx)^{3/2}}\,dx,x,\tanh^2(x)\right)\\
&= \frac{a}{b(a+b)\sqrt{a+b\tanh^2(x)}}+\frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx,x,\tanh^2(x)\right)}{2(a+b)}\\
&= \frac{a}{b(a+b)\sqrt{a+b\tanh^2(x)}}+\frac{\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\tanh^2(x)}\right)}{b(a+b)}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}+\frac{a}{b(a+b)\sqrt{a+b\tanh^2(x)}}
\end{aligned}
$$

Mathematica [A] time = 0.118817, size = 52, normalized size = 1.

$$
\frac{a}{b(a+b)\sqrt{a+b\tanh^2(x)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/(a + b)^(3/2) + a/(b*(a + b)*Sqr
t[a + b*Tanh[x]^2])
```

Maple [B] time = 0.023, size = 287, normalized size = 5.5

$$
\frac{1}{b}\frac{1}{\sqrt{a+b\left(\tanh(x)\right)^2}}-\frac{1}{2b+2a}\frac{1}{\sqrt{\left(1+\tanh(x)\right)^2b-2\left(1+\tanh(x)\right)b+a+b}}-\frac{b\left(2\left(1+\tanh(x)\right)b-2b\right)}{(a+b)\left(4b(a+b)-4b^2\right)}\frac{}{\sqrt{(1+\tan}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+b*tanh(x)^2)^(3/2),x)
```

```
[Out] 1/b/(a+b*tanh(x)^2)^(1/2)-1/2/(a+b)/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(
1/2)-b/(a+b)*(2*(1+tanh(x))*b-2*b)/(4*b*(a+b)-4*b^2)/((1+tanh(x))^2*b-2*(1+
tanh(x))*b+a+b)^(1/2)+1/2/(a+b)^(3/2)*ln((2*a+2*b-2*(1+tanh(x))*b+2*(a+b)^(
1/2)*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))/(1+tanh(x)))-1/2/(a+b)/((
tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)+b/(a+b)*(2*(tanh(x)-1)*b+2*b)/(4*
b*(a+b)-4*b^2)/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)+1/2/(a+b)^(3/2)*
ln((2*a+2*b+2*(tanh(x)-1)*b+2*(a+b)^(1/2)*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+
a+b)^(1/2))/(tanh(x)-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^3}{\left(b\tanh(x)^2 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^3/(b*tanh(x)^2 + a)^(3/2), x)
```

Fricas [B] time = 3.07382, size = 6730, normalized size = 129.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh(x)*sinh(x)^3 + (a*b + b^2
)*sinh(x)^4 + 2*(a*b - b^2)*cosh(x)^2 + 2*(3*(a*b + b^2)*cosh(x)^2 + a*b -
b^2)*sinh(x)^2 + a*b + b^2 + 4*((a*b + b^2)*cosh(x)^3 + (a*b - b^2)*cosh(x)
)*sinh(x))*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(
x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2
*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*
cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2
 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 +
 b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x
)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x
))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*co
sh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*
a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^
2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^
2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*
a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^
4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 +
 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh
(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*
(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*
a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 1
5*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6
*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*c
osh(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + 2*(a*b - b^2)*cosh(x)^2 + 2*(3*(
a*b + b^2)*cosh(x)^2 + a*b - b^2)*sinh(x)^2 + a*b + b^2 + 4*((a*b + b^2)*co
```

```
sh(x)^3 + (a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*log(-((a + b)*cosh(x)^4
 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(
a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
- b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 -
b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) +
4*sqrt(2)*((a^2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (a^2 + a
*b)*sinh(x)^2 + a^2 + a*b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
- b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^3*b + 3*a^2*b^2 + 3*
a*b^3 + b^4)*cosh(x)^4 + 4*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cosh(x)*sinh
(x)^3 + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*sinh(x)^4 + a^3*b + 3*a^2*b^2 +
 3*a*b^3 + b^4 + 2*(a^3*b + a^2*b^2 - a*b^3 - b^4)*cosh(x)^2 + 2*(a^3*b + a
^2*b^2 - a*b^3 - b^4 + 3*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cosh(x)^2)*sin
h(x)^2 + 4*((a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cosh(x)^3 + (a^3*b + a^2*b^
2 - a*b^3 - b^4)*cosh(x))*sinh(x)), -1/2*(((a*b + b^2)*cosh(x)^4 + 4*(a*b +
 b^2)*cosh(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + 2*(a*b - b^2)*cosh(x)^2 +
 2*(3*(a*b + b^2)*cosh(x)^2 + a*b - b^2)*sinh(x)^2 + a*b + b^2 + 4*((a*b +
b^2)*cosh(x)^3 + (a*b - b^2)*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*
(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a - b)*sqrt
(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sin
h(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3
 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2 + a*b)*c
osh(x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a
*b)*cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh(x))) + ((a*b + b^2)*cosh(
x)^4 + 4*(a*b + b^2)*cosh(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + 2*(a*b - b
^2)*cosh(x)^2 + 2*(3*(a*b + b^2)*cosh(x)^2 + a*b - b^2)*sinh(x)^2 + a*b + b
^2 + 4*((a*b + b^2)*cosh(x)^3 + (a*b - b^2)*cosh(x))*sinh(x))*sqrt(-a - b)*
arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)
*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x
)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 +
(a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*s
inh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) - 2*sq
rt(2)*((a^2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (a^2 + a*b)*
sinh(x)^2 + a^2 + a*b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)
/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^3*b + 3*a^2*b^2 + 3*a*b^
3 + b^4)*cosh(x)^4 + 4*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cosh(x)*sinh(x)^
3 + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*sinh(x)^4 + a^3*b + 3*a^2*b^2 + 3*a
*b^3 + b^4 + 2*(a^3*b + a^2*b^2 - a*b^3 - b^4)*cosh(x)^2 + 2*(a^3*b + a^2*b
^2 - a*b^3 - b^4 + 3*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cosh(x)^2)*sinh(x)
^2 + 4*((a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cosh(x)^3 + (a^3*b + a^2*b^2 -
a*b^3 - b^4)*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^3(x)}{\left(a + b\tanh^2(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+b*tanh(x)**2)**(3/2),x)
```

```
[Out] Integral(tanh(x)**3/(a + b*tanh(x)**2)**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.241 $\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=53

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}}-\frac{\tanh(x)}{(a+b)\sqrt{a+b\tanh^2(x)}}$$

```
[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(3/2) - Tanh[x
]/((a + b)*Sqrt[a + b*Tanh[x]^2])
```

Rubi [A] time = 0.0992793, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3670, 471, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}}-\frac{\tanh(x)}{(a+b)\sqrt{a+b\tanh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(3/2) - Tanh[x
]/((a + b)*Sqrt[a + b*Tanh[x]^2])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx &= \text{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)\left(a+bx^2\right)^{3/2}}\,dx, x, \tanh(x)\right)\\ &= -\frac{\tanh(x)}{(a+b)\sqrt{a+b\tanh^2(x)}}+\frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{a+b}\\ &= -\frac{\tanh(x)}{(a+b)\sqrt{a+b\tanh^2(x)}}+\frac{\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{a+b}\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}}-\frac{\tanh(x)}{(a+b)\sqrt{a+b\tanh^2(x)}}\end{aligned}$$

Mathematica [B] time = 1.45602, size = 112, normalized size = 2.11

$$\frac{\tanh(x)\left(\tanh^{-1}\left(\frac{\sqrt{\frac{(a+b)\tanh^2(x)}{a}}}{\sqrt{\frac{b\tanh^2(x)}{a}+1}}\right)\sqrt{\frac{(a+b)\tanh^2(x)}{a}}\left(a\coth^2(x)+b\right)-(a+b)\sqrt{\frac{b\tanh^2(x)}{a}+1}\right)}{(a+b)^2\sqrt{a+b\tanh^2(x)}\sqrt{\frac{b\tanh^2(x)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] (Tanh[x]*(ArcTanh[Sqrt[((a + b)*Tanh[x]^2)/a]/Sqrt[1 + (b*Tanh[x]^2)/a]]*(b
 + a*Coth[x]^2)*Sqrt[((a + b)*Tanh[x]^2)/a] - (a + b)*Sqrt[1 + (b*Tanh[x]^2
)/a]))/((a + b)^2*Sqrt[a + b*Tanh[x]^2]*Sqrt[1 + (b*Tanh[x]^2)/a])
```

Maple [B] time = 0.021, size = 289, normalized size = 5.5

$$-\frac{\tanh(x)}{a}\frac{1}{\sqrt{a+b\left(\tanh(x)\right)^2}}+\frac{1}{2b+2a}\frac{1}{\sqrt{\left(1+\tanh(x)\right)^2b-2\left(1+\tanh(x)\right)b+a+b}}+\frac{b\left(2\left(1+\tanh(x)\right)b-2b\right.}{(a+b)\left(4b(a+b)-4b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a+b*tanh(x)^2)^(3/2),x)
```

```
[Out] -tanh(x)/a/(a+b*tanh(x)^2)^(1/2)+1/2/(a+b)/((1+tanh(x))^2*b-2*(1+tanh(x))*b
+a+b)^(1/2)+b/(a+b)*(2*(1+tanh(x))*b-2*b)/(4*b*(a+b)-4*b^2)/((1+tanh(x))^2*
b-2*(1+tanh(x))*b+a+b)^(1/2)-1/2/(a+b)^(3/2)*ln((2*a+2*b-2*(1+tanh(x))*b+2*
(a+b)^(1/2)*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))/(1+tanh(x)))-1/2/(
a+b)/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)+b/(a+b)*(2*(tanh(x)-1)*b+2
```

```
*b)/(4*b*(a+b)-4*b^2)/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)+1/2/(a+b)
^(3/2)*ln((2*a+2*b+2*(tanh(x)-1)*b+2*(a+b)^(1/2)*((tanh(x)-1)^2*b+2*(tanh(x
)-1)*b+a+b)^(1/2))/(tanh(x)-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh (x)^2}{\left(b\tanh (x)^2 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^2/(b*tanh(x)^2 + a)^(3/2), x)
```

Fricas [B] time = 3.09922, size = 6395, normalized size = 120.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4
+ 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a +
 b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*log(-((a*b^2
+ b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x
)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cos
h(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(
x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^
3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2
)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3
+ (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*
b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(
x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*
b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*
cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 -
b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a
*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b
+ 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)
^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh
(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)
^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 -
a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(
x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^
3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) +
((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a
 - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*co
sh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*log(((a + b)*cosh(x
)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(
3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x)
 + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
 a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3
 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2))
```

```
 - 4*sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(
x)^2 - a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh
(x)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)*sinh(x)^3 + (a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*sinh(x)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 + a^2
*b - a*b^2 - b^3)*cosh(x)^2 + 2*(a^3 + a^2*b - a*b^2 - b^3 + 3*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 3*a^2*b + 3*a*b^2 + b^
3)*cosh(x)^3 + (a^3 + a^2*b - a*b^2 - b^3)*cosh(x))*sinh(x)), -1/2*(((a + b
)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*c
osh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3
 + (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(-a - b)*arctan(sqrt(2)*(b*cosh(x)
^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - a - b)*sqrt(-a - b)*sqrt(((a + b)*
cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sin
h(x)^2))/((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh(x)*sinh(x)^3 + (a*b +
b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^2)*cosh(x)^2 + (6*(a*b + b^2)*cosh(x)^2 +
 a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a*b + b^2)*cosh(x
)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x))) + ((a + b)*cosh(x)^4 + 4*(a +
b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a +
b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*s
inh(x) + a + b)*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x)
+ sinh(x)^2 + 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
 a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4
*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3
*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh
(x))*sinh(x) + a + b)) + 2*sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*s
inh(x) + (a + b)*sinh(x)^2 - a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(
x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)*sin
h(x)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(x)^4 + a^3 + 3*a^2*b + 3*a*b^
2 + b^3 + 2*(a^3 + a^2*b - a*b^2 - b^3)*cosh(x)^2 + 2*(a^3 + a^2*b - a*b^2
- b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 +
3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^3 + (a^3 + a^2*b - a*b^2 - b^3)*cosh(x))*s
inh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^2(x)}{\left(a + b\tanh^2(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+b*tanh(x)**2)**(3/2),x)
```

```
[Out] Integral(tanh(x)**2/(a + b*tanh(x)**2)**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.242 $\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=49

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{1}{(a+b)\sqrt{a+b\tanh^2(x)}}$$

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/(a + b)^(3/2) - 1/((a + b)*Sqrt[
a + b*Tanh[x]^2])
```

Rubi [A] time = 0.0856056, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3670, 444, 51, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{1}{(a+b)\sqrt{a+b\tanh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/(a + b)^(3/2) - 1/((a + b)*Sqrt[
a + b*Tanh[x]^2])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx &= \text{Subst}\left(\int \frac{x}{\left(1-x^2\right)\left(a+bx^2\right)^{3/2}}\,dx, x, \tanh(x)\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)^{3/2}}\,dx, x, \tanh^2(x)\right)\\ &= -\frac{1}{(a+b)\sqrt{a+b\tanh^2(x)}}+\frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)}{2(a+b)}\\ &= -\frac{1}{(a+b)\sqrt{a+b\tanh^2(x)}}+\frac{\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b(a+b)}\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{1}{(a+b)\sqrt{a+b\tanh^2(x)}}\end{aligned}$$

Mathematica [C] time = 0.0349182, size = 41, normalized size = 0.84

$$-\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\tanh^2(x)+a}{a+b}\right)}{(a+b)\sqrt{a+b\tanh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] -(Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tanh[x]^2)/(a + b)]/((a + b)*Sqrt[
a + b*Tanh[x]^2]))
```

Maple [B] time = 0.02, size = 273, normalized size = 5.6

$$-\frac{1}{2b+2a}\frac{1}{\sqrt{(1+\tanh(x))^2 b-2(1+\tanh(x))b+a+b}}-\frac{b(2(1+\tanh(x))b-2b)}{(a+b)\left(4b(a+b)-4b^2\right)}\frac{1}{\sqrt{(1+\tanh(x))^2 b-2(1+\text{ta}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*tanh(x)^2)^(3/2),x)
```

```
[Out] -1/2/(a+b)/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)-b/(a+b)*(2*(1+tanh(x
))*b-2*b)/(4*b*(a+b)-4*b^2)/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)+1/2
/(a+b)^(3/2)*ln((2*a+2*b-2*(1+tanh(x))*b+2*(a+b)^(1/2)*((1+tanh(x))^2*b-2*(
1+tanh(x))*b+a+b)^(1/2))/(1+tanh(x)))-1/2/(a+b)/((tanh(x)-1)^2*b+2*(tanh(x)
-1)*b+a+b)^(1/2)+b/(a+b)*(2*(tanh(x)-1)*b+2*b)/(4*b*(a+b)-4*b^2)/((tanh(x)-
1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)+1/2/(a+b)^(3/2)*ln((2*a+2*b+2*(tanh(x)-1)
*b+2*(a+b)^(1/2)*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))/(tanh(x)-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)}{\left(b\tanh(x)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)/(b*tanh(x)^2 + a)^(3/2), x)
```

Fricas [B] time = 2.98922, size = 6395, normalized size = 130.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4
+ 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a +
 b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*log(((a^3 + a
^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)
^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh
(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x
))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b
)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)
*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 +
 (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b
^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x
)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a
^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*c
osh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a
^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*
a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b
 - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^
3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(
x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^
2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 +
 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x
))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3
*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + (
(a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a
- b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cos
h(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*log(-((a + b)*cosh(x
)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(
```

```
3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x)
 + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
 a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3
 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2))
 - 4*sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(
x)^2 + a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh
(x)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)*sinh(x)^3 + (a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*sinh(x)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 + a^2
*b - a*b^2 - b^3)*cosh(x)^2 + 2*(a^3 + a^2*b - a*b^2 - b^3 + 3*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 3*a^2*b + 3*a*b^2 + b^
3)*cosh(x)^3 + (a^3 + a^2*b - a*b^2 - b^3)*cosh(x))*sinh(x)), -1/2*(((a + b
)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*c
osh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3
 + (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(-a - b)*arctan(sqrt(2)*(a*cosh(x)
^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a - b)*sqrt(((a + b)*
cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sin
h(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 +
a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 +
 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a*b)*cosh(x
)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh(x))) + ((a + b)*cosh(x)^4 + 4*(a +
b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a +
b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*s
inh(x) + a + b)*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x)
+ sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
 a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4
*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3
*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh
(x))*sinh(x) + a + b)) + 2*sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*s
inh(x) + (a + b)*sinh(x)^2 + a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(
x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)*sin
h(x)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(x)^4 + a^3 + 3*a^2*b + 3*a*b^
2 + b^3 + 2*(a^3 + a^2*b - a*b^2 - b^3)*cosh(x)^2 + 2*(a^3 + a^2*b - a*b^2
- b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 +
3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^3 + (a^3 + a^2*b - a*b^2 - b^3)*cosh(x))*s
inh(x))]
```

Sympy [A] time = 12.2585, size = 51, normalized size = 1.04

$$-\frac{1}{(a+b)\sqrt{a+b\tanh^2(x)}}-\frac{\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)**2)**(3/2),x)
```

```
[Out] -1/((a + b)*sqrt(a + b*tanh(x)**2)) - atan(sqrt(a + b*tanh(x)**2)/sqrt(-a -
 b))/(sqrt(-a - b)*(a + b))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.243 $\int \frac{1}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=56

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}}+\frac{b\tanh(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

```
[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(3/2) + (b*Tan
h[x])/(a*(a + b)*Sqrt[a + b*Tanh[x]^2])
```

Rubi [A] time = 0.042802, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3661, 382, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}}+\frac{b\tanh(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tanh[x]^2)^(-3/2),x]
```

```
[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(3/2) + (b*Tan
h[x])/(a*(a + b)*Sqrt[a + b*Tanh[x]^2])
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx=\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\left(a+bx^2\right)^{3/2}}\,dx,x,\tanh(x)\right)$$

$$=\frac{b\tanh(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}+\frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+bx^2}}\,dx,x,\tanh(x)\right)}{a+b}$$

$$=\frac{b\tanh(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}+\frac{\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx,x,\frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{a+b}$$

$$=\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}}+\frac{b\tanh(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

Mathematica [C] time = 4.21493, size = 223, normalized size = 3.98

$$-\frac{\sinh^2(x)\left(\sqrt{2}a^2(a+b)\tanh(x)\,{}_2F_1\left(2,2;\frac{7}{2};-\frac{(a+b)\sinh^2(x)}{a}\right)\left(-\frac{(a+b)\sinh^2(x)((a+b)\cosh(2x)+a-b)}{a^2}\right)^{3/2}+\frac{15}{4}a\operatorname{csch}(x)\operatorname{sech}(x)((3a\ldots}{15a^4\left(-\frac{(a+b)\sinh^2(x)}{a}\right)^{3/2}\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Tanh[x]^2)^(-3/2),x]
```

```
[Out] -(Sinh[x]^2*((15*a*(3*a - 2*b + (3*a + 2*b)*Cosh[2*x])*Csch[x]*Sech[x]*((a
- b)*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]] + (a + b)*ArcSin[Sqrt[-(((a + b
)*Sinh[x]^2)/a)]]*Cosh[2*x] - 2*a*Sqrt[-(((a + b)*(b + a*Coth[x]^2)*Sinh[x]
^4)/a^2)]))/4 + Sqrt[2]*a^2*(a + b)*Hypergeometric2F1[2, 2, 7/2, -(((a + b)
*Sinh[x]^2)/a)]*(-(((a + b)*(a - b + (a + b)*Cosh[2*x])*Sinh[x]^2)/a^2))^(3
/2)*Tanh[x]))/(15*a^4*(-(((a + b)*Sinh[x]^2)/a))^(3/2)*Sqrt[Cosh[x]^2 + (b*
Sinh[x]^2)/a]*Sqrt[a + b*Tanh[x]^2])
```

Maple [B] time = 0.026, size = 272, normalized size = 4.9

$$\frac{1}{2b+2a}\frac{1}{\sqrt{(1+\tanh(x))^2b-2(1+\tanh(x))b+a+b}}+\frac{b(2(1+\tanh(x))b-2b)}{(a+b)\left(4b(a+b)-4b^2\right)}\frac{1}{\sqrt{(1+\tanh(x))^2b-2(1+\tanh(\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tanh(x)^2)^(3/2),x)
```

```
[Out] 1/2/(a+b)/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)+b/(a+b)*(2*(1+tanh(x)
)*b-2*b)/(4*b*(a+b)-4*b^2)/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)-1/2/
(a+b)^(3/2)*ln((2*a+2*b-2*(1+tanh(x))*b+2*(a+b)^(1/2)*((1+tanh(x))^2*b-2*(1
```

```
+tanh(x))*b+a+b)^(1/2))/(1+tanh(x)))-1/2/(a+b)/((tanh(x)-1)^2*b+2*(tanh(x)-
1)*b+a+b)^(1/2)+b/(a+b)*(2*(tanh(x)-1)*b+2*b)/(4*b*(a+b)-4*b^2)/((tanh(x)-1
)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)+1/2/(a+b)^(3/2)*ln((2*a+2*b+2*(tanh(x)-1)*
b+2*(a+b)^(1/2)*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))/(tanh(x)-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\tanh\left(x\right)^2 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tanh(x)^2 + a)^(-3/2), x)
```

Fricas [B] time = 3.09694, size = 6730, normalized size = 120.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b
)*sinh(x)^4 + 2*(a^2 - a*b)*cosh(x)^2 + 2*(3*(a^2 + a*b)*cosh(x)^2 + a^2 -
a*b)*sinh(x)^2 + a^2 + a*b + 4*((a^2 + a*b)*cosh(x)^3 + (a^2 - a*b)*cosh(x)
)*sinh(x))*sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh
(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(
a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)
*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2
+ 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 +
6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(
x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(
x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*c
osh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a
^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)
^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b
^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3
*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)
^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2
+ 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sin
h(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cos
h(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3
*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a
^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) +
15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 +
6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*
cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + 2*(a^2 - a*b)*cosh(x)^2 + 2*(3*
(a^2 + a*b)*cosh(x)^2 + a^2 - a*b)*sinh(x)^2 + a^2 + a*b + 4*((a^2 + a*b)*c
osh(x)^3 + (a^2 - a*b)*cosh(x))*sinh(x))*sqrt(a + b)*log(((a + b)*cosh(x)^4
 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*(
a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
```

```
- b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 +
a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) +
4*sqrt(2)*((a*b + b^2)*cosh(x)^2 + 2*(a*b + b^2)*cosh(x)*sinh(x) + (a*b + b
^2)*sinh(x)^2 - a*b - b^2)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
- b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^4 + 3*a^3*b + 3*a^2*
b^2 + a*b^3)*cosh(x)^4 + 4*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)*sinh
(x)^3 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sinh(x)^4 + a^4 + 3*a^3*b + 3*a
^2*b^2 + a*b^3 + 2*(a^4 + a^3*b - a^2*b^2 - a*b^3)*cosh(x)^2 + 2*(a^4 + a^3
*b - a^2*b^2 - a*b^3 + 3*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^2)*sin
h(x)^2 + 4*((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^3 + (a^4 + a^3*b -
a^2*b^2 - a*b^3)*cosh(x))*sinh(x)), -1/2*(((a^2 + a*b)*cosh(x)^4 + 4*(a^2 +
 a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + 2*(a^2 - a*b)*cosh(x)^2 +
 2*(3*(a^2 + a*b)*cosh(x)^2 + a^2 - a*b)*sinh(x)^2 + a^2 + a*b + 4*((a^2 +
a*b)*cosh(x)^3 + (a^2 - a*b)*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*
(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - a - b)*sqrt(-a - b)*sqrt
(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sin
h(x) + sinh(x)^2))/((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh(x)*sinh(x)^3
 + (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^2)*cosh(x)^2 + (6*(a*b + b^2)*c
osh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a*b + b
^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x))) + ((a^2 + a*b)*cosh(
x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + 2*(a^2 - a
*b)*cosh(x)^2 + 2*(3*(a^2 + a*b)*cosh(x)^2 + a^2 - a*b)*sinh(x)^2 + a^2 + a
*b + 4*((a^2 + a*b)*cosh(x)^3 + (a^2 - a*b)*cosh(x))*sinh(x))*sqrt(-a - b)*
arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)
*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x
)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 +
(a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*s
inh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) - 2*sq
rt(2)*((a*b + b^2)*cosh(x)^2 + 2*(a*b + b^2)*cosh(x)*sinh(x) + (a*b + b^2)*
sinh(x)^2 - a*b - b^2)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)
/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^4 + 3*a^3*b + 3*a^2*b^2
+ a*b^3)*cosh(x)^4 + 4*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)*sinh(x)^
3 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sinh(x)^4 + a^4 + 3*a^3*b + 3*a^2*b
^2 + a*b^3 + 2*(a^4 + a^3*b - a^2*b^2 - a*b^3)*cosh(x)^2 + 2*(a^4 + a^3*b -
 a^2*b^2 - a*b^3 + 3*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^2)*sinh(x)
^2 + 4*((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^3 + (a^4 + a^3*b - a^2*
b^2 - a*b^3)*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + b \tanh^2(x)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tanh(x)**2)**(-3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.244 $\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=78

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{b}{a(a+b)\sqrt{a+b\tanh^2(x)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}$$

```
[Out] -(ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]]/a^(3/2)) + ArcTanh[Sqrt[a + b*Tanh
[x]^2]/Sqrt[a + b]]/(a + b)^(3/2) + b/(a*(a + b)*Sqrt[a + b*Tanh[x]^2])
```

Rubi [A] time = 0.146428, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3670, 446, 85, 156, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{b}{a(a+b)\sqrt{a+b\tanh^2(x)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]]/a^(3/2)) + ArcTanh[Sqrt[a + b*Tanh
[x]^2]/Sqrt[a + b]]/(a + b)^(3/2) + b/(a*(a + b)*Sqrt[a + b*Tanh[x]^2])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx &= \text{Subst}\left(\int \frac{1}{x\left(1-x^2\right)\left(a+bx^2\right)^{3/2}}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)x(a+bx)^{3/2}}\,dx, x, \tanh^2(x)\right)\\
&= \frac{b}{a(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{\text{Subst}\left(\int \frac{-a-b+bx}{(1-x)x\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)}{2a(a+b)}\\
&= \frac{b}{a(a+b)\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)}{2a} + \frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, \right.}{2(a+b)}\\
&= \frac{b}{a(a+b)\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{ab} + \frac{\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\right.}{}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} + \frac{b}{a(a+b)\sqrt{a+b\tanh^2(x)}}
\end{aligned}$$

Mathematica [C] time = 0.0765405, size = 70, normalized size = 0.9

$$\frac{(a+b)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\tanh^2(x)}{a}+1\right) - a\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\tanh^2(x)+a}{a+b}\right)}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] (-(a*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tanh[x]^2)/(a + b)]) + (a + b)*
Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Tanh[x]^2)/a])/(a*(a + b)*Sqrt[a + b
*Tanh[x]^2])
```

Maple [F] time = 0.115, size = 0, normalized size = 0.

$$\int \coth(x)\left(a+b(\tanh(x))^2\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*tanh(x)^2)^(3/2),x)
```

```
[Out] int(coth(x)/(a+b*tanh(x)^2)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{\left(b\tanh(x)^2+a\right)^{\frac{3}{2}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)/(b*tanh(x)^2 + a)^(3/2), x)
```

Fricas [B] time = 5.60527, size = 18423, normalized size = 236.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a^3 + a^2*b)*cosh(x)^4 + 4*(a^3 + a^2*b)*cosh(x)*sinh(x)^3 + (a^3 +
 a^2*b)*sinh(x)^4 + a^3 + a^2*b + 2*(a^3 - a^2*b)*cosh(x)^2 + 2*(a^3 - a^2*
b + 3*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + a^2*b)*cosh(x)^3 + (a^
3 - a^2*b)*cosh(x))*sinh(x))*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(
a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b
)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*
(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3
 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 +
 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a
^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a
*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 +
 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2
*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*c
osh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^
2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*
a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2
+ (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a
^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b -
b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x
)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2
*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^
3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cos
```

```
h(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(
x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 2*((a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)*sinh(x)^
3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(x)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b
^3 + 2*(a^3 + a^2*b - a*b^2 - b^3)*cosh(x)^2 + 2*(a^3 + a^2*b - a*b^2 - b^3
 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*cosh(x)^3 + (a^3 + a^2*b - a*b^2 - b^3)*cosh(x))*sinh(x
))*sqrt(a)*log(-((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a
 + b)*sinh(x)^4 + 2*(2*a - b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 + 2*a -
b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sq
rt(a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a + b)*cosh(x)^3 + (2*a - b)*cosh(x))*
sinh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh
(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1))
+ ((a^3 + a^2*b)*cosh(x)^4 + 4*(a^3 + a^2*b)*cosh(x)*sinh(x)^3 + (a^3 + a^2
*b)*sinh(x)^4 + a^3 + a^2*b + 2*(a^3 - a^2*b)*cosh(x)^2 + 2*(a^3 - a^2*b +
3*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + a^2*b)*cosh(x)^3 + (a^3 -
a^2*b)*cosh(x))*sinh(x))*sqrt(a + b)*log(-((a + b)*cosh(x)^4 + 4*(a + b)*co
sh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^
2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*
sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2
 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 - b*cosh(x))*sinh
(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*sqrt(2)*(a^2*
b + a*b^2 + (a^2*b + a*b^2)*cosh(x)^2 + 2*(a^2*b + a*b^2)*cosh(x)*sinh(x) +
 (a^2*b + a*b^2)*sinh(x)^2)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
 - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^5 + 3*a^4*b + 3*a^3*
b^2 + a^2*b^3 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^4 + 4*(a^5 +
3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)*sinh(x)^3 + (a^5 + 3*a^4*b + 3*a^3*b
^2 + a^2*b^3)*sinh(x)^4 + 2*(a^5 + a^4*b - a^3*b^2 - a^2*b^3)*cosh(x)^2 + 2
*(a^5 + a^4*b - a^3*b^2 - a^2*b^3 + 3*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)
*cosh(x)^2)*sinh(x)^2 + 4*((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^3
+ (a^5 + a^4*b - a^3*b^2 - a^2*b^3)*cosh(x))*sinh(x)), 1/4*(4*((a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)*s
inh(x)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(x)^4 + a^3 + 3*a^2*b + 3*a*
b^2 + b^3 + 2*(a^3 + a^2*b - a*b^2 - b^3)*cosh(x)^2 + 2*(a^3 + a^2*b - a*b^
2 - b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3
+ 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^3 + (a^3 + a^2*b - a*b^2 - b^3)*cosh(x))
*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2 + 1)*sqrt(-a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(
x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(
x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(
x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) +
 a + b)) + ((a^3 + a^2*b)*cosh(x)^4 + 4*(a^3 + a^2*b)*cosh(x)*sinh(x)^3 + (
a^3 + a^2*b)*sinh(x)^4 + a^3 + a^2*b + 2*(a^3 - a^2*b)*cosh(x)^2 + 2*(a^3 -
 a^2*b + 3*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + a^2*b)*cosh(x)^3
+ (a^3 - a^2*b)*cosh(x))*sinh(x))*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8
+ 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 +
a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6
 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (
6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*
a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(
14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*
b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*
a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3
+ a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b
^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5
 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 +
4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(
x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^
```

```
2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a
*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*s
inh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3
+ a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2
 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 +
6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*
cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a^3 + a^2*b)*co
sh(x)^4 + 4*(a^3 + a^2*b)*cosh(x)*sinh(x)^3 + (a^3 + a^2*b)*sinh(x)^4 + a^3
 + a^2*b + 2*(a^3 - a^2*b)*cosh(x)^2 + 2*(a^3 - a^2*b + 3*(a^3 + a^2*b)*cos
h(x)^2)*sinh(x)^2 + 4*((a^3 + a^2*b)*cosh(x)^3 + (a^3 - a^2*b)*cosh(x))*sin
h(x))*sqrt(a + b)*log(-((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (
a + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 +
sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((
a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x
) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(
x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*sqrt(2)*(a^2*b + a*b^2 + (a^2*b
+ a*b^2)*cosh(x)^2 + 2*(a^2*b + a*b^2)*cosh(x)*sinh(x) + (a^2*b + a*b^2)*si
nh(x)^2)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 -
2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 + (a^
5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^4 + 4*(a^5 + 3*a^4*b + 3*a^3*b^2
 + a^2*b^3)*cosh(x)*sinh(x)^3 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sinh(
x)^4 + 2*(a^5 + a^4*b - a^3*b^2 - a^2*b^3)*cosh(x)^2 + 2*(a^5 + a^4*b - a^3
*b^2 - a^2*b^3 + 3*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^2)*sinh(x)
^2 + 4*((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^3 + (a^5 + a^4*b - a^
3*b^2 - a^2*b^3)*cosh(x))*sinh(x)), -1/2*(((a^3 + a^2*b)*cosh(x)^4 + 4*(a^3
 + a^2*b)*cosh(x)*sinh(x)^3 + (a^3 + a^2*b)*sinh(x)^4 + a^3 + a^2*b + 2*(a^
3 - a^2*b)*cosh(x)^2 + 2*(a^3 - a^2*b + 3*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^
2 + 4*((a^3 + a^2*b)*cosh(x)^3 + (a^3 - a^2*b)*cosh(x))*sinh(x))*sqrt(-a -
b)*arctan(sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a + b)
*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)
*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^2
+ (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b
^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh(x))) +
((a^3 + a^2*b)*cosh(x)^4 + 4*(a^3 + a^2*b)*cosh(x)*sinh(x)^3 + (a^3 + a^2*b
)*sinh(x)^4 + a^3 + a^2*b + 2*(a^3 - a^2*b)*cosh(x)^2 + 2*(a^3 - a^2*b + 3*
(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + a^2*b)*cosh(x)^3 + (a^3 - a^
2*b)*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*s
inh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh
(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x
)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2
 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a -
b)*cosh(x))*sinh(x) + a + b)) - ((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^4
+ 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)*sinh(x)^3 + (a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*sinh(x)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 + a^2*b - a
*b^2 - b^3)*cosh(x)^2 + 2*(a^3 + a^2*b - a*b^2 - b^3 + 3*(a^3 + 3*a^2*b + 3
*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos
h(x)^3 + (a^3 + a^2*b - a*b^2 - b^3)*cosh(x))*sinh(x))*sqrt(a)*log(-((2*a +
 b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*
a - b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 + 2*a - b)*sinh(x)^2 - 2*sqrt(2
)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt(((a + b)*cos
h(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x
)^2)) + 4*((2*a + b)*cosh(x)^3 + (2*a - b)*cosh(x))*sinh(x) + 2*a + b)/(cos
h(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 -
2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)) - 2*sqrt(2)*(a^2*b + a*
b^2 + (a^2*b + a*b^2)*cosh(x)^2 + 2*(a^2*b + a*b^2)*cosh(x)*sinh(x) + (a^2*
b + a*b^2)*sinh(x)^2)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^5 + 3*a^4*b + 3*a^3*b^2 +
a^2*b^3 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^4 + 4*(a^5 + 3*a^4*
```

```
b + 3*a^3*b^2 + a^2*b^3)*cosh(x)*sinh(x)^3 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a
^2*b^3)*sinh(x)^4 + 2*(a^5 + a^4*b - a^3*b^2 - a^2*b^3)*cosh(x)^2 + 2*(a^5
+ a^4*b - a^3*b^2 - a^2*b^3 + 3*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(
x)^2)*sinh(x)^2 + 4*((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^3 + (a^5
 + a^4*b - a^3*b^2 - a^2*b^3)*cosh(x))*sinh(x)), 1/2*(2*((a^3 + 3*a^2*b + 3
*a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)*sinh(x)
^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(x)^4 + a^3 + 3*a^2*b + 3*a*b^2 +
b^3 + 2*(a^3 + a^2*b - a*b^2 - b^3)*cosh(x)^2 + 2*(a^3 + a^2*b - a*b^2 - b^
3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*cosh(x)^3 + (a^3 + a^2*b - a*b^2 - b^3)*cosh(x))*sinh(
x))*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
*sqrt(-a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 -
 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sin
h(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 +
 a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b
)) - ((a^3 + a^2*b)*cosh(x)^4 + 4*(a^3 + a^2*b)*cosh(x)*sinh(x)^3 + (a^3 +
a^2*b)*sinh(x)^4 + a^3 + a^2*b + 2*(a^3 - a^2*b)*cosh(x)^2 + 2*(a^3 - a^2*b
 + 3*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + a^2*b)*cosh(x)^3 + (a^3
 - a^2*b)*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(a*cosh(x)^2 + 2*a*
cosh(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2
 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/
((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh
(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 +
a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*
a^2 + a*b - b^2)*cosh(x))*sinh(x))) - ((a^3 + a^2*b)*cosh(x)^4 + 4*(a^3 + a
^2*b)*cosh(x)*sinh(x)^3 + (a^3 + a^2*b)*sinh(x)^4 + a^3 + a^2*b + 2*(a^3 -
a^2*b)*cosh(x)^2 + 2*(a^3 - a^2*b + 3*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^2 +
4*((a^3 + a^2*b)*cosh(x)^3 + (a^3 - a^2*b)*cosh(x))*sinh(x))*sqrt(-a - b)*a
rctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*
sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)
*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (
a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*si
nh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + 2*sqr
t(2)*(a^2*b + a*b^2 + (a^2*b + a*b^2)*cosh(x)^2 + 2*(a^2*b + a*b^2)*cosh(x)
*sinh(x) + (a^2*b + a*b^2)*sinh(x)^2)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sin
h(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^5 + 3*a^4*
b + 3*a^3*b^2 + a^2*b^3 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^4 +
 4*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)*sinh(x)^3 + (a^5 + 3*a^4*b
 + 3*a^3*b^2 + a^2*b^3)*sinh(x)^4 + 2*(a^5 + a^4*b - a^3*b^2 - a^2*b^3)*cos
h(x)^2 + 2*(a^5 + a^4*b - a^3*b^2 - a^2*b^3 + 3*(a^5 + 3*a^4*b + 3*a^3*b^2
+ a^2*b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*
cosh(x)^3 + (a^5 + a^4*b - a^3*b^2 - a^2*b^3)*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{\left(a + b\tanh^2(x)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*tanh(x)**2)**(3/2),x)
```

```
[Out] Integral(coth(x)/(a + b*tanh(x)**2)**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.245 $\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=85

$$-\frac{(a+2b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a^2(a+b)}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}}+\frac{b\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

```
[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(3/2) + (b*Cot
h[x])/(a*(a + b)*Sqrt[a + b*Tanh[x]^2]) - ((a + 2*b)*Coth[x]*Sqrt[a + b*Tan
h[x]^2])/(a^2*(a + b))
```

Rubi [A] time = 0.161239, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3670, 472, 583, 12, 377, 206}

$$-\frac{(a+2b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a^2(a+b)}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}}+\frac{b\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(3/2) + (b*Cot
h[x])/(a*(a + b)*Sqrt[a + b*Tanh[x]^2]) - ((a + 2*b)*Coth[x]*Sqrt[a + b*Tan
h[x]^2])/(a^2*(a + b))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
```

```
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx &= \text{Subst}\left(\int \frac{1}{x^2\left(1-x^2\right)\left(a+bx^2\right)^{3/2}}\,dx,x,\tanh(x)\right)\\
&= \frac{b\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}-\frac{\text{Subst}\left(\int \frac{-a-2b+2bx^2}{x^2\left(1-x^2\right)\sqrt{a+bx^2}}\,dx,x,\tanh(x)\right)}{a(a+b)}\\
&= \frac{b\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}-\frac{(a+2b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a^2(a+b)}+\frac{\text{Subst}\left(\int \frac{a^2}{\left(1-x^2\right)\sqrt{a+bx^2}}\,dx,x,\right.}{a^2(a+b)}\\
&= \frac{b\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}-\frac{(a+2b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a^2(a+b)}+\frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+bx^2}}\,dx,x,\right.}{a+b}\\
&= \frac{b\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}-\frac{(a+2b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a^2(a+b)}+\frac{\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx,x,\frac{}{\sqrt{a}}\right.}{a+b}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}}+\frac{b\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}-\frac{(a+2b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a^2(a+b)}
\end{aligned}$$

Mathematica [C] time = 7.81111, size = 230, normalized size = 2.71

$$-\frac{\sinh(2x)\text{sech}^2(x)\left(-\sqrt{2}a^2(a+b)\sqrt{\frac{\text{csch}^2(x)((a+b)\cosh(2x)+a-b)}{b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\text{csch}^2(x)((a+b)\cosh(2x)+a-b)}{b}}}{\sqrt{2}}\right),1\right)+(a+b)\text{csch}^2\right.}{2\sqrt{2}a^2(a+b)^2\sqrt{\text{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] -(((a + b)*(a^2 - 2*b^2 + (a^2 + 2*a*b + 2*b^2)*Cosh[2*x])*Csch[x]^2 - Sqrt
[2]*a^2*(a + b)*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*EllipticF[A
rcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1] + Sqrt[2
]*a^3*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*EllipticPi[b/(a + b),
 ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1])*Sech[
x]^2*Sinh[2*x])/(2*Sqrt[2]*a^2*(a + b)^2*Sqrt[(a - b + (a + b)*Cosh[2*x])*S
ech[x]^2])
```

Maple [F] time = 0.121, size = 0, normalized size = 0.

$$\int (\coth (x))^2 \left(a + b (\tanh (x))^2\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+b*tanh(x)^2)^(3/2),x)
```

```
[Out] int(coth(x)^2/(a+b*tanh(x)^2)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth (x)^2}{\left(b \tanh (x)^2 + a\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)^2/(b*tanh(x)^2 + a)^(3/2), x)
```

Fricas [B] time = 4.99433, size = 9975, normalized size = 117.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a^3 + a^2*b)*cosh(x)^6 + 6*(a^3 + a^2*b)*cosh(x)*sinh(x)^5 + (a^3 +
 a^2*b)*sinh(x)^6 + (a^3 - 3*a^2*b)*cosh(x)^4 + (a^3 - 3*a^2*b + 15*(a^3 +
a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^3 + a^2*b)*cosh(x)^3 + (a^3 - 3*a^2*b
)*cosh(x))*sinh(x)^3 - a^3 - a^2*b - (a^3 - 3*a^2*b)*cosh(x)^2 + (15*(a^3 +
 a^2*b)*cosh(x)^4 - a^3 + 3*a^2*b + 6*(a^3 - 3*a^2*b)*cosh(x)^2)*sinh(x)^2
+ 2*(3*(a^3 + a^2*b)*cosh(x)^5 + 2*(a^3 - 3*a^2*b)*cosh(x)^3 - (a^3 - 3*a^2
*b)*cosh(x))*sinh(x))*sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2
+ b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh
(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a
*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b
 + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b +
```

```
4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 +
b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6
*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2
 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*co
sh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)
^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh
(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*co
sh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*
b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2
*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*c
osh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*co
sh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cos
h(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5
*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*s
inh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a^3 + a^2*b)*cosh(x)^6 + 6
*(a^3 + a^2*b)*cosh(x)*sinh(x)^5 + (a^3 + a^2*b)*sinh(x)^6 + (a^3 - 3*a^2*b
)*cosh(x)^4 + (a^3 - 3*a^2*b + 15*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(5
*(a^3 + a^2*b)*cosh(x)^3 + (a^3 - 3*a^2*b)*cosh(x))*sinh(x)^3 - a^3 - a^2*b
 - (a^3 - 3*a^2*b)*cosh(x)^2 + (15*(a^3 + a^2*b)*cosh(x)^4 - a^3 + 3*a^2*b
+ 6*(a^3 - 3*a^2*b)*cosh(x)^2)*sinh(x)^2 + 2*(3*(a^3 + a^2*b)*cosh(x)^5 + 2
*(a^3 - 3*a^2*b)*cosh(x)^3 - (a^3 - 3*a^2*b)*cosh(x))*sinh(x))*sqrt(a + b)*
log(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 +
2*a*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2
+ 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 +
(a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4
*((a + b)*cosh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*si
nh(x) + sinh(x)^2)) - 4*sqrt(2)*((a^3 + 3*a^2*b + 4*a*b^2 + 2*b^3)*cosh(x)^
4 + 4*(a^3 + 3*a^2*b + 4*a*b^2 + 2*b^3)*cosh(x)*sinh(x)^3 + (a^3 + 3*a^2*b
+ 4*a*b^2 + 2*b^3)*sinh(x)^4 + a^3 + 3*a^2*b + 4*a*b^2 + 2*b^3 + 2*(a^3 + a
^2*b - 2*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(a^3 + a^2*b - 2*a*b^2 - 2*b^3 + 3*(a
^3 + 3*a^2*b + 4*a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 3*a^2*b +
4*a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 + a^2*b - 2*a*b^2 - 2*b^3)*cosh(x))*sinh(
x))*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cos
h(x)*sinh(x) + sinh(x)^2)))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^
6 + 6*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)*sinh(x)^5 + (a^5 + 3*a^
4*b + 3*a^3*b^2 + a^2*b^3)*sinh(x)^6 - a^5 - 3*a^4*b - 3*a^3*b^2 - a^2*b^3
+ (a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*cosh(x)^4 + (a^5 - a^4*b - 5*a^3*b^
2 - 3*a^2*b^3 + 15*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^2)*sinh(x)
^4 + 4*(5*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^3 + (a^5 - a^4*b -
5*a^3*b^2 - 3*a^2*b^3)*cosh(x))*sinh(x)^3 - (a^5 - a^4*b - 5*a^3*b^2 - 3*a^
2*b^3)*cosh(x)^2 - (a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3 - 15*(a^5 + 3*a^4*b
 + 3*a^3*b^2 + a^2*b^3)*cosh(x)^4 - 6*(a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)
*cosh(x)^2)*sinh(x)^2 + 2*(3*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^
5 + 2*(a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*cosh(x)^3 - (a^5 - a^4*b - 5*a^
3*b^2 - 3*a^2*b^3)*cosh(x))*sinh(x)), -1/2*(((a^3 + a^2*b)*cosh(x)^6 + 6*(a
^3 + a^2*b)*cosh(x)*sinh(x)^5 + (a^3 + a^2*b)*sinh(x)^6 + (a^3 - 3*a^2*b)*c
osh(x)^4 + (a^3 - 3*a^2*b + 15*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a
^3 + a^2*b)*cosh(x)^3 + (a^3 - 3*a^2*b)*cosh(x))*sinh(x)^3 - a^3 - a^2*b -
(a^3 - 3*a^2*b)*cosh(x)^2 + (15*(a^3 + a^2*b)*cosh(x)^4 - a^3 + 3*a^2*b + 6
*(a^3 - 3*a^2*b)*cosh(x)^2)*sinh(x)^2 + 2*(3*(a^3 + a^2*b)*cosh(x)^5 + 2*(a
^3 - 3*a^2*b)*cosh(x)^3 - (a^3 - 3*a^2*b)*cosh(x))*sinh(x))*sqrt(-a - b)*ar
ctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - a - b)*sqrt
(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 -
2*cosh(x)*sinh(x) + sinh(x)^2))/((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh
(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^2)*cosh(x)^2 + (6*
(a*b + b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 +
2*(2*(a*b + b^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x))) + ((a^3
 + a^2*b)*cosh(x)^6 + 6*(a^3 + a^2*b)*cosh(x)*sinh(x)^5 + (a^3 + a^2*b)*sin
```

```
h(x)^6 + (a^3 - 3*a^2*b)*cosh(x)^4 + (a^3 - 3*a^2*b + 15*(a^3 + a^2*b)*cosh
(x)^2)*sinh(x)^4 + 4*(5*(a^3 + a^2*b)*cosh(x)^3 + (a^3 - 3*a^2*b)*cosh(x))*
sinh(x)^3 - a^3 - a^2*b - (a^3 - 3*a^2*b)*cosh(x)^2 + (15*(a^3 + a^2*b)*cos
h(x)^4 - a^3 + 3*a^2*b + 6*(a^3 - 3*a^2*b)*cosh(x)^2)*sinh(x)^2 + 2*(3*(a^3
 + a^2*b)*cosh(x)^5 + 2*(a^3 - 3*a^2*b)*cosh(x)^3 - (a^3 - 3*a^2*b)*cosh(x)
)*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sin
h(x)^2 + 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a -
b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a +
 b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a +
 b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*
sinh(x) + a + b)) + 2*sqrt(2)*((a^3 + 3*a^2*b + 4*a*b^2 + 2*b^3)*cosh(x)^4
+ 4*(a^3 + 3*a^2*b + 4*a*b^2 + 2*b^3)*cosh(x)*sinh(x)^3 + (a^3 + 3*a^2*b +
4*a*b^2 + 2*b^3)*sinh(x)^4 + a^3 + 3*a^2*b + 4*a*b^2 + 2*b^3 + 2*(a^3 + a^2
*b - 2*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(a^3 + a^2*b - 2*a*b^2 - 2*b^3 + 3*(a^3
 + 3*a^2*b + 4*a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 3*a^2*b + 4*
a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 + a^2*b - 2*a*b^2 - 2*b^3)*cosh(x))*sinh(x)
)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(
x)*sinh(x) + sinh(x)^2)))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^6
+ 6*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)*sinh(x)^5 + (a^5 + 3*a^4*
b + 3*a^3*b^2 + a^2*b^3)*sinh(x)^6 - a^5 - 3*a^4*b - 3*a^3*b^2 - a^2*b^3 +
(a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*cosh(x)^4 + (a^5 - a^4*b - 5*a^3*b^2
- 3*a^2*b^3 + 15*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^2)*sinh(x)^4
 + 4*(5*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^3 + (a^5 - a^4*b - 5*
a^3*b^2 - 3*a^2*b^3)*cosh(x))*sinh(x)^3 - (a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*
b^3)*cosh(x)^2 - (a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3 - 15*(a^5 + 3*a^4*b +
 3*a^3*b^2 + a^2*b^3)*cosh(x)^4 - 6*(a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*c
osh(x)^2)*sinh(x)^2 + 2*(3*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^5
+ 2*(a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*cosh(x)^3 - (a^5 - a^4*b - 5*a^3*
b^2 - 3*a^2*b^3)*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2(x)}{\left(a + b\tanh^2(x)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+b*tanh(x)**2)**(3/2),x)
```

```
[Out] Integral(coth(x)**2/(a + b*tanh(x)**2)**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.246 $\int \frac{\tanh^6(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=118

$$\frac{a(a+2b)\tanh(x)}{b^2(a+b)^2\sqrt{a+b\tanh^2(x)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{b^{5/2}}+\frac{a\tanh^3(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}}$$

```
[Out] -(ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/b^(5/2)) + ArcTanh[(Sqrt
[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(5/2) + (a*Tanh[x]^3)/(3*b*
(a + b)*(a + b*Tanh[x]^2)^(3/2)) + (a*(a + 2*b)*Tanh[x])/(b^2*(a + b)^2*Sqr
t[a + b*Tanh[x]^2])
```

Rubi [A] time = 0.220451, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {3670, 470, 578, 523, 217, 206, 377}

$$\frac{a(a+2b)\tanh(x)}{b^2(a+b)^2\sqrt{a+b\tanh^2(x)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{b^{5/2}}+\frac{a\tanh^3(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^6/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/b^(5/2)) + ArcTanh[(Sqrt
[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(5/2) + (a*Tanh[x]^3)/(3*b*
(a + b)*(a + b*Tanh[x]^2)^(3/2)) + (a*(a + 2*b)*Tanh[x])/(b^2*(a + b)^2*Sqr
t[a + b*Tanh[x]^2])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\tanh^6(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{x^6}{(1-x^2)\left(a+bx^2\right)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= \frac{a\tanh^3(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{x^2(3a-3(a+b)x^2)}{(1-x^2)(a+bx^2)^{3/2}}\,dx, x, \tanh(x)\right)}{3b(a+b)}$$

$$= \frac{a\tanh^3(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{a(a+2b)\tanh(x)}{b^2(a+b)^2\sqrt{a+b\tanh^2(x)}} - \frac{\text{Subst}\left(\int \frac{3a(a+2b)-3(a+b)^2x^2}{(1-x^2)\sqrt{a+bx^2}}\,dx,\right.}{3b^2(a+b)^2}$$

$$= \frac{a\tanh^3(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{a(a+2b)\tanh(x)}{b^2(a+b)^2\sqrt{a+b\tanh^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tanh(x\right.}{b^2}$$

$$= \frac{a\tanh^3(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{a(a+2b)\tanh(x)}{b^2(a+b)^2\sqrt{a+b\tanh^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tanh(x}{\sqrt{a+b\tanh}}\right.}{b^2}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{b^{5/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}} + \frac{a\tanh^3(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{a(a}{b^2(a+b}$$

Mathematica [C] time = 1.86165, size = 231, normalized size = 1.96

$$\frac{\sqrt{\text{sech}^2(x)((a+b)\cosh(2x)+a-b)}\left(\frac{a(a+b)\sinh(2x)\left(\left(3a^2+10ab+7b^2\right)\cosh(2x)+3a^2+2ab-7b^2\right)}{((a+b)\cosh(2x)+a-b)^2} - \frac{3\sqrt{2}a\coth(x)\left(\left(a^2+3ab+2b^2\right)\text{EllipticF}\left(\sin^{-1}\right.\right.}{}\right.}{3\sqrt{2}b^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^6/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] (Sqrt[(a - b + (a + b)*Cosh[2*x])*Sech[x]^2]*((-3*Sqrt[2]*a*Coth[x]*((a^2 +
 3*a*b + 2*b^2)*EllipticF[ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^
2)/b]/Sqrt[2]], 1] + b^2*EllipticPi[b/(a + b), ArcSin[Sqrt[((a - b + (a + b
)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1]))/(b*Sqrt[((a - b + (a + b)*Cosh[2*
x])*Csch[x]^2)/b]) + (a*(a + b)*(3*a^2 + 2*a*b - 7*b^2 + (3*a^2 + 10*a*b +
7*b^2)*Cosh[2*x])*Sinh[2*x])/(a - b + (a + b)*Cosh[2*x])^2))/(3*Sqrt[2]*b^2
*(a + b)^3)
```

Maple [B] time = 0.032, size = 549, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^6/(a+b*tanh(x)^2)^(5/2),x)
```

```
[Out] 1/3*tanh(x)^3/b/(a+b*tanh(x)^2)^(3/2)+1/b^2*tanh(x)/(a+b*tanh(x)^2)^(1/2)-1
/b^(5/2)*ln(tanh(x)*b^(1/2)+(a+b*tanh(x)^2)^(1/2))+1/3*tanh(x)/b/(a+b*tanh(
x)^2)^(3/2)-1/3/a/b*tanh(x)/(a+b*tanh(x)^2)^(1/2)-1/3*tanh(x)/a/(a+b*tanh(x
)^2)^(3/2)-2/3/a^2*tanh(x)/(a+b*tanh(x)^2)^(1/2)+1/6/(a+b)/((1+tanh(x))^2*b
-2*(1+tanh(x))*b+a+b)^(3/2)+1/6*b/(a+b)/a/((1+tanh(x))^2*b-2*(1+tanh(x))*b+
a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1
/2)*tanh(x)+1/2/(a+b)^2/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)+1/2/(a+
b)^2/a/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)*b*tanh(x)-1/2/(a+b)^(5/2
)*ln((2*a+2*b-2*(1+tanh(x))*b+2*(a+b)^(1/2)*((1+tanh(x))^2*b-2*(1+tanh(x))*
b+a+b)^(1/2))/(1+tanh(x)))-1/6/(a+b)/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^
(3/2)+1/6*b/(a+b)/a/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(3/2)*tanh(x)+1/3
*b/(a+b)/a^2/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)*tanh(x)-1/2/(a+b)^
2/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)+1/2/(a+b)^2/a/((tanh(x)-1)^2*
b+2*(tanh(x)-1)*b+a+b)^(1/2)*b*tanh(x)+1/2/(a+b)^(5/2)*ln((2*a+2*b+2*(tanh(
x)-1)*b+2*(a+b)^(1/2)*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))/(tanh(x)
-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^6}{\left(b\tanh(x)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^6/(b*tanh(x)^2 + a)^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^6(x)}{\left(a+b\tanh^2(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**6/(a+b*tanh(x)**2)**(5/2),x)
```

```
[Out] Integral(tanh(x)**6/(a + b*tanh(x)**2)**(5/2), x)
```

<u>Giac</u> [F(-2)]   time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.247 $\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=84

$$-\frac{a^2}{3b^2(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}+\frac{a(a+2b)}{b^2(a+b)^2\sqrt{a+b\tanh^2(x)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}$$

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/(a + b)^(5/2) - a^2/(3*b^2*(a +
b)*(a + b*Tanh[x]^2)^(3/2)) + (a*(a + 2*b))/(b^2*(a + b)^2*Sqrt[a + b*Tanh[
x]^2])
```

Rubi [A] time = 0.180589, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3670, 446, 87, 63, 208}

$$-\frac{a^2}{3b^2(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}+\frac{a(a+2b)}{b^2(a+b)^2\sqrt{a+b\tanh^2(x)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^5/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/(a + b)^(5/2) - a^2/(3*b^2*(a +
b)*(a + b*Tanh[x]^2)^(3/2)) + (a*(a + 2*b))/(b^2*(a + b)^2*Sqrt[a + b*Tanh[
x]^2])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx &= \text{Subst}\left(\int \frac{x^5}{\left(1-x^2\right)\left(a+bx^2\right)^{5/2}}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(1-x)(a+bx)^{5/2}}\,dx, x, \tanh^2(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{b(a+b)(a+bx)^{5/2}} - \frac{a(a+2b)}{b(a+b)^2(a+bx)^{3/2}} - \frac{1}{(a+b)^2(-1+x)\sqrt{a+bx}}\right) dx, x, \text{ta}\right.\\
&= -\frac{a^2}{3b^2(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{a(a+2b)}{b^2(a+b)^2\sqrt{a+b\tanh^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+bx}}\,dx, x,\right.}{2(a+b)^2}\\
&= -\frac{a^2}{3b^2(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{a(a+2b)}{b^2(a+b)^2\sqrt{a+b\tanh^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{-1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a}\right.}{b(a+b)^2}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{a^2}{3b^2(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{a(a+2b)}{b^2(a+b)^2\sqrt{a+b\tanh^2(x)}}
\end{aligned}
$$

Mathematica [C] time = 0.110645, size = 68, normalized size = 0.81

$$
\frac{(a+b)\left(2a+3b\tanh^2(x)+b\right) - b^2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\tanh^2(x)+a}{a+b}\right)}{3b^2(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^5/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] (-(b^2*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tanh[x]^2)/(a + b)]) + (a +
b)*(2*a + b + 3*b*Tanh[x]^2))/(3*b^2*(a + b)*(a + b*Tanh[x]^2)^(3/2))
```

Maple [B] time = 0.027, size = 469, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a+b*tanh(x)^2)^(5/2),x)
```

```
[Out] tanh(x)^2/b/(a+b*tanh(x)^2)^(3/2)+2/3*a/b^2/(a+b*tanh(x)^2)^(3/2)+1/3/b/(a+
b*tanh(x)^2)^(3/2)-1/6/(a+b)/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(3/2)-1/
6*b/(a+b)/a/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(3/2)*tanh(x)-1/3*b/(a+b)
/a^2/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)*tanh(x)-1/2/(a+b)^2/((1+ta
nh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)-1/2/(a+b)^2/a/((1+tanh(x))^2*b-2*(1+t
anh(x))*b+a+b)^(1/2)*b*tanh(x)+1/2/(a+b)^(5/2)*ln((2*a+2*b-2*(1+tanh(x))*b+
2*(a+b)^(1/2)*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))/(1+tanh(x)))-1/6
/(a+b)/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(3/2)+1/6*b/(a+b)/a/((tanh(x)-
1)^2*b+2*(tanh(x)-1)*b+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/((tanh(x)-1)^2*b+
2*(tanh(x)-1)*b+a+b)^(1/2)*tanh(x)-1/2/(a+b)^2/((tanh(x)-1)^2*b+2*(tanh(x)-
1)*b+a+b)^(1/2)+1/2/(a+b)^2/a/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)*b
*tanh(x)+1/2/(a+b)^(5/2)*ln((2*a+2*b+2*(tanh(x)-1)*b+2*(a+b)^(1/2)*((tanh(x
)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))/(tanh(x)-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^5}{\left(b\tanh(x)^2 + a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^5/(b*tanh(x)^2 + a)^(5/2), x)
```

Fricas [B] time = 9.05503, size = 16629, normalized size = 197.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a^2*b^2 + 2*a*b^3 + b^4)*cosh(x)^8 + 8*(a^2*b^2 + 2*a*b^3 + b^4)
*cosh(x)*sinh(x)^7 + (a^2*b^2 + 2*a*b^3 + b^4)*sinh(x)^8 + 4*(a^2*b^2 - b^4
)*cosh(x)^6 + 4*(a^2*b^2 - b^4 + 7*(a^2*b^2 + 2*a*b^3 + b^4)*cosh(x)^2)*sin
h(x)^6 + 8*(7*(a^2*b^2 + 2*a*b^3 + b^4)*cosh(x)^3 + 3*(a^2*b^2 - b^4)*cosh(
x))*sinh(x)^5 + 2*(3*a^2*b^2 - 2*a*b^3 + 3*b^4)*cosh(x)^4 + 2*(35*(a^2*b^2
+ 2*a*b^3 + b^4)*cosh(x)^4 + 3*a^2*b^2 - 2*a*b^3 + 3*b^4 + 30*(a^2*b^2 - b^
4)*cosh(x)^2)*sinh(x)^4 + a^2*b^2 + 2*a*b^3 + b^4 + 8*(7*(a^2*b^2 + 2*a*b^3
 + b^4)*cosh(x)^5 + 10*(a^2*b^2 - b^4)*cosh(x)^3 + (3*a^2*b^2 - 2*a*b^3 + 3
*b^4)*cosh(x))*sinh(x)^3 + 4*(a^2*b^2 - b^4)*cosh(x)^2 + 4*(7*(a^2*b^2 + 2*
a*b^3 + b^4)*cosh(x)^6 + 15*(a^2*b^2 - b^4)*cosh(x)^4 + a^2*b^2 - b^4 + 3*(
3*a^2*b^2 - 2*a*b^3 + 3*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((a^2*b^2 + 2*a*b^3 +
 b^4)*cosh(x)^7 + 3*(a^2*b^2 - b^4)*cosh(x)^5 + (3*a^2*b^2 - 2*a*b^3 + 3*b^
4)*cosh(x)^3 + (a^2*b^2 - b^4)*cosh(x))*sinh(x))*sqrt(a + b)*log(((a^3 + a^
2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^
8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(
x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x)
)*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)
*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*
sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 +
(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^
2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)
```

```
^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^
2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*co
sh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^
2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a
*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b
- b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3
 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x
)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2
)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 +
4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x)
)/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*
sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 3*
((a^2*b^2 + 2*a*b^3 + b^4)*cosh(x)^8 + 8*(a^2*b^2 + 2*a*b^3 + b^4)*cosh(x)*
sinh(x)^7 + (a^2*b^2 + 2*a*b^3 + b^4)*sinh(x)^8 + 4*(a^2*b^2 - b^4)*cosh(x)
^6 + 4*(a^2*b^2 - b^4 + 7*(a^2*b^2 + 2*a*b^3 + b^4)*cosh(x)^2)*sinh(x)^6 +
8*(7*(a^2*b^2 + 2*a*b^3 + b^4)*cosh(x)^3 + 3*(a^2*b^2 - b^4)*cosh(x))*sinh(
x)^5 + 2*(3*a^2*b^2 - 2*a*b^3 + 3*b^4)*cosh(x)^4 + 2*(35*(a^2*b^2 + 2*a*b^3
 + b^4)*cosh(x)^4 + 3*a^2*b^2 - 2*a*b^3 + 3*b^4 + 30*(a^2*b^2 - b^4)*cosh(x
)^2)*sinh(x)^4 + a^2*b^2 + 2*a*b^3 + b^4 + 8*(7*(a^2*b^2 + 2*a*b^3 + b^4)*c
osh(x)^5 + 10*(a^2*b^2 - b^4)*cosh(x)^3 + (3*a^2*b^2 - 2*a*b^3 + 3*b^4)*cos
h(x))*sinh(x)^3 + 4*(a^2*b^2 - b^4)*cosh(x)^2 + 4*(7*(a^2*b^2 + 2*a*b^3 + b
^4)*cosh(x)^6 + 15*(a^2*b^2 - b^4)*cosh(x)^4 + a^2*b^2 - b^4 + 3*(3*a^2*b^2
 - 2*a*b^3 + 3*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((a^2*b^2 + 2*a*b^3 + b^4)*cos
h(x)^7 + 3*(a^2*b^2 - b^4)*cosh(x)^5 + (3*a^2*b^2 - 2*a*b^3 + 3*b^4)*cosh(x
)^3 + (a^2*b^2 - b^4)*cosh(x))*sinh(x))*sqrt(a + b)*log(-((a + b)*cosh(x)^4
 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(
a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
- b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 -
b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) +
8*sqrt(2)*((a^4 + 5*a^3*b + 7*a^2*b^2 + 3*a*b^3)*cosh(x)^6 + 6*(a^4 + 5*a^3
*b + 7*a^2*b^2 + 3*a*b^3)*cosh(x)*sinh(x)^5 + (a^4 + 5*a^3*b + 7*a^2*b^2 +
3*a*b^3)*sinh(x)^6 + 3*(a^4 + 3*a^3*b + a^2*b^2 - a*b^3)*cosh(x)^4 + 3*(a^4
 + 3*a^3*b + a^2*b^2 - a*b^3 + 5*(a^4 + 5*a^3*b + 7*a^2*b^2 + 3*a*b^3)*cosh
(x)^2)*sinh(x)^4 + a^4 + 5*a^3*b + 7*a^2*b^2 + 3*a*b^3 + 4*(5*(a^4 + 5*a^3*
b + 7*a^2*b^2 + 3*a*b^3)*cosh(x)^3 + 3*(a^4 + 3*a^3*b + a^2*b^2 - a*b^3)*co
sh(x))*sinh(x)^3 + 3*(a^4 + 3*a^3*b + a^2*b^2 - a*b^3)*cosh(x)^2 + 3*(5*(a^
4 + 5*a^3*b + 7*a^2*b^2 + 3*a*b^3)*cosh(x)^4 + a^4 + 3*a^3*b + a^2*b^2 - a*
b^3 + 6*(a^4 + 3*a^3*b + a^2*b^2 - a*b^3)*cosh(x)^2)*sinh(x)^2 + 6*((a^4 +
5*a^3*b + 7*a^2*b^2 + 3*a*b^3)*cosh(x)^5 + 2*(a^4 + 3*a^3*b + a^2*b^2 - a*b
^3)*cosh(x)^3 + (a^4 + 3*a^3*b + a^2*b^2 - a*b^3)*cosh(x))*sinh(x))*sqrt(((
a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x
) + sinh(x)^2)))/((a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6
+ b^7)*cosh(x)^8 + 8*(a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b
^6 + b^7)*cosh(x)*sinh(x)^7 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^
5 + 5*a*b^6 + b^7)*sinh(x)^8 + a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^
5 + 5*a*b^6 + b^7 + 4*(a^5*b^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^
6 - b^7)*cosh(x)^6 + 4*(a^5*b^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b
^6 - b^7 + 7*(a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7
)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^
5 + 5*a*b^6 + b^7)*cosh(x)^3 + 3*(a^5*b^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b
^5 - 3*a*b^6 - b^7)*cosh(x))*sinh(x)^5 + 2*(3*a^5*b^2 + 7*a^4*b^3 + 6*a^3*b
^4 + 6*a^2*b^5 + 7*a*b^6 + 3*b^7)*cosh(x)^4 + 2*(3*a^5*b^2 + 7*a^4*b^3 + 6*
a^3*b^4 + 6*a^2*b^5 + 7*a*b^6 + 3*b^7 + 35*(a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^
4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*cosh(x)^4 + 30*(a^5*b^2 + 3*a^4*b^3 + 2*a^3
*b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^5*b^2 + 5*
a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*cosh(x)^5 + 10*(a^5*b^2
+ 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7)*cosh(x)^3 + (3*a^5*b^2
 + 7*a^4*b^3 + 6*a^3*b^4 + 6*a^2*b^5 + 7*a*b^6 + 3*b^7)*cosh(x))*sinh(x)^3
```

```
+ 4*(a^5*b^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7)*cosh(x)^2
 + 4*(a^5*b^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7 + 7*(a^5*
b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*cosh(x)^6 + 15*(
a^5*b^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7)*cosh(x)^4 + 3*
(3*a^5*b^2 + 7*a^4*b^3 + 6*a^3*b^4 + 6*a^2*b^5 + 7*a*b^6 + 3*b^7)*cosh(x)^2
)*sinh(x)^2 + 8*((a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 +
 b^7)*cosh(x)^7 + 3*(a^5*b^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^6
- b^7)*cosh(x)^5 + (3*a^5*b^2 + 7*a^4*b^3 + 6*a^3*b^4 + 6*a^2*b^5 + 7*a*b^6
 + 3*b^7)*cosh(x)^3 + (a^5*b^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^
6 - b^7)*cosh(x))*sinh(x)), -1/6*(3*((a^2*b^2 + 2*a*b^3 + b^4)*cosh(x)^8 +
8*(a^2*b^2 + 2*a*b^3 + b^4)*cosh(x)*sinh(x)^7 + (a^2*b^2 + 2*a*b^3 + b^4)*s
inh(x)^8 + 4*(a^2*b^2 - b^4)*cosh(x)^6 + 4*(a^2*b^2 - b^4 + 7*(a^2*b^2 + 2*
a*b^3 + b^4)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^2*b^2 + 2*a*b^3 + b^4)*cosh(x)^
3 + 3*(a^2*b^2 - b^4)*cosh(x))*sinh(x)^5 + 2*(3*a^2*b^2 - 2*a*b^3 + 3*b^4)*
cosh(x)^4 + 2*(35*(a^2*b^2 + 2*a*b^3 + b^4)*cosh(x)^4 + 3*a^2*b^2 - 2*a*b^3
 + 3*b^4 + 30*(a^2*b^2 - b^4)*cosh(x)^2)*sinh(x)^4 + a^2*b^2 + 2*a*b^3 + b^
4 + 8*(7*(a^2*b^2 + 2*a*b^3 + b^4)*cosh(x)^5 + 10*(a^2*b^2 - b^4)*cosh(x)^3
 + (3*a^2*b^2 - 2*a*b^3 + 3*b^4)*cosh(x))*sinh(x)^3 + 4*(a^2*b^2 - b^4)*cos
h(x)^2 + 4*(7*(a^2*b^2 + 2*a*b^3 + b^4)*cosh(x)^6 + 15*(a^2*b^2 - b^4)*cosh
(x)^4 + a^2*b^2 - b^4 + 3*(3*a^2*b^2 - 2*a*b^3 + 3*b^4)*cosh(x)^2)*sinh(x)^
2 + 8*((a^2*b^2 + 2*a*b^3 + b^4)*cosh(x)^7 + 3*(a^2*b^2 - b^4)*cosh(x)^5 +
(3*a^2*b^2 - 2*a*b^3 + 3*b^4)*cosh(x)^3 + (a^2*b^2 - b^4)*cosh(x))*sinh(x))
*sqrt(-a - b)*arctan(sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)
^2 + a + b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a -
b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*
(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)
*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2
+ 2*a*b + b^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*s
inh(x))) + 3*((a^2*b^2 + 2*a*b^3 + b^4)*cosh(x)^8 + 8*(a^2*b^2 + 2*a*b^3 +
b^4)*cosh(x)*sinh(x)^7 + (a^2*b^2 + 2*a*b^3 + b^4)*sinh(x)^8 + 4*(a^2*b^2 -
 b^4)*cosh(x)^6 + 4*(a^2*b^2 - b^4 + 7*(a^2*b^2 + 2*a*b^3 + b^4)*cosh(x)^2)
*sinh(x)^6 + 8*(7*(a^2*b^2 + 2*a*b^3 + b^4)*cosh(x)^3 + 3*(a^2*b^2 - b^4)*c
osh(x))*sinh(x)^5 + 2*(3*a^2*b^2 - 2*a*b^3 + 3*b^4)*cosh(x)^4 + 2*(35*(a^2*
b^2 + 2*a*b^3 + b^4)*cosh(x)^4 + 3*a^2*b^2 - 2*a*b^3 + 3*b^4 + 30*(a^2*b^2
- b^4)*cosh(x)^2)*sinh(x)^4 + a^2*b^2 + 2*a*b^3 + b^4 + 8*(7*(a^2*b^2 + 2*a
*b^3 + b^4)*cosh(x)^5 + 10*(a^2*b^2 - b^4)*cosh(x)^3 + (3*a^2*b^2 - 2*a*b^3
 + 3*b^4)*cosh(x))*sinh(x)^3 + 4*(a^2*b^2 - b^4)*cosh(x)^2 + 4*(7*(a^2*b^2
+ 2*a*b^3 + b^4)*cosh(x)^6 + 15*(a^2*b^2 - b^4)*cosh(x)^4 + a^2*b^2 - b^4 +
 3*(3*a^2*b^2 - 2*a*b^3 + 3*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((a^2*b^2 + 2*a*b
^3 + b^4)*cosh(x)^7 + 3*(a^2*b^2 - b^4)*cosh(x)^5 + (3*a^2*b^2 - 2*a*b^3 +
3*b^4)*cosh(x)^3 + (a^2*b^2 - b^4)*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sq
rt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((a
 + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
 + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*s
inh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2
+ 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) - 4*sqrt(2)*((a
^4 + 5*a^3*b + 7*a^2*b^2 + 3*a*b^3)*cosh(x)^6 + 6*(a^4 + 5*a^3*b + 7*a^2*b^
2 + 3*a*b^3)*cosh(x)*sinh(x)^5 + (a^4 + 5*a^3*b + 7*a^2*b^2 + 3*a*b^3)*sinh
(x)^6 + 3*(a^4 + 3*a^3*b + a^2*b^2 - a*b^3)*cosh(x)^4 + 3*(a^4 + 3*a^3*b +
a^2*b^2 - a*b^3 + 5*(a^4 + 5*a^3*b + 7*a^2*b^2 + 3*a*b^3)*cosh(x)^2)*sinh(x
)^4 + a^4 + 5*a^3*b + 7*a^2*b^2 + 3*a*b^3 + 4*(5*(a^4 + 5*a^3*b + 7*a^2*b^2
 + 3*a*b^3)*cosh(x)^3 + 3*(a^4 + 3*a^3*b + a^2*b^2 - a*b^3)*cosh(x))*sinh(x
)^3 + 3*(a^4 + 3*a^3*b + a^2*b^2 - a*b^3)*cosh(x)^2 + 3*(5*(a^4 + 5*a^3*b +
 7*a^2*b^2 + 3*a*b^3)*cosh(x)^4 + a^4 + 3*a^3*b + a^2*b^2 - a*b^3 + 6*(a^4
+ 3*a^3*b + a^2*b^2 - a*b^3)*cosh(x)^2)*sinh(x)^2 + 6*((a^4 + 5*a^3*b + 7*a
^2*b^2 + 3*a*b^3)*cosh(x)^5 + 2*(a^4 + 3*a^3*b + a^2*b^2 - a*b^3)*cosh(x)^3
 + (a^4 + 3*a^3*b + a^2*b^2 - a*b^3)*cosh(x))*sinh(x))*sqrt(((a + b)*cosh(x
)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2
)))/((a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*cosh(x
```

```
)^8 + 8*(a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*cos
h(x)*sinh(x)^7 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 +
 b^7)*sinh(x)^8 + a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 +
 b^7 + 4*(a^5*b^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7)*cosh
(x)^6 + 4*(a^5*b^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7 + 7*
(a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*cosh(x)^2)*
sinh(x)^6 + 8*(7*(a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 +
 b^7)*cosh(x)^3 + 3*(a^5*b^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^6
- b^7)*cosh(x))*sinh(x)^5 + 2*(3*a^5*b^2 + 7*a^4*b^3 + 6*a^3*b^4 + 6*a^2*b^
5 + 7*a*b^6 + 3*b^7)*cosh(x)^4 + 2*(3*a^5*b^2 + 7*a^4*b^3 + 6*a^3*b^4 + 6*a
^2*b^5 + 7*a*b^6 + 3*b^7 + 35*(a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^
5 + 5*a*b^6 + b^7)*cosh(x)^4 + 30*(a^5*b^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*
b^5 - 3*a*b^6 - b^7)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^5*b^2 + 5*a^4*b^3 + 10*
a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*cosh(x)^5 + 10*(a^5*b^2 + 3*a^4*b^3 +
 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7)*cosh(x)^3 + (3*a^5*b^2 + 7*a^4*b^3
+ 6*a^3*b^4 + 6*a^2*b^5 + 7*a*b^6 + 3*b^7)*cosh(x))*sinh(x)^3 + 4*(a^5*b^2
+ 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7)*cosh(x)^2 + 4*(a^5*b^2
 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7 + 7*(a^5*b^2 + 5*a^4*b
^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*cosh(x)^6 + 15*(a^5*b^2 + 3*a
^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7)*cosh(x)^4 + 3*(3*a^5*b^2 +
7*a^4*b^3 + 6*a^3*b^4 + 6*a^2*b^5 + 7*a*b^6 + 3*b^7)*cosh(x)^2)*sinh(x)^2 +
 8*((a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*cosh(x)
^7 + 3*(a^5*b^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7)*cosh(x
)^5 + (3*a^5*b^2 + 7*a^4*b^3 + 6*a^3*b^4 + 6*a^2*b^5 + 7*a*b^6 + 3*b^7)*cos
h(x)^3 + (a^5*b^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7)*cosh
(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^5(x)}{\left(a + b\tanh^2(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**5/(a+b*tanh(x)**2)**(5/2),x)
```

```
[Out] Integral(tanh(x)**5/(a + b*tanh(x)**2)**(5/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.248 $\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=90

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}}-\frac{(a+4b)\tanh(x)}{3b(a+b)^2\sqrt{a+b\tanh^2(x)}}+\frac{a\tanh(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

```
[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(5/2) + (a*Tan
h[x])/(3*b*(a + b)*(a + b*Tanh[x]^2)^(3/2)) - ((a + 4*b)*Tanh[x])/(3*b*(a +
 b)^2*Sqrt[a + b*Tanh[x]^2])
```

Rubi [A] time = 0.137977, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3670, 470, 527, 12, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}}-\frac{(a+4b)\tanh(x)}{3b(a+b)^2\sqrt{a+b\tanh^2(x)}}+\frac{a\tanh(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(5/2) + (a*Tan
h[x])/(3*b*(a + b)*(a + b*Tanh[x]^2)^(3/2)) - ((a + 4*b)*Tanh[x])/(3*b*(a +
 b)^2*Sqrt[a + b*Tanh[x]^2])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)\left(a+bx^2\right)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= \frac{a\tanh(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{a+(-a-3b)x^2}{\left(1-x^2\right)\left(a+bx^2\right)^{3/2}}\,dx, x, \tanh(x)\right)}{3b(a+b)}$$

$$= \frac{a\tanh(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{(a+4b)\tanh(x)}{3b(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{3ab}{\left(1-x^2\right)\sqrt{a+bx^2}}\,dx, x, \text{ta}\right.}{3ab(a+b)^2}$$

$$= \frac{a\tanh(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{(a+4b)\tanh(x)}{3b(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+bx^2}}\,dx, x, \text{ta}\right.}{(a+b)^2}$$

$$= \frac{a\tanh(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{(a+4b)\tanh(x)}{3b(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\text{ta}}{\sqrt{a+b}}\right.}{(a+b)^2}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}} + \frac{a\tanh(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{(a+4b)\tanh(x)}{3b(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

Mathematica [A] time = 2.55557, size = 132, normalized size = 1.47

$$\frac{\tanh^3(x)\left(3\tanh^{-1}\left(\frac{\sqrt{\frac{(a+b)\tanh^2(x)}{a}}}{\sqrt{\frac{b\tanh^2(x)}{a}+1}}\right)\sqrt{\frac{(a+b)\tanh^2(x)}{a}}\left(a\coth^2(x)+b\right)^2-(a+b)\sqrt{\frac{b\tanh^2(x)}{a}+1}\left(3a\coth^2(x)+a+4b\right)\right)}{3(a+b)^3\left(a+b\tanh^2(x)\right)^{3/2}\sqrt{\frac{b\tanh^2(x)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] (Tanh[x]^3*(3*ArcTanh[Sqrt[((a + b)*Tanh[x]^2)/a]/Sqrt[1 + (b*Tanh[x]^2)/a]
]*(b + a*Coth[x]^2)^2*Sqrt[((a + b)*Tanh[x]^2)/a] - (a + b)*(a + 4*b + 3*a*
Coth[x]^2)*Sqrt[1 + (b*Tanh[x]^2)/a]))/(3*(a + b)^3*(a + b*Tanh[x]^2)^(3/2)
*Sqrt[1 + (b*Tanh[x]^2)/a])
```

Maple [B] time = 0.025, size = 491, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+b*tanh(x)^2)^(5/2),x)
```

```
[Out] 1/3*tanh(x)/b/(a+b*tanh(x)^2)^(3/2)-1/3/a/b*tanh(x)/(a+b*tanh(x)^2)^(1/2)-1
/3*tanh(x)/a/(a+b*tanh(x)^2)^(3/2)-2/3/a^2*tanh(x)/(a+b*tanh(x)^2)^(1/2)+1/
6/(a+b)/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(3/2)+1/6*b/(a+b)/a/((1+tanh(
x))^2*b-2*(1+tanh(x))*b+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/((1+tanh(x))^2*b
-2*(1+tanh(x))*b+a+b)^(1/2)*tanh(x)+1/2/(a+b)^2/((1+tanh(x))^2*b-2*(1+tanh(
x))*b+a+b)^(1/2)+1/2/(a+b)^2/a/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)*
b*tanh(x)-1/2/(a+b)^(5/2)*ln((2*a+2*b-2*(1+tanh(x))*b+2*(a+b)^(1/2)*((1+tan
h(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))/(1+tanh(x)))-1/6/(a+b)/((tanh(x)-1)^2
*b+2*(tanh(x)-1)*b+a+b)^(3/2)+1/6*b/(a+b)/a/((tanh(x)-1)^2*b+2*(tanh(x)-1)*
b+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^
(1/2)*tanh(x)-1/2/(a+b)^2/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)+1/2/(
a+b)^2/a/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)*b*tanh(x)+1/2/(a+b)^(5
/2)*ln((2*a+2*b+2*(tanh(x)-1)*b+2*(a+b)^(1/2)*((tanh(x)-1)^2*b+2*(tanh(x)-1
)*b+a+b)^(1/2))/(tanh(x)-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^4}{\left(b\tanh(x)^2 + a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^4/(b*tanh(x)^2 + a)^(5/2), x)
```

Fricas [B] time = 8.55704, size = 14436, normalized size = 160.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a^2 + 2*a*b + b^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(x)*sin
h(x)^7 + (a^2 + 2*a*b + b^2)*sinh(x)^8 + 4*(a^2 - b^2)*cosh(x)^6 + 4*(7*(a^
2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2
)*cosh(x)^3 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 - 2*a*b + 3*b^2)*
cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 30*(a^2 - b^2)*cosh(x)^2
+ 3*a^2 - 2*a*b + 3*b^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 1
0*(a^2 - b^2)*cosh(x)^3 + (3*a^2 - 2*a*b + 3*b^2)*cosh(x))*sinh(x)^3 + 4*(a
^2 - b^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^6 + 15*(a^2 - b^2)*c
osh(x)^4 + 3*(3*a^2 - 2*a*b + 3*b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2
 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*cosh(x)^7 + 3*(a^2 - b^2)*cosh(x)^5
 + (3*a^2 - 2*a*b + 3*b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))*sqrt(a
 + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (
a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 1
4*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a
*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^
4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2
 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2
+ 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a
^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*
(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*
b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2
*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*
(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sin
h(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(
x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x
)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)
*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x
)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*
cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*
b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh
(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)
^5 + sinh(x)^6)) + 3*((a^2 + 2*a*b + b^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2)
*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2)*sinh(x)^8 + 4*(a^2 - b^2)*cosh(x)^
6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^6 + 8*(7*(a^2 +
 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 - 2*a
*b + 3*b^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 30*(a^2 - b^2
)*cosh(x)^2 + 3*a^2 - 2*a*b + 3*b^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2)*c
osh(x)^5 + 10*(a^2 - b^2)*cosh(x)^3 + (3*a^2 - 2*a*b + 3*b^2)*cosh(x))*sinh
(x)^3 + 4*(a^2 - b^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^6 + 15*(
a^2 - b^2)*cosh(x)^4 + 3*(3*a^2 - 2*a*b + 3*b^2)*cosh(x)^2 + a^2 - b^2)*sin
h(x)^2 + a^2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*cosh(x)^7 + 3*(a^2 - b^
2)*cosh(x)^5 + (3*a^2 - 2*a*b + 3*b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sin
h(x))*sqrt(a + b)*log(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a
 + b)*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + s
qrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a
 + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
 + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x
)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 16*sqrt(2)*((a^2 + 2*a*b + b^2)*cos
h(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh
(x)^6 - 3*(a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 - a*b
- b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 - 3*(a*b + b^2)*cosh(
x))*sinh(x)^3 + 3*(a*b + b^2)*cosh(x)^2 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^
4 - 6*(a*b + b^2)*cosh(x)^2 + a*b + b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 6*
((a^2 + 2*a*b + b^2)*cosh(x)^5 - 2*(a*b + b^2)*cosh(x)^3 + (a*b + b^2)*cosh
(x))*sinh(x))*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2
*b^3 + 5*a*b^4 + b^5)*cosh(x)^8 + 8*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^
3 + 5*a*b^4 + b^5)*cosh(x)*sinh(x)^7 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2
*b^3 + 5*a*b^4 + b^5)*sinh(x)^8 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3
```

- 3*a*b^4 - b^5)*cosh(x)^6 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a
b^4 - b^5 + 7(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*co
sh(x)^2)*sinh(x)^6 + 8*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^
4 + b^5)*cosh(x)^3 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b
^5)*cosh(x))*sinh(x)^5 + a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4
+ b^5 + 2*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*cosh(
x)^4 + 2*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5 + 35*(a
^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)^4 + 30*(a^5
 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^2)*sinh(x)^4 +
8*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)^5 +
10*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^3 + (3*a
^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*cosh(x))*sinh(x)^3
+ 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^2 + 4*(
7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)^6 + a^5
 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5 + 15*(a^5 + 3*a^4*b + 2*
a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^4 + 3*(3*a^5 + 7*a^4*b + 6*a^3
*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*cosh(x)^2)*sinh(x)^2 + 8*((a^5 + 5*a^4*
b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)^7 + 3*(a^5 + 3*a^4*b +
 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^5 + (3*a^5 + 7*a^4*b + 6*a^
3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*cosh(x)^3 + (a^5 + 3*a^4*b + 2*a^3*b^2
 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x))*sinh(x)), -1/6*(3*((a^2 + 2*a*b + b^
2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2
)*sinh(x)^8 + 4*(a^2 - b^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^2
+ a^2 - b^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 - b^2)
*cosh(x))*sinh(x)^5 + 2*(3*a^2 - 2*a*b + 3*b^2)*cosh(x)^4 + 2*(35*(a^2 + 2*
a*b + b^2)*cosh(x)^4 + 30*(a^2 - b^2)*cosh(x)^2 + 3*a^2 - 2*a*b + 3*b^2)*si
nh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 10*(a^2 - b^2)*cosh(x)^3 + (
3*a^2 - 2*a*b + 3*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 - b^2)*cosh(x)^2 + 4*(7*
(a^2 + 2*a*b + b^2)*cosh(x)^6 + 15*(a^2 - b^2)*cosh(x)^4 + 3*(3*a^2 - 2*a*b
 + 3*b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 8*((a^2 +
2*a*b + b^2)*cosh(x)^7 + 3*(a^2 - b^2)*cosh(x)^5 + (3*a^2 - 2*a*b + 3*b^2)*
cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(b*co
sh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - a - b)*sqrt(-a - b)*sqrt(((a
+ b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
+ sinh(x)^2))/((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh(x)*sinh(x)^3 + (a
*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^2)*cosh(x)^2 + (6*(a*b + b^2)*cosh(x
)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a*b + b^2)*c
osh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x))) + 3*((a^2 + 2*a*b + b^2)*
cosh(x)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2)*s
inh(x)^8 + 4*(a^2 - b^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a
^2 - b^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 - b^2)*co
sh(x))*sinh(x)^5 + 2*(3*a^2 - 2*a*b + 3*b^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b
 + b^2)*cosh(x)^4 + 30*(a^2 - b^2)*cosh(x)^2 + 3*a^2 - 2*a*b + 3*b^2)*sinh(
x)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 10*(a^2 - b^2)*cosh(x)^3 + (3*a
^2 - 2*a*b + 3*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 - b^2)*cosh(x)^2 + 4*(7*(a^
2 + 2*a*b + b^2)*cosh(x)^6 + 15*(a^2 - b^2)*cosh(x)^4 + 3*(3*a^2 - 2*a*b +
3*b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 8*((a^2 + 2*a
*b + b^2)*cosh(x)^7 + 3*(a^2 - b^2)*cosh(x)^5 + (3*a^2 - 2*a*b + 3*b^2)*cos
h(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*sqrt(-a
- b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*co
sh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x)
+ (a + b)*sinh(x)^2 + a + b)) + 8*sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 +
6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 - 3
*(a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 - a*b - b^2)*si
nh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 - 3*(a*b + b^2)*cosh(x))*sinh(
x)^3 + 3*(a*b + b^2)*cosh(x)^2 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^4 - 6*(a*
b + b^2)*cosh(x)^2 + a*b + b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 6*((a^2 + 2
*a*b + b^2)*cosh(x)^5 - 2*(a*b + b^2)*cosh(x)^3 + (a*b + b^2)*cosh(x))*sinh
(x))*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*co

```
sh(x)*sinh(x) + sinh(x)^2)))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*
a*b^4 + b^5)*cosh(x)^8 + 8*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b
^4 + b^5)*cosh(x)*sinh(x)^7 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*
a*b^4 + b^5)*sinh(x)^8 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4
 - b^5)*cosh(x)^6 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^
5 + 7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)^2)*
sinh(x)^6 + 8*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*
cosh(x)^3 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(
x))*sinh(x)^5 + a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5 + 2
*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*cosh(x)^4 + 2*
(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5 + 35*(a^5 + 5*a^
4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)^4 + 30*(a^5 + 3*a^4*
b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^5
 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)^5 + 10*(a^5 +
 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^3 + (3*a^5 + 7*a^
4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*cosh(x))*sinh(x)^3 + 4*(a^5
+ 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^2 + 4*(7*(a^5 +
5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)^6 + a^5 + 3*a^4*
b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5 + 15*(a^5 + 3*a^4*b + 2*a^3*b^2 -
 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^4 + 3*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*
a^2*b^3 + 7*a*b^4 + 3*b^5)*cosh(x)^2)*sinh(x)^2 + 8*((a^5 + 5*a^4*b + 10*a^
3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)^7 + 3*(a^5 + 3*a^4*b + 2*a^3*b^
2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^5 + (3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6
*a^2*b^3 + 7*a*b^4 + 3*b^5)*cosh(x)^3 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*
b^3 - 3*a*b^4 - b^5)*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^4(x)}{\left(a + b\tanh^2(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+b*tanh(x)**2)**(5/2),x)
```

```
[Out] Integral(tanh(x)**4/(a + b*tanh(x)**2)**(5/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.249 $\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=74

$$\frac{a}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}-\frac{1}{(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/(a + b)^(5/2) + a/(3*b*(a + b)*(
a + b*Tanh[x]^2)^(3/2)) - 1/((a + b)^2*Sqrt[a + b*Tanh[x]^2])
```

Rubi [A] time = 0.144009, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3670, 446, 78, 51, 63, 208}

$$\frac{a}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}-\frac{1}{(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/(a + b)^(5/2) + a/(3*b*(a + b)*(
a + b*Tanh[x]^2)^(3/2)) - 1/((a + b)^2*Sqrt[a + b*Tanh[x]^2])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx &= \text{Subst}\left(\int \frac{x^3}{\left(1-x^2\right)\left(a+bx^2\right)^{5/2}}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(1-x)(a+bx)^{5/2}}\,dx, x, \tanh^2(x)\right)\\
&= \frac{a}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)^{3/2}}\,dx, x, \tanh^2(x)\right)}{2(a+b)}\\
&= \frac{a}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{1}{(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \tanh^2\right.}{2(a+b)^2}\\
&= \frac{a}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{1}{(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\,\mathrm{t}}\right.}{b(a+b)^2}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} + \frac{a}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{1}{(a+b)^2\sqrt{a+b\tanh^2(x)}}
\end{aligned}$$

Mathematica [C] time = 0.0866463, size = 63, normalized size = 0.85

$$\frac{a(a+b)-3b\left(a+b\tanh^2(x)\right){}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\tanh^2(x)+a}{a+b}\right)}{3b(a+b)^2\left(a+b\tanh^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] (a*(a + b) - 3*b*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tanh[x]^2)/(a + b)]
*(a + b*Tanh[x]^2))/(3*b*(a + b)^2*(a + b*Tanh[x]^2)^(3/2))
```

Maple [B] time = 0.021, size = 435, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+b*tanh(x)^2)^(5/2),x)
```

```
[Out] 1/3/b/(a+b*tanh(x)^2)^(3/2)-1/6/(a+b)/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)
^(3/2)-1/6*b/(a+b)/a/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(3/2)*tanh(x)-1/
3*b/(a+b)/a^2/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)*tanh(x)-1/2/(a+b)
^2/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)-1/2/(a+b)^2/a/((1+tanh(x))^2
*b-2*(1+tanh(x))*b+a+b)^(1/2)*b*tanh(x)+1/2/(a+b)^(5/2)*ln((2*a+2*b-2*(1+ta
nh(x))*b+2*(a+b)^(1/2)*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))/(1+tanh
(x)))-1/6/(a+b)/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(3/2)+1/6*b/(a+b)/a/(
(tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/((tanh(x
)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)*tanh(x)-1/2/(a+b)^2/((tanh(x)-1)^2*b+2*
(tanh(x)-1)*b+a+b)^(1/2)+1/2/(a+b)^2/a/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b
)^(1/2)*b*tanh(x)+1/2/(a+b)^(5/2)*ln((2*a+2*b+2*(tanh(x)-1)*b+2*(a+b)^(1/2)
*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))/(tanh(x)-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^3}{\left(b\tanh(x)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^3/(b*tanh(x)^2 + a)^(5/2), x)
```

Fricas [B] time = 8.75918, size = 15965, normalized size = 215.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a^2*b + 2*a*b^2 + b^3)*cosh(x)^8 + 8*(a^2*b + 2*a*b^2 + b^3)*cos
h(x)*sinh(x)^7 + (a^2*b + 2*a*b^2 + b^3)*sinh(x)^8 + 4*(a^2*b - b^3)*cosh(x
)^6 + 4*(a^2*b - b^3 + 7*(a^2*b + 2*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 8*(
7*(a^2*b + 2*a*b^2 + b^3)*cosh(x)^3 + 3*(a^2*b - b^3)*cosh(x))*sinh(x)^5 +
2*(3*a^2*b - 2*a*b^2 + 3*b^3)*cosh(x)^4 + 2*(35*(a^2*b + 2*a*b^2 + b^3)*cos
h(x)^4 + 3*a^2*b - 2*a*b^2 + 3*b^3 + 30*(a^2*b - b^3)*cosh(x)^2)*sinh(x)^4
+ 8*(7*(a^2*b + 2*a*b^2 + b^3)*cosh(x)^5 + 10*(a^2*b - b^3)*cosh(x)^3 + (3*
a^2*b - 2*a*b^2 + 3*b^3)*cosh(x))*sinh(x)^3 + a^2*b + 2*a*b^2 + b^3 + 4*(a^
2*b - b^3)*cosh(x)^2 + 4*(7*(a^2*b + 2*a*b^2 + b^3)*cosh(x)^6 + 15*(a^2*b -
 b^3)*cosh(x)^4 + a^2*b - b^3 + 3*(3*a^2*b - 2*a*b^2 + 3*b^3)*cosh(x)^2)*si
nh(x)^2 + 8*((a^2*b + 2*a*b^2 + b^3)*cosh(x)^7 + 3*(a^2*b - b^3)*cosh(x)^5
```

```
+ (3*a^2*b - 2*a*b^2 + 3*b^3)*cosh(x)^3 + (a^2*b - b^3)*cosh(x))*sinh(x))*s
qrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7
 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b
 + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 +
3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh
(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2
*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*
a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3
 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*
(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*
b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*
(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4
+ 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))
*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*c
osh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*co
sh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a
+ b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*co
sh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2
*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*
b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*
sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sin
h(x)^5 + sinh(x)^6)) + 3*((a^2*b + 2*a*b^2 + b^3)*cosh(x)^8 + 8*(a^2*b + 2*
a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a^2*b + 2*a*b^2 + b^3)*sinh(x)^8 + 4*(a^2
*b - b^3)*cosh(x)^6 + 4*(a^2*b - b^3 + 7*(a^2*b + 2*a*b^2 + b^3)*cosh(x)^2)
*sinh(x)^6 + 8*(7*(a^2*b + 2*a*b^2 + b^3)*cosh(x)^3 + 3*(a^2*b - b^3)*cosh(
x))*sinh(x)^5 + 2*(3*a^2*b - 2*a*b^2 + 3*b^3)*cosh(x)^4 + 2*(35*(a^2*b + 2*
a*b^2 + b^3)*cosh(x)^4 + 3*a^2*b - 2*a*b^2 + 3*b^3 + 30*(a^2*b - b^3)*cosh(
x)^2)*sinh(x)^4 + 8*(7*(a^2*b + 2*a*b^2 + b^3)*cosh(x)^5 + 10*(a^2*b - b^3)
*cosh(x)^3 + (3*a^2*b - 2*a*b^2 + 3*b^3)*cosh(x))*sinh(x)^3 + a^2*b + 2*a*b
^2 + b^3 + 4*(a^2*b - b^3)*cosh(x)^2 + 4*(7*(a^2*b + 2*a*b^2 + b^3)*cosh(x)
^6 + 15*(a^2*b - b^3)*cosh(x)^4 + a^2*b - b^3 + 3*(3*a^2*b - 2*a*b^2 + 3*b^
3)*cosh(x)^2)*sinh(x)^2 + 8*((a^2*b + 2*a*b^2 + b^3)*cosh(x)^7 + 3*(a^2*b -
 b^3)*cosh(x)^5 + (3*a^2*b - 2*a*b^2 + 3*b^3)*cosh(x)^3 + (a^2*b - b^3)*cos
h(x))*sinh(x))*sqrt(a + b)*log(-((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh
(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sin
h(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b
)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(
x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 - b*cosh(x))*sinh(x) + a +
b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*sqrt(2)*((a^3 - a^2*b -
 5*a*b^2 - 3*b^3)*cosh(x)^6 + 6*(a^3 - a^2*b - 5*a*b^2 - 3*b^3)*cosh(x)*sin
h(x)^5 + (a^3 - a^2*b - 5*a*b^2 - 3*b^3)*sinh(x)^6 + 3*(a^3 - a^2*b - a*b^2
 + b^3)*cosh(x)^4 + 3*(a^3 - a^2*b - a*b^2 + b^3 + 5*(a^3 - a^2*b - 5*a*b^2
 - 3*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^3 - a^2*b - 5*a*b^2 - 3*b^3)*cosh(
x)^3 + 3*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 - a^2*b - 5*a
*b^2 - 3*b^3 + 3*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^2 + 3*(5*(a^3 - a^2*b
- 5*a*b^2 - 3*b^3)*cosh(x)^4 + a^3 - a^2*b - a*b^2 + b^3 + 6*(a^3 - a^2*b -
 a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 6*((a^3 - a^2*b - 5*a*b^2 - 3*b^3)*cos
h(x)^5 + 2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^3 + (a^3 - a^2*b - a*b^2 + b
^3)*cosh(x))*sinh(x))*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^5*b + 5*a^4*b^2 + 10*a^3*
b^3 + 10*a^2*b^4 + 5*a*b^5 + b^6)*cosh(x)^8 + 8*(a^5*b + 5*a^4*b^2 + 10*a^3
*b^3 + 10*a^2*b^4 + 5*a*b^5 + b^6)*cosh(x)*sinh(x)^7 + (a^5*b + 5*a^4*b^2 +
 10*a^3*b^3 + 10*a^2*b^4 + 5*a*b^5 + b^6)*sinh(x)^8 + 4*(a^5*b + 3*a^4*b^2
+ 2*a^3*b^3 - 2*a^2*b^4 - 3*a*b^5 - b^6)*cosh(x)^6 + 4*(a^5*b + 3*a^4*b^2 +
 2*a^3*b^3 - 2*a^2*b^4 - 3*a*b^5 - b^6 + 7*(a^5*b + 5*a^4*b^2 + 10*a^3*b^3
+ 10*a^2*b^4 + 5*a*b^5 + b^6)*cosh(x)^2)*sinh(x)^6 + a^5*b + 5*a^4*b^2 + 10
*a^3*b^3 + 10*a^2*b^4 + 5*a*b^5 + b^6 + 8*(7*(a^5*b + 5*a^4*b^2 + 10*a^3*b^
3 + 10*a^2*b^4 + 5*a*b^5 + b^6)*cosh(x)^3 + 3*(a^5*b + 3*a^4*b^2 + 2*a^3*b^
3 - 2*a^2*b^4 - 3*a*b^5 - b^6)*cosh(x))*sinh(x)^5 + 2*(3*a^5*b + 7*a^4*b^2
```

```
+ 6*a^3*b^3 + 6*a^2*b^4 + 7*a*b^5 + 3*b^6)*cosh(x)^4 + 2*(3*a^5*b + 7*a^4*b
^2 + 6*a^3*b^3 + 6*a^2*b^4 + 7*a*b^5 + 3*b^6 + 35*(a^5*b + 5*a^4*b^2 + 10*a
^3*b^3 + 10*a^2*b^4 + 5*a*b^5 + b^6)*cosh(x)^4 + 30*(a^5*b + 3*a^4*b^2 + 2*
a^3*b^3 - 2*a^2*b^4 - 3*a*b^5 - b^6)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^5*b + 5
*a^4*b^2 + 10*a^3*b^3 + 10*a^2*b^4 + 5*a*b^5 + b^6)*cosh(x)^5 + 10*(a^5*b +
 3*a^4*b^2 + 2*a^3*b^3 - 2*a^2*b^4 - 3*a*b^5 - b^6)*cosh(x)^3 + (3*a^5*b +
7*a^4*b^2 + 6*a^3*b^3 + 6*a^2*b^4 + 7*a*b^5 + 3*b^6)*cosh(x))*sinh(x)^3 + 4
*(a^5*b + 3*a^4*b^2 + 2*a^3*b^3 - 2*a^2*b^4 - 3*a*b^5 - b^6)*cosh(x)^2 + 4*
(7*(a^5*b + 5*a^4*b^2 + 10*a^3*b^3 + 10*a^2*b^4 + 5*a*b^5 + b^6)*cosh(x)^6
+ a^5*b + 3*a^4*b^2 + 2*a^3*b^3 - 2*a^2*b^4 - 3*a*b^5 - b^6 + 15*(a^5*b + 3
*a^4*b^2 + 2*a^3*b^3 - 2*a^2*b^4 - 3*a*b^5 - b^6)*cosh(x)^4 + 3*(3*a^5*b +
7*a^4*b^2 + 6*a^3*b^3 + 6*a^2*b^4 + 7*a*b^5 + 3*b^6)*cosh(x)^2)*sinh(x)^2 +
 8*((a^5*b + 5*a^4*b^2 + 10*a^3*b^3 + 10*a^2*b^4 + 5*a*b^5 + b^6)*cosh(x)^7
 + 3*(a^5*b + 3*a^4*b^2 + 2*a^3*b^3 - 2*a^2*b^4 - 3*a*b^5 - b^6)*cosh(x)^5
+ (3*a^5*b + 7*a^4*b^2 + 6*a^3*b^3 + 6*a^2*b^4 + 7*a*b^5 + 3*b^6)*cosh(x)^3
 + (a^5*b + 3*a^4*b^2 + 2*a^3*b^3 - 2*a^2*b^4 - 3*a*b^5 - b^6)*cosh(x))*sin
h(x)), -1/6*(3*((a^2*b + 2*a*b^2 + b^3)*cosh(x)^8 + 8*(a^2*b + 2*a*b^2 + b^
3)*cosh(x)*sinh(x)^7 + (a^2*b + 2*a*b^2 + b^3)*sinh(x)^8 + 4*(a^2*b - b^3)*
cosh(x)^6 + 4*(a^2*b - b^3 + 7*(a^2*b + 2*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6
 + 8*(7*(a^2*b + 2*a*b^2 + b^3)*cosh(x)^3 + 3*(a^2*b - b^3)*cosh(x))*sinh(x
)^5 + 2*(3*a^2*b - 2*a*b^2 + 3*b^3)*cosh(x)^4 + 2*(35*(a^2*b + 2*a*b^2 + b^
3)*cosh(x)^4 + 3*a^2*b - 2*a*b^2 + 3*b^3 + 30*(a^2*b - b^3)*cosh(x)^2)*sinh
(x)^4 + 8*(7*(a^2*b + 2*a*b^2 + b^3)*cosh(x)^5 + 10*(a^2*b - b^3)*cosh(x)^3
 + (3*a^2*b - 2*a*b^2 + 3*b^3)*cosh(x))*sinh(x)^3 + a^2*b + 2*a*b^2 + b^3 +
 4*(a^2*b - b^3)*cosh(x)^2 + 4*(7*(a^2*b + 2*a*b^2 + b^3)*cosh(x)^6 + 15*(a
^2*b - b^3)*cosh(x)^4 + a^2*b - b^3 + 3*(3*a^2*b - 2*a*b^2 + 3*b^3)*cosh(x)
^2)*sinh(x)^2 + 8*((a^2*b + 2*a*b^2 + b^3)*cosh(x)^7 + 3*(a^2*b - b^3)*cosh
(x)^5 + (3*a^2*b - 2*a*b^2 + 3*b^3)*cosh(x)^3 + (a^2*b - b^3)*cosh(x))*sinh
(x))*sqrt(-a - b)*arctan(sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sin
h(x)^2 + a + b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4
+ 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b -
b^2)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 +
a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x
))*sinh(x))) + 3*((a^2*b + 2*a*b^2 + b^3)*cosh(x)^8 + 8*(a^2*b + 2*a*b^2 +
b^3)*cosh(x)*sinh(x)^7 + (a^2*b + 2*a*b^2 + b^3)*sinh(x)^8 + 4*(a^2*b - b^3
)*cosh(x)^6 + 4*(a^2*b - b^3 + 7*(a^2*b + 2*a*b^2 + b^3)*cosh(x)^2)*sinh(x)
^6 + 8*(7*(a^2*b + 2*a*b^2 + b^3)*cosh(x)^3 + 3*(a^2*b - b^3)*cosh(x))*sinh
(x)^5 + 2*(3*a^2*b - 2*a*b^2 + 3*b^3)*cosh(x)^4 + 2*(35*(a^2*b + 2*a*b^2 +
b^3)*cosh(x)^4 + 3*a^2*b - 2*a*b^2 + 3*b^3 + 30*(a^2*b - b^3)*cosh(x)^2)*si
nh(x)^4 + 8*(7*(a^2*b + 2*a*b^2 + b^3)*cosh(x)^5 + 10*(a^2*b - b^3)*cosh(x)
^3 + (3*a^2*b - 2*a*b^2 + 3*b^3)*cosh(x))*sinh(x)^3 + a^2*b + 2*a*b^2 + b^3
 + 4*(a^2*b - b^3)*cosh(x)^2 + 4*(7*(a^2*b + 2*a*b^2 + b^3)*cosh(x)^6 + 15*
(a^2*b - b^3)*cosh(x)^4 + a^2*b - b^3 + 3*(3*a^2*b - 2*a*b^2 + 3*b^3)*cosh(
x)^2)*sinh(x)^2 + 8*((a^2*b + 2*a*b^2 + b^3)*cosh(x)^7 + 3*(a^2*b - b^3)*co
sh(x)^5 + (3*a^2*b - 2*a*b^2 + 3*b^3)*cosh(x)^3 + (a^2*b - b^3)*cosh(x))*si
nh(x))*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
^2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(
cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*
cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*
cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh
(x) + a + b)) - 2*sqrt(2)*((a^3 - a^2*b - 5*a*b^2 - 3*b^3)*cosh(x)^6 + 6*(a
^3 - a^2*b - 5*a*b^2 - 3*b^3)*cosh(x)*sinh(x)^5 + (a^3 - a^2*b - 5*a*b^2 -
3*b^3)*sinh(x)^6 + 3*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^4 + 3*(a^3 - a^2*b
 - a*b^2 + b^3 + 5*(a^3 - a^2*b - 5*a*b^2 - 3*b^3)*cosh(x)^2)*sinh(x)^4 + 4
*(5*(a^3 - a^2*b - 5*a*b^2 - 3*b^3)*cosh(x)^3 + 3*(a^3 - a^2*b - a*b^2 + b^
3)*cosh(x))*sinh(x)^3 + a^3 - a^2*b - 5*a*b^2 - 3*b^3 + 3*(a^3 - a^2*b - a*
b^2 + b^3)*cosh(x)^2 + 3*(5*(a^3 - a^2*b - 5*a*b^2 - 3*b^3)*cosh(x)^4 + a^3
 - a^2*b - a*b^2 + b^3 + 6*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2
```

```
 + 6*((a^3 - a^2*b - 5*a*b^2 - 3*b^3)*cosh(x)^5 + 2*(a^3 - a^2*b - a*b^2 +
b^3)*cosh(x)^3 + (a^3 - a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x))*sqrt(((a + b
)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + s
inh(x)^2)))/((a^5*b + 5*a^4*b^2 + 10*a^3*b^3 + 10*a^2*b^4 + 5*a*b^5 + b^6)*
cosh(x)^8 + 8*(a^5*b + 5*a^4*b^2 + 10*a^3*b^3 + 10*a^2*b^4 + 5*a*b^5 + b^6)
*cosh(x)*sinh(x)^7 + (a^5*b + 5*a^4*b^2 + 10*a^3*b^3 + 10*a^2*b^4 + 5*a*b^5
 + b^6)*sinh(x)^8 + 4*(a^5*b + 3*a^4*b^2 + 2*a^3*b^3 - 2*a^2*b^4 - 3*a*b^5
- b^6)*cosh(x)^6 + 4*(a^5*b + 3*a^4*b^2 + 2*a^3*b^3 - 2*a^2*b^4 - 3*a*b^5 -
 b^6 + 7*(a^5*b + 5*a^4*b^2 + 10*a^3*b^3 + 10*a^2*b^4 + 5*a*b^5 + b^6)*cosh
(x)^2)*sinh(x)^6 + a^5*b + 5*a^4*b^2 + 10*a^3*b^3 + 10*a^2*b^4 + 5*a*b^5 +
b^6 + 8*(7*(a^5*b + 5*a^4*b^2 + 10*a^3*b^3 + 10*a^2*b^4 + 5*a*b^5 + b^6)*co
sh(x)^3 + 3*(a^5*b + 3*a^4*b^2 + 2*a^3*b^3 - 2*a^2*b^4 - 3*a*b^5 - b^6)*cos
h(x))*sinh(x)^5 + 2*(3*a^5*b + 7*a^4*b^2 + 6*a^3*b^3 + 6*a^2*b^4 + 7*a*b^5
+ 3*b^6)*cosh(x)^4 + 2*(3*a^5*b + 7*a^4*b^2 + 6*a^3*b^3 + 6*a^2*b^4 + 7*a*b
^5 + 3*b^6 + 35*(a^5*b + 5*a^4*b^2 + 10*a^3*b^3 + 10*a^2*b^4 + 5*a*b^5 + b^
6)*cosh(x)^4 + 30*(a^5*b + 3*a^4*b^2 + 2*a^3*b^3 - 2*a^2*b^4 - 3*a*b^5 - b^
6)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^5*b + 5*a^4*b^2 + 10*a^3*b^3 + 10*a^2*b^4
 + 5*a*b^5 + b^6)*cosh(x)^5 + 10*(a^5*b + 3*a^4*b^2 + 2*a^3*b^3 - 2*a^2*b^4
 - 3*a*b^5 - b^6)*cosh(x)^3 + (3*a^5*b + 7*a^4*b^2 + 6*a^3*b^3 + 6*a^2*b^4
+ 7*a*b^5 + 3*b^6)*cosh(x))*sinh(x)^3 + 4*(a^5*b + 3*a^4*b^2 + 2*a^3*b^3 -
2*a^2*b^4 - 3*a*b^5 - b^6)*cosh(x)^2 + 4*(7*(a^5*b + 5*a^4*b^2 + 10*a^3*b^3
 + 10*a^2*b^4 + 5*a*b^5 + b^6)*cosh(x)^6 + a^5*b + 3*a^4*b^2 + 2*a^3*b^3 -
2*a^2*b^4 - 3*a*b^5 - b^6 + 15*(a^5*b + 3*a^4*b^2 + 2*a^3*b^3 - 2*a^2*b^4 -
 3*a*b^5 - b^6)*cosh(x)^4 + 3*(3*a^5*b + 7*a^4*b^2 + 6*a^3*b^3 + 6*a^2*b^4
+ 7*a*b^5 + 3*b^6)*cosh(x)^2)*sinh(x)^2 + 8*((a^5*b + 5*a^4*b^2 + 10*a^3*b^
3 + 10*a^2*b^4 + 5*a*b^5 + b^6)*cosh(x)^7 + 3*(a^5*b + 3*a^4*b^2 + 2*a^3*b^
3 - 2*a^2*b^4 - 3*a*b^5 - b^6)*cosh(x)^5 + (3*a^5*b + 7*a^4*b^2 + 6*a^3*b^3
 + 6*a^2*b^4 + 7*a*b^5 + 3*b^6)*cosh(x)^3 + (a^5*b + 3*a^4*b^2 + 2*a^3*b^3
- 2*a^2*b^4 - 3*a*b^5 - b^6)*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+b*tanh(x)**2)**(5/2),x)
```

```
[Out] Integral(tanh(x)**3/(a + b*tanh(x)**2)**(5/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.250 $\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=88

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}}-\frac{(2a-b)\tanh(x)}{3a(a+b)^2\sqrt{a+b\tanh^2(x)}}-\frac{\tanh(x)}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

```
[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(5/2) - Tanh[x
]/(3*(a + b)*(a + b*Tanh[x]^2)^(3/2)) - ((2*a - b)*Tanh[x])/(3*a*(a + b)^2*
Sqrt[a + b*Tanh[x]^2])
```

Rubi [A] time = 0.131813, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3670, 471, 527, 12, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}}-\frac{(2a-b)\tanh(x)}{3a(a+b)^2\sqrt{a+b\tanh^2(x)}}-\frac{\tanh(x)}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(5/2) - Tanh[x
]/(3*(a + b)*(a + b*Tanh[x]^2)^(3/2)) - ((2*a - b)*Tanh[x])/(3*a*(a + b)^2*
Sqrt[a + b*Tanh[x]^2])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
```

```
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)\left(a+bx^2\right)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= -\frac{\tanh(x)}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1+2x^2}{\left(1-x^2\right)\left(a+bx^2\right)^{3/2}}\,dx, x, \tanh(x)\right)}{3(a+b)}$$

$$= -\frac{\tanh(x)}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{(2a-b)\tanh(x)}{3a(a+b)^2\sqrt{a+b\tanh^2(x)}} - \frac{\text{Subst}\left(\int -\frac{3a}{\left(1-x^2\right)\sqrt{a+bx^2}}\,dx, x,\right.}{3a(a+b)^2}$$

$$= -\frac{\tanh(x)}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{(2a-b)\tanh(x)}{3a(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+bx^2}}\,dx, x, \text{ta}\right.}{(a+b)^2}$$

$$= -\frac{\tanh(x)}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{(2a-b)\tanh(x)}{3a(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\text{ta}}{\sqrt{a+b}}\right.}{(a+b)^2}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}} - \frac{\tanh(x)}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{(2a-b)\tanh(x)}{3a(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

Mathematica [C] time = 7.51892, size = 193, normalized size = 2.19

$$\frac{\coth(x)\left(-\frac{35a\cosh^2(x)\left(-5a-2b\tanh^2(x)\right)\left(3\left(a+b\tanh^2(x)\right)^2\sin^{-1}\left(\sqrt{-\frac{(a+b)\sinh^2(x)}{a}}\right)-a\,\text{sech}^2(x)\left((a+4b)\tanh^2(x)+3a\right)\sqrt{-\frac{(a+b)\sinh^2(x)\cosh^2(x)\left(a+b\tanh^2(\right.}{a^2}}\right.}{\sqrt{-\frac{(a+b)\sinh^2(x)\cosh^2(x)\left(a+b\tanh^2(x)\right)}{a^2}}}\right.}{315a^3(a+b)^2\left(a+b\tanh^2(x)\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tanh[x]^2/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] (Coth[x]*(-12*(a + b)^3*Hypergeometric2F1[2, 2, 9/2, -(((a + b)*Sinh[x]^2)/
a)]*Sinh[x]^4*Tanh[x]^2*(a + b*Tanh[x]^2) - (35*a*Cosh[x]^2*(-5*a - 2*b*Tan
h[x]^2)*(3*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]]*(a + b*Tanh[x]^2)^2 - a*S
ech[x]^2*Sqrt[-(((a + b)*Cosh[x]^2*Sinh[x]^2*(a + b*Tanh[x]^2))/a^2)]*(3*a
+ (a + 4*b)*Tanh[x]^2)))/Sqrt[-(((a + b)*Cosh[x]^2*Sinh[x]^2*(a + b*Tanh[x]
^2))/a^2)]))/(315*a^3*(a + b)^2*(a + b*Tanh[x]^2)^(3/2))
```

Maple [B] time = 0.02, size = 454, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a+b*tanh(x)^2)^(5/2),x)
```

```
[Out] -1/3*tanh(x)/a/(a+b*tanh(x)^2)^(3/2)-2/3/a^2*tanh(x)/(a+b*tanh(x)^2)^(1/2)+
1/6/(a+b)/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(3/2)+1/6*b/(a+b)/a/((1+tan
h(x))^2*b-2*(1+tanh(x))*b+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/((1+tanh(x))^2
*b-2*(1+tanh(x))*b+a+b)^(1/2)*tanh(x)+1/2/(a+b)^2/((1+tanh(x))^2*b-2*(1+tan
h(x))*b+a+b)^(1/2)+1/2/(a+b)^2/a/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2
)*b*tanh(x)-1/2/(a+b)^(5/2)*ln((2*a+2*b-2*(1+tanh(x))*b+2*(a+b)^(1/2)*((1+t
anh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))/(1+tanh(x)))-1/6/(a+b)/((tanh(x)-1)
^2*b+2*(tanh(x)-1)*b+a+b)^(3/2)+1/6*b/(a+b)/a/((tanh(x)-1)^2*b+2*(tanh(x)-1
)*b+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b
)^(1/2)*tanh(x)-1/2/(a+b)^2/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)+1/2
/(a+b)^2/a/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)*b*tanh(x)+1/2/(a+b)^
(5/2)*ln((2*a+2*b+2*(tanh(x)-1)*b+2*(a+b)^(1/2)*((tanh(x)-1)^2*b+2*(tanh(x)
-1)*b+a+b)^(1/2))/(tanh(x)-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^2}{\left(b\tanh(x)^2 + a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^2/(b*tanh(x)^2 + a)^(5/2), x)
```

Fricas [B] time = 8.76281, size = 15741, normalized size = 178.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cos
h(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 4*(a^3 - a*b^2)*cosh(x
)^6 + 4*(a^3 - a*b^2 + 7*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 8*(
7*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(a^3 - a*b^2)*cosh(x))*sinh(x)^5 +
2*(3*a^3 - 2*a^2*b + 3*a*b^2)*cosh(x)^4 + 2*(35*(a^3 + 2*a^2*b + a*b^2)*cos
h(x)^4 + 3*a^3 - 2*a^2*b + 3*a*b^2 + 30*(a^3 - a*b^2)*cosh(x)^2)*sinh(x)^4
+ 8*(7*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(a^3 - a*b^2)*cosh(x)^3 + (3*
a^3 - 2*a^2*b + 3*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*a^2*b + a*b^2 + 4*(a^
3 - a*b^2)*cosh(x)^2 + 4*(7*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(a^3 - a
*b^2)*cosh(x)^4 + a^3 - a*b^2 + 3*(3*a^3 - 2*a^2*b + 3*a*b^2)*cosh(x)^2)*si
nh(x)^2 + 8*((a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(a^3 - a*b^2)*cosh(x)^5
+ (3*a^3 - 2*a^2*b + 3*a*b^2)*cosh(x)^3 + (a^3 - a*b^2)*cosh(x))*sinh(x))*s
qrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^
7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^
3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 -
 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cos
h(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(
a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a
*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^
3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2
*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2
 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)
*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4
 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x)
)*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*
cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*c
osh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a
 + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*
b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2
 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4
*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*si
nh(x)^5 + sinh(x)^6)) + 3*((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a
^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 4*(a^
3 - a*b^2)*cosh(x)^6 + 4*(a^3 - a*b^2 + 7*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2
)*sinh(x)^6 + 8*(7*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(a^3 - a*b^2)*cosh
(x))*sinh(x)^5 + 2*(3*a^3 - 2*a^2*b + 3*a*b^2)*cosh(x)^4 + 2*(35*(a^3 + 2*a
^2*b + a*b^2)*cosh(x)^4 + 3*a^3 - 2*a^2*b + 3*a*b^2 + 30*(a^3 - a*b^2)*cosh
(x)^2)*sinh(x)^4 + 8*(7*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(a^3 - a*b^2
)*cosh(x)^3 + (3*a^3 - 2*a^2*b + 3*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*a^2*
b + a*b^2 + 4*(a^3 - a*b^2)*cosh(x)^2 + 4*(7*(a^3 + 2*a^2*b + a*b^2)*cosh(x
)^6 + 15*(a^3 - a*b^2)*cosh(x)^4 + a^3 - a*b^2 + 3*(3*a^3 - 2*a^2*b + 3*a*b
^2)*cosh(x)^2)*sinh(x)^2 + 8*((a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(a^3 -
a*b^2)*cosh(x)^5 + (3*a^3 - 2*a^2*b + 3*a*b^2)*cosh(x)^3 + (a^3 - a*b^2)*co
sh(x))*sinh(x))*sqrt(a + b)*log(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh
(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sin
h(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b
)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(
x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 + a*cosh(x))*sinh(x) + a +
b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*((3*a^3 + 5*a^2
*b + a*b^2 - b^3)*cosh(x)^6 + 6*(3*a^3 + 5*a^2*b + a*b^2 - b^3)*cosh(x)*sin
h(x)^5 + (3*a^3 + 5*a^2*b + a*b^2 - b^3)*sinh(x)^6 + 3*(a^3 - a^2*b - a*b^2
 + b^3)*cosh(x)^4 + 3*(a^3 - a^2*b - a*b^2 + b^3 + 5*(3*a^3 + 5*a^2*b + a*b
^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(3*a^3 + 5*a^2*b + a*b^2 - b^3)*cosh(
x)^3 + 3*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 - 3*a^3 - 5*a^2*b -
 a*b^2 + b^3 - 3*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^2 + 3*(5*(3*a^3 + 5*a^
2*b + a*b^2 - b^3)*cosh(x)^4 - a^3 + a^2*b + a*b^2 - b^3 + 6*(a^3 - a^2*b -
 a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 6*((3*a^3 + 5*a^2*b + a*b^2 - b^3)*cos
h(x)^5 + 2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^3 - (a^3 - a^2*b - a*b^2 + b
```

```
^3)*cosh(x))*sinh(x))*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^6 + 5*a^5*b + 10*a^4*b^2
+ 10*a^3*b^3 + 5*a^2*b^4 + a*b^5)*cosh(x)^8 + 8*(a^6 + 5*a^5*b + 10*a^4*b^2
 + 10*a^3*b^3 + 5*a^2*b^4 + a*b^5)*cosh(x)*sinh(x)^7 + (a^6 + 5*a^5*b + 10*
a^4*b^2 + 10*a^3*b^3 + 5*a^2*b^4 + a*b^5)*sinh(x)^8 + 4*(a^6 + 3*a^5*b + 2*
a^4*b^2 - 2*a^3*b^3 - 3*a^2*b^4 - a*b^5)*cosh(x)^6 + 4*(a^6 + 3*a^5*b + 2*a
^4*b^2 - 2*a^3*b^3 - 3*a^2*b^4 - a*b^5 + 7*(a^6 + 5*a^5*b + 10*a^4*b^2 + 10
*a^3*b^3 + 5*a^2*b^4 + a*b^5)*cosh(x)^2)*sinh(x)^6 + a^6 + 5*a^5*b + 10*a^4
*b^2 + 10*a^3*b^3 + 5*a^2*b^4 + a*b^5 + 8*(7*(a^6 + 5*a^5*b + 10*a^4*b^2 +
10*a^3*b^3 + 5*a^2*b^4 + a*b^5)*cosh(x)^3 + 3*(a^6 + 3*a^5*b + 2*a^4*b^2 -
2*a^3*b^3 - 3*a^2*b^4 - a*b^5)*cosh(x))*sinh(x)^5 + 2*(3*a^6 + 7*a^5*b + 6*
a^4*b^2 + 6*a^3*b^3 + 7*a^2*b^4 + 3*a*b^5)*cosh(x)^4 + 2*(3*a^6 + 7*a^5*b +
 6*a^4*b^2 + 6*a^3*b^3 + 7*a^2*b^4 + 3*a*b^5 + 35*(a^6 + 5*a^5*b + 10*a^4*b
^2 + 10*a^3*b^3 + 5*a^2*b^4 + a*b^5)*cosh(x)^4 + 30*(a^6 + 3*a^5*b + 2*a^4*
b^2 - 2*a^3*b^3 - 3*a^2*b^4 - a*b^5)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^6 + 5*a
^5*b + 10*a^4*b^2 + 10*a^3*b^3 + 5*a^2*b^4 + a*b^5)*cosh(x)^5 + 10*(a^6 + 3
*a^5*b + 2*a^4*b^2 - 2*a^3*b^3 - 3*a^2*b^4 - a*b^5)*cosh(x)^3 + (3*a^6 + 7*
a^5*b + 6*a^4*b^2 + 6*a^3*b^3 + 7*a^2*b^4 + 3*a*b^5)*cosh(x))*sinh(x)^3 + 4
*(a^6 + 3*a^5*b + 2*a^4*b^2 - 2*a^3*b^3 - 3*a^2*b^4 - a*b^5)*cosh(x)^2 + 4*
(7*(a^6 + 5*a^5*b + 10*a^4*b^2 + 10*a^3*b^3 + 5*a^2*b^4 + a*b^5)*cosh(x)^6
+ a^6 + 3*a^5*b + 2*a^4*b^2 - 2*a^3*b^3 - 3*a^2*b^4 - a*b^5 + 15*(a^6 + 3*a
^5*b + 2*a^4*b^2 - 2*a^3*b^3 - 3*a^2*b^4 - a*b^5)*cosh(x)^4 + 3*(3*a^6 + 7*
a^5*b + 6*a^4*b^2 + 6*a^3*b^3 + 7*a^2*b^4 + 3*a*b^5)*cosh(x)^2)*sinh(x)^2 +
 8*((a^6 + 5*a^5*b + 10*a^4*b^2 + 10*a^3*b^3 + 5*a^2*b^4 + a*b^5)*cosh(x)^7
 + 3*(a^6 + 3*a^5*b + 2*a^4*b^2 - 2*a^3*b^3 - 3*a^2*b^4 - a*b^5)*cosh(x)^5
+ (3*a^6 + 7*a^5*b + 6*a^4*b^2 + 6*a^3*b^3 + 7*a^2*b^4 + 3*a*b^5)*cosh(x)^3
 + (a^6 + 3*a^5*b + 2*a^4*b^2 - 2*a^3*b^3 - 3*a^2*b^4 - a*b^5)*cosh(x))*sin
h(x)), -1/6*(3*((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^
2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 4*(a^3 - a*b^2)*
cosh(x)^6 + 4*(a^3 - a*b^2 + 7*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6
 + 8*(7*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(a^3 - a*b^2)*cosh(x))*sinh(x
)^5 + 2*(3*a^3 - 2*a^2*b + 3*a*b^2)*cosh(x)^4 + 2*(35*(a^3 + 2*a^2*b + a*b^
2)*cosh(x)^4 + 3*a^3 - 2*a^2*b + 3*a*b^2 + 30*(a^3 - a*b^2)*cosh(x)^2)*sinh
(x)^4 + 8*(7*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(a^3 - a*b^2)*cosh(x)^3
 + (3*a^3 - 2*a^2*b + 3*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*a^2*b + a*b^2 +
 4*(a^3 - a*b^2)*cosh(x)^2 + 4*(7*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(a
^3 - a*b^2)*cosh(x)^4 + a^3 - a*b^2 + 3*(3*a^3 - 2*a^2*b + 3*a*b^2)*cosh(x)
^2)*sinh(x)^2 + 8*((a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(a^3 - a*b^2)*cosh
(x)^5 + (3*a^3 - 2*a^2*b + 3*a*b^2)*cosh(x)^3 + (a^3 - a*b^2)*cosh(x))*sinh
(x))*sqrt(-a - b)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sin
h(x)^2 - a - b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a*b + b^2)*cosh(x)^4
+ 4*(a*b + b^2)*cosh(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*
b^2)*cosh(x)^2 + (6*(a*b + b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 +
a^2 + 2*a*b + b^2 + 2*(2*(a*b + b^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x
))*sinh(x))) + 3*((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*
b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 4*(a^3 - a*b^2
)*cosh(x)^6 + 4*(a^3 - a*b^2 + 7*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)
^6 + 8*(7*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(a^3 - a*b^2)*cosh(x))*sinh
(x)^5 + 2*(3*a^3 - 2*a^2*b + 3*a*b^2)*cosh(x)^4 + 2*(35*(a^3 + 2*a^2*b + a*
b^2)*cosh(x)^4 + 3*a^3 - 2*a^2*b + 3*a*b^2 + 30*(a^3 - a*b^2)*cosh(x)^2)*si
nh(x)^4 + 8*(7*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(a^3 - a*b^2)*cosh(x)
^3 + (3*a^3 - 2*a^2*b + 3*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*a^2*b + a*b^2
 + 4*(a^3 - a*b^2)*cosh(x)^2 + 4*(7*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*
(a^3 - a*b^2)*cosh(x)^4 + a^3 - a*b^2 + 3*(3*a^3 - 2*a^2*b + 3*a*b^2)*cosh(
x)^2)*sinh(x)^2 + 8*((a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(a^3 - a*b^2)*co
sh(x)^5 + (3*a^3 - 2*a^2*b + 3*a*b^2)*cosh(x)^3 + (a^3 - a*b^2)*cosh(x))*si
nh(x))*sqrt(-a - b)*arctan(sqrt(2)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (
a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a
```

```
+ b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)) +
2*sqrt(2)*((3*a^3 + 5*a^2*b + a*b^2 - b^3)*cosh(x)^6 + 6*(3*a^3 + 5*a^2*b +
 a*b^2 - b^3)*cosh(x)*sinh(x)^5 + (3*a^3 + 5*a^2*b + a*b^2 - b^3)*sinh(x)^6
 + 3*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^4 + 3*(a^3 - a^2*b - a*b^2 + b^3 +
 5*(3*a^3 + 5*a^2*b + a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(3*a^3 + 5*a
^2*b + a*b^2 - b^3)*cosh(x)^3 + 3*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x))*sinh
(x)^3 - 3*a^3 - 5*a^2*b - a*b^2 + b^3 - 3*(a^3 - a^2*b - a*b^2 + b^3)*cosh(
x)^2 + 3*(5*(3*a^3 + 5*a^2*b + a*b^2 - b^3)*cosh(x)^4 - a^3 + a^2*b + a*b^2
 - b^3 + 6*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 6*((3*a^3 + 5
*a^2*b + a*b^2 - b^3)*cosh(x)^5 + 2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^3 -
 (a^3 - a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x))*sqrt(((a + b)*cosh(x)^2 + (a
 + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^
6 + 5*a^5*b + 10*a^4*b^2 + 10*a^3*b^3 + 5*a^2*b^4 + a*b^5)*cosh(x)^8 + 8*(a
^6 + 5*a^5*b + 10*a^4*b^2 + 10*a^3*b^3 + 5*a^2*b^4 + a*b^5)*cosh(x)*sinh(x)
^7 + (a^6 + 5*a^5*b + 10*a^4*b^2 + 10*a^3*b^3 + 5*a^2*b^4 + a*b^5)*sinh(x)^
8 + 4*(a^6 + 3*a^5*b + 2*a^4*b^2 - 2*a^3*b^3 - 3*a^2*b^4 - a*b^5)*cosh(x)^6
 + 4*(a^6 + 3*a^5*b + 2*a^4*b^2 - 2*a^3*b^3 - 3*a^2*b^4 - a*b^5 + 7*(a^6 +
5*a^5*b + 10*a^4*b^2 + 10*a^3*b^3 + 5*a^2*b^4 + a*b^5)*cosh(x)^2)*sinh(x)^6
 + a^6 + 5*a^5*b + 10*a^4*b^2 + 10*a^3*b^3 + 5*a^2*b^4 + a*b^5 + 8*(7*(a^6
+ 5*a^5*b + 10*a^4*b^2 + 10*a^3*b^3 + 5*a^2*b^4 + a*b^5)*cosh(x)^3 + 3*(a^6
 + 3*a^5*b + 2*a^4*b^2 - 2*a^3*b^3 - 3*a^2*b^4 - a*b^5)*cosh(x))*sinh(x)^5
+ 2*(3*a^6 + 7*a^5*b + 6*a^4*b^2 + 6*a^3*b^3 + 7*a^2*b^4 + 3*a*b^5)*cosh(x)
^4 + 2*(3*a^6 + 7*a^5*b + 6*a^4*b^2 + 6*a^3*b^3 + 7*a^2*b^4 + 3*a*b^5 + 35*
(a^6 + 5*a^5*b + 10*a^4*b^2 + 10*a^3*b^3 + 5*a^2*b^4 + a*b^5)*cosh(x)^4 + 3
0*(a^6 + 3*a^5*b + 2*a^4*b^2 - 2*a^3*b^3 - 3*a^2*b^4 - a*b^5)*cosh(x)^2)*si
nh(x)^4 + 8*(7*(a^6 + 5*a^5*b + 10*a^4*b^2 + 10*a^3*b^3 + 5*a^2*b^4 + a*b^5
)*cosh(x)^5 + 10*(a^6 + 3*a^5*b + 2*a^4*b^2 - 2*a^3*b^3 - 3*a^2*b^4 - a*b^5
)*cosh(x)^3 + (3*a^6 + 7*a^5*b + 6*a^4*b^2 + 6*a^3*b^3 + 7*a^2*b^4 + 3*a*b^
5)*cosh(x))*sinh(x)^3 + 4*(a^6 + 3*a^5*b + 2*a^4*b^2 - 2*a^3*b^3 - 3*a^2*b^
4 - a*b^5)*cosh(x)^2 + 4*(7*(a^6 + 5*a^5*b + 10*a^4*b^2 + 10*a^3*b^3 + 5*a^
2*b^4 + a*b^5)*cosh(x)^6 + a^6 + 3*a^5*b + 2*a^4*b^2 - 2*a^3*b^3 - 3*a^2*b^
4 - a*b^5 + 15*(a^6 + 3*a^5*b + 2*a^4*b^2 - 2*a^3*b^3 - 3*a^2*b^4 - a*b^5)*
cosh(x)^4 + 3*(3*a^6 + 7*a^5*b + 6*a^4*b^2 + 6*a^3*b^3 + 7*a^2*b^4 + 3*a*b^
5)*cosh(x)^2)*sinh(x)^2 + 8*((a^6 + 5*a^5*b + 10*a^4*b^2 + 10*a^3*b^3 + 5*a
^2*b^4 + a*b^5)*cosh(x)^7 + 3*(a^6 + 3*a^5*b + 2*a^4*b^2 - 2*a^3*b^3 - 3*a^
2*b^4 - a*b^5)*cosh(x)^5 + (3*a^6 + 7*a^5*b + 6*a^4*b^2 + 6*a^3*b^3 + 7*a^2
*b^4 + 3*a*b^5)*cosh(x)^3 + (a^6 + 3*a^5*b + 2*a^4*b^2 - 2*a^3*b^3 - 3*a^2*
b^4 - a*b^5)*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+b*tanh(x)**2)**(5/2),x)
```

```
[Out] Integral(tanh(x)**2/(a + b*tanh(x)**2)**(5/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.251 $\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=70

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}-\frac{1}{(a+b)^2\sqrt{a+b\tanh^2(x)}}-\frac{1}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/(a + b)^(5/2) - 1/(3*(a + b)*(a
+ b*Tanh[x]^2)^(3/2)) - 1/((a + b)^2*Sqrt[a + b*Tanh[x]^2])
```

Rubi [A] time = 0.103077, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3670, 444, 51, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}-\frac{1}{(a+b)^2\sqrt{a+b\tanh^2(x)}}-\frac{1}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/(a + b)^(5/2) - 1/(3*(a + b)*(a
+ b*Tanh[x]^2)^(3/2)) - 1/((a + b)^2*Sqrt[a + b*Tanh[x]^2])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx &= \text{Subst}\left(\int \frac{x}{\left(1-x^2\right)\left(a+bx^2\right)^{5/2}}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)^{5/2}}\,dx, x, \tanh^2(x)\right)\\
&= -\frac{1}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)^{3/2}}\,dx, x, \tanh^2(x)\right)}{2(a+b)}\\
&= -\frac{1}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{1}{(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \text{ta}\right.}{2(a+b)^2}\\
&= -\frac{1}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{1}{(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+}\right.}{b(a+b)^2}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{1}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{1}{(a+b)^2\sqrt{a+b\tanh^2(x)}}
\end{aligned}$$

Mathematica [C] time = 0.0408918, size = 43, normalized size = 0.61

$$-\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\tanh^2(x)+a}{a+b}\right)}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] -Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tanh[x]^2)/(a + b)]/(3*(a + b)*(a
+ b*Tanh[x]^2)^(3/2))
```

Maple [B] time = 0.02, size = 420, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*tanh(x)^2)^(5/2),x)
```

```
[Out] -1/6/(a+b)/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(3/2)-1/6*b/(a+b)/a/((1+ta
nh(x))^2*b-2*(1+tanh(x))*b+a+b)^(3/2)*tanh(x)-1/3*b/(a+b)/a^2/((1+tanh(x))^
2*b-2*(1+tanh(x))*b+a+b)^(1/2)*tanh(x)-1/2/(a+b)^2/((1+tanh(x))^2*b-2*(1+ta
nh(x))*b+a+b)^(1/2)-1/2/(a+b)^2/a/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/
2)*b*tanh(x)+1/2/(a+b)^(5/2)*ln((2*a+2*b-2*(1+tanh(x))*b+2*(a+b)^(1/2)*((1+
tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))/(1+tanh(x)))-1/6/(a+b)/((tanh(x)-1
)^2*b+2*(tanh(x)-1)*b+a+b)^(3/2)+1/6*b/(a+b)/a/((tanh(x)-1)^2*b+2*(tanh(x)-
1)*b+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+
b)^(1/2)*tanh(x)-1/2/(a+b)^2/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)+1/
2/(a+b)^2/a/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)*b*tanh(x)+1/2/(a+b)
^(5/2)*ln((2*a+2*b+2*(tanh(x)-1)*b+2*(a+b)^(1/2)*((tanh(x)-1)^2*b+2*(tanh(x
)-1)*b+a+b)^(1/2))/(tanh(x)-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)}{\left(b\tanh(x)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)/(b*tanh(x)^2 + a)^(5/2), x)
```

Fricas [B] time = 8.39963, size = 14660, normalized size = 209.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a^2 + 2*a*b + b^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(x)*sin
h(x)^7 + (a^2 + 2*a*b + b^2)*sinh(x)^8 + 4*(a^2 - b^2)*cosh(x)^6 + 4*(7*(a^
2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2
)*cosh(x)^3 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 - 2*a*b + 3*b^2)*
cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 30*(a^2 - b^2)*cosh(x)^2
+ 3*a^2 - 2*a*b + 3*b^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 1
0*(a^2 - b^2)*cosh(x)^3 + (3*a^2 - 2*a*b + 3*b^2)*cosh(x))*sinh(x)^3 + 4*(a
^2 - b^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^6 + 15*(a^2 - b^2)*c
osh(x)^4 + 3*(3*a^2 - 2*a*b + 3*b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2
 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*cosh(x)^7 + 3*(a^2 - b^2)*cosh(x)^5
 + (3*a^2 - 2*a*b + 3*b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))*sqrt(a
 + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a
^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14
*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*
a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4
 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3
+ a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 +
 a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^
3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(
a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b
```

^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*
cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(
5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh
(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x
)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)
^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*
sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)
sinh(x) + sinh(x)^2)) + 4(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*c
osh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b
^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(
x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^
5 + sinh(x)^6)) + 3*((a^2 + 2*a*b + b^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2)*
cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2)*sinh(x)^8 + 4*(a^2 - b^2)*cosh(x)^6
 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^6 + 8*(7*(a^2 +
2*a*b + b^2)*cosh(x)^3 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 - 2*a*
b + 3*b^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 30*(a^2 - b^2)
*cosh(x)^2 + 3*a^2 - 2*a*b + 3*b^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2)*co
sh(x)^5 + 10*(a^2 - b^2)*cosh(x)^3 + (3*a^2 - 2*a*b + 3*b^2)*cosh(x))*sinh(
x)^3 + 4*(a^2 - b^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^6 + 15*(a
^2 - b^2)*cosh(x)^4 + 3*(3*a^2 - 2*a*b + 3*b^2)*cosh(x)^2 + a^2 - b^2)*sinh
(x)^2 + a^2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*cosh(x)^7 + 3*(a^2 - b^2
)*cosh(x)^5 + (3*a^2 - 2*a*b + 3*b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh
(x))*sqrt(a + b)*log(-((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a
 + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + s
qrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a
 + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
 + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x
)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 16*sqrt(2)*((a^2 + 2*a*b + b^2)*cos
h(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh
(x)^6 + 3*(a^2 + a*b)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2
+ a*b)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + a*b)*cosh(
x))*sinh(x)^3 + 3*(a^2 + a*b)*cosh(x)^2 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^
4 + 6*(a^2 + a*b)*cosh(x)^2 + a^2 + a*b)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 6*
((a^2 + 2*a*b + b^2)*cosh(x)^5 + 2*(a^2 + a*b)*cosh(x)^3 + (a^2 + a*b)*cosh
(x))*sinh(x))*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2
*b^3 + 5*a*b^4 + b^5)*cosh(x)^8 + 8*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^
3 + 5*a*b^4 + b^5)*cosh(x)*sinh(x)^7 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2
*b^3 + 5*a*b^4 + b^5)*sinh(x)^8 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3
 - 3*a*b^4 - b^5)*cosh(x)^6 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a
b^4 - b^5 + 7(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*co
sh(x)^2)*sinh(x)^6 + 8*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^
4 + b^5)*cosh(x)^3 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b
^5)*cosh(x))*sinh(x)^5 + a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4
+ b^5 + 2*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*cosh(
x)^4 + 2*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5 + 35*(a
^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)^4 + 30*(a^5
 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^2)*sinh(x)^4 +
8*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)^5 +
10*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^3 + (3*a
^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*cosh(x))*sinh(x)^3
+ 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^2 + 4*(
7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)^6 + a^5
 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5 + 15*(a^5 + 3*a^4*b + 2*
a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^4 + 3*(3*a^5 + 7*a^4*b + 6*a^3
*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*cosh(x)^2)*sinh(x)^2 + 8*((a^5 + 5*a^4*
b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)^7 + 3*(a^5 + 3*a^4*b +
 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^5 + (3*a^5 + 7*a^4*b + 6*a^
3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*cosh(x)^3 + (a^5 + 3*a^4*b + 2*a^3*b^2

```
 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x))*sinh(x)), -1/6*(3*((a^2 + 2*a*b + b^
2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2
)*sinh(x)^8 + 4*(a^2 - b^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^2
 + a^2 - b^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 - b^2)
*cosh(x))*sinh(x)^5 + 2*(3*a^2 - 2*a*b + 3*b^2)*cosh(x)^4 + 2*(35*(a^2 + 2*
a*b + b^2)*cosh(x)^4 + 30*(a^2 - b^2)*cosh(x)^2 + 3*a^2 - 2*a*b + 3*b^2)*si
nh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 10*(a^2 - b^2)*cosh(x)^3 + (
3*a^2 - 2*a*b + 3*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 - b^2)*cosh(x)^2 + 4*(7*
(a^2 + 2*a*b + b^2)*cosh(x)^6 + 15*(a^2 - b^2)*cosh(x)^4 + 3*(3*a^2 - 2*a*b
 + 3*b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 8*((a^2 +
2*a*b + b^2)*cosh(x)^7 + 3*(a^2 - b^2)*cosh(x)^5 + (3*a^2 - 2*a*b + 3*b^2)*
cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(a*co
sh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a - b)*sqrt(((a
 + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
 + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a
^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x
)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a*b)*c
osh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh(x))) + 3*((a^2 + 2*a*b + b^2)*
cosh(x)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2)*s
inh(x)^8 + 4*(a^2 - b^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a
^2 - b^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 - b^2)*co
sh(x))*sinh(x)^5 + 2*(3*a^2 - 2*a*b + 3*b^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b
 + b^2)*cosh(x)^4 + 30*(a^2 - b^2)*cosh(x)^2 + 3*a^2 - 2*a*b + 3*b^2)*sinh(
x)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 10*(a^2 - b^2)*cosh(x)^3 + (3*a
^2 - 2*a*b + 3*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 - b^2)*cosh(x)^2 + 4*(7*(a^
2 + 2*a*b + b^2)*cosh(x)^6 + 15*(a^2 - b^2)*cosh(x)^4 + 3*(3*a^2 - 2*a*b +
3*b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 8*((a^2 + 2*a
*b + b^2)*cosh(x)^7 + 3*(a^2 - b^2)*cosh(x)^5 + (3*a^2 - 2*a*b + 3*b^2)*cos
h(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)
^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^
2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))
/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(
a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*c
osh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + 8*sqrt(2)*((a^2 + 2*a*b + b
^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^
2)*sinh(x)^6 + 3*(a^2 + a*b)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2
 + a^2 + a*b)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + a*b
)*cosh(x))*sinh(x)^3 + 3*(a^2 + a*b)*cosh(x)^2 + 3*(5*(a^2 + 2*a*b + b^2)*c
osh(x)^4 + 6*(a^2 + a*b)*cosh(x)^2 + a^2 + a*b)*sinh(x)^2 + a^2 + 2*a*b + b
^2 + 6*((a^2 + 2*a*b + b^2)*cosh(x)^5 + 2*(a^2 + a*b)*cosh(x)^3 + (a^2 + a*
b)*cosh(x))*sinh(x))*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(
cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^5 + 5*a^4*b + 10*a^3*b^2 +
 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)^8 + 8*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10
*a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)*sinh(x)^7 + (a^5 + 5*a^4*b + 10*a^3*b^2 +
 10*a^2*b^3 + 5*a*b^4 + b^5)*sinh(x)^8 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a
^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^6 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^
3 - 3*a*b^4 - b^5 + 7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 +
b^5)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 +
 5*a*b^4 + b^5)*cosh(x)^3 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*
b^4 - b^5)*cosh(x))*sinh(x)^5 + a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5
*a*b^4 + b^5 + 2*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5
)*cosh(x)^4 + 2*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5
+ 35*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)^4 +
30*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^2)*sinh(
x)^4 + 8*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(
x)^5 + 10*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^3
 + (3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*cosh(x))*sin
h(x)^3 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^
2 + 4*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)^
```

```
6 + a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5 + 15*(a^5 + 3*a^4
*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^4 + 3*(3*a^5 + 7*a^4*b
+ 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*cosh(x)^2)*sinh(x)^2 + 8*((a^5 +
 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)^7 + 3*(a^5 + 3*
a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^5 + (3*a^5 + 7*a^4*b
 + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*cosh(x)^3 + (a^5 + 3*a^4*b + 2*
a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x))*sinh(x))]
```

Sympy [A] time = 20.6808, size = 73, normalized size = 1.04

$$-\frac{1}{3(a+b)\left(a+b\tanh^2(x)\right)^{\frac{3}{2}}}-\frac{1}{(a+b)^2\sqrt{a+b\tanh^2(x)}}-\frac{\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}\,(a+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)**2)**(5/2),x)
```

```
[Out] -1/(3*(a + b)*(a + b*tanh(x)**2)**(3/2)) - 1/((a + b)**2*sqrt(a + b*tanh(x)
**2)) - atan(sqrt(a + b*tanh(x)**2)/sqrt(-a - b))/(sqrt(-a - b)*(a + b)**2)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.252 $\int \frac{1}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=93

$$\frac{b(5a+2b)\tanh(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}}+\frac{b\tanh(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

```
[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(5/2) + (b*Tan
h[x])/(3*a*(a + b)*(a + b*Tanh[x]^2)^(3/2)) + (b*(5*a + 2*b)*Tanh[x])/(3*a^
2*(a + b)^2*Sqrt[a + b*Tanh[x]^2])
```

Rubi [A] time = 0.0852452, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3661, 414, 527, 12, 377, 206}

$$\frac{b(5a+2b)\tanh(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}}+\frac{b\tanh(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tanh[x]^2)^(-5/2),x]
```

```
[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(5/2) + (b*Tan
h[x])/(3*a*(a + b)*(a + b*Tanh[x]^2)^(3/2)) + (b*(5*a + 2*b)*Tanh[x])/(3*a^
2*(a + b)^2*Sqrt[a + b*Tanh[x]^2])
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx &= \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\left(a+bx^2\right)^{5/2}}\,dx, x, \tanh(x)\right)\\
&= \frac{b\tanh(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{b-3(a+b)+2bx^2}{\left(1-x^2\right)\left(a+bx^2\right)^{3/2}}\,dx, x, \tanh(x)\right)}{3a(a+b)}\\
&= \frac{b\tanh(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(5a+2b)\tanh(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{3a^2}{\left(1-x^2\right)\sqrt{a+bx^2}}\,dx,\right.}{3a^2(a+b)^2}\\
&= \frac{b\tanh(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(5a+2b)\tanh(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+bx^2}}\,dx,\right.}{(a+b)^2}\\
&= \frac{b\tanh(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(5a+2b)\tanh(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x,\right.}{(a+b)^2}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}} + \frac{b\tanh(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(5a+2b)\tanh(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}}
\end{aligned}
$$

Mathematica [C] time = 7.48803, size = 976, normalized size = 10.49

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Tanh[x]^2)^(-5/2),x]
```

```
[Out] (Cosh[x]*Sinh[x]*(1575*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]] + (3150*(a +
b)*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]]*Sinh[x]^2)/a + (1575*(a + b)^2*Ar
```

```
cSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]]*Sinh[x]^4)/a^2 + (2100*b*ArcSin[Sqrt[-
(((a + b)*Sinh[x]^2)/a)]]*Tanh[x]^2)/a + (4200*b*(a + b)*ArcSin[Sqrt[-(((a
+ b)*Sinh[x]^2)/a)]]*Sinh[x]^2*Tanh[x]^2)/a^2 + (2100*b*(a + b)^2*ArcSin[Sq
rt[-(((a + b)*Sinh[x]^2)/a)]]*Sinh[x]^4*Tanh[x]^2)/a^3 + (840*b^2*ArcSin[Sq
rt[-(((a + b)*Sinh[x]^2)/a)]]*Tanh[x]^4)/a^2 + (1680*b^2*(a + b)*ArcSin[Sqr
t[-(((a + b)*Sinh[x]^2)/a)]]*Sinh[x]^2*Tanh[x]^4)/a^3 + (840*b^2*(a + b)^2*
ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]]*Sinh[x]^4*Tanh[x]^4)/a^4 + 2100*(-((
(a + b)*Sinh[x]^2)/a))^(3/2)*Sqrt[(Cosh[x]^2*(a + b*Tanh[x]^2))/a] + 96*Hyp
ergeometric2F1[2, 2, 9/2, -(((a + b)*Sinh[x]^2)/a)]*(-(((a + b)*Sinh[x]^2)/
a))^(7/2)*Sqrt[(Cosh[x]^2*(a + b*Tanh[x]^2))/a] + 24*HypergeometricPFQ[{2,
2, 2}, {1, 9/2}, -(((a + b)*Sinh[x]^2)/a)]*(-(((a + b)*Sinh[x]^2)/a))^(7/2)
*Sqrt[(Cosh[x]^2*(a + b*Tanh[x]^2))/a] + (2800*b*(-(((a + b)*Sinh[x]^2)/a))
^(3/2)*Tanh[x]^2*Sqrt[(Cosh[x]^2*(a + b*Tanh[x]^2))/a])/a + (168*b*Hypergeo
metric2F1[2, 2, 9/2, -(((a + b)*Sinh[x]^2)/a)]*(-(((a + b)*Sinh[x]^2)/a))^(
7/2)*Tanh[x]^2*Sqrt[(Cosh[x]^2*(a + b*Tanh[x]^2))/a])/a + (48*b*Hypergeomet
ricPFQ[{2, 2, 2}, {1, 9/2}, -(((a + b)*Sinh[x]^2)/a)]*(-(((a + b)*Sinh[x]^2
)/a))^(7/2)*Tanh[x]^2*Sqrt[(Cosh[x]^2*(a + b*Tanh[x]^2))/a])/a + (1120*b^2*
(-(((a + b)*Sinh[x]^2)/a))^(3/2)*Tanh[x]^4*Sqrt[(Cosh[x]^2*(a + b*Tanh[x]^2
))/a])/a^2 + (72*b^2*Hypergeometric2F1[2, 2, 9/2, -(((a + b)*Sinh[x]^2)/a)]
*(-(((a + b)*Sinh[x]^2)/a))^(7/2)*Tanh[x]^4*Sqrt[(Cosh[x]^2*(a + b*Tanh[x]^
2))/a])/a^2 + (24*b^2*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, -(((a + b)*Sin
h[x]^2)/a)]*(-(((a + b)*Sinh[x]^2)/a))^(7/2)*Tanh[x]^4*Sqrt[(Cosh[x]^2*(a +
 b*Tanh[x]^2))/a])/a^2 - 1575*Sqrt[-(((a + b)*Cosh[x]^2*Sinh[x]^2*(a + b*Ta
nh[x]^2))/a^2)] - (2100*b*Tanh[x]^2*Sqrt[-(((a + b)*Cosh[x]^2*Sinh[x]^2*(a
+ b*Tanh[x]^2))/a^2)])/a - (840*b^2*Tanh[x]^4*Sqrt[-(((a + b)*Cosh[x]^2*Sin
h[x]^2*(a + b*Tanh[x]^2))/a^2)])/a^2))/(315*a^2*(-(((a + b)*Sinh[x]^2)/a))^
(5/2)*Sqrt[a + b*Tanh[x]^2]*Sqrt[(Cosh[x]^2*(a + b*Tanh[x]^2))/a]*(1 + (b*T
anh[x]^2)/a))
```

Maple [B] time = 0.024, size = 420, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tanh(x)^2)^(5/2),x)
```

```
[Out] 1/6/(a+b)/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(3/2)+1/6*b/(a+b)/a/((1+tan
h(x))^2*b-2*(1+tanh(x))*b+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/((1+tanh(x))^2
*b-2*(1+tanh(x))*b+a+b)^(1/2)*tanh(x)+1/2/(a+b)^2/((1+tanh(x))^2*b-2*(1+tan
h(x))*b+a+b)^(1/2)+1/2/(a+b)^2/a/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2
)*b*tanh(x)-1/2/(a+b)^(5/2)*ln((2*a+2*b-2*(1+tanh(x))*b+2*(a+b)^(1/2)*((1+t
anh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))/(1+tanh(x)))-1/6/(a+b)/((tanh(x)-1)
^2*b+2*(tanh(x)-1)*b+a+b)^(3/2)+1/6*b/(a+b)/a/((tanh(x)-1)^2*b+2*(tanh(x)-1
)*b+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b
)^(1/2)*tanh(x)-1/2/(a+b)^2/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)+1/2
/(a+b)^2/a/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)*b*tanh(x)+1/2/(a+b)^
(5/2)*ln((2*a+2*b+2*(tanh(x)-1)*b+2*(a+b)^(1/2)*((tanh(x)-1)^2*b+2*(tanh(x)
-1)*b+a+b)^(1/2))/(tanh(x)-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\tanh\left(x\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tanh(x)^2 + a)^(-5/2), x)
```

Fricas [B] time = 8.35041, size = 16405, normalized size = 176.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^8 + 8*(a^4 + 2*a^3*b + a^2*b^2)
*cosh(x)*sinh(x)^7 + (a^4 + 2*a^3*b + a^2*b^2)*sinh(x)^8 + 4*(a^4 - a^2*b^2
)*cosh(x)^6 + 4*(a^4 - a^2*b^2 + 7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^2)*sin
h(x)^6 + 8*(7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^3 + 3*(a^4 - a^2*b^2)*cosh(
x))*sinh(x)^5 + 2*(3*a^4 - 2*a^3*b + 3*a^2*b^2)*cosh(x)^4 + 2*(35*(a^4 + 2*
a^3*b + a^2*b^2)*cosh(x)^4 + 3*a^4 - 2*a^3*b + 3*a^2*b^2 + 30*(a^4 - a^2*b^
2)*cosh(x)^2)*sinh(x)^4 + a^4 + 2*a^3*b + a^2*b^2 + 8*(7*(a^4 + 2*a^3*b + a
^2*b^2)*cosh(x)^5 + 10*(a^4 - a^2*b^2)*cosh(x)^3 + (3*a^4 - 2*a^3*b + 3*a^2
*b^2)*cosh(x))*sinh(x)^3 + 4*(a^4 - a^2*b^2)*cosh(x)^2 + 4*(7*(a^4 + 2*a^3*
b + a^2*b^2)*cosh(x)^6 + 15*(a^4 - a^2*b^2)*cosh(x)^4 + a^4 - a^2*b^2 + 3*(
3*a^4 - 2*a^3*b + 3*a^2*b^2)*cosh(x)^2)*sinh(x)^2 + 8*((a^4 + 2*a^3*b + a^2
*b^2)*cosh(x)^7 + 3*(a^4 - a^2*b^2)*cosh(x)^5 + (3*a^4 - 2*a^3*b + 3*a^2*b^
2)*cosh(x)^3 + (a^4 - a^2*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*log(-((a*b^2 +
 b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)
^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh
(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x
))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3
)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)
*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 +
 (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b
^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x
)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b
 + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*c
osh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b
^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*
b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b +
 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^
3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(
x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^
2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a
^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x
))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3
*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 3
*((a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^8 + 8*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)
*sinh(x)^7 + (a^4 + 2*a^3*b + a^2*b^2)*sinh(x)^8 + 4*(a^4 - a^2*b^2)*cosh(x
)^6 + 4*(a^4 - a^2*b^2 + 7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^2)*sinh(x)^6 +
 8*(7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^3 + 3*(a^4 - a^2*b^2)*cosh(x))*sinh
(x)^5 + 2*(3*a^4 - 2*a^3*b + 3*a^2*b^2)*cosh(x)^4 + 2*(35*(a^4 + 2*a^3*b +
a^2*b^2)*cosh(x)^4 + 3*a^4 - 2*a^3*b + 3*a^2*b^2 + 30*(a^4 - a^2*b^2)*cosh(
x)^2)*sinh(x)^4 + a^4 + 2*a^3*b + a^2*b^2 + 8*(7*(a^4 + 2*a^3*b + a^2*b^2)*
cosh(x)^5 + 10*(a^4 - a^2*b^2)*cosh(x)^3 + (3*a^4 - 2*a^3*b + 3*a^2*b^2)*co
sh(x))*sinh(x)^3 + 4*(a^4 - a^2*b^2)*cosh(x)^2 + 4*(7*(a^4 + 2*a^3*b + a^2*
b^2)*cosh(x)^6 + 15*(a^4 - a^2*b^2)*cosh(x)^4 + a^4 - a^2*b^2 + 3*(3*a^4 -
```

```
2*a^3*b + 3*a^2*b^2)*cosh(x)^2)*sinh(x)^2 + 8*((a^4 + 2*a^3*b + a^2*b^2)*co
sh(x)^7 + 3*(a^4 - a^2*b^2)*cosh(x)^5 + (3*a^4 - 2*a^3*b + 3*a^2*b^2)*cosh(
x)^3 + (a^4 - a^2*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*log(((a + b)*cosh(x)^4
 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*(
a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
- b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 +
a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) +
8*sqrt(2)*((3*a^3*b + 7*a^2*b^2 + 5*a*b^3 + b^4)*cosh(x)^6 + 6*(3*a^3*b + 7
*a^2*b^2 + 5*a*b^3 + b^4)*cosh(x)*sinh(x)^5 + (3*a^3*b + 7*a^2*b^2 + 5*a*b^
3 + b^4)*sinh(x)^6 + 3*(a^3*b - a^2*b^2 - 3*a*b^3 - b^4)*cosh(x)^4 + 3*(a^3
*b - a^2*b^2 - 3*a*b^3 - b^4 + 5*(3*a^3*b + 7*a^2*b^2 + 5*a*b^3 + b^4)*cosh
(x)^2)*sinh(x)^4 - 3*a^3*b - 7*a^2*b^2 - 5*a*b^3 - b^4 + 4*(5*(3*a^3*b + 7*
a^2*b^2 + 5*a*b^3 + b^4)*cosh(x)^3 + 3*(a^3*b - a^2*b^2 - 3*a*b^3 - b^4)*co
sh(x))*sinh(x)^3 - 3*(a^3*b - a^2*b^2 - 3*a*b^3 - b^4)*cosh(x)^2 + 3*(5*(3*
a^3*b + 7*a^2*b^2 + 5*a*b^3 + b^4)*cosh(x)^4 - a^3*b + a^2*b^2 + 3*a*b^3 +
b^4 + 6*(a^3*b - a^2*b^2 - 3*a*b^3 - b^4)*cosh(x)^2)*sinh(x)^2 + 6*((3*a^3*
b + 7*a^2*b^2 + 5*a*b^3 + b^4)*cosh(x)^5 + 2*(a^3*b - a^2*b^2 - 3*a*b^3 - b
^4)*cosh(x)^3 - (a^3*b - a^2*b^2 - 3*a*b^3 - b^4)*cosh(x))*sinh(x))*sqrt(((
a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x
) + sinh(x)^2)))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^
2*b^5)*cosh(x)^8 + 8*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 +
 a^2*b^5)*cosh(x)*sinh(x)^7 + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*
a^3*b^4 + a^2*b^5)*sinh(x)^8 + a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*
a^3*b^4 + a^2*b^5 + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 -
a^2*b^5)*cosh(x)^6 + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 -
 a^2*b^5 + 7*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5
)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*
a^3*b^4 + a^2*b^5)*cosh(x)^3 + 3*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3
*a^3*b^4 - a^2*b^5)*cosh(x))*sinh(x)^5 + 2*(3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6
*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)*cosh(x)^4 + 2*(3*a^7 + 7*a^6*b + 6*a^5*b^
2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5 + 35*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10
*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*cosh(x)^4 + 30*(a^7 + 3*a^6*b + 2*a^5*b^2 -
 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^7 + 5*a^6*
b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*cosh(x)^5 + 10*(a^7 + 3*
a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*cosh(x)^3 + (3*a^7 + 7
*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)*cosh(x))*sinh(x)^3
+ 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*cosh(x)^2
 + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5 + 7*(a^7
+ 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*cosh(x)^6 + 15*(
a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*cosh(x)^4 + 3*
(3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)*cosh(x)^2
)*sinh(x)^2 + 8*((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2
*b^5)*cosh(x)^7 + 3*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^
2*b^5)*cosh(x)^5 + (3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3
*a^2*b^5)*cosh(x)^3 + (a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 -
a^2*b^5)*cosh(x))*sinh(x)), -1/6*(3*((a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^8 +
8*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)*sinh(x)^7 + (a^4 + 2*a^3*b + a^2*b^2)*s
inh(x)^8 + 4*(a^4 - a^2*b^2)*cosh(x)^6 + 4*(a^4 - a^2*b^2 + 7*(a^4 + 2*a^3*
b + a^2*b^2)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^
3 + 3*(a^4 - a^2*b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^4 - 2*a^3*b + 3*a^2*b^2)*
cosh(x)^4 + 2*(35*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^4 + 3*a^4 - 2*a^3*b + 3
*a^2*b^2 + 30*(a^4 - a^2*b^2)*cosh(x)^2)*sinh(x)^4 + a^4 + 2*a^3*b + a^2*b^
2 + 8*(7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^5 + 10*(a^4 - a^2*b^2)*cosh(x)^3
 + (3*a^4 - 2*a^3*b + 3*a^2*b^2)*cosh(x))*sinh(x)^3 + 4*(a^4 - a^2*b^2)*cos
h(x)^2 + 4*(7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^6 + 15*(a^4 - a^2*b^2)*cosh
(x)^4 + a^4 - a^2*b^2 + 3*(3*a^4 - 2*a^3*b + 3*a^2*b^2)*cosh(x)^2)*sinh(x)^
2 + 8*((a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^7 + 3*(a^4 - a^2*b^2)*cosh(x)^5 +
(3*a^4 - 2*a^3*b + 3*a^2*b^2)*cosh(x)^3 + (a^4 - a^2*b^2)*cosh(x))*sinh(x))
```

```
*sqrt(-a - b)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)
^2 - a - b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a -
b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a*b + b^2)*cosh(x)^4 + 4*
(a*b + b^2)*cosh(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^2)
*cosh(x)^2 + (6*(a*b + b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2
+ 2*a*b + b^2 + 2*(2*(a*b + b^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*s
inh(x))) + 3*((a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^8 + 8*(a^4 + 2*a^3*b + a^2*
b^2)*cosh(x)*sinh(x)^7 + (a^4 + 2*a^3*b + a^2*b^2)*sinh(x)^8 + 4*(a^4 - a^2
*b^2)*cosh(x)^6 + 4*(a^4 - a^2*b^2 + 7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^2)
*sinh(x)^6 + 8*(7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^3 + 3*(a^4 - a^2*b^2)*c
osh(x))*sinh(x)^5 + 2*(3*a^4 - 2*a^3*b + 3*a^2*b^2)*cosh(x)^4 + 2*(35*(a^4
+ 2*a^3*b + a^2*b^2)*cosh(x)^4 + 3*a^4 - 2*a^3*b + 3*a^2*b^2 + 30*(a^4 - a^
2*b^2)*cosh(x)^2)*sinh(x)^4 + a^4 + 2*a^3*b + a^2*b^2 + 8*(7*(a^4 + 2*a^3*b
 + a^2*b^2)*cosh(x)^5 + 10*(a^4 - a^2*b^2)*cosh(x)^3 + (3*a^4 - 2*a^3*b + 3
*a^2*b^2)*cosh(x))*sinh(x)^3 + 4*(a^4 - a^2*b^2)*cosh(x)^2 + 4*(7*(a^4 + 2*
a^3*b + a^2*b^2)*cosh(x)^6 + 15*(a^4 - a^2*b^2)*cosh(x)^4 + a^4 - a^2*b^2 +
 3*(3*a^4 - 2*a^3*b + 3*a^2*b^2)*cosh(x)^2)*sinh(x)^2 + 8*((a^4 + 2*a^3*b +
 a^2*b^2)*cosh(x)^7 + 3*(a^4 - a^2*b^2)*cosh(x)^5 + (3*a^4 - 2*a^3*b + 3*a^
2*b^2)*cosh(x)^3 + (a^4 - a^2*b^2)*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sq
rt(2)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(co
sh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^2 + 2*(a + b)*co
sh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)) - 4*sqrt(2)*((3*a^3*b + 7*a^2*b
^2 + 5*a*b^3 + b^4)*cosh(x)^6 + 6*(3*a^3*b + 7*a^2*b^2 + 5*a*b^3 + b^4)*cos
h(x)*sinh(x)^5 + (3*a^3*b + 7*a^2*b^2 + 5*a*b^3 + b^4)*sinh(x)^6 + 3*(a^3*b
 - a^2*b^2 - 3*a*b^3 - b^4)*cosh(x)^4 + 3*(a^3*b - a^2*b^2 - 3*a*b^3 - b^4
+ 5*(3*a^3*b + 7*a^2*b^2 + 5*a*b^3 + b^4)*cosh(x)^2)*sinh(x)^4 - 3*a^3*b -
7*a^2*b^2 - 5*a*b^3 - b^4 + 4*(5*(3*a^3*b + 7*a^2*b^2 + 5*a*b^3 + b^4)*cosh
(x)^3 + 3*(a^3*b - a^2*b^2 - 3*a*b^3 - b^4)*cosh(x))*sinh(x)^3 - 3*(a^3*b -
 a^2*b^2 - 3*a*b^3 - b^4)*cosh(x)^2 + 3*(5*(3*a^3*b + 7*a^2*b^2 + 5*a*b^3 +
 b^4)*cosh(x)^4 - a^3*b + a^2*b^2 + 3*a*b^3 + b^4 + 6*(a^3*b - a^2*b^2 - 3*
a*b^3 - b^4)*cosh(x)^2)*sinh(x)^2 + 6*((3*a^3*b + 7*a^2*b^2 + 5*a*b^3 + b^4
)*cosh(x)^5 + 2*(a^3*b - a^2*b^2 - 3*a*b^3 - b^4)*cosh(x)^3 - (a^3*b - a^2*
b^2 - 3*a*b^3 - b^4)*cosh(x))*sinh(x))*sqrt(((a + b)*cosh(x)^2 + (a + b)*si
nh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^7 + 5*a^
6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*cosh(x)^8 + 8*(a^7 + 5
*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*cosh(x)*sinh(x)^7 +
 (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*sinh(x)^8
+ a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5 + 4*(a^7 +
3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*cosh(x)^6 + 4*(a^7 +
 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5 + 7*(a^7 + 5*a^6*b +
 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*cosh(x)^2)*sinh(x)^6 + 8*(7
*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*cosh(x)^3
+ 3*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*cosh(x))*
sinh(x)^5 + 2*(3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*
b^5)*cosh(x)^4 + 2*(3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3
*a^2*b^5 + 35*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^
5)*cosh(x)^4 + 30*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*
b^5)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 +
 5*a^3*b^4 + a^2*b^5)*cosh(x)^5 + 10*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3
 - 3*a^3*b^4 - a^2*b^5)*cosh(x)^3 + (3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^
3 + 7*a^3*b^4 + 3*a^2*b^5)*cosh(x))*sinh(x)^3 + 4*(a^7 + 3*a^6*b + 2*a^5*b^
2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*cosh(x)^2 + 4*(a^7 + 3*a^6*b + 2*a^5*b
^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5 + 7*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a
^4*b^3 + 5*a^3*b^4 + a^2*b^5)*cosh(x)^6 + 15*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2
*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*cosh(x)^4 + 3*(3*a^7 + 7*a^6*b + 6*a^5*b^2
+ 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)*cosh(x)^2)*sinh(x)^2 + 8*((a^7 + 5*a^6
*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*cosh(x)^7 + 3*(a^7 + 3*
a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*cosh(x)^5 + (3*a^7 + 7
*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)*cosh(x)^3 + (a^7 +
```

```
3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + b\tanh^2(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)**2)**(5/2),x)
```

```
[Out] Integral((a + b*tanh(x)**2)**(-5/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.253 $\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=108

$$\frac{b(2a+b)}{a^2(a+b)^2\sqrt{a+b\tanh^2(x)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{b}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}$$

```
[Out] -(ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]]/a^(5/2)) + ArcTanh[Sqrt[a + b*Tanh
[x]^2]/Sqrt[a + b]]/(a + b)^(5/2) + b/(3*a*(a + b)*(a + b*Tanh[x]^2)^(3/2))
 + (b*(2*a + b))/(a^2*(a + b)^2*Sqrt[a + b*Tanh[x]^2])
```

Rubi [A] time = 0.208374, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3670, 446, 85, 152, 156, 63, 208}

$$\frac{b(2a+b)}{a^2(a+b)^2\sqrt{a+b\tanh^2(x)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{b}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]]/a^(5/2)) + ArcTanh[Sqrt[a + b*Tanh
[x]^2]/Sqrt[a + b]]/(a + b)^(5/2) + b/(3*a*(a + b)*(a + b*Tanh[x]^2)^(3/2))
 + (b*(2*a + b))/(a^2*(a + b)^2*Sqrt[a + b*Tanh[x]^2])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{1}{x\left(1-x^2\right)\left(a+bx^2\right)^{5/2}}\,dx, x, \tanh(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)x(a+bx)^{5/2}}\,dx, x, \tanh^2(x)\right)$$
$$= \frac{b}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-a-b+bx}{(1-x)x(a+bx)^{3/2}}\,dx, x, \tanh^2(x)\right)}{2a(a+b)}$$
$$= \frac{b}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(2a+b)}{a^2(a+b)^2\sqrt{a+b\tanh^2(x)}} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}(a+b)^2+\frac{1}{2}b(2a+b)x}{(1-x)x\sqrt{a+bx}}\right.}{a^2(a+b)^2}$$
$$= \frac{b}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(2a+b)}{a^2(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \tan\right.}{2a^2}$$
$$= \frac{b}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(2a+b)}{a^2(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a}\right.}{a^2b}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} + \frac{b}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{}{a^2(a}$$

Mathematica [C] time = 0.0716538, size = 73, normalized size = 0.68

$$\frac{(a+b)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\tanh^2(x)}{a}+1\right) - a\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\tanh^2(x)+a}{a+b}\right)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] (-(a*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tanh[x]^2)/(a + b)]) + (a + b)
*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Tanh[x]^2)/a])/(3*a*(a + b)*(a + b
*Tanh[x]^2)^(3/2))
```

Maple [F] time = 0.122, size = 0, normalized size = 0.

$$\int \coth(x)\left(a+b(\tanh(x))^2\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*tanh(x)^2)^(5/2),x)
```

```
[Out] int(coth(x)/(a+b*tanh(x)^2)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{\left(b\tanh(x)^2 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)/(b*tanh(x)^2 + a)^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*tanh(x)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.254 $\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=131

$$-\frac{(3a+2b)(a+4b)\coth(x)\sqrt{a+b\tanh^2(x)}}{3a^3(a+b)^2}+\frac{b(7a+4b)\coth(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}}+\frac{b\,\mathrm{co}}{3a(a+b)\left(a+\right.}$$

```
[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(5/2) + (b*Cot
h[x])/(3*a*(a + b)*(a + b*Tanh[x]^2)^(3/2)) + (b*(7*a + 4*b)*Coth[x])/(3*a^
2*(a + b)^2*Sqrt[a + b*Tanh[x]^2]) - ((3*a + 2*b)*(a + 4*b)*Coth[x]*Sqrt[a
+ b*Tanh[x]^2])/(3*a^3*(a + b)^2)
```

Rubi [A] time = 0.241572, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {3670, 472, 579, 583, 12, 377, 206}

$$-\frac{(3a+2b)(a+4b)\coth(x)\sqrt{a+b\tanh^2(x)}}{3a^3(a+b)^2}+\frac{b(7a+4b)\coth(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}}+\frac{b\,\mathrm{co}}{3a(a+b)\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(5/2) + (b*Cot
h[x])/(3*a*(a + b)*(a + b*Tanh[x]^2)^(3/2)) + (b*(7*a + 4*b)*Coth[x])/(3*a^
2*(a + b)^2*Sqrt[a + b*Tanh[x]^2]) - ((3*a + 2*b)*(a + 4*b)*Coth[x]*Sqrt[a
+ b*Tanh[x]^2])/(3*a^3*(a + b)^2)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
```

```
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{1}{x^2(1-x^2)(a+bx^2)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= \frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-3a-4b+4bx^2}{x^2(1-x^2)(a+bx^2)^{3/2}}\,dx, x, \tanh(x)\right)}{3a(a+b)}$$

$$= \frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(7a+4b)\coth(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{(3a+2b)(a+4b)-2b(7}{x^2(1-x^2)\sqrt{a+b}}\right.}{3a^2(a+}$$

$$= \frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(7a+4b)\coth(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}} - \frac{(3a+2b)(a+4b)\coth(x)}{3a^3(a+b)}$$

$$= \frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(7a+4b)\coth(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}} - \frac{(3a+2b)(a+4b)\coth(x)}{3a^3(a+b)}$$

$$= \frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(7a+4b)\coth(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}} - \frac{(3a+2b)(a+4b)\coth(x)}{3a^3(a+b)}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}} + \frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(7a+4b)\coth(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}} -$$

Mathematica [C] time = 7.45596, size = 246, normalized size = 1.88

$$\frac{\sqrt{\text{sech}^2(x)((a+b)\cosh(2x)+a-b)}\left(\frac{3\sqrt{2}a^3\coth(x)\left((a+b)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\text{csch}^2(x)((a+b)\cosh(2x)+a-b)}{b}}}{\sqrt{2}}\right),1\right)-a\Pi\left(\frac{b}{a+b};\sin^{-1}\left(\frac{\sqrt{\frac{(a-b+(a+b)\cosh}{b}}}{\sqrt{2}}\right.\right.\right.}{b\sqrt{\frac{\text{csch}^2(x)((a+b)\cosh(2x)+a-b)}{b}}}\right.}{3\sqrt{2}a^3(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] (Sqrt[(a - b + (a + b)*Cosh[2*x])*Sech[x]^2]*((3*Sqrt[2]*a^3*Coth[x]*((a +
b)*EllipticF[ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]
], 1] - a*EllipticPi[b/(a + b), ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Cs
ch[x]^2)/b]/Sqrt[2]], 1]))/(b*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/
b]) - ((a + b)*(3*(a + b)^2*(a - b + (a + b)*Cosh[2*x])^2*Coth[x] + 2*a*b^3
*Sinh[2*x] + b^2*(9*a + 5*b)*(a - b + (a + b)*Cosh[2*x])*Sinh[2*x]))/(a - b
 + (a + b)*Cosh[2*x])^2))/(3*Sqrt[2]*a^3*(a + b)^3)
```

Maple [F] time = 0.119, size = 0, normalized size = 0.

$$\int (\coth(x))^2\left(a+b(\tanh(x))^2\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+b*tanh(x)^2)^(5/2),x)
```

```
[Out] int(coth(x)^2/(a+b*tanh(x)^2)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)^2}{\left(b\tanh(x)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)^2/(b*tanh(x)^2 + a)^(5/2), x)
```

Fricas [B] time = 18.786, size = 25307, normalized size = 193.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^10 + 10*(a^5 + 2*a^4*b + a^3*b^
2)*cosh(x)*sinh(x)^9 + (a^5 + 2*a^4*b + a^3*b^2)*sinh(x)^10 + (3*a^5 - 2*a^
4*b - 5*a^3*b^2)*cosh(x)^8 + (3*a^5 - 2*a^4*b - 5*a^3*b^2 + 45*(a^5 + 2*a^4
*b + a^3*b^2)*cosh(x)^2)*sinh(x)^8 + 8*(15*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x
)^3 + (3*a^5 - 2*a^4*b - 5*a^3*b^2)*cosh(x))*sinh(x)^7 + 2*(a^5 - 2*a^4*b +
 5*a^3*b^2)*cosh(x)^6 + 2*(a^5 - 2*a^4*b + 5*a^3*b^2 + 105*(a^5 + 2*a^4*b +
 a^3*b^2)*cosh(x)^4 + 14*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*cosh(x)^2)*sinh(x)^6
 + 4*(63*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^5 + 14*(3*a^5 - 2*a^4*b - 5*a^3*
b^2)*cosh(x)^3 + 3*(a^5 - 2*a^4*b + 5*a^3*b^2)*cosh(x))*sinh(x)^5 - a^5 - 2
*a^4*b - a^3*b^2 - 2*(a^5 - 2*a^4*b + 5*a^3*b^2)*cosh(x)^4 + 2*(105*(a^5 +
2*a^4*b + a^3*b^2)*cosh(x)^6 - a^5 + 2*a^4*b - 5*a^3*b^2 + 35*(3*a^5 - 2*a^
4*b - 5*a^3*b^2)*cosh(x)^4 + 15*(a^5 - 2*a^4*b + 5*a^3*b^2)*cosh(x)^2)*sinh
(x)^4 + 8*(15*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^7 + 7*(3*a^5 - 2*a^4*b - 5*
a^3*b^2)*cosh(x)^5 + 5*(a^5 - 2*a^4*b + 5*a^3*b^2)*cosh(x)^3 - (a^5 - 2*a^4
*b + 5*a^3*b^2)*cosh(x))*sinh(x)^3 - (3*a^5 - 2*a^4*b - 5*a^3*b^2)*cosh(x)^
2 + (45*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^8 + 28*(3*a^5 - 2*a^4*b - 5*a^3*b
^2)*cosh(x)^6 - 3*a^5 + 2*a^4*b + 5*a^3*b^2 + 30*(a^5 - 2*a^4*b + 5*a^3*b^2
)*cosh(x)^4 - 12*(a^5 - 2*a^4*b + 5*a^3*b^2)*cosh(x)^2)*sinh(x)^2 + 2*(5*(a
^5 + 2*a^4*b + a^3*b^2)*cosh(x)^9 + 4*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*cosh(x)
^7 + 6*(a^5 - 2*a^4*b + 5*a^3*b^2)*cosh(x)^5 - 4*(a^5 - 2*a^4*b + 5*a^3*b^2
)*cosh(x)^3 - (3*a^5 - 2*a^4*b - 5*a^3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*l
og(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 +
 b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^
2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 +
2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70
*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^
3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3
)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*
```

a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2
+ b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3
*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x
)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*
cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3
- (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 -
a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6
*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt((
(a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(
x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)
^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*co
sh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 +
20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + si
nh(x)^6)) + 3*((a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^10 + 10*(a^5 + 2*a^4*b + a
^3*b^2)*cosh(x)*sinh(x)^9 + (a^5 + 2*a^4*b + a^3*b^2)*sinh(x)^10 + (3*a^5 -
2*a^4*b - 5*a^3*b^2)*cosh(x)^8 + (3*a^5 - 2*a^4*b - 5*a^3*b^2 + 45*(a^5 +
2*a^4*b + a^3*b^2)*cosh(x)^2)*sinh(x)^8 + 8*(15*(a^5 + 2*a^4*b + a^3*b^2)*c
osh(x)^3 + (3*a^5 - 2*a^4*b - 5*a^3*b^2)*cosh(x))*sinh(x)^7 + 2*(a^5 - 2*a^
4*b + 5*a^3*b^2)*cosh(x)^6 + 2*(a^5 - 2*a^4*b + 5*a^3*b^2 + 105*(a^5 + 2*a^
4*b + a^3*b^2)*cosh(x)^4 + 14*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*cosh(x)^2)*sinh
(x)^6 + 4*(63*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^5 + 14*(3*a^5 - 2*a^4*b - 5
*a^3*b^2)*cosh(x)^3 + 3*(a^5 - 2*a^4*b + 5*a^3*b^2)*cosh(x))*sinh(x)^5 - a^
5 - 2*a^4*b - a^3*b^2 - 2*(a^5 - 2*a^4*b + 5*a^3*b^2)*cosh(x)^4 + 2*(105*(a
^5 + 2*a^4*b + a^3*b^2)*cosh(x)^6 - a^5 + 2*a^4*b - 5*a^3*b^2 + 35*(3*a^5 -
2*a^4*b - 5*a^3*b^2)*cosh(x)^4 + 15*(a^5 - 2*a^4*b + 5*a^3*b^2)*cosh(x)^2)
sinh(x)^4 + 8(15*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^7 + 7*(3*a^5 - 2*a^4*b
- 5*a^3*b^2)*cosh(x)^5 + 5*(a^5 - 2*a^4*b + 5*a^3*b^2)*cosh(x)^3 - (a^5 -
2*a^4*b + 5*a^3*b^2)*cosh(x))*sinh(x)^3 - (3*a^5 - 2*a^4*b - 5*a^3*b^2)*cos
h(x)^2 + (45*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^8 + 28*(3*a^5 - 2*a^4*b - 5*
a^3*b^2)*cosh(x)^6 - 3*a^5 + 2*a^4*b + 5*a^3*b^2 + 30*(a^5 - 2*a^4*b + 5*a^
3*b^2)*cosh(x)^4 - 12*(a^5 - 2*a^4*b + 5*a^3*b^2)*cosh(x)^2)*sinh(x)^2 + 2*
(5*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^9 + 4*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*co
sh(x)^7 + 6*(a^5 - 2*a^4*b + 5*a^3*b^2)*cosh(x)^5 - 4*(a^5 - 2*a^4*b + 5*a^
3*b^2)*cosh(x)^3 - (3*a^5 - 2*a^4*b - 5*a^3*b^2)*cosh(x))*sinh(x))*sqrt(a +
b)*log(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^
4 + 2*a*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x
)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^
2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))
+ 4*((a + b)*cosh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x
)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*((3*a^5 + 15*a^4*b + 39*a^3*b^2 + 53*a^
2*b^3 + 34*a*b^4 + 8*b^5)*cosh(x)^8 + 8*(3*a^5 + 15*a^4*b + 39*a^3*b^2 + 53
*a^2*b^3 + 34*a*b^4 + 8*b^5)*cosh(x)*sinh(x)^7 + (3*a^5 + 15*a^4*b + 39*a^3
*b^2 + 53*a^2*b^3 + 34*a*b^4 + 8*b^5)*sinh(x)^8 + 4*(3*a^5 + 9*a^4*b + 6*a^
3*b^2 - 14*a^2*b^3 - 22*a*b^4 - 8*b^5)*cosh(x)^6 + 4*(3*a^5 + 9*a^4*b + 6*a
^3*b^2 - 14*a^2*b^3 - 22*a*b^4 - 8*b^5 + 7*(3*a^5 + 15*a^4*b + 39*a^3*b^2 +
53*a^2*b^3 + 34*a*b^4 + 8*b^5)*cosh(x)^2)*sinh(x)^6 + 8*(7*(3*a^5 + 15*a^4
*b + 39*a^3*b^2 + 53*a^2*b^3 + 34*a*b^4 + 8*b^5)*cosh(x)^3 + 3*(3*a^5 + 9*a
^4*b + 6*a^3*b^2 - 14*a^2*b^3 - 22*a*b^4 - 8*b^5)*cosh(x))*sinh(x)^5 + 3*a^
5 + 15*a^4*b + 39*a^3*b^2 + 53*a^2*b^3 + 34*a*b^4 + 8*b^5 + 6*(3*a^5 + 7*a^
4*b + 3*a^3*b^2 + 9*a^2*b^3 + 18*a*b^4 + 8*b^5)*cosh(x)^4 + 2*(9*a^5 + 21*a
^4*b + 9*a^3*b^2 + 27*a^2*b^3 + 54*a*b^4 + 24*b^5 + 35*(3*a^5 + 15*a^4*b +
39*a^3*b^2 + 53*a^2*b^3 + 34*a*b^4 + 8*b^5)*cosh(x)^4 + 30*(3*a^5 + 9*a^4*b
+ 6*a^3*b^2 - 14*a^2*b^3 - 22*a*b^4 - 8*b^5)*cosh(x)^2)*sinh(x)^4 + 8*(7*(
3*a^5 + 15*a^4*b + 39*a^3*b^2 + 53*a^2*b^3 + 34*a*b^4 + 8*b^5)*cosh(x)^5 +
10*(3*a^5 + 9*a^4*b + 6*a^3*b^2 - 14*a^2*b^3 - 22*a*b^4 - 8*b^5)*cosh(x)^3
+ 3*(3*a^5 + 7*a^4*b + 3*a^3*b^2 + 9*a^2*b^3 + 18*a*b^4 + 8*b^5)*cosh(x))*s
inh(x)^3 + 4*(3*a^5 + 9*a^4*b + 6*a^3*b^2 - 14*a^2*b^3 - 22*a*b^4 - 8*b^5)*
cosh(x)^2 + 4*(7*(3*a^5 + 15*a^4*b + 39*a^3*b^2 + 53*a^2*b^3 + 34*a*b^4 + 8
*b^5)*cosh(x)^6 + 3*a^5 + 9*a^4*b + 6*a^3*b^2 - 14*a^2*b^3 - 22*a*b^4 - 8*b

^5 + 15*(3*a^5 + 9*a^4*b + 6*a^3*b^2 - 14*a^2*b^3 - 22*a*b^4 - 8*b^5)*cosh(
x)^4 + 9*(3*a^5 + 7*a^4*b + 3*a^3*b^2 + 9*a^2*b^3 + 18*a*b^4 + 8*b^5)*cosh(
x)^2)*sinh(x)^2 + 8*((3*a^5 + 15*a^4*b + 39*a^3*b^2 + 53*a^2*b^3 + 34*a*b^4
 + 8*b^5)*cosh(x)^7 + 3*(3*a^5 + 9*a^4*b + 6*a^3*b^2 - 14*a^2*b^3 - 22*a*b^
4 - 8*b^5)*cosh(x)^5 + 3*(3*a^5 + 7*a^4*b + 3*a^3*b^2 + 9*a^2*b^3 + 18*a*b^
4 + 8*b^5)*cosh(x)^3 + (3*a^5 + 9*a^4*b + 6*a^3*b^2 - 14*a^2*b^3 - 22*a*b^4
 - 8*b^5)*cosh(x))*sinh(x))*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
 - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^8 + 5*a^7*b + 10*a^
6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*cosh(x)^10 + 10*(a^8 + 5*a^7*b +
10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*cosh(x)*sinh(x)^9 + (a^8 + 5
*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*sinh(x)^10 + (3*a^8
 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*cosh(x)^8 + (
3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5 + 45*(a^8
 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*cosh(x)^2)*sinh
(x)^8 - a^8 - 5*a^7*b - 10*a^6*b^2 - 10*a^5*b^3 - 5*a^4*b^4 - a^3*b^5 + 8*(
15*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*cosh(x)^
3 + (3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*cos
h(x))*sinh(x)^7 + 2*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*
a^3*b^5)*cosh(x)^6 + 2*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 +
 5*a^3*b^5 + 105*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3
*b^5)*cosh(x)^4 + 14*(3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4
 - 5*a^3*b^5)*cosh(x)^2)*sinh(x)^6 + 4*(63*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10
*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*cosh(x)^5 + 14*(3*a^8 + 7*a^7*b - 2*a^6*b^2
 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*cosh(x)^3 + 3*(a^8 + a^7*b + 2*a^6*
b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*cosh(x))*sinh(x)^5 - 2*(a^8 + a^
7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*cosh(x)^4 - 2*(a^8 +
 a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5 - 105*(a^8 + 5*a^7
*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*cosh(x)^6 - 35*(3*a^8 +
 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*cosh(x)^4 - 15*
(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*cosh(x)^2)*
sinh(x)^4 + 8*(15*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^
3*b^5)*cosh(x)^7 + 7*(3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4
 - 5*a^3*b^5)*cosh(x)^5 + 5*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*
b^4 + 5*a^3*b^5)*cosh(x)^3 - (a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4
*b^4 + 5*a^3*b^5)*cosh(x))*sinh(x)^3 - (3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^
5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*cosh(x)^2 + (45*(a^8 + 5*a^7*b + 10*a^6*b^2
 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*cosh(x)^8 - 3*a^8 - 7*a^7*b + 2*a^6*b^
2 + 18*a^5*b^3 + 17*a^4*b^4 + 5*a^3*b^5 + 28*(3*a^8 + 7*a^7*b - 2*a^6*b^2 -
 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*cosh(x)^6 + 30*(a^8 + a^7*b + 2*a^6*b
^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*cosh(x)^4 - 12*(a^8 + a^7*b + 2*a
^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*cosh(x)^2)*sinh(x)^2 + 2*(5*(
a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*cosh(x)^9 +
4*(3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*cosh(
x)^7 + 6*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*co
sh(x)^5 - 4*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)
*cosh(x)^3 - (3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3
*b^5)*cosh(x))*sinh(x)), -1/6*(3*((a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^10 + 10
*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)*sinh(x)^9 + (a^5 + 2*a^4*b + a^3*b^2)*si
nh(x)^10 + (3*a^5 - 2*a^4*b - 5*a^3*b^2)*cosh(x)^8 + (3*a^5 - 2*a^4*b - 5*a
^3*b^2 + 45*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^2)*sinh(x)^8 + 8*(15*(a^5 + 2
*a^4*b + a^3*b^2)*cosh(x)^3 + (3*a^5 - 2*a^4*b - 5*a^3*b^2)*cosh(x))*sinh(x
)^7 + 2*(a^5 - 2*a^4*b + 5*a^3*b^2)*cosh(x)^6 + 2*(a^5 - 2*a^4*b + 5*a^3*b^
2 + 105*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^4 + 14*(3*a^5 - 2*a^4*b - 5*a^3*b
^2)*cosh(x)^2)*sinh(x)^6 + 4*(63*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^5 + 14*(
3*a^5 - 2*a^4*b - 5*a^3*b^2)*cosh(x)^3 + 3*(a^5 - 2*a^4*b + 5*a^3*b^2)*cosh
(x))*sinh(x)^5 - a^5 - 2*a^4*b - a^3*b^2 - 2*(a^5 - 2*a^4*b + 5*a^3*b^2)*co
sh(x)^4 + 2*(105*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^6 - a^5 + 2*a^4*b - 5*a^
3*b^2 + 35*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*cosh(x)^4 + 15*(a^5 - 2*a^4*b + 5*
a^3*b^2)*cosh(x)^2)*sinh(x)^4 + 8*(15*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^7 +

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 7*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*cosh(x)^5 + 5*(a^5 - 2*a^4*b + 5*a^3*b^2)*
cosh(x)^3 - (a^5 - 2*a^4*b + 5*a^3*b^2)*cosh(x))*sinh(x)^3 - (3*a^5 - 2*a^4
*b - 5*a^3*b^2)*cosh(x)^2 + (45*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^8 + 28*(3
*a^5 - 2*a^4*b - 5*a^3*b^2)*cosh(x)^6 - 3*a^5 + 2*a^4*b + 5*a^3*b^2 + 30*(a
^5 - 2*a^4*b + 5*a^3*b^2)*cosh(x)^4 - 12*(a^5 - 2*a^4*b + 5*a^3*b^2)*cosh(x
)^2)*sinh(x)^2 + 2*(5*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^9 + 4*(3*a^5 - 2*a^
4*b - 5*a^3*b^2)*cosh(x)^7 + 6*(a^5 - 2*a^4*b + 5*a^3*b^2)*cosh(x)^5 - 4*(a
^5 - 2*a^4*b + 5*a^3*b^2)*cosh(x)^3 - (3*a^5 - 2*a^4*b - 5*a^3*b^2)*cosh(x)
)*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) +
 b*sinh(x)^2 - a - b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x
)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a*b + b^2)*cosh
(x)^4 + 4*(a*b + b^2)*cosh(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + (a^2 - a*
b - 2*b^2)*cosh(x)^2 + (6*(a*b + b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x
)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a*b + b^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*
cosh(x))*sinh(x))) + 3*((a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^10 + 10*(a^5 + 2*
a^4*b + a^3*b^2)*cosh(x)*sinh(x)^9 + (a^5 + 2*a^4*b + a^3*b^2)*sinh(x)^10 +
 (3*a^5 - 2*a^4*b - 5*a^3*b^2)*cosh(x)^8 + (3*a^5 - 2*a^4*b - 5*a^3*b^2 + 4
5*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^2)*sinh(x)^8 + 8*(15*(a^5 + 2*a^4*b + a
^3*b^2)*cosh(x)^3 + (3*a^5 - 2*a^4*b - 5*a^3*b^2)*cosh(x))*sinh(x)^7 + 2*(a
^5 - 2*a^4*b + 5*a^3*b^2)*cosh(x)^6 + 2*(a^5 - 2*a^4*b + 5*a^3*b^2 + 105*(a
^5 + 2*a^4*b + a^3*b^2)*cosh(x)^4 + 14*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*cosh(x
)^2)*sinh(x)^6 + 4*(63*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^5 + 14*(3*a^5 - 2*
a^4*b - 5*a^3*b^2)*cosh(x)^3 + 3*(a^5 - 2*a^4*b + 5*a^3*b^2)*cosh(x))*sinh(
x)^5 - a^5 - 2*a^4*b - a^3*b^2 - 2*(a^5 - 2*a^4*b + 5*a^3*b^2)*cosh(x)^4 +
2*(105*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^6 - a^5 + 2*a^4*b - 5*a^3*b^2 + 35
*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*cosh(x)^4 + 15*(a^5 - 2*a^4*b + 5*a^3*b^2)*c
osh(x)^2)*sinh(x)^4 + 8*(15*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^7 + 7*(3*a^5
- 2*a^4*b - 5*a^3*b^2)*cosh(x)^5 + 5*(a^5 - 2*a^4*b + 5*a^3*b^2)*cosh(x)^3
- (a^5 - 2*a^4*b + 5*a^3*b^2)*cosh(x))*sinh(x)^3 - (3*a^5 - 2*a^4*b - 5*a^3
*b^2)*cosh(x)^2 + (45*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^8 + 28*(3*a^5 - 2*a
^4*b - 5*a^3*b^2)*cosh(x)^6 - 3*a^5 + 2*a^4*b + 5*a^3*b^2 + 30*(a^5 - 2*a^4
*b + 5*a^3*b^2)*cosh(x)^4 - 12*(a^5 - 2*a^4*b + 5*a^3*b^2)*cosh(x)^2)*sinh(
x)^2 + 2*(5*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^9 + 4*(3*a^5 - 2*a^4*b - 5*a^
3*b^2)*cosh(x)^7 + 6*(a^5 - 2*a^4*b + 5*a^3*b^2)*cosh(x)^5 - 4*(a^5 - 2*a^4
*b + 5*a^3*b^2)*cosh(x)^3 - (3*a^5 - 2*a^4*b - 5*a^3*b^2)*cosh(x))*sinh(x))
*sqrt(-a - b)*arctan(sqrt(2)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)
*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*c
osh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)) + 2*sqrt
(2)*((3*a^5 + 15*a^4*b + 39*a^3*b^2 + 53*a^2*b^3 + 34*a*b^4 + 8*b^5)*cosh(x
)^8 + 8*(3*a^5 + 15*a^4*b + 39*a^3*b^2 + 53*a^2*b^3 + 34*a*b^4 + 8*b^5)*cos
h(x)*sinh(x)^7 + (3*a^5 + 15*a^4*b + 39*a^3*b^2 + 53*a^2*b^3 + 34*a*b^4 + 8
*b^5)*sinh(x)^8 + 4*(3*a^5 + 9*a^4*b + 6*a^3*b^2 - 14*a^2*b^3 - 22*a*b^4 -
8*b^5)*cosh(x)^6 + 4*(3*a^5 + 9*a^4*b + 6*a^3*b^2 - 14*a^2*b^3 - 22*a*b^4 -
 8*b^5 + 7*(3*a^5 + 15*a^4*b + 39*a^3*b^2 + 53*a^2*b^3 + 34*a*b^4 + 8*b^5)*
cosh(x)^2)*sinh(x)^6 + 8*(7*(3*a^5 + 15*a^4*b + 39*a^3*b^2 + 53*a^2*b^3 + 3
4*a*b^4 + 8*b^5)*cosh(x)^3 + 3*(3*a^5 + 9*a^4*b + 6*a^3*b^2 - 14*a^2*b^3 -
22*a*b^4 - 8*b^5)*cosh(x))*sinh(x)^5 + 3*a^5 + 15*a^4*b + 39*a^3*b^2 + 53*a
^2*b^3 + 34*a*b^4 + 8*b^5 + 6*(3*a^5 + 7*a^4*b + 3*a^3*b^2 + 9*a^2*b^3 + 18
*a*b^4 + 8*b^5)*cosh(x)^4 + 2*(9*a^5 + 21*a^4*b + 9*a^3*b^2 + 27*a^2*b^3 +
54*a*b^4 + 24*b^5 + 35*(3*a^5 + 15*a^4*b + 39*a^3*b^2 + 53*a^2*b^3 + 34*a*b
^4 + 8*b^5)*cosh(x)^4 + 30*(3*a^5 + 9*a^4*b + 6*a^3*b^2 - 14*a^2*b^3 - 22*a
*b^4 - 8*b^5)*cosh(x)^2)*sinh(x)^4 + 8*(7*(3*a^5 + 15*a^4*b + 39*a^3*b^2 +
53*a^2*b^3 + 34*a*b^4 + 8*b^5)*cosh(x)^5 + 10*(3*a^5 + 9*a^4*b + 6*a^3*b^2
- 14*a^2*b^3 - 22*a*b^4 - 8*b^5)*cosh(x)^3 + 3*(3*a^5 + 7*a^4*b + 3*a^3*b^2
 + 9*a^2*b^3 + 18*a*b^4 + 8*b^5)*cosh(x))*sinh(x)^3 + 4*(3*a^5 + 9*a^4*b +
6*a^3*b^2 - 14*a^2*b^3 - 22*a*b^4 - 8*b^5)*cosh(x)^2 + 4*(7*(3*a^5 + 15*a^4
*b + 39*a^3*b^2 + 53*a^2*b^3 + 34*a*b^4 + 8*b^5)*cosh(x)^6 + 3*a^5 + 9*a^4*
b + 6*a^3*b^2 - 14*a^2*b^3 - 22*a*b^4 - 8*b^5 + 15*(3*a^5 + 9*a^4*b + 6*a^3
*b^2 - 14*a^2*b^3 - 22*a*b^4 - 8*b^5)*cosh(x)^4 + 9*(3*a^5 + 7*a^4*b + 3*a^
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3*b^2 + 9*a^2*b^3 + 18*a*b^4 + 8*b^5)*cosh(x)^2)*sinh(x)^2 + 8*((3*a^5 + 15
*a^4*b + 39*a^3*b^2 + 53*a^2*b^3 + 34*a*b^4 + 8*b^5)*cosh(x)^7 + 3*(3*a^5 +
 9*a^4*b + 6*a^3*b^2 - 14*a^2*b^3 - 22*a*b^4 - 8*b^5)*cosh(x)^5 + 3*(3*a^5
+ 7*a^4*b + 3*a^3*b^2 + 9*a^2*b^3 + 18*a*b^4 + 8*b^5)*cosh(x)^3 + (3*a^5 +
9*a^4*b + 6*a^3*b^2 - 14*a^2*b^3 - 22*a*b^4 - 8*b^5)*cosh(x))*sinh(x))*sqrt
(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sin
h(x) + sinh(x)^2)))/((a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 +
 a^3*b^5)*cosh(x)^10 + 10*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*
b^4 + a^3*b^5)*cosh(x)*sinh(x)^9 + (a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3
 + 5*a^4*b^4 + a^3*b^5)*sinh(x)^10 + (3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*
b^3 - 17*a^4*b^4 - 5*a^3*b^5)*cosh(x)^8 + (3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18
*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5 + 45*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5
*b^3 + 5*a^4*b^4 + a^3*b^5)*cosh(x)^2)*sinh(x)^8 - a^8 - 5*a^7*b - 10*a^6*b
^2 - 10*a^5*b^3 - 5*a^4*b^4 - a^3*b^5 + 8*(15*(a^8 + 5*a^7*b + 10*a^6*b^2 +
 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*cosh(x)^3 + (3*a^8 + 7*a^7*b - 2*a^6*b^2
 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*cosh(x))*sinh(x)^7 + 2*(a^8 + a^7*b
 + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*cosh(x)^6 + 2*(a^8 + a^
7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5 + 105*(a^8 + 5*a^7*b
+ 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*cosh(x)^4 + 14*(3*a^8 + 7*
a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*cosh(x)^2)*sinh(x)
^6 + 4*(63*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*
cosh(x)^5 + 14*(3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a
^3*b^5)*cosh(x)^3 + 3*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 +
5*a^3*b^5)*cosh(x))*sinh(x)^5 - 2*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 1
3*a^4*b^4 + 5*a^3*b^5)*cosh(x)^4 - 2*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3
+ 13*a^4*b^4 + 5*a^3*b^5 - 105*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5
*a^4*b^4 + a^3*b^5)*cosh(x)^6 - 35*(3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^
3 - 17*a^4*b^4 - 5*a^3*b^5)*cosh(x)^4 - 15*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^
5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*cosh(x)^2)*sinh(x)^4 + 8*(15*(a^8 + 5*a^7*b
 + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*cosh(x)^7 + 7*(3*a^8 + 7*
a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*cosh(x)^5 + 5*(a^8
 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*cosh(x)^3 - (a^
8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*cosh(x))*sinh(
x)^3 - (3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*
cosh(x)^2 + (45*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*
b^5)*cosh(x)^8 - 3*a^8 - 7*a^7*b + 2*a^6*b^2 + 18*a^5*b^3 + 17*a^4*b^4 + 5*
a^3*b^5 + 28*(3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3
*b^5)*cosh(x)^6 + 30*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5
*a^3*b^5)*cosh(x)^4 - 12*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4
 + 5*a^3*b^5)*cosh(x)^2)*sinh(x)^2 + 2*(5*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*
a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*cosh(x)^9 + 4*(3*a^8 + 7*a^7*b - 2*a^6*b^2 -
 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*cosh(x)^7 + 6*(a^8 + a^7*b + 2*a^6*b^
2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*cosh(x)^5 - 4*(a^8 + a^7*b + 2*a^6
*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*cosh(x)^3 - (3*a^8 + 7*a^7*b -
2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*cosh(x))*sinh(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+b*tanh(x)**2)**(5/2),x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.255 $\int \frac{1}{\sqrt{1+\tanh^2(x)}} \, dx$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{\tanh^2(x)+1}}\right)}{\sqrt{2}}$$

```
[Out] ArcTanh[(Sqrt[2]*Tanh[x])/Sqrt[1 + Tanh[x]^2]]/Sqrt[2]
```

Rubi [A] time = 0.0185679, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3661, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{\tanh^2(x)+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + Tanh[x]^2],x]
```

```
[Out] ArcTanh[(Sqrt[2]*Tanh[x])/Sqrt[1 + Tanh[x]^2]]/Sqrt[2]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{1+\tanh^2(x)}}\,dx = \text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{1+x^2}}\,dx, x, \tanh(x)\right)$$
$$= \text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{1+\tanh^2(x)}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{1+\tanh^2(x)}}\right)}{\sqrt{2}}$$

Mathematica [A] time = 0.0249142, size = 35, normalized size = 1.4

$$\frac{\sinh^{-1}\left(\sqrt{2}\sinh(x)\right)\sqrt{\cosh(2x)}\,\mathbf{sech}(x)}{\sqrt{2}\sqrt{\tanh^2(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + Tanh[x]^2],x]
```

```
[Out] (ArcSinh[Sqrt[2]*Sinh[x]]*Sqrt[Cosh[2*x]]*Sech[x])/(Sqrt[2]*Sqrt[1 + Tanh[x
]^2])
```

Maple [B] time = 0.047, size = 62, normalized size = 2.5

$$-\frac{\sqrt{2}}{4}\text{Artanh}\left(\frac{(2-2\tanh(x))\sqrt{2}}{4}\frac{1}{\sqrt{(1+\tanh(x))^2-2\tanh(x)}}\right)+\frac{\sqrt{2}}{4}\text{Artanh}\left(\frac{(2\tanh(x)+2)\sqrt{2}}{4}\frac{}{\sqrt{(\tanh(x)-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+tanh(x)^2)^(1/2),x)
```

```
[Out] -1/4*2^(1/2)*arctanh(1/4*(2-2*tanh(x))*2^(1/2)/((1+tanh(x))^2-2*tanh(x))^(1
/2))+1/4*2^(1/2)*arctanh(1/4*(2*tanh(x)+2)*2^(1/2)/((tanh(x)-1)^2+2*tanh(x)
)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\tanh(x)^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(tanh(x)^2 + 1), x)
```

Fricas [B] time = 2.54609, size = 1831, normalized size = 73.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log(-2*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + (28*cosh(
x)^2 - 3)*sinh(x)^6 - 3*cosh(x)^6 + 2*(28*cosh(x)^3 - 9*cosh(x))*sinh(x)^5
+ 5*(14*cosh(x)^4 - 9*cosh(x)^2 + 1)*sinh(x)^4 + 5*cosh(x)^4 + 4*(14*cosh(x
)^5 - 15*cosh(x)^3 + 5*cosh(x))*sinh(x)^3 + (28*cosh(x)^6 - 45*cosh(x)^4 +
30*cosh(x)^2 - 4)*sinh(x)^2 - 4*cosh(x)^2 + 2*(4*cosh(x)^7 - 9*cosh(x)^5 +
10*cosh(x)^3 - 4*cosh(x))*sinh(x) + (sqrt(2)*cosh(x)^6 + 6*sqrt(2)*cosh(x)*
sinh(x)^5 + sqrt(2)*sinh(x)^6 + 3*(5*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x)^4
 - 3*sqrt(2)*cosh(x)^4 + 4*(5*sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x))*sinh(x
)^3 + (15*sqrt(2)*cosh(x)^4 - 18*sqrt(2)*cosh(x)^2 + 4*sqrt(2))*sinh(x)^2 +
 4*sqrt(2)*cosh(x)^2 + 2*(3*sqrt(2)*cosh(x)^5 - 6*sqrt(2)*cosh(x)^3 + 4*sqr
t(2)*cosh(x))*sinh(x) - 4*sqrt(2))*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2
- 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4)/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 1
5*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6
*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 1/8*sqrt(2)*log(2*(cosh(x)^4 + 4*cosh(x)
*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(2*cos
h(x)^3 + cosh(x))*sinh(x) + (sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x)
+ sqrt(2)*sinh(x)^2 + sqrt(2))*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*
cosh(x)*sinh(x) + sinh(x)^2)) + 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
^2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\tanh^2(x) + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(tanh(x)**2 + 1), x)
```

Giac [B] time = 1.30805, size = 78, normalized size = 3.12

$$-\frac{1}{4}\sqrt{2}\left(\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}+1\right)+\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)-\log\left(-\sqrt{e^{(4x)}+1}+e^{(2x)}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(log(sqrt(e^(4*x) + 1) - e^(2*x) + 1) + log(sqrt(e^(4*x) + 1)
- e^(2*x)) - log(-sqrt(e^(4*x) + 1) + e^(2*x) + 1))
```

3.256 $\int \frac{1}{\sqrt{-1-\tanh^2(x)}}\,dx$

Optimal. Leaf size=27

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)}{\sqrt{2}}$$

```
[Out] ArcTan[(Sqrt[2]*Tanh[x])/Sqrt[-1 - Tanh[x]^2]]/Sqrt[2]
```

Rubi [A] time = 0.0201954, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3661, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-1 - Tanh[x]^2],x]
```

```
[Out] ArcTan[(Sqrt[2]*Tanh[x])/Sqrt[-1 - Tanh[x]^2]]/Sqrt[2]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{-1-\tanh^2(x)}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{-1-x^2}\,(1-x^2)}\,dx, x, \tanh(x)\right)$$
$$= \text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right)}{\sqrt{2}}$$

Mathematica [A] time = 0.0208512, size = 37, normalized size = 1.37

$$\frac{\sinh^{-1}\left(\sqrt{2}\sinh(x)\right)\sqrt{\cosh(2x)}\,\text{sech}(x)}{\sqrt{2}\sqrt{-\tanh^2(x)-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 - Tanh[x]^2],x]
```

```
[Out] (ArcSinh[Sqrt[2]*Sinh[x]]*Sqrt[Cosh[2*x]]*Sech[x])/(Sqrt[2]*Sqrt[-1 - Tanh[
x]^2])
```

Maple [B] time = 0.049, size = 66, normalized size = 2.4

$$\frac{\sqrt{2}}{4}\arctan\left(\frac{(2\tanh(x)-2)\sqrt{2}}{4}\frac{1}{\sqrt{-(1+\tanh(x))^2+2\tanh(x)}}\right)-\frac{\sqrt{2}}{4}\arctan\left(\frac{(-2-2\tanh(x))\sqrt{2}}{4}\frac{1}{\sqrt{-(\tanh(x)-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1-tanh(x)^2)^(1/2),x)
```

```
[Out] 1/4*2^(1/2)*arctan(1/4*(2*tanh(x)-2)*2^(1/2)/(-(1+tanh(x))^2+2*tanh(x))^(1/
2))-1/4*2^(1/2)*arctan(1/4*(-2-2*tanh(x))*2^(1/2)/(-(tanh(x)-1)^2-2*tanh(x)
)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\tanh(x)^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-tanh(x)^2 - 1), x)
```

Fricas [C] time = 2.32529, size = 551, normalized size = 20.41

$$\frac{1}{8}i\sqrt{2}\log\left(\frac{1}{2}\left(i\sqrt{2}\sqrt{-2\,e^{(4\,x)}-2}+2\,e^{(2\,x)}+2\right)e^{(-2\,x)}\right)-\frac{1}{8}i\sqrt{2}\log\left(\frac{1}{2}\left(-i\sqrt{2}\sqrt{-2\,e^{(4\,x)}-2}+2\,e^{(2\,x)}+2\right)e^{(-2\,x)}\right)-\frac{1}{8}i\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*I*sqrt(2)*log(1/2*(I*sqrt(2)*sqrt(-2*e^(4*x) - 2) + 2*e^(2*x) + 2)*e^(-
2*x)) - 1/8*I*sqrt(2)*log(1/2*(-I*sqrt(2)*sqrt(-2*e^(4*x) - 2) + 2*e^(2*x)
+ 2)*e^(-2*x)) - 1/8*I*sqrt(2)*log((sqrt(-2*e^(4*x) - 2)*(e^(2*x) - 2) + I*
sqrt(2)*e^(4*x) - I*sqrt(2)*e^(2*x) + 2*I*sqrt(2))*e^(-4*x)) + 1/8*I*sqrt(2
)*log((sqrt(-2*e^(4*x) - 2)*(e^(2*x) - 2) - I*sqrt(2)*e^(4*x) + I*sqrt(2)*e
^(2*x) - 2*I*sqrt(2))*e^(-4*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\tanh^{2}(x)-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-tanh(x)**2 - 1), x)
```

Giac [C] time = 1.33031, size = 96, normalized size = 3.56

$$\frac{1}{4}\sqrt{2}\left(i\log\left(-\left(\sqrt{e^{(4\,x)}+1}+1\right)e^{(-2\,x)}\right)-i\log\left(-\left(-i\sqrt{e^{(4\,x)}+1}-i\right)e^{(-2\,x)}+i\right)+i\log\left(-\left(-i\sqrt{e^{(4\,x)}+1}-i\right)e^{(-2\,x)}-i\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(I*log(-(sqrt(e^(4*x) + 1) + 1)*e^(-2*x)) - I*log(-(-I*sqrt(e^(
4*x) + 1) - I)*e^(-2*x) + I) + I*log(-(-I*sqrt(e^(4*x) + 1) - I)*e^(-2*x) -
 I))
```

3.257 $\int \left(a+b\tanh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=89

$$x\left(a^2+b^2\right)-\frac{ab\tanh^2(c+dx)}{d}+\frac{2ab\log(\cosh(c+dx))}{d}-\frac{b^2\tanh^5(c+dx)}{5d}-\frac{b^2\tanh^3(c+dx)}{3d}-\frac{b^2\tanh(c+dx)}{d}$$

```
[Out] (a^2 + b^2)*x + (2*a*b*Log[Cosh[c + d*x]])/d - (b^2*Tanh[c + d*x])/d - (a*b
*Tanh[c + d*x]^2)/d - (b^2*Tanh[c + d*x]^3)/(3*d) - (b^2*Tanh[c + d*x]^5)/(
5*d)
```

Rubi [A] time = 0.0749499, antiderivative size = 112, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3661, 1810, 633, 31}

$$-\frac{ab\tanh^2(c+dx)}{d}-\frac{(a+b)^2\log(1-\tanh(c+dx))}{2d}+\frac{(a-b)^2\log(\tanh(c+dx)+1)}{2d}-\frac{b^2\tanh^5(c+dx)}{5d}-\frac{b^2\tanh^3(c+}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] -((a + b)^2*Log[1 - Tanh[c + d*x]])/(2*d) + ((a - b)^2*Log[1 + Tanh[c + d*x
]])/(2*d) - (b^2*Tanh[c + d*x])/d - (a*b*Tanh[c + d*x]^2)/d - (b^2*Tanh[c +
 d*x]^3)/(3*d) - (b^2*Tanh[c + d*x]^5)/(5*d)
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \left(a+b\tanh^3(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{(a+bx^3)^2}{1-x^2} dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-b^2-2abx-b^2x^2-b^2x^4+\frac{a^2+b^2+2abx}{1-x^2}\right) dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{b^2\tanh(c+dx)}{d}-\frac{ab\tanh^2(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}-\frac{b^2\tanh^5(c+dx)}{5d}+\frac{\text{Subs}}{}$$
$$= -\frac{b^2\tanh(c+dx)}{d}-\frac{ab\tanh^2(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}-\frac{b^2\tanh^5(c+dx)}{5d}-\frac{(a-}{}$$
$$= -\frac{(a+b)^2\log(1-\tanh(c+dx))}{2d}+\frac{(a-b)^2\log(1+\tanh(c+dx))}{2d}-\frac{b^2\tanh(c+dx)}{d}$$

Mathematica [A] time = 0.673195, size = 95, normalized size = 1.07

$$-\frac{30ab\tanh^2(c+dx)+15\left((a+b)^2\log(1-\tanh(c+dx))-(a-b)^2\log(\tanh(c+dx)+1)\right)+6b^2\tanh^5(c+dx)+10}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] -(15*((a + b)^2*Log[1 - Tanh[c + d*x]] - (a - b)^2*Log[1 + Tanh[c + d*x]])
+ 30*b^2*Tanh[c + d*x] + 30*a*b*Tanh[c + d*x]^2 + 10*b^2*Tanh[c + d*x]^3 +
6*b^2*Tanh[c + d*x]^5)/(30*d)
```

Maple [A] time = 0.006, size = 163, normalized size = 1.8

$$-\frac{b^2(\tanh(dx+c))^5}{5d}-\frac{b^2(\tanh(dx+c))^3}{3d}-\frac{ab(\tanh(dx+c))^2}{d}-\frac{b^2\tanh(dx+c)}{d}-\frac{a^2\ln(\tanh(dx+c)-1)}{2d}-\ln($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tanh(d*x+c)^3)^2,x)
```

```
[Out] -1/5*b^2*tanh(d*x+c)^5/d-1/3*b^2*tanh(d*x+c)^3/d-a*b*tanh(d*x+c)^2/d-b^2*ta
nh(d*x+c)/d-1/2*a^2/d*ln(tanh(d*x+c)-1)-1/d*ln(tanh(d*x+c)-1)*a*b-1/2/d*ln(
tanh(d*x+c)-1)*b^2+1/2/d*ln(tanh(d*x+c)+1)*a^2-1/d*ln(tanh(d*x+c)+1)*a*b+1/
2/d*ln(tanh(d*x+c)+1)*b^2
```

Maxima [B] time = 1.52232, size = 262, normalized size = 2.94

$$\frac{1}{15}b^2\left(15x+\frac{15c}{d}-\frac{2\left(70e^{(-2dx-2c)}+140e^{(-4dx-4c)}+90e^{(-6dx-6c)}+45e^{(-8dx-8c)}+23\right)}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1\right)}\right)+2ab\left(x+\frac{c}{d}+\frac{\log}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 1/15*b^2*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 9
0*e^(-6*d*x - 6*c) + 45*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
10*c) + 1))) + 2*a*b*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x -
 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a^2*x
```

Fricas [B] time = 2.67561, size = 5516, normalized size = 61.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/15*(15*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^10 + 150*(a^2 - 2*a*b + b^2)
*d*x*cosh(d*x + c)*sinh(d*x + c)^9 + 15*(a^2 - 2*a*b + b^2)*d*x*sinh(d*x +
c)^10 + 15*(5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b + 6*b^2)*cosh(d*x + c)^8 + 15
*(45*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^2 + 5*(a^2 - 2*a*b + b^2)*d*x +
4*a*b + 6*b^2)*sinh(d*x + c)^8 + 120*(15*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x +
 c)^3 + (5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b + 6*b^2)*cosh(d*x + c))*sinh(d*x
 + c)^7 + 30*(5*(a^2 - 2*a*b + b^2)*d*x + 6*a*b + 6*b^2)*cosh(d*x + c)^6 +
30*(105*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + 5*(a^2 - 2*a*b + b^2)*d*x
 + 14*(5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b + 6*b^2)*cosh(d*x + c)^2 + 6*a*b +
 6*b^2)*sinh(d*x + c)^6 + 60*(63*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^5 +
14*(5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b + 6*b^2)*cosh(d*x + c)^3 + 3*(5*(a^2
- 2*a*b + b^2)*d*x + 6*a*b + 6*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 10*(15
*(a^2 - 2*a*b + b^2)*d*x + 18*a*b + 28*b^2)*cosh(d*x + c)^4 + 10*(315*(a^2
- 2*a*b + b^2)*d*x*cosh(d*x + c)^6 + 105*(5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b
 + 6*b^2)*cosh(d*x + c)^4 + 15*(a^2 - 2*a*b + b^2)*d*x + 45*(5*(a^2 - 2*a*b
 + b^2)*d*x + 6*a*b + 6*b^2)*cosh(d*x + c)^2 + 18*a*b + 28*b^2)*sinh(d*x +
c)^4 + 40*(45*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^7 + 21*(5*(a^2 - 2*a*b
+ b^2)*d*x + 4*a*b + 6*b^2)*cosh(d*x + c)^5 + 15*(5*(a^2 - 2*a*b + b^2)*d*x
 + 6*a*b + 6*b^2)*cosh(d*x + c)^3 + (15*(a^2 - 2*a*b + b^2)*d*x + 18*a*b +
28*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 15*(a^2 - 2*a*b + b^2)*d*x + 5*(15
*(a^2 - 2*a*b + b^2)*d*x + 12*a*b + 28*b^2)*cosh(d*x + c)^2 + 5*(135*(a^2 -
 2*a*b + b^2)*d*x*cosh(d*x + c)^8 + 84*(5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b +
 6*b^2)*cosh(d*x + c)^6 + 90*(5*(a^2 - 2*a*b + b^2)*d*x + 6*a*b + 6*b^2)*co
sh(d*x + c)^4 + 15*(a^2 - 2*a*b + b^2)*d*x + 12*(15*(a^2 - 2*a*b + b^2)*d*x
 + 18*a*b + 28*b^2)*cosh(d*x + c)^2 + 12*a*b + 28*b^2)*sinh(d*x + c)^2 + 46
*b^2 + 30*(a*b*cosh(d*x + c)^10 + 10*a*b*cosh(d*x + c)*sinh(d*x + c)^9 + a*
b*sinh(d*x + c)^10 + 5*a*b*cosh(d*x + c)^8 + 5*(9*a*b*cosh(d*x + c)^2 + a*b
)*sinh(d*x + c)^8 + 10*a*b*cosh(d*x + c)^6 + 40*(3*a*b*cosh(d*x + c)^3 + a*
b*cosh(d*x + c))*sinh(d*x + c)^7 + 10*(21*a*b*cosh(d*x + c)^4 + 14*a*b*cosh
(d*x + c)^2 + a*b)*sinh(d*x + c)^6 + 10*a*b*cosh(d*x + c)^4 + 4*(63*a*b*cos
h(d*x + c)^5 + 70*a*b*cosh(d*x + c)^3 + 15*a*b*cosh(d*x + c))*sinh(d*x + c)
^5 + 10*(21*a*b*cosh(d*x + c)^6 + 35*a*b*cosh(d*x + c)^4 + 15*a*b*cosh(d*x
+ c)^2 + a*b)*sinh(d*x + c)^4 + 5*a*b*cosh(d*x + c)^2 + 40*(3*a*b*cosh(d*x
+ c)^7 + 7*a*b*cosh(d*x + c)^5 + 5*a*b*cosh(d*x + c)^3 + a*b*cosh(d*x + c))
*sinh(d*x + c)^3 + 5*(9*a*b*cosh(d*x + c)^8 + 28*a*b*cosh(d*x + c)^6 + 30*a
*b*cosh(d*x + c)^4 + 12*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^2 + a*b +
10*(a*b*cosh(d*x + c)^9 + 4*a*b*cosh(d*x + c)^7 + 6*a*b*cosh(d*x + c)^5 + 4
*a*b*cosh(d*x + c)^3 + a*b*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c
)/(cosh(d*x + c) - sinh(d*x + c))) + 10*(15*(a^2 - 2*a*b + b^2)*d*x*cosh(d*
x + c)^9 + 12*(5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b + 6*b^2)*cosh(d*x + c)^7 +
 18*(5*(a^2 - 2*a*b + b^2)*d*x + 6*a*b + 6*b^2)*cosh(d*x + c)^5 + 4*(15*(a^
2 - 2*a*b + b^2)*d*x + 18*a*b + 28*b^2)*cosh(d*x + c)^3 + (15*(a^2 - 2*a*b
+ b^2)*d*x + 12*a*b + 28*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c
```

```
)^10 + 10*d*cosh(d*x + c)*sinh(d*x + c)^9 + d*sinh(d*x + c)^10 + 5*d*cosh(d
*x + c)^8 + 5*(9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 40*(3*d*cosh(d*x
+ c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^7 + 10*d*cosh(d*x + c)^6 + 10*(21*d
*cosh(d*x + c)^4 + 14*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 4*(63*d*cosh
(d*x + c)^5 + 70*d*cosh(d*x + c)^3 + 15*d*cosh(d*x + c))*sinh(d*x + c)^5 +
10*d*cosh(d*x + c)^4 + 10*(21*d*cosh(d*x + c)^6 + 35*d*cosh(d*x + c)^4 + 15
*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 40*(3*d*cosh(d*x + c)^7 + 7*d*cos
h(d*x + c)^5 + 5*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^3 + 5*d
*cosh(d*x + c)^2 + 5*(9*d*cosh(d*x + c)^8 + 28*d*cosh(d*x + c)^6 + 30*d*cos
h(d*x + c)^4 + 12*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 10*(d*cosh(d*x +
 c)^9 + 4*d*cosh(d*x + c)^7 + 6*d*cosh(d*x + c)^5 + 4*d*cosh(d*x + c)^3 + d
*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [A] time = 0.772513, size = 100, normalized size = 1.12

$$\begin{cases} a^2x + 2abx - \frac{2ab\log(\tanh(c+dx)+1)}{d} - \frac{ab\tanh^2(c+dx)}{d} + b^2x - \frac{b^2\tanh^5(c+dx)}{5d} - \frac{b^2\tanh^3(c+dx)}{3d} - \frac{b^2\tanh(c+dx)}{d} & \text{for } d \neq 0 \\ x\left(a + b\tanh^3(c)\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)**3)**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*x - 2*a*b*log(tanh(c + d*x) + 1)/d - a*b*tanh(c +
 d*x)**2/d + b**2*x - b**2*tanh(c + d*x)**5/(5*d) - b**2*tanh(c + d*x)**3/(
3*d) - b**2*tanh(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c)**3)**2, True))
```

Giac [A] time = 1.34766, size = 196, normalized size = 2.2

$$\frac{2\,ab\log\left(e^{(2\,dx+2\,c)}+1\right)}{d} + \frac{\left(a^2-2\,ab+b^2\right)(dx+c)}{d} + \frac{2\left(23\,b^2+15\left(2\,ab+3\,b^2\right)e^{(8\,dx+8\,c)}+90\left(ab+b^2\right)e^{(6\,dx+6\,c)}+10\,(\cdots\right.}{15\,d\left(e^{(2\,dx+2\,c)}+1\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 2*a*b*log(e^(2*d*x + 2*c) + 1)/d + (a^2 - 2*a*b + b^2)*(d*x + c)/d + 2/15*(
23*b^2 + 15*(2*a*b + 3*b^2)*e^(8*d*x + 8*c) + 90*(a*b + b^2)*e^(6*d*x + 6*c
) + 10*(9*a*b + 14*b^2)*e^(4*d*x + 4*c) + 10*(3*a*b + 7*b^2)*e^(2*d*x + 2*c
))/(d*(e^(2*d*x + 2*c) + 1)^5)
```

3.258 $\int \frac{1}{1+\tanh^3(x)} dx$

Optimal. Leaf size=38

$$\frac{x}{2} - \frac{1}{6(\tanh(x)+1)} - \frac{2\tan^{-1}\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] x/2 - (2*ArcTan[(1 - 2*Tanh[x])/Sqrt[3]])/(3*Sqrt[3]) - 1/(6*(1 + Tanh[x]))
```

Rubi [A] time = 0.0662628, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3661, 2074, 207, 618, 204}

$$\frac{x}{2} - \frac{1}{6(\tanh(x)+1)} - \frac{2\tan^{-1}\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Tanh[x]^3)^(-1),x]
```

```
[Out] x/2 - (2*ArcTan[(1 - 2*Tanh[x])/Sqrt[3]])/(3*Sqrt[3]) - 1/(6*(1 + Tanh[x]))
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{1+\tanh^3(x)}\,dx &= \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\left(1+x^3\right)}\,dx, x, \tanh(x)\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{6(1+x)^2}-\frac{1}{2\left(-1+x^2\right)}+\frac{1}{3\left(1-x+x^2\right)}\right)dx, x, \tanh(x)\right)\\
&= -\frac{1}{6(1+\tanh(x))}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \tanh(x)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \tanh(x)\right)\\
&= \frac{x}{2}-\frac{1}{6(1+\tanh(x))}-\frac{2}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\tanh(x)\right)\\
&= \frac{x}{2}-\frac{2\tan^{-1}\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6(1+\tanh(x))}
\end{aligned}
$$

Mathematica [A] time = 0.0652679, size = 40, normalized size = 1.05

$$
\frac{1}{2}\tanh^{-1}(\tanh(x))-\frac{1}{6(\tanh(x)+1)}-\frac{2\tan^{-1}\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Tanh[x]^3)^(-1),x]
```

```
[Out] (-2*ArcTan[(1 - 2*Tanh[x])/Sqrt[3]])/(3*Sqrt[3]) + ArcTanh[Tanh[x]]/2 - 1/(
6*(1 + Tanh[x]))
```

Maple [A] time = 0.024, size = 41, normalized size = 1.1

$$
-\frac{1}{6+6\tanh(x)}+\frac{\ln(1+\tanh(x))}{4}+\frac{2\sqrt{3}}{9}\arctan\left(\frac{(2\tanh(x)-1)\sqrt{3}}{3}\right)-\frac{\ln(\tanh(x)-1)}{4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+tanh(x)^3),x)
```

```
[Out] -1/6/(1+tanh(x))+1/4*ln(1+tanh(x))+2/9*3^(1/2)*arctan(1/3*(2*tanh(x)-1)*3^(
1/2))-1/4*ln(tanh(x)-1)
```

Maxima [B] time = 1.50688, size = 99, normalized size = 2.61

$$
\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)}+3^{\frac{1}{4}}\sqrt{2}\right)\right)-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)}-3^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{1}{2}x-\frac{1}{12}e^{(-2x)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tanh(x)^3),x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) + 3^(1/4)*sqrt(2))
) - 2/9*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) - 3^(1/4)*sqrt
```

```
(2))) + 1/2*x - 1/12*e^(-2*x)
```

Fricas [B] time = 2.46051, size = 340, normalized size = 8.95

$$\frac{18\,x\cosh\left(x\right)^{2} + 36\,x\cosh\left(x\right)\sinh\left(x\right) + 18\,x\sinh\left(x\right)^{2} - 8\left(\sqrt{3}\cosh\left(x\right)^{2} + 2\,\sqrt{3}\cosh\left(x\right)\sinh\left(x\right) + \sqrt{3}\sinh\left(x\right)^{2}\right)\arctan}{36\left(\cosh\left(x\right)^{2} + 2\,\cosh\left(x\right)\sinh\left(x\right) + \sinh\left(x\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tanh(x)^3),x, algorithm="fricas")
```

```
[Out] 1/36*(18*x*cosh(x)^2 + 36*x*cosh(x)*sinh(x) + 18*x*sinh(x)^2 - 8*(sqrt(3)*c
osh(x)^2 + 2*sqrt(3)*cosh(x)*sinh(x) + sqrt(3)*sinh(x)^2)*arctan(-1/3*(sqrt
(3)*cosh(x) + sqrt(3)*sinh(x))/(cosh(x) - sinh(x))) - 3)/(cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2)
```

Sympy [B] time = 0.822486, size = 102, normalized size = 2.68

$$\frac{9x\tanh\left(x\right)}{18\tanh\left(x\right)+18} + \frac{9x}{18\tanh\left(x\right)+18} + \frac{4\sqrt{3}\tanh\left(x\right)\operatorname{atan}\left(\frac{2\sqrt{3}\tanh\left(x\right)}{3} - \frac{\sqrt{3}}{3}\right)}{18\tanh\left(x\right)+18} + \frac{4\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}\tanh\left(x\right)}{3} - \frac{\sqrt{3}}{3}\right)}{18\tanh\left(x\right)+18} - \frac{3}{18\tanh(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tanh(x)**3),x)
```

```
[Out] 9*x*tanh(x)/(18*tanh(x) + 18) + 9*x/(18*tanh(x) + 18) + 4*sqrt(3)*tanh(x)*a
tan(2*sqrt(3)*tanh(x)/3 - sqrt(3)/3)/(18*tanh(x) + 18) + 4*sqrt(3)*atan(2*s
qrt(3)*tanh(x)/3 - sqrt(3)/3)/(18*tanh(x) + 18) - 3/(18*tanh(x) + 18)
```

Giac [A] time = 1.29784, size = 34, normalized size = 0.89

$$\frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}e^{\left(2\,x\right)}\right) + \frac{1}{2}\,x - \frac{1}{12}\,e^{\left(-2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tanh(x)^3),x, algorithm="giac")
```

```
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*e^(2*x)) + 1/2*x - 1/12*e^(-2*x)
```

3.259 $\int \tanh(x)\left(a+b\tanh^4(x)\right)^{3/2}\,dx$

Optimal. Leaf size=124

$$\frac{1}{2}(a+b)^{3/2}\tanh^{-1}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)-\frac{1}{6}\left(a+b\tanh^4(x)\right)^{3/2}-\frac{1}{4}\sqrt{b}(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right)-\frac{1}{4}$$

```
[Out] -(Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[x]^2)/Sqrt[a + b*Tanh[x]^4]])/4
 + ((a + b)^(3/2)*ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]
^4])])/2 - ((2*(a + b) + b*Tanh[x]^2)*Sqrt[a + b*Tanh[x]^4])/4 - (a + b*Tan
h[x]^4)^(3/2)/6
```

Rubi [A] time = 0.241808, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {3670, 1248, 735, 815, 844, 217, 206, 725}

$$\frac{1}{2}(a+b)^{3/2}\tanh^{-1}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)-\frac{1}{6}\left(a+b\tanh^4(x)\right)^{3/2}-\frac{1}{4}\sqrt{b}(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right)-\frac{1}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]*(a + b*Tanh[x]^4)^(3/2),x]
```

```
[Out] -(Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[x]^2)/Sqrt[a + b*Tanh[x]^4]])/4
 + ((a + b)^(3/2)*ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]
^4])])/2 - ((2*(a + b) + b*Tanh[x]^2)*Sqrt[a + b*Tanh[x]^4])/4 - (a + b*Tan
h[x]^4)^(3/2)/6
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \tanh(x)\left(a+b\tanh^4(x)\right)^{3/2}\,dx = \text{Subst}\left(\int \frac{x\left(a+bx^4\right)^{3/2}}{1-x^2}\,dx, x, \tanh(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{1-x}\,dx, x, \tanh^2(x)\right)$$

$$= -\frac{1}{6}\left(a+b\tanh^4(x)\right)^{3/2} - \frac{1}{2}\text{Subst}\left(\int \frac{(-a-bx)\sqrt{a+bx^2}}{1-x}\,dx, x, \tanh^2(x)\right)$$

$$= -\frac{1}{4}\left(2(a+b)+b\tanh^2(x)\right)\sqrt{a+b\tanh^4(x)} - \frac{1}{6}\left(a+b\tanh^4(x)\right)^{3/2} - \frac{\text{Subst}\left(\int \frac{-a\text{i}}{}\right.}{}$$

$$= -\frac{1}{4}\left(2(a+b)+b\tanh^2(x)\right)\sqrt{a+b\tanh^4(x)} - \frac{1}{6}\left(a+b\tanh^4(x)\right)^{3/2} + \frac{1}{2}(a+b)^2\,\text{Su}$$

$$= -\frac{1}{4}\left(2(a+b)+b\tanh^2(x)\right)\sqrt{a+b\tanh^4(x)} - \frac{1}{6}\left(a+b\tanh^4(x)\right)^{3/2} - \frac{1}{2}(a+b)^2\,\text{Su}$$

$$= -\frac{1}{4}\sqrt{b}(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right) + \frac{1}{2}(a+b)^{3/2}\tanh^{-1}\left(\frac{a+b\tanh^2(}{\sqrt{a+b}\sqrt{a+b\tan}}\right.$$

Mathematica [A] time = 4.39531, size = 166, normalized size = 1.34

$$\frac{1}{12}\left(6(a+b)^{3/2}\tanh^{-1}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)-6\sqrt{b}(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right)-\sqrt{a+b\tanh^4(x)}\left(8a+2\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]*(a + b*Tanh[x]^4)^(3/2),x]
```

```
[Out] (-6*Sqrt[b]*(a + b)*ArcTanh[(Sqrt[b]*Tanh[x]^2)/Sqrt[a + b*Tanh[x]^4]] + 6*
(a + b)^(3/2)*ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])
] - Sqrt[a + b*Tanh[x]^4]*(8*a + 6*b + 3*b*Tanh[x]^2 + 2*b*Tanh[x]^4) - (3*
Sqrt[a]*Sqrt[b]*ArcSinh[(Sqrt[b]*Tanh[x]^2)/Sqrt[a]]*Sqrt[a + b*Tanh[x]^4])
/Sqrt[1 + (b*Tanh[x]^4)/a])/12
```

Maple [C] time = 0.082, size = 620, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)*(a+b*tanh(x)^4)^(3/2),x)
```

```
[Out] -1/6*b*tanh(x)^4*(a+b*tanh(x)^4)^(1/2)-1/4*b*tanh(x)^2*(a+b*tanh(x)^4)^(1/2
)-2/3*(a+b*tanh(x)^4)^(1/2)*a-1/2*b*(a+b*tanh(x)^4)^(1/2)-1/2*(-5/3*a*b-b^2
)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)*(1+I/a^(1
/2)*b^(1/2)*tanh(x)^2)^(1/2)/(a+b*tanh(x)^4)^(1/2)*EllipticF(tanh(x)*(I/a^(
1/2)*b^(1/2))^(1/2),I)-3/4*ln(2*b^(1/2)*tanh(x)^2+2*(a+b*tanh(x)^4)^(1/2))*
b^(1/2)*a-1/2*ln(2*b^(1/2)*tanh(x)^2+2*(a+b*tanh(x)^4)^(1/2))*b^(3/2)-1/2*I
*(7/5*a*b+b^2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tanh(
x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)/(a+b*tanh(x)^4)^(1/2)/b^(
1/2)*(EllipticF(tanh(x)*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(tanh(x)*(I/a
^(1/2)*b^(1/2))^(1/2),I))+1/2*a^2/(a+b)^(1/2)*arctanh(1/2*(2*b*tanh(x)^2+2*
a)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))+a*b/(a+b)^(1/2)*arctanh(1/2*(2*b*tanh
(x)^2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))+1/2*b^2/(a+b)^(1/2)*arctanh(1
/2*(2*b*tanh(x)^2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))-1/2*(5/3*a*b+b^2)
/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)*(1+I/a^(1/
2)*b^(1/2)*tanh(x)^2)^(1/2)/(a+b*tanh(x)^4)^(1/2)*EllipticF(tanh(x)*(I/a^(1
/2)*b^(1/2))^(1/2),I)-1/2*I*(-7/5*a*b-b^2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2
)*(1-I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/
2)/(a+b*tanh(x)^4)^(1/2)/b^(1/2)*(EllipticF(tanh(x)*(I/a^(1/2)*b^(1/2))^(1/
2),I)-EllipticE(tanh(x)*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\tanh(x)^4+a\right)^{\frac{3}{2}}\tanh(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)*(a+b*tanh(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tanh(x)^4 + a)^(3/2)*tanh(x), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)*(a+b*tanh(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b \tanh^{4}(x)\right)^{\frac{3}{2}} \tanh(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)*(a+b*tanh(x)**4)**(3/2),x)
```

```
[Out] Integral((a + b*tanh(x)**4)**(3/2)*tanh(x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \tanh(x)^{4} + a\right)^{\frac{3}{2}} \tanh(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)*(a+b*tanh(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tanh(x)^4 + a)^(3/2)*tanh(x), x)
```

3.260 $\int \tanh(x)\sqrt{a+b\tanh^4(x)}\,dx$

Optimal. Leaf size=89

$$-\frac{1}{2}\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right)+\frac{1}{2}\sqrt{a+b}\tanh^{-1}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)-\frac{1}{2}\sqrt{a+b\tanh^4(x)}$$

```
[Out] -(Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[x]^2)/Sqrt[a + b*Tanh[x]^4]])/2 + (Sqrt[a +
 b]*ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])])/2 - Sqr
t[a + b*Tanh[x]^4]/2
```

Rubi [A] time = 0.128402, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3670, 1248, 735, 844, 217, 206, 725}

$$-\frac{1}{2}\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right)+\frac{1}{2}\sqrt{a+b}\tanh^{-1}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)-\frac{1}{2}\sqrt{a+b\tanh^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]*Sqrt[a + b*Tanh[x]^4],x]
```

```
[Out] -(Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[x]^2)/Sqrt[a + b*Tanh[x]^4]])/2 + (Sqrt[a +
 b]*ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])])/2 - Sqr
t[a + b*Tanh[x]^4]/2
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \tanh(x)\sqrt{a+b\tanh^4(x)}\,dx = \text{Subst}\left(\int \frac{x\sqrt{a+bx^4}}{1-x^2}\,dx, x, \tanh(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{1-x}\,dx, x, \tanh^2(x)\right)$$
$$= -\frac{1}{2}\sqrt{a+b\tanh^4(x)} - \frac{1}{2}\text{Subst}\left(\int \frac{-a-bx}{(1-x)\sqrt{a+bx^2}}\,dx, x, \tanh^2(x)\right)$$
$$= -\frac{1}{2}\sqrt{a+b\tanh^4(x)} - \frac{1}{2}(-a-b)\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx^2}}\,dx, x, \tanh^2(x)\right) - \frac{1}{2}b\,\text{Sub}$$
$$= -\frac{1}{2}\sqrt{a+b\tanh^4(x)} - \frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right) - \frac{1}{2}(a+b)\,\text{Subst}$$
$$= -\frac{1}{2}\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right) + \frac{1}{2}\sqrt{a+b}\tanh^{-1}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right) - \frac{1}{2}\sqrt{}$$

Mathematica [A] time = 0.0572947, size = 86, normalized size = 0.97

$$\frac{1}{2}\left(-\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right) + \sqrt{a+b}\tanh^{-1}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right) - \sqrt{a+b\tanh^4(x)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]*Sqrt[a + b*Tanh[x]^4],x]
```

```
[Out] (-(Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[x]^2)/Sqrt[a + b*Tanh[x]^4]]) + Sqrt[a + b
]*ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])] - Sqrt[a +
 b*Tanh[x]^4])/2
```

Maple [A] time = 0.052, size = 116, normalized size = 1.3

$$-\frac{1}{2}\sqrt{a+b\left(\tanh\left(x\right)\right)^{4}}-\frac{1}{2}\sqrt{b}\ln\left(2\sqrt{b}\left(\tanh\left(x\right)\right)^{2}+2\sqrt{a+b\left(\tanh\left(x\right)\right)^{4}}\right)+\frac{a}{2}\mathrm{Artanh}\left(\frac{2b\left(\tanh\left(x\right)\right)^{2}+2a}{2}\frac{1}{\sqrt{a+b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)*(a+b*tanh(x)^4)^(1/2),x)
```

```
[Out] -1/2*(a+b*tanh(x)^4)^(1/2)-1/2*b^(1/2)*ln(2*b^(1/2)*tanh(x)^2+2*(a+b*tanh(x
)^4)^(1/2))+1/2*a/(a+b)^(1/2)*arctanh(1/2*(2*b*tanh(x)^2+2*a)/(a+b)^(1/2)/(
a+b*tanh(x)^4)^(1/2))+1/2*b/(a+b)^(1/2)*arctanh(1/2*(2*b*tanh(x)^2+2*a)/(a+
b)^(1/2)/(a+b*tanh(x)^4)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\tanh\left(x\right)^{4}+a}\tanh\left(x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)*(a+b*tanh(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tanh(x)^4 + a)*tanh(x), x)
```

Fricas [B] time = 4.128, size = 15323, normalized size = 172.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)*(a+b*tanh(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*si
nh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)*sqrt(b)*log(-(
(a + 2*b)*cosh(x)^8 + 8*(a + 2*b)*cosh(x)*sinh(x)^7 + (a + 2*b)*sinh(x)^8 +
 4*(a - 2*b)*cosh(x)^6 + 4*(7*(a + 2*b)*cosh(x)^2 + a - 2*b)*sinh(x)^6 + 8*
(7*(a + 2*b)*cosh(x)^3 + 3*(a - 2*b)*cosh(x))*sinh(x)^5 + 6*(a + 2*b)*cosh(
x)^4 + 2*(35*(a + 2*b)*cosh(x)^4 + 30*(a - 2*b)*cosh(x)^2 + 3*a + 6*b)*sinh
(x)^4 + 8*(7*(a + 2*b)*cosh(x)^5 + 10*(a - 2*b)*cosh(x)^3 + 3*(a + 2*b)*cos
h(x))*sinh(x)^3 + 4*(a - 2*b)*cosh(x)^2 + 4*(7*(a + 2*b)*cosh(x)^6 + 15*(a
- 2*b)*cosh(x)^4 + 9*(a + 2*b)*cosh(x)^2 + a - 2*b)*sinh(x)^2 - 2*sqrt(2)*(
cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2
 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)*sqrt(b)*sqrt(((a + b)
*cosh(x)^4 + (a + b)*sinh(x)^4 + 4*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)
^2 + 2*a - 2*b)*sinh(x)^2 + 3*a + 3*b)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6
*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + 8*((a + 2*b)*cos
h(x)^7 + 3*(a - 2*b)*cosh(x)^5 + 3*(a + 2*b)*cosh(x)^3 + (a - 2*b)*cosh(x))
*sinh(x) + a + 2*b)/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cos
h(x)^2 + 1)*sinh(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*sinh(x)^5
 + 2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(
x)^5 + 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x)^4
+ 9*cosh(x)^2 + 1)*sinh(x)^2 + 4*cosh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5 + 3
```

```
*cosh(x)^3 + cosh(x))*sinh(x) + 1)) + (cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + si
nh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh
(x))*sinh(x) + 1)*sqrt(a + b)*log(((a^2 + 2*a*b + b^2)*cosh(x)^8 + 8*(a^2 +
 2*a*b + b^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2)*sinh(x)^8 + 4*(a^2 -
b^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^6
+ 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^5 + 2
*(3*a^2 + 2*a*b + 3*b^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(x)^4 +
30*(a^2 - b^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 3*b^2)*sinh(x)^4 + 8*(7*(a^2 + 2
*a*b + b^2)*cosh(x)^5 + 10*(a^2 - b^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 3*b^2)*
cosh(x))*sinh(x)^3 + 4*(a^2 - b^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cos
h(x)^6 + 15*(a^2 - b^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b + 3*b^2)*cosh(x)^2 + a
^2 - b^2)*sinh(x)^2 + sqrt(2)*((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x
)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a
- b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)*s
qrt(a + b)*sqrt(((a + b)*cosh(x)^4 + (a + b)*sinh(x)^4 + 4*(a - b)*cosh(x)^
2 + 2*(3*(a + b)*cosh(x)^2 + 2*a - 2*b)*sinh(x)^2 + 3*a + 3*b)/(cosh(x)^4 -
 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x
)^4)) + a^2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*cosh(x)^7 + 3*(a^2 - b^2
)*cosh(x)^5 + (3*a^2 + 2*a*b + 3*b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh
(x))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*s
inh(x)^3 + sinh(x)^4)) - 2*sqrt(2)*sqrt(((a + b)*cosh(x)^4 + (a + b)*sinh(x
)^4 + 4*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + 2*a - 2*b)*sinh(x)^2 +
 3*a + 3*b)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*co
sh(x)*sinh(x)^3 + sinh(x)^4)))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4
 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*si
nh(x) + 1), -1/4*(2*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cos
h(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)*
sqrt(-a - b)*arctan(sqrt(2)*((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^
3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a -
b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)*sqr
t(-a - b)*sqrt(((a + b)*cosh(x)^4 + (a + b)*sinh(x)^4 + 4*(a - b)*cosh(x)^2
 + 2*(3*(a + b)*cosh(x)^2 + 2*a - 2*b)*sinh(x)^2 + 3*a + 3*b)/(cosh(x)^4 -
4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)
^4))/((a^2 + 2*a*b + b^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)
^7 + (a^2 + 2*a*b + b^2)*sinh(x)^8 + 4*(a^2 - b^2)*cosh(x)^6 + 4*(7*(a^2 +
2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2)*co
sh(x)^3 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^
4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 30*(a^2 - b^2)*cosh(x)^2 + 3*a^2
+ 6*a*b + 3*b^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 10*(a^2 -
 b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 - b^2)*
cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^6 + 15*(a^2 - b^2)*cosh(x)^4 +
 9*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2
 + 8*((a^2 + 2*a*b + b^2)*cosh(x)^7 + 3*(a^2 - b^2)*cosh(x)^5 + 3*(a^2 + 2*
a*b + b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))) - (cosh(x)^4 + 4*cosh
(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4
*(cosh(x)^3 + cosh(x))*sinh(x) + 1)*sqrt(b)*log(-((a + 2*b)*cosh(x)^8 + 8*(
a + 2*b)*cosh(x)*sinh(x)^7 + (a + 2*b)*sinh(x)^8 + 4*(a - 2*b)*cosh(x)^6 +
4*(7*(a + 2*b)*cosh(x)^2 + a - 2*b)*sinh(x)^6 + 8*(7*(a + 2*b)*cosh(x)^3 +
3*(a - 2*b)*cosh(x))*sinh(x)^5 + 6*(a + 2*b)*cosh(x)^4 + 2*(35*(a + 2*b)*co
sh(x)^4 + 30*(a - 2*b)*cosh(x)^2 + 3*a + 6*b)*sinh(x)^4 + 8*(7*(a + 2*b)*co
sh(x)^5 + 10*(a - 2*b)*cosh(x)^3 + 3*(a + 2*b)*cosh(x))*sinh(x)^3 + 4*(a -
2*b)*cosh(x)^2 + 4*(7*(a + 2*b)*cosh(x)^6 + 15*(a - 2*b)*cosh(x)^4 + 9*(a +
 2*b)*cosh(x)^2 + a - 2*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^4 + 4*cosh(x)*sin
h(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(
x)^3 - cosh(x))*sinh(x) + 1)*sqrt(b)*sqrt(((a + b)*cosh(x)^4 + (a + b)*sinh
(x)^4 + 4*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + 2*a - 2*b)*sinh(x)^2
 + 3*a + 3*b)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*
cosh(x)*sinh(x)^3 + sinh(x)^4)) + 8*((a + 2*b)*cosh(x)^7 + 3*(a - 2*b)*cosh
(x)^5 + 3*(a + 2*b)*cosh(x)^3 + (a - 2*b)*cosh(x))*sinh(x) + a + 2*b)/(cosh
```

(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 + 1)*sinh(x)^6 + 4
cosh(x)^6 + 8(7*cosh(x)^3 + 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 + 30*c
osh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 + 10*cosh(x)^3 + 3*c
osh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x)^4 + 9*cosh(x)^2 + 1)*sinh(x
)^2 + 4*cosh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5 + 3*cosh(x)^3 + cosh(x))*sin
h(x) + 1)) + 2*sqrt(2)*sqrt(((a + b)*cosh(x)^4 + (a + b)*sinh(x)^4 + 4*(a -
b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + 2*a - 2*b)*sinh(x)^2 + 3*a + 3*b)/
(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x
)^3 + sinh(x)^4)))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh
(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1),
1/4*(2*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*s
inh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)*sqrt(-b)*arct
an(sqrt(2)*sqrt(-b)*sqrt(((a + b)*cosh(x)^4 + (a + b)*sinh(x)^4 + 4*(a - b)
cosh(x)^2 + 2(3*(a + b)*cosh(x)^2 + 2*a - 2*b)*sinh(x)^2 + 3*a + 3*b)/(co
sh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3
+ sinh(x)^4))/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 - 2*b*cos
h(x)^2 + 2*(3*b*cosh(x)^2 - b)*sinh(x)^2 + 4*(b*cosh(x)^3 - b*cosh(x))*sinh
(x) + b)) + (cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 +
1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)*sqrt(a +
b)*log(((a^2 + 2*a*b + b^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(x)*sinh
(x)^7 + (a^2 + 2*a*b + b^2)*sinh(x)^8 + 4*(a^2 - b^2)*cosh(x)^6 + 4*(7*(a^2
+ 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2)
cosh(x)^3 + 3(a^2 - b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 3*b^2)*c
osh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 30*(a^2 - b^2)*cosh(x)^2 +
3*a^2 + 2*a*b + 3*b^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 10
*(a^2 - b^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 3*b^2)*cosh(x))*sinh(x)^3 + 4*(a^
2 - b^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^6 + 15*(a^2 - b^2)*co
sh(x)^4 + 3*(3*a^2 + 2*a*b + 3*b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + sqrt
(2)*((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 +
2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b
)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*sqrt(((a + b)*c
osh(x)^4 + (a + b)*sinh(x)^4 + 4*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2
+ 2*a - 2*b)*sinh(x)^2 + 3*a + 3*b)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*c
osh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2
+ 8*((a^2 + 2*a*b + b^2)*cosh(x)^7 + 3*(a^2 - b^2)*cosh(x)^5 + (3*a^2 + 2*a
*b + 3*b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x
)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) - 2
*sqrt(2)*sqrt(((a + b)*cosh(x)^4 + (a + b)*sinh(x)^4 + 4*(a - b)*cosh(x)^2
+ 2*(3*(a + b)*cosh(x)^2 + 2*a - 2*b)*sinh(x)^2 + 3*a + 3*b)/(cosh(x)^4 - 4
*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^
4)))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sin
h(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1), -1/2*((cosh(x)
^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*co
sh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)*sqrt(-a - b)*arctan(sqrt(2)*
((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a
- b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*co
sh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(-a - b)*sqrt(((a + b)*cosh
(x)^4 + (a + b)*sinh(x)^4 + 4*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 +
2*a - 2*b)*sinh(x)^2 + 3*a + 3*b)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh
(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/((a^2 + 2*a*b + b^2)*co
sh(x)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2)*sin
h(x)^8 + 4*(a^2 - b^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2
- b^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 - b^2)*cosh
(x))*sinh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2
)*cosh(x)^4 + 30*(a^2 - b^2)*cosh(x)^2 + 3*a^2 + 6*a*b + 3*b^2)*sinh(x)^4 +
8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 10*(a^2 - b^2)*cosh(x)^3 + 3*(a^2 + 2
*a*b + b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 - b^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*
b + b^2)*cosh(x)^6 + 15*(a^2 - b^2)*cosh(x)^4 + 9*(a^2 + 2*a*b + b^2)*cosh(
x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*co

```
sh(x)^7 + 3*(a^2 - b^2)*cosh(x)^5 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (a^2
- b^2)*cosh(x))*sinh(x))) - (cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 +
2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(
x) + 1)*sqrt(-b)*arctan(sqrt(2)*sqrt(-b)*sqrt(((a + b)*cosh(x)^4 + (a + b)*
sinh(x)^4 + 4*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + 2*a - 2*b)*sinh(
x)^2 + 3*a + 3*b)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2
- 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 +
b*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 - b)*sinh(x)^2 + 4*(b*cosh(x
)^3 - b*cosh(x))*sinh(x) + b)) + sqrt(2)*sqrt(((a + b)*cosh(x)^4 + (a + b)*
sinh(x)^4 + 4*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + 2*a - 2*b)*sinh(
x)^2 + 3*a + 3*b)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2
- 4*cosh(x)*sinh(x)^3 + sinh(x)^4)))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sin
h(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(
x))*sinh(x) + 1)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\tanh^{4}(x)}\tanh(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)*(a+b*tanh(x)**4)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tanh(x)**4)*tanh(x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tanh(x)^{4} + a}\tanh(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)*(a+b*tanh(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tanh(x)^4 + a)*tanh(x), x)
```

3.261 $\int \frac{\tanh(x)}{\sqrt{a+b\tanh^4(x)}}\,dx$

Optimal. Leaf size=40

$$\frac{\tanh^{-1}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{2\sqrt{a+b}}$$

```
[Out] ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])]/(2*Sqrt[a +
b])
```

Rubi [A] time = 0.0782411, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3670, 1248, 725, 206}

$$\frac{\tanh^{-1}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{2\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/Sqrt[a + b*Tanh[x]^4],x]
```

```
[Out] ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])]/(2*Sqrt[a +
b])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^4(x)}}\,dx = \text{Subst}\left(\int \frac{x}{(1-x^2)\sqrt{a+bx^4}}\,dx, x, \tanh(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx^2}}\,dx, x, \tanh^2(x)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{-a-b\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right)\right)$$
$$= \frac{\tanh^{-1}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{2\sqrt{a+b}}$$

Mathematica [A] time = 0.0157533, size = 40, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{2\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/Sqrt[a + b*Tanh[x]^4],x]
```

```
[Out] ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])]/(2*Sqrt[a +
b])
```

Maple [A] time = 0.054, size = 37, normalized size = 0.9

$$\frac{1}{2}\text{Artanh}\left(\frac{2\,b\,(\tanh(x))^2+2\,a}{2}\frac{1}{\sqrt{a+b}}\frac{1}{\sqrt{a+b\,(\tanh(x))^4}}\right)\frac{1}{\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*tanh(x)^4)^(1/2),x)
```

```
[Out] 1/2/(a+b)^(1/2)*arctanh(1/2*(2*b*tanh(x)^2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^4)
^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)}{\sqrt{b\tanh(x)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)/sqrt(b*tanh(x)^4 + a), x)
```

Fricas [B] time = 4.29379, size = 3553, normalized size = 88.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(((a^2 + 2*a*b + b^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(x)*sin
h(x)^7 + (a^2 + 2*a*b + b^2)*sinh(x)^8 + 4*(a^2 - b^2)*cosh(x)^6 + 4*(7*(a^
2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2
)*cosh(x)^3 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 3*b^2)*
cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 30*(a^2 - b^2)*cosh(x)^2
+ 3*a^2 + 2*a*b + 3*b^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 1
0*(a^2 - b^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 3*b^2)*cosh(x))*sinh(x)^3 + 4*(a
^2 - b^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^6 + 15*(a^2 - b^2)*c
osh(x)^4 + 3*(3*a^2 + 2*a*b + 3*b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + sqr
t(2)*((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 +
 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a +
b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*sqrt(((a + b)*
cosh(x)^4 + (a + b)*sinh(x)^4 + 4*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^
2 + 2*a - 2*b)*sinh(x)^2 + 3*a + 3*b)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*
cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2
 + 8*((a^2 + 2*a*b + b^2)*cosh(x)^7 + 3*(a^2 - b^2)*cosh(x)^5 + (3*a^2 + 2*
a*b + 3*b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(
x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/sq
rt(a + b), -1/2*sqrt(-a - b)*arctan(sqrt(2)*((a + b)*cosh(x)^4 + 4*(a + b)*
cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*
cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh
(x) + a + b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^4 + (a + b)*sinh(x)^4 + 4*(
a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + 2*a - 2*b)*sinh(x)^2 + 3*a + 3*
b)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sin
h(x)^3 + sinh(x)^4))/((a^2 + 2*a*b + b^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2)
*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2)*sinh(x)^8 + 4*(a^2 - b^2)*cosh(x)^
6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^6 + 8*(7*(a^2 +
 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^5 + 6*(a^2 + 2*a*b
 + b^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 30*(a^2 - b^2)*co
sh(x)^2 + 3*a^2 + 6*a*b + 3*b^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(
x)^5 + 10*(a^2 - b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3
+ 4*(a^2 - b^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^6 + 15*(a^2 -
b^2)*cosh(x)^4 + 9*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a
^2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*cosh(x)^7 + 3*(a^2 - b^2)*cosh(x)
^5 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x)))/(a +
b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)}{\sqrt{a + b\tanh^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)**4)**(1/2),x)
```

```
[Out] Integral(tanh(x)/sqrt(a + b*tanh(x)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)}{\sqrt{b\tanh(x)^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(x)/sqrt(b*tanh(x)^4 + a), x)
```

3.262 $\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=74

$$\frac{\tanh^{-1}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{2(a+b)^{3/2}}-\frac{a-b\tanh^2(x)}{2a(a+b)\sqrt{a+b\tanh^4(x)}}$$

```
[Out] ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])]/(2*(a + b)^(
3/2)) - (a - b*Tanh[x]^2)/(2*a*(a + b)*Sqrt[a + b*Tanh[x]^4])
```

Rubi [A] time = 0.122871, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3670, 1248, 741, 12, 725, 206}

$$\frac{\tanh^{-1}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{2(a+b)^{3/2}}-\frac{a-b\tanh^2(x)}{2a(a+b)\sqrt{a+b\tanh^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(a + b*Tanh[x]^4)^(3/2),x]
```

```
[Out] ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])]/(2*(a + b)^(
3/2)) - (a - b*Tanh[x]^2)/(2*a*(a + b)*Sqrt[a + b*Tanh[x]^4])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{3/2}}\,dx &= \text{Subst}\left(\int \frac{x}{\left(1-x^2\right)\left(a+bx^4\right)^{3/2}}\,dx,x,\tanh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)\left(a+bx^2\right)^{3/2}}\,dx,x,\tanh^2(x)\right)\\
&= -\frac{a-b\tanh^2(x)}{2a(a+b)\sqrt{a+b\tanh^4(x)}}+\frac{\text{Subst}\left(\int \frac{a}{(1-x)\sqrt{a+bx^2}}\,dx,x,\tanh^2(x)\right)}{2a(a+b)}\\
&= -\frac{a-b\tanh^2(x)}{2a(a+b)\sqrt{a+b\tanh^4(x)}}+\frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx^2}}\,dx,x,\tanh^2(x)\right)}{2(a+b)}\\
&= -\frac{a-b\tanh^2(x)}{2a(a+b)\sqrt{a+b\tanh^4(x)}}-\frac{\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx,x,\frac{-a-b\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right)}{2(a+b)}\\
&= \frac{\tanh^{-1}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{2(a+b)^{3/2}}-\frac{a-b\tanh^2(x)}{2a(a+b)\sqrt{a+b\tanh^4(x)}}
\end{aligned}
$$

Mathematica [A] time = 0.523797, size = 73, normalized size = 0.99

$$
\frac{1}{2}\left(\frac{\tanh^{-1}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{(a+b)^{3/2}}-\frac{a-b\tanh^2(x)}{a(a+b)\sqrt{a+b\tanh^4(x)}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + b*Tanh[x]^4)^(3/2),x]
```

```
[Out] (ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])]/(a + b)^(3/
2) - (a - b*Tanh[x]^2)/(a*(a + b)*Sqrt[a + b*Tanh[x]^4]))/2
```

Maple [C] time = 0.041, size = 431, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*tanh(x)^4)^(3/2),x)
```

```
[Out] b*(-1/4/a/(a+b)*tanh(x)^3+1/4/a/(a+b)*tanh(x)^2-1/4/a/(a+b)*tanh(x)-1/4/(a+
b)/b)/((tanh(x)^4+a/b)*b)^(1/2)-1/2/(a+b)*(-1/2/(a+b)^(1/2)*arctanh(1/2*(2*
b*tanh(x)^2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))+1/(I/a^(1/2)*b^(1/2))^(
1/2)*(1-I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tanh(x)^2)^
(1/2)/(a+b*tanh(x)^4)^(1/2)*EllipticPi(tanh(x)*(I/a^(1/2)*b^(1/2))^(1/2),-I
*a^(1/2)/b^(1/2),(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)))+b*(
1/4/a/(a+b)*tanh(x)^3+1/4/a/(a+b)*tanh(x)^2+1/4/a/(a+b)*tanh(x)-1/4/(a+b)/b
)/((tanh(x)^4+a/b)*b)^(1/2)-1/2/(a+b)*(-1/2/(a+b)^(1/2)*arctanh(1/2*(2*b*ta
nh(x)^2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))-1/(I/a^(1/2)*b^(1/2))^(1/2)
*(1-I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2
)/(a+b*tanh(x)^4)^(1/2)*EllipticPi(tanh(x)*(I/a^(1/2)*b^(1/2))^(1/2),-I*a^(
1/2)/b^(1/2),(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)}{\left(b\tanh(x)^4 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)/(b*tanh(x)^4 + a)^(3/2), x)
```

Fricas [B] time = 5.85996, size = 10031, normalized size = 135.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a^2 + a*b)*cosh(x)^8 + 8*(a^2 + a*b)*cosh(x)*sinh(x)^7 + (a^2 + a*b
)*sinh(x)^8 + 4*(a^2 - a*b)*cosh(x)^6 + 4*(7*(a^2 + a*b)*cosh(x)^2 + a^2 -
a*b)*sinh(x)^6 + 8*(7*(a^2 + a*b)*cosh(x)^3 + 3*(a^2 - a*b)*cosh(x))*sinh(x
)^5 + 6*(a^2 + a*b)*cosh(x)^4 + 2*(35*(a^2 + a*b)*cosh(x)^4 + 30*(a^2 - a*b
)*cosh(x)^2 + 3*a^2 + 3*a*b)*sinh(x)^4 + 8*(7*(a^2 + a*b)*cosh(x)^5 + 10*(a
^2 - a*b)*cosh(x)^3 + 3*(a^2 + a*b)*cosh(x))*sinh(x)^3 + 4*(a^2 - a*b)*cosh
(x)^2 + 4*(7*(a^2 + a*b)*cosh(x)^6 + 15*(a^2 - a*b)*cosh(x)^4 + 9*(a^2 + a*
b)*cosh(x)^2 + a^2 - a*b)*sinh(x)^2 + a^2 + a*b + 8*((a^2 + a*b)*cosh(x)^7
+ 3*(a^2 - a*b)*cosh(x)^5 + 3*(a^2 + a*b)*cosh(x)^3 + (a^2 - a*b)*cosh(x))*
sinh(x))*sqrt(a + b)*log(((a^2 + 2*a*b + b^2)*cosh(x)^8 + 8*(a^2 + 2*a*b +
b^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2)*sinh(x)^8 + 4*(a^2 - b^2)*cosh
(x)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^6 + 8*(7*(a
^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 +
```

2*a*b + 3*b^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 30*(a^2 - b^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 3*b^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 10*(a^2 - b^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 3*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 - b^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^6 + 15*(a^2 - b^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b + 3*b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + sqrt(2)*((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^4 + (a + b)*sinh(x)^4 + 4*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + 2*a - 2*b)*sinh(x)^2 + 3*a + 3*b)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*cosh(x)^7 + 3*(a^2 - b^2)*cosh(x)^5 + (3*a^2 + 2*a*b + 3*b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) - 2*sqrt(2)*((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*cosh(x)*sinh(x)^3 + (a^2 - b^2)*sinh(x)^4 + 2*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 2*(3*(a^2 - b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(x)^3 + (a^2 + 2*a*b + b^2)*cosh(x))*sinh(x))*sqrt(((a + b)*cosh(x)^4 + (a + b)*sinh(x)^4 + 4*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + 2*a - 2*b)*sinh(x)^2 + 3*a + 3*b)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^8 + 8*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)*sinh(x)^7 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sinh(x)^8 + 4*(a^4 + a^3*b - a^2*b^2 - a*b^3)*cosh(x)^6 + 4*(a^4 + a^3*b - a^2*b^2 - a*b^3 + 7*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^3 + 3*(a^4 + a^3*b - a^2*b^2 - a*b^3)*cosh(x))*sinh(x)^5 + 6*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^4 + 2*(35*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^4 + 3*a^4 + 9*a^3*b + 9*a^2*b^2 + 3*a*b^3 + 30*(a^4 + a^3*b - a^2*b^2 - a*b^3)*cosh(x)^2)*sinh(x)^4 + a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 + 8*(7*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^5 + 10*(a^4 + a^3*b - a^2*b^2 - a*b^3)*cosh(x)^3 + 3*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x))*sinh(x)^3 + 4*(a^4 + a^3*b - a^2*b^2 - a*b^3)*cosh(x)^2 + 4*(7*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^6 + 15*(a^4 + a^3*b - a^2*b^2 - a*b^3)*cosh(x)^4 + a^4 + a^3*b - a^2*b^2 - a*b^3 + 9*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^2)*sinh(x)^2 + 8*((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^7 + 3*(a^4 + a^3*b - a^2*b^2 - a*b^3)*cosh(x)^5 + 3*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^3 + (a^4 + a^3*b - a^2*b^2 - a*b^3)*cosh(x))*sinh(x)), -1/2*(((a^2 + a*b)*cosh(x)^8 + 8*(a^2 + a*b)*cosh(x)*sinh(x)^7 + (a^2 + a*b)*sinh(x)^8 + 4*(a^2 - a*b)*cosh(x)^6 + 4*(7*(a^2 + a*b)*cosh(x)^2 + a^2 - a*b)*sinh(x)^6 + 8*(7*(a^2 + a*b)*cosh(x)^3 + 3*(a^2 - a*b)*cosh(x))*sinh(x)^5 + 6*(a^2 + a*b)*cosh(x)^4 + 2*(35*(a^2 + a*b)*cosh(x)^4 + 30*(a^2 - a*b)*cosh(x)^2 + 3*a^2 + 3*a*b)*sinh(x)^4 + 8*(7*(a^2 + a*b)*cosh(x)^5 + 10*(a^2 - a*b)*cosh(x)^3 + 3*(a^2 + a*b)*cosh(x))*sinh(x)^3 + 4*(a^2 - a*b)*cosh(x)^2 + 4*(7*(a^2 + a*b)*cosh(x)^6 + 15*(a^2 - a*b)*cosh(x)^4 + 9*(a^2 + a*b)*cosh(x)^2 + a^2 - a*b)*sinh(x)^2 + a^2 + a*b + 8*((a^2 + a*b)*cosh(x)^7 + 3*(a^2 - a*b)*cosh(x)^5 + 3*(a^2 + a*b)*cosh(x)^3 + (a^2 - a*b)*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^4 + (a + b)*sinh(x)^4 + 4*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + 2*a - 2*b)*sinh(x)^2 + 3*a + 3*b)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/((a^2 + 2*a*b + b^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2)*sinh(x)^8 + 4*(a^2 - b^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 30*(a^2 - b^2)*cosh(x)^2 + 3*a^2 + 6*a*b + 3*b^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 10*(a^2 - b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*si

```
nh(x)^3 + 4*(a^2 - b^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^6 + 15
*(a^2 - b^2)*cosh(x)^4 + 9*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(
x)^2 + a^2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*cosh(x)^7 + 3*(a^2 - b^2)
*cosh(x)^5 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x)
)) + sqrt(2)*((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*cosh(x)*sinh(x)^3 + (a^
2 - b^2)*sinh(x)^4 + 2*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 2*(3*(a^2 - b^2)*cos
h(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(x)^
3 + (a^2 + 2*a*b + b^2)*cosh(x))*sinh(x))*sqrt(((a + b)*cosh(x)^4 + (a + b)
*sinh(x)^4 + 4*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + 2*a - 2*b)*sinh
(x)^2 + 3*a + 3*b)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2
 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*
cosh(x)^8 + 8*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)*sinh(x)^7 + (a^4
+ 3*a^3*b + 3*a^2*b^2 + a*b^3)*sinh(x)^8 + 4*(a^4 + a^3*b - a^2*b^2 - a*b^3
)*cosh(x)^6 + 4*(a^4 + a^3*b - a^2*b^2 - a*b^3 + 7*(a^4 + 3*a^3*b + 3*a^2*b
^2 + a*b^3)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)
*cosh(x)^3 + 3*(a^4 + a^3*b - a^2*b^2 - a*b^3)*cosh(x))*sinh(x)^5 + 6*(a^4
+ 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^4 + 2*(35*(a^4 + 3*a^3*b + 3*a^2*b^2
 + a*b^3)*cosh(x)^4 + 3*a^4 + 9*a^3*b + 9*a^2*b^2 + 3*a*b^3 + 30*(a^4 + a^3
*b - a^2*b^2 - a*b^3)*cosh(x)^2)*sinh(x)^4 + a^4 + 3*a^3*b + 3*a^2*b^2 + a*
b^3 + 8*(7*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^5 + 10*(a^4 + a^3*b
- a^2*b^2 - a*b^3)*cosh(x)^3 + 3*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x
))*sinh(x)^3 + 4*(a^4 + a^3*b - a^2*b^2 - a*b^3)*cosh(x)^2 + 4*(7*(a^4 + 3*
a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^6 + 15*(a^4 + a^3*b - a^2*b^2 - a*b^3)*c
osh(x)^4 + a^4 + a^3*b - a^2*b^2 - a*b^3 + 9*(a^4 + 3*a^3*b + 3*a^2*b^2 + a
*b^3)*cosh(x)^2)*sinh(x)^2 + 8*((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)
^7 + 3*(a^4 + a^3*b - a^2*b^2 - a*b^3)*cosh(x)^5 + 3*(a^4 + 3*a^3*b + 3*a^2
*b^2 + a*b^3)*cosh(x)^3 + (a^4 + a^3*b - a^2*b^2 - a*b^3)*cosh(x))*sinh(x))
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)**4)**(3/2),x)
```

```
[Out] Integral(tanh(x)/(a + b*tanh(x)**4)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)}{\left(b\tanh(x)^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(x)/(b*tanh(x)^4 + a)^(3/2), x)
```

3.263 $\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=118

$$-\frac{3a^2-b(5a+2b)\tanh^2(x)}{6a^2(a+b)^2\sqrt{a+b\tanh^4(x)}}+\frac{\tanh^{-1}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{2(a+b)^{5/2}}-\frac{a-b\tanh^2(x)}{6a(a+b)\left(a+b\tanh^4(x)\right)^{3/2}}$$

```
[Out] ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])]/(2*(a + b)^(
5/2)) - (a - b*Tanh[x]^2)/(6*a*(a + b)*(a + b*Tanh[x]^4)^(3/2)) - (3*a^2 -
b*(5*a + 2*b)*Tanh[x]^2)/(6*a^2*(a + b)^2*Sqrt[a + b*Tanh[x]^4])
```

Rubi [A] time = 0.198754, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3670, 1248, 741, 823, 12, 725, 206}

$$-\frac{3a^2-b(5a+2b)\tanh^2(x)}{6a^2(a+b)^2\sqrt{a+b\tanh^4(x)}}+\frac{\tanh^{-1}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{2(a+b)^{5/2}}-\frac{a-b\tanh^2(x)}{6a(a+b)\left(a+b\tanh^4(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(a + b*Tanh[x]^4)^(5/2),x]
```

```
[Out] ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])]/(2*(a + b)^(
5/2)) - (a - b*Tanh[x]^2)/(6*a*(a + b)*(a + b*Tanh[x]^4)^(3/2)) - (3*a^2 -
b*(5*a + 2*b)*Tanh[x]^2)/(6*a^2*(a + b)^2*Sqrt[a + b*Tanh[x]^4])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{5/2}}\,dx &= \text{Subst}\left(\int \frac{x}{\left(1-x^2\right)\left(a+bx^4\right)^{5/2}}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(1-x)\left(a+bx^2\right)^{5/2}}\,dx, x, \tanh^2(x)\right)\\
&= -\frac{a-b\tanh^2(x)}{6a(a+b)\left(a+b\tanh^4(x)\right)^{3/2}}-\frac{\text{Subst}\left(\int \frac{-3a-2b+2bx}{(1-x)\left(a+bx^2\right)^{3/2}}\,dx, x, \tanh^2(x)\right)}{6a(a+b)}\\
&= -\frac{a-b\tanh^2(x)}{6a(a+b)\left(a+b\tanh^4(x)\right)^{3/2}}-\frac{3a^2-b(5a+2b)\tanh^2(x)}{6a^2(a+b)^2\sqrt{a+b\tanh^4(x)}}+\frac{\text{Subst}\left(\int \frac{3a^2b}{(1-x)\sqrt{a+bx^2}}\,dx,\right.}{6a^2b(a+b)}\\
&= -\frac{a-b\tanh^2(x)}{6a(a+b)\left(a+b\tanh^4(x)\right)^{3/2}}-\frac{3a^2-b(5a+2b)\tanh^2(x)}{6a^2(a+b)^2\sqrt{a+b\tanh^4(x)}}+\frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx^2}}\,dx,\right.}{2(a+b)^2}\\
&= -\frac{a-b\tanh^2(x)}{6a(a+b)\left(a+b\tanh^4(x)\right)^{3/2}}-\frac{3a^2-b(5a+2b)\tanh^2(x)}{6a^2(a+b)^2\sqrt{a+b\tanh^4(x)}}-\frac{\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x,\right.}{2(a+b)^2}\\
&= \frac{\tanh^{-1}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{2(a+b)^{5/2}}-\frac{a-b\tanh^2(x)}{6a(a+b)\left(a+b\tanh^4(x)\right)^{3/2}}-\frac{3a^2-b(5a+2b)\tanh^2(x)}{6a^2(a+b)^2\sqrt{a+b\tanh^4(x}}
\end{aligned}
$$

Mathematica [A] time = 0.785528, size = 113, normalized size = 0.96

$$\frac{1}{6}\left(\frac{-3a^2b\tanh^4(x)-a^2(4a+b)+b^2(5a+2b)\tanh^6(x)+3ab(2a+b)\tanh^2(x)}{a^2(a+b)^2\left(a+b\tanh^4(x)\right)^{3/2}}+\frac{3\tanh^{-1}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{(a+b)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + b*Tanh[x]^4)^(5/2),x]
```

```
[Out] ((3*ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])])/(a + b)
^(5/2) + (-(a^2*(4*a + b)) + 3*a*b*(2*a + b)*Tanh[x]^2 - 3*a^2*b*Tanh[x]^4
+ b^2*(5*a + 2*b)*Tanh[x]^6)/(a^2*(a + b)^2*(a + b*Tanh[x]^4)^(3/2)))/6
```

Maple [C] time = 0.053, size = 637, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*tanh(x)^4)^(5/2),x)
```

```
[Out] -1/2*(1/6/a/(a+b)/b*tanh(x)^3-1/6/a/(a+b)/b*tanh(x)^2+1/6/a/(a+b)/b*tanh(x)
+1/6/(a+b)/b^2)*(a+b*tanh(x)^4)^(1/2)/(tanh(x)^4+a/b)^2+b*(-1/8*(3*a+b)/a^2
/(a+b)^2*tanh(x)^3+1/12*(5*a+2*b)/a^2/(a+b)^2*tanh(x)^2-1/24*(11*a+5*b)/a^2
/(a+b)^2*tanh(x)-1/4/(a+b)^2/b)/((tanh(x)^4+a/b)*b)^(1/2)-1/2/(a+b)^2*(-1/2
/(a+b)^(1/2)*arctanh(1/2*(2*b*tanh(x)^2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1
/2))+1/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)*(1+I
/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)/(a+b*tanh(x)^4)^(1/2)*EllipticPi(tanh(x)*
(I/a^(1/2)*b^(1/2))^(1/2),-I*a^(1/2)/b^(1/2),(-I/a^(1/2)*b^(1/2))^(1/2)/(I/
a^(1/2)*b^(1/2))^(1/2)))-1/2*(-1/6/a/(a+b)/b*tanh(x)^3-1/6/a/(a+b)/b*tanh(x
)^2-1/6/a/(a+b)/b*tanh(x)+1/6/(a+b)/b^2)*(a+b*tanh(x)^4)^(1/2)/(tanh(x)^4+a
/b)^2+b*(1/8*(3*a+b)/a^2/(a+b)^2*tanh(x)^3+1/12*(5*a+2*b)/a^2/(a+b)^2*tanh(
x)^2+1/24*(11*a+5*b)/a^2/(a+b)^2*tanh(x)-1/4/(a+b)^2/b)/((tanh(x)^4+a/b)*b)
^(1/2)-1/2/(a+b)^2*(-1/2/(a+b)^(1/2)*arctanh(1/2*(2*b*tanh(x)^2+2*a)/(a+b)^
(1/2)/(a+b*tanh(x)^4)^(1/2))-1/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/
2)*tanh(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)/(a+b*tanh(x)^4)^(
1/2)*EllipticPi(tanh(x)*(I/a^(1/2)*b^(1/2))^(1/2),-I*a^(1/2)/b^(1/2),(-I/a^
(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)}{\left(b\tanh(x)^4+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)/(b*tanh(x)^4 + a)^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)**4)**(5/2),x)
```

```
[Out] Integral(tanh(x)/(a + b*tanh(x)**4)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)}{\left(b\tanh(x)^4+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(x)/(b*tanh(x)^4 + a)^(5/2), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
              print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
              return "A";
            else
              return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
              print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
    not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                         #since this estimate of leaf count is bit lower than
```

```
                      #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): # is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```